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OF

A. I. E. E. TRANSACTIONS

Vol. XXXVIII, Parts I and II

The main headings under which these synopses are classified were arrived at by a careful study of all the papers contributed since the organization of the Institute.

The method of making this classification may be called the automatic method, since it is created by sorting the papers themselves into groups and then naming the groups.

Many papers fall naturally into several different groups and in such cases they are inserted under as many different heads as it is thought they rightfully belong.

The classified synopses are designed for those searching for comprehensive information on any given topic, while the subject index is intended for those looking up specific and definite data or information.

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2. GENERAL THEORY

THE GENERAL EQUATIONS OF THE ELECTRIC CIRCUIT—III. VARIATION OF CONSTANTS, r, L, C AND g, AND ITS EFFECTS

C. P. Steinmetz **Vol. xxxviii—1919, pp. 191-260**

In the usual theory of transients the assumption is made, that resistance, inductance, capacity and conductance are constant. This however, is not correct, and as the result thereof, it was not possible to theoretically investigate, and numerically calculate the dissipation of high-frequency disturbances, the flattening of the wave fronts of impulses, the rounding off of steep waves, etc., with the time and the distance of travel, and therefrom to determine the distance, to which the danger from such disturbances extends, and to investigate the conditions of line construction, which limit the danger zone of such phenomena to the smallest local extent.

In the following, two of the foremost causes of change of the line constants with the equivalent frequency are investigated, the unequal current distribution in the conductor, and the electric radiation from the conductor.

Equations of the line constants as function of the equivalent frequency are derived, and applications thereof made to a few problems.

It is shown that in high-frequency conduction the section of the conductor is of little importance, but the circumference is the determining factor, except at very high frequencies, where size, shape, and material—within certain limits— becomes of secondary importance.

The conditions, which cause a flattening of steep wave fronts and impulses, and a rounding of irregular waves, are investigated.

For convenience, the theoretical part has been separated and placed in an appendix, giving in the text the discussion, with numerous tables derived from the theoretical equations, and curves illustrating these tables.

Discussion incorporated with that of paper by H. S. Osborne on "Review of Work of Sub-Committee on Wave Shape Standard of the Standards Committee."

THEORY OF THE TRANSIENT OSCILLATIONS OF ELECTRICAL NETWORKS AND TRANSMISSION SYSTEMS

J. R. Carson **Vol. xxxviii—1919, pp. 345-427**

The purpose of this research was to make a broad theoretical study of transient phenomena with a view to developing methods of calculation directly applicable to engineering problems. The investigation starts with the problem of formulating the current in an electrical network or transmission system in response to a suddenly applied e. m. f. of arbitrary form. A simple formula is derived which expresses this current in terms of two independent functions: one, the applied e. m. f. expressed as a time function, and the other a characteristic function of the constants and connections of the system, this latter being termed the "indicial admittance" of the system. A systematic investigation of methods for solving and computing the indicial admittance follows, in the course of which original solutions for transmission and artificial lines are derived and a new method involving integral equations is developed.

Discussion incorporated with that of paper by W. W. Crawford on "Telephone Circuits with Zero Mutual Induction."

ORDER AND AMPLITUDE OF HARMONICS IN VOLTAGE WAVE FORMS WITH INDICATING INSTRUMENTS

Leslie F. Curtis **Vol. xxxviii—1919, pp. 1179-1190**

The author presents a method for the determination of the order and percentage of the various components of an alternating wave of e. m. f., using indicating meters and other inexpensive apparatus.

Two examples are given. Oscillograms are included to show interesting phenomena and to check the results of the calculations.

The value of so-called standards for the indication of wave form is questioned.

Discussion, pages 1191-1198, by Messrs. N. S. Diamant, J. C. Albert, D. J. Cone, H. A. Barre and L. F. Curtis.

A general discussion on determination of harmonics.

PREDETERMINATION OF SYNCHRONOUS PHASE-MODIFIER PERFORMANCE

Hubert V. Carpenter **Vol. xxxviii—1919, pp. 1223-1229**

The author reviews the method for showing the behavior of transmission lines first given by Perrine and Baum and then shows how it can be used in determining the effect of the use of a synchronous motor operating without load for improving the power factor.

The diagram given shows both the improvement in voltage regulation and the change in power factor due to the phase-modifier for any assumed condition of loading for the transmission line.

The errors of the method are discussed with methods for determining their magnitude, and the advantages of the graphical treatment pointed out.

Discussion, pages 1230-1235, by Messrs. H. B. Dwight, M. O. Bosler, J. A. Lighthipe, P. M. Downing, J. H. Anderton, L. F. Curtis, J. F. Wilson and H. L. Melvin.

Discussion of various methods of determination including use of circle diagram.

A REPORT ON ELECTROMAGNETIC INDUCTION

S. J. Barnett **Vol. xxxviii—1919, pp. 1495-1513**

This report discusses briefly the chief fundamental results obtained from the days of Faraday to the present time in studying the electromotive forces ordinarily referred to the domain of electromagnetic induction.

Self-induction is first taken up.

The motional electromotive force, developed when matter movse in a magnetic field, is next considered and is derived from Ampere's law on the electron theory.

The induced electromotive force in fixed conductors and insulators arising from the motion or alteration of other systems is next considered.

The report closes with a treatment of unipolar induction in both so-called open and closed circuits, including brief descriptions of some of the principal experiments, a discussion of the theories involved, and their application to the unipolar generator.

No discussion of this paper.

SOME NEW FORMULAS FOR REACTANCE COILS

H. B. Dwight **Vol. xxxviii—1919, pp. 1675-1696**

Formulas are presented and derived, which have not been previously published, for mutual inductance of coils with parallel axes, repulsion of coils with parallel axes, and self-inductance of long cylindrical coils.

These formulas apply to practically all cases of reactance coils in common use. They are very convergent and accurate, and will give results to a given degree of accuracy with a minimum amount of labor.

Sets of curves are given from which approximate readings may be taken.

No discussion.

INHERENT LIMITATIONS ON TRANSFORMATIONS POSSIBLE BY STATIONARY APPARATUS

Joseph Slepian **Vol. xxxviii—1919, pp. 1697-1711**

Expressions for electrostatic and electromagnetic energies, Joulian heat dissipation and power are given in complex quantities. The pure imaginary part of the expression for power in a static network is shown to be equal to 2 ω times the difference between the mean electromagnetic energy and mean electrostatic energy. Use is made of this new principle in considering the problems of power-factor correction and phase splitting. It is shown that in general for phase transforming by static apparatus both magnetic and electrostatic storage of energy are necessary, and it is shown how the minimum amounts of each are determined by the load.

The symmetry of the coefficients in the general equations for the steady state in a static network is demonstrated, and it is shown that limitations upon voltage and current transformations follow. The voltage regulation of any phase-splitting arrangement is considered.

No discussion.

STARTING CONDITIONS OF SYNCHRONOUS MACHINES

A. Hay and F. N. Mowdawalla **Vol. xxxviii—1919, pp. 1712-1755**

A detailed study of the behavior of the synchronous motor during what has always been regarded as a critical period—*viz.* the period when the motor is being accelerated from rest to synchronous speed.

Discussion, **pages 1756-1757, by Q. Graham.**

3. UNITS MEASUREMENTS AND INSTRUMENTS

THE ABSOLUTE MEASUREMENT OF THE INTENSITY OF SOUND

Arthur G. Webster **Vol. xxxviii—1919, pp. 701-712**

This paper includes a description of a series of acoustical researches extending over a period of twenty-eight years. The properties of vibrating bodies and the subject of elastic hysteresis are discussed. Two fundamentally important instruments for the absolute measurement of sound have been developed and the theory given. The first is the standard of sound, called the phone, which is capable of reproducing at any time a sound of the simplest character and which permits the output of sound to be measured in watts of energy. The second is an instrument called the phonometer for measuring a sound in absolute measure. This instrument is now practically as sensitive as the human ear. The determination of the space distribution of sound and of the effect of disturbing bodies, and the measurement of the reflecting coefficient of surfaces have been accomplished. The phonotrope is a third instrument designed and used to find the direction of a source of sound, for example a fog signal.

Discussion, (including that of paper by A. E. Kennelly and H. Nukiyama), pages 713-723, by Messrs. J. B. Taylor, T. E. Shea, B. A. Behrend, R. L. Jones, A. G. Webster, H. S. Osborne, A. Press and A. E. Kennelly.

A general discussion including the algebraic theory of the receiver and the application of the circle diagram.

UTILIZING THE TIME CHARACTERISTICS OF ALTERNATING CURRENT

Henry E. Warren **Vol. xxxviii—1919, pp. 767-781**

By the invention of a very small, simple and reliable self-starting synchronous motor, in conjunction with convenient means for regulating the average frequency of an alternating circuit, there has been developed a new field of usefulness for electric power. It is now feasible to drive all kinds of timing devices such as clocks, graphic instrument movements, time recorders, etc., directly from the lighting circuits. Remarkable accuracy is obtained, and the amount of care is minimized by the elimination of winding and regulating.

No discussion.

THE DIELECTRIC FIELD IN AN ELECTRIC POWER CABLE

R. W. Atkinson **Vol. xxxviii—1919,pp. 971-1016**

The data given pertain particularly to the field of three-conductor three-phase cables when supplied with three-phase voltage, and are primarily the solution by physical measurements of some of the geometric problems of the three-conductor three-phase cable.

Data are given so that it is possible, from electrical measurements on three-conductor cable, to determine certain specific quantities as permittivity, resistivity, etc. of the dielectric of three-conductor cables in the same way as can readily be done for single-conductor cables, from geometric considerations.

Also, there is shown the potential and stress distribution in a three-conductor cable.

Discussion (including that of paper by W. S. Clark and G. B. Shanklin), pages 1017-1036, by Messrs. H. W. Fisher, C. W. Davis, W. A. Del Mar, D. W. Roper, P. Torchio, H. L. Wallau, W. R. Atkinson, E. B. Meyer and G. B. Shanklin.

A general discussion with particular emphasis on comparison of stresses in various types of cables.

ORDER AND AMPLITUDE OF HARMONICS IN VOLTAGE WAVE FORMS WITH INDICATING INSTRUMENTS

Leslie F. Curtis **Vol. xxxviii—1919, pp. 1179-1190**

The author presents a method for the determination of the order and percentage of the various components of an alternating wave of e. m. f., using indicating meters and other inexpensive apparatus.

Two examples are given. Oscillograms are included to show interesting phenomena and to check the results of the calculations.

The value of so-called standards for the indication of wave form is questioned.

Discussion, pages 1191-1198, by Messrs. N. S. Diamant, J. C. Albert, D. J. Cone, H. A. Barre and L. F. Curtis.

A general discussion on determination of harmonics.

4. INSULATION AND DIELECTRIC PHENOMENA

IONIZATION OF OCCLUDED GASES IN HIGH-TENSION INSULATION

G. B. Shanklin and J. J. Matson **Vol. xxxviii—1919, pp. 489-536**

This paper deals with the ionization of occluded gases in solid insulation from the standpoint that these gas spaces are the weakest part of an insulation design and should receive first consideration. The stress at which ionization starts in different types of built-up insulation, such as used in cables and coils, is measured and from these measurements a safe working stress determined.

Discussion incorporated with that of paper by F. Dubsky on "The Dielectric Strength of Air Films Entrapped in Solid Insulation and A Practical Application to the Problem for Alternator Coils and Cables."

THE DIELECTRIC STRENGTH OF AIR FILMS ENTRAPPED IN SOLID INSULATION AND A PRACTICAL APPLICATION OF THE PROBLEM FOR ALTERNATOR COILS AND CABLES

F. Dubsky **Vol. xxxviii—1919, pp. 537-556**

An experimental investigation of the strength of air films of various thicknesses between glass plates. In the arrangement used, the breakdown of the air or the starting point of corona could be readily observed.

Tests were also made of the dielectric strength of air films between other solid insulations.

It was found that the dielectric strength of air films between insulations was practically the same as the dielectric strength of air films between conductors.

The second part of the paper is devoted to the practical application of the data to the design of armature coils and cables, and several specific examples are given.

Discussion (including that of paper by G.B. Shanklin and J. J. Matson), pages 559-575, by Messrs. H. G. Reist, L. W. Chubb, J. B. Whitehead, C. A. Adams, W. J. Foster, H. W. Fisher, R. E. Argersinger, C. F. Harding, R. W. Atkinson, D. Du Bois, F. W. Peek, Jr., E. E. F. Creighton, J. J. Matson, W. I. Middleton, E. W. Davis and F. Dubsky.

A general discussion.

HIGH-TENSION SINGLE-CONDUCTOR CABLE FOR POLYPHASE SYSTEMS

W. S. Clark and G. B. Shanklin **Vol. xxxviii—1919, pp. 917-969**

In this paper the dielectric, inductive and general line characteristics of three-conductor and single-conductor cable are compared. The advantages and disadvantages of each type of cable are brought out in a way that will aid in deciding the merits of individual problems.

Discussion incorporated with that of paper by R. W. Atkinson on "The Dielectric Field in an Electric Power Cable."

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A general discussion with particular emphasis on comparison of stresses in various types of cables.

THE EFFECT OF TRANSIENT VOLTAGES ON DIELECTRICS—II

The Effect of Lightning Voltages on Arrester Gaps, Insulators and Bushings on Transmission Lines

F. W. Peek, Jr. **Vol. xxxviii—1919, pp. 1137-1164**

This paper treats of some of the practical applications resulting from an investigation of the effect of lightning voltages on insulators, bushings and protective gaps.

The relative protective values of various gaps for steep and slanting wave fronts and high frequency are shown graphically in Figs. 14, 15, 16 and 17.

Data are given on the steepness of lightning waves actually occurring on transmission lines in practise.

Discussion, pages 1165-1177, by Messrs. W. A. Del Mar, F. W. Peek, Jr. and C. T. Allcutt.

A general discussion.

THEORY OF PROBABILITIES APPLIED TO FAILURES OF SUSPENSION INSULATORS

L. M. Klauber **Vol. xxxviii—1919, pp. 1199-1212**

There are a wide variety of operating conditions which affect the amount of over-insulation required, and after having found the minimum number of insulators per string required for any given operating conditions the author points out a method of determining the amount of extra insulation desirable from an insurance standpoint according to the law of probabilities. Equations are developed from which the probability of failure for any given case or the ratio between such probabilities for any pair of cases may be determined directly. A numerical example is also given which shows the development of the theory of minimum annual cost for combined mechanical and electrical failures.

Discussion, pages 1213-1222, by Messrs. W. D. Peaslee, C. O. Poole, J. A. Lighthipe, J. H. Anderton, H. A. Barre, J. B. Fisken, R. W. Shoemaker, E. F. Scattergood, L. C. Williams and L. M. Klauber.

A general discussion on the practicability of the method described.

5. ELECTRIC CONDUCTORS

THE GENERAL EQUATIONS OF THE ELECTRIC CIRCUIT—III

VARIATION OF CONSTANTS, r, L, C AND g, AND ITS EFFECTS

C. P. Steinmetz **Vol. xxxviii—1919, pp. 191-260**

In the usual theory of transients the assumption is made, that resistance, inductance, capacity and conductance are constant. This however, is not correct, and as the result thereof, it was not possible to theoretically investigate, and numerically calculate the dissipation of high-frequency disturbances, the flattening of the wave fronts of impulses, the rounding off of steep waves, etc., with the time and the distance of travel, and therefrom to determine the distance, to which the danger from such disturbances extends, and to investigate the conditions of line construction, which limit the danger zone of such phenomena to the smallest local extent.

In the following, two of the foremost causes of change of the line constants with the equivalent frequency are investigated, the unequal current distribution in the conductor, and the electric radiation from the conductor.

Equations of the line constants as function of the equivalent frequency are derived, and applications thereof made to a few problems.

It is shown that in high-frequency conduction the section of the conductor is of little importance, but the circumference is the determining factor, except at very high frequencies, where size, shape, and material—within certain limits—becomes of secondary importance.

The conditions, which cause a flattening of steep wave fronts and impulses, and a rounding of irregular waves, are investigated.

For convenience, the theoretical part has been separated and placed in an appendix, giving in the text the discussion, with numerous tables derived from the theoretical equations, and curves illustrating these tables.

Discussion incorporated with that of paper by H. S. Osborne on "Review of Work of Sub-Committee on Wave Shape Standard of the Standards Committee."

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W. S. Clark and G. B. Shanklin **Vol. xxxviii—1919, pp. 917-966**

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Discussion (including that of paper by W. S. Clark and G. B. Shanklin), pages 1017-1036, by Messrs. H. W. Fisher, C. W. Davis, W. A. Del Mar, D. W. Roper, P. Torchio, H. L. Wallau, W. R. Atkinson, E. B. Meyer and G. B. Shanklin.

A general discussion with particular emphasis on comparison of stresses in various types of cables.

8. TRANSFORMERS

ABNORMAL VOLTAGES WITHIN TRANSFORMERS

L. F. Blume and A. Boyajian **Vol. xxxviii—1919, pp. 577-614**

Mathematical analysis is made of a rectangular wave impinging upon a transformer winding and quantitative values deduced of the resulting internal voltage stresses in terms of transformer constants. It is shown that the conclusions also apply in part to abrupt impulses and approximate idea is given of the reaction of transformer to high frequencies. The difference between operating transformer with isolated and grounded neutral is shown. Energy losses are not considered in the mathematics although the manner in which the results are affected is pointed out. Finally, theoretical results are compared with impulse and high frequency tests made in the laboratory.

Discussion, pages 615-620, by Messrs. H. R. Summerhayes, G. Faccioli, C. A. Adams, J. F. Peters, F. Dubsky, F. W. Peek, Jr., A. E. Kennelly and L. F. Blume.

A general discussion.

9. ELECTRICAL MACHINERY AND APPARATUS

TRANSMISSION LINE RELAY PROTECTION

H. R. Woodrow, D. W. Roper, O. C. Traver and P. Macgahan

Vol. xxxviii—1919, pp. 795-826

The Protective Devices Committee made an analysis of the transmission line protective relay situation throughout the country and prepared a summary of the experiences of the operating companies with this protective apparatus. Questionnaires were sent out to sixty-one operating companies asking for their experience and present practise.

Replies were received from 32 and from these a general analysis was made of the practise of relay protection for transmission lines.

The Protective Devices Committee undertook the work of standardizing the nomenclature which it is hoped will be adopted by all parties.

Discussion incorporated with that of paper by W. E. Richards on "Grounded Neutral Transmission Line."

PREDETERMINATION OF SYNCHRONOUS PHASE-MODIFIER PERFORMANCE

Hubert V. Carpenter **Vol. xxxviii—1919, pp. 1223-1229**

The author reviews the method for showing the behavior of transmission lines first given by Perrine and Baum and then shows how it can be used in determining the effect of the use of a synchronous motor operating without load for improving the power factor.

The diagram given shows both the improvement in voltage regulation and the change in power factor due to the phase-modifier for any assumed condition of loading for the transmission line.

The errors of the method are discussed with methods for determining their magnitude, and the advantages of the graphical treatment pointed out.

Discussion, pages 1230-1235, by Messrs. H. B. Dwight, M. O. Bosler, J. A. Lighthipe, P. M. Downing, J. H. Anderton, L. F. Curtis, J. F. Wilson and H. L. Melvin.

Discussion of various methods of determination including use of circle diagram.

THE VACUUM TUBE AS A GENERATOR OF ALTERNATING-CURRENT POWER

John H. Morecroft and H. Trap Friis **Vol. xxxviii—1919, pp. 1415-1444**

The first part of this article deals with the operation of the tube when separately excited, the variation of power with the amount of excitation, the load impedance, etc., and also gives an analysis of the forms and phases of voltages and currents in the different parts of the circuit.

The second part deals with the efficiency of the tube as a generator; the action is analyzed in detail and the conditions for maximum efficiency deduced, the theoretically deduced conclusions being substantiated by experimental data. Oscillograms are given to show the action of the tube under practically all the conditions which are likely to occur.

Discussion incorporated with that of paper by A. M. Nicolson on "The Piezo-Electric Effect in the Composite Rochelle Salt Cyrstal."

PRESENT LIMITS OF SPEED AND POWER OF SINGLE-SHAFT STEAM TURBINES

J. F. Johnson **Vol. xxxviii—1919, pp. 1515-1525**

This paper restricted to a discussion of some of the factors which influence limits as applying particularly to turbines of the reaction type. With the employment of high vacua the limit of power will be determined largely by the area obtainable through the last stage.

Fig. 6 shows maximum capacity at various speeds which are physically possible without exceeding present limits of stresses. It is valuable chiefly as showing the physical relation between speed and capacity with given limiting stress values.

Discussion incorporated with that of paper by F. D. Newbury on "Present Limits of Speed and Output of Single-Shaft Turbo Generators."

PRESENT LIMITS OF SPEED AND POWER OF SINGLE-SHAFT CURTIS STEAM TURBINES

Eskil Berg **Vol. xxxviii—1919, pp. 1527-1585**

This paper starts by showing that the limit of a single-unit turbo generator does not lie in the generator but is confined to the steam turbine, and that the last wheel of the turbine is the limiting feature.

The author therefore takes the last wheel of an 1800-rev. per min. turbine, giving dimension stresses, kind of material used etc., and then designs two turbines, one having 23 stages and the other 13 stages.

A 5000-kw., five-stage, 3600-rev. per min. turbine load curve is also given and discussed.

Discussion incorporated with that of paper by F. D. Newbury on "Present Limits of Speed and Output of Single-Shaft Turbo Generators."

PRESENT LIMITS OF SPEED AND OUTPUT OF SINGLE-SHAFT TURBO GENERATORS

F. D. Newbury **Vol. xxxviii—1919, pp. 1537-1546**

Output is determined broadly by rotor or stator dimensions. With speeds of 1200 rev. per min. and lower, the stator is the limiting member, while with higher speeds, the rotor is the limiting member.

The most effective rotor diameter is not necessarily the largest diameter. To obtain maximum output at a given speed the rotor proportions must be chosen to properly balance mechanical stresses, rotor ampere turns and flux. American design practise has established 400 ft. per sec. as an upper limit of rotor peripheral speed.

The maximum length of core is determined by such factors as ventilation, bearing temperatures, critical speed and limits to weight imposed by forging and transportation facilities.

No opinion is expressed as to the wisdom of installing very large single-ft units.

Discussion, (including that of papers by J. F. Johnson and E. Berg), pages 1547-1565, by Messrs. P. Torchio, B. A. Behrend, W. L. R. Emmet, W. J. Foster, A. M. Gray, R. B. Williamson, F. Hodgkinson, B. G. Fernald, C. A. Adams, Farley Osgood, E. Berg and F. D. Newbury.

A general discussion of factors limiting capacity and reliability with particular emphasis on metal fatigue due to vibration.

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Discussion, pages 1756-1757, by Q. Graham.

10. PRIME MOVERS AND STEAM BOILERS

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A general discussion of factors limiting capacity and reliability with particular emphasis on metal fatigue due to vibration.

11. POWER PLANTS AND CENTRAL STATIONS

SOME PROBLEMS IN THE OPERATION OF POWER PLANTS IN PARALLEL

E. C. Stone **Vol. xxxviii—1919, pp. 1651-1674**

In order to operate two power plants satisfactorily in parallel, the transmission line which ties them together must have sufficient synchronizing power, as well as sufficient carrying capacity. The "synchronizing power" of a line depends upon its resistance and reactance, the bus voltages maintained at its ends, and the maximum kilovolt-amperes it must transmit. Limiting values for "synchronizing power" of lines under various operating conditions are given.

The design of a transmission line involves a consideration of load to be transmitted, voltage, reactance, resistance, losses, and charging current of the line, and of wattless generating capacity at the receiving end of the line. The wattless generating capacity at the end of the line determines how many kilowatts will be transmitted for each ampere of line current, by fixing the power factor of the load transmitted and the voltage at the receiving end of the line. When a line is to be designed for paralleling two plants, it must have sufficient "synchronizing power" to hold the two plants together.

No discussion.

12. PARALLEL OPERATION

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No discussion.

13. TRANSMISSION LINES

THEORY OF THE TRANSIENT OSCILLATIONS OF ELECTRICAL NETWORKS AND TRANSMISSION SYSTEMS

J. R. Carson **Vol. xxxviii—1919, pp. 345-427**

The purpose of this research was to make a broad theoretical study of transient phenomena with a view to developing methods of calculation directly applicable to engineering problems. The investigation starts with the problem of formulating the current in an electrical network or transmission system in response to a suddenly applied e. m. f. of arbitrary form. A simple formula is derived which expresses this current in terms of two independent functions: one, the applied e. m. f. expressed as a time function, and the other a characteristic function of the constants and connections of the system, this latter being termed the "indicial admittance" of the system. A systematic investigation of methods for solving and computing the indicial admittance follows, in the course of which original solutions for transmission and artificial lines are derived and a new method involving integral equations is developed.

Discussion incorporated with that of paper by W. W. Crawford on "Telephone Circuits with Zero Mutual Induction."

HIGH-TENSION SINGLE-CONDUCTOR CABLE FOR POLYPHASE SYSTEMS

W. S. Clark and G. B. Shanklin **Vol. xxxviii—1919, pp. 917-969**

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Discussion incorporated with that of paper by R. W. Atkinson on "The Dielectric Field in an Electric Power Cable."

PROBLEMS OF 220-KV. POWER TRANSMISSION

A. E. Silver **Vol. xxxviii—1919, pp. 1037-1100**

Two hundred and twenty kv. is suggested as a logical voltage for high capacity, long-distance transmission, and the important problem-introduced by large concentrations of power, high voltage and high service standards are discussed. The economic and technical considerations underlying design of a 220-kv. system are outlined, and general designs are developed for a typical 220-kv. transmission line.

Discussion, pages 1101-1135, by Messrs. W. S. Murray, W. M. Dann, P. Torchio, S. W. Mauger, R. M. Spurck, J. C. Parker, J. C. Clark, F. C. Hanker, H. R. Summerhayes, R. P. Jackson, H. B. Dwight, J. F. Peters, F. F. Brand, C. F. Harding, F. W. Peek, Jr., J. A. Koontz, H. G. Mac Donald, L. B. Chubbuck, J. N. Mahoney, E. B. Meyer, C. E. Howell, T. B. Parker, J. B. Crane and A. E. Silver.

A general discussion of the super-power problem particularly with reference to the N. Y. and New England section.

THE EFFECT OF TRANSIENT VOLTAGES ON DIELECTRICS—II

The Effect of Lightning Voltages on Arrester Gaps, Insulators and Bushings on Transmission Lines

F. W. Peek, Jr. **Vol. xxxviii—1919, pp. 1137-1164**

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Data are given on the steepness of lightning waves actually occurring on transmission lines in practise.

Discussion, pages 1165-1177, by Messrs. W. A. Del Mar, F. W. Peek, Jr. and C. T. Allcutt.

A general discussion.

CALIFORNIA 220,000-VOLT—1100-MILE—1,500,000-KW. TRANSMISSION BUS

R. W. Sorensen, H. H. Cox and G. E. Armstrong **Vol. xxxviii—1919, pp. 1237-1248**

This paper summarizes the power resources of California and the probable loads to be supplied within the next six or seven years. For the purpose of economically distributing the necessary power and supplying the load, a long high-voltage transmission line is proposed. As this line would interconnect a number of different companies, it assumes the nature of a bus bar. The authors show how the proposed line may link with some of the lines now in service and enumerate the advantages of such interconnection. A comparison is made between the 240-mile Big Creek line now operating at 150,000 volts, 50 cycles, and the operation of this line at 220,000 volts, 60 cycles. Operating data on the Big Creek line are shown to indicate the character of the construction necessary for California conditions. Conclusions are drawn as to the particular features to be observed for successful operation of 220,000-kv. lines.

Discussion, pages 1249-1268, by Messrs. R. Bennett, P. D. Jennings, C. O. Poole, J. D. Ross, L. F. Curtis, J. B. Fisken, H. A. Barre, L. M. Klauber, J. A. Lighthipe, P. M. Downing, J. F. Stevens, C. W. Koiner, L. S. Ready, G. A. Damon, L. J. Corbett, H. H. Cox and G. E. Armstrong.

A general discussion of the advantages and disadvantages of the super-power line.

14. ELECTRIC SERVICE DISTURBANCES AND PROTECTION

REVIEW OF WORK OF SUB-COMMITTEE ON WAVE SHAPE STANDARD OF THE STANDARDS COMMITTEE

H. S. Osborne **Vol. xxxviii—1919, pp. 261-288**

The paper gives a review of the work done during the past three years by a sub-committee of the Standards Committee of the Institute appointed to make recommendations for changes in the Institute's rules regarding the wave shape of alternators. The Sub-Committee last spring made recommendations that for the present the ten per cent deviation rule should be retained without change (except in wording), and that trial use should be made of a supplementary wave shape factor. The new factor, based on the relation between voltage wave shape and interfering effect in telephone circuits, when power and telephone lines parallel each other, is called the "telephone interference factor."

Discussion (including that of paper by C. P. Steinmetz), pages 289-303, by Messrs. C. F. Scott, C. R. Underhill, A. E. Kennelly, C. P. Steinmetz, J. B. Whitehead, W. J. Foster, J. B. Taylor, J. J. Linebaugh, E. E. F. Creighton, L. W. Chubb, C. A. Adams, L. T. Robinson, C. F. Harding and H. S. Osborne.

A general discussion.

THEORY OF THE TRANSIENT OSCILLATIONS OF ELECTRICAL NETWORKS AND TRANSMISSION SYSTEMS

J. R. Carson **Vol. xxxviii—1919, pp. 345-427**

The purpose of this research was to make a broad theoretical study of transient phenomena with a view to developing methods of calculation directly applicable to engineering problems. The investigation starts with the problem of formulating the current in an electrical network or transmission system in response to a suddenly applied e. m. f. of arbitrary form. A simple formula is derived which expresses this current in terms of two independent functions: one, the applied e. m. f. expressed as a time function, and the other a characteristic function of the constants and connections of the system, this latter being termed the "indicial admittance" of the system. A systematic investigation of methods for solving and computing the indicial admittance follows, in the course of which original solutions for transmission and artificial lines are derived and a new method involving integral equations is developed.

Discussion incorporated with that of paper by W. W. Crawford on "Telephone Circuits with Zero Mutual Induction."

TELEPHONE CIRCUITS WITH ZERO MUTUAL INDUCTION

W. W. Crawford **Vol. xxxviii—1919, pp. 429-457**

The paper deals with the reduction of inductive interference in telephone circuits.

Various relative positions of two or more circuits, in which the mutual inductance is zero, and the mutual capacitance unbalance is approximately zero, are discussed. The most important case is that when the two wires of one circuit occupy opposite ends of one diagonal of a square, and the other circuit, the ends of the other diagonal.

Several forms of construction embodying this arrangement, and built largely with standard parts, are illustrated.

Calculations and tentative designs are presented to show that the use of these forms of construction will give greatest refinement of balance against induction from power circuits, and possibly also against cross-talk, increased flexibility in coordinating with the variations in exposure to power circuits, a simplification of the transpositions system, fewer transposition poles and transpositions, and when desired, the realization of a part of these advantages with the lead compressed into less than the normal space.

The cost of initial construction of the proposed forms and the cost of conversion of an existing lead to one of the proposed forms.

Discussion (including that of papers by E. B. Craft and E. H. Colpitts, and J. R. Carson), pages 458-488, by Messrs. G. O. Squier, M. I. Pupin, A. H. Cowles, A. S. Dana, A. Pinto, V. Bush, J. H. Morecroft, A. G. Chapman, H. S. Osborne and J. B. Carson.

A general discussion including a description of the theory of functioning of vacuum tubes.

ABNORMAL VOLTAGES WITHIN TRANSFORMERS

L. F. Blume and A. Boyajian **Vol. xxxviii—1919, pp. 577-614**

Mathematical analysis is made of a rectangular wave impinging upon a transformer winding and quantitative values deduced of the resulting internal voltage stresses in terms of transformer constants. It is shown that the conclusions also apply in part to abrupt impulses and approximate idea is given of the reaction of transformer to high frequencies. The difference between operating transformer with isolated and grounded neutral is shown. Energy losses are not considered in the mathematics although the manner in which the results are affected is pointed out. Finally, theoretical results are compared with impulse and high frequency tests made in the laboratory.

Discussion, pages 615-620, by Messrs. H. R. Summerhayes, G. Faccioli,

C. A. Adams, J. F. Peters, F. Dubsky, F. W. Peek, Jr., A. E. Kennelly and L. F. Blume.

A general discussion.

TRANSMISSION LINE RELAY PROTECTION

H. R. Woodrow, D. W. Roper, O. C. Traver and P. Macgahan

Vol. xxxviii—1919, pp. 795-826

The Protective Devices Committee made an analysis of the transmission line protective relay situation throughout the country and prepared a summary of the experiences of the operating companies with this protective apparatus. Questionnaires were sent out to sixty-one operating companies asking for their experience and present practise.

Replies were received from 32 and from these a general analysis was made of the practise of relay protection for transmission lines.

The Protective Devices Committee undertook the work of standardizing the nomenclature which it is hoped will be adopted by all parties.

Discussion incorporated with that of paper by W. E. Richards on "Grounded Neutral Transmission Line."

GROUNDING THE NEUTRAL OF GENERATING AND TRANSMISSION SYSTEMS

H. R. Woodrow **Vol. xxxviii—1919, pp. 827-831**

Four different high-tension transmission systems in the New York district have been grounded at different times during the past four years, all of which are operating with the grounded neutral at the present time. These systems were all grounded in a different manner and the results obtained from the effect of grounding showed that the troubles have been reduced.

In addition to the grounding, special ground relays have been installed on three of the systems which disconnected the feeder very quickly on a ground and the results show that the addition of this quick acting relay further reduces the stresses on a system.

Discussion incorporated with that of paper by W. E. Richards on "Grounded Neutral Transmission Line."

GROUNDED NEUTRAL TRANSMISSION LINE

W. E. Richards **Vol. xxxviii—1919, pp. 833-838**

The paper describes the conditions on the system in Toledo which was originally delta-connected. Serious trouble was experienced when a short circuit occurred especially with synchronous apparatus. This trouble was overcome by changing the transmission to a Y system with the neutral grounded.

Discussion (including that of papers by Woodrow, Roper Traver and Macgahan and Woodrow), pages 839-869, by Messrs. J. R. Craighead, J. A. Johnson, A. H. Lawton, E. E. F. Creighton, H. T. Conover, O. C. Traver, P. Torchio, D. W. Roper, P. H. Adams, R. N. Conwell, A. W. Copley, C. P. Osborne, P. M. Lincoln, C. P. Steinmetz, E. T. Moore, N. L. Pollard, R. F. Schuchardt, J. C. Parker, H. C. Don Carlos, D. A. McKenzie, F. L. Hunt, F. C. Hanker, H. C. Albrecht, E. A. Hester,

E. G. Merrick, N. A. Carle, H. R. Woodrow, H. B. Vincent, F. D. Newbury and A. E. Silver.

A general discussion of present practise.

THE EFFECT OF TRANSIENT VOLTAGES ON DIELECTRICS—II

The Effect of Lightning Voltages on Arrester Gaps, Insulators and Bushings on Transmission Lines

F. W. Peek, Jr. **Vol. xxxviii—1919, pp. 1137-1164**

This paper treats of some of the practical applications resulting from an investigation of the effect of lightning voltages on insulators, bushings and protective gaps.

The relative protective values of various gaps for steep and slanting wave fronts and high frequency are shown graphically in Figs. 14, 15, 16 and 17.

Data are given on the steepness of lightning waves actually occurring on transmission lines in practise.

Discussion, pages 1165-1177, by Messrs. W. A. Del Mar, F. W. Peek, Jr. and C. T. Allcutt.

A general discussion.

THEORY OF PROBABILITIES APPLIED TO FAILURES OF SUSPENSION INSULATORS

L. M. Klauber **Vol. xxxviii—1919, pp. 1199-1212**

There are a wide variety of operating conditions which affect the amount of over-insulation required, and after having found the minimum number of insulators per string required for any given operating conditions the author points out a method of determining the amount of extra insulation desirable from an insurance standpoint according to the law of probabilities. Equations are developed from which the probability of failure for any given case or the ratio between such probabilities for any pair of cases may be determined directly. A numerical example is also given which shows the development of the theory of minimum annual cost for combined mechanical and electrical failures.

Discussion, pages 1213-1222, by Messrs. W. D. Peaslee, C. O. Poole, J. A. Lighthipe, J. H. Anderton, H. A. Barre, J. B. Fisken, R. W. Shoemaker, E. F. Scattergood, L. C. Williams and L. M. Klauber.

A general discussion on the practicability of the method described.

16. CONTROL, REGULATION AND SWITCHING

PREDETERMINATION OF SYNCHRONOUS PHASE-MODIFIER PERFORMANCE

Hubert V. Carpenter **Vol. xxxviii—1919, pp. 1223-1229**

The author reviews the method for showing the behavior of transmission lines first given by Perrine and Baum and then shows how it can be used in determining the effect of the use of a synchronous motor operating without load for improving the power factor.

The diagram given shows both the improvement in voltage regulation and the change in power factor due to the phase-modifier for any assumed condition of loading for the transmission line.

The errors of the method are discussed with methods for determining their magnitude, and the advantages of the graphical treatment pointed out.

Discussion, pages 1230-1235, by Messrs. H. B. Dwight, M. O. Bosler, J. A. Lighthipe, P. M. Downing, J. H. Anderton, L. F. Curtis, J. F. Wilson and H. L. Melvin.

Discussion of various methods of determination including use of circle diagram.

STARTING CONDITIONS OF SYNCHRONOUS MACHINES

A. Hay and F. N. Mowdawalla **Vol. xxxviii—1919, pp. 1713-1755**

A detailed study of the behavior of the synchronous motor during what has always been regarded as a critical period—*viz.* the period when the motor is being accelerated from rest to synchronous speed.

Discussion, pages 1756-1757, by Q. Graham.

17. TRACTION

SOME POSSIBILITIES OF STEAM RAILROAD ELECTRIFICATION AS AFFECTING FUTURE POLICIES

Calvert Townley **Vol. xxxviii—1919, pp. 621-627**

Electricity fills every requirement of railroad service, but as it involves a large investment, electrification has proceeded slowly. Electrification has also been retarded because the problem has been largely considered one of replacing the steam locomotive by the electric locomotive whereas in reality the problem is much broader. It really offers a fundamentally different method of train propulsion because the limitations of the steam locomotive disappear and the strictly limited motive power is replaced by one that is practically unlimited, thereby, opening up many possibilities in the methods of railroad operation. A brief review is given of electrified sections of railways showing the advantages which have been realized in both the freight and passenger service.

Discussion, pages 628-649, by Messrs. F. H. Shepard, W. S. Murray, C. Schwartz, J. B. Taylor, N. W. Storer, W. B. Potter, G. F. Sever, C. F. Scott, G. Gibbs, J. Murphy, H. D. Jackson and C. Townley.

Discussion deals chiefly with coal saving incident to electrification particularly that saving involved in the proposed super-power transmission line, etc.

18. LIGHTING AND LAMPS

PRESENT STATUS OF INDUSTRIAL LIGHTING CODES

G. H. Stickney **Vol. xxxviii—1919, pp. 725-742**

Industrial lighting codes have been adopted in four states and in Federal establishments. Similar action is under consideration in several other states and there is prospect of extension throughout the country.

Investigation and experience indicate the need of government regulation of factory lighting. When adopted by industrial commissions under authority granted by Legislatures, the codes become in effect state law. The existing codes correspond in essentials to the Illuminating Engineering Society code, on which they are based.

Discussion, pages 743-765, by Messrs. C. A. Adams, C. E. Clewell, E. B. Rosa, C. B. Auel, M. G. Lloyd, W. Harrison, J. Vogt, W. P. Hurley, W. T. Blackwell, E. G. Perrott, T. F. Foltz, J. R. Cravath and G. H. Stickney.

A general discussion of the code as established in the various States.

THE SEARCHLIGHT IN THE U. S. NAVY

Ralph Kelly **Vol. xxxviii—1919, pp. 1605-1620**

The types and uses of searchlights and signaling lights on naval ships are briefly described.

A changed form of 12-inch incandescent searchlight is suggested.

The introduction of the high-power searchlight revolutionized the application of the searchlight to naval ships.

It is believed there is also a possibility of a considerable improvement in the electrodes.

The use of the dome glass door enables a searchlight to operate in close proximity to large calibre rifles and makes it possible to successfully build even larger sizes of searchlights than those at present in use.

The star shell has great possibilities, but it is doubtful if it will ever supersede the high-power searchlight.

Discussion, pages 1621-1634, by Messrs. D. McNicol, G. A. DeGraaf, M. L. Patterson, E. J. Murphy, J. C. Ledbetter, R. A. Beekman, B. P. Beehler, C. Lichtenberg and R. Kelly.

A general discussion of searchlights including a comprehensive discussion of army practise.

19. ELECTRICITY IN THE ARMY AND NAVY

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20. MISCELLANEOUS APPLICATIONS OF ELECTRICITY

WELDING MILD STEEL

H. M. Hobart **Vol. xxxviii—1919, pp. 63-111**

The Welding Research Sub-Committee, formed in 1918, was a subcommittee of the Metallurgical and Electrical Sections of the Engineering Division of the National Research Council, and the Welding Committee, (under the chairmanship of Professor C. A. Adams) came under the direction of the Emergency Fleet Corporation. The paper is in large part based on the work done by the Welding Research Sub-Committee up to January of this year.

Discussion incorporated with that of paper by O. H. Eschholz on "Fusion in Arc Welding."

WELDING AS A PROCESS IN SHIP CONSTRUCTION

S. V. Goodall **Vol. xxxviii—1919, pp. 113-119**

The paper points to the necessity for a reduction in the cost of shipbuilding, and as riveting is one of the most expensive items of construction the substitution of welding for riveting would decrease the cost of construction considerably. A brief review is given of what has been done in substituting welding for riveting, and to the limited extent to which electric welding has been tried it has been found successful. Lloyd's Register is prepared to classify electrically-welded vessels subject to certain provisions, but shipbuilders have not as yet adopted welding to a large extent for the reason, in the author's opinion, that they know that welds are lacking in uniformity and it is impossible to tell when a joint is good or bad.

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O. H. Eschholz **Vol. xxxviii—1919, pp. 121-129**

This paper calls attention to such characteristics as penetration and overlap, peculiar to this process, which facilitate visual inspection and discusses briefly the effect of arc length, welding procedure, electrode material, arc current and electrode diameter upon these characteristics.

Discussion (including that of papers by H. M. Hobart and G. V. Goodall), pages 130-177, by Messrs. W. H. Hill, H. A. Hornor, C. A. Adams, Capt. Corbett, A. M. Candy, C. J. Holslag, W. L. Merrill, D. C. Alexander, Jr., W. Spraragen, J. C. Armor, S. V. Goodall, R. P. Jackson, F. M. Farmer, W. S. Andrews, J. C. Lincoln, H. Lemp, H. D.

Morton, R. E. Wagner, T. T. Heaton, C. R. Darling, J. Churchward, C. H. Kicklighter, Z. Jeffries, A. S. Kinsey, H. G. Knox, J. Martin and O. H. Eschholz.

A general discussion of electric welding and its practical application.

UTILIZING THE TIME CHARACTERISTICS OF ALTERNATING CURRENT

Henry E. Warren **Vol. xxxviii—1919, pp. 767-781**

By the invention of a very small, simple and reliable self-starting synchronous motor, in conjunction with convenient means for regulating the average frequency of alternating circuit, there has been developed a new field of usefulness for electric power. It is now feasible to drive all kinds of timing devices such as clocks, graphic instrument movements, time recorders, etc., directly from the lighting circuits. Remarkable accuracy is obtained, and the amount of care is minimized by the elimination of winding and regulating.

No discussion.

THE PIEZO-ELECTRIC EFFECT IN THE COMPOSITE ROCHELLE SALT CRYSTAL

A. McL. Nicolson **Vol. xxxviii—1919, pp. 1467-1485**

The piezo-electric effect is an electro-elastic property of certain crystals. It involves the conversion of mechanical into electrical energy, and also the converse effect. The paper presents an exposition of piezo-electricity and related optical and other properties belonging to these crystals.

Applications of the piezo-electric effect in such crystals to the transmission and reception of sound, are described and demonstrated.

Discussion, (including that of papers by A. W. Hull and Morecroft and Friis), pages 1486-1493, by Messrs. J. B. Taylor, A. W. Hull, A. M. Nicolson, E. A. Eckhardt, J. P. Wintringham, Irving Langmuir, J. E. Schroder, D. M. Therrell and J. H. Morecroft.

A general discussion.

21. TELEPHONY AND TELEGRAPHY

RADIO TELEPHONY

E. B. Craft and E. H. Colpitts **Vol. xxxviii—1919, pp. 305-343**

This paper is divided into two parts. The first part describes the development of the art of radio telephony by the American Telephone and Telegraph Company and the Western Electric Company to the accomplishment of trans-atlantic telephony, followed by demonstrations of the use of radio telephony between ships and of methods of connecting radio and wire telephone systems.

The second part is concerned almost entirely with the work of producing radio telephone and allied apparatus for the Army and Navy in the late war.

Discussion incorporated with that of paper by W. W. Crawford on "Telephone Circuits with Zero Mutual Induction."

TELEPHONE CIRCUITS WITH ZERO MUTUAL INDUCTION

W. W. Crawford **Vol. xxxviii—1919, pp. 429-457**

The paper deals with the reduction of inductive interference in telephone circuits.

Various relative positions of two or more circuits, in which the mutual inductance is zero, and the mutual capacitance unbalance is approximately zero, are discussed. The most important case is that when the two wires of one circuit occupy opposite ends of one diagonal of a square, and the other circuit, the ends of the other diagonal.

Several forms of construction embodying this arrangement, and built largely with standard parts, are illustrated.

Calculations and tentative designs are presented to show that the use of these forms of construction will give greatest refinement of balance against induction from power circuits, and possibly also against cross-talk, increased flexibility in coordinating with the variations in exposure to power circuits, a simplification of the transpositions system, fewer transposition poles and transpositions, and when desired, the realization of a part of these advantages with the lead compressed into less than the normal space.

The cost of initial construction of the proposed forms and the cost of conversion of an existing lead to one of the proposed forms.

Discussion (including that of papers by E. B. Craft and E. H. Colpitts, and J. R. Carson), pages 458-488, by Messrs. G. O. Squier, M. I. Pupin, A. H. Cowles, A. S. Dana, A. Pinto, V. Bush, J. H. Morecroft, A. G. Chapman, H. S. Osborne and J. B. Carson.

A general discussion including a description of the theory of functioning of vacuum tubes.

ELECTROMAGNETIC THEORY OF THE TELEPHONE RECEIVER WITH SPECIAL REFERENCE TO MOTIONAL IMPEDANCE

A. E. Kennelly and H. Nukiyama **Vol. xxxviii—1919, pp. 651-699**

The theory of the telephone receiver here offered is based upon the motional impedance circle which has been published in various chapters during the last few years. The new theory, which is stated under definite limitations, is a further development; taking into account the m. m. f. produced by the vibration of the diaphragm in the permanent magnetic field.

Discussion incorporated with that of paper by A. G. Webster on "The Absolute Measurement of the Intensity of Sound.".

TRANSOCEANIC RADIO COMMUNICATION

E. F. W. Alexanderson **Vol. xxxviii—1919, pp. 1269-1285**

The paper defines the state of the art of today which is the result of developments during the war. Transoceanic radio communication is at present maintained by five first class stations, two in America and three in Europe. New developments indicate three methods for increasing the radio traffic without interference between the different messages.

The second part of the paper describes the radio transmitting system for the development of which the author is responsible. This system is represented by the naval radio station, New Brunswick, N. J. and comprises new means for generating, modulating, and radiating the continuous wave energy. The general theory and figures for the increased radiation efficiency are given. The author also predicts that the multiple antenna will make possible directive radiation on a large scale.

No discussion.

TELEPHONE REPEATERS

Bancroft Gherardi and Frank B. Jewett **Vol. xxxviii—1919, pp. 1287-1345**

In this paper the authors have endeavored to set forth briefly but clearly the history of the research and development work which has led up to the final production of successful telephone repeaters. The various forms of amplifiers which have been suggested are described and their possibilities and limitations pointed out. The essential properties of repeater networks together with the necessary line conditions for successful repeater operation are described and illustrated. Tandem operation of repeaters is discussed as is also the use of repeaters in four-wire circuits. Illustrations are given showing a few of the more important repeater installations now in regular commercial service in the United States.

No discussion.

PRINCIPLES OF RADIO TRANSMISSION AND RECEPTION WITH ANTENNA AND COIL AERIALS

J. H. Dellinger **Vol. xxxviii—1919, pp. 1347-1402**

Coil aerials are coming to replace the large antennas in radio work. The advantage of the coil aerial as a direction finder, interference preventer, reducer of strays and submarine aerial, make it important to know how effective such an aerial is as a transmitting and receiving device in comparison with the ordinary antenna. In this article the mathematical theory is presented and as a result, the answer to this question is obtained. The fundamental principles of design of aerials are given in this paper.

Discussion, pages 1403-1414, by Messrs. F. W. Grover and A. Press. A criticism of assumptions made by the author.

APPLICABILITY OF AUTOMATIC SWITCHING TO ALL CLASSES OF TELEPHONE SERVICE

A. B. Smith **Vol. xxxviii—1919, pp. 1567-1600**

This paper is an attempt to place before engineers general information in regard to automatic telephone switching.

The automatic switching of telephone lines is adaptable to all classes of telephone service and offers a flexible means of solving problems.

Discussion, pages 1601-1604, by Messrs. Selby Haar, L. F. Morehouse, P. S. Clapp, F. L. Baer and A. B. Smith.

UNIFICATION OF THE MANUAL AND AUTOMATIC TELEPHONE SYSTEMS

D. E. Wiseman **Vol. xxxviii—1919, pp. 1635-1650**

A description is given of the physical consolidation of the Bell manual and the Automatic Electric telephone systems of Los Angeles, Cal., which previously to June 1, 1918 operated as separate systems.

No discussion.

22. MISCELLANEOUS TOPICS AND INSTITUTE AFFAIRS

AERONAUTICS IN THE UNITED STATES, 1918

G. O. Squier **Vol. xxxviii—1919, pp. 1-62**

Major General George O. Squier, Chief Signal Officer of the United States Army, reviews in this address the development of Military Aeronautics in the United States up to the date of the armistice, November 11, 1918.

No discussion.

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H. M. Hobart **Vol. xxxviii—1919, pp. 63-111**

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A general discussion of electric welding and its practical application.

ENGINEERING AND INDUSTRIAL STANDARDIZATION

C. A. Adams **Vol. xxxviii—1919, pp. 179-189**

An address presented at Midwinter Convention.

REVIEW OF WORK OF SUB-COMITTEE ON WAVE SHAPE STANDARD OF THE STANDARDS COMMITTEE

H. S. Osborne **Vol. xxxviii—1919, pp. 261-288**

The paper gives a review of the work done during the past three years by a sub-committee of the Standards Committee of the Institute appointed to make recommendations for changes in the Institute's rules regarding the wave shape of alternators. The Sub-Committee last spring made recommendations that for the present the ten per cent deviation rule should be retained without change (except in wording), and that trial use should be made of a supplementary wave shape factor. The new factor, based on the relation between voltage wave shape and interfering effect in telephone circuits, when power and telephone lines parallel each other, is called the "telephone interference factor."

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A general discussion.

IONIZATION OF OCCLUDED GASES IN HIGH-TENSION INSULATION

G. B. Shanklin and J. J. Matson **Vol. xxxviii—1919, pp. 489-536**

This paper deals with the ionization of occluded gases in solid insulation from the standpoint that these gas spaces are the weakest part of an insulation design and should receive first consideration. The stress at which ionization starts in different types of built-up insulation, such as used in cables and coils, is measured and from these measurements a safe working stress determined.

Discussion incorporated with that of paper by F. Dubsky on "The Dielectric Strength of Air Films Entrapped in Solid Insulation and A Practical Application to the Problem for Alternator Coils and Cables."

THE DIELECTRIC STRENGTH OF AIR FILMS ENTRAPPED IN SOLID INSULATION AND A PRACTICAL APPLICATION OF THE PROBLEM FOR ALTERNATOR COILS AND CABLES

F. Dubsky **Vol. xxxviii—1919, pp. 537-558**

An experimental investigation of the strength of air films of various thicknesses between glass plates. In the arrangement used, the breakdown of the air or the starting point of corona could be readily observed. Tests were also made of the dielectric strength of air films between other solid insulations.

It was found that the dielectric strength of air films between insulations was practically the same as the dielectric strength of air films between conductors.

The second part of the paper is devoted to the practical application of the data to the design of armature coils and cables, and several specific examples are given.

Discussion (including that of paper by G. B. Shanklin and J. J. Matson) pages 559-575, by Messrs. H. G. Reist, L. W. Chubb, J. B. Whitehead, C. A. Adams, W. J. Foster, H. W. Fisher, R. E. Argersinger, C. F. Harding, R. W. Atkinson, D. Du Bois, F. W. Peek, Jr., E. E. F. Creighton, J. J. Matson, W. I. Middleton, E. W. Davis and F. Dubsky.

A general discussion.

THE ABSOLUTE MEASUREMENT OF THE INTENSITY OF SOUND

Arthur G. Webster **Vol. xxxviii—1919, pp. 701-712**

This paper includes a description of a series of acoustical researches extending over a period of twenty-eight years. The properties of vibrating bodies and the subject of elastic hysteresis are discussed. Two fundamentally important instruments for the absolute measurement of sound have been developed and the theory given. The first is the standard of sound, called the phone, which is capable of reproducing at any time a sound of the simplest character and which permits the output of sound to be measured in watts of energy. The second is an instrument called the phonometer for measuring a sound in absolute measure. This instrument is now practically as sensitive as the human ear. The determination of the space distribution of sound and of the effect of disturbing bodies, and the measurement of the reflecting coefficient of surfaces have been accomplished. The phonotrope is a third instrument designed and used to find the direction of a source of sound, for example a fog signal.

Discussion, (including that of paper by A. E. Kennelly and H. Nukiyama), pages 713-723, by Messrs. J. B. Taylor, T. E. Shea, B. A.

Behrend, R. L. Jones, A. G. Webster, H. S. Osborne, A. Press and A. E. Kennelly.

A general discussion including the algebraic theory of the receiver and the application of the circle diagram.

COOPERATION

C. A. Adams **Vol. xxxviii—1919, pp. 783-793**

President's Address.

TECHNICAL COMMITTEE REPORTS

Vol. xxxviii—1919, pp. 871-916

THE VACUUM TUBE AS A GENERATOR OF ALTERNATING-CURRENT POWER

John H. Morecroft and H. Trap Friis **Vol. xxxviii—1919, pp. 1415-1444**

The first part of this article deals with the operation of the tube when separately excited, the variation of power with the amount of excitation, the load impedance, etc., and also gives an analysis of the forms and phases of voltages and currents in the different parts of the circuit.

The second part deals with the efficiency of the tube as a generator; the action is analyzed in detail and the conditions for maximum efficiency deduced, the theoretically deduced conclusions being substantiated by experimental data. Oscillograms are given to show the action of the tube under practically all the conditions which are likely to occur.

Discussion incorporated with that of paper by A. M. Nicolson on "The Piezo-Electric Effect in the Composite Rochelle Salt Crystal."

THE POSITIONS OF ATOMS IN METALS

A. W. Hull **Vol. xxxviii—1919, pp. 1445-1466**

When a narrow beam of X-rays passes through a fine powder of any crystalline material, it produces on a photographic plate placed just behind the powder a pattern of concentric circles. These circles are produced by the reflection of the X-rays from the planes of atoms in the crystal, and their diameters are a measure of the distances between these planes of atoms. By measuring the diameters of the circles the exact positions of the atoms can be determined. The results of this analysis are given for twenty common metals and several salts, with examples and brief description of the method, and a discussion of the results.

Discussion incorporated with that of paper by A. M. Nicolson on "The Piezo-Electric Effect in the Composite Rochelle Salt Crystal."

THE PIEZO-ELECTRIC EFFECT IN THE COPOSITE ROCHELLE SALT CRYSTAL

A. McL. Nicolson **Vol. xxxviii—1919, pp. 1467-1485**

The piezo-electric effect is an electro-elastic property of certain crystals. It involves the conversion of mechanical into electrical energy, and also the converse effect. The paper presents an exposition of piezo-electricity and related optical and other properties belonging to these crystals.

Applications of the piezo-electric effect in such crystals to the transmission and reception of sound, are described and demonstrated.

Discussion, (including that of papers by A. W. Hull and Morecroft and Friis), pages 1486-1493, by Messrs. J. B. Taylor, A. W. Hull, A. M. Nicolson, E. A. Eckhardt, J. P. Wintringham, Irving Langmuir, J. E. Schroder, D. M. Therrell and J. H. Morecroft.

A general discussion.

A REPORT ON ELECTROMAGNETIC INDUCTION

S. J. Barnett **Vol. xxxviii—1919, pp. 1495-1513**

This report discusses briefly the chief fundamental results obtained from the days of Faraday to the present time in studying the electromotive forces ordinarily referred to the domain of electromagnetic induction.

Self-induction is first taken up.

The motional electromotive force, developed when matter moves in a magnetic field, is next considered and is derived from Ampere's law on the electron theory.

The induced electromotive force in fixed conductors and insulators arising from the motion or alteration of other systems is next considered.

The report closes with a treatment of unipolar induction in both so-called open and closed circuits, including brief descriptions of some of the principal experiments, a discussion of the theories involved, and their application to the unipolar generator.

No discussion of this paper.

INHERENT LIMITATIONS ON TRANSFORMATIONS POSSIBLE BY STATIONARY APPARATUS

Joseph Slepian **Vol. xxxviii—1919, pp. 1697-1711**

Expressions for electrostatic and electromagnetic energies, Joulian heat dissipation and power are given in complex quantities. The pure imaginary part of the expression for power in a static network is shown to be equal to 2 ω times the difference between the mean electromagnetic energy and mean electrostatic energy. Use is made of this new principle in considering the problems of power-factor correction and phase splitting. It is shown that in general for phase transforming by static apparatus both magnetic and electrostatic storage of energy are necessary, and it is shown how the minimum amounts of each are determined by the load.

The symmetry of the coefficients in the general equations for the steady state in a static network is demonstrated, and it is shown that limitations upon voltage and current transformations follow. The voltage regulation of any phase-splitting arrangement is considered.

No discussion.

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AERONAUTICS IN THE UNITED STATES, 1918

BY GEORGE O. SQUIER

ABSTRACT OF PAPER

Major General George O. Squier, Chief Signal Officer of the United States Army, reviews in this address the development of Military Aeronautics in the United States up to the date of the armistice, November 11, 1918. The War Department's first heavier-than-air flying machine was produced approximately ten years ago, but for the eight years that followed, the development lagged—in fact less than a million dollars was appropriated for aeronautics in the entire eight years. Then under the pressure of war, the United States by necessity plunged into a gigantic aircraft program—and the accomplishments today are numbered by the score. The Liberty Engine, acknowledged now to be the standard for the world, was produced; an industry new to the United States was developed, and other tremendous strides taken in the science of aerial navigation.

I—Introduction

ALMOST exactly ten years ago (December, 1908) the undersigned had the honor of addressing, from this same platform, the American Society of Mechanical Engineers on the subject of Military Aeronautics, at its annual meeting. The preceding summer months had witnessed the first public flights of the Wright airplane at Fort Myer, Virginia.

The Signal Corps of the United States Army, under date of December 23, 1907, had issued an advertisement and specification for a heavier-than-air flying machine, and the sealed proposals under this specification were opened formally at Washington at twelve o'clock noon on February 1, 1908. The main requirement before acceptance, by the Government, was an endurance test in which the flying machine must remain continuously in the air for one hour without landing.

We all remember the world-wide interest which was aroused by the performance of this new and promising realization of the dream of the ages. During the continuance of the experiments, the eyes of the world were centered upon the little flying field just outside of Washington. The President and Cabinet Ministers were in frequent attendance, and the Congress adjourned from day to day whenever a flight was in prospect.

And then an almost inconceivable thing occurred. The Messrs. Wright having thoroughly demonstrated and fulfilled all of the conditions required by that specification, and the first machine having been duly purchased and paid for, during the eight years following the entire appropriations by this government for military aeronautics amounted to less than a million dollars.

European Powers, however, in their quest for military advantage, quickly interpreted what had happened at Fort Myer, and France, in particular, during these years was responsible for the principal advances in construction and design.

I shall not enter here into the causes, some military, some civil, of this utter unpreparedness, but the outbreak of war found the United States with but a handful of fliers and very few training planes. There was no aviation industry in this country, and the number of professional men trained as aeronautical engineers and designers was so small as to be practically negligible. In this respect the problem of developing the air program was unique. The United States had built ships before, had manufactured clothing, guns, munitions, built cantonments, etc.; and had a splendid body of men trained in these professions and employments, but, outside of a few men there was no one in the United States with experience in the design or building of even training planes.

Once the United States actually had entered the war, the pressure from our Allies and a sudden realization of our real situation in aeronautics led Congress to grant for this purpose, in the Act of May 12, 1917, $10,800,000; the Act of June 15, 1917, $31,846,000, and finally the appropriation of $640,000,000, the largest ever made by Congress for one specific purpose, which was put through the House of Representatives' Military Affairs Committee in two sittings, the House itself in one, the Senate Military Affairs Committee in forty-five minutes; and the Senate itself a week later, becoming a law on July 24, 1917, three months and a half after the outbreak of war.

The task you have given me of presenting within the present hour even an outline of the great undertaking which this nation has accomplished during the last nineteen months in developing the Air Service, is indeed impossible to meet.

I hasten to say at the outset that I can only hope to present to you some of the salient features and accomplishments of this effort. Of the hundreds and thousands of men and women who wrought valiantly and loyally to achieve these results, it will be impossible to mention specific names, or to give anything like just credit or mention to individuals or particular corporations.

Where names are mentioned in connection with certain features of the work, it must not be interpreted that there are not many others who are equally meritorious.

The Honor Roll of this Institute comprises no less than 1,317 men who participated in the Allied forces in the Great War. Of these, 976 were officers, 728 were officers in the American Army, 110 in the Signal Corps, and 52 in the Air Service.

Indeed, it is only in the long reaches of future history that we can form a just estimate of the many lines of endeavor which were set in motion in carrying out this enterprise, and which will, it is hoped, go forward into the future to bear their fruits in times of peace.

II—Statistical Resume of Aircraft Production and Training

THE work involved in training aviators and obtaining aeronautical equipment in the United States has been of such magnitude as to preclude the possibility of presenting here anything more than a brief résumé of the major accomplishments.

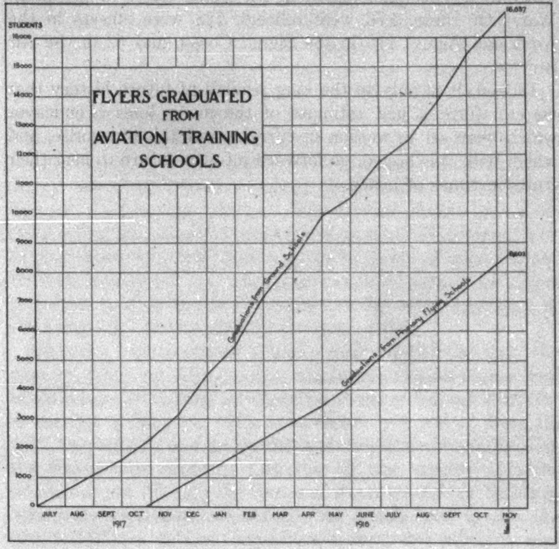


FIG. 1

The data is presented as of six o'clock a. m., November 11, 1918 (the date of the signing of the armistice).

PERSONNEL

More than 8600 fliers have been trained in the United States since its entry into the war. Monthly graduations from ground and flying schools are shown on Fig. 1. The gradually increasing deviation between the two curves on this chart is caused largely by failure of graduates of ground schools to develop into fliers.

There have been training fatalities in the United States, as in all other countries where training has been conducted on a large scale. However, when the fact is realized that our students have flown more than 880,000 hours, which is the equivalent to more than 66,000,000 miles, it will be seen that our training casualties have been astonishingly few. Statistics show that the United States has a lower percentage of fatalities than any other of the allied countries. The monthly average in the United States has been only one fatality for each 3200 hours flown. Training fatalities by months are shown in Fig. 2, and in Fig. 3 are shown the analyzed causes of these fatalities. Two independent investigations made in connection with this paper have developed the fact that more than 90 per cent of training casualties are attributed to the aviator himself. Fig. 4 shows the actual distribution of our fliers overseas.

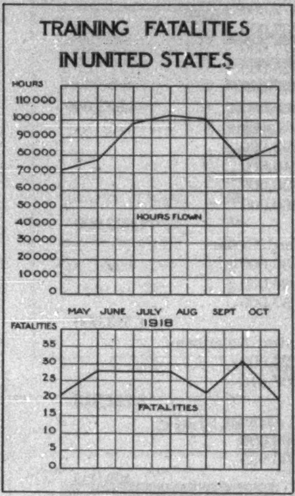


Fig. 2

Training Equipment

Herewith is presented a tabulation showing the actual production of training material up to November 11, 1918:

Planes		Engines			Propellers
Type	Number furnished	Name	H.P.	Number furnished	Number furnished
Elementary					
JN–4D	3,762	OX–5		83,18	17,830
SJ–1	1,600	A–7–A		2,250	5,173
Advanced					
JN–4H	929				
JN–6H	803	Hispano	150	3,497	5,295
SE–5	6				
S–4B	100	Gnome	100	278	500
S–4C	384				
E–1	18	La Rhone	80	1,107	358
Total	7,602			15,450	29,156

RAW MATERIALS

Some idea of the quantity of wood necessary for airplane and propeller construction may be obtained from the following table of actual deliveries:

	Spruce Board Feet	Totals
U. S. Army	25,472,000	
U. S. Navy	8,667,000	
England	36,877,000	
France	22,929,000	
Italy	9,147,000	
		103,092,000
	Douglas Fir	
U. S. Army	19,193,000	
U. S. Navy	21,746,000	
England	21,226,000	
France	9,460,000	
		71,625,000
	Port Orford Cedar	
U. S. Army	30,000	
U. S. Navy	1,926,000	
England	2,557,000	
		4,513,000
	Mahogany, Central American	
U. S. Army & Navy	4,524,000	
Allies	4,197,815	
		8,721,815
	Mahogany, African	
U. S. Army & Navy	269,000	
Allies	200,935	
		469,935
	American Black Walnut	
U. S. Army & Navy	2,408,000	
Allies	2,096,876	
		4,504,876
	Quartered White Oak	
U. S. Army & Navy	308,000	308,000
	Cherry	
U. S. Army & Navy	618,000	618,000
	Ash	
U. S. Army & Navy	87,000	
Allies	33,565	
		120,565
	Birch	
U. S. Army & Navy	663,000	663,000

In addition to airplane wood, it was found necessary to maintain strict supervision over the production of airplane and balloon fabrics. Of this class of material there had been produced to November 11, 1918, 3,187,000 yards of linen; 7,000,000 yards of cotton airplane fabric; 2,647,000 yards of balloon fabric.

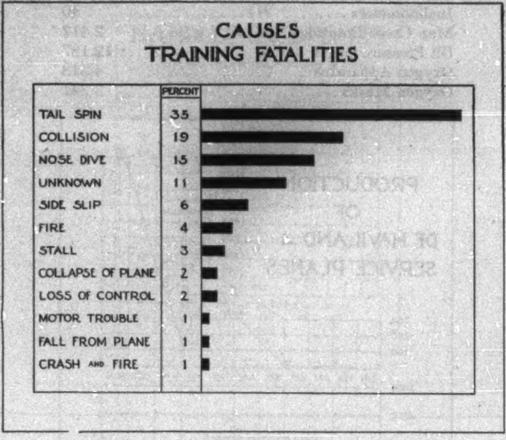


Fig. 3

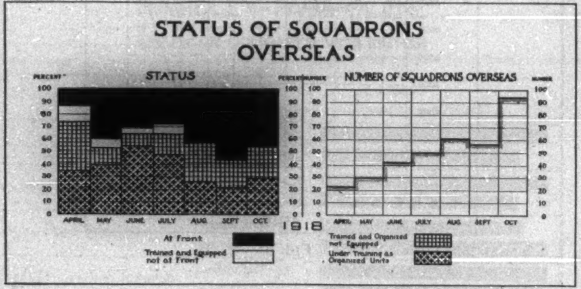


Fig. 4

Accessories

The production of airplane accessories to November 11, 1918 was as follows:

Air Pressure Gages	9,994
Air Speed Indicators	9,051
Altimeters	19,657

Clocks	17,593
Compasses Type "B"	10,179
Fire Extinguishers	12,209
Gasoline Gages	550
Inclinometers	40
Map Cases Rotating	2,417
Oil Pressure Gages	12,187
Oxygen Apparatus	4,318
Oxygen Masks	3,341

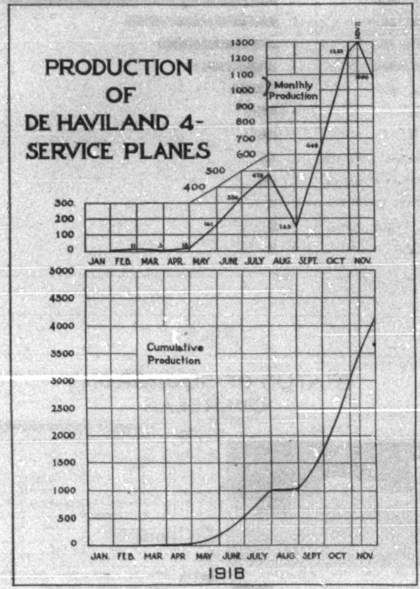


FIG. 5

Oxygen Tanks	12,000
Panel Switches	15,466
Radiator Thermometers	11,984
Tachometers	20,549

Ordnance for use in connection with the air service, such as aircraft machine guns, bombs, etc. were produced under the direction of the Air Service of the Army. Of this type of

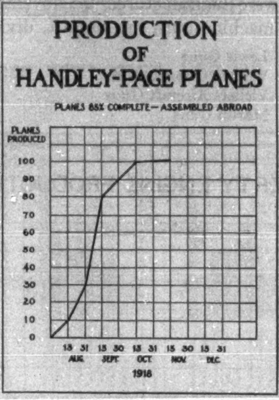
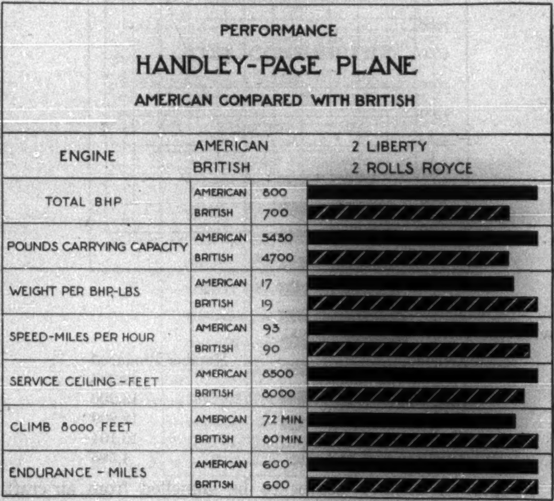


FIG. 6


PERFORMANCE

HANDLEY-PAGE PLANE

AMERICAN COMPARED WITH BRITISH

ENGINE	AMERICAN BRITISH		2 LIBERTY 2 ROLLS ROYCE
TOTAL BHP	AMERICAN	800	
	BRITISH	700	
POUNDS CARRYING CAPACITY	AMERICAN	5430	
	BRITISH	4700	
WEIGHT PER BHP-LBS	AMERICAN	17	
	BRITISH	19	
SPEED-MILES PER HOUR	AMERICAN	93	
	BRITISH	90	
SERVICE CEILING-FEET	AMERICAN	8500	
	BRITISH	8000	
CLIMB 8000 FEET	AMERICAN	72 MIN.	
	BRITISH	80 MIN.	
ENDURANCE - MILES	AMERICAN	600	
	BRITISH	600	

FIG. 7

material there were manufactured to November 11, 1918, a total of 81,754 machine guns for aircraft, divided as follows:

31,671	Lewis Guns
11,904	Vickers Ground Guns
411	Vickers Aircraft Guns
37,768	Marlin Guns

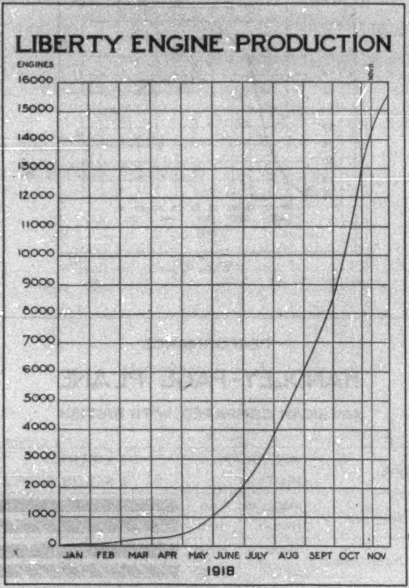


Fig. 8

with the following accessories:

Article	Number Produced
Flexible ring mounts	12,336
Ring sights	13,200
Wing vane sights	12,999
Unit sights	10,104
Auxiliary post and ring	4,585

A total of 510,271 bombs to be launched from air craft was produced to November 11, 1918, of which 114,809 were

LIBERTY ENGINE

COMPARED WITH OTHER AIRPLANE ENGINES

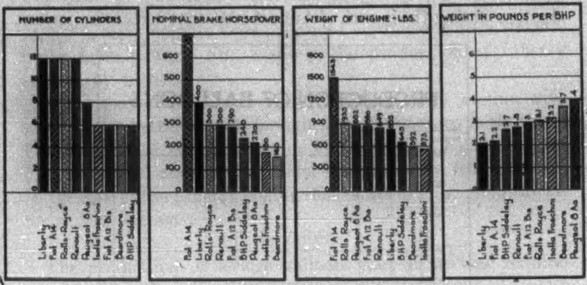


FIG. 9

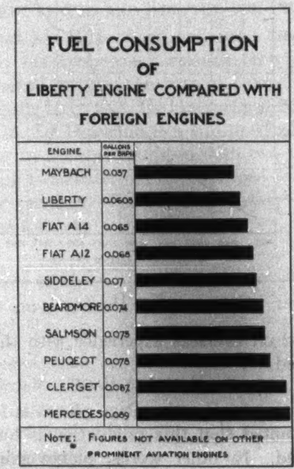


FIG. 10

incendiary bombs; 386,543 were high-capacity demolition bombs, and 8,919 were high-capacity fragmentation bombs.

SERVICE PLANES

DE HAVILAND 4

The production of the De Haviland 4 service plane is shown by months in Fig. 5. It was found necessary to delay production during the month of August 1918 that certain important changes, based on overseas experience with this plane, might be incorporated.

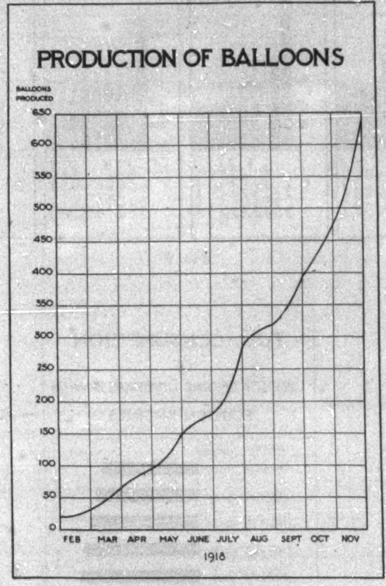


FIG. 11

HANDLEY-PAGE

The production of Handley-Page planes is found in Fig. 6. It should be noted that this production is for planes 85 per cent completed. In other words, planes shipped from this country were not assembled. Fifteen per cent was allowed for assembling abroad. The reason for shipping these planes abroad unassembled is obvious when the size of the assembled plane is taken into consideration. An interesting comparison of the American Handley-Page plane with its British prototype will be found in Fig. 7.

SERVICE ENGINES

More than 16,000 Liberty engines were produced during the calendar year 1918. To November 11, 1918, more than 14,000 Liberty engines were produced, equivalent to 5,700,000 brake horse-power. Fig. 8 shows the production of Liberty engines by months.

A comparison of the weight of the Liberty engine with some of the better known foreign airplane engines is shown in Fig. 9, and fuel consumption in Fig. 10. The basic data for Figs. 9 and 10 came from official communiques from abroad. In the case of the Liberty engine the results of more than thirty tests were averaged.

BALLOONS

The production of service balloons met all service requirements. Fig. 11 presents the production of balloons by months.

TYPES OF AMERICAN MACHINES DEVELOPED, TESTED AND ADOPTED FOR PRODUCTION

On November 11, 1918, there had been developed, tested and adopted by the Army four airplanes, on which production would have started early in the present calendar year. They were the Lepere or L. U. S. A. C.-11 equipped with the Liberty engine, the U. S. De Haviland 9-A equipped with the Liberty engine, the Martin Bomber equipped with two Liberty engines and the Loening two-seater fighter equipped with the 300-h.p. Hispano-Suiza engine. The characteristics of these new airplanes are as follows:

Lepere or L. U. S. A. C. 11

Type: Two seater fighter — Engine: One Liberty

Weight empty with water	2468	pounds
Fuel and oil	475	"
Crew	360	"
Guns, ammunition, etc.	352	"
Gross weight	3655	"
Pounds per b. h. p.	10.15	"
Pounds per square foot	9.33	"
Speed at ground	136	mi. per hr.
Climb to 6500 feet	6	minutes

U. S. De Haviland 9-A

Type: Day bombing and reconnaissance Engine: One Liberty

As a Day Bomber

Weight empty with water	2815	pounds
Fuel and oil	933	"
Crew	360	"
Guns, ammunition, etc.	764	"
Gross weight	4872	"
Pounds per b. h. p.	13.5	"
Pounds per square foot	9.5	"
Speed at ground	121.5	mi. per hr.
Climb to 6500 feet	11	minutes 40 seconds

As Reconnaissance Plane

Weight empty with water	2815	pounds
Fuel and oil	933	"
Crew	360	"
Guns, ammunition, etc.	214	"
Gross weight	4322	"
Pounds per b. h. p.	12	"
Pounds per square foot	8.4	"
Speed at ground	126.2	mi. per hr.
Climb to 6500 feet	7	minutes 30 seconds

Loening

Type: Two seater fighter Engine: One 300 HP Hispano

Weight empty with water	1130	pounds
Fuel and oil	290	"
Crew	360	"
Guns, ammunition, etc.	828	"
Gross weight	2608	"
Pounds per b. h. p.	8.7	pounds
Pounds per square foot	12.1	"
Speed at ground	143.5	mi. per hr.
Climb to 6500 feet	5	minutes 12 seconds

Martin Bomber

Type: Night Bomber Engine: Two Libertys

Weight empty with water	5862	pounds
Fuel and oil	1492	"
Crew	540	"
Guns, ammunition, etc.	1769	"
Gross weight	9663	"
Pounds per b. h. p.	13.4	"
Pounds per square foot	9	"
Speed at ground	113.3	mi. per hr.
Climb to 6500 feet	10	minutes 45 seconds

In order that intelligent comparisons may be made, there are submitted below characteristics of the De Haviland 4 plane:

Engine	One Liberty	
Weight empty with water	2391	pounds
Fuel and oil	457	"
Crew	360	"
Guns, ammunition, etc.	374	"
Gross weight	3582	"
Pounds per b. h. p.	8.95	"
Pounds per square foot	7.12	"
Speed at ground	124.7	mi. per hr.
Climb to 6500 feet	8	minutes

THE NAVAL SEA-PLANE NC-1

One of the most striking accomplishments of the United States in airplane design has been the development by the Navy of the Naval Seaplane or flying boat NC-1. This plane has a central float 46 feet long, a wing span of 126 feet and a total wing area of 2400 square feet. This plane is equipped with three Liberty engines with tractor screws. Fully loaded the plane weighs 22,000 pounds. The weight empty with water is 14,000 pounds.

This plane is the largest seaplane in the world, at the moment, and is an American product throughout. The hull was designed after tank experiments on models. On a recent test this plane made a flight at Rockaway Beach, New York, with fifty-one passengers aboard, getting off in fifty-three seconds. Upon landing it was discovered that the fifty-first passenger was a "stowaway" on board the plane.

III—Remarks on Instruments, Raw Materials and the Training of Personnel

INSTRUMENTS

THE facilities for the manufacture of instruments necessary to give airplanes, pilots and observers their greatest usefulness were extremely limited at the outbreak of the war. The making of these instruments involved practically the formation of an industry new to this country. By June, 1918, twenty types of instruments were in production and being supplied for installation in training, bombing and fighting planes. The list included airspeed indicators, altimeters, compasses, safety belts, oxygen apparatus, radio sets and all other necessary adjuncts for the navigation of the air, and for the safety and comfort of the pilot.

Looking into the future, a reliable and simple "turning indicator" for airplanes is much desired. Furthermore, the development of some form of gyroscopic compass to meet the severe conditions of aircraft service is a problem of great importance. It has been said that the magnetic needle has sense but no power, and the gyroscope has power but no sense.

SPRUCE PRODUCTION

Some idea of the magnitude of the task involved in the procurement of the principal raw material used in the construction of the plane proper, namely spruce, may be obtained from the following figures on production and personnel. On June 30, 1918, the spruce production personnel consisted of 18,305 officers, enlisted men and civilians. Railroads were constructed into the forests of the West Coast and a cut-up plant having a maximum capacity of 9,000,000 board feet of lumber a month was erected at Vancouver Barracks near Portland, Oregon. To November 11, 1918, approximately 174,000,000 board feet of spruce and fir, as shown above, had been shipped of which more than two-thirds went to our Allies.

LINEN, DOPE AND OIL

As it was impossible to produce the millions of yards of linen fabric required for airplane wing and body covering, a suitable cotton substitute was developed and manufactured in this country in quantities sufficient to meet the needs of both

America and the Allies, and at a cost slightly more than half that of the linen fabric formerly used. This accomplishment, theretofore thought impossible, ranks among achievements of the first importance. The dope used on the fabric of airplanes presented many difficulties not only to us but to our Allies. This and other chemicals required soon were being produced in this country in quantities sufficient for their requirements as well as our own.

To meet the extensive demands for a high-grade lubricating oil, castor bean seeds were imported from India and about 108,000 acres planted in this country. Meanwhile, research work with mineral oils was carried on intensively, with the result that lubricant was developed which proved satisfactory in practically every type of airplane engine, except the rotary engine, in which castor oil still is preferred.

TRAINING OF PERSONNEL

The personnel side of the Air Service, including the selection, training, organization, and operation of the flying forces, developed within the fiscal year 1917-18 into an educational system on a scale infinitely larger and more diverse than anyone had anticipated. Teaching men to fly, to send messages by wireless, to operate machine guns in the air, to know artillery fire by its bursts, and to travel hundreds of miles by compass, teaching other men to read the enemy's strategy from aerial photographs, and still others to repair instruments, ignition systems, propellers, airplane wings, and motors, has required a network of flying fields and schools, a large instructional force, and a maze of equipment and curricula.

IV—Some Remarks on the Physical Conception of the Air.

IN a very condensed form I have endeavored to present in the above outline, the salient facts and figures of actual production and deliveries in connection with the air program in the United States up to the moment of signing the armistice. These figures are the cold facts of actual accomplishment of finished product.

I have made no exterpolations to these production curves which would show what flow of Liberty engines and combat planes for instance, we reasonably could expect would result in the succeeding months following the signing of the armistice. I leave this exterpolation to those of you who may be curious.

Behind the production figures of November 11 was mobilized in the United States an industrial army of about 350 firms and corporations employing more than 200,000 men and women. A huge imaginary conduit leading from the factories and assembly shops of America to the fighting front in France was in operation. At any given moment, the number of engines, planes and accessories distributed along the 3000 miles of this imaginary conduit were known accurately and organized in such a way that the flow throughout should be continuous, or that minimum congestion should occur at any point, be it factory door, railway train, embarkation depot, steamship, or debarkation point in Europe.

In order, however, to give an adequate idea of our whole effort, it will be necessary to refer now to certain lines of development, some physical, some chemical and some physiological, which have been set in motion during the progress of this work.

I think we will admit that no world-movement such as we now are witnessing in the development of aerial navigation demands such a close union between science and art. To make a machine to go into the air to transport man is a task set for the hand and brain as never before.

Fifteen years seems a short time when we contemplate what has been accomplished; yet, it was only fifteen years ago, on December 17, 1903, that Wilbur Wright made the first successful flight in a heavier-than-air machine at Kitty Hawk, N. C. To-day Government airplanes have carried the mail between

Washington and New York for seven months, and few of us take the trouble even to watch for these daily flights.

The advent of the first heavier-than-air flying machine begat, as a consequence, a period of intensive interest and study of the physical and dynamical properties of the medium, which are only now in the early beginning, and whose ultimate extensions we can only dimly perceive.

In November, 1916, I had the honor of addressing the National Academy of Sciences at Boston and outlining at that time some outstanding problems for research in connection with aeronautics. Although this was only two years ago, such has been the activity in research in studying air problems that fully two-thirds of the subjects suggested at that time now have been satisfactorily solved and are in actual use in the art.

Technical courses have been established at universities and colleges, and the new profession of Air Engineering is at the moment vying with the older engineering branches.

Everyone knows, of course, that if there were no air there could be no life, but probably very few fully realize its immense importance in almost everything we do. In one condition it is invigorating and gives us a zest for hard work whether mental or physical; in another it leaves us depressed and incapacitated for efficient labor. Numerous manufacturing processes are radically affected by the amount of moisture in the air, and many others by its temperature. Power is transmitted by it; we communicate our thoughts to one another by vibrations of the air; and by its aid we recently have acquired our swiftest mode of travel. Obviously then, a knowledge of the composition, structure, and physical properties of this universal medium is of such vital importance as to justify most painstaking study and research.

In the last few years, for instance, several elements: helium, argon, neon, krypton, xenon, have been found in the atmosphere that previously were unknown and even unsuspected, for they were not required by the Mendeleoff table of the elements as then understood. One of these, argon, amounts to nearly one part in a hundred of the whole atmosphere, and yet through decade after decade of chemical investigations involving countless thousands of air analyses, it, and all its family of gases, remained undiscovered.

Recently, too, means have been found for drawing directly on the atmosphere for an inexhaustible supply of nitrogen compounds used in the production of powerful explosives, fertilizers, and many other things of industrial value.

The first eighteen years of the present century have witnessed a wholly new and startling flood of physical knowledge which has resulted in profoundly modifying our conceptions of electricity and matter, as developed in the nineteenth century. These two great subjects have been brought together and now are recognized as one and the same subject.

What we called self-induction now is considered to be only inertia. Mass is supposed to be wholly electrical and a vector instead of a scalor quantity. The most fundamental of all physical constants, the electron, has been isolated and its charge measured. Electricity from whatever source is made up of these electrons and even metallic conduction consists of the actual projection of these granular units along the conductor.

A gas like the air, on the present view, is not a continuous medium, but is composed of many individual structures which preserve their indentity and individuality during the normal life of the gas. These largest units are known as molecules, and they are each moving with considerable velocity on the average, the path of each being for the greater part a straight line until two approach near to each other. The forces that these two exert upon each other reach out into the medium which surround them both, creating a region of intense field at sufficient distance away from each so that their integrity is not disturbed.

When the two molecules approach each other, they are deflected off without collision, preventing a catastrophe much as a comet coming into the solar system is merely deflected by the action of the sun, and does not fall into it unless it makes a direct hit, the chances of which are extremely small. The gas pressure of the air upon the walls of the containing vessel merely is the reaction of these deflected molecules that come into close proximity with the more fixed molecules of the container.

If we consider for a moment a cubic centimeter of air under standard conditions, we know that the number of the largest structures therein, the molecules, is a constant, and is about 2.705×10^{19}. It does not matter what kind of a gas we have, whether hydrogen, oxygen, nitrogen or compounds like methane carbonic acid, etc; the number of molecules remains fixed. In fact, we probably can determine the number of molecules in a

PLATE I.
A. I. E. E.
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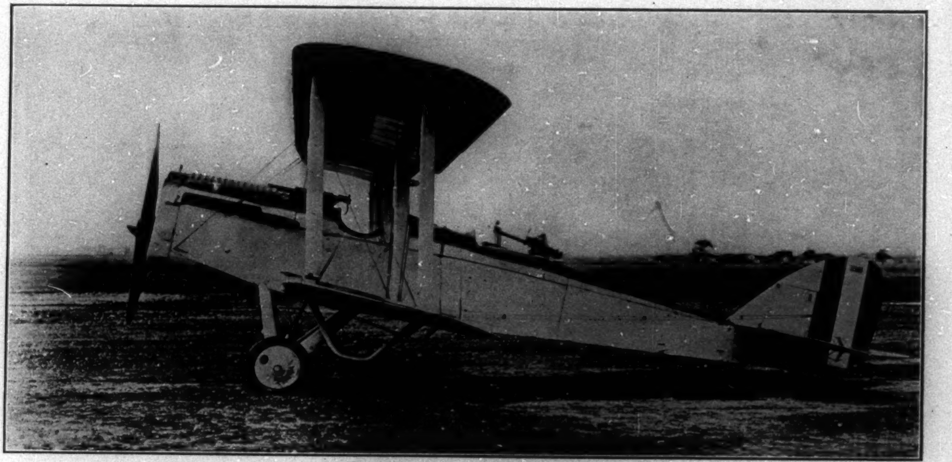
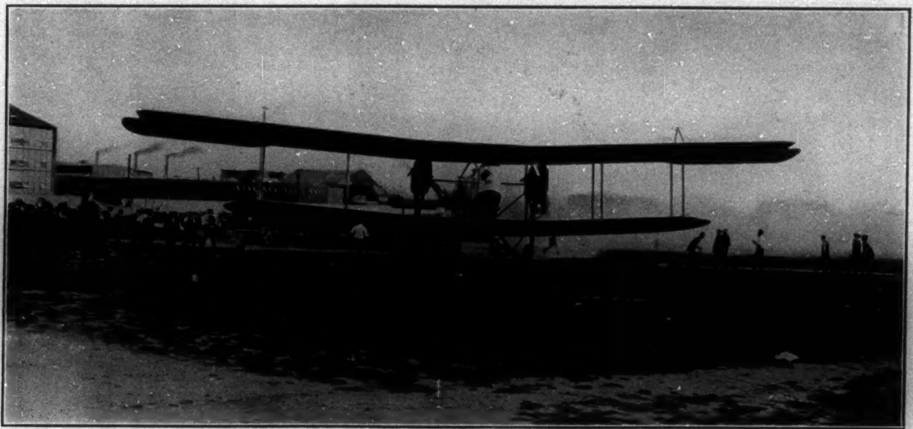


DE HAVILAND 4—LIBERTY ENGINE

[SQUIER]

PLATE II.
A. I. E. E.
VOL. XXXVIII, 1919



Handley-Page Bombing Plane—Two Liberty Engines [Squier] cubic centimeter of a gas more accurately than we can determine the number of inhabitants in New York City, in spite of the fact that the number is slightly more than twenty-seven billion billion.

By measuring the weight of the cubic centimeter of air and dividing this by the number of molecules contained therein, we obtain the weight of the largest structure composing the air, namely, the molecule. The structure of the molecule, while sufficiently enduring to remain together as a unit in the normal condition of the gas, yet is not strong enough to withstand forces that may be easily brought to bear upon it by chemical means to cause it to separate into still smaller units known as atoms. In the nitrogen and the oxygen of the air there are but two atoms making up each molecule, so that each structure in air can only be divided into two by any means at the disposal of the chemist.

The atoms, however, have been broken up into still smaller units, but it can be done only with great difficulty, comparatively speaking, and until recently, the difficulty was so great that they earned the name atom, signifying that they could not be cut in two. They are far more enduring structures than the molecule.

The only two kinds of things that ever have been obtained thus far by the breaking up of the atoms, of all kinds, are the so-called positive ions and negative electrons, and these remain today as things which never have been broken up into smaller components. Whereas the negative electron has been isolated from gross matter the positive charge never has been dissociated from matter. The present idea of physicists is that we have found an ultimate constituent of all gross matter. The mass of this negative electron has been measured, and the mass of the smallest atom, hydrogen, is 1845 times greater than that of the electron.

We regard the atom as consisting of a positive nucleus about which negative electrons are ceaselessly revolving. The dimensions of the negative and positive constituents of atoms, in comparison with the dimensions of the atoms themselves are like the dimensions of the planets and asteroids in comparison with the size of the solar system. Here then the master physicists with consummate skill have built up for us an alluring conception of miniature solar systems of which all gross matter is composed "A mighty maze! but not without a plan."

In the light of the foregoing conception of a cubic centimeter of air, imagine if you can, what a compilcated thing is the result of an orchestra playing perhaps one hundred instruments and two hundred voices singing in chorus, accompanied by the orchestra. The cubic centimeter of atomic worlds, in this case, becomes subjected from without to a great number of superimposed frequencies representing each and all of the individual instruments, as well as the innumerable harmonics required to indicate the qualities of the human voices; yet our medium is sensitive enough to respond accurately to each and every one of these frequencies simultaneously, and synthetically adds them together and produces a resultant envelope curve of pressure which the marvels of the human ear are able to analyze and interpret into its elements. A medium which has such qualities as to be capable of acting and reacting in this delicate manner, and at the same time when moved bodily with the velocity of a tornado may cut down trees and buildings as if by a giant steel knife, is a medium the possibilities of which for future research we, at present, only dimly perceive.

V—The Production of Helium on a Commercial Scale

ONE of the greatest scientific achievements of the present war from a technical standpoint is the production of helium in balloon quantities. This gas is non-inflammable and has about 92 per cent of the buoyant effect of hydrogen. Its name is due to its having been discovered in the sun's atmosphere through a characteristic line in the solar spectrum, before its presence on the earth, or any of its properties were known. It first was obtained in minute quantities by Ramsay in England some twenty years ago by heating certain radioactive minerals, in which it occurs because it is a disintegration product of radium. Its pre-war scarcity may be appreciated from the fact that, up to two years ago, not more than 100 cubic feet ever had been obtained, and the usual selling price was about $1700 a cubic foot.

Notwithstanding so discouraging an outlook someone in the British Admiralty had imagination enough to propose the large scale separation of helium from certain natural gases in Canada, that contain about one-third of one per cent of it, and experiments were undertaken at the University of Toronto. Soon after the entry of the United States into the war, the Bureau of Mines, learning of the problem from a British confidential memorandum, persuaded the Signal Corps and the Bureau of Steam Engineering of the Navy to approve and finance jointly an experimental program on a large scale. Thanks partly to the unusually rich sources of supply in this country, and partly to the skill of the two commercial companies whose services were enlisted, and to the enthusiasm of the Bureau of Mines Staff and of Mr. Carter, of the Navy, who for a time represented the Army as well in the project, such success was achieved that, at the cessation of hostilities, there was compressed and on the dock ready for floating 147,000 cu. ft. of nearly pure helium, and plants were under construction to give at least 50,000 cu. ft. a day at an estimated cost of not more than ten cents a cubic foot.

The production of a balloon gas that assures safety from fire opens up a new era for the dirigible balloon. In November, 1917, a Zeppelin made the trip from Bulgaria to German East Africa with twenty five tons of medicines and munitions, only to find that the German forces already had been dispersed, and returned safely to its base without landing. With a non-inflammable gas, not only comfortable and expeditious, but also safe transcontinental and transatlantic travel in dirigibles will, it is believed, soon be commonplace.

VI—Meteorological Service of the Army

IN August, 1917, the Chief Signal Officer directed Lieut. Colonel R. A. Millikan to organize an Army Meteorological and Aerological Service, the purpose of which was three-fold:

a. To provide the American Expeditionary Forces with all the meteorological and aerological information needed.

b. To supply the aviation fields, the coast artillery stations, the ordnance proving grounds, and the gas warfare service within the United States with such meteorological and aerological data as might be useful to them.

c. To undertake for the first time in the history of the world the problem of mapping the upper air currents over the United States, the Atlantic, and Western Europe in aid of aviation, and particularly with reference to transatlantic flight.

In carrying out the first of these projects, there were selected approximately 550 men of high qualifications, most of them physicists or engineers who were given a two months' course in meteorological and aerological theory and observations at College Station, Texas. Three hundred and fourteen of these men were sent overseas where they have been operating as an effective and well organized branch of the work of the A. E. F. In addition to furnishing a general weather forecast for the A. E. F., this service has supplied a meteorological unit to each aviation post, each gas service post, each artillery post, and each sound-ranging post of the American Army in France. Under the effective direction of Major W. R. Blair, one of the most experienced aerologists of the United States, commissioned for the service from the Weather Bureau, about twenty upper-air stations were established in France and England and a forecast based on data furnished by these stations made regularly to the A. E. F.

For the accomplishment of the second element of the program, twenty eight stations manned by 150 men, all carefully picked at the start and well trained at College Station, have been established for furnishing local data as to either surface or upper-air conditions, or both, to the flying fields, artillery posts, and proving grounds in this country. The largest of these stations is that at the Aberdeen Proving Ground, which is manned by twenty two men and furnished to the Bureau of Ordnance all necessary data for the determination of ballistic wind which, in view of the development of high angle fire, has become altogether indispensable for the construction of range tables needed for obtaining accuracy in the work of the artillery.

In carrying out the third element of the program, twenty-six meteorological stations were established, placed at carefully selected points over the whole of the United States, which stations have been manned by trained observers who telegraph to Washington each day observations on wind velocities at all altitudes up to 35,000 feet. In one instance these observations have been carried to 65,000 feet. On the basis of these observations, a daily forecast of upper-air winds is now being issued. The use which such forecasts may serve, both in connection with the aviation mail service and ultimately with the trans-atlantic service, may be seen from the fact that above the level of 10,000 feet, 95 per cent of the winds in both the United States and Europe are from West to East and often attain velocities in excess of 100 miles an hour. On November 6, 1918, at Chattanooga, Tennessee, a velocity of 154 miles an hour at an altitude of 28,000 feet was observed. It is because of this easterly direction of these upper air currents that all of the long flights thus far made have been from West to East. The importance of a forecast of such currents for the purpose of long flights will be appreciated as soon as the foregoing facts are understood. An airplane capable of a velocity of 154 miles an hour in still air either would remain stationary or travel at 308 miles an hour depending on whether it was headed into or with a wind of the velocity of that observed at Chattanooga.

All of the aerological work so far mentioned has been done with the aid of theodolites especially designed by Major W. R. Blair for this service. Sixty of these have been built for the work in this country and twenty shipped abroad.

The problem of exploring the upper-air currents over the Atlantic was at first thought insoluble on account of the absence of fixed bases, but the Meterological Service has developed propaganda balloons which already have flown at an average altitude of 18,000 feet from Omaha to New Jersey, a distance of more than a thousand miles. The success of the project now has made possible the mapping of the upper-air highways across the Atlantic; for arrangements are being made to send up from both coastal stations and from transatlantic steamers, these long-range balloons designed for from two to three-thousand-mile flights, and adjusted to maintain a constant altitude and to drop in Western Europe their records of average winds in these heretofore unchartable regions. The importance of this work for the future of aviation needs no emphasis.

The success which the Meteorological Service has attained would have been wholly impossible had it not been for the intimate and effective cooperation which has been extended to it in all of its projects by Director C. F. Marvin and the entire staff of the United States Weather Bureau.

VII—Physiological Study of the Flier

UNTIL a year and a half ago, interest centered in the development of aircraft and not in the flier. There was little regard for the special fitness of the man on the day and hour he drove his machine.

This was true, not only in the United States, but also in the Allied countries. The pilot was not selected because of any peculiar fitness for flying. It simply was a question of whether or not he had the nerve.

But, about the time the United States entered the war, it became clear that the science of physiology could be applied with advantage to the selection, classification and maintenance of the aviator. Each of the countries in the war now has a fully established air medical service, as an integral part of the air forces. In the British Army there is a separate air medical service with a Surgeon General of Aeronautics. In the American Army this work has been handled effectively by a division of the Surgeon General's office, assigned as part of the Air Service.

The early view that any one who "had the nerve" could fly caused enormous avoidable wastage of life and material. The lesson learned from bitter experience was that it is essential to obtain fliers who are especially fitted for particular work and to keep them in condition to perform their duties at all times.

Nature never intended man to fly in the same sense that she did not intend him for life in a submarine. Conditions are unnatural from the time he leaves the ground until he returns. There are many obstacles to overcome. He flies in an atmosphere deficient in that oxygen which is the "breath of life;" he is subjected in war to the shells of anti-aircraft guns and enemy aircraft; he travels through space high above the ground at rates well in excess of 100 miles an hour. In attaining altitudes and breathing rarefied air, the flier is defying nature.

The pilot is the heart and brain of the whole flying apparatus. Parts of the airplane may break without serious result, but when the pilot breaks, even momentarily, nothing is left to direct the flight. The man and the machine come crashing to the earth.

All this the Air Medical Service had to consider. The service had the advantage of a series of reports of the air medical services of our Allies, which enabled it to attain self-footing in a remarkably brief time. A medical research laboratory, fully equipped, was established by the Army Air Service at the flying fields on Long Island, N. Y.

The keynote of the American Air Medical Service is the handling of the flier as an individual, which naturally brings the work into three main divisions:

First, the selection of the flier:

Second, the classification of the flier, and

Third, the maintenance of the physical efficiency of the flier.

The Selection of the Flier

The outset of the war made it necessary to obtain a large number of military aviators in the shortest possible space of time. The medical problem consisted of selecting men who were physically fit.

It is possible for men to fly in spite of one or more physical handicaps, such as having only one leg or one eye or even being cross-eyed, but such men are not desirable fliers, since men with complete endowments naturally have superior advantages. The man in the flying service would be called upon to negotiate critical emergencies in the air. Instant decision and action, therefore, would be essential for success. So it was seen that only those men with exceptional mental and physical capabilities should be chosen.

In order to accomplish the best results, a comprehensive program was undertaken, providing for the standardization of both tests and examiners. Sixty-seven military units were established, each examining from ten to sixty applicants a day. The figures show that 70.7 per cent of the applicants were qualified. The large number of applicants made it possible to maintain the highest standard in selecting men.

In the same spirit with which the United States determined to supply the American air fighters with as good, if not better, planes than those used by the enemy, it was decided that the American aviators must be as good if not better than those of the enemy. The outcome of any encounter might easily depend upon which combatant possessed the better vision and other special senses, the better nervous system, and the better mental and physical equipment in general.

Each applicant received a complete physical examination embracing all the features ordinarily required of men entering the military service, and, in addition, comprehensive and extensive tests of the special senses of vision, hearing and motion sensing.

CLASSIFICATION OF THE FLIER

It is important to classify the flier for the kind of work he is physically capable of performing. Some men are not able to fly at higher levels than a few thousand feet without serious deleterious effects, while others may operate at much higher altitudes. It is necessary to know a flier's limitations before his training is specialized, for the saving of time and money, and, in fact, the flier himself. It would be an evident waste to train a pilot for combat work and then find he was physically capable of doing only bombing at a relatively low level. To accomplish this classification, branch medical research laboratories were placed in the flying fields, which by appropriate tests on the rebreather machine, developed at the parent institution at Mineola, classified the fliers.

The work of these laboratories demonstrated that sixty-one per cent are capable of flying to 20,000 feet or more, twenty-five per cent should not fly above 15,000 feet, and fourteen per cent were unsafe above 8,000 feet.

The re-breather is used also to determine temporary unfitness for altitudes, due to some intercurrent illness, dietary indiscretion and staleness from prolonged flying.

The necessity for altitude classification is readily understood from the fact that whereas in 1915 flying rarely exceeded 8000 or 10,000 feet, through improved design, scouts of today climb to altitudes of 25,000 feet within a very short time.

Night bombing is carried out at altitudes as low as 300 feet. Day bombing, in order not to reveal the objective of the flight and to guard against concentrated anti-aircraft fire, may call for flights at extremely high altitudes. Reconnaissance machines rarely get to high altitudes, owing to the necessity for more or less close observation of the ground, and machines doing this work accomplish low flying even in the face of highly concentrated anti-aircraft fire, and other enemy activity. Machines cooperating with the artillery making range corrections for batteries, do not often work above 6000 or 8000 feet.

Owing to the fact that all this work is widely diversified, it is essential that each man be placed in that division for which he is best suited. A knowledge of the limitations of a valuable man spells increased efficiency. Just as the pilot is provided with a certain type of plane adapted to the work in hand, so the plane must be provided with a pilot adapted to the work.

Fliers who are forced to undergo abrupt changes in atmospheric pressure and oxygen supply must be singled out. Atmospheric pressure plays an unimportant role; the whole problem resolves itself into a deprivation of the normal oxygen supply.

The Flack bag was the prototype of the rebreathing apparatus which has been developed in the medical research laboratory. By means of this apparatus the aviator rebreathes air confined in a tank, from which he gradually consumes the oxygen. As the percentage of oxygen decreases, the flier, in effect, is slowly ascending to higher altitudes. In the course of twenty-five to thirty minutes he lowers the oxygen content of the air in this tank to 8 or 7 per cent, which is equivalent to attaining altitudes of 25,000 to 28,000 feet.

Another means of attaining the same result is by the diluting apparatus, which supplies directly to a mask over the face a mixture of air and nitrogen in whatever proportion is desired. All of these tests have been standardized and confirmed by the low-pressure tank, in which the air is rarefied to correspond to any given altitude.

By a comparison of the percentage of oxygen to which the flier succumbs when on the low-oxygen test, it is possible to determine the altitude at which he would fail were he in the air. This determination is made on the ground, without danger either to the flier or to his machine.

The effect of low oxygen upon the mental processes varies greatly in the individual. He usually becomes mentally inefficient at an altitude at which there is as yet no serious failure of his vital bodily functions. By simple tests of mental alertness during rebreathing it is easy to determine that one flier becomes mentally inefficient at 15,000 feet, in sharp contrast to another who has his full mental powers up to and beyond an altitude of 25,000 feet.

Oxygen-want exaggerates any latent defect of the eyes. Crash reports have demonstrated that a large proportion are due to eye defects.

"Stunting" essentially is an internal-ear problem. During and after rapid turning the flier's brain is receiving impulses from his semicircular canals. In a normal individual nothing

PLATE III.
A. I. E. E.
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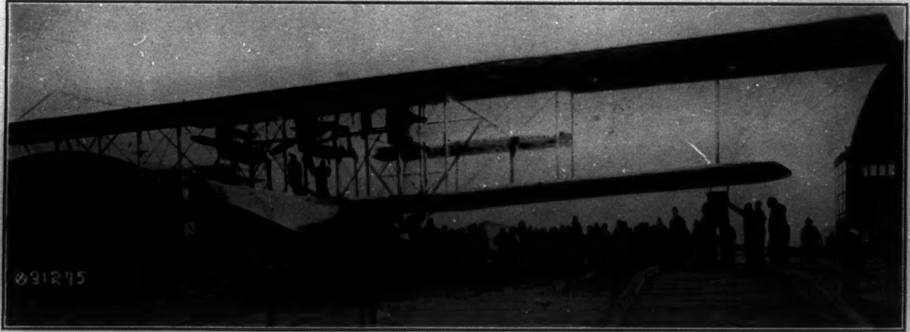
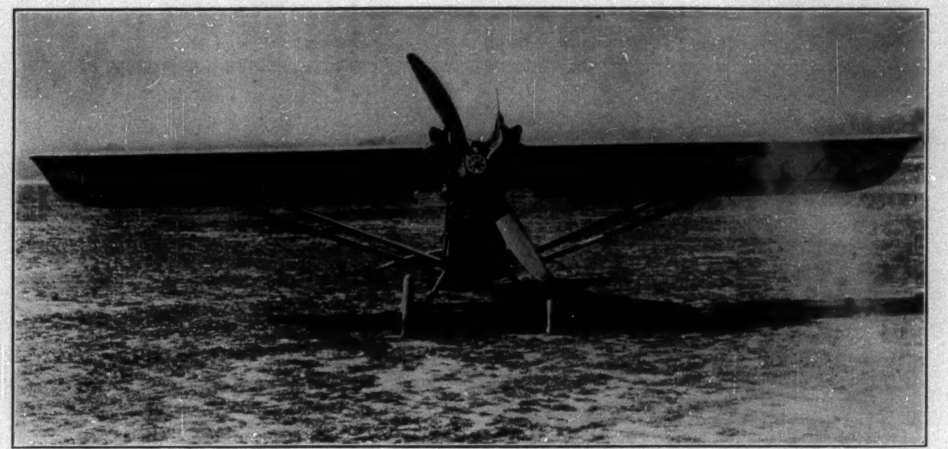


N. C. 1 Navy Seaplane (or Flying Boat)—Three Liberty Engines

[Squier]

PLATE IV
A. I. E. E.
VOL. XXXVIII, 1919



LEONING MONOPLANE—300 H. P. HISPANA-SUIZA ENGINE

[SQUIER]

can control or alter the sending or receiving of these impulses, which produce sensations of motion. Fliers vary greatly in their ability to interpret correctly the significance of these impulses. Experience alone enables them to familiarize themselves with their meaning. Those who develop the greatest ability in their interpretation naturally fall into the scout-pursuit class. Those who, in spite of training, are still disturbed or bewildered by stunting are reserved exclusively for straight flying.

The Maintenance of Efficiency

When the work of selection and classification is completed, the great problem of the Air Medical Service is to maintain the fliers on a basis of high physical efficiency.

After a certain amount of continuous service the flier begins to show unmistakable signs of deterioration. Until the Air Medical Service went to work, fliers were permitted to continue until they broke. Their breaking was signalized sometimes by simple failure to return from behind the enemy lines; sometimes by becoming mentally and nervously exhausted to the extent of permanently unfitting them for flying.

The rebreathing test also is valuable in determining this staleness in fliers, caused by frequent exposure to high altitudes. Incipient cases of the deterioration of efficiency caused by oxygen-want are detected easily by means of the rebreather. Then it is possible to ground a man for a certain period to enable him to recover entirely, whereas if this condition were not detected, it might progress to a point where it would be impossible for the man to regain his former efficiency. When staleness becomes marked the flier is liable to faint in the air, thus losing his life and wrecking his machine. Periodical examinations are made to detect staleness.

There are three and only three means by which the flier's usefulness may be terminated; first, by the enemy; second, by the failure of the machine; and third, by the failure of the flier himself.

While it was not possible to arrive at exact percentages, estimates made at the time of our entrance into the war, based upon information from Italy, France and Great Britain, indicated that not more than two per cent. of the aviation losses in active service were caused by the enemy. Failures of the airplane were responsible for only slight losses, due, of course, to the rigid inspection of the machines. Statements from all sources agreed that of the total number of fliers permanently out of the flying service, not more than 8 per cent could be attributed to mechanical shortcomings of the airplane or engine.

The remaining 90 per cent loomed large when it was realized that this proportion represented trouble in the flier himself.

Therefore the "Flight Surgeon" was authorized to keep aviators physically fit and to study the causes of accidents attributed to the failure of the man.

Keeping the flier fit embraces the amount of physical exercise necessary; the provision of proper recreation; the state of fatigue of the individual; the amount of sleep; the provisions for leave or furlough; the food problems; the field conditions bearing on his welfare, and his reexamination at frequent intervals.

It was observed that more than half the injuries in crashes were caused by the flier striking his head against the cowl. The cowl was cut out to give the flier more room in front, and this change has practically eliminated the head injuries. A safety belt was lashed to the machine by a simple rubber shock absorber, and since this has been done the number and extent of injuries to the upper abdomen and ribs have been reduced decidedly. The problem of protecting the flier against the extreme cold of high altitudes in winter was solved by designing electrically warmed clothing. The problem of enabling the flier to withstand the glare of reflected sunlight above cloud banks was solved by furnishing him with the "Noviol" type of goggles. The Dreyer oxygen apparatus is used to compensate for his lack of sufficient oxygen in high altitudes. This apparatus has been so modified and perfected that it is now possible to automatically supply a flier with the correct amount of oxygen for the altitude at which he is flying.

All this has been done toward reducing this "ninety per cent," and much more is being developed. Within the last few months an apparatus has been perfected whereby students may acquire flying experience and training without leaving the ground. This machine, known as the Ruggles Orientator, is a modification of the universal joint, composed of three concentric rings so pivoted as to permit the fuselage, which is pivoted within the innermost ring, to be put through every possible evolution experienced in actual flying, except forward progression.

An analysis of "crash reports" shows that a large number are due solely to failure to come out of the spinning nose dive or tail spin. The student failed in these maneuvers because he had not learned to compensate by sufficient previous experience for his dizziness. The new apparatus gives the flier experience until he becomes familiar with the various sensations.

Another method of educating the student is by means of flying calisthenics. By daily turning and tumbling exercises he becomes accustomed to positions and movements in which at first he is awkward and bewildered.

It has become evident during the last nine months through activities of nutritional survey parties of the Food Division, Surgeon General's Office, that there is a great need in each aviation camp for a nutrition officer. The nervous system is more highly differentiated than the muscular system, and more easily upset by improper food. It has been shown that a nutrition officer with special knowledge of food values should supervise the messes of all cadets and officer fliers in order to keep up efficiency.

Under date of November 20, 1918, General Pershing says in a report to the Secretary of War: "Our aviators have no equals in daring or in fighting ability, and have left a record of courageous deeds that will ever remain a brilliant page in the annals of our Army."

The work of the Air Service Division of the Surgeon General's Office was instituted by Major, now Brigadier General T. C. Lyster, with the assistance of such specialists as Colonel George H. Crabtree, Colonel William H. Wilmer, Lt. Col. E. G. Seibert, Lt. Col. Isaac H. Jones, Lt. Col. Eugene R. Lewis, and Lt. Col. Ralph H. Goldthwaite.

VIII—Activities of the Bureau of Standards for the Air Service

RADIO INVESTIGATIONS

A THOROUGH study of vacuum tubes for use in transmitting, receiving, and amplifying is in progress; a study of insulating materials used in radio apparatus; the development of a permanent contact crystal detector; the development of a high-frequency oscillographic equipment; the furnishing of standards for use in radio development are among the subjects handled in connection with the production of radio apparatus for military purposes.

The facilities of the laboratory and working quarters have been provided for the Research Laboratory of the Science and Research Division of the Signal Corps, and the Intelligence Division of the Signal Corps.

AERODYNAMIC INVESTIGATIONS

The wind tunnel of the Bureau of Standards has been employed in a wide variety of military problems, among which may be mentioned the following:

Bomb-dropping devices have been tested and compared. The head resistance and other characteristics of airplane bombs have been measured in order to provide data for computing the trajectories. Machine gun sights for airplane use, with automatic adjustments for speed have been tested and adjusted in the wind tunnel. Hundreds of Pitot tubes and Pitot-Venturi combinations for measuring the speed of the airplane have been calibrated. The head resistance of many types of engine-radiator sections has been measured. Small variable-pitch propellers for driving at constant speed the generator used in radio signaling, independently of the speed of the airplane or engine were perfected through wind tunnel tests and later tested in large numbers. Finally, the characteristics of new wing-sections and new airplane designs have been determined from wind tunnel tests on models.

AERONAUTIC POWER PLANTS

An altitude laboratory for the study of airplane engine characteristics and performance at all flying altitudes was begun in August, 1917, completed in January, 1918, and has been in service since that time.

Studies have been made of the Liberty "8" and Liberty "12", Hispano-Suiza 150 h. p., Hispano-Suiza 180 h. p., and Hispano-Suiza 300 h. p. models.

The results include complete data as to the power developed, fuel consumptions and heat balances at all practicable engine speeds and altitudes up to 30,000 feet; the demonstration of causes of bad carburetion at high altitudes and of the requirements for the best results, together with the development of means for producing these results automatically; the analysis of the effect of fuel composition on engine performance, the results of which served as the basis of specifications for aviation gasolines; the securing of necessary data for the establishment of standard flight test specifications for engines, including variations of horse power with the pressure, density and temperature of the air supplied the carburetor; the measurements of the effect of different compression ratios on engine power at different altitudes; the increase in power possible when using supercharging devices designed to supply the engine with air at pressures greater than that at which the airplane is flying.

AIRPLANE RADIATORS

The necessary apparatus has been designed and constructed to study the heat transfer characteristics of radiator cores used in airplanes. The performance of more than 85 different types of cores has been studied in this apparatus at reduced pressure and high rates of air flow. Head resistance measurements have been made on these cores in the 54-inch wind tunnel. The results of these experiments have shown that certain types of cores in use today may absorb twice or three times as much of the engine power as other types and as much as 25 per cent of the total power developed by the engine.

Experiments on the head resistance of a model fuselage fitted with a nose radiator have shown that the power absorbed by the resistance of a nose radiator may be as much as 40 per cent greater than that absorbed by a radiator more suitably placed. To afford maximum efficiency, a radiator must be designed for its particular air speed and position on the airplane. The results of this investigation give the necessary data for the design or selection of radiators for all air speeds and for various locations.

IGNITION PROBLEMS

Investigations have been made on various problems connected with spark plugs and other ignition apparatus used on airplane engines. This work has resulted in the development of a new porcelain for spark plug insulators which is definitely superior electrically and mechanically to any previous commercial product. Methods of test for spark plugs have been developed as a basis for Bureau of Aircraft Production specifications and several thousand spark plugs have been tested by these methods. The results have led to the use for aviation work of several high-grade commercial porcelains not previously considered for this purpose.

CARBURETION

In the carburetor testing plant the work has included a detail study of the changes in mixture proportions that occur with throttle manipulation under the conditions of atmospheric pressure and temperature existing at and between ground level and 33,000 feet altitude. This work has resulted in the development of suitable control of the mixture under throttle manipulation at any level above the earth's surface; and in the development of methods and devices for automatic correction of the variations in mixture proportions ordinarily occurring in passing from ground level to an altitude of 33,000 feet.

LIGHT ALLOYS

An important group of problems which engaged the attention of the Bureau was that relating to the production of light aluminum alloys and the improvement of their mechanical properties. Thus, new manufacturing methods were developed for a well-known rolling and forging alloy by which both the ease of its production was increased and its physical properties improved. Studies were made of the corrodibility of various alloys, while mechanical and metallurgical tests were made of a variety of different compositions of alloys giving most valuable information for the guidance of the designer of engines and planes.

A light alloys committee was organized to act as a clearing house for information along these lines and to oversee and direct the research work proceeding throughout the country.

AIRPLANE DOPES AND VARNISHES

Specifications for acetate and nitrate dopes were prepared at the Bureau with the cooperation of dope manufacturers, airplane constructors, and representatives of the government. Simultaneously, dopes manufactured in the country were examined in accordance with the specifications and list of products prepared which were approved for use on government aircraft. After much controversy, the use of acetate dopes containing tetrachlorethane was forbidden because of the toxicity of the above compound. Many proposed methods of fireproofing dopes and fabrics have also been examined. A commerical method of preparing and stabilizing lactic acid esters was devised for use in acetate dopes, in order to conserve the products of acetate of lime. A transparent silk fabric for possible use in obtaining low visibility has recently been perfected. Lately work has been in progress on the pigmentation of dopes with a view to protecting the dope film from the deteriorating action of sunlight, and thus prolonging its life, at the same time dispensing with the use of enamels, thus saving one step in production. The use of pigmented dopes will also conserve cellulose acetate.

Investigations of methods of making wooden parts of airplanes water resistant showed that a good grade of spar varnish was satisfactory, and a specification for such a varnish that could be produced at low cost was prepared. The same varnish has been found to be a satisfactory coating for doped linen or cotton, used either as a transparent varnish or as the basis of a vehicle for pigmented coatings. These may be used on both wood and metal.

AIRPLANE FABRICS

Previous to 1916 linen fabric was used exclusively in the covering of airplane wings and found to be satisfactory in every respect. During the present crisis, it became evident that the available supply of linen would not suffice for the demands of the military programs of the Allies, and it became necessary to find a material which would form a satisfactory substitute for linen. The urgency was emphasized by the conditions in Russia, from which the larger portion of the flax formerly had been derived.

As early as January, 1916, the Bureau started investigating the possibilities of substituting cotton for linen airplane fabric and found that the general consensus of opinion among manufacturers and investigators here and abroad was that cotton fabric could not be used for wing coverings. Their difficulties were due to the fact that they attempted to substitute structure of fabric rather than physical properties. The linen fiber has radically different properties from the cotton fiber, and the only hope for successful cotton airplane fabrics was so to change their structure that the ultimate fabric had the same properties as the linen fabric.

The investigation was confined to the study of stresses and stress distribution in fabrics, together with the factors covering the properties of cotton fabric as related to wing covering.

This phase of cotton manufacturing was an entirely new one, and great difficulty was experienced in studying the manufacture from a new angle, on account of the limited time available. Upon the entrance of this country into the war all the facilities of the manufacturers were made available to the Bureau's experts.

In March, 1917, the Bureau was in a position to issue instructions covering the construction of cotton fabrics, some of which proved quite successful. The next important step was to determine the actual performance of these fabrics and to this end samples were placed on Army planes at Langley Field and Navy planes at Pensacola during August, 1917. Later these fabrics were placed on planes manufactured by the Canadian Aeroplane Company of Toronto about the middle of October, 1917, in all cases with the most satisfactory results.

The Grade A cotton fabric now being supplied to the Air Service compares favorably with the linen regarding weight, has a much higher factor of safety, a greater tear resistance, and dopes up to satisfactory tautness. The life of any fabric is dependent entirely upon the life of the dope, and therefore the cotton has as long a life as the linen.

The English Government became concerned about its linen supply, and also adopted the fabrics designed by the Bureau.

As a result of more recent investigations by the Bureau of Standards, another distinct fabric has been evolved which is 25 per cent lighter than any linen fabric now in use, and is materially stronger.

Director S. W. Stratton has placed the services of the entire Bureau of Standard's staff at all times at the disposal of the Air Services of both the Army and Navy in attacking and solving new problems which have constantly been presented, and in fact the Bureau has been during the war practically an adjunct of the War and Navy Departments.

IX—National Advisory Committee for Aeronautics

THE National Advisory Committee for Aeronautics was established by Congress by Act approved March 3, 1915. Under the law the Committee is charged with the supervision and direction of the scientific study of the problems of flight with a view to their practical solution, the determination of the problems which should be experimentally attacked, their investigation and application to practical questions of aeronautics. The Committee is also authorized to direct and conduct research and experiment in aeronautics in such laboratory or laboratories, in whole or in part, as may be placed under its direction.

In addition to its specific duties, the Committee has acted as a clearing house for inventions submitted to the War and Navy Departments with a view to their possible usefulness in the field of aeronautics. These inventions have been studied carefully by the Committee's technical staff and those deemed worthy of development have been called to the attention of the military branches of the Government. During the last year, the Committee has examined approximately seven thousand such inventions and suggestions.

The Committee is recognized by the Aircraft industry as an authoritative source of technical information and as a court of arbitration whose weighed judgment obtains. The National Advisory Committee for Aeronautics is today the only official agency for the development of aeronautics which numbers among its members officers of both the Army and the Navy thus assisting in the coordination of the air activities of these services.

The scope of activity for aeronautical research to be carried out by the Committee in the future has been outlined to include the following investigations:

1. Propellers; 2. Aerofoils, or elementary supporting and controlling surfaces; 3. Combinations of supporting and controlling surfaces; 4. Reaction between the airplane and the propeller system; 5. Structure; 6. Investigations to improve the present instruments for use in airplanes and to develop new instruments to meet future needs; 7. In connection with aviation engines, the problem of supercharging to maintain power at different altitudes; 8. Improvement of engine details, radiation, carburetors, ignition, and fuels and lubricants;

9. Standardization and investigation of materials for aircraft construction; 10. Free flight tests; 11. The study of the relation of the atmosphere to aeronautics.

Some of the more important reports rendered during the last year are:

1. Behavior of Airplanes in Gusts. 2. General Theory of the Blade Screw. 3. The Testing of Balloon Fabrics. 4. Aluminum and its Light Alloys. 5. Carbureting Conditions Characteristic of Aircraft Engines. 6. General Analysis of Airplane Radiator Problems. 7. Effect of Compression Ratio, Pressure, Temperature, and Humidity on Power. 8. Properties and Preparation of Ceramic Insulators for Spark Plugs. 9. Heat Energy of Various Ignition Sparks. 10. A New Process for the Production of Aircraft Engine Fuels.

With the completion of the Committee's aeronautical laboratory at Langley Field, the Army Experimental Air Station, the work of aeronautical research will be expanded, it is hoped, to cover present and future problems vital to the Science and Art of Aeronautics.

The National Advisory Committee for Aeronautics gave valuable service to the Army and Navy both before and especially during the early months of the war. This body of selected men, reporting directly to the Congress, furnished to the Chief Signal Officer a medium for counsel and advice. This Committee inaugurated and recommended the establishment of the Aircraft Board. Prior to the establishment of the Aircraft Board, the Chief Signal Officer enjoyed the counsel of the Advisory Committee which was always ready to assemble without regard to time or place.

Although the statute creating this Committee contemplated a general supervision of aeronautical research, all manner of questions were submitted to its deliberation and judgment during the early period of the war. Whether it was the founding of a permanent aeronautical experimental station which should be a sort of West Point for aeronautics, the assembling of University Presidents to establish curricula for our ground schools, or the search for a practical solution to the stifling situation confronting the Government as a result of the airplane patent controversy, the National Advisory Committee was at all times available to the Army and Navy authorities for counsel and decisions.

X—Work of the Science and Research Division of the Signal Corps

THE Science and Research Division of the Signal Corps was established by order of the Chief Signal Officer on October 22, 1917, and Lieut. Colonel R. A. Millikan placed at its head.

Its personnel, exclusive of the Meteorological Section, consisted on November 11, 1918, of twenty-one commissioned officers, 118 enlisted men, and sixteen civilian employes of scientific and technical qualifications. Four of these officers are abroad and seventeen in the United States.

At the time of the signing of the armistice, the Science and Research Division had in progress sixty-four problems.

A summary of the most important results actually achieved up to that time is given herewith:

1. The Aeronautic Instruments Section, in addition to fulfilling the function specifically assigned to it of writing all the initial specifications for aeronautical instruments in use on planes, and in working continually with the instruments in production in order to detect imperfections and make suggestions for improvement, designed and developed wholly within its organization, through the activity of its chief, Major C. E. Mendenhall, a new and improved Pitot-Venturi tube for use in the determinations of airplane air speed. This instrument was actually put into production and thirty-seven thousand ordered.

2. Major Mendenhall and Lieut. R. C. Williamson have cooperated with the General Electric Company in the development of a new and improved compass, ten thousand of which were produced. This compass is now in use on planes for the American Army.

3. The Science & Research Division cooperated with the Eastman Kodak Company in the development of an entirely new film camera which is in production and which is the only film camera in existence capable of taking the standard size 18 by 24-cm. pictures. This camera is entirely automatic and capable of taking one hundred pictures without refilling.

4. The Science and Research Division with the assistance of the Burke & James Company of Chicago developed a new plate camera which is a modification of the French De Ram. It is

semi-automatic, carrying fifty plates. It has been accepted and is in production now. Both this photographic development and the one mentioned under (3) were made possible solely by the bringing together of a highly trained scientific group in the Photographic Section of the Science & Research Division. There would have been practically no possibility of the attainment of these results without such a group. This photographic group constituted one of the Sections of the Science & Research Division until about July 1, 1918, when it was transferred to the Production Division. The development of photography during the present war, due to the advent of the airplane has placed it permanently among the indispensable agencies for successful military operations. The older methods of recording personal observations have been superseded entirely by the far more accurate and reliable photographic record. A complete detailed photographic map must now be made daily of each sector immediately in front of an army, and by carefully matching these maps and by intensive study of each individual area, the maze of trenches, entanglements, machine-gun nests and shell-holes can be accurately analyzed. It is worthy of notice that 17,000 photographs were taken by the British army before the operations at St. Quentin in order that a relief map of the sector might be made before the drive against the Germans. By means of this map, every detail of the work to be performed by the British troops was planned. By a series of consecutive photographs taken at regular intervals by a moving-picture camera an accurate and reliable picture of a military road, for instance, showing all details to scale may now be made in a few minutes. With the aid of the camera the detection of camouflage is possible where the human eye would fail. This highly specialized photographic work has now taken its permanent place in the waging of modern warfare.

5. Dr. Gordon S. Fulcher, working in collaboration with the Miller Rubber Company, developed a leak proof tank simultaneously with the development of a similar tank in England—achievements of the utmost importance for the lives of Allied aviators. The Fulcher Tank was ordered placed upon all fighting planes.

6. Major C. E. Mendenhall and Lieut. John T. Tate, with the assistance of the General Electric Company, perfected a trench signalling lamp which, after test at the front, was

ordered sent over in considerable numbers and will probably constitute a standard American projector for the United States Army.

7. Major R. W. Wood developed a telescopic signaling device using a six-volt, two-ampere lamp. This lamp was tested at the front and favorably passed upon by the A.E.F. It has made possible light signaling in broad daylight over a distance of eighteen miles. General Pershing ordered a considerable number of these sent to the A.E.F.

8. Major Wood also developed a secret daylight signaling lamp which has a range of five miles. This also was ready for shipment abroad at the time of the signing of the armistice.

9. Secret signaling at night, with the aid of ultra-violet light, was perfected by this department, working in collaboration with Mr. Norman Marshall. With simple signaling telescopes of the sort mentioned above, using only a six-volt, two-ampere lamp, secret signals have easily been sent six miles. Major Wood also developed new means of adapting this method to the problems of signaling to airplanes, and secret signaling between convoys, and obtained thereby a device which will be of much use in peace as well as in war.

10. W. J. Lester, S. R. Williams, Capt B. J. Sherry and Sergt. W. H. Redman of this Department developed propaganda balloons which have a range of more than a thousand miles. This is an accomplishment which is invaluable to the future development of aviation, particularly with reference to transatlantic flights, whether in peace or in war.

11. Dr. W. F. G. Swann and Dr. Gordon S. Fulcher of the Science and Research Division gave invaluable service to the Balloon Section by making an elaborate and highly important study of the causes of fires in balloons and sending to the balloon officers in the Balloon Section directions for the prevention of such fires.

12. Dr. H. N. Russell of the Science and Research Division and Capt. J. P. Ault developed means for navigating airplanes with the aid of the sextant and an artificial horizon. They also developed means of speedy reduction of observations so that an observer in a plane can locate himself with an average error of not more than ten nautical miles within five minutes after he makes his observation. This achievement is of great value for the problem of long flights.

13. Lieut. John T. Tate and Otto Mohr developed a port-

able landing light for use on aviation fields. This device was officially tested and favorably reported upon by the division of Military Aeronautics. It is now in use at some fields and recommendation has been made by the Division of Military Aeronautics that two complete units be made a part of the permanent equipment of all aviation fields.

14. Dr. Harvey N. Davis of this Division represented the army in the development of the helium program and contributed in a large measure to the great success which has been attained in that work.

15. The Chemical Section under Dr. H. D. Gibbs developed new methods of producing acetone, which will make it possible to obtain this substance for one-fourth the price which the Government is now paying.

16. The Chemical Section, through the activity and energy of Dr. L. E. Wise, developed sensitizing dyes which previously had not been obtainable at all in the United States and which were urgently needed in airplane photography.

17. Capt. Herbert E. Ives and Dr. F. A. Saunders of this Department, in collaboration with I. W. Priest of the Bureau of Standards, made notable contributions in the development of color filters for detecting camouflage and increasing visibility. Forty thousand of these devices now are in use in the Army and Navy.

18. Dr. Wilmer A. Duff's Section worked out the only method which has been developed thus far for the accurate, experimental determination of bomb-trajectories. This is of primary importance in obtaining precision in bombing. It will have peace uses as well as war uses.

19. Dr. Duff's section also developed a bombsight stabilizer which has reduced the main error now made in bombing, namely, the error in the determination of the vertical, by more than three-fold. When it is remembered that a three-fold increase in the accuracy of bombing is precisely equivalent to the multiplication by three of the production of bombing planes, the importance of work of this kind scarcely needs comment.

There are probably a dozen of the nineteen developments mentioned above, each one of which is worth more to this country than the total amount spent upon the establishment and maintenance of the Science and Research Division from its inception up to the present time.

XI—Radio Development Work

IN the question of the engineering achievements of the Signal Corps during the war, the development of radio apparatus forms a large part. Inasmuch as the vacuum tube occupies so prominent a role in almost every kind of radio apparatus, an outline of its development logically precedes discussion of the radio sets.

VACUUM TUBES

The application to radio inter-communication of the vacuum tube—perhaps more properly called the thermionic tube or bulb—is one of the most interesting developments in the whole field of applied science. For not only has it made possible what has been justly heralded as one of the most spectacular achievements of the whole war—the airplane radiophone—but the confidence growing out of the extensive experience with the vacuum tube in warfare, coupled with its extreme adaptability, have resulted in a rapidly increasing amount of radio development involving its use.

PRE-WAR HISTORY

The vacuum tube was known in various forms before the war. Following extensive experiments with the so-called "Edison effect", Fleming, some years ago, produced the well known Fleming valve—a current rectifying device, capable therefore of being used as a detector of radio signals. This device contains two elements: an incandescent filament emitting electrons, and a plate upon which an alternating voltage is impressed, both placed within an evacuated bulb. Later Dr. Lee DeForest introduced an important modification by placing a wire mesh or "grid" between the filament and the plate. A small voltage variation on this grid produces the same current change through the tube as would a much larger voltage variation on the plate, thus adding amplifying properties to the detector characteristics of the Fleming valve. DeForest called his device the "audion". Later, with superior facilities for evacuation available and with a more intimate knowledge of the laws of thermionic emission from hot bodies, improvements and modifications were made in the audion or vacuum tube by both the General Electric Company and the Western Electric

Company, the latter designating their product as "vacuum tube", and the former the "pliotron".

In addition to acting as detectors and amplifiers, as mentioned above, vacuum tubes can function in two other important ways:

1. As Oscillators. In properly designed circuits containing inductance and capacity they will act as radio frequency generators, for use in transmitting or receiving radio signals.
2. As Modulators. By suitable connection to an oscillator circuit or antenna, they can be made to vary the power radiated so that the envelopes of the waves transmitted shall have any desired wave form, as for example, the speech waves from an ordinary telephone transmitter.

The most striking use made of vacuum tubes prior to the time we entered the war was the transmission of speech by radio from Washington to Paris and Honolulu, during the experiments carried out by the American Telephone and Telegraph Company and the Navy Department. Vacuum tubes were used as the radio frequency generator for transmitting, and for detector and amplifier in receiving.

When the United States entered the war, vacuum tubes already were in use by the Allied forces for various signaling purposes. The French particularly had been quick to recognize the military value of vacuum tubes and had, previous to June, 1917, developed very creditable tubes and apparatus. In America, tubes were in limited use as "repeaters" on telephone lines, and as detectors and amplifiers in laboratories and radio stations. The total production, however, in this country did not exceed three or four hundred a week.

DEVELOPMENTS DURING THE WAR

Early in our participation in the war, it became evident that vacuum tubes would be required in very large quantities in order to meet the growing demands for radio communication and signaling. It was equally evident that service conditions, not hitherto anticipated, would require great mechanical strength, freedom from disturbance under extreme vibration, and uniformity of product sufficient to make possible absolute interchangeability of the tubes in sets, without the necessity of readjusting when changing tubes. To these conditions must be added that of minimum size consistent with dependable operation.

To make such a device, with its complicated, yet accurately constructed metallic system, within a practically perfect vaccum, is no small problem even when made in the laboratory, on the individual unit basis, by a skilled operator who appreciates the delicacy of the job. To turn out tubes by the thousands by factory methods involves almost infinitely greater difficulties. How well certain companies, in collaboration with the Signal Corps, have succeeded in solving these difficulties is indicated by the fact that recently the total rate of production in the United States of high quality standardized tubes was considerably in excess of one million a year. This rate of production could be made many times greater on short notice.

As an example of the difficulties which this quantity production has involved may be mentioned that of evacuation. The degree of vacuum required is such that unusual methods of exhaust are necessary. The heating of the tubes in electric ovens is supplemented by heating of the elements of the tube by excessive filament and plate electrical power input. Molecular pumps are employed, necessitating an extremely large number of pumps to handle quantity production. Special treatment of metal parts prior to assembly is employed to reduce the gas given off by them during the exhaust process.

Another problem is that of making the complicated metallic structure of all tubes exactly alike, in order to insure identical electrical properties. As an indication of progress in this direction, it may be stated that one company is prepared to manufacture, in quantity, a certain tube in which the clearance between filament and grid is only three hundredths of an inch, the allowable variation being of course only a small percentage of this.

Manufacturing in quantity involves careful inspection. The problem of specifying definitely the required performance of tubes, the development of adequate testing specifications, the placing of standardized testing and inspection methods, personnel, and equipment in the various factories so that tubes manufactured at different times and places would, after passing inspection, be uniform and interchangeable—these questions were entirely new and have been solved almost entirely by the Signal Corps Engineers.

PRESENT STATE OF THE ART

Tubes developed by the Signal Corps may be divided into two general classes: the tungsten filament types as developed and manufactured by the General Electric Company and the De Forest Radio Telephone and Telegraph Company, and the coated filament or Wehnelt Cathode types as developed and manufactured by the Western Electric Company. The coated filament tubes so far have proven superior to the tungsten filament tubes for Signal Corps use. Both classes have been standardized as regards base, exterior dimensions, filament current and voltage, and in addition, plate voltage and output for transmitting tubes; and amplifying power and detecting power for receiving tubes. Except in certain special cases, the Signal Corps uses two types of tubes, one for transmitting, and another for receiving. The French and the British have been using one type for both transmitting and receiving, but present tendencies of the British are toward different tubes for different duties.

Vacuum tubes are now employed for electric wave detection, radio frequency and audio frequency amplification, radio telephony, particularly in the airplane radiophone, continuous wave radio telegraphy, voltage and current regulators on generators, and for other miscellaneous purposes. However, varied as are the applications at present, the uses, actual and potential, growing out of war development work have proved that the art of Vacuum Tube Engineering, and the application of its products to radio engineering, telephone and telegraph engineering, and particularly to electrical engineering in general, are still in their early infancy. That vacuum tubes in various forms and sizes will, within a few years, become widely used in every field of electrical development and application is not to be denied.

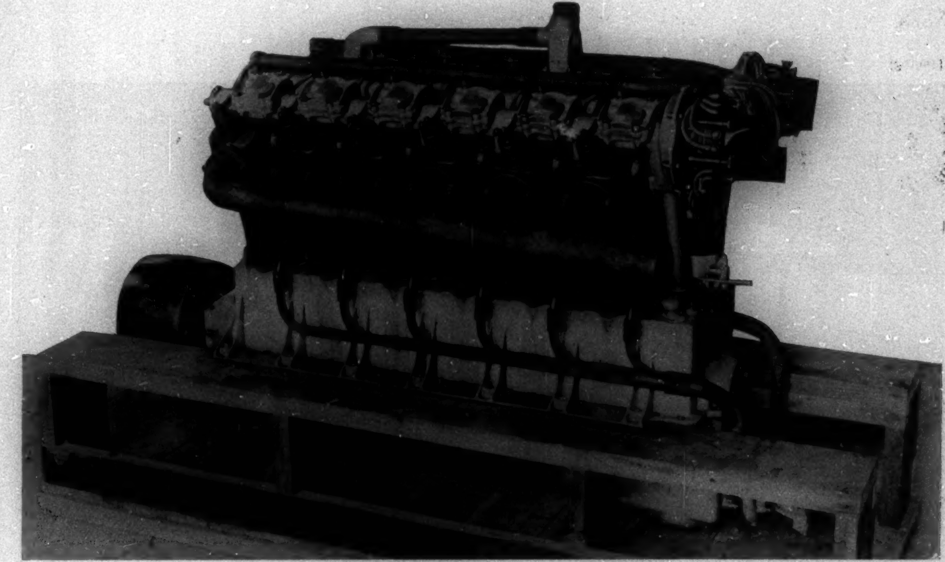
The engineering advancement accomplished in less than two years represents at least a decade under the normal conditions of peace, and our profession will, it is hoped, profit by this particular salvage of war which offers perhaps the most striking example extant of a minimum "time-lag" between the advanced "firing line" of so-called pure physics, and applied engineering.

The Chief Signal Officer considers that the work of standardization and quantity production of vacuum tubes accomplished during the last eighteen months under the pressure of military necessity, represents an advance in the art of electrical engineering which will prove of inestimable industrial and scientific value to this country, and to the engineering world at large.

PLATE V.
A. I. E. E.
VOL. XXXVIII, 1919

MARTIN BOMBING PLANE—TWO LIBERTY ENGINES

[SQUIER]



LIBERTY ENGINE—12 CYLINDERS

[SQUIER]

AIRPLANE RADIO TELEPHONE SET

Prior to April, 1917, a few experiments had been made, in which speech had been transmitted from airplane to ground by radio methods; but the apparatus involved was hopelessly crude. On May 22, 1917, the Chief Signal Officer called a conference in his office at which the project of evolving a "voice command" equipment for airplanes, which should meet all the severe requirements of the military service, and which should be thoroughly standardized for quantity production, was definitely set in motion against time. The plan for accomplishing this was the same as in the evolution of the Liberty engine, and from the beginning this project was regarded as one of the major creative efforts in the development of the American Air Service. The present airplane radiophone, therefore, is the result of a period of intensive development work begun shortly after we entered the war. The services of the Western Electric engineers were enlisted, and under the direction of the Signal Corps, rapid development resulted in successful tests as early as August, 1917. Speech was exchanged between airplanes twenty-five miles apart in October, and sample sets were sent at once to the Army in France for trial. Several thousands sets were ordered and have been completed and distributed to flying fields here and to the Air Service in France.

The satisfactory performance of this apparatus has resulted in a new type of military unit known as a voice-commanded squadron. The commander of an air fleet directs the movements of the individual units in any manner desired; the effectiveness of the squadron as a military machine is thereby enormously increased.

Other uses are in communicating information from airplanes to ground stations, and in directing one or more airplanes from a ground station. Innumerable applications will be evolved as the possibilities are realized.

The essential elements of the Airplane Radiophone are the power equipment, the radio equipment, and the antenna.

a. The power equipment includes a double-voltage direct-current generator driven by an air fan, with a vacuum-tube voltage regulator.

b. The radio equipment consists of the vacuum tube transmitting and receiving set, and the special telephone transmitters and receivers. Those of you who have heard the terrific roar of a Liberty engine will realize the difficulty of talking in an airplane in flight. The development of a transmitter which is affected by the human voice, and not by the enormously greater engine and wind noises, is one of

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the principal features of this set. Similarly, to shield the ears of the aviator from the same noises required a special combination of sound-insulating materials surrounding the telephone receivers and suitable for use within an aviator's helmet.

c. The antenna originally consisted of a flexible copper wire several hundred feet long, unreeled by the aviator and trailing almost horizontally behind the airplane. Modified antenna using much shorter wires fixed to the framework are now used.

The operation of the sets is extremely simple, all adjustments being made before leaving the ground. The only manipulation required of the aviator is that of the change-over switch to change from talking to listening.

AIRPLANE RADIO TELEGRAPH SET

The principal use of radio communication made during the war was in sending radio telegraph signals from observation airplaines, for controlling artillery fire. The French developed a set which consisted of several units, making the installation and operation complicated. The Signal Corps developed a self-contained set, which has been demonstrated to be far superior to any other airplane set.

It consists of three units—first, the 200-watt, 900-cycle alternator, driven by a regulating air fan, and containing in a stream line case attached to the generator, all the elements of the radio set. This radio apparatus is of the synchronous spark type, with four spark tones and nine wave lengths. The weight of the complete unit is only twenty three pounds, and size only six inches by six inches by twenty-six inches. The regulating air fan maintains the speed of the generator within four per cent of 4500 rev. per min., with air velocities between 60 and 200 miles an hour.

The remaining units in the complete set are a variometer or tuning coil, with antenna ammeter attached, and the antenna system, comprising a reel, insulated bushing, and training antenna. When it is realized that voltages of thirty thousand or more are produced by this set, the difficulties of insulation in such restricted spaces will be appreciated.

Ranges of communication of one hundred miles have been accomplished with this set.

AIRPLANE DIRECTION FINDER

One of the principal problems of airplane navigation has been the evolution of a suitable compass, particularly for night bombing work. Magnetic and gyroscopic compasses have limitations at present which make impossible reliable air navigation by dead reckoning.

The use of directional effects of loops or coils for receiving radio signals have resulted in the development of a radio compass for airplanes which gives positive information to the aerial navigator, and enables him either to locate his position by triangulation with respect to two beacon land stations, or to fly at any given angle with respect to a certain beacon station.

The apparatus consists of two principal parts—the antenna coils, and the tuning and amplifying apparatus. The antenna coils are mounted in the fuselage of the Handley-Page airplane, with suitable means for rotating in azimuth. The amplifier is extremely sensitive, consisting of a detector and six-stage amplifier. A novel feature of the amplifier is the use of iron-core transformers for frequencies of 100,000 cycles.

The direction of the beacon land radio station is determined by maximum strength of signals, in a highly ingenious manner developed originally by the British. The precission of the directional effect is remarkable. In fact the radio direction finder may well be called a radio eye, by which the aerial navigator sees one or more radio lighthouses which are sending identifying signals to guide him on this way. These lighthouses, furthermore, have certain advantages over the normal light-house in that their ranges may be much greater, and they are not invisible in the day time nor obscured by fog and mist.

The remarkable advances made during the last eighteen months have resulted in the application of radio communication to practically every phase of military aviation. Commercial and military possibilities have, however, hardly been touched as yet. It is believed that radio apparatus soon will be as essential on aircraft as it now is on ocean going steamships, and that its use will enormously increase the effectiveness of aircraft for all purposes.

XII—The Liberty Aircraft Engine

FOREWORD

AS we look back on the record of accomplishment in the problem of obtaining large numbers of high powered aviation engines for our Army and Navy Air Services both in this country and abroad, it seems to those of us who were in close contact with the work and the difficulties, more like a fairy tale than the statement of hard facts which it is in reality. On the face of things, it certainly would seem to be the height of presumption to assume that this country could, following its almost total neglect of aviation development in previous years, hope to design, develop and produce in unprecedented quantities an acceptable aircraft engine of greater power than had yet been evolved by any of the European nations, even under their spurs of governmental encouragement and tremendous war demands. Yet just that and nothing else was the only thing to do, and the story of its doing is one of the most brilliant chapters in the history of our Country's part in the Great War.

INITIAL SITUATION

At the time this country entered the war, in April 1917, there were being built in the United States only four makes of engines that were developed so far as to be considered of any military value, and even these were useful only for primary training. We had no engines at all suitable for services on the battle front, or even for the advanced training of pilots. The largest engine of domestic manufacture developed about 220 horse power and had not proved satisfactory when judged from the standpoint of combat service requirements. The others ranged in power from 90 h. p. to 135 h. p. It, therefore, was evident that the existing American engines could be used for preliminary instruction purposes only, and that their further manufacture should be limited to the training requirements. This was done, with the result that by far the greater portion of the primary training of pilots has been conducted with the Curtis OX-5 90-h. p. engine, the quantity production of which was early obtained. This engine was particularly valuable, owing to the very satisfactory training plane which had been designed around it. The Hall-Scott A7-A 100-h. p. engine was also extensively used at first until the production of the Curtiss engine could be brought to a point to meet all demands for primary training.

Two European engines, the Gnome 100-h. p. and the Hispano-Suiza 150-h. p., were being put into production in this country early in 1917, by the General Vehicle Company and the Wright-Martin Aircraft Corporation respectively. These concerns had obtained contracts with Great Britain and France, but were experiencing considerable difficulty in getting production under way. The production of the first Gnome engine was not completed until a period of more than nine months had elapsed, during all of which time English experts were present in the contractor's plant and aiding the manufacture. In the case of the Hispano-Suiza, notwithstanding the assistance of a group of French experts sent over to help get it into production in the minimum time, thirteen months were required to get the first production engine on the test block, and another month before the first delivery was made. Preparation periods were filled with the most costly experiments and the development of methods and tools for performing the intricate operations required by the design of these engines. All of which expense and delay had cost our Allies dearly.

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The Gnome and Hispano-Suiza engines represented the highest product of European design and were in a perfected and standardized state, according to foreign practise and conditions, when their production was undertaken in this country. Nevertheless, the changes involved in adapting them to manufacture by American methods, and the development of expert workers for those operations which could not readily be so adapted, required so much time that the advances made in aeronautical engineering rendered such engines largely obsolete for service at the front, by the time they could be produced in sufficient numbers to supply any material portion of the requirements.

These two engines were, however, of unquestioned value for advanced training purposes, the Hispano-Suiza in particular being a dominant factor in this work. Later the Le Rhone 80-h. p. was put into production by the Union Switch & Signal Company and, by proper utilization of the lessons learned in the case of the other two foreign engines, reasonably satisfactory progress was made in manufacture. This engine, too, was used for advanced training work.

LIBERTY ENGINE INCEPTION AND EXPERIMENTAL DEVELOPMENT

One of the serious mistakes which the Allies had fallen into at the time the United States entered the war was the development of a multiplicity of types of engines and planes which made it impossible to have a large number of any one of these types. As a further consequence, the trained personnel on the ground to operate and repair the machines had grown to such a proportion that it was estimated that from thirty to fifty men were required on the ground to keep each one of the many types of planes in the air on the fighting line. Manifestly, unless this large number of trained men per fighting plane could be reduced by some means it would be hopeless to expect within a reasonable time to put into the air thousands of flying planes, because a single thousand planes on this basis would require from 30,000 to 50,000 men in attendance.

The experience obtained in getting two foreign engines into production in this country and under our manufacturing methods, so thoroughly demonstrated the futility of attempting any such solution of our service engine problem as to at once eliminate its continued consideration. It was realized that copies of foreign designs could not be available in time and in adequate size or numbers to answer the demand for an overwhelming air force at the front. American air performance would have been very small indeed if limited by such a handicap as this.

Moreover, in spite of the fact that a technical commission was at once organized and despatched for the purpose of getting first hand information on the front and in the aircraft centers of Europe, it was unthinkable that this country should sit idly by and wait perhaps months for the final definite report which should decide the nature and extent of our part in the Allied aircraft program. Since the most successful airplanes are designed around specific engines, and the engine involves the greatest expenditure of time and effort in development, it was apparent to a few of us who were in close contact with the situation that it would in all likelihood be possible to design, develop and produce an entirely new American engine, embodying characteristics which would render it particularly adapted to manufacture under American conditions, in less time than would be required by the commission to determine the particular European engines that offered the best approximation to the various exacting requirements of service and production, plus the time to get it in production in this country. It was decided, therefore, that our efforts should be directed along both these channels simultaneously and in addition to purchase in Europe whatever service equipment might be available, to tide over the interval while we were getting into production.

It is apparent that the fundamental unit of engine design or construction is the cylinder, and that the evolution of engine power rested mainly with the unit power capacity of that cylinder which could be taken as representing the largest practicable size sanctioned by the state of the art at that time. Starting with the foundation of this cylinder then, it was the most direct reasoning to conclude that all requirements of the service for engines of varying power capacities could be most logically met by combining these unit cylinders into groups of whatever number were required to produce the several sizes of engines desired. This was done, and the cylinder size of five inches bore by seven inches stroke was selected, after a careful examination of the performance of both American and European engines of the then most modern design, as being the largest that could be relied upon to give satisfactory service. While originally designed to produce approximately 28 h. p., this cylinder has been so developed as to yield at present more than 40 h. p. as the result of somewhat increasing the speed and altering the functioning characteristics. The standard unit cylinders were to be used in engines having four or more cylinders each and yielding the following power output:

Number of Cylinders.	Original Rated h.p.
4	110
6	165
8	225
12	335

Thus, for the first time in the history of the aeronautic engine for military uses, a truly comprehensive design plan was evolved which in a simple and direct manner provided for the production of a whole line of engines of wide power capacities, but composed of units that were highly standardized and therefore could be really manufactured, instead of being merely built. This was, and is, the only way in which this country could meet the requirements of this most vital part of the program.

It must not be thought, however, that such a revolutionary decision was easy to make, or adhere to when made, in the face of all kinds of adverse criticism, some of which seemed to be based upon adequate grounds. The development of new types of aircraft engines heretofore had been such a time-consuming matter that it was generally regarded as a thing to be avoided if any ready-developed engine could be found anywhere which would at all answer the requirements. Thus the numerical preponderance of opinion was against the possibility of a standardized American engine being designed, developed, and produced in large numbers, in anything like as short a time as would be required to duplicate some European engine. The nation may well render thanks that its destinies in this particular were guided by such a far-seeing and courageous group of men, who had the ability to formulate such plans and then to enforce their realization.

You are all familiar to some extent with the history of the design and construction of this engine:—how Lieutenant Colonels J. G. Vincent, of the Packard Co., and E. J. Hall, of the Hall-Scott Motor Car Co., laid down the general features and got out the first assembly drawings personally between mid-day of May 29 and the afternoon of May 31, 1917, working in Col. E. A. Deed's apartment in a Washington hotel in response to a request for a report on the aircraft engine situation, which came from Howard E. Coffin, Chairman of the Aircraft Production Board; the Chief Signal Officer of the Army; Colonel Edward A. Deeds, Chief of the Equipment Division, Signal Corps, and Colonel Sidney D. Waldon, Assistant Chief:—how the order to build ten sample eights and twelves was given as the result of the approval by the joint conference of the Army and Navy Technical Board and the Aircraft Production Board:—how the first engine, an eight-cylinder, was built in one month as a result of the enthusiastic cooperation of some ten manufacturers, each of whom produced those parts for which they were best fitted:—how the first sample twelve-cylinder finished its official fifty-hour endurance test eighty-two days from the time the order for samples was given, and that the total elapsed time during this test was only about fifty five hours, a record-breaking performance:—and how the success of this endurance test definitely removed the engine from the experimental stage to the realm of proved engines.

A very gratifying endorsement of this standardized engine project came from the late Colonel R. C. Bolling, whose untimely death in France cost the American Air Service one of its most valuable officers, and Col. V. E. Clark, and Lieut. Col. Howard C. Marmon, members of the commission sent abroad to ascertain the requirements, and which returned about this time, to the effect that a 400 h. p. engine was absolutely demanded at the front for the types of airplanes which it had been decided this country should supply, and that no engine of this size then existed in Europe.

Those of us who are familiar with the difficulties and disappointments involved in the design, development and perfection of any form of intricate mechanical device can readily appreciate the really remarkable accomplishment represented in the Liberty. Therefore, it is not at all surprising that the representatives of the Allies were for some time unable to believe the full truth of this accomplishment. They never had been able to obtain such action and were, naturally, only fully convinced after many varied and exhaustive tests. So well recognized did the value of the Liberty engine become, however, that the Allies had on order at the time of signing the armistice 16,741 Liberty engines, and were constantly endeavoring each to increase their rate of monthly delivery. Airplanes were being designed around this engine in all Allied countries and it was fast becoming the predominating aeronautical engine of the Allied cause.

It is of interest in this connection to note that this standardized engine already has been tested in the twenty-four cylinder model, and shown results which prove that the original basic idea will provide for engines of any size which would have been required for any probable increase in airplane size during years of continuation of the war. The sixteen cylinder was also proved by the success of the larger engine.

The experimental development of the Liberty has been in charge of a department entirely separate from that dealing with its production, the Airplane Engineering Department under Lieut. Col. J. C. Vincent and Lieut. Col. Howard C. Marmon. The work of this department has resulted in a continuous improvement of the power output and performance characteristics of the Liberty twelve cylinder, to such an extent that 526 h. p. have been obtained with special fuel and detail changes; certainly a remarkable increase from the

335 h. p. which the original design was intended to yield. While the weight of the service engine per horse power has remained at approximately two pounds, the maximum present development had reduced this figure to one and two thirds pounds.

LIBERTY PRODUCTION—DEVELOPMENT AND PRODUCTION

The record of production and production-development of the Liberty conclusively proves the wisdom of the decision to concentrate all efforts on this one engine for the major part of our program of combat engines. In common with all similar machines, many possible improvements and cost reductions become evident as the manufacturing processes and tools are being evolved and as experience is being acquired in the actual production. Also, experience in the operating and adjustment of these engines led to alterations being made which resulted in increasing the power output; so that when we were advised that more power was desired than the 335 h. p. which the original Liberty was designed to produce, the necessary steps already were known and the delay incidental to putting them into effect was small. The resulting power increase to about 375 h. p. answered requirements for several months when advices were again received that more power was needed, and we then again made such alterations as were required to increase the power to around 420 h. p. The weight of some of the parts was increased at this time in order that the reliability might not be reduced.

The problem of production was placed in the hands of a separate department in charge of Lieut. H. H. Emmons, U.S.N., and the continuous assistance of such men as Henry M. Leland, of the Lincoln Motors Co.; C. Harold Wills, of the Ford Motor Co.; F. F. Beall, O. E. Hunt and Edward Roberts, all of the Packard Motor Co., were obtained by Major J. G. Heaslet, in charge of the Detroit district. The work of Mr. Wills which has the greatest value was the perfection of a new process for the forging of cylinder blanks in sufficient quantities to supply all manufacturers, and which was so very effective that the problem of cylinder forging supply and cost was reduced to a minor consideration. The wisdom of the action taken is evidenced by the production of 1100 Liberty 12s in one year from the day when Messrs. Vincent and Hall first met and started a preliminary drawing, and over 15,000 by the end of another six months.

It is perhaps to be expected that many criticisms would be leveled at an engine evolved under the conditions obtaining at that time, and registering the large success which it represents, but all such have proved to be the result of misinformation regarding the conditions to be met, the fundamental ideas of design, or of lack of appreciation of the difficulties encountered in creating at a single stroke and without previous experience, an aeronautical power plant so much larger than was then in existence.

The Liberty engine stands today as an achievement which for daring, constructive imagination and farsightedness will ever be a cause of pride to the American people.

XIII—A Granular Theory of Armies

IN these days we are face to face with the so-called granular theory of electricity and matter, whose fundamental unit is the electron. From one angle of view, as I shall hope to show, there is also a granular theory of armies.

The history of the military operations of the past disclose the fact that the actual combatant troops are specially selected men most fit physically for the hardships of war. In general terms the actual fighting has been restricted to selected persons from the male population with definite maximum and minimum limits for age and also definite requirements for the vital physical functions.

In the language of the physicist we may say that the male population of a nation is carefully segregated by a sort of "band filter" which picks out for combatant warfare only those whose age cycles fall between say eighteen and thirty years. These selected men are then brought to a further common standard by intensive physical training before they are permitted actual combatant service. This elevation of the military unit to a common physical standard makes possible, as will be readily seen, the formation of larger units such as companies, battalions, brigades, divisions, corps and armies and their direction by a centralized command in the tactics of war.

Invoking the law of averages, we may say furthermore that,—*the one unchanging factor in warfare is the individual physical strength of a man.*

The soldiers of Ceasar's army were physically no stronger nor weaker than our own. On the average, a battalion or company of soldiers from one nation can march, for instance, just as far in a day as a battalion or company from any other nation, and this statement was equally true before the Christian era, and is not likely to change in the future.

In consequence of this fact, military supremacy must be looked for primarily in the weapons and agencies provided by scientists and engineers and placed in the hands of these combatant units to multiply their military strength.

To illustrate, the day in the fourteenth century, when Berthold Schwartz or whoever actually invented gunpowder, put together charcoal, sulphur and saltpeter, he, by a scientific act, and at a single stroke, exerted more influence upon the development of warfare and indeed the history of the world since that time, than many armies could accomplish by any mere physical qualities which they possessed.

Stated in another way, if our enemies in the recent great conflict had been made up wholly of civilizations like those of Turkey and Bulgaria, nearly all of the agencies which I have been describing during the past hour would have been utterly impossible of either development or production by them. There were no Liberty engines nor airplane radio sets evolved or made in this quarter, nor could there be such without outside aid.

It follows from this, that those civilizations which by their scientific and engineering training can create and manufacture these agencies, will and must control the fortunes of war in the future, or far better, apply these same agencies, in a potential way, to banish war from the world for all time.

The present war has made aviation in four years what it would have taken two decades at least to accomplish in times of peace, and has multiplied the reach and capabilities of the physical military unit to an extent which is only now in the beginning. May we not fervently hope that the further development of aerial navigation will place in our hands such potential destructive powers as to go a long way toward making war impossible again?

I can say no more at the present time.

* * * * * *

IN conclusion, the courteous invitation of the Board of Directors of this Institute to address you on this occasion, suggested that I may have some message to deliver to the American Engineer.

The organization of the American Air Service requiring, *inter alia*, the foundation of a new industry under the adverse conditions above outlined, involved a creative project worthy of the broad conceptions for which the United States is noted.

When the United States entered the war, it was evident that the time was fast approaching when the reservoirs of raw material for the allies were to be found only in the United States and that America's effort should be so organized from the beginning as to furnish a continuous flow of this necessary raw material, not only for our own air program but for those of our Allies as well. It was clear, for instance, that the spruce, the fabric, the dope and the oil must be produced as a part of our program on a scale to supply also the air programs of our Allies. It was always the general Allied cause therefore which controlled the decisions in founding this industry, rather than the needs of the United States alone, and this is obviously the only correct point of view.

It is believed that the major decisions which had to be made by those in authority were accurately made, and promptly executed, and that as a consequence the record of the United States Air Service during the war shows a creditable performance. If America will but press on into the future, building upon the sound foundations now erected, she may lead the world in the development and utilization of aerial navigation for the triumphs of peace.

By a wise policy of readjustment, utilizing immediately our machines and our surplus aviators for the rapid expansion of the aerial mail and special passenger services, it will be possible to salvage for the nation a greater percentage of the money and energy invested for strictly war purposes than from any other feature of our war activities.

As a steward of the people, and in all humility therefore, I must not shrink from answering, as best I may, your request for a message which applies equally to peace and war.

The general principles which governed in the making of this enterprise in all of its ramifications of material and personnel, and to which the success attained must be attributed, may perhaps be formulated as follows:

VICTORY CREED

To foster individual talent, imagination and initiative, to couple with this a high degree of cooperation, and to subject these to a not too minute direction; the whole vitalized by a supreme purpose which serves as the magic key to unlock the upper strata of the energies of men.

Presented at the Seventh Midwinter Convention of the American Institute of Electrical Engineers, at a Joint Session with the American Institute of Mining Engineers, New York, February 19, 1919.

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WELDING MILD STEEL

BY H. M. HOBART

ABSTRACT OF PAPER

The probability that welding could, with advantage, be more widely used in ship construction than had heretofore been the case, led to the formation, in 1917, of a Welding Committee, which at first was conducted under the auspices of the Standards Committee of the Institute. The Welding Research Sub-Committee, formed in 1918, was a sub-committee of the Metallurgical and Electrical Sections of the Engineering Division of the National Research Council, and the Welding Committee, (under the chairmanship of Professor C. A. Adams) came under the direction of the Emergency Fleet Corporation. The paper is in large part based on the work done by the Welding Research Sub-Committee up to January of this year.

THIS paper deals principally with investigations undertaken by the Welding Research Sub-committee of the Welding Committee of the Emergency Fleet Corporation. The general object of the investigations has been to extend the use of welding in the construction of merchant ships and, specifically, to provide a definite basis for obtaining the best economy and efficiency in employing welding in place of riveting in the construction of the hulls of such ships.

Composition of Ship-Plate Steel. The chemical composition of the steel employed in such hull construction varies with the thickness of the plates. Through the courtesy of Mr. H. Jasper Cox, of Lloyd's Register of Shipping, the following information may be given concerning the kind of steel plate employed in American Shipyards in 1918 for the hull construction of merchant ships.

Lloyd's requirements do not relate to the chemical composition. They require a tensile strength of 58,000 lb. per square inch (40.75 kg. per sq. mm.) for their lower limit and 72,000 lb. per square inch (50.59 kg. per sq. mm.) for their upper limit. For the information of the Committee, Lloyd's obtained from their surveyors at various works data of the carbon content, which are as follows:

Works		Carbon Content for Plates ¼ in. thick	 1 in. thick
A		0.14	0.23
B		0.14	0.25
C		0.19	0.25
D		0.20	0.30
E	Upper Limit	0.30	0.35
	Lower Limit	0.24	0.29
F	Upper Limit	0.25	0.30
	Lower Limit	0.21	0.27
G	Upper Limit	0.25	0.35
	Lower Limit	0.22	0.28

For shapes, Works H employ:

Shapes about ½ in. thick, 0.24 per cent to 0.30 per cent carbon.

Shapes about 1 in. thick, 0.28 per cent to 0.35 per cent carbon.

Small shapes such as:

2½×2¼×¼-in. angles, about 0.15 per cent carbon.

4×4×⅞-in. angles, about 0.20 per cent carbon.

From several tons of half-inch thick (12.7 mm.) plate from the yard of the Chester Shipbuilding Company, which was employed in making many sample welds in an investigation designated the Wirt-Jones Tests, seven analyses were made at the Bureau of Standards. The maximum and minimum percentages of each of the impurities for these seven samples were as follows:

	Maximum per cent.	Minimum per cent.
Carbon	0.25	0.24
Manganese	0.46	0.45
Phosphorus	0.043	0.039
Sulphur	0.031	0.027
Silicon	0.052	0.024

For this material the Bureau of Standards reports:

Yield point, 37,900 lb. per sq. in.

Ultimate tensile strength, 63,600 lb. per sq. in.

Elongation in 2 in., 38.6 per cent.

The following manufacturer's data apply to about ten tons of half-inch ship plate supplied by the Worth Steel Company, of Claymont, Del., and to be used for testing electrodes:

Chemical Analysis (Ladle Analysis)	
Carbon	0.29 per cent.
Manganese	0.37 per cent.
Phosphorus	0.015 per cent.
Sulphur	0.032 per cent.
Physical Properties	
Tensile strength—lb. per sq. inch	67,400
Elongation, per cent in 8 inches	25.25 per cent.

PLATE VII.
A. I. E. E.
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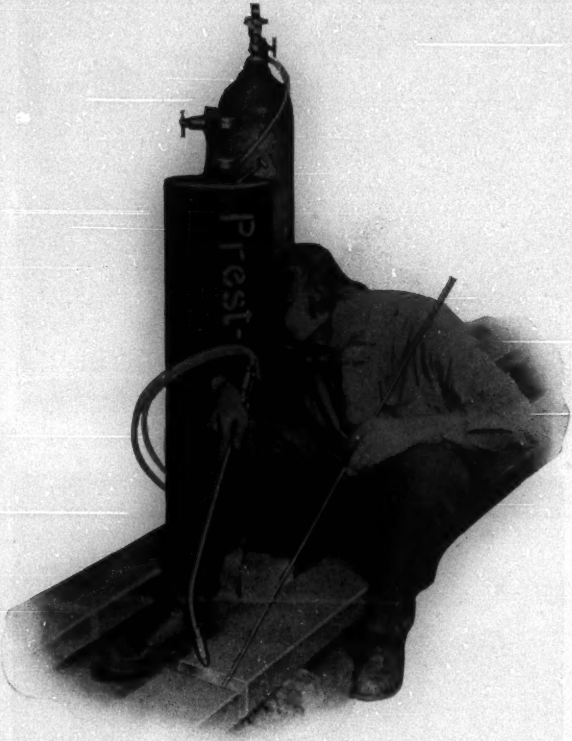

FIG. 1—GAS WELDING [HOBART]

FIG. 2—CARBON-ARC WELDING

FIG. 3—METAL-ARC WELDING

[HOBART]

Another lot of about 1½ tons of ½-in. and 1-in. ship plate from another source was analyzed by the Electrical Testing Laboratories with the following result, four analyses being made for each thickness:

	Maximum per cent		Minimum per cent	
	½ in.	1 in.	½ in.	1 in.
Carbon	0.24	0.28	0.22	0.26
Manganese	0.44	0.53	0.40	0.47
Phosphorus	0.033	0.033	0.028	0.027

The specifications of the American Society for Testing Materials for structural steel for ships (serial designation A 12–16, p. 98, A. S. T. M. *standard,* 1918) are in abstract as follows:

Phosphorus (acid steel), Not over 0.06 per cent.
Phosphorus (basic steel), Not over 0.04 per cent.
Sulphur, Not over 0.05 per cent.
Tensile strength, between 58,000 and 68,000 lb. per sq. inch.
Elongation, min. per cent in 8 in. 1,500,000/tensile strength.

From the above data we have a good idea of the kind of steel in connection with which it was the Committee's first and specific task to investigate welding.

Two kinds of welding are under investigation at present:

1. Fusion welding.
2. Spot welding.

These are totally different kinds of welding. The fundamental difference is that while in fusion welding no pressure is employed, the success of spot welding is entirely dependent upon the application of both heat and pressure. For the spot welding of thick plates, the required pressure is very great.

The main features of each of these two kinds of welding will now be stated:

Fusion Welding

The term fusion welding is employed to cover gas welding and electric-arc welding.

Gas welding is usually effected by simultaneously fusing with an oxyacetylene flame (1) the material at and near the surfaces which it is desired to join, and (2) some material (which is usually similar in composition) in the form of a rod, the tip of which is subjected to the heat of the flame. The oxy-acetylene flame is directed with one hand and the welding rod is manipulated with the other hand. The operation is illustrated in Fig. 1.

Electric-arc welding may be subdivided into several classes. The two broadest classes are·

a. Carbon-arc welding
b. Metal-arc-welding

In carbon-arc welding, an arc is established between a carbon or graphite electrode (usually a *graphite* electrode) and the two pieces of steel which it is desired to join. This graphite electrode is manipulated with one hand and a welding rod is fed into the weld by the other hand. The operation of carbon-arc welding is illustrated in Fig. 2. The manual activities in carbon-arc welding are seen to be quite similar to those in gas welding. In neither case is it necessary for the material of the welding rod to traverse the arc.[1]

In metal-arc welding, we find a fundamental difference in this latter respect, since in metal-arc welding of mild steel, the arc, instead of having a graphite electrode for one terminal of the circuit, is established between a steel welding rod (or welding electrode) and the two steel parts requiring to be joined. The operator in Fig. 3 is employing metal-arc welding to build up an incorrectly machined crank-shaft journal. There is always a distance of a matter of a tenth of an inch (2.5 mm.) or more between the end of the welding rod and the work. This distance is bridged by an electric arc. The form in which the steel exists during its passage from one end of the arc to the other is at present the subject of investigation by several independent experimenters.

Their conclusions are awaited with interest. The material cannot pass as a continuous liquid stream, since then there could be no interruptions in the metallic circuit and hence there could be no arc. It can pass as a series of liquid drops, and these can even momentarily short-circuit the arc, the duration of the short-circuit being too brief to be apparent to the operator or ordinary observer unaided by special apparatus. Or the drops can be so minute as to be incapable of effecting a short-circuit. If this should be the case, we can conceive of the metal passing as a stream of finely-divided liquid. Still another possibility is that the steel may pass as a highly-heated gas and condense on the opposite surfaces. It is suggested by physicists that, in its passage through the arc, the steel may undergo instantaneous transformations of which no human knowledge at present exists.

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[1] Both for carbon-arc welding and gas welding, the edges of the parts to be joined sometimes may be so designed as to obviate the need for any additional material; in other words, no welding rod is necessary in such cases.

There would appear to be more of these complex possibilities in metal-arc welding than in gas welding or in carbon-arc welding. Nevertheless, it is precisely metal-arc welding which is at present proving very attractive to engineers. It is too early to return a verdict as to whether this wide-spread tendency toward metal-arc welding is based on sound premises or whether there ultimately may not be a reaction (for certain kinds of work) back to carbon-arc welding. It may be that there has been undue precipitancy in the general stampede which has taken place from carbon-arc welding (which was the first to be developed) to metal-arc welding, which is a later development.

SPOT WELDING

Spot welding, as developed for use in ship construction, consists in bringing into good contact, by hydraulic or pneumatic pressure, over-lapping portions of the plates or parts requiring to be joined, and in sending through the spot of contact a sufficiently large current to heat the plates or parts at this point to a welding temperature. The weld is effected by the combination of pressure and heat.

Several large spot welders have been built. With one of these (which was an experimental machine), sufficient pressure and current were available to weld together three one-inch thick plates. The usual construction of commercial spot welders for use in shipbuilding is similar in general appearance to so-called bull-riveters. The largest spot welder yet built for actual use in ship fabrication has a six-foot (1.8 m.) gap. This outfit is a large stationary machine to which the steel plates and shapes must be brought. It is planned that bulkheads, frames, floors, and other parts shall be constructed with it and shall then be transported by cranes to their places in the ship. This six-foot-gap machine is designed with capacity to weld two three-quarter-inch-thick plates. It provides a pneumatic pressure of 60,000 lb. and a current of 50,000 amperes and welds simultaneously two spots, each of some 1½ inch diameter, in about 30 seconds. With less current a longer time is required, and vice-versa. This particular spot welder, which is shown in Fig. 4, is known as a Duplex Welder. This name is due to the feature that *two* spots are *simultaneously* welded, the current crossing the plate in one direction between two electrodes, and then back again between two other electrodes.

Two transformers, one located on each side of the plate, are comprised in the outfit. The arrangement is indicated, diagrammatically in Fig. 5, in which:

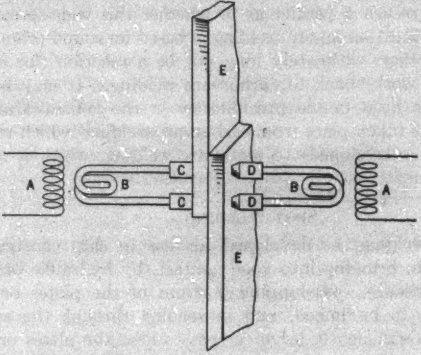


Fig. 5—Wiring Diagram of Duplex Spot Welder

A A represents the two primaries.

B B represents the two secondaries (which, in the actual construction, have only one turn each.)

C C and *D D* represent the electrodes between which the current flows and between which the pressure is exerted.

E E represents the two plates to be joined.

The chief object of this duplex feature is to eliminate the large reactance drop of a conducting loop some six feet long and one foot wide when traversed by some 50,000 amperes of 60-cycle current. This amounts to approximately 25 volts. Mr. J. M. Weed, the designer of the machine, reports the interesting fact that the presence in the gap of the plates to be

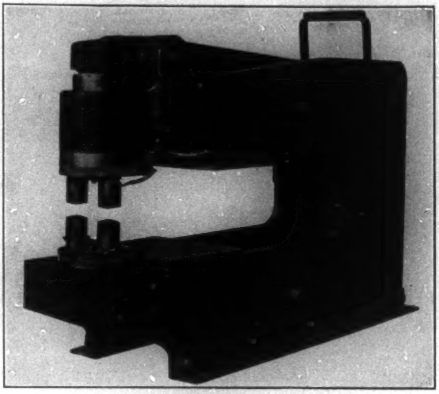
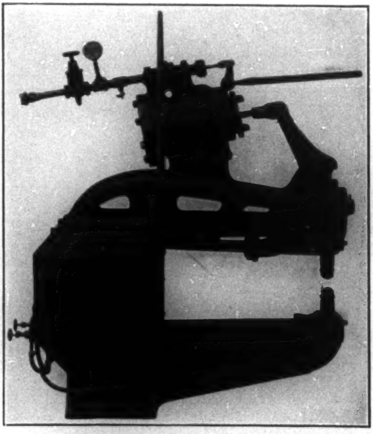


FIG. 4—DUPLEX SPOT WELDER



[HOBART]

FIG. 6—PORTABLE SPOT WELDER WITH 27-INCH GAP

welded, only decreased the current some ten per cent. This six-foot-gap machine weighs six tons.

In some other large spot welders of somewhat reduced size and capacity, the duplex feature is not employed and only one spot is welded at each application of the current. A portable welder of this type and having a 27-inch (68.5 cm.) gap, is shown in Fig. 6. This machine weighs only 2800 lb. (1271 kg.). In this case only one transformer is employed, and the circuit connections are those shown in Fig. 7, in which:

A represents the primary of the transformer.

B represents its secondary.

C and *D* represent the electrodes.

E and *F* represent the two plates to be welded.

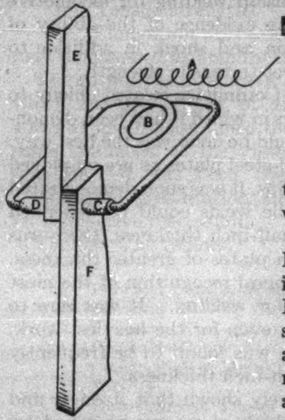


Fig. 7—Wiring Diagram of Portable Spot Welder with 27-inch Gap

In all spot welders for welding thick plates, the electrodes are water-cooled.[2]

When, in the Spring of 1918, Prof. C. A. Adams, of the Welding Committee of the Emergency Fleet Corporation, appointed several of us to be members of a Welding Research Sub-committee, we found ourselves facing a task of great importance and of enormous magnitude. It was desired that our investigations should be directed chiefly to the application of welding in the construction of the hulls of merchant ships. Much interior work on ships was already being performed very successfully with fusion welding and it appeared strongly indicated that the time was ripe for the extension of the application to the hulls of ships, with the prospect of producing work not only fully equal (and probably superior) to that obtained by riveting, but also distinctly quicker and cheaper. At the time a welded barge was already nearly completed in England. The Welding Research Sub-committee ascertained that several American railways had for some time employed fusion welding extensively in routine repairs of locomotives and that a matter of possibly a couple of thousand arc welders were at that time employed by American Railways. The extensive and successful use of fusion welding for locomotive repairs, in itself constituted strong evidence of the ability of such welds to withstand vibration and shock in addition to their proved excellence with respect to tensile strength.

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[2] Excellent discussions of the subject of spot welding and descriptions of several spot welders built for use in ship construction are given in the four following papers in the *General Electric Review*, December, 1918: Research in Spot Welding of Heavy Plates by W. L. Merrill, p. 919; Spot Welding and Some of its Applications to Ship Construction by H. A. Winne, p. 923; An Electrically Welded Freight Car by Jos. A. Osborne, p. 912; Some Recent Developments in Machines for Electric Spot Welding as a Substitute for Riveting by J. M. Weed, p. 928.

Any doubts entertained by the Committee related chiefly to the question of which of many ways in which it had been demonstrated that good fusion welding could be done, was the best way. Furthermore, as regards such mild-steel plates as are employed in the construction of merchant ships, it was soon demonstrated that while sound and quite ductile welds could be depended upon for plates of not over one-half-inch thickness, there was less certainty of good results with plates of greater thickness. But at that time there was no general recognition of the most suitable current to be employed for welding. It was rare to find more than 150 amperes used, even for the heaviest work, and as low as 100 to 125 amperes was found to be frequently employed for welding plates of half-inch thickness.

It now has been quite conclusively shown that stronger and more ductile welds of half-inch-thick plates are obtained by using at least 200 amperes. The author believes that fully 300 amperes should be used for butt-welding three-quarter-inch-thick plates and a matter of at least 400 amperes for one-inch-thick plates. These are some twice as great currents as have heretofore usually been employed in arc-welding plates of these thicknesses.

In view of this subsequent experience, it is clear that the disappointing lack of strength and ductility in certain welds of thick plates made nearly a year ago was a practically certain consequence of using such small currents.

It would be easy to yield to the temptation to enter discursively upon comments and opinions regarding the many points on which experienced welding specialists hold widely divergent opinions. All these specialists are producing thoroughly reliable work, but this is not saying that they are all producing nearly as good work as could be produced under the most appropriate conditions for each case. Indeed, the author's observations lead him to the conclusion that while excellent arc welding is being done on a wide scale, there is a margin for improvement over the present average quality, which, so far as it can be expressed by a sort of resultant of such physical characteristics as:

a. Bending and torsion tests
b. Tensile strength
c. Elongation at fracture

may be assessed as amounting to at least 25 per cent.

The author has attempted to make a list of some of the points which are the subject of discussion, and (while usually not going at much length into the questions), to make reference, in some instances, to the views and evidence on each side of a question. It has been the author's thought that such a summarized presentation might constitute the foundation for an instructive discussion.

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1. BARE AND COVERED WELDING WIRE

The kinds of electrodes advocated and actually used in the art range all the way from the cheapest fence wire, costing but a few cents per pound (say at present prices, some 10 cents per lb.) up to carefully treated and covered electrodes, certain types of which cost over five times as much as bare electrodes. Some makes of covered electrodes are, however, obtainable at reasonable cost (such as, at present prices, some 20 cents to 25 cents per lb.). One type of electrode must yield results uniformly superior to those obtained with another type in order to afford economic justification for a five-times greater price.

2. SALVAGING WELDING WIRE

As to bare electrodes it is generally considered that uniformity is very essential. An operator may be getting along very nicely but will suddenly come to bad places in the welding wire. Heretofore it has been considered necessary to reject such wire. The claim is now made by some people that by merely dipping the electrode wire in suitable material, it may be salvaged. Thus in the Welding Committee's Specification for Electrode Wire (given at page 75 of this paper) occurs a note to the effect that "If electrodes to the above specification sputter or flow unevenly, they may be dipped in milk of lime (whitewash) before welding. This dipping may be done in quantity on stock on hand and allowed to dry, or the welder may keep a pot of solution on hand into which the electrode may be dipped immediately before welding." This method of salvaging electrode wire was developed by the General Electric Research Laboratory.

Also, it has been demonstrated by Mr. E. Wanamaker that the application, by dipping, of a kind of coating which he has developed (and the precise composition of which he will doubtless contribute to the discussion of this paper), permits of doing good work with electrodes which would otherwise be useless.

3. Preferable Kind of Covering for Welding Wire

With regard to covered electrodes, while some claim that a thin covering obtained by dipping, accomplishes the desired purpose, others contend that it is desirable to provide a thick covering of appropriate material, which, in turn, is suitably impregnated. Moreover, even for covered electrodes, the usual belief is that the greatest care should be given to the composition and quality of the welding wire to which the covering is applied. In other words, it is not generally held that the use of inferior wire salvaged as indicated under (2), will permit of obtaining the best quality of welds. It is important that the covering shall be so designed as to be consumed at a definite rate as compared with the rate of consumption of the enclosed welding wire. A consequence is that any particular gage of *covered* welding wire must be used within rather close current limits.

For *overhead welding* one firm exploiting covered electrodes supplies a special (and additionally high-priced) grade in which the covering is impregnated with a more viscous material than is used for the electrodes which the firm supplies for other welding operations.

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4. Preferable Composition for Bare Welding Wire

There is a great diversity of practise as to the preferred composition of bare electrodes suitable for welding mild steel plates. As instances of extremes it may be said that amongst widely used electrodes, while one type consists of almost pure iron, other types have nearly 0.2 per cent of carbon and 0.5 per cent of manganese, and still other types run very much higher than this in manganese. This is quite aside from the subject of special compositions for welding high-carbon steel and for welding cast iron. It is anticipated that quantitative measurements will indicate superiority in tensile strength for some

TABLE I.

COMPOSITION OF WELDING ELECTRODES FOR METAL-ARC WELDING

Trade designation of electrode	Carbon	Manganese	Phosphorus	Sulphur	Silicon	Remarks
Page Steel & Wire Co.						
Armco	0.01	0.025	0.005	0.025	0.005	
Wilson Welders & Metals Co.						
Grade, No. 6	0.15 to 0.23	0.60 to 0.75	less than 0.04	less than 0.04		Also 0.25 per cent copper.
Grade, No. 9	0.30 to 0.40	about 1.00	less than 0.04	less than 0.04		
Grade, No. 8	0.17 to 0.22	0.30 to 0.45	less than 0.04	less than 0.04		
Grade, No. 17	0.10	0.30 to 0.45	0.06	0.06		
Quasi Arc Co.	0.08 to 0.12	0.45 to 0.55	0.00 to 0.06	0.00 to 0.06	0.05 to 0.08	Flux covering of blue asbestos fibre (Croeodilite) enclosing percentage of aluminum or other metal in form of fine wire capable of giving strong reducing action.
Roebling Co.	0.16	0.56	0.032	0.024	0.016	
Toncan wire	0.10	0.16	0.01	0.046	trace	
Electric Arc Cutting & Welding Co.	0.25	0.30	0.05	0.05	0.05	
Siemund Wenzel Co.	0.10 and under	0.30 to 0.50	0.05 and under	0.05 and under	trace	
Norway-iron Wire	0.05	0.02	0.025	0.007	0.08	
Double Arc Co., of England	0.085	0.35	.....	0.054	0.108	Flux covered.
T. Scott Anderson Co., of England	0.057	0.32	.....	0.026	0.014	Flux covered.
E.A.Jones & Co.,of England	0.22	0.25	0.001	0.026	0.024	Nickle-plated and flux covered.
Engineering and Equipment Co., of England	0.12	0.51	.....	0.08	0.016	Flux covered.
Central Steel & Wire Co.						
Swedox	0.05	0.18	0.04			
The Spencer Wire Co.						
Basic open-hearth steel electrode	00.06	0.12 to .20	0.013 and under	0.03 and under		

compositions and superiority in ductility for other compositions. Mr. R. E. Wagner has exhibited some very ductile welds made with electrodes containing small percentages of magnesium and of boron sub-oxide.

It is only within the last few months that there have been available any specifications for use in establishing the merits of welding wire. These are now available in the Welding Committee's specification setting forth a "Standard Procedure for Testing Welding Electrodes." This specification, which is given in Appendix A to this paper, was prepared by the Welding Research Sub-committee in collaboration with Prof. H. L. Whittemore representing the Bureau of Standards and with representatives of manufacturers of welding electrodes.

In Table I are given the compositions of various electrodes in current use. Prior to publication, the data in this Table was submitted to the manufacturers concerned, who, in six instances, improved the opportunity to correct the data to conform with their latest practise. The American Steel and Wire Co. has requested the omission from the accompanying Table of any analyses of electrodes which it has furnished for arc welding. This is for the reason that material has been supplied to a large number of users, and varies considerably in analyses in accordance with the ideas of the purchasers. There does not as yet seem to be an agreement as to the most advantageous chemical composition for electrodes, and the company is not prepared either from observations of the results obtained by its customers or from its own experimental work to make a definite recommendation.

The Welding Committee has issued the following Specification for Electrode Wire for Electric Welding. The specification was prepared under the immediate direction of Mr. Herman Lemp.

SPECIFICATION FOR ELECTRODE WIRE FOR METAL-ARC WELDING IN CONNECTION WITH MILD STEEL

Welding Committee Emergency Fleet Corporation

REVISED TO DECEMBER 20, 1918

(Note:—This wire may or may not be covered)

1. *Chemical Composition*

Carbon	Not over 0.18 per cent.
Manganese	" " 0.55 " "
Phosphorus	" " 0.05 " "
Sulphur	" " 0.05 " "
Silicon	" " 0.08 " "

2. *Sizes and Weights*

Diameter, in mils	Diameter, in fraction of an inch	Pounds per 100 ft.	Feet per 100 lb.
125	1/8	4.16	2400
156	5/32	6.51	1535
188	3/16	9.37	1066

(Allowable tolerance 6 mils plus or minus)

3. *Material.* The material from which the wire is manufactured shall be made by any approved process. Material made by *puddling process not permitted.*

4. *Physical Properties.* Wire to be of uniform homogeneous structure, free from oxides, pipes, seams, etc., as proved by photomicrographs.

5. *Workmanship and Finish.* (*a*) Electric welding wire shall be of the quality and finish known as the "Bright Hard" or "Bright Soft" finish—"Black Annealed" or "Bright Annealed" wire shall not be supplied.

(*b*) The surface shall be free from rust, oil or grease; a slight amount due to lubrication during last drawing is permissible.

6. *Tests.* Electrodes must, before shipment or after delivery, show good commercial weldability when tested by an experienced arc welder. The electrode material shall flow smoothly in relatively small particles through the arc without any detrimental phenomena.

Note: If electrodes to above specifications sputter or flow unevenly, they may be dipped in milk of lime (whitewash) before welding. This dipping may be done in quantity on stock on hand and allowed to dry, or welder may keep a pot of solution on hand into which the electrode is dipped immediately before welding.

7. *Delivery, Packing and Shipping.* Electrodes shall be furnished in straight lengths of either 14 inches or 28 inches, put up in bundles of 50 pounds or 100 pounds as ordered. Each bundle shall be wrapped in heavy paper securely wired and marked on one end showing diameter in mils, trade name and grade of wire.

5. Composition of Metal Deposited in Weld

A few analyses have been made of chemical compositions of the metal deposited in the weld. Results of the analyses of four sets of electrodes before and after the metal was deposited are quoted below from the Westinghouse chapter in Major Caldwell's report, published by the Emergency Fleet Corporation in 1918. To these results are added analyses of Toncan wire as supplied to the author by Mr. R. E. Wagner.

ANALYSES OF ELECTRODE—PER CENT OF IMPURITIES

	Carbon	Manganese	Phosphorus	Sulphur	Silicon
Roebling	0.16	0.56	0.032	0.024	0.016
Norway	0.049	0.021	0.025	0.007	0.08
C. R. S.	0.11	0.72	0.097	0.123	0.011
H. R. S.	0.13 to 0.17	0.50	0.012	0.045	0.011
Toncan	0.10	0.16	0.010	0.046	trace

ANALYSES OF DEPOSITED METAL—PER CENT OF IMPURITIES

	Carbon	Manganese	Phosphorus	Sulphur	Silicon
Roebling	0.05	0.18	0.031	0.036	0.011
Norway	0.05	0.018	0.020	0.072	0.011
C. R. S.	0.05	0.11	0.086	0.072	0.011
H. R. S.	0.14	0.14	0.012	0.039	0.011
Toncan	0.042	0.081	0.019	0.026	0.000

It is notable that most of the carbon and manganese is burned out in traversing the arc.

6. POLARITY

For carbon-arc welding, the standard practise is to connect the graphite electrode to the negative terminal. Mr. Wagner states as his experience that it is very difficult to weld with the carbon arc when the polarity of the carbon is positive. He states it to be almost impossible to direct the heat to the point desired and the welding qualities of the arc under this condition are very poor. He concludes: "Our experience has taught us that it is next to impossible to weld with a carbon arc unless the work is positive and the electrode negative."

For metal arc welding with bare wire, the electrode is usually connected to the negative terminal, but instances occur of bare welding wire which works best when the opposite polarity is employed. Also for some particular sizes and sorts of welds best results are sometimes obtained by a reversal of the polarity. With electrodes heavily covered with flux, the positive terminal is almost always connected to the electrode. Plenty of more or less plausible reasons for these differences have been offered on various occasions. On careful reflection none of these reasons prove particularly satisfying. Amongst other considerations the fact of the entire practicability of arc welding from an alternating-current circuit and of overhead welding have to be taken into account in judging some of these explanations. As yet, we have no satisfactory hypothesis as to what goes on in the welding arc.

7. Direct Current Versus Alternating Current for Arc Welding

While up to rather recently it had usually been contended that arc welding required a direct-current supply, there are now many advocates of alternating current.

Mr. E. H. Jones in the course of the discussion of Major James Caldwell's paper entitled "Notes on Welding Systems" read on Jan. 22, 1918, before the Institution of Engineers and Shipbuilders in Scotland, stated: "He would like to take this opportunity of drawing attention to the undoubted merits of alternating current for arc welding. For some reason which he was unable to fathom, the general impression was that direct current was superior to alternating for arc welding, but as a matter of fact he found that alternating current was far superior to direct current, and he would recommend the use of alternating current on every possible occasion. Apart entirely from the capital outlay needed, which was vastly higher in the case of direct current, the control of the current was much easier to effect. . . . He estimated that the amount of current which would be necessary to feed 20 operators with direct current would suffice to feed 28 with alternating current."

Mr. R. E. Wagner's experience is as follows: "Electric welding may be done with alternating current as well as with direct current. It is a little more difficult to hold the arc, but this simply resolves itself into a matter of practise. Men who have been regularly doing arc welding with direct current, very quickly learn how to handle the alternating-current arc."

At present there is no agreement as to the applicability of alternating current to carbon arc welding.

8. Periodicity for Alternating-current Arc Welding

Amongst the advocates of the use of alternating current, there is no agreement with reference to the periodicity. Although it is generally maintained that arc welding is only thoroughly practicable with as high a periodicity as fifty or sixty cycles per second, there is, on the other hand, expression given to the opinion that the use of twenty-five cycles, or less, is equally satisfactory. In October, 1918, Mr. R. E. Wagner reported to the Welding Research Sub-committee, that he had found from his tests that alternating current for arc welding could be used with a frequency as low as 12½ cycles and as high as 500 cycles. Mr. Wagner states that while there is no difficulty at either of these extreme periodicities, the arc is more readily held at 500 cycles than at 12½ cycles.

9. BARE OR COVERED ELECTRODES FOR ALTERNATING-CURRENT ARC WELDING

While some maintain that arc welding with alternating current is only at its best when flux-covered electrodes are used, it appears to have been conclusively demonstrated by others that excellent results are being obtained under commercial conditions with bare electrodes and an alternating-current supply. A novice can more quickly learn to weld from an alternating-current supply if he employs flux-covered electrodes. But if he can ultimately learn to weld just as rapidly and successfully with bare electrodes, the difficulties in the initial stages of his education should not be regarded as being of much consequence. Mr. Wagner finds that when welding with alternating current, "manipulation may be simplified in many cases by treating the electrode with a thin coating of ordinary lime."

10. RELATIVE SPEEDS OF ALTERNATING-CURRENT AND DIRECT-CURRENT ARC WELDING

Some contend that alternating-current welding is slower. As an instance of a diametrically opposite experience, the following recently received record of test of a certain electrode may be quoted:

"Its operation on 140 amperes, 115 volts alternating current is very good. It also works satisfactorily on 130 amperes, 75 volts direct current, but the metal flows more slowly on direct current than on alternating current."

The record concerning another type of electrode tested on the same occasion, reads as follows:

"This electrode was tried on 5/16 in. plate, 120 amperes, direct current, 75 volts. Its operation is satisfactory. It also works satisfactorily at 140 amperes, alternating current, 115 volts, but its operation on alternating current is not quite as good as on direct current."

Mr. R. E. Wagner, who has had much to do with the development of both kinds of welding, states that "on the average the speed of welding with alternating current and direct current is about the same. We have had cases where alternating current is faster, and vice versa."

11. COMPARATIVE QUALITY OF ARC WELDS MADE WITH ALTERNATING CURRENT AND WITH DIRECT CURRENT

It is contended that the greater difficulty of maintaining an alternating-current arc (involving the necessity of acquiring the skill to hold a very short arc,) entails as a consequence that an alternating-current weld is of superior quality. An experienced observer reports his experience as follows:

"Tests made have demonstrated conclusively that it is possible to do as good, or perhaps better, welding with alternating current as with direct current. No very decided difference has been noticed between welds made with alternating current and those made with direct current, but the welder who did most of the alternating-current welding says that in his opinion the alternating-current welds are better than the direct-current welds. This same opinion has been expressed by the machinist who made repairs on a small tank welded with alternating current. He said that the weld metal was better, more dense, and had fewer blowholes than a direct-current weld."

But an opinion from another authority of great experience in arc welding is as follows:

"As regards the strength of an alternating-current weld there is not the slightest doubt that a greater strength can be gotten on a test piece if that is all the work the man is going to do for some time. The facts, however, are that as the man's hand becomes fatigued in holding the alternating-current arc, his consequent breaking of the arc becomes more frequent, which means less strength in the weld, because every time the arc is broken, a bad spot is left in the weld."

12. CONSIDERATION OF THE POWER FACTOR FOR ALTERNATING-CURRENT ARC WELDING

A view presented with considerable persistency is that the low power-factor associated with alternating-current welding leads to capital and operating costs offsetting any advantages. One answer made is to the effect that since for ship welding on an extensive scale, motor-generators are required, this only affects the generator and its circuit and does not affect conditions as regards the motor or the circuit from which it is supplied.

13. Consideration of the Circumstance that Alternating-current Arc Welding is Essentially a Single-phase Load

Similar considerations are involved in regard to the necessity of providing for the characteristics of a single-phase load. It is well known that single-phase motors and generators are much heavier, more expensive and less efficient than polyphase motors and generators. With 30 or 40 arc welding outfits distributed fairly evenly on the different phases of a polyphase system, the load would be sufficiently balanced to be satisfactory, but this would correspond to an unusually large welding installation. In most cases it will be necessary to arrange for the welding to constitute a single-phase load and to make adequate provision to obtain satisfactory service with this condition.

Regarding the possibility of improving the power factor, Mr. W. S. Moody makes the following very suggestive statement.[3]

"Where a number of arcs are to be used within a reasonable distance of each other, the series system may be used. In this arrangement the secondary of an ordinary constant-current transformer supplies current to the primary of all the welding transformers in series. The individual transformers insulate the welding apparatus from the series circuit and transform from the series current to current of proper value for the arc. In this case the inherent reactance of the series transformer is low, but other features of the design are the same as those discussed above. The power factor of such a system can be safely made much higher than where individual arcs are operated in multiples from constant potential circuits."

14. Spot Welding is a Single-phase Load

Mr. J. M. Weed, who has had a great deal of experience with large spot welders, has kindly written the following paragraph on this subject:

"For welding plates from 3/8 to 3/4 in. in thickness, the single-phase currents required would be from 30,000 to 50,000 amperes and the kv-a. required at 60 cycles would range between 300 and 900 at power factors of from 0.35 to 0.50. These low power factors, combined with the fact that this load would be for short periods at very frequent intervals would make it decidedly undesirable from the central station standpoint. The condition would be much improved at 25 cycles, as the same machine would operate equally as well at 25 cycles as at 60 cycles, with about half the kv-a. and about double the power factor. The intervals of operation would however, be the same as for 60 cycles. If, however, a motor generator set with suitable flywheel attached, be provided for operating these machines, these disadvantages are all practically eliminated, this arrangement being such that the motor stores up energy in the flywheel during the interval of no load, the flywheel supplying a large part of the energy during the period of welding. By this means, for instance, a single-phase load of 900 kv-a. at 0.50 power factor for 30 second periods and with intervals of 1½ minutes between periods would be converted to a practically continuous 3-phase load of approximately 200 kv-a. at about 0.85 power factor."

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[3] *General Electric Review* (December, 1918) p. 937.

15. DUCTILITY OF ARC WELDS

Attention has been pertinaciously drawn to results of a very few tests which have appeared to indicate that metal-arc welds are inherently utterly deficient in ductility, yet the Committee has had also before it the results of many well-authenticated tests of ductile metal-arc welds.

It has been claimed that gas welds are more ductile. On this matter Mr. R. E. Wagner writes:

"At several meetings of the Welding Committee, special stress has been brought to bear on the bending qualities of acetylene and gas welds. We have done some experimenting with average acetylene and arc welders, and our impression is, that the acetylene and arc welds are in the same class with respect to bending. I submit herewith (See Fig. 8) a photograph showing comparative bends in acetylene and arc-welded joints, both welds were taken from half-inch plate and both samples were bent under the same conditions, that is, the sharp edge of an angle iron was placed along the weld and pressure applied to the angle iron to make a sharp bend. These I think are average comparative results. * * * * * * As far as our experiments are concerned, we feel, as regards physical characteristics, that acetylene and arc welds are in the same class."

16. RESPECTIVE FIELDS OF GAS AND ELECTRIC ARC WELDING

On this subject, under date of Oct. 22, 1918, Mr. R. P. Jackson, reports to the Welding Research Sub-Committee as follows:

"With reference to the comparative uses or fields of gas and electric-arc welding which came up at the last meeting, it was thought it might be well for some of us to express our opinions on the matter based on our experience with both kinds of welding. In general, we have found gas welding to be more satisfactory for thin material, say ⅛ inch and under, and for general repair work, particularly where various kinds of steel and cast iron are involved. For example, if repairs have to be made on broken machinery, lugs rebuilt, pieces attached to high-carbon steel and work of this character, then the gas-welding methods are superior and the extra cost not ordinarily prohibitive. When it comes, however, to depositing a large amount of metal and welding up structural steel or plates of ¼ inch thickness and upward, the results obtained by the ordinary direct-current arc with the metal electrode are at least equal to the gas welding work and certainly cheaper. In general, too, the finish of gas welding is more regular and better looking and where that is a consideration it may give a preference to gas. In fact, in the Westinghouse factory at East Pittsburgh, there has been considerably more gas work done than electric, but the electric arc welding is on the increase, not so much in displacing gas as in displacing riveting."

A view taken from a gas welding publication is as follows:

"The arc process is chiefly used for filling up blow holes in large steel or iron castings and building up worn surfaces which have not to be machined. With this process the results obtained are somewhat uncertain, and it is generally conceded, apart from the vital question of cost, that fusion produced by the burning of gases is to be preferred to the electric process. Welds made by the electric process are sometimes rough, hard, brittle and unworkable—in most cases this is highly objectionable, but not always so. With any fusion method of welding, annealing of the metal adjacent to the weld is desirable. It is impossible to do this annealing with an electric welder, but with gas welding the blow-pipe flame can be used for heating up the metal surrounding the welded part, and also for heating metal away from the weld, so as to counteract any strains that may be set up in the piece as the weld cools off. There are certain classes of work for which electric welding is the most suitable system, and, on the other hand, there are many classes of work where it would be most impractical, and which can be done satisfactorily only with gas welding. For general workshop use, a gas welding outfit is far better, not only because of its greater economy in installation and operation, but also because of its wider range of usefulness."

In the absence of any experience to the contrary this latter view appears fairly plausible, and it is natural that it should have received wide acceptance. But an enormous volume of experience in arc welding has gradually accumulated and it controverts the correctness of the view. Unfortunately the experimental data available on the subject of *Gas Welding* is surprisingly meagre. The Welding Research Sub-Committee has concluded thatthere are practically no test data from which it can draw any safe generalizations as to the mechanical characteristics of *gas welds*, and that it will be necessary to embark upon its own investigations to obtain suitable data.

Gas welding was an established art before there was any large amount of electric welding. This was still the state of affairs in England until shortly before the author was there in the autumn of 1917. But the war conditions had occasioned in England such a shortage of supplies of oxygen and carbide that the Government, as a war measure, practically forced the wide substitution of arc welding for gas welding. The British Government, in entering upon this policy, had relatively little concern as to the comparative merits of the two methods except in so far as that any merit or advantages found to be associated with arc welding would naturally assist in bringing about its use in place of gas welding.

It was, however, with considerable surprise that it was ascertained that the true economic field for arc welding as compared with gas welding was a very wide one, and that, simply due to inertia and tradition, engineers had been continuing in the contrary belief. Major James Caldwell, of the Admiralty Controller's Department, had wide responsibilities in this task of substituting arc welding as rapidly and generally as possible. Major Caldwell provided the author with the results of his investigations into the relative cost of gas and metal-arc welding. These results, which correspond to conditions in December, 1917, are set forth in Table II.

From Table II it is seen that metal-arc welding was found

TABLE II.—COMPARATIVE COSTS OF WELDING BY OXYACETYLENE AND METAL-ARC WELDING

Thickness of metal, inches	Oxyacetylene					Metal-Arc					Oxyacetylene		Metal-Arc	
	Gas per hour		Cost for gas per foot run pence	Iron wire for filling, pence	Labor per foot run	Power per foot run			Labor per foot run, pence	Cost of electrodes per foot run, pence	Feet run per hour	Total cost per foot run, pence	Feet run per hour	Total cost per foot run, pence
	Oxygen, cubic feet	Acetylene, cubic feet				Volts	Amp.	Kw.						
1/16	3	2.0	0.116	0.131	0.40	100	30	0.075	0.3	12 in. No. 12 = 1.2	30	0.647	40.0	1.575
1/8	9	6.3	0.77	0.196	0.856	100	75	0.312	0.5	18 in. No. 12 = 1.8	14	1.822	24.0	2.612
3/16	13	9.0	1.72	0.262	1.33	100	100	0.50	0.6	24 in. No. 10 = 2.64	9	3.312	20.0	3.74
1/4	17	13.0	2.70	0.250	1.50	100	140	0.70	0.6	22 in No. 8 = 3.6	8	4.45	20.0	4.90
5/16	27	16.0	4.21	0.327	1.714	100	110	1.10	1.2	12 in. No. 10 } 18 in. No. 8 } = 4.02	7	6.25	10.0	6.32
3/8	34	24.0	6.82	0.458	2.00	100	120	1.61	1.61	18 in. No. 10 } 24 in. No. 8 } = 5.58	6	9.278	7.4	8.802
7/16	41	29.0	9.90	0.655	2.40	100	120	2.00	2.0	24 in. No. 10 } 36 in. No. 8 } = 8.04	5	12.955	6.0	12.04
1/2	48	34.0	14.50	0.786	3.00	100	120	2.40	2.4	30 in No 10 } 42 in. No. 8 } = 9.6	4	18.286	5.0	14.40

Labor taken at 1 shilling per hour.
Oxygen taken at ½ pence per cu. ft.
Acetylene taken at 1 pence per cu. ft.
Iron for filling taken at 0.131 pence per ft.
The above figures are based on the British Oxygen Co. standards.

Labor taken at 1 shilling per hour.
Current taken at 1 pence per B.O.T. unit.
Electrodes, No. 10, 1.32 pence per ft.; No. 8, 1.8 pence per ft.; No. 12, 1.2 pence per ft.

to be a faster process for all thicknesses of steel. The British Admiralty results furthermore indicate the economic field for the two methods. The verdict from the data in the Table is in favor of gas welding for thin plates and of electric-arc welding for thick plates. But the comparison is based on the very high cost of electrodes set forth below:

Standard wire gauge	Cost in cents per foot	Ft. per pound of contained iron wire	Cost in cents per pound
Number 8	3.6	15	54
Number 10	2.6	23	60
Number 12	2.4	35	84

By substituting a typical American price for labor and substituting the cost of bare electrodes, such as are used with entire success in America, in place of the cost of flux-covered electrodes of the expensive type employed in arriving at the results set forth in the Table, the revised results show a lower cost for arc welding than for gas welding for all thicknesses above $^1\backslash_{16}$ inch. The question of the quality of the weld is another matter, but judging from the general reputation of the work of all sorts done by gas welding and by metal-arc welding, they are both thoroughly reliable. No more exact comparison can be made till we have carried through to completion, really elaborate tests of gas welds in order to permit of making a sound comparison with the large amount of research data already obtained with metal-arc welds.

In response to a request for his opinion as to the respective fields for gas and metal-arc welding, Mr. R. E. Wagner, writes as follows: "The present well-tried field for metal-arc welding is confined entirely to welding plates and forms, and a great deal of work has been done on plates varying in thickness from 1/16 in. to ¾ in. Up to 1/8 in. plates, the cost of gas and electric welding is about the same. Beyond this, the cost is in favor of the electric process. No difficulty is experienced in machining electric welds made with the metallic electrode. While it is recognized that the electric-welded-in material will not stand bending equal to that of the plate in which it is deposited, it is on the average equal to gas-deposited material in this respect."

17. RELATIVE DUCTILITY OF ARC WELDS MADE RESPECTIVELY WITH BARE AND COVERED ELECTRODES

By some authorities, ductility is believed to be most readily obtained by employing flux-covered electrodes. On the other hand, the Committee has knowledge of several kinds of bare electrodes of various compositions which, in competent hands, make reasonably ductile welds.

18. SPEED OF ARC WELDING

All sorts of values are given for the speed, in feet per hour, with which various types of joints can be welded. Operators making equally good welds have widely varying degrees of proficiency as regards speed. Any quantitative statement must consequently be of so guarded a character as to be of relatively small use. In general, and within reasonable limits, the speed of welding will increase considerably when larger currents are employed. It appears reasonable to estimate that this increase in speed will probably be about 25 to 35 per cent, for high values of current. This increase is not directly proportional to the current employed because a greater proportion of time is taken to insert new electrodes and the operator is working under more strenuous conditions. Incidentally, the operator who employs the larger current will not only weld quicker but the weld will have also better strength and ductility.

On this point Mr. Wagner writes as follows: "I would not say that speed in arc welding was proportional to the current used. Up to a certain point ductility and strength improve with increased current, but when these conditions are met, we do not obtain the best speed due to increased heating zone and size of weld puddle. Speed may fall off when current is carried beyond certain points."

In a research made by Mr. William Spraragen for the Welding Research Sub-Committee on several tons of half-inch-thick (12.7 mm.) ship plate, the average rate of welding was only two feet (0.6 m.) per hour. Highly skilled welders were employed but they were required to do the best possible work, and the kinds of joints and the particular matters under comparison were very varied and often novel.

However, in the researches carried on by Mr. Spraragen it was found that about 1.9 pounds (0.8 kg.) of metal were deposited per hour when using a 5/32 in. (3.9 mm.) bare electrode and with the plates in a flat position. The amount of electrodes used up was about 2.7 pounds per hour, of which approximately 16.5 per cent was wasted as short ends and 13 per cent burnt or vaporized, the remainder being deposited at the speed of 1.9 pounds per hour mentioned above.

For a 12-foot-cube tank of half-inch-thick steel welded at Pittsfield, the speed of welding was 3-feet per hour. The weight of the steel in this tank was 16,000 lb. and the weight of electrode used up was 334 lb. of which 299 lb. was deposited in the welds. The total welding time was 165 hours corresponding to using up electrodes at the rate of just 2 lb. per hour. The total length of weld was 501 feet, the weight of electrode used up per foot of weld thus being 0.60 lb. The design of this tank comprised eighteen different types of welded joint. Several different operators worked on this job and the average current per operator was 150 amperes.

For the British 125-foot-long Cross-Channel Barge for which the shell plating was composed of ¼-in. and 5/16-in. thick plates, in Mr. H. Jasper Cox's paper read before the Society of Naval Architects on Nov. 15, 1918, and entitled "The Application of Electric Welding to Ship Construction" it is stated that: "After a few initial difficulties had been overcome, an average speed of welding of seven feet per hour was maintained including overhead work which averaged from three to six feet per hour."

In a report appearing on page 67 of the Minutes and Records of the Welding Research Sub-Committee for June 28, 1918, Mr. O. A. Payne, of the British Admiralty, states: "A good welder could weld on about one pound of metal in one hour with the No. 10 Quasi-Arc electrode, using direct current at 100 volts. An electrode containing about 1½ ounces of metal is used up in about three minutes, but this rate cannot be kept up continuously."

The makers of the Quasi-Arc electrode publish the following data for the speed of arc welding in flat position with butt joints, a 60 degree angle and a free distance of ⅛ inch.

Thickness of Plates	Speed in Feet per Hour
⅛ in.	30
¼ in.	18
½ in.	6
1 in.	1.3

I cannot, however, reconcile the high speed of welding ½-inch plate published in this report as 6 ft. per hour, with the report given above by the British Admiralty that a good welder deposits one pound of metal per hour with Quasi-Arc electrode. If the rate given by the manufacturer is correct, it would mean that about four pounds of metal were deposited per hour. On this basis the rate must have been computed on the time taken to melt a single electrode and not the rate at which a welder could operate continuously, allowing for his endurance and for the time taken to insert fresh electrodes in the electrode holder and the time taken for cleaning the surface of each layer before commencing the next layer.

From his observations the author is of the opinion that a representative rate for a good welder lies about midway between these values given respectively by Mr. Payne and by the makers of the Quasi-Arc electrode, say for half-inch plates some two pounds per hour. This, it will be observed, agrees with Mr. Spraragen's experience in welding up some six tons of half-inch ship plate with a dozen or more varieties of butt joint and Mr. Wagner's results with the eight-ton tank. Even this rate of two pounds per hour is only the actual time of the welding operator after his plates are clamped in position. This preliminary work and the preparation of the edges which is quite an undertaking, and requires other kinds of artisans, accounts for a large amount of time and should not be underestimated.

The practise heretofore customary of stating the speed of welding in feet per hour has led to endless confusion as it depends on type of joint, height of weld and various details. A much better basis is to express the speed of welding in pounds of metal deposited per hour. Data for the pounds of metal deposited per hour are gradually becoming quite definite. The pounds of metal per foot of weld required to be deposited can be readily calculated from the drawings or specifications. With the further available knowledge of the average waste in electrode ends and from other causes, the required amount of electrode material for a given job can be estimated.

19. SUITABLE CURRENT FOR GIVEN CASES

For a given type of weld, for example, a double Vee weld in a one-half-inch-thick ship plate, it was found that in the summer of 1918, while some operators employed as low as 100 amperes, others worked with over 150 amperes. Some, in making such a weld, employed electrodes of only ⅛ in. diameter and others preferred electrodes of twice as great cross-section. For the particular size and design of weld above mentioned, the Welding Research Sub-Committee has since had welds made with from 200 to 300 amperes. The conclusion appears justified that the preferable current for such a weld is at least 200 amperes. If the weld of the half-inch-thick plate is of the double-bevel type, some 50 amperes less current should be used for the bottom layer than is used for the second layer, if two layers are used. For three-quarter-inch-thick plates, the most suitable welding current is some 300 amperes. This is of the order of twice the current heretofore usually employed for such a weld.

Mr. Wagner writes: "We have made a number of tests to determine the effect of varying current on the strength of the weld. Tests were made on a ½-in. plate with current values as follows: 80, 125, 150, 180, 220, 275, and 300 amperes. These tests show improvement in the tensile strength and bending qualities of welds as the current increases. The speed of welding increases up to a certain point and then decreases."

20. Effect on Arc Welding of Voltage Employed

Mr. Wagner reports as follows: "We have made a number of tests to determine the influence of variable voltages on the strength and character of electric welds. The experiments were made welding ½ in. plate with 150 amperes held constant and voltage varying as follows: 40, 75, 100, 125, 150, 200 and 225 volts.

"This test demonstrates that there is no material difference in the tensile strength, bending qualities or the appearance of the welded-in material. There is this advantage, however, in the higher voltage, that variations in the strength of the arc do not materially affect the value of the current.

"A curve-drawing ammeter was installed on the welding circuit which showed variations in current at 75 volts, but at 150 volts the current curve was practically a straight line."

21. Preferable Size of Electrode

On certain railways, a single diameter of electrode is employed independently of the size or shape of the plates or parts being welded. The experience of other people leads them to make use of several different sizes of electrodes according to the size of the job and the type of joint. Present British practise appears to be to use such a size of electrode as to have a current density of some 4000 to 6000 amperes per square inch. The investigations of the Welding Research Sub-Committee indicate that at least 10,000 to 12,000 amperes per square inch is suitable for electrodes of ⅛ in. and 5/32 in. diameter and well up toward 10,000 amperes per square inch for electrodes of 3/16 in. and ¾ in. diameter.

22. Automatic Machinery for Arc Welding

Several firms are developing machinery for feeding the electrode automatically. Such machinery appears to be capable of making excellent welds at higher speeds than are attainable by hand feeding.

23. Carbon-arc Welding

With the advent of metal-arc welding there has been a tendency to neglect the carbon-arc method. It is quite possible that this attitude is not justified for not only is there now a definite field where the carbon-arc method is advantageous but developments in the art may greatly extend its application.

It is generally agreed that the carbon-arc method is not applicable to vertical and overhead welding, which is, of course, a serious handicap in ship-hull work. The majority opinion of competent observers (with, however, some emphatic dissenting views) appears to indicate that carbon-arc welding is not as reliable as metal-arc welding in ordinary welding, because:

a. Carbon is carried into the deposited material thus reducing its ductility.

b. It is more difficult to obtain good fusion on account of overlapping of deposited metal on the original metal.

c. It is more difficult to manipulate and thus requires greater skill.

d. It is a much hotter arc which means greater discomfort to the operator and therefore lower efficiency.

e. Greater cooling stresses are developed because larger areas of adjacent metal are heated.

On the other hand, it is contended by some that carbon-arc welding can be developed to the point where these objections will no longer exist and thus gain the advantages of this method, the principals of which are:

a. No preparation of the abutting edges is necessary.

b. Greater rate of deposition of metal and therefore greater speed of welding, particularly in heavy work.

c. Probable greater adaptability to automatic welding. It should be stated that there is very general agreement as to the superiority of the carbon arc over the metal arc for heavy work where strength is not so important, especially cast-iron welding and the filling of holes in iron and steel castings.

24. PREPARATION OF WELDING EDGES

British practise permits the use of smaller angles when the edges of the plates are Veed, than accords with American traditions. If the smaller angles give welds which are equally satisfactory in all respects, the decreased amount of electrode material required, the decreased consumption of electricity, and the increased speed are advantages not to be overlooked; but obviously the matter requires careful investigation. American practise which, up to recently, has been with a very wide angle, appears to have required the consumption of about twice as great a weight of electrode as British practise with the smaller angle. The urgent importance of determining whether the use of the smaller angle involves any sacrifice in quality is evident. There is already considerable basis for the belief that actually better results attend the employment of a smaller angle of bevel when a suitably large current is used. A shoulder in place of the heretofore commonly used sharp bottom edge of the bevel, also constitutes a material gain not only in the saving in welding material, but also in the quality of the weld.

TABLE III.—TIME, METAL, AND CURRENT USED WITH WELDS OF DIFFERENT BEVELS

	Angle of Bevel Used, in Degrees			
	15	30	45	60
Amperes	160.	145.	118.	125.
Weight of electrode used up (lb.)	2.56	3.83	4.63	6.63
Weight of metal deposited (lb.)	1.70	2.55	3.65	5.08
Weight of metal wasted (lb.)	0.86	1.28	0.98	1.55
Pounds deposited per hour	1.82	1.61	1.82	1.81
Feet welded per hour	3.22	1.90	1.50	1.07
Circuit kilowatts	9.91	9.00	7.68	8.25
Kilowatt-hours per foot of weld	3.10	4.70	5.10	7.70

Mr. Wagner states that at Pittsfield they have long adopted the practise of using a 30-degree bevel for plate edgings and that they find it satisfactory for all thicknesses up to ¾ in. He states that this angle gives sufficient room for depositing the metal, reduces the time to weld and the amount of metal deposited.

In one of Mr. Spraragen's researches, various angles of bevel were used. Although the physical tests have not yet been made we can gain from Table III valuable lessons on the time, amount of metal, and electricity consumed for these different angles of bevel. The "free distance" in each case was ⅛ in. and the welding was done in a flat position with 5/32-in. bare electrodes. In each case the weld had a length of three feet.

25. QUALITY OF OVERHEAD ARC WELDING

The British Admiralty regards overhead welding as too inferior and too expensive to be employed when it can possibly be avoided. In America a large amount of overhead welding is done in railway shops and it is claimed that it is simply a matter of training operators to the required degree of proficiency.

26. NUMBER OF LAYERS TO BE EMPLOYED

Good progress is being made in obtaining knowledge of the relative characteristics of welds made with different number of layers and of the most suitable current and the most suitable size and type of electrodes to employ for each layer. The tendency is toward the use of at least two layers for half-inch-thick plates, and three layers for three-quarter-inch-thick plate.

27. RIGID VS. NON-RIGID METHODS OF WELDING

On this question it is more a matter of determining the conditions essential to obtaining good results with whichever of the two methods is most appropriate for each particular purpose.

The term *rigid* is applied to the process of arc welding, in which the two parts to be joined by welding are, prior to welding, held rigidly by bolting or clamping or by a series of preliminary tack-welds distributed at various points. The rigid plan is the most obvious for welding the hull plates of ships but its critics claim that the resultant joints are deficient in ductility due to the presence of internal stresses. It is considered that by suitably arranging the order of welding it is practicable to so distribute the heat as to avoid these stresses. At any rate, there are many alternative orders of procedure in making welds by the rigid method and elaborate researches should be made to ascertain the procedure which will yield the best result.

The non-rigid method consists in placing at a slight angle to each other the two plates to be welded. As the welding operation progresses along the seam the angle gradually closes and when the weld is completed the width of the welded seam is equal throughout its extent. Such welds are generally considered to be very free from internal stresses, and hence more ductile.

28. Consequences of Different Lengths of Arc

The metal arc is much shorter than the carbon arc. As a result the metal arc weld has the advantage that there is less opportunity for oxygen and nitrogen to gain access to the weld and so far as relates to this feature the metal arc weld should be better. But with the carbon arc the added metal does not traverse the arc, the tip of the welding rod being held down close to the surface on which it is to be deposited. This may render the deposited material less subject to contamination in carbon arc welding than in metal arc welding since it has not traversed the arc.

Coming to the exclusive consideration of metal-arc welding, the greater the welding current the less is the area represented by the cylindrical surface of the arc per pound of metal traversing the arc, and consequently the less should be the contamination by oxygen and nitrogen from the surrounding air. So far as this circumstance is concerned, the greater the welding current, for a given case, the greater should be the ductility of the joint. On the other hand, it seems probable that even the most skilful operators will be unable to hold quite so short an arc with the larger current.

29. Spot and Arc Welding

A good deal of progress is being made in America in the use of spot welding for the joining of thick plates. It is believed that spot welding has a great future as applied to shipbuilding and several large spot welders have been built for shipyards.

In some of its applications, spot welding affords a method of preliminarily joining the hull plates, after which the required additional strength is provided by arc welding. The Welding Research Sub-Committee has already made some progress in comparing combined spot and arc welds and combined rivet and arc welds with riveted, spot-welded, and arc-welded joints.

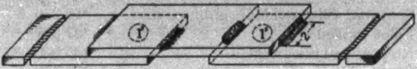

Fig. 9—Fillet and Spot Welded

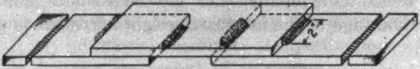

Fig. 10—Fillet Welded

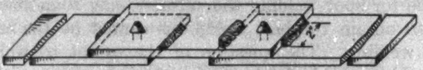

Fig. 11—Riveted and Fillet Welded

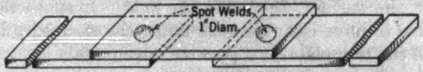

Fig. 12—Spot Welded

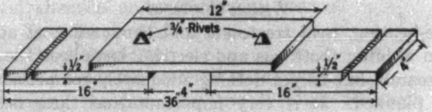

Fig. 13—Riveted Joint

It is not a question in such an investigation of spot *versus* arc welding, but of spot *and* arc welding.

In the tests mentioned, the specimens were made up of the following combination:

1. Spot and fillet welded Fig. 9 (2 samples made).
2. Fillet welded—made by welding fillets about two inches in length at the ends of the plates Fig. 10 (2 samples made).
3. Riveted and fillet welded Fig. 11 (1 sample made.)

4. Spot welded—made by welding two spots approximately one inch in diameter, on the plates Fig. 12 (2 samples made).

5. Riveted joint, made by riveting a ½-inch by 4-inch by 12-inch plate with two plates ½-inch by 4-inch by 16-inch, using two ¾-inch rivets and a 4-inch lap. (One sample made).

The results of the test show the comparative strength of the joints as follows:

Spot and fillet welded—	ultimate	load . . .	*50,350	lb.
Fillet welded—	"	" . . .	*37,000	"
Riveted and fillet welded—	"	" . . .	†35,000	"
Spot welded—	"	" . . .	*28,000	"
Riveted joint—	"	" . . .	†13,000	"

Spot welds, as compared with arc-welded butt joints, have the disadvantage of the increased weight corresponding to the overlap.

30. CONDITION OF SURFACES TO BE WELDED

While for spot welding the surfaces may sometimes be *too* clean to obtain the best weld, this cannot be the case with fusion welding. The question of the extent to which it is practicable to go in freeing the surfaces from impurities prior to making the fusion weld is entirely a commercial one. The cleaner the surface, the better the weld. In spot welding it is desirable to have clean surfaces under the electrodes, but scale *between* the two plates is a positive advantage.

31. PRE-HEATING AND HEAT TREATMENT AND HAMMERING WHILE COOLING

Pre-heating, heat treatment and hammering, as applied to fusion welding (both gas and electric) have been the subjects of research, but as yet nothing adequately comprehensive has been planned. It is very important that these deficiencies should be recognized and remedied.

32. TIME REQUIRED TO TRAIN WELDERS

As to the training of welders, there are on the one hand those who are so ill-informed as to assert that good welding can be done after a very few weeks of training, while on the other hand, others assert that an operator can hardly acquire any considerable degree of skill in much less than a year. Still others advocate the substitution of automatic feeds and the complete elimination of dependence upon manual dexterity. For arc welding work requiring more than 100 amperes this is, in the author's opinion, not a trade for women when the arc is manipulated by hand as distinguished from the use of automatic or semi-automatic machinery.

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*Average of tests on two samples.

†Only one sample made.

33. Question of Need for Special Machines for Welding

A great variety of machinery for supplying and controlling the current for welding is on the market. Some of this machinery comprises elaborate mechanisms in virtue of which it is claimed that it would be very difficult for even a novice to make a bad weld. Some advocate the use of simple resistance to be inserted in series with the arc on any available circuit, and claim that any additional machinery is superfluous. The capital outlay for the equipment of a welder (at the point of consumption) when the first kind of equipment is used, may be a matter of over $1000.00, while in the second case, well below half of that sum is sufficient.

34. Technique of Testing Welds

The ideal weld should presumably be at least as strong and as durable as the metal of the members joined together. In other words, the section containing the weld should have the same chemical and physical characteristics as adjacent sections in the original metal. A weld is therefore measured by the degree of approximation to this condition as dertermined by mechanical, chemical and metallurgical tests of:

a. The parent metal
b. The welded joint
c. The deposited material in the weld.

While during the last year the Welding Research Sub-Committee has made a great deal of progress in establishing standard procedures for the mechanical testing of welds, much still remains to be done. Obviously, the procedure for testing the original metal should follow standard practise as recommended by the American Society for Testing Materials, but there is considerable difference of opinion and uncertainty as to just how and what mechanical tests should be made of the welded joint and of the deposited metal. For instance

a. Should all the usual observations be taken when making a tensile test of a welded joint? Obviously the strength of the union between the two pieces of metal should be determined but in view of the non-homogeneity of the specimen, does not a very different significance attach to yield point, elongation and reduction of area? Where a series of welds having the same ratio of deposited material to original metal is concerned, such data are undoubtedly important for comparison purposes but for evaluating a weld in terms of the original metal, questions are repeatedly being raised as to just what extent these data have value.

b. Would not more reliable information as to the ductility of the weld be obtained if elongation and reduction of area measurements were made on specimens prepared from the deposited metal or from specimens cut lengthwise of the weld instead of crosswise?

c. Similarly with the bending test, which is a test for ductility. There are some (including the author) who would make the bending with the axis of the mandrel *normal* to the weld instead of parallel thereto, which latter position is the one usually employed. It may be that both tests should be made; the normal position as test of the ductility of the deposited material and the parallel position as an additional test of the union between the deposited material and the original metal.

d. How important are torsion tests and impact or shock tests in measuring welded joints?

e. Fatigue tests of welded joints are generally conceded to be vital and the importance of obtaining reliable information as to how this test should be made probably transcends (at present at least) that attached to any other research in the field of fusion welding. The researches should be made:

1. With the Moore bending fatigue machine
2. With rod samples rotated at high speed, as employed by Lloyd's Register in England
3. With the Strohmenger torsion-fatigue machine
4. With the Cammell-Laird bending fatigue apparatus
5. By the Upton-Lewis test.

After the necessary research work has been done to solve these and other similar questions pertaining to the testing of welds, standard specifications for the testing procedure can be prepared which will be properly balanced between the cost of making the tests and the amount of testing necessary to insure a reliable estimate of the weld.

CONCLUSION

The extent of the field of application for fusion welding and spot welding is but little appreciated by engineers other than those who have been directly connected with welding developments. It is evident that this field is an enormous one, including as it does all structures where steel is employed, such as bridges, building structures, tanks of all types and kinds, railway rolling stock, and ships, in addition to numberless miscellaneous applications in industry in general.

However, engineers associated with welding research should be on their guard that their enthusiasm over this great field of application shall not lead them into prematurely endorsing the use of fusion welding or spot welding in constructions where the consequences of failure involve serious menace to life and property, as may often be the case. For example, a particularly important case is that of pressure vessels and especially large high-pressure containers. The success in one hundred installations will not excuse failure (accompanied possibly by fatalities), in the one hundred and first installation. It is the opinion amongst the best informed engineers that before fusion welding can advisedly be employed for large high-pressure vessels, much vigorous and elaborate research work should be carried out on the fatigue characteristics of fusion welds of long seams, and that this research work must comprise full-sized structures since the conditions cannot be reproduced in test samples.

In fact, if the general acceptance of welding, particularly by Inspection boards, underwriters, and classification societies, is to be accomplished in a reasonably short time, such extensive research work on a large scale is absolutely essential in order to demonstrate conclusively that welded joints are equal to or better than joints made by other methods. Obviously the development of the art could proceed along the lines of the usual order of evolution as in the cases of previous arts, but this would, as in those cases, involve the lapse of years.

For structures subjected to less extreme stresses, such as the hulls of ships, the adequacy of fusion welding as a substitute for riveting is in process of being thoroughly demonstrated in actual practise in Great Britain. It is recognized that the hulls of ocean-going ships are exposed to very great stresses, nevertheless there is a clear distinction between the magnitude of those stresses and the stresses to which many large, high-pressure containers are subjected.

The author hopes this paper will aid in focusing attention on the vast importance of the welding art, particularly by occasioning discussion of the many problems in welding research some of which have been mentioned in the paper.

The author cannot undertake to give adequate acknowledgment of his indebtedness to his many associates in the preparation of this paper. The most generous assistance has been given him on every hand. Mr. William Spraragen has extended much assistance in preparing data and in many useful ways. Mr. F. M. Farmer, chief engineer of the Electrical Testing Laboratories, has given very generously of his time in advising the author in detail about many points which arose in the course of the preparation of this paper.

Four Appendices accompany this paper:

Appendix A: Standard Procedure for Testing Welding Electrodes.

Appendix B: The Wirt-Jones Tests of Arc Welded Half-Inch Ship Plates.

Appendix C: Bending Tests of Gas Welds.

Appendix D: Arc-Welded Steel Box.

Appendix A

STANDARD PROCEDURE FOR TESTING WELDING ELECTRODES.

WELDING COMMITTEE, EMERGENCY FLEET CORPORATION

NOVEMBER, 1918

1. *Purposes.* The purpose of this specification is to provide a standard procedure for testing welding electrodes for metal-arc welding which are submitted for the information of, and with the view to the approval by, the Welding Committee of the Emergency Fleet Corporation.

Note.—This specification describes a test *of electrodes* and not a combination of an electrode and of an apparatus. The fact that the applicant is given the option of selecting the system with which the test is made, does not make it a test of that system, any more than the employment of a particular welding operator could be said to make it a test of a welding operator. The system used in making these tests may or may not prove to be of importance. It is sought to minimize the influence of the individuality of the operator by requiring the test to include welds made by at least two operators.

The Welding Committee has two other Sub-Committees into whose province falls the approving and certifying of operators and approving and certifying of systems. The admission that these three tasks overlap does not involve any reason for delaying the important task of proceeding at once with the standardizing of the testing *of electrodes*.

2. *General Conditions*. a. Each applicant shall provide at his factory or elsewhere as he may elect, the necessary facilities and at least two operators for making the test welds prescribed by these specifications. Except when otherwise indicated in these specifications, the applicant shall select the apparatus and other conditions for making test welds which he considers most suitable for his electrodes.

b. The test welds shall be made in the presence of an *authorized representative* of the Welding Committee of the Emergency Fleet Corporation who will be empowered to certify as to the compliance with the conditions prescribed in the specifications. Also representatives of Lloyd's Register of Shipping and the American Bureau of Shipping shall have the opportunity to witness the making of the test welds.

c. Until otherwise declared, the *authorized representative* of the Welding Committee shall be the Electrical Testing Laboratories, 80th Street and East End Avenue, New York, N. Y.

d. The Welding Committee shall assume charge of the completed test welds and have them tested by the *authorized representative* in accordance with these specifications.

e. The *authorized representative* shall render to the Welding Committee a complete, detailed report of each test of an electrode.

f. The cost of carrying out the prescribed tests shall be borne by the applicant.

3. *Sample Electrodes*. About 100 pounds of each electrode to be tested, which will be known as the *sample*, shall be furnished without charge. It shall be accompanied by an affidavit giving the following information:

a. The *trade name* under which the electrode is marketed together with certification that all electrodes bearing this trade name will be substantially the same as the sample submitted.

b. The manufacturer of the complete electrode.

c. The location of the factory in which the electrode was made.

d. The manufacturer of the welding wire.

e. The location of the mill in which the welding wire was produced.

f. The chemical analysis of the welding wire.

The *authorized representative* shall retain the remainder of the sample for reference purposes.

4. *Plate Material.* For these tests one-half inch ship plate shall be used. This material shall comply with the "*standard specification for structural steel for ships*" as adopted by the American Society for Testing Materials, serial designation A 12–16 (see page 98, A. S. T. M. Standards, 1918).

An abstract of this specification is as follows:

1. Open hearth steel.

2. Phosphorous $\begin{cases}\text{acid steel, not over 0.06 per cent.}\\ \text{basic steel, not over 0.04 per cent.}\end{cases}$

Sulphur, not over 0.05 per cent.

5. Tensile strength, pounds per square inch, 58000–68000

Yield point, minimum pounds per square inch, 0.5 tensile strength.

Elongation in 8 inches, minimum per cent., $\dfrac{1{,}500{,}000}{\text{tensile strength}}$

6. Yield point by drop of beam method.

7. Cold bend test; no cracking on outside of bent portion when bent 180 degrees around a pin the diameter of which is equal to the thickness of the plate.

5. *Cutting of Plates.* a. The method of cutting a plate 5 feet by 20 feet into pieces for welding is shown in the accompanying reproduction of drawing No. 102 of the Bureau of Standards.

b. Each plate shall be given a distinguishing letter, A. B. etc. This letter shall appear upon each piece cut from it.

c. Pieces shall be plainly marked with the letter and number arranged as shown in the drawing so that the location of the piece in the plate may be determined.

d. The plates from which tensile, cold bend and fatigue specimens are to be made, shall be cut into pieces 9 inches by 30 inches (see Fig. 14.)

e. The plates from which impact specimens are to be made shall be cut into pieces 30 inches by 30 inches (see Fig. 17).

Notes.—A power shear may be employed for cutting these plates if they are left flat. Pieces whose dimensions are within ¼ inch of the required size will be satisfactory.

6. *Arrangement of Piece for Welding.* a. All welds are to be made across the grain, that is, the welded joint is to be at right angles to the direction of rolling. Fig. 15 shows the correct arrangement of the pieces for the test welds to be used for the physical tests, but in Fig. 17, which shows the arrangement of pieces for the impact test, pieces such as J–1 and J–2 should be taken for a test weld instead of J–1 and J–9.

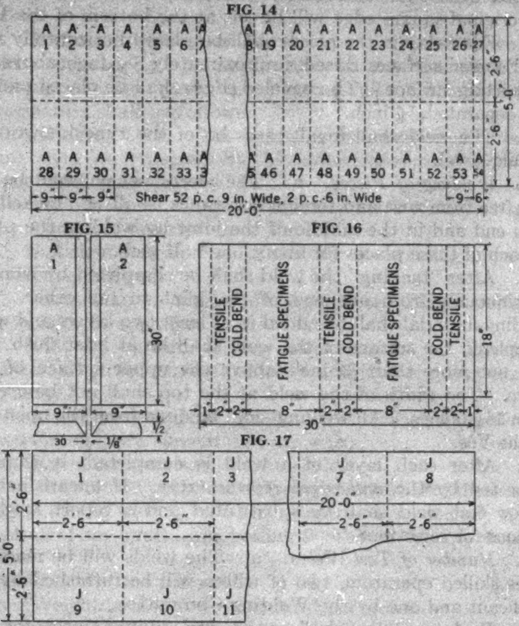
FIG. 14
A 1
A 2
A 3
A 4
A 5
A 6
A 7
A 8
A 19
A 20
A 21
A 22
A 23
A 24
A 25
A 26
A 27
A 28
A 29
A 30
A 31
A 32
A 33
A 3
A 5
A 46
A 47
A 48
A 49
A 50
A 51
A 52
A 53
A 54
2'-6"
5'-0"
2'-6"
9" 9" 9"
Shear 52 p. c. 9 in. Wide, 2 p. c. 6 in. Wide
9" 6"
20'-0"
FIG. 15
A 1
A 2
30"
9"
9"
1/2
30
1/8"
FIG. 16
TENSILE
COLD BEND
FATIGUE SPECIMENS
TENSILE
COLD BEND
FATIGUE SPECIMENS
COLD BEND
TENSILE
18"
1 2 2 8" 2 2 8" 2 2 1
30"
FIG. 17
J 1
J 2
J 3
J 7
J 8
20'-0"
2'-6"
2'-6"
2'-6"
2'-6"
2'-6"
5'-0"
2'-6"
J 9
J 10
J 11

Specimens for Tests of Welding Electrodes. Welding Research Sub-committee Emergency Fleet Corp. Bureau of Standards, Wash., D. C.
No. 102—VII-I Nov. 9, 1918
Changed Nov. 25, 1918

b. Preparatory to making a weld, one 30-inch edge of each piece will be beveled at the angle preferred by the applicant.

If he has no preference, the angle of the bevel shall be 30 degrees as shown in Fig. 15. The bevel is not to be carried through to the bottom edge but only to within 1/16 inch of the bottom edge. In other words, the ends of the pieces to be welded are not to be beveled to a sharp point but 1/16 inch of the original square edge will be left at the bottom of the Vee.

c. For welding, the pieces shall be placed horizontally with their under surfaces raised approximately ⅛ inch above the supporting surface. The beveled edges shall be placed parallel and separated ⅛ inch.

d. The weld shall in all cases be of the type known as a double bevel.

7. *Welding of Pieces.* a. The pieces, which shall be dry and free from rust and foreign substance, shall be "tacked" at each end and in the middle of the joint by welding the plates at each of these places for about one-half inch.

b. After "tacking" the weld shall be completed by working continuously from one end of the joint to the other. The welding material shall be added in at least two layers and when completed the surface of the weld shall be at least flush with and not more than ⅛ inch above the upper surface of the plate. The width of the weld at the top shall not be greater than 1¼ inches. All welding shall be done from the open side of the Vee.

c. After each layer of a weld is completed, it shall be inspected by the *authorized representative.* If unsatisfactory, a new test weld shall be substituted and a report made of reasons for rejection.

8. *Number of Test Welds.* a. The welds will be made by three skilled operators, two of whom will be furnished by the applicant and one by the Welding Committee.

b. Each operator shall make one 30-inch weld for the tensile, cold bend and fatigue tests (see Figure 15) using direct current and a similar weld using alternating current. For these tests, therefore, six (6) test welds will be required for each sample electrode.

c. Each operator shall also make one 30-inch test weld for the impact test (see Fig. 17) using direct current and a similar weld using alternating current. For impact tests, therefore, six (6) test welds will be required for each sample electrode.

d. If, however, the applicant intends his electrode for use with only one kind of current, that kind only shall be used in making the welds. In that case only three (3) test welds will be required for (b) and for (c) instead of six (6).

9. *Arrangement of Conductors.* a. One of the conductors shall be attached to a clamp which will be fastened to both pieces at the end of the joint opposite that from which work is started.

b. For direct-current work, the electrode shall have the polarity desired by the applicant. In the absence of any preference on his part, the instructions of the *authorized representative* shall be followed.

10. *Welding Data.* The following information and observations shall be reported by the *authorized representative* during the welding operations.

a. Trade name or other identification mark of the electrodes.

b. Complete description of the electrode.

c. Sufficient description to identify the welding apparatus or system employed.

d. Identification marks on the pieces being welded.

e. Name of operator.

f Kind of current (*i. e.*, direct current or alternating current); if direct current, polarity and if alternating current, frequency.

g. Electrical quantities as follows: arc volts (both open and closed circuit), arc current and arc watts.

h. Room temperature and humidity.

i. The opinion of the *authorized representative* of the working quality of the electrode. This statement to include a description of any sputtering, boiling or other noticeable peculiarities.

j Elapsed time per weld.

k. Weight of electrode consumed per weld.

l. Any other information which will assist in determining the performance of the electrode, such as a photograph of at least one weld, etc.

11. *Preparation of Welded Plate and of Specimens for Physical Tests* (see Fig. 16).

a. Each test weld shall be machined down on both sides to about the surface of the plate.

b. Specimens shall be cut from *each* test weld reserved for physical tests as follows:

1. *Three Tensile Specimens.* These shall be machined to a uniform width of 1.5 inches unless a weld of great

strength makes it necessary to leave shoulders at the ends, in which case the standard A. S. T. M. test specimens for sheet iron and steel shall be prepared.

2. *Three Cold Bend Specimens.* These shall be machined to a uniform width of 1.5 inches.

3. *Six Fatigue Specimens.*—These shall be machined to about ½ inch diameter and 10 inches long. (The exact dimensions are to be determined by experiment.)

12. *Physical Tests.* a. *Tensile Strength.* Each of the three specimens shall be tested in accordance with the practise recommended by the A. S. T. M. and shall include the determination of the tensile strength, yield point (by drop-of-beam), method), reduction of area and total elongation after rupture in two inches and in eight inches.

b. *Cold-bend Test.* This test shall be made by placing the specimen on two rollers with the apex of the Vee upward and midway between the rollers and loaded at the center of the span thus formed by a cylindrical surface having a diameter of one-half inch. This surface shall bend the specimen downward between the rollers until a fracture appears on the lower side of the specimen when loading shall be stopped and the angle noted through which the specimen has been bent.

c. *Fatigue Test.* Each of the six specimens shall be tested in a special rotating type of machine similar to that used by Lloyd's Register of shipping. (Exact details to be determined by experiment.)

d. *Impact Test.* Each impact-test specimen shall be placed on supports 18 inches high and 4½ feet apart. A spherical weight of 500 pounds shall be allowed to fall freely through a distance of 10 feet striking the weld which shall be at the center of the span. The apex of the Vee shall be upward.

e. *Test of Original Plate.* In order to establish the physical properties of the unwelded plate, tensile, cold-bend and fatigue tests shall be made on a sample selected at random from the pieces used for the test welds but before such welds are made.

OPTIONAL TESTS

The following tests are optional with the applicant but the welding Committee considers that they give information of great importance and recommends that they be made in order to make the report on the electrode more complete.

13. *Chemical Analysis.* A chemical analysis shall be made of:

a. The original plate in one test weld selected at random.

b. The metal at the center of one test weld selected at random. If test welds are made with both direct current and alternating current, an analysis shall be made of one weld of each set.

14. *Photomicrographs.* Photomicrographs shall be made of one specimen weld selected at random as follows: (If test welds are made with both direct current and alternating current, photomicrographs shall be made of one weld of each set.)

a. At center of weld.
b. At juncture of weld and original metal.
c. In adjacent original metal.
d. Cross section of electrode.
e. Longitudinal section of electrode.

Appendix B

WIRT-JONES TESTS OF ARC-WELDED HALF-INCH SHIP PLATE

In Tables IV and V are set forth the results of the tests carried out in 1918 on arc-welded half-inch ship plate by the Welding Research Sub-Committee. The plates with which the welds were made had their edges prepared with a 45-degree double-Vee. The welds were made by a dozen different concerns. In each instance the operator employed whatever conditions were considered most suitable. Several types and sizes of electrodes, including both bare and covered, were used. While in the majority of cases direct current was employed, alternating currents of 25 and 60 cycles are also represented in the series. The number of layers and the current employed and, indeed, all the conditions, were left to the discretion of the individual operator. The length of the weld was eight inches. After the welds were made, the plates were cut up and tests were made at the Bureau of Standards under the direction of Prof. H. L. Whittemore. The tensile tests were made on specimens with a cross-section of one in. by one-half in. Specimens with the weld machined have the projecting metal planed off so that the welded portion is smooth and of approximately the same cross-section as the remainder of the specimen. For the specimens with the weld not machined, no record was made of the added cross-section due to the projecting material.

The cold bend specimens were one in. by one-half in. and were bent around a mandrel of 1.5 in. diameter. The results

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TABLE IV
WIRT-JONES TESTS OF ONE-HALF-INCH ARC-WELDED SHIP PLATES
(Machined Samples)

Test number	Current in amperes		Diam. of elec. in in.	Current density, amp. per sq. in.	Tensile tests		Bending	Electrode		
	a-c.	d-c.			Ult. lb. per sq. in.	Per cent. elon. in 2 in.	Angle at which crack starts, degrees	Cov.	Bare	Make
20	175	...	0.125	14,227	62,700	9.0	34	C	..	E.A.C.& W.Co.
1	...	175	0.166	9,118	59,800	9.0	42	..	B	Roebling
22	160	...	0.125	13,008	50,000	6.0	78	C	..	E.A.C.& W.Co.
2	...	155	0.156	8,075	62,600	11.5	44	..	B	Roebling
19	150	...	0.125	12,195	62,800	12.0	32	C	..	Quasi
31	...	150	0.166	6,930	60,900	8.0	..	..	B	Roebling
29	...	150	0 156	7,815	56,400	6.5	..	..	B	W.W.& M.Co.
38	150	...	0.156	7,815	54,500	7.0	45	C	..	E.A.C.& W.Co.
29	...	150	0.156	7,815	54,400	6.0	47	..	B	W.W.& M.Co.
9	...	150	0.156	7,815	44,200	4.0	26	..	B	Toncan
9A	150	...	0.187	5,430	42,500	4.0	30	..	B	Armco
10	...	150	0.156	7,815	41,900	4.5	25	..	B	Toncan
10	...	150	0.156	7,815	41,300	4.5	24	..	B	Toncan
9	...	150	0.156	7,815	38,100	4.0	15	..	B	Toncan
9A	150	...	0.187	5,430	36,300	4.0	..	..	B	Armco
5	...	145	0.156	7,554	61,100	8.5	55	..	B	Am.S.& W.Co.
25	136	...	0.156	7,085	43,700	3.5	26	..	B	Roebling
30	...	135	0.156	7,033	53,200	11.0	..	..	B	W.W.& M.Co.
30	...	135	0.156	7,033	52,000	10.5	50		B	W.W.& M.Co.
37	125	...	0.125	10,162	56,400	7.0	42	C	..	E.A.C.& W.Co.
7	...	120	0.125	9,756	57,600	8.5	50	..	B	Roebling
7	...	120	0.125	9,756	53,700	7.0	45	..	B	Roebling
8	...	115	0.125	9,349	59,400	13.5	60	..	B	Roebling
8	...	115	0.125	9,349	58,200	14.0	..	..	B	Roebling
3	...	115	0.156	5,891	50,600	5.0	34	..	B	Am.S.& W.Co.
16	...	110	0.134	7,799	50,250	7.0	35	C	..	Quasi
27	...	110	0.125	8,943	45,800	8.0	42	..	B	Roebling
21	105	...	0.134	7,444	42,200	5.0	42	C	..	E.A.C.& W.Co.
26	94	...	0.125	7,642	34,200	3.0	15	..	B	Roebling
18	90	...	0.134	6,381	54,000	6.0	65	C	..	Quasi
15	....	90	0.134	6,381	39,400	4.0	30	C	..	Quasi
24		86	0.125	7,154	45,400	6.0	15	..	B	Roebling
23	...	85	0.125	6,930	51,900	4.0	..	..	B	Roebling

PLATE X.
A. I. E. E.
VOL. XXXVIII, 1919

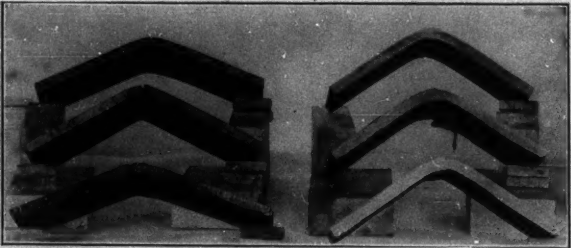


FIG. 8—GAS AND ELECTRIC BENT SAMPLES (GAS WELDS AT LEFT AND ELECTRIC WELDS AT RIGHT)

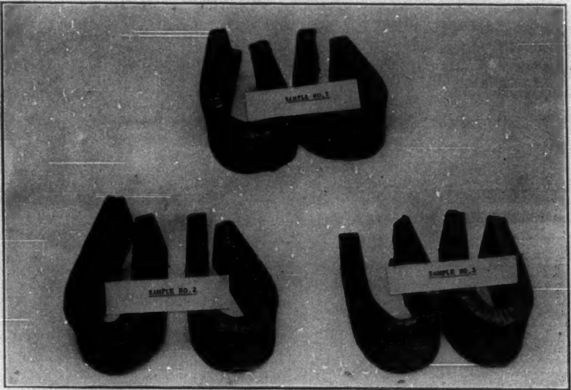

SAMPLE NO. 1
SAMPLE NO. 2
SAMPLE NO. 3

FIG. 18

[HOBART]

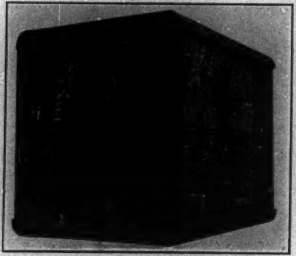
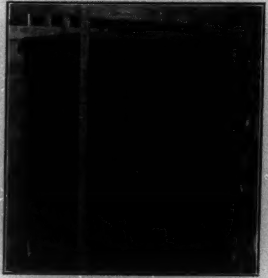


FIG. 20—ELECTRIC-ARC WELDED STEEL BOX OF ONE-HALF INCH TANK STEEL

FIG, 21—ELECTRIC-ARC WELDED BOX UNDER 22-INCH VACUUM—SHOWING TOP SIDE DEPRESSED



[HOBART]

FIG. 22—ELECTRIC-ARC WELDED STEEL BOX UNDER 15 LB. HYDROSTATIC PRESSURE—ONE PHASE OF THE BREATHING TEST

of the tensile and bending tests for the machined samples are given in Table IV. Those for the samples which were not machined are given in Table V.

TABLE V.—WIRT JONES TESTS OF ONE-HALF INCH ARC-WELDED SHIP PLATES

(Samples Not Machined)

Test No.	Current in amperes		Diam. of elec. in inches	Current density, amp. per sq. in.	Tensile tests		Electrode		
	A. C.	D. C.			Ult. lb. per sq. in.	Per cent elon. in 2 in.	Cov.	Bare	Make of Electrode
20	175	...	0.125	14,275	66,480	11.0	C	..	E.A.C.&W.Co.
22	160	...	0.125	13,008	66,400	9.0	C	..	E.A.C.&W.Co.
19	150	...	0.125	12,195	65,400	6.0	C	..	Quasi
5	...	145	0.156	7,554	52,280	7.0	..	B	Am.S.&W.Co.
25	136	...	0.156	7,085	47,800	4.0	..	B	Roebling
3	...	115	0.156	5,891	65,470	13.0	..	B	Roebling
27	...	110	0.125	8,943	48,200	7.0	..	B	Am.S.&W.Co.
16	...	90	0.134	6,381	58,740	5.2	C	..	Quasi
18	90	...	0.134	6,381	61,760	11.0	C	..	Quasi
15	..	90	0.134	6,381	57,340	8.0	C	..	Quasi
23	...	85	0.125	6,910	57,060	5.0	..	B	Roebling

Appendix C

BENDING TESTS OF GAS WELDS

Too late for inclusion in the body of this paper there has been brought to the author's attention some gas welds made under the direction of Mr. H. I. Walsh, which have withstood very severe bending tests before failing. The specimens are shown in the illustration, Fig. 18. Three pairs of specimens welded with three kinds of electrodes are shown. These specimens show much greater ductility than those shown in Fig. 8.

Appendix D

ARC-WELDED STEEL BOX

Mr. R. E. Wagner has prepared the following description of a large steel box, arc welded with bare electrodes at Pittsfield. The box was built and tested in the latter half of 1918. It was made of half-inch-thick tank steel and its dimensions were 12 ft. by 10 ft. by 9 ft. The total weight was 16,000 lb. The welding was done with 75-volt direct current. This box contained eleven different varieties of welds. Its general dimensions and the types of welds used are shown in Fig. 19. No rivets were used in its construction.

The principal objects to be ascertained in the construction of this box were:

1. Could a structure of this character be made close to drawing dimensions and without excessive distortions and warping of plates and parts?
2. Would the structure be strong and capable of withstanding severe shocks and distortions without serious ruptures?
3. What are the detail costs and time required to build such a structure?

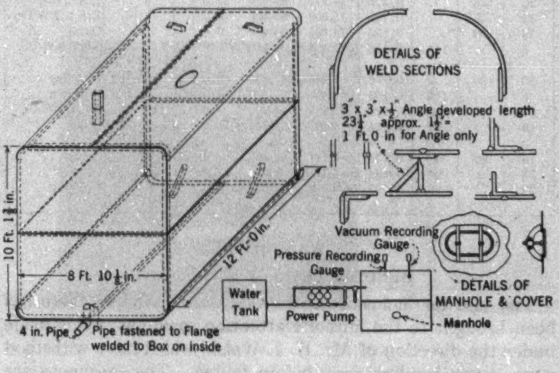

DETAILS OF
WELD SECTIONS
3" x 3" x ½" Angle developed length
23½" approx. 1½" =
1 Ft. 0 in for Angle only
10 Ft. 1½ in.
8 Ft. 10½ in.
12 Ft.-0 in.
Vacuum Recording
Gauge
Pressure Recording
Gauge
Water
Tank
Power Pump
DETAILS OF
MANHOLE & COVER
Manhole
4 in. Pipe
Pipe fastened to Flange
welded to Box on inside

Fig. 19—Outline and Dimensions of Arc-Welded Steel Box

It was found that if proper allowances are made for contraction and expansion, especially where plates are butted and welded, the resulting plate will show very little warping. The maximum variation of the sides from a plane was 1.5 in. The general appearance of the box when completed is shown in Fig. 20.

The box, when finished, was strong and well built. All joints had a smooth workmanlike finish. A number of incipient leaks developed when the box was first filled with water, but these were closed by hammering. The box was subjected to a hydrostatic pressure test of 43 lb. per square inch, and, after it had been severely stressed and distorted, it ruptured at a corner, the break starting at the top and opening the entire corner. This corner was afterward repaired (as shown in Fig. 21), and the box was subjected to a "breathing" test consisting of alternate applications of 15 lb. pressure and 22 in. of vacuum.

During each cycle of this "breathing" test, the top of the tank had a maximum movement of 8 in. from bulge to depression. At the sides the movement was 6 in. Every time that the side, bottom and top plates moved from bulge to depression, there was a sharp snap of the plates as they moved from "arch in" to "arch out." These sudden changes in form subjected the welded joints not only to bending stresses but to severe snapping stresses. At the end of the twelfth cycle a break occurred in a double-bevel weld. At one end the break left the weld and went into the solid plate. This break was patched and the "breathing" tests were resumed. The next break which occurred after the 235th cycle of operation extended 24 in. into the solid end plate as well as 17 in. along a weld. This break will be repaired and the tests will be resumed.

The following cost data are of interest:

Welding	$ 151.00
Fabricating and assembling	134.00
Blacksmithing	24.00
Testing and supervision	50.00
Welding wire	30.00
Steel plate	651.00
Total cost	$1040.00

Total pounds of wire used	334
Total pounds of wire deposited and lost	299
Total scrap ends of wire	35
Total feet of welding	501
Total hours of welding	165
Total kilowatt hours	1011
Per cent scrap ends of wire	11
Wire deposited per foot (pounds)	0.6
Kilowatt hours per pound of wire deposited	3.4
Average feet welded per hour	3.0
Average kilowatt hours per foot of welding	2.0
Per cent. of time welder is welding	60
Diameter of welding wire, inch	$^3/_{16}$

Six different welders obtained the results noted, which are representative for a job of this character.

Presented at the 7th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 19, 1919.

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WELDING AS A PROCESS IN SHIP CONSTRUCTION

BY S. V. GOODALL

Abstract of Paper

The paper points to the necessity for a reduction in the cost of shipbuilding, and as riveting is one of the most expensive items of construction the substitution of welding for riveting would decrease the cost of construction considerably. A brief review is given of what has been done in substituting welding for riveting, and to the limited extent to which electric welding has been tried it has been found successful. Lloyd's Register is prepared to classify electrically-welded vessels subject to certain provisions, but shipbuilders have not as yet adopted welding to a large extent for the reason, in the author's opinion, that they know that welds are lacking in uniformity and it is impossible to tell when a joint is good or bad. The author believes that the correct method of approaching the problem of electric welding is first to obtain workmen who are thoroughly trained and skilled and the suggestion is made that only certified men should be employed at this work. From the welding of minor parts the process could be advanced to more important members until eventually a ship would be produced in which both the processes of welding and riveting would be utilized in whatever proportions would attain maximum economy of construction.

DURING the war, economy of time in the production of all that was needed to terminate hostilities successfully was of paramount importance. Economy in first cost and maintenance was relatively unimportant. If the production of a ship, an aeroplane, a tank or any implement of war could hasten the end by a single day it was more economical, in the long run, to place that implement on service today instead of tomorrow, even at a somewhat higher cost, for the shortening of the struggle by that single day meant a saving of many lives and millions of dollars. But besides being called upon to save time the engineer was forced to economize in labor and to use unskilled workmen. During the war gas welding and electric welding have been enormously developed as processes in the manufacture of munitions of all kinds. It goes without saying, therefore, that in these processes engineers found a means of saving time and labor.

Now that hostilities have ceased, economy in first cost and maintenance becomes once again of primary importance, economy of time and labor being of value only in so far as it results in cheaper production. This is particularly true of shipbuilding. Every shipbuilder should now carefully review the various items of cost in the production of a ship, and while he may devote his energies to a reduction all round, he will, if he be wise, give greatest attention to a reduction in the cost of the most expensive of those items. Riveting is this most expensive item. It has been estimated that the labor cost of riveting is about 40 per cent of the total labor cost of building the structure of a steel cargo carrier, while the labor cost of shopwork on structural material is about 15 to 20 per cent. During the past few years considerable attention has been given to improvements in shopwork and it is unlikely that much economy can be effected in this direction. Can anything be done to lower the cost of riveting? The advocates of welding reply "Yes, by a wide substitution of welding for riveting." This is a claim that should not be lightly placed on one side but carefully and critically examined by all shipbuilders. To offer some slight assistance in that examination this paper has been prepared.

Gas welding has been used in shipyards for many years, resulting in economy in the production of staple angles and similar smith work. It has found a wide field in the construction of light fittings, being more particularly of value for thin work than for heavy plating. As it may be considered that gas welding has now found its level as a shipbuilding process, this subject is not enlarged upon; the little that is said must not be taken as a measure of its importance.

Electric welding as a science and art has been dealt with at this meeting. As a process in shipbuilding much literature on the subject already exists, the most comprehensive papers known to the author being that by Mr. W. S. Abell, Chief Ship Surveyor of Lloyd's Register, read before the British North East Coast Institution of Shipbuilders and Engineers at Newcastle in November, 1918, and that by Mr. H. Jasper Cox, read before the Society of Naval Architects and Marine Engineers at Philadelphia in the same month. The author makes no attempt, even if he were capable of doing so, of giving such detailed particulars of the subjects as can be found in these and other papers, but he does attempt to speak, as a shipbuilder to shipbuilders, plainly and impartially, without extravagant claims, and with some little knowledge of experience so far obtained, on the present position of the process and the policy which should be adopted, looking towards the future.

It is very well known that electric welding has been of great value on repair work, particularly during the war. Ships that would otherwise have been laid up for weeks while new castings were obtained, old plates removed and replaced by new, &c., &c., have been rapidly repaired and returned to service. If the ship repairer finds a process of such value the shipbuilder should at least inquire whether it would not be of some value to him.

The Classification Societies have sanctioned the use of electric welding for a considerable amount of work of little importance so far as structural strength is concerned. The Welding Committee of the Emergency Fleet Corporation has found by actual experience that a labor saving of at least 60 per cent results from the substitution of arc welding for riveting on these minor parts of a ship; and Mr. J. H. Anderton has estimated that a saving in time of 70 per cent was effected at Hog Island.

Lloyd's Register of Shipbuilding is prepared to classify an electrically welded vessel, subject to the notations, "Experimental" and "Electrically Welded," and provided the builders conform to certain rules. This decision was reached after an extensive and exhaustive series of trials. Before any electric welding system can be employed on a vessel which Lloyd's will classify, specimens welded according to that system must satisfy tests, which, at first sight, appear almost prohibitive in their severity, but an electrically welded ship is now under construction at Cammell Laird's, England, to be classified by Lloyd's, who have approved the employment of the Quasi-Arc Company's system in its construction. This is a small vessel about 150 feet long, with plating generally $\frac{3}{8}$ to $\frac{1}{2}$ inch in thickness.

Electric welding work carried out by the British Admiralty has been described in a paper read by the author before the Engineers' Club of Philadelphia, in July, 1918. Since that time valuable experience with this work on service has been obtained. The arc welded barge has been satisfactory. The cost of material, electric current and labor for this vessel was £301 ($1463) as compared with £389 ($1892) for the cost of riveting, caulking and drilling on a similar barge constructed at the same yard. The vessel was designed to be rivetless as a demonstration of arc welding; it was found that greater economy would result if certain parts such as beams to frames, floor plates to angles, etc., were riveted.

As the plating of this barge is thin and the stresses to which it is subjected are not comparable to those which a large seagoing ship is called upon to withstand, greater interest attaches to experience with other British ships in which arc welding has been employed by the Admiralty, particularly to those where the work has been comparatively heavy. It must be said at once that failures of welded joints have been reported, although the number of such failures has not been large considering the amount of work that has been done, the lack of experience, the dearth of skilled operators, and the radical change in method. The Admiralty authorities, however, consider it wise, pending further experience, to defer the adoption of electric welding for those parts of the structure subjected to high complex stresses unless the work can be so tested as to demonstrate exactly that such stresses can be safely carried.

Lap welding has been more satisfactory than butt welding and the latter has been, for the present, discarded for all important work where the plating is $\frac{3}{8}$-inch or more in thickness. It is hoped that this situation will not last long, for it is in butt welding that the greatest gain from the use of this process may be expected, and lap joints necessitate a certain amount of overhead welding which is difficult and unreliable.

While the above briefly sums up the present position of electric welding as a process in shipbuilding it may be fairly asked why—if the Classification Societies permit the process on minor parts; if Lloyd's are prepared, subject to certain conditions to classify an electrically welded ship; if the British Admiralty has employed the process to a considerable extent; if American railroads have used welding extensively on locomotives—why has not electric welding been adopted in American shipyards to a greater extent than it has been up to the present? There is no doubt that during the war, shipbuilders in the United States were too hard pressed to be able to give much time and thought to the development of an entirely new process in ship construction. That is one reason. Inate conservatism is another. But the real reason, in the author's opinion, is that shipbuilders know that, at present, welds are lacking in uniformity, and it is not yet possible to tell when the welded joint is good or bad. Hence they hang back and the author commends their wisdom.

It is far better that the first ocean-going electrically welded ship should be on service three years hence with every joint sufficiently sound—universally accepted as a thorough success—than that it should be at the bottom of the sea tomorrow because, while 999 joints were sound, one went astray.

How can this certainty that every joint will be efficient be ensured? While the electrical engineer and the metallurgist can assist the shipbuilder to solve this problem, there is much that he himself can do.

In the first place he must realize the importance of employing only thoroughly skilled and experienced workmen and he should call in the manufacturers of electric welding apparatus to assist him. The latter, for their own protection and the welfare of the industry on which they rely, should see to it that no ship should be allowed to be built with some of the thoroughly rotten work done by absolutely unskilled welders that the author has seen in this country. It is strongly urged that steps should be taken at once to establish welding as a skilled trade and only certified men should be allowed to be employed. All the reports that have been seen on good and bad welds lay stress upon the fact that, in general, defects are mechanical and not an inherent feature of the process. Skilled men working under an experienced foreman would know, for example, that clean surfaces are necessary for good work, they would quickly learn what types of joint are made with ease, in what portions of a ship welding is cheap or where it is unreliable, and from their ranks should be drawn the draughtsmen who will design in detail the welded vessel of the future. The Electric Welding Committee of the Emergency Fleet Corporation has spent much time and thought on the attempt to discover a practical non-destructive method of testing welds. While such a method of testing would be very valuable, it is the author's opinion, that when only welders of proved skill are employed under experienced supervision, such tests would be no more necessary than a shear test on every rivet of a riveted ship; but, so long as such welders are not available the shipbuilder will look askance at a process that may be good generally but may have weak spots of which he is ignorant.

Secondly, after he has trained a reliable staff of welders on work of minor importance the shipbuilder must devote his attention to the best method of assembly of large parts, so that he may feel reasonably sure that every joint of the finished structure possesses the qualities which his welders can with certainty produce in test pieces. From the very outset it has been realized that this is a difficult problem which can be solved only by patient trial and experience. Present opinion at the British Admiralty is that the only satisfactory method is by the use of closing bolts as for riveted joints. In the case of the Richborough barge a certain amount of buckling resulted on the outboard edge of the garboard strakes after these plates had been welded to the keel plates, but this was removed when the bilge plates were put into position and bolted up. In the heavier work it may be that, buckling of plates being impossible, the internal stresses set up in welding were the causes of the failures that occurred. It is understood that this method is being adopted at Cammell Laird's but, again the plating of this vessel is not very heavy and it is definitely known that this feature received most careful consideration from the very beginning. In the author's opinion this difficulty is by no means insuperable. Its existence should spur on the progressive shipbuilder, for when he has solved the problem, by that much will he be ahead of his competitors.

In the third place it appears to be the fact that shipbuilding steel in Great Britain is more adapted to welding than shipbuilding steel in this country, and the former is also more regular in its qualities. If shipbuilders are satisfied that such is the case and are also convinced that electric welding has a great future in ship construction, they should take concerted action to ensure that steel producers supply the material best suited to their purpose.

So far the author has urged a careful survey of the possibilities of welding as a substitute for riveting only on the ground that the latter is the most costly item of labor on structural work and it behooves the shipbuilder to reduce that cost. But there are other advantages incidental to welding that must not be overlooked. A sound arc-welded joint is necessarily watertight and once such a joint with known efficiency can be made with certainty caulking is eliminated and the work of water-testing made easier. It must be admitted that, so far, the advantages anticipated in this direction have not been fully realized, but that they will be realized is only a matter of time, and it has already been fully demonstrated that welded joints, being stiffer, stand up better under test and there is no breaking away of the caulk such as is sometimes experienced with riveted connections under water pressure. Another advantage is a saving in steel due to the absence of rivet heads and the smaller width of laps and butt straps. Some advocates of welding claim that the labor of marking off, punching and assembling will be much reduced, more particularly by the adoption of spot-welding, but the author is bound to say that at present he is not a convert to the adoption of spot-welding for heavy ship work, it is his opinion that there are many difficulties ahead for those shipbuilders who attempt this form of welding and until those difficulties have been fairly and squarely met he hesitates to affirm that spot-welding will be found a valuable process in ship construction.

In conclusion, it is desired once again to place the subject before the shipbuilder in this light. The cost of ship construction must be reduced. A reduction in riveting is the most fruitful field for economy. Electric welding as a substitute has been tried to a limited extent and found successful. Classification Societies are prepared to accept the process, though at present the only vessels built or under construction are small. There are difficulties ahead but they do not appear insuperable. The best line of approach appears to be to build up a staff of thoroughly skilled welders, to gain experience by the adoption of welding on minor parts, to proceed cautiously in the extension of the process to more important members, not to expect a great saving in time and cost immediately, but to persevere, not necessarily towards the rivetless ship but towards a vessel in which the employment of both processes of riveting and welding is so adjusted that more riveting or more welding could only be done at greater cost.

Presented at the 7th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 19, 1919.

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FUSION IN ARC WELDING

BY O. H. ESCHHOLZ

ABSTRACT OF PAPER

Although welding literature is replete with discussions of applications, little information is available descriptive either of non-destructive inspection methods or of the process of metallic electrode arc welding. This paper calls attention to such characteristics as penetration and overlap, peculiar to this process, which facilitate visual inspection and discusses briefly the effect of arc length, welding procedure, electrode material, arc current and electrode diameter upon these characteristics.

THE physical characteristics of a weld are admittedly determined chiefly by the degree of fusion secured between adjacent metals. In the production of welds by the metallic electrode process, a fair estimate of the fusion may be obtained by an examination of the surface of the deposited metal and the depth of the arc crater formed when welding at normal speed. These observations give an adequate indication of the two prime characteristics of fusion—penetration and over-lap —which, however, are definitely revealed only on exposing a section through the zone fusion.

Fig. 1 illustrates the union between a layer of metal deposited from a low carbon steel electrode upon a low carbon steel plate. The line of demarcation between the plate and deposit indicates the zone of fusion, the polished and etched surface of the deposited material presenting a different appearance, due to its crystalline structure, from that of the plate material. Two characteristics are at once apparent; a penetration of the deposited metal into the plate metal over almost the entire width of the layer, and a slight overlapping upon the plates without fusion of the extreme edges of the deposit. The penetration is due to liquefying of the plate metal by the energy absorbed from the arc as well as that absorbed from the molten deposit.

At the center of the deposit, where both effects are present, the penetration is greatest; at the extreme edges, however, where the deposit has overflowed the area included by the arc crater, the energy liberated on cooling is insufficient to melt the plate metal and the deposit congeals without fusing to it. It is typical of such overlapping that the edges of the deposit are well rounded, a distinct gap often existing between the plate and edge of deposit. The convex surface of the deposit thus forms a re-entrant angle with the plate edge. To insure satisfactory fusion a proper degree of penetration should be obtained with no overlap.

PENETRATION

Fig. 2 illustrates a section through the crater obtained on breaking an arc during the process of depositing a layer of metal on a steel plate. It may be observed that the depth of the crater depression is comparable to the depth of penetration of the deposited metal below the plate surface. A simple visual indication is, therefore, available during the welding operation for gaging the penetration.

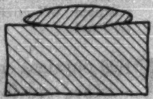

FIG. 4 — EXTREME OVERLAP OBTAINED ON WELDING WITH AN ELECTRODE OF LOWER MELTING POINT THAN THAT OF THE PLATE METAL

OVERLAP

The convex edges of the deposit shown in Fig. 2, particularly evident at the right of each view, would lead one to expect an appreciable overlap. This is disclosed on a careful inspection of the sections. Fig. 3 illustrates a surface contour with no overlap. This condition is usually typified by the absence of even the smallest re-entrant angle, an intimate contact between plate and deposit being secured, the junction between the two metals often forming a concave surface.

A condition of extreme overlap is shown in Fig. 4. This is typical of a low melting point electrode or of too great speed of arc travel. The convex deposit edge and the re-entrant angle between it and the plate are pronounced.

ARC LENGTH

The effect on fusion of arc length has been discussed at considerable length by welding engineers. Some assert that better fusion is secured for a given arc current with a long arc than with a short arc, due to the greater amount of energy

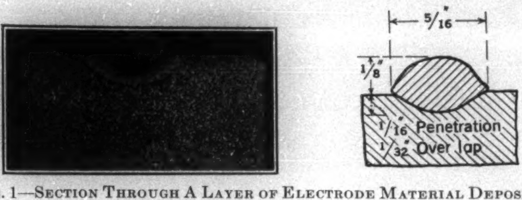

Fig. 1—Section Through A Layer of Electrode Material Deposited on a Mild Steel Plate—When Using a Short Arc and a Proper Welding Current—Showing Degree of Penetration and Extent of Overlap

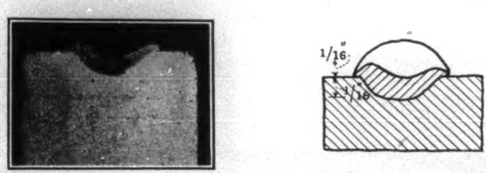

Fig. 2—Section Through the Crater Remaining on Interrupting Arc Current Showing the Comparable Depth of Crater Depression and Deposit Penetration

Fig. 3—Absence of Overlap When Depositing Metal of a High Melting Point on a Plate of Low Melting Point

Fig. 5—Characteristic Section of Deposits Obtained with a Long Arc—Indicating Excessive Overlap with Practically no Penetration

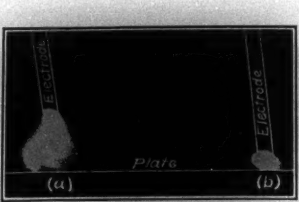

Fig. 6

A—Illustrating the Exposure of New Metal Resulting on Holding a Long Arc and the Protection Obtained

B—From the Enveloping Gases with a Short Arc

[ESCHHOLZ]

Fig. 9—Section through the Center of a Deposited Layer in the Direction of Forming Deposit—Showing Degree of Penetration with no Unfused Areas

absorbed by the arc; others have stated that the degree of fusion is dependent solely on the arc current and, of course, the thermal characteristics of the metal welded. For a given arc current it has been the author's experience that fusion is not improved with increase in arc length, that the best weld characteristics are obtained when holding a short arc, not longer than 1/8 in. and preferably about 1/16 in. With a given arc length the ease of securing complete fusion increases with arc current until a maximum value is reached beyond which further increase results in a weakened weld structure.

Figs. 1 and 5 are typical of sections through deposits formed with a short and a long arc, respectively, the arc current being approximately the same in each instance. With a short arc the plate metal is liquefied by the energy required to maintain the arc crater, by that conducted from the arc flame in constant intimate contact with the plate and by the energy radiated from the arc stream and electrode terminals, as well as by that absorbed from the molten electrode deposit. When the molten material passes through a short arc it is quite completely enveloped by hot gases and suffers but a slight loss in temperature before being deposited. Upon drawing a long arc the air currents in its vicinity blow the arc stream and arc flame about violently, so that it is difficult to maintain the plate terminal of the arc in a given position long enough to effect sufficient penetration. Moreover, the energy absorbed from the arc stream and electrode terminal becomes practically negligible, due to the greater diffusion of the radiation in the region of the arc plate terminal, while the energy absorbed from the molten metal is decreased because of the loss in temperature the deposit has sustained in transition. Some conception of the appearance of short and long arc lengths may be obtained from Fig. 6.

WELDING PROCEDURE

The extreme condition of no overlap is difficult to secure when depositing a single layer of electrode metal upon the usual low-carbon steel plate. For most purposes a slight overlap, such as is indicated in Fig. 1, is permissible in single layers as a simple procedure of depositing the subsequent layers will fuse the overlapped areas.

Fig. 7 illustrates the characteristic penetration and overlap when parallel abutting layers are formed. The minimum overlap for this condition will approach 1/16 in. per layer. If the width of a layer is increased by weaving the electrode back and forth the percentage overlap will be decreased. However, it may be eliminated by the simple expedient of always overlapping adjacent layers as shown in Fig. 8. On forming the second tier of deposits the same precaution should be taken to prevent unfused zones. Fig. 9 shows a typical section through the center of a layer. It may be noted that although the depth of penetration varies somewhat, due to the speed of arc travel and unavoidable variation in arc length, good fusion has been secured between the plate and deposit along the entire length of the deposit.

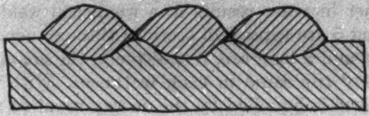

FIG. 7—FUSION OBTAINED ON ABUTTING PARALLEL DEPOSITS

Since the choice of electrode material, arc current and electrode diameter largely determine, with a given manipulative skill and welding procedure, the depth of arc crater and deposit contour, a brief discussion follows of such characteristics which affect fusion between the deposit and plate metals.

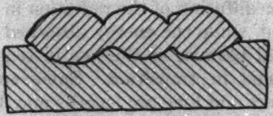

FIG. 8—FUSION SECURED WHEN PARALLEL DEPOSITS ARE OVERLAPPED

ELECTRODE MATERIAL

To minimize the occurrence of unfused areas, the melting point of the electrode metal should exceed that of the plate metal. Fig. 1 illustrates the fusion obtained with the melting points approximately equal, (0.18 per cent carbon, 1510 deg. cent. approx.), Fig. 3 that secured with a greater melting point for the electrode (0.18 per cent carbon, 1510 deg. cent. approx.) than for the plate (3 per cent carbon, 1280 deg. cent. approx.)

and Fig. 4 that obtained with a lower electrode melting point (1.1 per cent carbon, 1440 deg. cent. approx.) The melting point of most weldable steels may be gaged with sufficient accuracy for welding purposes by the carbon content as shown in Fig. 10. The usual additions of other alloying ingredients reduce the melting point a negligible amount.

While pure iron possesses the highest melting point, its use is not advisable since excessive oxidation of the vaporized metal occurs. However, a commercially pure iron alloyed with about 0.18 per cent carbon and 0.5 per cent manganese greatly reduces the oxidation, and improves the ease of manipulation without an appreciable sacrifice in melting point. For most purposes the use of this electrode gives a reasonable minimum overlap.

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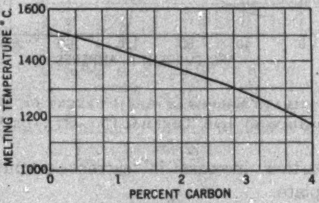


Fig. 10—Change in Melting Point of Iron Alloyed with Various Amounts of Carbon

Under exceptional conditions such as high thermal capacity of weld, or overhead welding with a low current, the overlap may be excessive even when using this electrode, it is customary then to preheat the object to be welded or to adopt a welding procedure which utilizes the thermal energy from preliminary welding operations to raise the temperature of the object welded.

Arc Current

For a given plate thickness the welding current will vary with the type of weld scarf, cleanliness of surface, thermal capacity, and conductivity of the metal, and position of plate with respect to the welder. The values shown in Fig. 11 apply only to clean, butt-welded plates, single V, scarfed to 90 deg., supported in a horizontal position, below welder, in poor thermal contact with any metal of high thermal capacity. The fusion obtained with too low and too high arc currents is shown in Fig. 12. In Fig. 12A the overlap is excessive and the penetration unsatisfactory because of the small amount of energy applied at the arc crater. In Fig. 12B the excessive energy consumption at the plate arc terminal produces excessive penetration, a further increase in current permitting cutting through the plate.

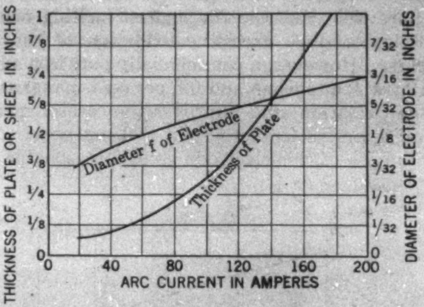
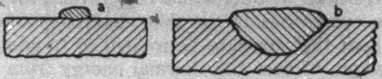


Fig. 11—Approximate Values of Arc Current on Welding Under Conditions of Low Thermal Capacity for Weld

Fig. 13 indicates approximately the effect of variation of arc current on fusion and therefore on weld strength for (a) skilled and (b) unskilled welders. Weld structures are occasionally encountered in which variations in thermal capacity and plate temperature necessitate changes in arc current if complete fusion is to be secured. Observations of the deposit contour and crater depth usually serve to indicate to the operator when such changes are advisable.



A–Fusion Secured with too Low an Arc Current–Excessive Overlap
B–With too High an Arc Current—Excessive Penetration
Fig. 12

Electrode Diameter

A close adjustment of electrode diameter to arc current is not essential. For miscellaneous direct-current welding the densities usually fall within a range of 7000 to 10,000 amperes per square inch, if an 0.18 per cent carbon steel electrode is used. However, whenever practicable, the author prefers a closer range of 8000 to 9000 amperes per square inch. With too great an electrode diameter for a given current, the arc becomes more unstable, increasing the manipulative skill required and decreasing the rate of depositing electrode material. Too small a diameter causes a deposition of electrode material before the plate has been properly liquefied; it also results in overheating and softening of electrode which increases the skill required to prevent the electrode from "freezing" to the plate. With the correct diameter the electrode material is sprayed in a regular stream at a rate which will insure maximum welding speed consistent with good fusion.

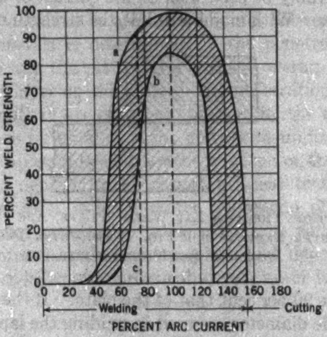


Fig. 13—Approximate Change in Weld Strength with Change in Arc Current, for
A—Skilled, and B—Unskilled Welders

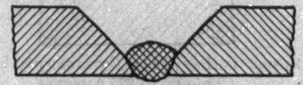


Fig. 14—Section Through Butt Welded Plate Showing Good Fusion at 150 Arc Amperes

Example. The characteristics of fusion and the effect of a

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control of the conditions which govern them may be summarized in comparing the procedure for making butt and lap welds.

In butt welding ½-in. low-carbon steel plates, scarfed as shown in Fig. 14, a $^{5}/_{32}$-in. diameter welding electrode containing approximately 0.18 per cent carbon, 0.5 per cent manganese, would preferably be used with an arc current of approximately 150 amperes. With moderate skill, the strength of weld should approximate 90 per cent that of the plate. If the same current and electrode diameter are used in welding the lap joint formed from ½ in. plates shown in Fig. 15, poor fusion will be obtained owing to the increased thermal capacity of the abutting sections. As shown in Fig. 15 the penetration will be too slight and the overlap excessive, similar to the case shown in Fig. 12. In fact, sections through completed lap joints most invariably show that in forming the first deposit, which is the most important, even expert welders tend to penetrate only one side of the weld scarf with resultant overlap on the other. Breakdown tests of lap joints have plainly shown failure due to this cause.

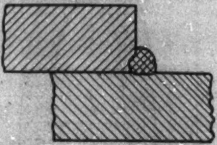

Fig. 15—Section Through Lap Weld—Plate, Arc Current, Electrode Diameter Same as for Fig. 11—Showing Poor Fusion due to Increased Thermal Capacity

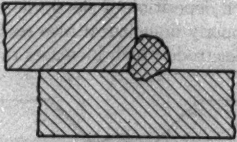

Fig. 16—Good Fusion in Lap Weld Obtained on Increasing Arc Current to 225 Amperes

To secure complete fusion on both faces of the joint it is necessary to increase the arc current and with it the electrode diameter. For the condition shown in Fig. 16 an arc current of 225 amperes and an electrode diameter of $\frac{3}{16}$ in. was found to produce complete fusion, facilitate electrode manipulation and increase rate of electrode deposition.

CONCLUSION

The fusion obtained in arc-welded joints between the plate and deposit metals is determined by the penetration of the deposit metal into the plate metal and the overlap of the edge of the deposit layer on the plate. The penetration is comparable to the depth of arc crater as observed by the welder when moving the arc terminal over the plate with uniform velocity, and the overlap is indicated by the contour of the surface of the congealed deposit in the region adjacent to the material welded. For most welds a crater depression of $\frac{1}{16}$ in. will give the desired penetration while a contour which does not show a re-entrant angle between plate and deposit indicates an overlap per layer of approximately $^1/_{32}$ in. or less.

DISCUSSION ON "WELDING MILD STEEL" (HOBART), "ELECTRIC WELDING IN SHIPBUILDING" (GOODALL) AND "FUSION IN ARC WELDING" (ESCHHOLZ), NEW YORK, N. Y., FEBRUARY 19, 1919.

W. H. Hill: We have fully sympathized with Commander Goodall in his conception of the dangers of using welding in making the joints on ships; and the picture that he gives, with the possibility of one weld in a thousand failing in a stormy sea, perhaps isn't a very encouraging one.

We have been developing another form of ship, in which we do not have to use welding for the tensile strength joints, but which is a welded ship nevertheless. The ship is built in cast studs. It is a system of ship building which is even more rapid than that of the roll plate ship, and it has joints which upon investigation seem perfect. The joints are of that kind which are necessary for ships, before welded ships will be generally satisfactory for carrying.

There is one other point I would like to bring to your attention in connection with the strength of welds, particularly with regard to the filament in crystals. This point has been developed in connection with electric furnace work. In making our castings for ship plates I have been fortunate to come in contact with Mr. Snyder, and he has developed a furnace during the last year or two which melts steel in the absence of oxygen; that is, the furnace is so tight, the air has no chance of getting in. The pressure inside is slightly in excess of that outside, so that the gases come out rather than go in.

Now, he found something very interesting in testing the castings made from such steel. The tensile strength is increased, the ductility is increased and the elastic limit, and his explanation of it is the fact that the film is reduced in thickness and thereby permits a stronger adhesion of one grain to the other.

I am in harmony with Mr. Eschholz' suggestion of less contact with the air. It is in that direction that we have to look for greater strength in welds.

H. A. Hornor: Commander Goodall was extremely wise in bringing out the requirements for a skilled welder. After months of investigation of arc welding, we have found undoubtedly that the crux of the situation rests in the skilled welder. How are you going to get the skilled welder? It might be of interest to know what has been the experience of the Fleet Corporation regarding this question.

The system introduced by the Fleet Corporation permitted the shipbuilder to select the men whom he wished trained as skilled welders. You can see, therefore, that we met with a miscellaneous type.

Now, I wish to put myself on record as saying that anyone who undertakes to drill or to use the laboring type of man on arc welding might just as well forget electric welding. I am merely giving you this as a belief from an experience of eight months in this connection.

The Fleet Corporation gave out as an award a certificate as soon as a man had completed his course, and I think the proportion of certificates that were issued, Mr. President, were very, very few. It was utterly impossible to drill the majority of the men who were sent to the school for training.

To be sure, our training had to be intensive. The shipbuilder would naturally complain if we couldn't return a man to him within a reasonable course of time. We, therefore, cut our courses as short as possible. We didn't put limits, however, upon the course. There were some remarkable records made, but those remarkable records were never made by the laboring type of man, and the laboring type of man, I am sure, returned from the school as inadequate to do arc welding as when he came in.

I believe that a great many men have blamed the arc welding process quite improperly, due to the fact that we were endeavoring to make the application in a most improper way. If the conditions are not fitted for a special job, a skilled operator will stop and I doubt very much whether you will ever find a skilled operator who can make a perfect arc welded joint when he is being annoyed by things flying over in his vicinity, etc. I assure you a skilled arc welder has to concentrate with all his senses absolutely on the weld. Give him what he requires and he will return to you a proper weld, one that is good and sure. In the majority of cases where a skilled operator can make a good weld, I think the researches of this Committee will prove that when you test that weld, you will have a difficult time to break it.

C. A. Adams: It may interest you to know that there is pretty good evidence that the prejudice of riveters against arc welding, a very natural one, has caused intentional disturbance of the arc welders to the extent of throwing hot things in their vicinity.

Captain Corbett: Why cannot the ship men and the steel manufacturers get together and devise some type of plate with an interlocking shape, which will give them a much higher factor of safety?

W. H. Hill: That is exactly what the cast steel ship is doing.

Captain Corbett: My suggestion differs from that of Mr. Hill in this respect: If I understand him correctly, his plan consists in having cast steel plates with interlocking edges. My suggestion was to use our ordinary rolled steel plates with the ends cut in such a manner that they would dovetail together. Then by means of buck welding, the stresses will be distributed more efficiently by being in different lines.

A. M. Candy: According to the tables at the end of Section 5 of Mr. Hobart's paper the deposit obtained with

Roebling cold-rolled steel, and Toncan electrodes contained considerably less carbon and manganese than the original electrodes; whereas very little carbon is lost in the case of the hot-rolled steel electrode. When using the hot-rolled steel electrodes, the metal apparently has a tendency, probably due to greater surface tension, to pass through the arc in larger particles than the other electrode materials. This affords greater protection against oxidation, and thereby prevents much loss of carbon. The larger globules tend to increase the arc fluctuations due to greater partial short-circuiting of the arc. Actual oscillographic records show that with the hot-rolled steel electrodes the current peaks were 70 per cent above normal whereas the peaks for the other electrodes were only about 25 per cent above normal. Probably due to the larger globules, a greater proportion of slag in the electrode was carried into the weld. Some four or five test pieces made with the hot-rolled steel electrodes were all considerably below the tensile strength of the samples welded with the other electrodes. These facts indicate that an electrode may not be satisfactory even though of approximately the proper chemical composition. For example, the hot-rolled steel and Roebling electrodes are quite similar in chemical analysis.

SPEED OBTAINED IN ARC-WELDING SHIP PLATES

Test No.	Speed, in feet per hour	Current used	
		Amperes	Cycle
1	3.33	170–180	..
2	2.86	150–160	..
19	1.90	150	60
20	1.48	175	60
21	1.59	105	25
22	0.97	160	25

In Section 10, the relative speeds of alternating and direct-current arc welding are mentioned. By referring to Table IV we find that tests 1 and 2 were made with direct current, that tests 19, 20, 21 and 22 were made with alternating current. The speeds of welding are shown in the accompanying table. Of course these are isolated cases, but it is interesting to see what is obtained from actual tests.

In Section 19, Mr. Hobart has brought out the question of suitable current values. This is probably one of the most important and least understood problems in the entire arc-welding category. In fact, it is the problem of proper fusion in arc welding, which can only be secured by the use of the proper current volumes, depending on the characteristics and temperature of the material to be welded, and by the use of the proper diameter of electrodes for the current. A visual inspection of the deposited metal will almost unfailingly indicate when the desired effects are obtained. The method of determination is very ably and clearly pointed out in Mr. Eschholz's paper; namely, judging the penetration metal by the depth of the arc craters, measured from a straight line across the crater top to the bottom of the crater, and an examination of the edge of the deposits for re-entrant angles, to see how much of an angle there is.

Mr. Hobart also made reference to the effect on arc welding of voltage employed. I do not believe that the question of voltage is quite as simple as inferred, unless coated or wrapped metal electrodes or carbon electrodes are used. We will grant that for most open work an experienced conscientious welder can produce results with circuit potentials as high as 250 volts; however, an inexperienced or a careless operator will follow the course of least effort and maintain a long arc, resulting in an unconcentrated porous deposit, which will be poorly fused and does not penetrate into the shank metal. An inexperienced operator certainly ought not be to trusted with such high open-circuit voltages, as he is sure to produce unsatisfactory welds.

C. J. Holslag: Speed in welding may be advantageous or disadvantageous according as to whether the metal is deposited faster than the operator can make sure it is being deposited in the crater made fluid by the arc, or whether some of the metal is getting ahead or outside of the crater. Speed at which good welding can be done is an advantage and alternating current provides a greater speed than direct-current welding with the electrode negative, because with the electrode negative approximately 60 per cent of the heat is in the work, and with alternating current the heat is naturally evenly divided between the electrode and work.

Direct current with the electrode positive provides faster welding, but generally too fast to be of any use, as mentioned, unless the arc is closed in by a very heavy coating so that the metal is enclosed in a viscous sleeve, or where the work is very rough, such as filling in with a large electrode on a casting where strength is not required. In a test recently made with direct-current and alternating-current machines, the rate of melting was taken; that is, the number of pounds per hour that could be melted with no stops was determined. In each case under the same conditions, *i. e.*, different currents and voltages were taken, but comparisons made only when they were the same, alternating-current melting was from 20 to 30 per cent faster. For instance, at 150 amperes and 25 volts, 4 lb. (1.8 kg.) an hr. were deposited with alternating current and 3 lb. (1.3 kg.) an hr. with direct current; and at 175 amp. and 25 volts, 5 lb. (2.2 kg.) an hr. with alternating current and 4 lb. (1.8 kg.) an hr. with direct current. With a 5/32-in. (3.9 mm.) electrode, from 4 to 5 lb. an hr. is the maximum amount that can be deposited in electric arc welding, for with higher current densities good welding is lost.

Speed also depends on the length of the arc because the rate of melting depends on the rate of heating and the rate of heating naturally depends on the voltage across the arc, as well as the current through it. With a long arc, this rate can be raised, and hence the welding speeded up; but as pointed out in Mr. Eschholz's paper, everything is in favor of a short arc for good welding, *i. e.*, less air radiation, less contact with the air for oxidation and forming of nitrides, greater chance that metal of the electrode will deposit in the crater and nowhere else. With alternating-current apparatus, the greater possibilities for automatically holding a short arc, and constant rate of heating are generally taken advantage of. Hence any test of speed must be accompanied by voltage across the arc and amperes through it, which means by definition, the same rate of heating; and with the same rate of heating, the only factor left is the disposition of the heat at the positive and negative electrodes, for as explained before, the amount of heat at the electrode end determines the rate of its melting.

W. L. Merrill: Mr. Hornor touched on the training of arc welders. Perhaps when you realize that all of the Liberty motors had some kind of welding, either arc or acetylene, that for about twelve years locomotive frames have been repaired by arc welding, and that the majority of these operators have been workmen, you will recognize that his statement should have been qualified.

These men haven't been particularly chosen, but were men who have been picked for that trade and trained for it, and I wish to record the statement that with suitable training and instruction, it is my belief that any good, live, honest working man, be he a boiler maker, or a mechanic of any kind, can be trained to be a successful welder. The evidence of this is noted above. Also, the particular man who is doing the welding, and who is the most insistent welder that this Research Committee has ever found, is a working man.

Until about two years ago, spot welding was limited to very thin sections, say up to ¼ or ⅜ in. (6.3 or 9.5 mm.) thick; beyond that the field had not been thoroughly explored. It was thought that it might be advantageous to use this method in the construction of ships, so tests were made and a machine built, the results of which have been published in Capt. Caldwell's report and in one issue of the *General Electric Review*, which has been referred to by Mr. Hobart. As a result of those tests, it was found possible economically to build machinery capable of welding two or more pieces of structural steel up to a total of 3 in. (7.6 cm.) that being the maximum thickness we tried to weld.

Unlike arc welding, spot welding has very few variables; and a surface inspection in practically all cases is all that is necessary to determine a 100 per cent weld. Also, like the arc weld that we have been hearing about today, spot welding can be done to more than 100 per cent strength of the plate. As an example, Section 29 of Mr. Hobart's paper contains some test samples that were made up of different kinds of welding. The two-spot weldings, which would be used for these thicknesses of plates, gave a breaking strength of 28,000 lb. (12,700 kg.) as against 13,000 lb. (5896 kg.) for the standard riveting for that same plate.

As a result of experiments carried on in the large machine, which by the way was 2000-kv-a. capacity, although we did not reach the capacity of the machine, three separate and distinct machines were designed for use in fabricating of ships; that is work to be done in the shop, not on the ships. Pictures and explanations of these are given in Mr. Hobart's paper. These machines have been completed, and two of them are in the possession of the Emergency Fleet Corporation at the present time, working out the practicability of adapting them to the fabrication of ship parts in place of laying out, drilling, and riveting. Only experience will tell what saving or gain, if any, there will be in the use of this type of machinery. One thing, however, we know can be gained, and that is for a given strength of a given member, the joints will not be limiting features, and if 60 per cent is figured for riveted joints, the thickness of the plate can be reduced proportionately because you are always sure of 100 or more per cent strength if spot welding is done with proper care.

D. C. Alexander, Jr.: In mining work, manganese steel is used for rails, crossovers, and, I believe in some cases, wheels on mining cars. Now, for that sort of work it has been customary, when the manganese steel parts wore out, to scrap them because there seemed to be no way of repairing them. A covered electrode, if it is properly covered, will deposit the slag on the molten metal so that the constituents of the original metal can be retained in the positive metal. So by starting with a high manganese-steel wire and putting the suitable covering on that wire, it is possible to deposit on a plate, or manganese specimen of some sort, 12 per cent manganese steel. That in itself justifies the covered electrode for that particular kind of work. I imagine mining engineers may have occasion to build up worn parts composed of high-carbon steel. The same effect can be produced by using a 0.7 per cent carbon steel wire, and depositing 0.5 per cent carbon steel, to build up carbon steel parts.

The remarks made this afternoon about covered electrodes have been a little bit vague in some cases, and possibly some of you have the idea that any kind of covering on an electrode produces the same result. I notice in Mr. Hobart's paper that, in some instances where it did not give satisfactory results, a great improvement could be made by dipping the wire into the melted iron. That may be correct, and some people may think they will get a big improvement in that way. But when you think about it, you must realize that in steel-furnace practise you must have suitable slide, you can not have any kind of slide; similarly the material used as electrodes must not be selected because it is cheap or common, but because it is the only one found to contain the proper constituents. What we use is blue asbestos, which is not like white asbestos at all. The coating fuses with the steel and covers the molten steel with a thin slag, which keeps out the oxygen and prevents the formation of oxide. It also maintains in the molten metal the constituents of the steel that we want to keep there.

W. Spraragen: The angle referred to in Table III of Mr. Hobart's paper is the angle on each plate, therefore the total opening in each weld will be just double that given. For instance, the total opening for the 15 deg. bevel listed in this table would be 30 deg. The statement is in connection with Section 24, and Mr. Hobart states that no physical test has been made on these samples. I have, however, received from the Electric Testing Laboratories some results of tensile and bending tests on these specimens; each result given below is an average of three test specimens.

Angle of bevel on plates, in degrees	Total opening, in degrees	Ultimate strength, in lb. per sq. in.	Angle of bevel at which crack starts, in degrees
15	30	44,000	9
30	60	51,000	15
45	90	45,300	13
60	120	48,800	14

These results seem to indicate that the advantages claimed by the American traditions for the very large opening are rather questionable if we also take into account the time, electrode material, and power saved by the smaller angle of bevel, as shown in Table III. Obviously one could have too small an opening so that the operator would not have sufficient room in which to properly manipulate his electrode. The results of the above test would indicate that this point was reached when the total opening was 30 deg.

J. C. Armor: (communicated after adjournment). Mr. Hobart has presented a comprehensive list of points that are of vital interest in fusion welding. In connection with his 4th point of the summary directly preceding Section I, "Preferable Composition for Bare Welding Wires," would say that in alternating-current arc welding of steel plates with bare electrodes good sound welds can be produced with wire that is composed of practically pure iron alloyed with from 0.02 to 0.65 per cent manganese. The brands of welding wire that contain 0.30 to 0.60 per cent manganese will generally produce the strongest welds, while those carrying a very small amount of manganese will, in general, produce the softest welds. I have not had much success with wires carrying more than 0.65 per cent manganese or with alloying elements other than manganese.

In connection with the 7th, 8th, 10th, and 11th points, would say that there is very little, if any, difference in the quality of work done with alternating current and direct current, provided, of course, that proper conditions are maintained in both cases. It is as easy to work with alternating current as with direct current, provided the alternating-current "striking" voltage is sufficiently high. Any commercial frequency is suitable for arc welding if sufficient open-circuit voltage is employed. The lower frequencies require somewhat higher voltages. My experience indicates that the speed of welding is about the same with alternating current and direct current. An operator who is experienced in direct-current welding usually adjusts his alternating current to a value that allows him to manipulate the fused metal in the way he has always done with direct current, with the result that he melts in about the same quantity in a given time.

In connection with Mr. Hobart's 20th point," Effect on Arc Welding of Voltage Employed," it is, of course, a well-known fact that the alternating-current arc is much more easily extinguished than the direct-current arc and also that the lower the frequency the more easily is the arc interrupted. These facts must be taken into account in welding with alternating current. The general tendency has been to employ alternating potentials of too low a value for effective work, with the result that the operator cannot manipulate his arc properly and therefore has difficulty in producing good work. This question of open-circuit potential is extremely important; and if it is desired to be free to use all welding wires now on the market, the "striking" voltage for 60 cycles should be at least 150 volts. Good welds can be made with less voltage when using certain brands of wire but other brands will require 150 volts. When the voltage is too low, there is a very strong tendency to hold the arc between the electrode and the top, or outside, of the weld, with the result that the metal at the bottom of the weld is not fused in as well as it should be. Arc welding, even under the best possible conditions, is very trying to a conscientious operator and for that reason the work should be done in the easiest practicable manner, as it is impossible for an operator to do uniformly good work if he is under a continuous strain. While an expert may make a few very good test welds, using a potential so low that he has the greatest difficulty in working, he will not be willing to work that way day after day; and if required to do so he will tend to become discontented and careless. For the reason just given, the alternating-current potential employed for most commercial wires now on the market should, in the writer's opinion, be about 140 or 150 volts.

In regard to Mr. Hobart's 28th point, "Consequences of Different Lengths of Arc," the writer's experience has been that the shortest practicable arc produces the best welds. A long arc, on the other hand, deposits inferior and weak metal in the weld. No doubt this is due to the absorption of larger amounts of oxygen and nitrogen from the air. The question of arc length is certainly as important as the electrode composition and the secret of good welding with most of the commercial wires now on the market is found in the maintenance of a very short arc manipulated so that the deposited metal fuses into the plate over the whole surface to be welded.

S. V. Goodall: (by letter). Mr. Hobart omits to refer to the investigations carried out by the Welding Research Sub-committee to ascertain whether any trouble is to be expected from excessive corrosion of a welded joint exposed to sea water. The point was raised, early in 1918, that on account of the difference in chemical composition and electrolytic characteristics of mild-steel ship plate and the metal laid on at the weld, it is possible that excessive corrosion would set in at the joint if the protective and anti-fouling compositions were accidentally removed or otherwise rendered ineffective, and the joint might fail in consequence. As welding had already been employed for effecting repairs to under-water parts of ships, the Welding Research Sub-committee instituted inquiries to ascertain whether excessive corrosion had been experienced in these cases; and while few replies were received, owing to the fact that the history of the weld after the repair was effected had not been kept, no actual cases were cited where excessive corrosion had occurred.

The British Admiralty and the U. S. Navy Department are carefully watching the effect of sea water on welded joints, and under the superintendence of Commander H. G. Knox of the U. S. Construction Corps, a series of tests on the corrosion of welded joints made with various types of electrodes is being carried out at the Norfolk Navy Yard. The Bureau of Standards is also investigating this matter. So far the experience at present available and the opinions obtained lead to the conclusion that no danger need be feared from excessive corrosion of a weld on service; further experience, however, is considered necessary before this question can be regarded as definitely settled.

With regard to the application of spot welding to shipbuilding, Mr. Hobart states that it is planned that bulkheads, frames, floors, and other parts shall be spot-welded and then transported to their places in the ship. This sounds attractive; but the shipbuilder will at once ask how the cost of such spot welding compares with that of riveting. The riveting of parts such as those mentioned can be done in shops or sheds, and is comparatively inexpensive. If it is to be displaced by spot welding, this latter process must be considerably cheaper than I understand it to be; and if Mr. Hobart could add particulars as to the relative cost of spot welding and riveting such members, it would add to the value of this part of the paper. With regard to the adoption of spot welding for work at the ship, there are many difficulties ahead for those shipbuilders who attempt this work; this opinion is based on experience with portable hydraulic riveters in ship construction and is altogether apart from the practicability, or otherwise, of efficiently connecting two heavy plates by spot welding. Even in this latter respect it must be remembered that there is considerable difference between the spot welding of two small thick plane plates in a shop and the the spot welding of two large nonplanar plates forming the complex structure of a ship. Moreover, spot welding, compared with line welding, possesses the disadvantage that economy of steel is not likely to be so great.

Mr. Hobart, in presenting the paper, rather ineffectively trailed the coat-tails of the bare-wire advocates in the face of those who believe in covered electrodes. In Great Britain, shipbuilders and engineers are not paying the additional price of covered electrodes for philanthropic reasons; as soon as they are convinced that the bare electrode will give as good and as uniform results as the more costly covered wire, Mr. Hobart may rest assured that the bare electrode will be freely used in that country. But I am bound to say that present experience in Great Britain indicates that the covered electrode gives superior results and gives such results more uniformly, the superiority being particularly indicated in fatigue tests, which is an extremely important matter if welding is to be employed on parts of ships contributing to general structural strength. Mr. Hobart states that one type of electrode must yield results uniformly superior to those obtained with another type in order to afford economic justification for a five times greater price. It is not considered that this matter can be summed up in this manner in regard to shipbuilding if electric welding is employed on such parts that the resulting joint is of importance in the general structural strength. If, by using a high-priced electrode, the desired minimum standard of strength can be always attained with certainty, while the lower priced electrode can not be relied on to attain such a standard at all times, the shipbuilder would not be justified in using the cheaper article, unless it were possible to subject all the joints to some test that would imitate in quantity and quality the forces that will be brought upon the joints on service. The cost of such tests might offset the lower cost of bare electrodes. Moreover, if two different types of electrodes were used on a ship—the cheaper where strength is not of much importance and the more expensive where structural strength is vital—extremely careful supervision would be necessary, and it would probably be more practicable for the shipbuilder to adopt the higher priced article throughout.

It is desired to emphasize point 30 of Mr. Hobart's paper, on the necessity of a clean surface for fusion welding. Some of the unsatisfactory work that I have seen is due to the attempt to weld together plates, etc., having surfaces that are oily or are otherwise insufficiently clean.

R. P. Jackson (by letter): Both the varied and somewhat conflicting information set forth in Mr. Hobart's paper and that obtained by observation and experience indicate that in a large measure electric welding is an art rather than a science. While it is obvious that a great deal can be done in the way of eliminating the useless and fake elements, after all that a scientific investigation can contribute has been taken into account, the actual production of safe welds is essentially a personal problem. A skillful welder will do some very creditable work with a variety of materials and equipment and an unskillful one will do poor and unreliable work with the most perfect equipment.

It is this feature of the essential dependence on the skill of the individual welder that has been such a handicap to the more extended use of welding. The difficulty is that a designing engineer laying out his computations and drawings cannot know with any degree of certainty what he can depend on in the way of a joint as compared with riveted work put up with a very moderate degree of skill but capable of being checked by inspection.

While this comment does not disparage the necessity of research work and particularly of a greater degree of publicity as to the actual facts, its purpose is to accentuate the necessity of training welders and inspectors. The writer feels that no extensive undertaking like shipbuilding can be worked out except as a result of the building up of a welding organization, comprising a superintendent, inspectors and welders who are conscientious and know their job. Without such an organization, no cautious engineer would specify the use of welding for the obvious reason that he could not be sure what he would get. With such an organization, on the other hand, there is almost no engineering structure that could not be undertaken, with the only limitation as to the amount of welding to be the accessibility of the assembled structure for the proper welding operation.

F. M. Farmer (by letter): Mr. Hobart's paper presents many phases of the welding art concerning which there are very divergent views. In many cases, the wide differences of opinion are based on the results of mechanical tests, but an examination of the mechanical test data discloses a wide variation in character of the tests made and in the details of procedure, so that it is frequently not only difficult to analyze and check the basis of the conclusions but impossible to correlate various groups of test results. It would seem, therefore, that the technique of testing welds is one of the phases of the welding art that should be studied and standardized in order that the results of mechanical tests made by different people may be strictly comparable and have the same significance to all.

The vital difference between testing a specimen of steel that includes a welded joint and testing an ordinary specimen is the non-homogeneity of the weld specimen. The weld specimen has at its center a section composed of material that usually has physical, chemical, and metallurgical characteristics distinctly different from the adjoining metal. Furthermore, the section of added metal is more or less irregular and the ratio of added metal to original metal in the section is quite variable. Consequently the details of procedure and of recording data should be more minutely prescribed than in the testing of ordinary specimens.

A conspicuous example of a mechanical test that requires standardization is the cold bending test, a standart test for mild steel. With the usual method, the specimen is bent around a pin of prescribed diameter and, as all specifications usually require bending to 180 deg. without fracture, the manner of applying the load is immaterial because the specimen is ultimately completely wrapped around the pin. But a specimen containing a welded joint at the center will usually not withstand bending to 180 deg. because of the much lower ductility of the deposited metal in the weld. It is therefore obvious that the farther apart the points of support are, the greater will be the angle attained before cracking occurs, simply because most of the bending will take place in the original metal. It is therefore highly desirable that there be prescribed a standard test that fixes exactly the distance between the supports, the curvature of the surface of supports, and the relation between this curvature, that of the loading surface, and the thickness of the specimen.

The Research Sub-Committee of the Welding Committee of the National Research Council is using, in connection with some of its research work, a bending test that tests a specimen 1½ in. (38.1 mm.) wide as a simple beam with two supports and with the load applied at the center of the weld and midway between the supports. The load is applied to a cylindrical surface, about which the specimen is bent, that has a diameter equal to the thickness of the specimen, which corresponds to the specifications of the American Society for Testing Materials for cold bending tests of steel plate. The supporting surfaces are placed as close together as practicable so that the deposited metal in the weld will occupy the largest possible proportion of the beam. This minimum distance is obviously that which will permit the bending of a specimen to 180 deg. The corners of the supports are machined and polished to a radius equal to the diameter of the loading surface and to the thickness of the specimen. The initial distance between the points of support is then the sum of twice the radius of the corners of the supports, the diameter of the loading surface, twice the thickness of the specimen and ⅛ in. (3.1 mm.) which gives 1/16 in. (1.5 mm.) clearance on each side. Such a test is very easily made in a standard testing machine by attaching a suitably prepared die to the moving head of the machine and setting two blocks with rounded corners on the bed of the machine with the proper space between them. The first crack on the under side of the specimen is readily detected with the aid of a mirror.

In a considerable number of tests of weld specimens made by this method, the maximum angle obtained was of the order of 15 deg. while the angles usually reported in the literature are of the order of 30 deg. to 60 deg. It seems very certain that the relatively small angle found is due to the greater severity of the test and not to any inferiority of the welds. It should be stated that the parent metal can be bent to 180 deg. by this method without a sign of a crack.

Similar standardization is required in tensile tests, particularly the gage length. With long gage lengths, the elongation in the weld itself is obscured by the stretch in the parent metal.

Mr. Hobart referred to the feeling that has been frequently expressed that more information in regard to the durability of the welded joints under the action of repeated loading is needed before entire confidence in welding in certain fields can be obtained. A special fatigue testing machine was developed in connection with the work of the Research Sub-Committee of the Welding Committee for testing specimens of welds in ½-in. (12.7 mm.) plate. The machine employs the rotating-beam principle, the beam having two free supports at the ends and two loads at the middle. The arrangement is such that the stress is absolutely uniform throughout the length of the specimen occupied by the weld and the exact maximum fibre stress can be calculated. The principle is not at all new but the particular form of apparatus developed is very small, simple, and inexpensive. The results of tests recently made indicate that where standard ship-plate steel will withstand 4,000,000 or 5,000,000 reversals of stress at 25,000 lb. per sq. in. before failure, electrically welded joints will withstand of the order of 700,000 or 800,000 repetitions of the stress. All the failures occur in the deposited metal, indicating that there is no difficulty in getting a thorough bond between the added metal and the original metal, but that the added metal has not the fatigue strength of the original metal. This simply confirms the general view that greater ductility in the deposited metal of electric welds must be had before the process can be applied to a maximum extent in those very large fields where this characteristic is absolutely essential.

W. S. Andrews (by letter): Mr. Hobart has omitted to make any mention of colored glasses to be used by welders, in connection with goggles, shields, and masks for protecting their eyes and skin from the dangerous radiations produced by the electric arc. The iron arc is especially prolific in generating ultra-violet radiation, which will induce painful and dangerous inflammation in the unprotected eye and a powerful arc will produce a sunburn effect on the bare skin in a very short time. Leather gloves on the hands and shields or masks of light material, such as thin wood or vulcanized fibre, afford a perfect protection to the face and neck from these harmful rays. The protection of the eyes from these rays and the intense glare of the arc, however, is not so easy, as it involves the necessity of preserving clear vision for the operator.

It is well known that the human eye is more sensitive to yellow and green light than it is to red and blue light; so, in order to secure the clearest vision with the least amount of glare, densely colored glasses should be used. These should exclude most of the red rays and practically all blue and violet rays, while the orange, yellow, and green rays should be toned down to the lowest degree consistent with clear observation of the work in hand. Owing to individual difference in visual acuteness, no absolute rule can be given as to the exact amount of obscuration necessary for various operations. This may be decided best by each operator for himself, for one man may be able to see clearly through a glass that would obscure the vision of another. On the other hand, the fundamental color transmission of all safety glasses should be determined by an expert in accordance with spectroscopic analysis. There are many safety glasses on the market that fully conform with these requirements. Glasses dense enough to tone down the intense glare of the electric arc to an extent comfortable to the eye will always afford sure protection against ultra-violet radiation.

Infra-red radiation or heat rays naturally produce a drying effect on the skin; but as the latter is sensitive to them, they give their own danger signal. The shields and masks worn as a protection against the ultra-violet rays will usually be found a sufficient guard against heat rays. Most of the reliable dense-colored safety glasses on the market are also good shields against heat rays, so there is very little danger, if any, to be apprehended from this source. All harmful effects to the eye from the radiations of the iron arc may indeed be entirely avoided by the exercise of a little common sense and intelligent caution.

J. C. Lincoln (by letter): The general problem of electric welding, as it presents itself to the writer, is the problem of substituting electric welding for riveting. This substitution should take place in the construction of ships, boilers, bridges, and in the industrial arts. It is a large problem but a solution of the electric welding problem will eventually very largely replace the art of riveting for patching together of steel members. In the construction of a ship's hull, smooth steel plates from ⅜ in. (9.5 mm.) to ¾ in. thick, 6 ft. (1.8 m.) wide, and from 15 ft. (4.5 m.) to 30 ft. long must be joined to one another and to the framework. The plates must be welded to one another in such a way that the builder can be certain that the joint is equal to or similar in strength and ductility to the present riveted construction. Experience has shown that the ordinary electric weld, using the metallic electrode process, for samples 2 in. (5 cm.) or 3 in. (7.6 cm.) wide and from ⅜ to ¾ in. thick will produce a weld that, when made with sufficient care and tested in the pulling machine is very much stronger than the best riveted joint. The writer, however, does not think that this statement can be made when large sheets are welded together, as in the construction of a ship's hull. The heating, which is necessarily a part of the electric welding process, introduces contraction stresses between the plates that are welded, and these internal stresses, in some cases, are practically equal to the strength of the weld. It is impossible by any means now known to determine the amount of these stresses and, consequently, the use of the riveting process for welding the plates, which is much more expensive and not so strong, is universally used because it eliminates these stresses. In some cases these internal stresses can be relieved by annealing the whole sheet, including the joints, after it is welded. This process has been used with entire success for welding some tanks for holding ammonia under a pressure of 200 lb. per sq. in. It is obvious that in the construction of a ship's hull such an annealing process is out of the question. The conclusion, therefore, is that if the construction of ship's hulls by electric or gas welding is to be successful, the weld must be of such a nature that the welded material can stretch and relieve the internal or contraction stresses and this stretching must not crack the joint at any time nor weaken the strength of the welded joint.

One or two hundred tests on metallic welded joints with the best electrode that could be secured convinced the writer that the metallic electrode welding process will not produce a joint that will fulfill the above specifications; consequently, his attention has been directed toward the use of the carbon arc. He has developed the practise to a point where it is possible to say that the metal in the weld fulfills the specifications necessary to take care of the internal stresses outlined above. In other words, the practise of carbon arc welding has been advanced to the point where it is easy to make joints that are past the elastic limit of ordinary steel plate. The metal in these welds at the same time is ductile so that a test piece of metal cut from the weld itself and pulled in a testing machine will show satisfactory elongation and reduction of area. The writer hopes within a short time to be able to present the details of this process.

Herman Lemp (by letter): This paper is a very good resumé of the art of fusion and spot welding of mild steel, but it has entirely overlooked the well-known process of resistance welding, originated by Prof. Elihu Thomson. The Thomson process, the sire of spot welding, is so well established in our industries and has been so thoroughly discussed in the technical press and the records of the A. I. E. E. that it has almost passed the stage of debate, but may well be called to the attention of the younger engineers·

When one considers the vast quantities of mild-steel products daily welded in the United States alone by that process, and this, with unskilled persons operating automatic and semi-automatic machinery, with a speed unexcelled by the fusion process, the writer may be pardoned for drawing attention to one characteristic feature of the process; namely, that parts to be welded are substantially protected against contact with the oxygen of the air, and what few oxides may be formed near the edges are effectively squeezed out of the joint by the pressure exerted on completing the weld. In addition, all kinds of metals and alloys, some of which are termed unweldable, are successfully welded thereby. Crane chains, harness rings, automobile rims, bicycle tubing, wire fences, carriage tires, metal wheels, etc., are welded daily in such quantities as would astonish any one when confronted with actual figures and in the welding of valve flanges to cylinders of Liberty aviation motors at Detroit, the Thomson process easily surpassed the gas and electric-arc processes, as regards both speed and excellence of finished product.

Spot welding was the first process considered by the Welding Committee created by the A. I. E. E. in the summer of 1917, and its adaptability to ship-plate welding was clearly recognized at that time in respect to fabricated parts that could be brought to a stationary spot welder in place of having the spot welder brought to the work. A portable spot welder was designed for ½ in. (12.7 mm.) ship plates. The long secondary leads required for spanning the gap, and consequently reduced output by reactance, failed to give satisfactory performance with the transformers originally designed for ½ in. plates. The increased capacity of the transformer required to overcome the effects of reactance has always been a bugaboo in connection with spot welders or electric welders of large dimensions, and the method of overcoming this reactance by making two welds in series by two transformers placed on opposite sides of the plates suggested itself to the writer in August of 1917, and was communicated to Mr. Hobart at that time. This statement in no way is meant to reflect upon the able work done by Mr. Weed, who independently developed and is responsible for the practical and successful design of the Duplex spot welder referred to in Mr. Hobart's paper. As the current flowing through both welds is substantially the same, the heating effect will depend entirely on the local resistance, which, in turn, depends on contact pressure. It is this difficulty that has been overcome in so satisfactory a manner by Mr. Weed by utilizing independent contact pressure for the two welds made simultaneously.

Spot welding, as the name implies, may be carried out in two ways. The electrodes through which the current is passed to the plates may be of so small a section that only the metal directly under the contacts or in its immediate vicinity becomes heated; or the plates to be welded together may be provided with raised surfaces, or teats, which form a restricted contact with each other and are heated to welding temperature. In the latter case, the electrodes supplying the current need not be so small in area as in the former modification of the process. As the thickness of the plates to be welded is increased, both the current density and the pressures are materially increased, and the problem of producing a satisfactory electrode with good wearing qualities becomes more difficult.

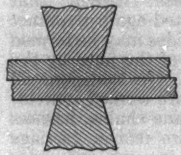
Fig. 1

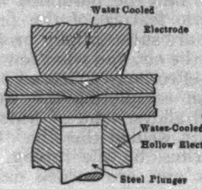

Fig. 2

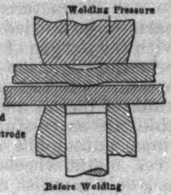

Fig. 3

From his experience with an electric armor-plate annealer, which he designed in 1894, the writer has learned that tremendous quantities of current can be passed through relatively small contact surfaces of copper, provided they are backed up by an increasing mass of copper properly water-cooled; 50,000 to 60,000 amperes were taken through contacts 1 in. (25.4 mm.) square, rounded off so as to make almost a point contact at times, and these contacts would last almost indefinitely, but in that particular application the pressure on the contacts was relatively light. I have been assured by Mr. Merrill, who was connected with the heavy spot-welding work referred to in Mr. Hobart's paper, that the problem of contacts is not as serious as one may think. It may be well, however, to consider the modification shown in the accompanying sketches, when large energies for spot welding are not available.

Fig. 1 shows the present method, in which two plates are placed in contact with each other and current passed through them through contact surfaces of restricted area. In this case the pressures on the contact surfaces have to be considerable and the surfaces themselves are relatively small.

Fig. 2 shows one of the plates provided with a series of teats produced by a punch press, similar to the one used for punching holes for riveting but in which the punch is rounded and is allowed to enter a small distance into the metal. Such a plate, when placed in contact with a water-cooled electrode of liberal dimensions, will make contact only on a ring surface, not in the middle. The other plate, which is without teats, rests on a hollow electrode, also of liberal dimensions, as far as contact surface is concerned. The teat in the upper plate insures a contact with the other plate at the apex of the teat and will be heated at the point of contact with the other plate.

On Completion of Heating before Welding Pressure is Applied

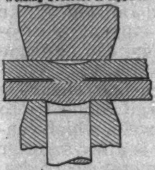

Fig. 4

Weld Completed and Pressure Applied

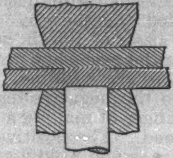

Fig. 5

After Completion by Arc Welding, for Calking Purpose

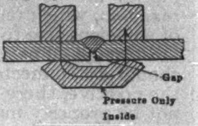

Fig. 6

Owing to the fact that the lower plate rests on a circular electrode unsupported and uncooled in the center, the center will be heated evenly with the teat in the upper plate. In the center of the lower electrode, but normally not in contact therewith, is a steel plunger, which may be separately operated by either hydraulic or pneumatic pressure.

Fig. 3 shows the plates in contact under ordinary pressure suitable for proper heating at the beginning of the weld. Fig. 4 shows the plates on the completion of welding; they are still under contact pressure but the hydraulic pressure has not been applied. It will be noted that the plates have been pushed together and a cup-shaped depression formed at the bottom of the lower plate that fits into the space in the hollow electrode. It will also be noted that the hollow depression in the top of the upper plate, formed at the time when the teat was made, still exists. Fig. 5 shows the last part of the process. After cessation of current, pressure on the central plunger is applied, leaving the upper plate restored to its original form with a spot weld in the center and a cup-shaped depression in the bottom.

By this method we have, first, the certainty of concentrating the heat always at the teat, even if the plates should be slightly bent; second, lasting copper contacts; and, third, the heated metal is worked, reducing its loose structure to a denser fibre. The drawbacks of the method are that one of the plates, at least, will require an additional mechanical operation to provide the teats, and once these teats are formed, welds can be made at no other place unless loose metal buttons are inserted.

Welding Ship Plates with One Transformer. A method of welding ship plates by spot welding, which requires only one transformer in close contact with the plates, and hence a relatively low resistance, is shown in Fig. 6. The two plates to be welded are butted together without actually touching, leaving a narrow gap. A butt strap, or individual strap with teats, is then pressed against the inner side of the plates. The transformer is on the outside of the hull of the ship, so the current passes from plate to plate through the strap, and pressure is applied from the inside. After the welds are completed, the gap is closed by arc welding for calking purposes only. The welder and pressure devices, both inside and outside of the plates, are to be pulled together by direct-current electromagnets, which on the basis of 100 lb. per sq. in. can be made of sufficient size to give the necessary pressure.

Polarity. For carbon arc welding, the carbon electrode should be connected to the negative terminal; if connected to the positive, there is danger of making a hard weld, which cannot be properly machined. The positive pole is always hotter than the negative, hence, when welding with metal electrodes, it is best to connect the positive to the plates as they are usually of greater volume than the electrode and should be heated sufficiently to produce fusion. Under certain conditions only as when very thin plates are to be welded with the metallic arc, it might be well to connect the positive to the electrode. When welding with alternating current, graphite electrodes may be used when the plates to be welded have turned up edges to be united together and where no machining is required after welding. On general principles, metallic electrodes are more suitable for alternating currents.

Harry D. Morton (by letter): *Theory of Electric Arc.* The generally accepted theory of the electric arc is that part of the electrode material is vaporized, and that this vaporous tube or column forms a path for the electric current. As a result of the vaporous character of the current path, all arcs are inherently unstable; and the maximum of instability is no doubt found in that form of arc employed for metallic-electrode welding purposes. We here have, in conjunction with the natural instability characteristic of all arcs, rapidly fusing electrode materials and the disturbing

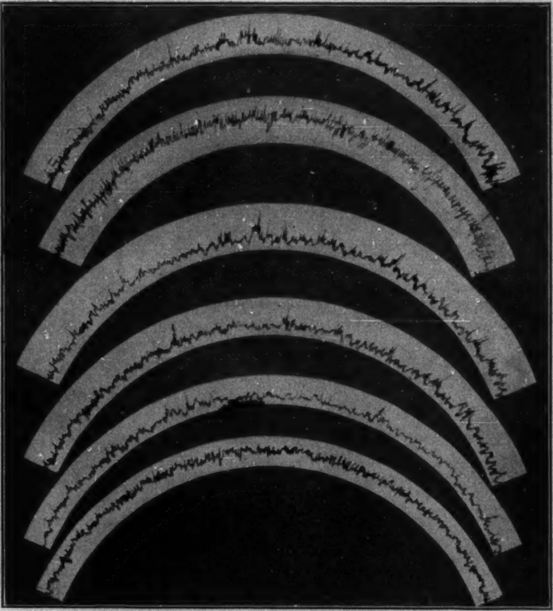

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FIG. 7—TYPICAL AMMETER CHARTS OF OPERATION OF AUTOMATIC METALLIC-ELECTRODE ARC-WELDING MACHINE. AVERAGE TIME ABOUT 1 MIN. 45 SEC.

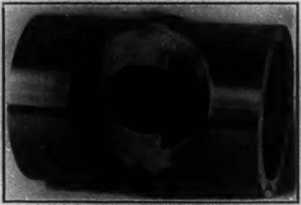

Fig. 8—Piloted Cup Automatically Welded by Metallic-Electrode Arc Process to Tube to Form 75 mm. Shrapnel Shell

Analysis of Electrode Material: Silicon, 0.02 per cent; Sulphur, 0.013 per cent; Phosphorous, 0.07 per cent; Manganese, Trace; Carbon, 0.07 per cent; Aluminum, 0.038 per cent.

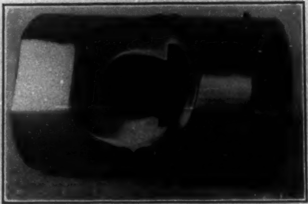

Fig. 9—Piloted Cup Automatically Welded by Metallic-Electrode Arc Process to Tube to Form 75 mm. Shrapnel Shell

Analysis of Electrode Material: Silicon, 0.03 per cent; Sulphur, 0.049 per cent; Phosphorous, 0.008 per cent; Manganese, 0.31 per cent; Carbon, 0.28 per cent.

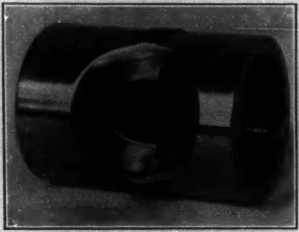

Fig. 10—Piloted Cup Automatically Welded by Metallic-Electrode Arc Process to Tube to Form 75 mm. Shrapnel Shell

Analysis of Electrode Material: Silicon, 0.02 per cent; Sulphur, 0.032 per cent, Phosphorous, 0.008 per cent; Manganese, 0.20 per cent, Carbon, 0.18 per cent.

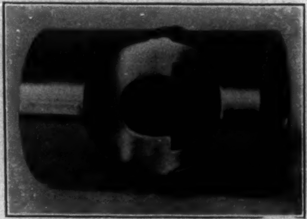

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Fig. 11—Piloted Cup Automatically Welded by Metallic-Electrode Arc Process to Tube to Form 75 mm. Shrapnel Shell

Analysis of Electrode Material: Silicon, 0.04 per cent; Sulphur, 0.016 per cent; Phosphorous, 0.058 per cent; Manganese, None; Carbon, 0.24 per cent.

effect of the constant passage through the arc of a large quantity of molten metal to form the weld. This molten metal must pass through the arc so rapidly that it will not be injured or materially contaminated, otherwise the weld will be useless. Prima facie, the combination of these unfavorable conditions would seem to justify fully the skepticism of most electrical engineers as to the possibility of effecting such control of the metallic arc as to permit of uniformity and continuity in welding results. In addition, there is another and more important factor, and one that seriously mitigates against this desired uniformity and continuity, namely, the personal equation of the operator. The consensus of opinion, so far as is known to the writer, seems to be that about 95 per cent of the welding result is dependent on the skill of the operator and that at least six months' practise is necessary to acquire reasonably satisfactory proficiency.

As the result of thousands of observations of welds produced automatically (wherein the personal equation is entirely eliminated), the writer inclines toward the theory that the molten electrode material passes through the arc in the form of globules; and that where ⅛-in. electrode material is employed with a current of about 150 amperes these globules are deposited at the rate of approximately two per second. The passage through the arc of each globule apparently constitutes a specific cause of instability in addition to those existent with slowly consumed electrodes. This hypothesis seems to be borne out by ammeter records (typical specimens of which appear in Fig. 7), together with the fact that the electrode fuses at the rate of 0.2 in. (5.08 mm.) per sec. Moreover, the globules appear to be approximately equal in volume to a piece of wire 0.125 in. (3.07 mm.) in diameter and 0.1 in. (2.54 mm.) long.

Essentials for Maintaining Uniform Length of Arc. Assuming this theory to be correct, to maintain a uniform arc length in manual welding, the operator must feed the electrode toward the work at the rate of 0.1 in. upon the deposition of each globule, in other words, 0.1 in. twice per second, a synchronism beyond human attainment. Simultaneously with such feeding, the arc must be moved over the work to melt the work material, distribute the molten electrode material, and form the weld. Inasmuch as the effect of the arc is highly localized, it is reasonable to suppose that different parts of the welding area present relatively wide variations in respect to temperature, fluidity, and conductivity of the molten mass—controlling factors not within the ken of the human mind. The situation is further complicated by the facts that neither the welding wire nor the work material is uniform in fusibility or in conductivity, and that the contour of the work varies continually as its surface is fused and the molten metal is caused to flow. The belief is general that a very short arc is

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productive of the best welding results; but it is an arc of this character that makes the greatest demands on the skill of the operator, for there is always the danger that the electrode will actually contact with the work and destroy the arc.

Effect of Fluctuations of Arc Length on Fusion Energy. As the fusing energy of the arc varies widely with fluctuations in the arc length and as the uniformity of the weld depends on the constancy and correctness of this fusing energy, it seems remarkable that operators are able ever to acquire such a degree of skill as to enable them to produce welds that are even commercially satisfactory. Further, so far as the writer is informed, there is no means, other than such as would be destructive, for determining whether a completed weld is good or bad. The logical solution appeared, to the writer, to be the elimination of the personal equation and the substitution therefor of means whereby tendencies toward variations in the arc would be caused automatically to correct themselves, just as a steam engine, through the action of its governor, is caused to control its own speed.

Methods of Mechanically Stabilizing and Controlling the Arc. The efforts of the writer, for a number of years, have been directed toward stabilizing and controlling the metallic arc, and applying such stabilizing and controlling means to two general lines of welding machinery: (1) machines wherein the feeding of the electrode wire, with reference to the work, and the producing simultaneously therewith of relative movement between the wire and the work are both automatically effected; and (2) what, for lack of a better term, might be called a semi-automatic machine, in which the feeding of the electrode and the control of the arc are accomplished automatically but the traversing of the electrode with reference to the work is manually effected by the operator, permitting him the exercise of judgment with reference to the quantity of metal to be deposited in various parts of the groove. The automatic machine has been in successful operation for a long period and the semi-automatic machine for about five months. While the goal was not attained without many difficulties and a great expenditure of time and money, the results have been surprisingly successful.

Because of the lack of any definite data as to what actually occurs in this form of arc, or why it occurs, due, no doubt, to the impossibility of differentiating between phenomena that are characteristic of the arc and phenomena due to the personal equation of the welder, it seemed logical that the initial step should be to so environ the arc that it would not be subject to erratic extraneous influences, to the end that reasonably definite determinations might be substituted for scientific speculation. In the design and construction of the machines, great care was exercised to minimize the possibility of mechanical defects that might lead to erroneous conclusions. Starting with the assumption that the work could only be based on open-minded observation of the behavior of the arc under machine control, an automatic welding machine was built in which was incorporated the greatest possible number of adjustable features, in order that, if necessary, it might be possible to wander far afield in the investigations. This adjustability has proved invaluable in that it has permitted logical, consistent, and consequential experimenting over a very wide range of conditions. Working under these favorable circumstances, there were soon segregated a few clearly demonstrable facts to serve as a foundation for the structure, which has since been added to, brick by brick, as it were.

Efforts have been directed toward the practical rather than the scientific aspect of the subject. The operation of the automatic machines has brought to light many curious and interesting phenomena, some of which appear to negative conclusions heretofore formed which have been predicated upon observations made in connection with manual welding. It is hoped that these and other phenomena, which can thus be identified as purely arc characteristics, will be the subject of profitable scientific investigation when time is available for this purpose.

Development of Automatic Machines. In the five forms of machines made in the course of the development, the welding wire is automatically fed to the arc; and, in the first four machines, the relative movement between the work and the welding wire is automatically and simultaneously effected. Early in his investigations, the writer concluded that a substantial equilibrium must be maintained between the fusing energy of the arc and the feeding rate of the welding strip; and it soon became evident that if the welding strip is mechanically fed forward at a uniform rate equal to the average rate of consumption with the selected arc energy, this equilibrium is actually maintained by the arc itself, which seems to have, within certain circumscribed limits, a compensatory action as follows: When the arc shortens, the resistance decreases and the current rises. This rise in current causes the welding strip to fuse more rapidly than it is fed, thereby causing the arc to lengthen. Conversely, when the arc lengthens, the resistance increases, the current falls, the welding strip is fused more slowly than it is fed, and the moving strip restores the arc to its normal length.

While this compensatory action of the arc will maintain the necessary equilibrium between the fusing energy and the feeding rate under very carefully adjusted conditions, this takes place only within relatively narrow limits. It was very apparent that, due to variations in the contour of the work, and, perhaps, to differences in the fusibility or conductivity of the welding strip or of the work, the range of this self-compensatory action of the arc was frequently insufficient to prevent either contacting of the welding strip with the work or a rupture of the arc due to its becoming too long. The problem that arose was to devise means whereby the natural self-compensatory action of the arc could be so greatly accentuated as to preclude, within wide limits, the occurrence of marked arc abnormalities. There was ultimately evolved, by experiment, such a relation between the fusing energy of the arc and the feeding rate of the welding strip as to give the desired arc length under normal conditions; and tendencies toward abnormalities in arc conditions, no matter how produced, were caused to bring into operation compensatory means for automatically, progressively, and correctively varying this relation between fusing energy and feeding rate, such compensatory means being under the control of a dominant characteristic of the arc. In their ultimate forms, the devices for effecting the control of the arc are simple and entirely positive in action, making discrepancies between fusing energy and feeding rate self-compensatory throughout widely varying welding conditions. For instance, the shrapnel shell shown in Fig. 8 was automatically welded with wire differing greatly in chemical constitution from that used on the shell shown in Fig. 9 (see analyses), yet no change was made in either the mechanical or the electrical adjustments. The radically different welding conditions were compensated for solely by the operation of the automatic control. The electrode materials used for the shells shown in Figs. 10 and 11 differed so greatly from those employed respectively in welding the shells shown in Figs. 8 and 9 that a change in the relation between fusing energy and feeding rate had to be made manually. After this adjustment was made, the shells were welded with their respective electrodes, which varied widely in their chemical constitution, without further manually changing either the mechanical or the electrical conditions.

In a recent test of the semi-automatic machine, shown in Fig. 12, successful welds were made under the condition that the impressed voltage of the welding generator was changed throughout a range of from 50 to 65 volts, without necessitating any manual adjustment. The only observable effects of the wide variations in the supply voltage were slight differences in the arc length.

In short, the compensatory action of the control has proved effective over so wide a range of welding conditions, not only as to the electrical supply and chemical constitution of both electrode and work materials, but as to extensive variations in the contour of the work and in many other particulars, as to make it seem apparent that the machines do not represent merely successful laboratory experiments but are suited to the requirements of actual commercial welding.

Angle of Electrode to Work on Insertion of Extra Resistance. One particularly interesting observation resulting from the experiments is that the angle of inclination of the electrode with reference to the work is very important. An angular variation of 5 deg. will sometimes determine the difference between

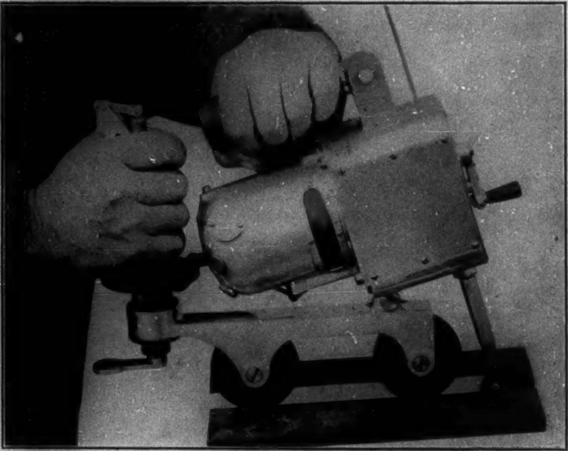

FIG. 12—SEMI-AUTOMATIC METALLIC-ELECTRODE ARC-WELDING MACHINE

The electrode is automatically fed to the arc, which is automatically maintained, while the machine is manually moved along the grove to be welded.

FIG. 13—TUNGSTEN HIGH-SPEED RING AUTOMATICALLY WELDED BY METALLIC-ELECTRODE ARC PROCESS TO COLD-ROLLED CORE TO FORM MILLING-CUTTER BLANK

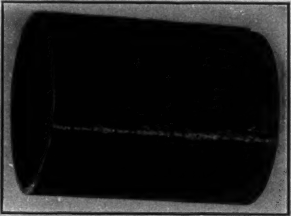

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FIG. 14—NO. 11 GAGE STEEL TUBING AUTOMATICALLY WELDED BY METALLIC-ELECTRODE ARC PROCESS AT THE RATE OF 1 FOOT (30.4 CM.) PER MIN.

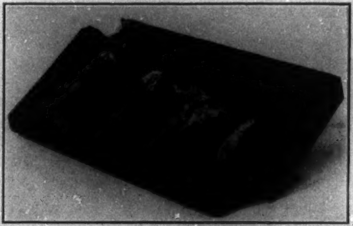
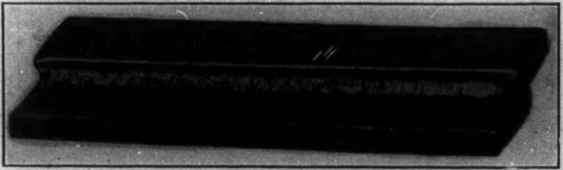


FIG. 15—TWO ½-IN. SHIP PLATES AUTOMATICALLY WELDED BY METALLIC-ELECTRODE ARC PROCESS TO FORM LAP JOINT

FIG. 16—TWO ½-IN. SHIP PLATES AUTOMATICALLY WELDED BY METALLIC-ELECTRODE ARC PROCESS TO FORM BUTT JOINT



[MORTON]

FIG. 17—TWO ½-IN. SHIP PLATES AUTOMATICALLY WELDED BY METALLIC-ELECTRODE ARC PROCESS, SHOWING FIRST OF THREE LAYERS TO FORM LAP JOINT

success and failure in a weld. About 15 deg. from the perpendicular works well in many cases. In welding some materials, the electrode should drag, that is, point toward the part already welded rather than toward the unwelded part of the seam.

While it has been customary in some welding systems to provide means whereby extra resistance is inserted in series with the arc at the instant of the initial contact which starts the flow of current, which resistance is automatically cut out upon the striking of the arc, experience with the automatic machines indicates that this is quite unnecessary.

Relation of Electrode to Work Materials. Early in the experiments, it was noted that in many cases there was a decidedly marked affinity between particular electrode materials and particular work materials. A slight change in either element affects the degree of this affinity. While it has invariably been possible to control and maintain the arc and weld continuously, in some instances incompatibility between electrode material and work material has been productive of interesting phenomena. For instance, the combination of work material (steel of about 0.45 per cent carbon content) and the particular electrode material shown in Fig. 8 produced an arc that was remarkably quiet and free from sputtering. Throughout the weld, this arc was suggestive of the quiet flame of a candle or lamp, the erratic behavior that we are accustomed to associate with the ordinary metallic arc being absent. The effect is reflected in the uniform deposition of the welding material.

On some classes of work material Bessemer wire, which some authorities claim cannot be used in metallic-electrode arc welding, produces an arc and a weld very satisfactory in appearance. On other work material, the Bessemer wire arc is violently explosive. These explosions are accompanied by quite sharp reports and the scattering over some considerable distance of globules of molten metal frequently 3/32 in. (2.38 mm.) or more in diameter. Under certain other conditions, apparently growing out of incompatibility between the work material and the electrode material, the oxygen flame accompanying the arc gyrates very rapidly about the arc, producing an effect suggestive of the "whirling dervish."

From both the practical and the scientific points of view, the writer has experimented quite extensively with varying combinations of work material and electrode material. Throughout all the differences in arc conditions, many of which palpably accentuate the natural inclination toward instability, the control has so operated as to justify the expression "the arc persists".

Generally speaking, the Swedish and Norway iron wires seem to produce more quiet arcs and, possibly, a more uniform deposition of electrode material, than do wires of other classes. These welds may perhaps be found to be slightly more ductile than those made with wires of other chemical composition. On the other hand, these soft wires, although undoubtedly of relatively high fusibility, do not, for some reason, seem to produce an arc that cuts into some work material as deeply as might be desired, nor as deeply as do the arcs formed with certain other kinds of wire. Considered from every angle, the writer is disposed to regard the Roebling welding wire as the best he has thus far tested for use on mild steel. This wire produces a reasonably quiet arc which seems to cut into the work to more than ordinary depth, while, at the same time, the electrode material is fused with more than average rapidity—thus increasing the welding rate.

While scientists will no doubt ultimately arrive at the correct hypothesis for solving the problem of why one combination of electrode material and work material is productive of better results than can be obtained with another combination, the writer's conclusion is that, with the data at present available, the determinations must be made by actual experimenting—having in mind the qualities desired in the particular weld, such as ductility, tensile strength, elongation, and elastic limit. Inasmuch as it is possible, with the automatic machine, to maintain arc uniformity with practically any kind of electrode material and to produce welds which, under low magnification, at least, appear to be perfect, and which respond favorably to ordinary tests such as bending, cutting and filing, it is reasonable to conclude that proper selection of electrode material will be productive of perfect welds on any kind of work material. To date, no steel has been tested on which apparently satisfactory welds could not be made. High-speed tungsten steel has been successfully welded to cold-rolled shafting, using Bessemer wire as electrode material, as is shown in Fig. 13 Ordinary steels varying in carbon content from perhaps 0.10 to 0.55 per cent have been welded with entire success.

Condition of Metal in Crater. Because of the fact that the complete welding operation has been automatic and may be continued for a considerable length of time, say 5 min., an exceptional opportunity has been afforded for close concentration upon and study of the appearance of the arc. What seems to occur is that the molten metal in the crater is in a state of violent surging, suggestive of a small lake lashed by a terrific storm. The waves are dashed against the sides of the crater, where the molten metal of which they are composed quickly solidifies. The surgings do not seem to synchronize with nor to be caused by the falling of the globules of molten metal into the crater, but seem rather to be continuous. They give the impression that the molten metal is subjected to an action arising from the disturbance of some powerful force associated with the arc—such, for instance, as might result from the violent distortion of a strong magnetic field. Altogether, the crater phenomena are very impressive; and the writer hopes ere long to be able to have motion pictures made which, when enlarged, should not only afford material for most fascinating study, but also throw light upon some of the mysterious happenings in the arc.

Electrodes and Currents Used. So far, electrode wires ⅛ in. (3.17 mm.) in diameter have been chiefly used in the machines. Successful welds have been made with current values ranging from below 90 to above 200 amperes at impressed voltages of 40, 45, 50, 55, 60, 65, and 80. Under these varying conditions, the voltage across the arc has been roughly from 16 to 22. The machines have thus far been run only on direct current. Inasmuch as it is possible, by electrical and mechanical adjustments, to establish nearly any arc length that may be found to be most desirable for a particular class of work, and as the control system will maintain substantially that arc length indefinitely, the fully automatic type of machine is nearly as certain in operation as a lathe, drill press, or other machine tool.

Automatic and Semi-automatic Welding Machine. Upon the writer's appointment to membership on the Welding Committee of the Emergency Fleet Corpn., Prof. Adams and other members of the Committee urged upon him the advisability, for purposes of ship welding, of incorporating in a small and readily portable welding tool the features of automatic wire feed and arc control. The tool designed is shown in Fig. 12. In welding with this so-called semi-automatic machine, which weighs about 10¼ lb. (4.6 kg.), the operator draws the tool along the groove to be welded at such a rate as will result in the deposition of the quantity of metal required to satisfactorily effect the weld. This tool is intended for use in the many restricted spaces encountered in ship welding, which would be relatively inacessible to a fully automatic machine. In its use, the skill required by the operator is reduced to a minimum. After having practised with the welding tool for not more than two hours, the writer demonstrated it to Mr. E. L. Hirt, of Bethlehem Shipbuilding Corpn. Mr. Hirt expressed it as his opinion that it would require six months to train a welder to such a degree of proficiency as to enable him to make a weld equally good in appearance.

Mr. Hobart, under "Fusion Welding," says "There is always a matter of 0.1 in. (2.5 mm.) or more between the end of the welding rod and the work." While undoubtedly it is difficult, if not impossible, to maintain in manual welding an arc shorter than this, the writer has frequently, with the automatic machines, made continuous and strikingly good welds with arcs of much less length. In fact, in some cases there has been continuously maintained an arc so short that there hardly seemed to be any actual separation. The writer has even wondered whether, under these conditions, there was not a close approach to casting with a continuous stream of fluid metal acting as the current conveyor in lieu of or in parallel with the usual assumed vapor path. The work that has been done indicates that under automatic control much shorter arcs can be utilized than have hitherto been deemed possible, and with probable marked gain in quality of work in some instances; also, that there is much to be learned as to the mode of current action and current conduction in such an arc.

Results Obtained with Automatic Welding Machine. With the automatic machine, black drawing steel 0.109 in. (2.76 mm.) thick has been welded at the rate of 22 in. (55.8 cm.) per min. A Detroit manufacturer welded manually with oxyacetylene at the rate of four per hour a large number of mine floats 10 in. (254 mm.) in diameter, made of this material. The automatic machine made the welds at the rate of forty per hour. Liberty motor valve cages 2¾ in. (69.8 mm.) in diameter have been welded to cylinders in 36 sec., as against about 5 min. required for manual welding. No. 11 gage steel tubing, shown in Fig. 14, has been welded, with an unnecessarily heavy deposit of metal, at the rate of 1 ft. (30.4 cm.) per min. The productive capacity of the machines so far made has been from three to ten times that of manual welding methods, depending on the thickness of the work material; the difference in favor of automatic welding varies inversely as such thickness. The writer is now designing an improved type of machine for use especially on heavy work, with which machine it is expected to be able automatically to lapweld ½ in. (12.7 mm.) ship plates, in the manner shown in Fig. 15, at the rate of 15 ft. (4.5 m.) per hr. One of the largest shipbuilding concerns in the United States reports that the general average of all its manual welders on this class of work is from 1 ft. (30.4 cm.) to 18 in. (45.7 cm.) per hr. Other specimens of automatic welding on ship plates are shown in Figs. 16 and 17.

Bare wire only has been used in the automatic machines; and the results obtained seem to indicate that the covering of the electrodes is an expensive superfluity. If the chief advantage of the covered electrode lies in the ability of the operator to maintain a very short arc, an arc equally short and possibly shorter can be continuously maintained by the automatic machine using bare electrodes.

No attempt has thus far been made to use the automatic machines on over head work. The welds made with the fully automatic machine have been of three kinds: the usual longitudinal form, annular about a horizontal axis, and annular about a vertical axis. The experience gained indicates that there is no difficulty in dealing with any ordinary contour.

As far as the maintenance of arc uniformity and the apparent character of the welds are concerned, they do not seem to be greatly influenced by any of the considerations referred to in the specifications appearing in Section 4 of Mr. Hobart's paper. The writer has repeatedly welded with wire showing evidence of pipes and seams, as well as with rusty wire and with wire covered with dirt and grease. In this connection it may be said that no pains is ever taken to remove rust, scale, or slag from the work material—even where welds are superimposed. Apparently under uniform conditions of work traverse, arc length, and electrode angle of inclination, such as are possible in the automatic machine, impurities vanish before the portion of the work on which they occur reaches the welding area of the arc.

Referring to Section 15 entitled, "Ductility of Arc Welds:" the writer is fully convinced that with the use of the automatic machine, ductility, like other physical properties in the weld, can be controlled by proper selection of electrode wire, in conjunction with electrical and mechanical adjustments best suited to the particular purpose in view. Automatic welds have repeatedly been made on $\frac{5}{16}$ in. (7.93 mm.) mild steel which, when subjected to a 90 deg. bend, showed a marked extrusion of the welded material but no sign of fracture. When the welded pieces are cut with a hack saw, it is very unusual to be able to note any difference in cutting qualities between the unwelded and the welded parts.

Welding Thin Material. While the automatic machine has not been used on metal less than 0.109 in. thick, it is fair to presume that, with proper adjustments, entirely satisfactory results can be obtained on much thinner work—particularly if the nature of the work is such as to permit of the use of a chill. The best method in welding very light metal seems to be to use a small electrode, a relatively low current, and a high rate of work traverse. In this way welding conditions may be controlled to almost any desired extent, because the heating action of the arc can be modified, its effect intensely localized, and the edges to be welded subjected to the fusing action for as brief a time as might be found necessary to prevent burning of the metal. These conditions, which seem to the writer to be requisite in order to successfully weld very thin material, cannot be met by the manual welder. It is here that the deficiencies incident to the personal equation become most apparent. A very slight variation in arc length or the least hesitancy in moving the arc over the work will almost certainly result in its being burned through. In short, this class of welding calls for a coordination of faculties and a delicacy of manipulation beyond the capabilities of the most skillful manual electric welder. Therefore this work is usually done with the oxy-acetylene flame, wherein fusing conditions are far more easily controlled than is possible in manual metallic-electrode arc welding.

R. E. Wagner: (by letter): The practise of arc welding, as it exists today, consists largely of the results of experiments and practical experience; the study of the underlying scientific principles has not kept pace with its practical application. This situation, however, does not differ materially from that of many other arts where practise leads the way to the laws and principles that govern their operations.

Arc welding has been widely used for a number of years and, where properly employed, is a reliable and economical process by which steel parts may be fused together. Excellent welding is being done due largely to the skill of the operator.

Many systems have been developed, all based on some particular feature that the inventor has considered as being important. Very little was done to bring out the advantages of these systems until the Welding Committee of the Emergency Fleet Corpn. began its work of research; then it was found that few of the fundamental principles of the art were known. It was generally admitted that a short arc should be used, that the size of electrodes and amount of current required would change for different thicknesses of pieces to be welded, that pieces would have different preparations for welding depending on their thickness and strength of weld desired, etc. But the best values in any of these cases could not be definitely specified. It is, therefore, necessary that a great deal of research work be done in order to establish the fundamentals of the art.

Among the fundamentals to be investigated, for example, are: (1) How is the character of the weld affected by varying values of current with a constant voltage supply? (2) How is the character of the weld affected by varying values of supply voltage at constant current? In determining these conditions we must assume a thickness and kind of steel plate, size and kind of electrode, a preparation of plate edges for welding, a method of welding, a good operator, and a source of current.

In an effort to answer these questions, tests were made with ½-in. tank-steel plates that contained 0.25 per cent carbon, 0.40 per cent manganese, 0.025 per cent phosphorus, 0.000 per cent silicon, and 0.028 per cent sulphur. The electrodes used contained 0.185 per cent carbon, 0.561 per cent manganese, 0.037 per cent phosphorus, a trace of silicon, and 0.038 per cent sulphur. The plate edges were prepared with a double *V* and had 45 deg. and 30 deg. bevels; they were 10 in. (254 mm.) by 4 in. (101.6 mm.) in size and the length of the weld was 10 in. Two sizes of Roebling electrodes were used, the 5/32-in. (3.96 mm.) and the 3/32 in. (4.76 mm.). All the welds were made by the same operator, who was a good welder; direct current was used. Later, the raised part of the welds was ground off even with the plate surface and the welded plates were cut into strips 1 in. (25.4 mm.) wide for the tensile and bending tests.

With varying direct current, and a constant 75-volts, the following average values were obtained:

Current amperes	Tensile strength, lb. per sq. in.	Brinnell hardness value	
		Plate	Weld
80	37,000	106	114
125	50,000	114	114
150	57,000	122	122
180	63,000	122	114
220	66,000	114	114
275	62,000	114	114
300	58,000	122	114

The curve given in Fig. 18 shows the plotting of values obtained in the test. The 5/32-in. electrode was used up to 220 amperes where it became so difficult to manipulate that a change was made to 3/16-in. The tests at 180 and 220 amperes were repeated with the 3/16-in. electrode in the place of the 5/32-in. so that the observations would overlap and then the test was carried on to 300 amperes. This curve indicates that the highest average tensile strength, in a weld of this particular kind and under the conditions specified, will be obtained with a 5/32- or 3/16-in. electrode at 220 amperes. Fig. 19 shows the broken ends of the same welds covered by the curve. Better fusion conditions may be readily observed by the gradual disappearance of the vertex of the *V*. At 220 amperes the line of the *V* has disappeared. Reduction in tensile strength beyond 220 amperes is due apparently to the increased porosity of the deposited metal and difficulty in manipulation.

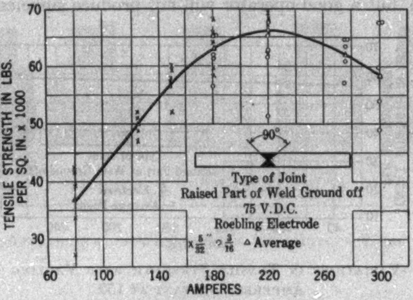


Fig. 18—Variations in Tensile Strength with Varying Current

The results of tests of welds made at a constant direct current of 150 amperes and voltage varying from 40 to 225 are as follows:

Volts	Tensile strength, lb. per sq. in.	Volts	Tensile strength, lb. per sq. in.
40	50,000	150	57,000
75	57,000	200	60,000
100	54,000	225	58,000
125	57,000		

The curve given in Fig. 20 shows that the tensile strength is little affected by the change in voltage. The only advantage of a higher voltage is to produce a more steady current, as slight variations in the length of the arc produced by the operator will affect the total resistance of the circuit less as the resistance of the circuit increases with the higher voltage.

Fig. 21 shows the variations in current at low and high voltages during welding operations. The higher voltage gives the more steady current. Various systems have been invented to maintain a constant current in the arc with a low supply voltage, but a good operator will not produce sufficient variations in his current to materially affect the strength of his weld at any voltage down to 40 volts direct current. These observations indicate that for any particular condition, the selection of the proper current is important if the strongest weld is to be obtained; and that if the current is right, the voltage of the supply system, if it is between 40 and 225, may be disregarded.

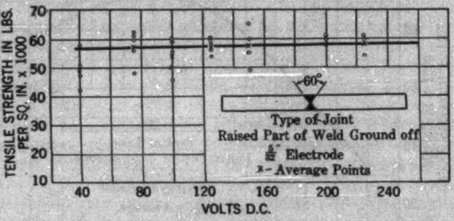


FIG. 20—VARIATIONS IN TENSILE STRENGTH WITH VARYING VOLTAGE, AMPERES CONSTANT AT 150

Bending tests were made for each condition of current and voltage, but it was not observed that the bending characteristics of the welds were in any way altered by the variations in current and voltage noted.

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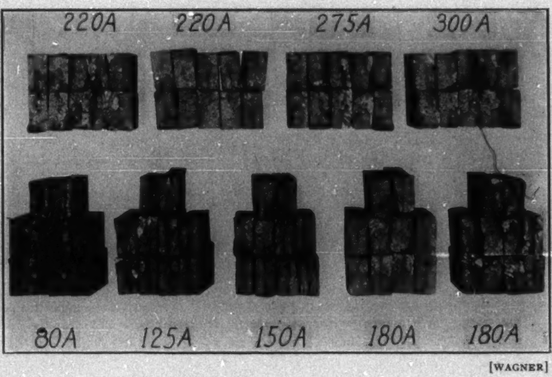

220A
220A
275A
300A
80A
125A
150A
180A
180A

[WAGNER]

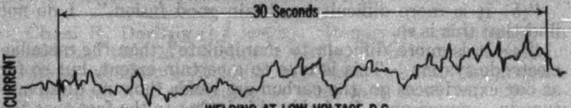
FIG. 19—APPEARANCE OF WELDS WHEN TEST PIECES WERE BROKEN

PLATE XVII.
A. I. E. E.
VOL. XXXVIII. 1919

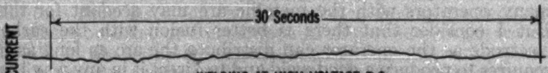
[KNOX]

FIG. 27—ARC-WELDED BATTLE TOWING TARGET UNDER CONSTRUCTION AT NORFOLK NAVY YARD

These observations are only a very small part of the broad field that is open for investigation and research. It is hoped that many investigators will submit data bearing on these two and other essential features of welding.



WELDING AT LOW VOLTAGE D.C.



WELDING AT HIGH VOLTAGE D.C.

Fig. 21—Comparative Stabilities of Current Value During Welding at Low and High Voltages Direct Current

T. T. Heaton (by letter): For the past 27 years the writer has been using electric arc welding with a carbon electrode. Fig. 2 of Mr. Hobart's paper shows the operator holding the welding rod in his left hand. This is not our practise. If metal is to be added to the weld, it is laid over the seam and fused by the direct action of the arc. But in many cases no metal is added, the parts to be united are simply joined together by fusing. A good deal of importance must be attached to the design of the parts to be welded so as to get good results in the welds and to avoid, as far as possible, any distortion due to the effect of contraction.

In Section 6 of Mr. Hobart's papers Mr. Wagner states, from his experience, that it is very difficult to weld with a carbon arc when the polarity of the carbon is positive. In welding mild steel or iron, the writer has found this to be quite true. For various reasons it is best to weld with the work positive and the carbon negative. Any deposit from one pole to the other takes place from the positive to the negative; therefore it is not possible to carry any foreign matter into the weld, so no carbon from the electrode can be mixed with the weld. Besides, the positive pole is considerably hotter than the negative, which is an advantage. It was stated recently that the temperature of the electric arc is approximately 3650 deg. or more cent., and that the positive pole is about 1000 deg. in excess of the negative.

In Section 23, certain criticisms are given: "(*a*) Carbon is carried into the deposited material thus reducing its ductility." With the arrangement of poles in the manner adopted by us this does not occur. Analyses show that the result of the welding is to remove more or less all impurities from mild steel leaving a very considerably increased proportion of pure iron, as compared with the metal before welding. This applies to welds made in sheet steel where added metal was simply scrap of the same material.

"(*b*) It is more difficult to obtain good fusion." I do not find that this is so.

"(*c*) It is more difficult to manipulate" than the metallic-electrode system. This is true to a certain extent, but so far as our experiences go, the carbon-arc welds show no disadvantage in comparison. Probably this is due to the fact that the operator with the carbon arc possessed, in the tests made, greater ability through long practise. The shorter training of many operators with the metallic arc may account for this. But I consider that there is better fusion with the carbon electrode as the operator can distribute the arc as long as he requires to produce a good weld and to direct it in any way that may be suitable, whereas with the metal electrode, he is limited to the time required to melt it.

With regard to criticism (*d*) we do not find that there is any discomfort to the operator.

"(*e*) Greater cooling stresses are developed because larger areas of adjacent metal are heated." For some classes of joints it is an advantage to spread the heat over a fairly wide area, as the metal surrounding the weld is improved thereby.

At the end of Section 23 certain counter-statements are made: "(*a*) No preparation of the abutting edges is necessary." I scarcely agree with this unless the metal is thin.

"(*b*) Greater rate of deposition of metal and greater speed of welding" is possible. This again depends on the thickness of metal.

"(c) Probable greater adaptability to automatic welding." So far as my experience goes, attempts at automatic welding with the carbon electrode have not been successful and in any case can only be limited to very special examples.

There is no doubt whatever that electric welding by this method is exceedingly successful, both technically and commercially, but our experience is that it is best to specialize the operators in certain kinds of work, as it is very difficult to find welders capable of doing a variety of work with success. To do this requires not only practise but a high order of intelligence in the workman.

Some time ago, we made some experiments in the speed of welding by this system with the following results:

A straight weld uniting two sheets 3½ mm. thick took 2¾ min. per ft. run to make a perfectly sound hammered weld. A stamped steel boss was welded into a mild steel sheet, also 3½ mm. thick. The circumference of weld, 17¼ in. (43.7 cm.) was made first by welding one side and turning over and welding the opposite side. The outside weld occupied 2½ min.

and the inside 2 min. These are, of course, good times and the welds were made by a practised workman.

The writer has followed the work done by the Welding Research Sub-committee in America with very great interest and is of the opinion that its labor will confer a benefit to engineering science.

Chas. R. Darling (by letter): It may be of interest, in connection with Mr. Hobart's paper, to indicate the present position of electric welding in Great Britain. In general, the progress of spot welding is somewhat slow, but the development of arc welding has been rapid during the past two years. Among the larger structures to which arc welding has been applied are: a rivetless barge with a displacement of 225 tons that has given excellent service during the last nine months, pontoons, and experimental ships under construction. Details of the last, with respect to cost of production and seaworthiness, are awaited with interest. In shipyard and general engineering repairs, arc welding is growing rapidly in popularity. Mention may be made of the successful deposition of manganese steel on mild steel on the rims of dredger buckets and in other cases where a hard wearing surface is required.

All who have experimented, in this country, are agreed that a thorough investigation of the destructive iron arc is necessary in order to arrive at the best method of working. At present, it is not possible to insure that two consecutive welds, made under apparently identical conditions, will give even approximately similar results under test. An inspection of the Wirt-Jones tests, Table IV, makes this clear; in my own experience it has been quite common to obtain results varying between 62,000 (28,132 kg.) and 52,000 lb. (23,586 kg.) in ultimate tensile strength, and 8.0 to 4.5 per cent of elongation, without any obvious reason for the difference. Until we know what is going on in the arc during the process of welding, it is unlikely that a standard method of working can be formulated.

One notable difference in the practise in America and in England is the general adoption of the coated, or fluxed, electrode by British welders, in contrast with the wide use of bare electrodes in America. Workmen claim that it is much easier to work with a fluxed electrode, as it is less difficult to maintain the arc; also that a better lateral spreading of the deposit is secured by the use of flux. It is difficult to overcome the prejudice of the welders, so most employers use fluxed electrodes in spite of the greatly increased cost. My impression is that much of the trouble experienced with bare electrodes has been due to improper chemical composition, and might be overcome by the use of rods of uniform and correct chemical content. There is no doubt that the general use of costly fluxed electrodes will seriously impede the extension of arc welding in many directions.

Direct current is generally preferred to alternating owing to the greater ease of striking and maintaining the arc, and also to the fact that the welders' hand soon becomes unsteady when alternating current is used. Good welders prefer direct current for overhead work. These factors have combined to make the use of direct current general in British practise. Good work has been accomplished with alternating current, but it is not recommended by makers of equipment.

The results of experiments made by members of the British Research Committee regarding the effect of voltage agree with those of Mr. Wagner. Some earlier outfits were constructed on the assumption that a minimum of 100 volts was essential; but it is now customary to employ 60 to 70 volts at the generator. The question of the effect of current is more debatable. From an analysis of the Wirt-Jones tests, the conclusion has been drawn that "the stronger the current the better the weld." This interpretation does not appear entirely justified, in view of the discrepancies presented by the results. For example, No. 31, with 150 amperes direct current, shows an ultimate tensile strength of 60,900 lb. (27, 623 kg.), while No. 9, also with 150 amperes direct current, shows only 38,100 lb. (17,281 kg.) Until greater uniformity is obtained, it does not appear possible to generalize on this point. In some recent work, Dr. J. H. Paterson of the British Committee found that the effective maximum deposition was obtained with 140 amperes when 20 gr. per min. were deposited. Beyond this strength, owing to splashing, the effective deposit diminished. Moreover, analysis showed that the minimum amount of oxygen entered the deposit at 140 amperes. This point is urgently in need of settlement, with a view to guiding the design of equipment; and a series of tests under identical conditions as far as procurable, should be conducted to decide the question. At least six welds, using identical plates and electrodes, should be made at each current strength to enable a correct decision to be arrived at.

With regard to the speed of arc welding, the maximum, according to Dr. Patterson's figures, would be 1200 gr. or about 2.6 lb. per hr., not allowing for intervals. Experiments confirming this would be useful, so as to settle this point definitely.

J. Churchward (by letter): Mr. Hobart states that various kinds of electrodes are advocated, running from the cheapest fence wire at 10 cents per lb. up to covered electrode costing five times that price. The importance of the electrode question has not been covered extensively enough. It is of the utmost importance to have an electrode that will be absolutely uniform in its welding qualities; that the deposited metal be as near the same chemical composition of the parent metal as it is possible to have in electric arc welding. If these things are accomplished, the price, within reasonable limits, should be disregarded.

It has been suggested that, when welding wire shows bad welding properties, it be dipped into whitewash to insure its uniformity of flow. The welder must take extra precautions to see that the slag introduced by this coating is puddled out of the weld. It is better to buy a good bare electrode than to coat a bad one, as with the latter there is always the danger of introducing slag into the weld.

The specifications for electrodes from the Welding Committee of the Emergency Fleet Corpn. are: Carbon not over 0.18 per cent, manganese not over 0.55 per cent, phosphorus and sulphur under 0.05 per cent, silicon under 0.08 per cent. The high limits are covered by patents in which an excess of manganese was allowed. The average manganese content for old welding wire was not over 0.35 per cent. If I understand these specifications correctly, they do not demand the rejection of any wire that would give satisfactory results even though not of this chemical composition.

Ductility of Welds Made with Covered and Bare Electrodes. Relative to the ductility of welds made with covered and bare electrode welds, our data show that normally coated electrodes produce less ductile welds than bare electrodes due to the slag inclusions made with the former.

Effect of Voltage on Arc Welding. Increased voltage, say from 40 volts up, enables the inexperienced operator to weld with greater ease on account of the length of the arc this voltage allows; but it sacrifices the strength of the weld. Experienced operators will not vary the length of the arc more than 3 or 4 volts, regardless of the open-circuit voltage of the machine. The voltage used across the arc by expert welders varies from 16 to 19 volts. With an inexperienced welder or learner, records show that a high open-circuit voltage has a tendency to make the operator careless and thereby cause bad welds.

Overhead Welding. The quality of overhead arc welding depends entirely on the experience of the operator, the quality of the wire, and the characteristics of system used. With excellent wire, an expert operator, and a machine of the proper characteristics, overhead welding may be very easily as strong if not stronger than flat welding.

Different Lengths of Arc. It is essential in metallic arc welding that the arc be kept as short as possible. This introduces two variables, the quality of the wire and the expertness of the welder. An expert welder with a good wire will maintain a 16-volt arc; with a poor wire he will try to maintain the same length of arc with many interruptions due to the breaking of the arc. An inexperienced welder will maintain the length of arc that will enable him to deposit the metal with the least possible exertion. If the electrode is good, the arc will be short. If the electrode is bad the arc will be long, if the design of the machine so allows. With a long arc, there is less heat per unit area in the parent metal and therefore less concentration of welding heat than with a short arc. If the voltage is so high that the operator is enabled to draw a long arc, although it may be easier for him, the weld will suffer. One would draw the conclusion from the above that with the personal element, the current and type of electrode constant, the quality of the weld depends on the length of the arc.

Charles H. Kicklighter (by letter): The disk-depression method of electric welding was developed primarily as a simple and effective means for forming a highly efficient joint or seam between heavy commercial plates. After the preparation of the parts, by making flat-bottom depressions in the surfaces of the plates and introducing therein thin flat disks, the parts are brought into the proper position and subjected to pressure and heating electric current between the terminals of an electric welding machine. The intermediate bodies, or disks, are welded to both plates and are softened and forced into the depressions so as to permit the faces of the plates to come into immediate contact.

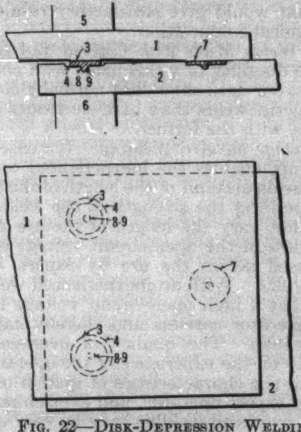


Fig. 22—Disk-Depression Welding

The diameter of the disk is slightly less than the diameter of the depression, but the thickness of the disk is greater than the depth of the depression, so as to hold the plates out of initial contact. Flat disks are used as the best results are obtained by having the areas of initial contact substantially the same as the areas of the desired weld and apply the pressure normal to these surfaces. The surfaces of contact are thereby brought to a welding heat at all points at about the same instant and a thorough fusion of the metals at all points is obtained by the time the intermediate metal is softened sufficiently to be forced into conformity with the depression. A plurality of such welds, properly staggered, makes an efficient joint.

Description of Process. Simple flat disks may be placed loose in flat bottom depressions, as in Fig. 22. Here disks *3*, in depressions *4*, between plates *1* and *2* are subjected to pressure and the heating electric current between terminals *5* and *6* of an electric welding machine. In order to hold the disks at the centers of the depressions during the progress of the work, small register projections *8*, on the disks fit snugly into register holes *9* in the bottoms of the depressions. A special drill for cutting the depressions and register holes is provided and a bank of, say, six, eight, or twelve such drills, operating simultaneously, will cut the depressions rapidly and economically. A special attachment limits the depth of the cut.

The shifting of the disks may be prevented by sticking them to the bottoms of the depressions; by fusing them at one or more small projecting points of the contacting surfaces. Depressions may be cut in each plate near the edge and the plates brought into position with the depressions overlapping or falling beyond each other, so that, in the welded joint, the full stresses can never come on a plate where it has been weakened in the least by such depression.

Disk-depression Welding versus Riveting. The advantages of the disk-depression method of electric welding over riveting are: a joint of greater efficiency and greater economy in production. A joint of very high efficiency is obtained, for the plates are not weakened by the cutting away of material in punching rivet holes. The small amount of metal removed in making the depressions is negligible and even this is restored by the disk material, which is fused into and completely fills the depressions. By cutting depressions in both plates near their edges and overlapping these, the full stress never comes on a plate where depressions have been cut. The only reason why the joint may not be made as strong if not stronger than the solid plate is a slight tendency of the metal of the plates to crystallize when heated, but due to the small region heated and the low temperature attained in the greater part of this region, this weakening effect is very slight.

With proper means of bracing or mechanical framing, the seam can be completed for the present cost of rivets alone. The amount of heating current is relatively small and the heating is limited to the immediate position of the weld. As the welds can be made rapidly, the conduction of heat by the plates causes very little heating of other parts.

Disk-depression Welding versus Arc and Autogenous Welding. The advantages of the disk-depression method of electric welding over arc or autogenous welding are: greater efficiency of joint; greater saving in time, labor, and electric power; and elimination of distortion troubles during heating and resulting stresses after cooling. Arc-welded or autogenous-welded joints are never as strong as the solid plate unless they are built up of considerably greater thickness than the plate. The great saving in time, labor, and power will be appreciated when one considers the length of time, labor, and the amount of heat necessary to weld a seam between thick plates by the arc or autogenous method. With the disk-depression method, a few seconds per weld is sufficient.

Disk-depression Welding versus Spot Welding. The advantage of the disk-depression method of electric welding over present methods of spot welding are: greater efficiency of joint; greater economy of welding current; smaller capacity and less cost of welding machines; practically no trouble because of distortion of work during operation due to heat; greater thicknesses of plates that can be welded; and simplicity and ease of preparing work prior to welding operation.

A high efficiency of the joint is insured by having clean surfaces free from scale. The operator is then certain that a perfect weld has been made when the disk becomes sufficiently fused to let the plates come into immediate contact. If the contacting surfaces are covered with scale, he cannot be absolutely sure of a perfect weld. The disks are of sufficient thickness to hold the plates out of immediate contact before the welding operation and thus serve to narrow the path of the welding current and concentrate its heating effect. The resistance to the welding current is small, thus requiring a minimum amount of electric power and a welding machine of relatively small capacity. With clean surfaces, the mechanical pressure does not have to be so great as where there is scale or the plates have to be forced into contact about an intermediate body with no depression in connection therewith. For this reason, a welding machine of lighter construction and smaller cost will perform the work.

The greater part of the resistance to the welding current is at the surfaces where the disk contacts with the plates. Here the heat is concentrated and in a few seconds it becomes so intense as to fuse the thin disk to both plates before the surrounding parts of the plates become heated by the conduction of heat. The seam does not become nearly so hot as in other methods of welding, the distortion of the work due to expansion is not nearly so great, and the stresses set up in the plates due to cooling and unequal contraction are small. As most of the heat is developed and concentrated at the contacting surfaces of the disk, plates of considerable thickness may be easily welded. As to the thickness of the material, there does not appear to be other drawback than the matter of forcing the plates through a very small distance into close contact. As the region of intense heat is so small and the weld is performed before the heat has spread much by conduction, the region affected by any tendency to crystallize is very limited and the strength of the joint is not appreciably affected thereby.

The slight increase in cost and the slight loss of simplicity occasioned by cutting the depressions is far outweighed by such considerations as the insuring of a perfect weld, a highly efficient joint, a large saving in electric power, the use of lighter and less expensive welding machines, rapidity of the welding operation, freedom from heat-distortion and stresses set up by the subsequent cooling and contraction, less crystallization of the metal due to heating, etc. It is preferable to cut the depressions before the plates are bent into shape.

Theory of Method. Fig. 23 illustrates the contracted path of the heating electric current as it passes between the broad flat-end terminals *5* and *6* through plates *1* and *2*. Most of the resistance to this current is along the surfaces of contact *aa* and *bb*, and most of the heat is developed in the disk *3* and in plates *1* and *2* in immediate contact therewith. Fig. 24 shows how it is often attempted to concentrate the pressure by means of conical terminals *5* and *6* in order to obtain a weld between plates *1* and *2*. Due to the rigidity of the thick plates, this pressure is distributed over a wide range between the plates *1* and *2*. The heating current also spreads considerably at *cc* where plates *1* and *2* are in contact. The path of the current is contracted most at *dd* and *ee*, causing considerable loss of heat, marring of the surfaces of plates *1* and *2*, and damage to terminals *5* and *6*.

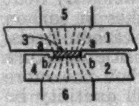


FIG. 23

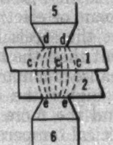


FIG. 24

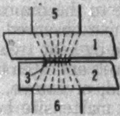


FIG. 25

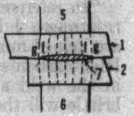


FIG. 26

Fig. 25 illustrates a disk *3* placed between two plates *1* and *2* and welded thereto by current passing between terminals *5* and *6*; if it is now attempted to force the plates into immediate contact, as in Fig. 26, the disk flattens and spreads out as at *7*. The area *ff* may be welded if the surfaces are clean, but the areas *fg* where the molten metal has been squeezed out against the cold plate surfaces is not welded; there is also some difficulty in forcing the plates into immediate contact around the weld. There is considerable widening of the path *gg* of the current between the plates; this causes considerable heat in the joint, resulting in heat-distortion and subsequent cooling stresses. The region of partial crystallization is greatly enlarged.

Tests. In order to determine the efficiency of the disk-depression method of electric welding, commercial bars $\frac{3}{8}$ in. (9.5 mm.) thick and 1 in. (25.4 mm.) wide were lapped and welded, some in two places and others in three. Depressions were cut in one of the bars $\frac{7}{16}$ in. (11.1 mm.) in diameter and $\frac{3}{64}$ in. (1.1 mm.) deep. Disks $\frac{1}{16}$ in. (1.5 mm.) thick cut from a $\frac{3}{8}$ in. commercial round bar were placed between blunt flat welding terminals of an ordinary spot-welding machine. The capacity of this machine in ordinary spot-welding work was $\frac{1}{8}$ in. (3.1 mm.) plates or less, but we were enabled to get splendid welds on stock $\frac{3}{8}$ in. and $\frac{1}{2}$ in. thick by the disk depression method. Some samples were grasped in the jaws of a vise and hammered over until they tore apart, revealing a perfect weld and a fibrous structure. Others were tested in a tensile testing machine until they broke. Bars with two welds broke at 13,090 lb. (5,937 kg.) and those with three welds at 19,405 lb. (8,801 kg.). The solid stock broke at 19,275 lb. and had an elastic limit of 13,125 lb.

Zay Jeffries (by letter): The following is quoted from a letter, dated Sept. 9, 1918, to Mr. Hobart.

"In reference to the improvement of electric welds by heat treatment, I have observed that the steels higher in carbon are helped more by heat treatment than the low-carbon steels. High-carbon steels are often the cause of a martensitic structure in the weld and, of course, the higher the carbon the more brittle will the martensite be, and the more will ductility be restored by heating above the critical temperature and cooling slowly. I have observed well-defined martensite in welds made with piano wire and with a nickel steel containing about 0.2 per cent carbon and 3.5 per cent nickel. As the carbon decreases, the martensitic needles become farther apart and the steel becomes more ductile.

"In addition to the martensitic needles, there are other needles to which Mr. Ruder has referred, which he seems to have proved to be nitride of iron, or at least the presence of the needles if they are not iron nitride seems to be due to the presence of nitrogen. We must not conclude prematurely, however, that the needles themselves are composed of nitride of iron because Timofeeff shows the development of the same microstructure by straining commercially pure zinc at room temperature. There is no sign of this structure in the annealed zinc so that the needles must be a direct result of the straining. At the same time, Howe has produced needles of a similar nature in manganese steel and there is no reason to suppose that the composition of the needles is any different from that of the remaining mass of the metal.

"But it has not been demonstrated that welds containing needles are always brittle and that welds not containing these needles are always ductile. In fact, I have examined many welds under the microscope that did not show the needles, but which were brittle.

"Recently J. C. Lincoln gave me four samples of electrically welded steel for microscopic examination. These were made with a modified carbon-arc process. Two steel plates ⅜ in. thick were placed edge to edge on a flat surface and the first side welded, then the plates were turned over and the other side welded. Bending tests were made in such a manner that the two plates bent parallel with the weld. The bending was stopped when the first crack appeared in the weld. Weld No. 1 was made in an atmosphere of carbon dioxide and a crack appeared after bending 90 deg. Weld No. 2 was made in an atmosphere consisting of about 90 per cent nitrogen and 10 per cent carbon dioxide and bent 60 deg. before cracking. Weld No. 3 was made under flux and it bent 50 deg. before cracking. Weld No. 4 was made in the air and a wide crack opened up after bending 45 deg. Welds Nos. 1 and 2 are unusually ductile for electric welds and they also contain the needles. Weld No. 3 contains the least number of needles, but it is not the most ductile. Weld No. 4 contains needles and it is the most brittle of the four. This weld, by the way, is an unusually good air weld.

"Having found, therefore, that electric welds containing needles may be ductile and electric welds containing no needles may be brittle, it seems evident that the needles are not the main cause of brittle welds. In this connection I should like to present J. C. Lincoln's opinion, namely, that oxygen is the "embrittling element" in electric welds. He has reached this conclusion after many and varied tests. The same idea is strongly suggested by the most interesting results reported by Mr. Wagner with magnesium and boron suboxide. Both of these would be oxygen removers. That the magnesium did remove oxygen seems apparent from Mr. Wagner's remark that the liquid metal in the weld "boiled" when the magnesium came in contact with it.

"Iron oxide can be seen in the welds under the microscope in the form of globules. In a microscopic examination we cannot hope to see particles of iron oxide containing less than about a million molecules. Owing to the nature of the transfer of metal in the making of an electric weld and the rapidity of solidification, it would not be surprising if much oxygen was present in the form of iron oxide in particles too small to be seen under the microscope. So far as the microscopic examination is concerned, these particles might actually be in solution. It is, therefore, possible that the "embrittling element," or at least the portion of the element which is the most harmful, is present in such small particles that it cannot be seen under the microscope.

"The weld made by Mr. Wagner with Armco iron weld rod coated with magnesium powder is the most ductile electric weld I have seen.

"It seems to me that this lead is so promising that it should be followed with the utmost speed. If it works out commercially, as well as in the sample submitted, it is my opinion that the matter of the quality of electric welds will be quite satisfactory. If the weld is ductile, it need not have as high tensile strength as when it is brittle."

It seems probable that Mr. Lincoln's experiments in no case entirely excluded nitrogen, but it is probable that there was a smaller percentage of oxygen in the atmosphere surrounding the welds where gases other than air were used than in the weld made in the open air. The weld made in an atmosphere of 90 per cent nitrogen should have less oxide between the grains than that made in air, but there seems to be no reason why it should have less nitride present than the one made in air. The weld made in the 90 per cent nitrogen atmosphere, however, was more ductile than that made in air, which would seem to bear out Mr. Lincoln's conclusion that oxygen is the "embrittling element."

It may be definitely concluded that nitrogen is the element that embrittles the grains. If the grains contained less nitride and hence had lower elastic limits, the grain-boundary strength might remain unchanged (that is the same as now observed) and the weld would show considerable increase in ductility on tensile test, without necessarily decreasing in strength. It would, therefore, seem advisable to keep the nitrogen content low in electric-steel fusion welds.

It also seems that the grain-boundary weakness is due to either oxygen or nitrogen or both. It remains to be decided how much each of these elements contributes to the weakness of the grain boundaries and to eliminate one or both from the welding atmosphere if sufficiently beneficial.

A. S. Kinsey (by letter): In paragraph 4 of Section 16 Mr. Hobart states that "an enormous volume of experience in arc welding has gradually accumulated and it controverts the correctness of the view" of a gas welding publication quoted which shows the advantages of the oxyacetylene over the electric-arc process. I presume the author refers only to the statement of the limits of the arc weld, and that even though there may be an absence of acceptable test data on gas welds on which to base his judgment, he will agree that oxyacetylene welds have, when carefully made, proved highly satisfactory to the best engineering practise, both in ductility and tensile efficiency. Also, there surely can be no doubt in the mind of the author as to the accuracy of the statement of the quoted article, Section 16, as to the value and adaptability of the oxyacetylene torch for annealing a steel weld. The reference to the absence of satisfactory laboratory tests of oxyacetylene welds need bear no undue significance, as the real situation has been that the oxyacetylene welds were so much stronger and more economical than the forged welds that tests of the physical value of gas welds were never necessary. They have stood the test of actual practise. On the other hand, is it not true that in the development of the electric-arc weld for use on steel plates for ships, it has been necessary to know exactly what are the physical limits of the arc weld in order to determine the risk connected with its use on ship hulls? This is what caused laboratory tests to be made of arc welds.

H. G. Knox: (by letter.) There is one subject that Mr. Hobart seems to have inadvertently omitted from his very valuable paper, and that is corrosion. The shipbuilder, perhaps more than anyone else, has constantly to guard against it, our chief reliance being paint and galvanizing, where practicable.

The corrodibility of a welded joint in salt water must be determined before deciding whether this type of construction is to supersede riveting. At the present time, this question cannot be answered, although considerable work has been started in an attempt to arrive at a conclusion. The British Admiralty and several British shipbuilders have studied it extensively. Their experiments and conclusions have been very carefully followed and reported in this country by Naval Constructor Goodall of the British Navy. To date, these show that no undue fear need be felt from corrosion. At the Norfolk Navy Yard a series of plates welded with various electrodes were submerged in dilute sulphuric acid, such as is used in cleaning prior to galvanizing. While this test may not be conclusive in any quantitative way, it does show a marked difference in the corrodibility of welds made with different electrodes under different conditions. To draw any conclusions from this accelerated test would be, however, ill advised.

In riveted joints, the exposed heads and points of the rivets always show the greatest tendency to corrode. This is due to two reasons: first, as they are raised above the surrounding surface the paint is more susceptible to abrasion; and, second, electrolytic action is set up between rivet and plate. These same reasons lead us to expect a tendency of the welded joint to corrode.

Various methods have been suggested for decreasing the probable inherent corrodibility of the weld, in addition to the customary coat of paint. These generally embody the introduction of alloys, principally copper, into the electrode material. Plating the electrode with copper is not effective in that the deposited copper segregates in the weld and actually increases the electrolytic action. As far as is known, no extensive test on the so-called copper-bearing alloy steels, which might be made up into electrodes, has been carried out, but when obtainable, electrodes of this alloy will be subjected to service tests.

About 9 months ago, we started attaching small plates, welded with different manufacturers' electrodes, to the under-water hulls of steel ships placed in dry dock. Up to the present time none of these ships have redocked in the Norfolk Yard, so no results from this source are available. We have also used several makes of electrodes in the construction of an arc-welded battle towing target, which will soon be launched. See Fig. 27.

It can readily be seen from the foregoing that little definite information on the subject of corrosion is available, but for the time being, at least, I do not believe any special precautions need be taken in the protection of welded joints beyond that employed for the surrounding plates.

John Martin (by letter): It was the writer's privilege to represent the American Bureau of Shipping on the Welding Research Sub-committee over which Mr. Hobart presided. The foreword that the paper deals principally with investigations undertaken by the Welding Research Sub-committee relieves the author of any save editorial responsibility for certain irreconcilable statements, some of which might be expected in the experimental research data of any new industry and especially in one like metallic-arc welding, which introduces so many variable factors and conditions essential to the process that a long acquaintance with the technic and data records is required before one can detect and discard an irreconcilable result.

The tables of Section 5 state that a Norway bare electrode wire contains 0.007 per cent sulphur and that the deposited metal of weld contains 0.072 per cent sulphur. As the ½-in. ship plate used for testing electrodes held 0.032 per cent sulphur, where did the increased sulphur come from? The metallic arc is a reducing agent and no flux was used. This illustration is used to show how carefully all test reports should be scrutinized and compared before insertion in finished records and in no sense to detract from the value of the author's compilation.

Composition of Ship-plate Steel. Where ship-hull plates are united metallurgically (welded) instead of physically (riveted), the chemical composition of the plate will become as important as the physical performance of its test specimen prior to installation. A certain amount of slag (reduction of foreign matter, carbon, manganese, phosphorus, sulphur, and silicon) is driven off in all fusions whatsoever the welding process used (smallest, however, in spot welding) and the chemical composition of the ship plate must be such that, after welding, stresses will be transmitted through the joint as efficiently as through the plate, that is, the joint shall be 100 per cent efficient the same as many riveted joints. This condition is impossible of accomplishment by arc welding from our present knowledge of the art as the deposited metal in the weld is always pure iron whatever the chemical composition of the electrode may be. In the spot weld, although the writer has no knowledge of their chemical composition, the "spots" more nearly approach the original composition of plate as the slag driven off (a measure of the chemical change within metal) is inconsiderable compared with that of arc welds.

Fusion Welding and Spot Welding. The author distinguishes between welds as fusion welds and spot welds. Spot welding is fusion welding and to differentiate by grouping under fusion is apt to create a false picture in the uninitiated reader's mind It is suggested, starting with the premise that a weld with heat means fusion, to classify welds processively as spot, gas, carbon-arc, metallic-arc welds in which the physical difference is that of continuity of weld.

Metallic-arc Welding. Inasmuch as overhead vertical and horizontal welds are equally well performed with the metallic arc, it may be assumed that the flow (or bombardment of metallic molecules) of metal from the electrode to the joint follows Newton's law of attraction of matter (directly as the mass and inversely as the square of the distance). Molecular cohesion of the electrode molecules, lessened by the heat of the arc, is overcome by the attraction of like mass in the plates; the result is the passage of electrode molecules to the plate, whence adhesion ensues, which becomes cohesion upon cooling contraction and consequent crystallization of deposited material.

Spot Welding. The author's description of spot welding is satisfactory save that no initial voltage is specified. In one test performed in the writer's presence 32,000 amperes with a voltage of 4 successfully spot-welded pairs of ⅝ in. to 2½ in. (15 to 63 mm.) ship steel plates together, the diameter of the spots being 1 in., the pitch 3 in., and the hydraulic pressure 7½ tons, the time of the current was 15 sec. and 15 ft. (4.5 m.) was welded in 30 min. The spots sheared at 56,000 lb. or partly pulled and sheared at 60,000 lb. Water- and oil-tight joints resulted without calking. For plates of the same pitch calking or arc fillet welds would be required for water-tight joints; though by lessening the pitch and decreasing the diameters of the spots, practically the same water-tight results could be had.

Summarization of Points for Discussion. From the fact that the author has found 34 points, the discussion of which he believes may advance welding technic but only one of which concerns spot welding, leads one to infer that much is to be learned about arc and little about spot welding. The writer believes that the greatest obstacle to the adaptation of spot welding to hull construction is the difficulty of manipulation of spot-welding apparatus in and about the hulls of ships, and that is but a mechanical detail to be worked out. If a year ago it had been realized that there were 33 variables in the arc-welding and but three in the spot-welding problem, the latter would have determined perhaps the more fruitful field for experimental investigation in seeking a satisfactory weld for ship construction.

The writer differs with the author's contentions that the best basis to express the speed of arc welding is in pounds of metal deposited per hour. Feet of joint per hour welded would better assist the ship builder in his estimates and cost sheets and certainly would afford a simpler basis for comparison between welding methods. The writer heartily endorses the author's belief that spot welding has a great future as applied to shipbuilding but does not agree with the statement that spot welding affords a method of preliminarily jointing the hull plates after which the required strength is furnished by arc welding. The writer would have expressed the thought thus: spot welding affords a method of joining the hull plates, after which the required water tightness may be secured by calking or calking arc welding.

The author points to the future application of spot welding to hull construction when he finds that the strength of the spot-welded joint, with calking arc weld, is 50,000 lb. (22,679 kg.), of the arc-welded joint, with calking arc weld, is 37,000 lb. (16,782 kg.); and of the riveted welded joint, with calking arc weld, is 13,000 lb. (5896 kg.). In other words, the joints are as 4 : 3 : 1, respectively. That is, the spot-welded joint with calking arc fillet is one and one-third times as strong as the arc-welded joint and four times as strong as the riveted joint.

Conclusion. The writer agrees with the author in his conclusion that welding should be applied in the general fields where the hazard to life would be minimum before applying it to constructions in which joint failures would jeopardize human lives. But he deprecates the author's under-rating the stresses in ships as being less severe on welded joints than those in high-pressure containers. The stresses in ships are dynamic, tensile and compressive, torsional, alternating, and an admixture of all these, whereas those in a high-pressure container, even in marine boilers, are largely static and it is the alternations of stresses that fatigue ship members and require the ever watchful eye of the classification society to conserve property and preserve life.

O. H. Eschholz: (communicated after adjournment.) In discussing the quality of welds made with a-c. and d-c. arcs, Section 11, a number of authorities are quoted as saying that a-c. welds are better than d-c. welds. As a review of the data given in the Wirts-Jones tests shows about equal maximum strength for d-c. and a-c. welds, I would be interested in knowing what basis exists for this opinion.

In the last paragraph of Section 19 Mr. Wagner is quoted as saying that "Tests were made on a ½ in. plate with current values as follows: 80, 125, 150, 180, 220, 275 and 300 amperes. These tests show improvement in the tensile strength and bending qualities of welds as the current increases." How is this statement reconciled with the information in the following table compiled from the data in the Wirts-Jones tests?

Characteristics of D-C. Arc Welds.

PLATE ULTIMATE TENSILE STRENGTH = 64,700.

Test No.	Weld U. T. S. in % of plate U. T. S.	Current	Amperes per sq. in.	Per cent Elong. in 2 in.	Angle at which break started.
2	97 %	155	8,075	11.5	42°
5	94.5	145	7,554	8.5	55°
31	94.2	150	6,930	8.0	..
1	92.5	175	9,118	9.	42
8	92.0	115	9,756	13.5	60
8	90.	115	9,349	14.	..

These results show that a maximum strength is obtained at 155 amperes with a decrease on either increasing or decreasing the arc current from this value.

In Section 21 the author states that "The investigations of the Welding Research Subcommittee are indicating that at least 10,000 or 12,000 amperes per square inch are suitable for electrodes of 1/8 in. and 5/32 in. diameter................." This conclusion can hardly apply to d-c. arc welding as the Wirts-Jones tests show a falling off in weld strength on increasing or decreasing the current density from 8000 amperes per sq. in., which confirms my experience.

a—(Good fusion) b—(Poor fusion)

FIG. 28

The use of a smaller bevel angle, as discussed in Section 24, undoubtedly leads to welding economies. However, it should be recognized that the use of this refinement is only justified when coupled with a proper "free" distance and a highly skilled operator. Fig. 28a illustrates the possible excellent fusion that may be obtained with 60 deg. scarf, while Fig. 28b illustrates the probable incomplete fusion resulting if the welder is fatigued or indifferent to the factors that produce fusion.

An address presented at the 7th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 20, 1919.

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ENGINEERING AND INDUSTRIAL STANDARDIZATION

BY C. A. ADAMS

INTRODUCTION

MOST of us speak of standardization in a vague sort of way, usually having in mind some particular corner of the field, but without sensing the comprehensiveness of the word or the magnitude and importance of the field covered, not realizing that the cooperation without which our modern society would be impossible, would in turn be impossible without a large measure of standardization. It might almost be said that the degree of standardization in any nation is a measure of its civilization, certainly in the material or industrial sense of the word.

Even in the tribe a common language was necessary, namely a standardization of the meaning of words. A dictionary is merely a recorded standardization of the spelling, pronunciation and meaning of words, as determined by usage and common consent. To be sure we still have dialects and colloquialism, but these are gradually decreasing. In civilized intercourse the accurate use of words is absolutely essential to mutual understanding, however rare it may be, and although new words are being slowly added, the meaning of each must be stable, that is standardized, if it is to serve its purpose.

As between nations we still have many different languages, although most of the civilized world now uses substantially the same alphabet and there is a persistent effort to establish a universal language.

After language, followed standardization in coinage, weights and measures. Our present Bureau of Standards is the outgrowth of the old Bureau of Weights and Measures, which was largely the repository for standards of length, weight, volume, etc. Imagine the hopeless confusion in this country if each state had its own language as well as its own system of coinage and of weights and measures. Even as between nations differences in language, coinage, weights and measures, constitute serious barriers to intercourse, social and commercial, which tend to prevent interchange of ideas and commodities and thus to retard that mutual understanding which makes for peace.

But standardization has already extended far beyond these fundamental fields and is now growing with leaps and bounds.

It is my purpose in presenting this subject to discuss chiefly standardization in these less fundamental fields; to point out its importance and its dangers; to indicate the comparative chaos which still reigns in most parts of the field of engineering and industrial standardization, not only in the standards themselves but underlying this in the methods of creating them; and finally to outline the machinery proposed by the American Engineering Standards Committee to remedy this condition.

Before undertaking the general discussion a brief classification and review of each field may be of interest, excluding the alphabet and coinage as outside of the engineering and industrial group.

CLASSIFICATION OF STANDARDS

Language. Definitions of scientific and technical words, terms, phrases, abbreviations, symbols and diagrams.

Standardization of these elements of our technical language is just as fundamental to intelligent intercourse in this field as in the case of ordinary word intercourse; in fact it is more fundamental since we are here dealing mostly with accurately measurable quantities.

Some progress has been made in this field even to the extent of international standardization of terminology, but chiefly in connection with very fundamental matters, there being still in the outlying fields a very wide diversity of usage in the matter of symbols, abbreviations and diagrams, and in a lesser degree in the use of words. There are at least fourteen different organizations in this country alone, each of which has its own system of diagrammatic representation covering electrical wiring and devices. Many other similar illustrations are available of the frightful inefficiency of the present haphazard uncoordinated work in this field.

Standards of Quantity. Length, area, volume, mass, weight density, pressure, heat, light, illumination, electricity, magnetism, radioactivity, physical constants and their relation to each other.

Some of the fundamental standards of this group are of such importance in the every-day life and trade of a nation that they are prescribed by law and each city or county has the necessary machinery for enforcing the law.

Nearly all European nations as well as the Latin-American nations have adopted the rational metric system, whereas the English speaking nations still cling to the English system with its confusion of oddly related weights and measures.

However since the electrical and magnetic units are the same the world over, since they are based upon the metric system, and since practically all scientific work is handled by that system, a vast amount of time is wasted by the English speaking nations not only by the necessity of becoming familiar with both systems, but also in their use due to the frequent necessity of transformations from one to the other. This latter loss is particularly serious in the field of electrical engineering. It is my own conviction that in the case of a real electrical engineer, a full year of his professional life is wasted on this account. This is partly during his educational work and involves an immense loss of efficiency due to the diversion of his attention from the real understanding of the subject.

Standards of Quality. Specifications for materials together with standard methods for testing their qualities.

Certain standards of this type are also of such importance to the welfare of society that they are prescribed by law notably in the food field, and a bill is now before Congress for the standardization of fertilizers.

Standard specifications and tests for the materials of engineering and construction are almost wholly prepared by the American Society for Testing Materials, which society was organized solely for this purpose. In no other field is the work of standardization better organized.

Standards of Performance. Operating characteristics of machines and devices, output, rating, speed, efficiency, durability, etc., as specified in terms susceptible to measurement, together with the methods of making these measurements.

The most notable example of comprehensive work in this field is to be found in the Standardization Rules of this Institute. Another is in the Power Test Code of the A. S. M. E.

Standards of Practise. Regulations or codes dealing with

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construction, installation and operation, based upon considerations of safety, quality, economy, convenience, etc.

The oldest code in the electrical field is the National Electric Code of the N. F. P. A. first issued in 1897 and since frequently revised. This has been largely copied in the local municipal regulations of most large cities.

The National Electrical Safety Code was issued about three year ago by the Bureau of Standards after long and careful collaboration with the most interested organizations. This covers safety to life rather than to fire.

The Boiler Code covering standards of construction for boilers was issued by the A. S. M. E. in 1914 after some years of laborious work involving cooperation with the interested organizations.

There is now under consideration the preparation of some fifty odd industrial safety codes. This work will probably be carried out according to the procedure prescribed by the American Engineering Standards Committee, outlined below, with the appropriate organizations acting as sponsors. The Bureau of Standards will doubtless serve as sponsor for some of these codes.

Dimensional and Type Standards. In any product the standardization of types, sizes or sets of dimensions in order to reduce the number of types and sizes and hence the cost of production.

A few illustrations will indicate the tremendous scope of this group and the comparative chaos that still reigns.

Among the earliest and most fundamental attempts in this group are:

Screw Threads, Bolts, Nuts, etc.

In 1864 a Committee of the Franklin Institute recommended the adoption of a system devised by William Sellers, now known as U. S. Standard.

In 1884 Charles Bower, of Warder, Bushnell & Glessner, Springfield, Ohio, adopted standard sizes of screw threads for bolts and taps, based upon the U. S. Standard screw thread.

In 1895 the American, Hartford and Worcester Machine Companies issued standards for set-screws and cap-screws, also based upon the U. S. Standard screw thread.

In 1906 the Association of Licensed Automobile Manufacturers adopted standards for automobile screws and nuts.

In 1907 the American Society of Mechanical Engineers accepted the report of its committee on standard proportions for machine screws. This Committee was appointed in 1902, and had devoted five years to the task. As far as the screw threads themselves are concerned, the American Society of Mechanical Engineers standard differs only in very minor details from the Sellers' or U. S. Standard, but the A. S. M. E. report gives standards also for taps, special screws, special taps, and screw heads of various types, together with tolerances.

In 1912, the Society of Automobile Engineers was organized and has adopted fine screw thread standards, as well as many other size standards of automobile materials and parts.

In the fall of 1918 a screw thread commission was appointed by act of congress and is still at work. In this work some consideration is being had of the international situation, and although there are obviously great difficulties in adapting our threads to those of the metric system nations, the differences between the U. S. and the British threads are slight, though sufficient to prevent interchangeability without excessive tolerances.

I have been told on good authority that there are fifteen different types of screw thread standards in use by the War Department alone.

Wire, Drill and Sheet Metal Gages

Of these there are several varieties still in use, although for copper wires, the Brown & Sharpe or American Wire gage is most commonly used for the smaller sizes, and the Edison or circular mil gage for all sizes above 0000 B. & S. All these gages were originally adopted by some corporation such as the twist drill gage of the Morse Twist Drill & Machine Company, the Stubbs steel wire gage, etc.

An act of Congress in 1893, established the Standard Gage for sheet iron and steel. Previous to this time there were many gages in use, and the addition of this one seemed only to add to the confusion.

In 1895 a joint Committee of the A. S. M. E. and the American Railway Master Mechanics Association agreed to recommend the use of the decimal gage,—that is a gage whose number is the thickness in thousandths of an inch, and to recommend the abandonment and disuse of the various other gages then in use.

In 1904 the Westinghouse Electric and Manufacturing Company abandoned the use of gage numbers referring to wire, sheet metal, etc.

There are now at least eight different sheet metal gages in use in this country.

Other fundamental standards in this group are,—pipe threads and flanges; pipe unions; and special threads for fixtures and fittings, *e. g.* incandescent lamp sockets. Consider the enormous saving in expense and the convenience resulting from our almost universal standard lamp socket.

In the less fundamental part of this field, namely the standardization of types and designs, most of the work has been done until comparatively recently, within individual corporations as a matter of production economy. A notable example of this is the Ford automobile.

But within the past few years much has been accomplished in the automobile industry as a whole by the S. A. E.

This also is true in certain parts of the railway field in the standard freight and passenger cars, through the Master Car Builders Association. The variety is however still enormous.

There are at least 500 different types of locomotives, although the U. S. Railroad Administration has recently reduced the standard types of passenger locomotives to twelve.

One of the most striking examples of the possibilities in this field is the attempt only just started by one of our most progressive manufacturers of electrical machinery, a member of this Institute, to develop standard electric motor designs. Nine manufacturers are working together to this end. Consider the immense advantage to the consumer resulting from the general interchangeability of parts, and the reduction in the number of spare parts carried by a user who has motors of several of these cooperating manufacturers. Moreover the designs will contain the best features of all these makes, and there will still remain ample opportunity for competition in production cost, and quality of workmanship.

NEED AND VALUE OF STANDARDIZATION

This would hardly seem to need argument, but a few illustrations may not be out of place.

Not many years ago the average automobile had nearly as many sizes and types of bolts as there were bolts on the car, with a resulting inconvenience almost insufferable. Since that time the Society of Automobile (now Automotive) Engineers has carried on a tremendous campaign of standardization which has not only vastly reduced the annoyance of repair work to the purchaser but has greatly reduced the cost of automobiles. It is also largely responsible for the unquestioned primacy of this country in that field.

Another illustration from the same field was when that same Society started to standardize the sizes of steel tubing. All the manufacturers agreed except one who persisted in his odd sizes. Shortly thereafter this manufacturer, wishing to purchase some steel tubing, asked for prices and shipment. He was told that the price would be about 30 per cent higher than for the standard sizes, and that the shipment would be made in three months as against immediate shipment of the standard sizes from stock. It is needless to say that this manufacturer soon adopted the standard sizes.

An illustration from the electrical field has to do with the rating of electrical machinery. In the early days there was no agreement as to what was meant, *e. g.*, by a ten horse-power motor. As a result fair competition was impossible. The ratings of the several manufacturers varied as much as 30 per cent and the customer was at the mercy of the persuasive talents of the salesman. It took the Standards Committee of the A. I. E. E. five years to revise its old system of rating of electrical machinery in such a way as to be rational, scientific, satisfactory to all concerned, and capable of reasonably accurate checking by commercial tests. The results of this work have proved to be of world-wide value. They have placed competition on a far more equitable and generally satisfactory basis; they have encouraged more careful and thorough designing; they have encouraged a more careful selection of size on the part of the user, and have generally resulted in better machines.

DANGERS OF STANDARDIZATION

In certain fields notably in standardization of types and in dimensional standardization when the latter is carried into complete designs, there is an obvious danger that standardization may develop into crystallization and serve as a brake on progress; but this is merely an obstacle and not an argument, since it is obviously possible to keep the standards abreast of progress. There is certainly no more difficulty in improving

12 standard types of locomotives than in making the same improvements in the 500 types which exist today. If the twelve types are not sufficient to meet effectively the variety of demands of service, make it 20 or 30 or 50; but the maximum real need will certainly be so far below the present number as to constitute an annual saving of many millions.

The more fundamental a field the greater the need of standardization, and the greater the difficulty of making a change.

A notable example of this latter difficulty is in connection with our abominable English system of weights and measures which just "happened" in those early days when the importance of a rational or simply related system was not appreciated. Now each generation is unwilling to shoulder the expense of a change which will be of chief benefit to ensuing generations. But this would hardly constitute a sufficient reason for never adopting a standard system of weights and measures.

Similarly but in quite a different field, it could hardly be claimed by the most pessimistic that standardization in the manufacture of automobiles had been anything but a striking success.

MACHINERY OF STANDARDIZATION

At present the work of standardization is carried on by numerous organizations each in its own field and with its own machinery, usually through a standards committee. But the fields overlap and the machinery differs largely in the different organizations. In many cases the committees are made up of men who do not realize that a standard must be not only rational and technically correct but also acceptable to all those interested in its production and use. The mere promulgation of a standard by any society however powerful, does not make it a real standard.

Moreover the fields of the several organizations overlap in large degree; there is much duplication of effort and confusion of result, and many of the standards promulgated are practically useless.

Cooperation between two or more organizations has been tried, sometimes successfully, but there was no recognized machinery of cooperation, no standard procedure, and confusion still reigns.

To meet this need there has been recently created an American Engineering Standards Committee whose purpose, organization and method of operation are outlined below.

AMERICAN ENGINEERING STANDARDS COMMITTEE*

The American Engineering Standards Committee was formed by joint action of five national engineering societies (American Society of Civil Engineers, American Institute of Mining Engineers, American Society of Mechanical Engineers, American Institute of Electrical Engineers and the American Society for Testing Materials), called the *Founder Societies*, to meet a long felt need of some available and satisfactory machinery for the development of engineering and industrial standards, by the operation of which duplication would be avoided and cooperation between all interested organizations and Government departments secured; so that when a standard or group of standards is developed it will be acceptable to all concerned and therefore a real American standard. Moreover as international standardization develops, there is imperative need for an authoritative national body to deal with the corresponding foreign bodies. National Engineering Standards Committees are already in operation in England, Canada, France and Holland, and others will undoubtedly be organized in the near future.

The A. E. S. Committee machinery now proposed for the development of standards is made up as follows:

a. The Committee proper or the "*Main Committee*" (with three representatives from each of the five "Founder Societies" and of three Government departments), whose functions are chiefly those of an organizing, coordinating and steering Committee.

The War Department, the Navy Department and the Department of Commerce (Bureau of Standards) have already been invited to join in this movement and to appoint representatives on the Main Committee. The War Department has already accepted the invitation and is now appointing representatives. Assurances are at hand that the other departments will also accept.

b. *Sectional Committees* one for each group of standards (with representatives from all organizations or Government departments vitally interested in that particular group of standards) whose function is to prepare the standards in question under the direction of the most vitally interested organization known as the "Sponsor Body".

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*In discussions with prospective cooperating societies, certain proposals for changes in the method of selecting the members of the Main Committee have been proposed and are under consideration; but these changes will not affect the general procedure or interrupt the continuity of the work of the Committee.

c. The *Sponsor Society* or *body* may be one of the Founder Societies, a Government department, or one of the "Cooperating Societies" or organizations.

d. Under *Cooperating Societies* it is intended to include all organizations interested in the production of standards and willing to cooperate.

PROCEDURE

1. When the development of a particular group of standards is proposed, the Main Committee assigns the work to the appropriate organization as "Sponsor", or, if the situation seems to indicate that more than one organization is equally interested, to these organizations as "Joint Sponsor."

2. The sponsor then appoints the Sectional Committee subject to the approval of the main committee. The purpose of this approval is merely to assure the comprehensive representation of all the interests involved. Complete records of all interested organizations and of their standardization work will be kept on file and properly classified in the office of the Main Committee. The Main Committee or its Secretary will thus be able, either promptly to suggest the proper representation to a sponsor on request, or to approve or amplify the representation as provisionally proposed by the Sponsor.

3. After a group of standards has been prepared and accepted by a Sectional Committee, it is submitted to the Sponsor Body for its approval and then to the Main Committee with a full report of its history. When approved by both the Sponsor Body and the Main Committee, the standards in question become American Standards.

4. When the report of any sectional committee is being considered by the Main Committee, three members of that sectional committee are invited to sit with the Main Committee to report, discuss and vote on the standards in question as if they were regular members of the Committee. Thus each sectional committee, (and therefore usually each Sponsor Body), will be represented on the Main Committee when standards in which they are interested are being discussed.

5. The scrutiny of a standard by the Main Committee is to make sure that the proper procedure was pursued, that it was prepared by a comprehensively representative sectional

committee, that the vote of acceptance was nearly enough unanimous, and that the standard is consistent with other related standards. Consideration is also given to international relations; but the Main Committee is not supposed to pass upon the details as it is obviously impossible for a moderate sized workable committee to cover the details of all parts of the field.

6. After approval by the Main Committee the standard is published by the Sponsor Body with the statement that it has been approved by the A. E. S. Committee, and labelled "American Standard" with the appropriate descriptive title.

Briefly summarized this procedure is as follows:

Standard assigned by Main Committee to Sponsor Body.

Sponsor Body appoints a thoroughly representative Sectional Committee, subject to approval of Main Committee.

Sectional Committee prepares standard and submits to Sponsor Body which then submits the standard with its approval to the Main Committee.

The Standard is then published by the Sponsor Body and labelled "American Standard."

The machinery thus provides for absolutely comprehensive cooperation, eliminates overlapping and duplication of effort, and yet does not undesirably restrict the initiative of the several cooperating societies.

With proper support and the cooperation of all interested organizations, it should contribute largely to the industrial development of the country.

Moreover it should be a potent factor in promoting international standardization and foreign commerce.

In the broad sense international standardization means a common industrial language and the removal of one of those barriers which tend to separate nations and to give rise to misunderstandings.

The more of these barriers we can remove, the more likely we are to attain that lasting peace which the world longs for.

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THE GENERAL EQUATIONS OF THE ELECTRIC CIRCUIT—III

Variation of Constants r, L, C, and g, and its Effects

BY CHARLES P. STEINMETZ

Abstract of Paper

In the usual theory of transients the assumption is made, that resistance, inductance, capacity and conductance are constant. This however, is not correct, and as the result thereof, it was not possible to theoretically investigate, and numerically calculate the dissipation of high-frequency disturbances, the flattening of the wave fronts of impulses, the rounding off of steep waves, etc., with the time and the distance of travel, and therefrom to determine the distance, to which the danger from such disturbances extends, and to investigate the conditions of line construction, which limit the danger zone of such phenomena to the smallest local extent.

In the following, two of the foremost causes of change of the line constants with the equivalent frequency are investigated, the unequal current distribution in the conductor, and the electric radiation from the conductor, and shown, that within the range of frequencies which may be met in industrial circuits, the effective resistance of the conductor, and its attenuation constant, may increase more than a million fold.

Equations of the line constants as function of the equivalent frequency are derived, and applications thereof made to a few problems:

(1) The laws of conduction of high-frequency currents, such as produced by lightning discharges and similar disturbances, and the conclusions resulting therefrom on the nature of the conductor.

(2) The decay of high-frequency sine waves in transmission lines.

(3) The attenuation of rectangular waves.

(4) The flattening of the wave front of steep impulses.

It is shown that in high-frequency conduction the section of the conductor is of little importance, but the circumference is the determining factor, except at very high frequencies, where size, shape, and material—within certain limits—becomes of secondary importance. The inductance and the capacity of the conductor remain constant up to very high frequencies, but the effective resistance may increase many thousand fold. At high frequencies, not all the energy consumed by the effective resistance of the conductor is converted into heat, but the major part may be radiated into space and thereby affect other circuits.

The conditions, which cause a flattening of steep wave fronts and impulses, and a rounding of irregular waves, are investigated, and it is shown, that within the range of distances in which

danger may result from a steep wave front or high frequency, the steepness of the wave front decreases with the square root of the distance traveled by the wave; that size and material of the conductor have little influence, but the distance between conductor and return conductor is the main factor in flattening the wave front and thereby limiting the danger zone.

For convenience, the theoretical part has been separated and placed in an appendix, giving in the text the discussion, with numerous tables derived from the theoretical equations, and curves illustrating these tables.

The investigation is not general, but rather limited to special applications, as the field is too new to permit fargoing generalization.

I.—General

IN 1908, in a paper under the above title, the general equations of the electric circuit were presented and discussed, and in a second paper in 1916, under the title "Outline of a Theory of Impulse Currents," the application of these general equations to the special cases of direct-current circuits, alternating-current circuits, etc., was discussed, and more particularly a theory of impulse currents outlined, that is, transient currents which are non-periodic in time, and either non-periodic or periodic in space.

In both of these papers however, and in all general theoretical investigations on these subjects, as far as I know, the assumption is made, that the circuit constants; resistance r, inductance L, capacity C and shunted conductance g, are constant. While this is true, with sufficient approximation, for the usual machine frequencies and for moderately high frequencies, experience shows that it is not even approximately true for very high frequencies and for very sudden circuit changes, as steep wave front impulses, etc.

If r, L, C and g are assumed as constant, it follows that the attenuation is independent of the frequency, that is, waves of all frequencies decay at the same rate, and as the result, a complex wave or an impulse traversing a circuit dies out without changing its wave shape or the steepness of its wave front.

Experience, however, has conclusively shown that steep wave fronts are dangerous only near their origin, and rapidly lose their destructiveness by the flattening of the wave front when running along the circuit. Experimentally, small inductances shunted by a spark gap, inserted in transmission lines for testing for high frequencies or steep wave fronts, have shown appreciable spark lengths, that is, high-voltage gradients, only near the origin of the disturbance, but no appreciable spark length beyond a few miles from the origin of the disturbance.

The rectangular wave of starting a transmission line by connecting it to a source of voltage, which is given by the theory under the assumption of constant r, L, C and g, and thus shown in most text books, has never been shown in any of the oscillograms of transmission lines, as far as I know.

If r and L are constant, the power factor of the line conductor, $\frac{r}{\sqrt{r^2 + (2\pi fL)}}$, should with increasing frequency continuously decrease, and reach extremely low values, at very high frequencies, so that at these, an oscillatory disturbance should be sustained over very many cycles, and show with increasing frequency an increasing liability to become a sustained or cumulative oscillation. Experience, however, shows that high-frequency oscillations die out much more rapidly than accounted for by the standard theory, and show at very high frequency practically no tendency to become cumulative.

This does not mean that the present theory of transients, which is based on the assumption of constant r, L, C and g, is useless by not correctly representing the actual phenomena; but it means that it correctly represents the transient only in its initial stage and near its origin, but does not satisfactorily represent its course after its initial stage and at some distance from the origin, at least not for high-frequency transients or steep wave fronts.

As the destructive effect of voltage does not depend only on the value of the voltage, but also on the duration of its application, this is a limitation, though not as serious as it may appear, since it tends to understate the danger of high-frequency transients.

However, it makes it impossible to determine such important questions as; how far the danger zone of a high-frequency disturbance or steep wave front extends; how the destructiveness decreases with the distance; how it depends on line construction, conductor shape and materials, etc., and to approach the problem of so designing the circuit as to limit the danger from any such disturbance to the smallest local extent.

In the following, therefore, the attempt is made to investigate and calculate theoretically some of the factors, which cause a change of the line constants r, L, C, and g, with increasing frequency or steepness of wave front, and to investigate theoretically the effect produced on the course of the transient with regard to duration and wave shape, by the variation of the line constants.

As this field is rather new, no very general treatment is yet possible, but rather, an outline of the problem, and applications to some of the most important phenomena, such as the conduction of high-frequency currents, the changes of wave shape with time and distance of wave travel, and the flattening of the wave front, and its bearing on circuit construction.

The two most important factors in the variation of the circuit constants r, L, C, and g, seem to be the *unequal current distribution in the conductor* and the *finite velocity of the electric field*.

UNEQUAL CURRENT DISTRIBUTION IN THE CONDUCTOR

The magnetic field of the current surrounds this current and fills all the space outside thereof, up to the return current. Some of the magnetic field due to the current in the interior and in the center of a conductor carrying current, thus is inside of the conductor, while all the magnetic field of the current in the outer layer of the conductor is outside of it. Therefore, more magnetic field surrounds the current in the interior of the conductor than the current in its outer layer, and the inductance therefore increases from the outer layer of the conductor towards its interior, by the "internal magnetic field." In the interior of the conductor the reactance voltage thus is higher than on the outside.

At low frequency, with moderate size of conductor, this difference is inappreciable in its effect. At higher frequencies however, the higher reactance in the interior of the conductor, due to this internal magnetic field, causes the current density to decrease towards the interior of the conductor, and the current to lag, until finally the current flows practically only through a thin layer of the conductor surface.

As the result thereof, the effective resistance of the conductor is increased, due to the uneconomical use of the conductor material caused by the lower current density in the interior, and due to the phase displacement, which results in the sum of the currents in the successive layers of the conductor being larger than the resultant current. Due to this unequal current distribution, the internal reactance of the conductor is decreased, as less current penetrates to the interior of the conductor, and thus produces less magnetic field inside of the conductor.

The derivation of the equations of the effective resistance of unequal current distribution in the conductor, r_1, and of the internal reactance x_1 under these conditions, is given in appendix A. It is interesting to note, that effective resistance and internal reactance, with increasing frequency, approach the same limit, and become proportional to the square root of the frequency, the square root of the permeability, and the square root of the resistivity of the conductor material, while at low frequencies the resistance is independent of the frequency and directly proportional to the resistivity, and the internal reactance is independent of the resistivity and directly proportional to the frequency.

FINITE VELOCITY OF THE ELECTRIC FIELD

In calculating the inductance or the capacity of a conductor, usually the assumption is made, that the electric field is instantaneous, that is, the magnetic field appears and disappears simultaneously with the current, and therefore, is in phase with the current, and the dielectric field appears and disappears simultaneously with the voltage. In reality, however, this is not strictly correct, but the electric field moves with a finite velocity from the conductor outwards. While this velocity is inconceivably high: $S = 3 \times 10^{10}$ cm. per sec., or 300,000 km. per sec., its effect is appreciable at high frequencies. Consider, for instance, a circuit representing average transmission line conditions with 6 ft. = 182 cm., between conductors, traversed by a current of $f = 10^6$, or one million cycles, such as may be produced by a nearby lightning discharge. The wave length of this current and thus of its magnetic field then would be $\frac{S}{f} = \frac{3 \times 10^{10}}{10^6} = 30{,}000$ cm. and the distance of 182 cm. between the line conductors would be $\frac{182}{30{,}000}$ or $\frac{1}{165}$ of a wave length or $\frac{360}{165} = 2.2$ deg. That is, the magnetic field of the current, when it reaches the return conductor, would not be in phase with the current, but $\frac{1}{165}$ of a wave length or 2.2 deg. behind the current. The voltage induced by the magnetic field would not be in quadrature with the current, or wattless, but lag $90 + 2.2 = 92.2$ deg. behind the current, thus have an energy component equal to cos 92.2 deg. = 3.8 per cent, giving rise to an effective resistance r_3, equal to 3.8 per cent of the reactance x. Even if at normal frequencies of 60 cycles the reactance is equal only to the ohmic resistance r_0 — usually it is larger—the reactance x to 10^6 cycles would be $\frac{10^6}{60}$, = 16,700 times the ohmic resistance, and the effective resistance of magnetic radiation, r_3, being 3.8 per cent of this reactance, thus would be 630 times the ohmic resistance: $r_3 = 630\, r_0$. Thus, the ohmic resistance would be entirely negligible compared with the effective resistance resulting from the finite velocity of the magnetic field.

Considering however, a high-frequency oscillation of 10^6 cycles, not between the line conductors, but between line conductor and ground, and assume, under average transmission line conditions, 30 ft. as the average height of the conductor above ground. The magnetic field of the conductor then can be represented as that between the conductor and its image conductor, 30 ft. below ground, and the distance between conductor and return conductor would be 2×30 ft. = 1820 cm. The lag of the magnetic field, due to the finite velocity of propagation, then becomes 22 deg. thus quite appreciable, and the energy component of the voltage induced by the magnetic field is $\cos (90 + 22^{\circ}) = 37$ per cent. This would give rise to an effective or radiation resistance $r_3 = 0.37\, x$. As in this case, the 60-cycle reactance usually is much larger than the ohmic resistance, assuming it as twice would make the radiation resistance $r_3 = 12{,}600\, r_0$, or more than ten thousand times the true ohmic resistance. It is true, that at these high frequencies, the ohmic resistance would be very greatly increased by unequal current distribution in the conductor. But the effective resistance of unequal current distribution increases only proportionally to the square root of the frequency, and, assuming for instance conductor No. 00 B. & S. gage, the effective resistance of unequal current distribution, r_1, would at 10^6 cycles be about 36 times the low-frequency ohmic resistance r_0. Thus the effective resistance of magnetic radiation would still be $\frac{12{,}600}{36} = 350$ times the effective resistance of unequal current distribution: $r_3 = 350\ r_1$, and the latter, therefore, is negligible compared with the former.

It is interesting to note, that the effective resistance of radiation, r_3, does not represent energy dissipation in the conductor by conversion into heat, but energy dissipation by radiation into space, and in distinction from the "radiation resistance," which dissipates energy into space, the effective resistance of unequal current distribution may be called a "thermal resistance," as it converts electric energy into heat

Thus in this instance of a 10^6-cycle high-frequency discharge between transmission line conductor and ground, the heating of the conductor would be increased 36 fold over that produced by a low-frequency current of the same amperage, by the increase of resistance by unequal current distribution in the conductor; but the total energy dissipation by the conductor would be increased by magnetic radiation still 350 times more, so that the total energy dissipation by the 10^6-cycle current would be 12,600 times greater than it would be with a low-frequency current of the same value, and the attenuation or rate of decay of the current thus would be increased many thousand times, over that calculated on the assumption of constant resistance at all frequencies.

The derivation of the equations of the effective resistance of magnetic radiation, and in general of the effects of the finite velocity of the electric field on the line constants, are given in appendix B.

The magnetic radiation resistance is proportional to the square of the frequency (except at extremely high frequencies). It therefore, is negligible at low and medium frequencies, but becomes the dominating factor at high frequencies. It is proportional to the distance of the return conductor, but entirely independent of size, shape, or material of the conductor, as is to be expected, since it represents the energy dissipated into space. Only at extremely high frequencies, the rise of radiation resistance becomes less than proportional to the square of the frequency. It becomes practically independent of the distance of the return conductor, when the latter becomes of the magnitude of the quarter wave length.

The same applies to the capacity. Due to the finite velocity of propagation, the dielectric or electrostatic field lags behind the voltage which produces it, by the same angle by which the magnetic field lags behind the current, and the capacity current or charging current thus is not in quadrature with the voltage, or reactive, but displaced in phase by more than 90 deg. thus contains a negative energy component, which gives rise to a shunted conductance of dielectric radiation *g*. This gives rise to an energy dissipation by the conductor, at high frequencies, by dielectric radiation into space, of the same magnitude as the energy dissipation by magnetic radiation, above considered.

The term "shunted conductance" *g* has been introduced into the general equations of the electric circuit largely from theoretical reasons, as representing the power consumption proportional to the voltage. Most theoretical investigations of transmission circuits consider only *r*, *L* and *C* as the circuit constants, and omit *g*, since under average transmission line conditions, at low and moderate frequencies, *g* usually is negligible. In communication circuits, as telegraph and telephone, there is a "leakage," which would be represented by a shunted conductance, and in underground cables there is a considerable energy consumption by dielectric losses in the insulation, as the investigations of the last years have shown, which gives a shunted conductance. In overhead power lines however, energy losses depending on the voltage,—and leading to a term *g*—have been known only at those high voltages where corona appears. It is interesting therefore to note, that at high frequencies, "shunted conductance" *g* may reach very formidable values even in transmission lines, due to electrostatic radiation.

In investigating the effect of the finite velocity of the electric field on the inductance *L* and the capacity *C*, in appendix B, it is seen, that the equations of external inductance and of capacity are not affected, but remain the same as the usual values derived by neglecting the velocity of the electric field, except at extremely high frequencies, when the distance of the return conductor approaches quarter wave length.

EQUATIONS OF ELECTRICAL CONSTANTS, AND NUMERICAL VALUES

In appendix C are given, compiled from appendices A and B, the equations of the components of the electrical constants, as functions of the frequency, for conductors with return conductor, and also for conductors without return conductor (as approximated by lightning strokes or wireless antennae):

Resistance: true ohmic resistance or effective resistance of unequal current distribution, and magnetic radiation resistance.

Reactance: low frequency internal reactance or internal reactance of unequal current distribution, and external reactance.

Inductance: low frequency internal inductance or internal inductance of unequal current distribution, and external inductance.

Shunted conductance and capacity are not so satisfactorily represented, and therefore, instead of representing energy storage and power dissipation depending on the voltage by a conductance g and a capacity C or susceptance b, in shunt with each other, it is more convenient to represent them by an effective resistance, the dielectric radiation resistance r_e, and a capacity reactance x_e, in series with each other.

As seen from appendix C, four successive stages may be distinguished in the expressions of the circuit constants as functions of the frequency.

1. Low frequencies, such as the machine frequencies of 25 and 60 cycles. The resistance is the true ohmic resistance, the internal reactance and inductance that corresponding to uniform current density throughout the conductor, with conductors of moderate size, and of non-magnetic material.

2. Medium frequencies, of the magnitude of a thousand to ten thousand cycles. Resistance and internal reactance or inductance are those due to unequal current distribution in the conductor, that is, the resistance is rapidly increasing, and the internal inductance decreasing. The conductance g is still negligible, radiation effects still absent, and all the energy loss that of thermal resistance.

3. High frequencies, of the magnitude of one hundred thousand to one million cycles. The radiation resistance is appreciable and becomes the dominating factor in the energy dissipation. The internal inductance has practically disappeared, due to the current penetrating only a thin surface layer. A considerable shunted conductance exists due to the dielectric radiation.

4. Extremely high frequencies. Of the magnitude of many millions of cycles, when the quarter wave length has become of the same magnitude or less than the distance of the return conductor. Radiation effects entirely dominate, and the usual expressions of inductance and of capacity have ceased to apply.

This last case is of little industrial importance, as such extremely high frequencies propagate only over short distances.

It would come into consideration only in calculating the flattening of the wave front of a rectangular impulse in the immediate neighborhood of its origin, and similar problems.

Thus far, a general investigation does not seem feasible. Substituting the equations of the circuit constants, as functions of the frequency, into the general equations of the electric circuit, leads to expressions too complex for general utility, and the investigation thus must largely be made by numerical calculations.

Only when the frequencies which are of importance in the problem, lie fairly well in one of the four ranges above discussed —as is the case in the investigation of the flattening of a steep wave front in moderate distances from its origin,—a more general theoretical investigation becomes possible.

As instances of industrial importance may be considered in the following:

The conduction of high frequency currents such as produced by lightning and similar disturbances.

The attenuation of waves in overhead transmission circuits.

The rounding of rectangular waves.

The flattening of steep wave fronts.

II.—Lightning Conductors

It has been known for many years that currents of high frequency, very sudden impulses, etc., such as are given by lightning discharges and similar disturbances in electric circuits, do not follow the laws of low-frequency currents. The conduction of such currents however, is of very great industrial importance; in the discharge path of lightning arresters and other protective devices, their ground connections, their connections with the circuit, etc., it is of fundamental importance to design and select the conductor so as to offer minimum impedance to the discharge. Inversely, in the connection of the station to the transmission line, it is of importance to design the conductor for maximum impedance against such currents. The protection of buildings by the lightning rod depends on the design of the conductor system, etc.

However, the high-frequency impedance of the conductor is not the only quantity of importance. At high frequencies, the thermal resistance, which dissipates energy in the conductor by conversion into heat, may be a small part only of the total effective resistance, and the radiation resistance dominates.

That is, most of the power consumed in the conductor is radiated into space. Thus it is not sufficient to merely afford a discharge path for lightning, because while discharging through such path, most of the energy of the lightning may be communicated by radiation to other bodies. This accounts for the power exerted upon bodies near the path of a lightning stroke, the destructive effects of a "side stroke." Hence it is of importance also, to study the extent of the electric field of the high-frequency conductor, to ascertain that nothing is within the electric field, which might be damaged by intercepting the electric field, or may lead the power, communicated to it by the radiation from the discharge path, into other circuits where it may be destructive. Thus a lightning arrester, while functioning perfectly and supposedly giving perfect protection, may be a source of danger, by concentrating the energy of lightning into its discharge path, if this discharge path is carried in close inductive relation to conductors of the circuit which is to be protected.

To show the relative magnitude of the different line constants and their components, in Table I are given their numerical values, in ohms per meter length of conductor, for frequencies from one cycle to one hundred million cycles per second, of a copper wire No. 00 B. & S. gage, for the three conditions:

a. Return conductor at $l' = 6$ ft. $= 182$ cm. corresponding to a discharge between transmission line conductors.

b. With the ground as return conductor, at 30 ft. distance, that is, $l' = 2 \times 30$ ft. $= 1820$ cm. corresponding to a discharge between transmission line and ground.

c. No return conductor, corresponding to the vertical discharge path of a lightning arrester ground, a lightning stroke, etc.

Some of these values are plotted in Fig. 1.

The low frequency values of resistance r_0 and external and internal reactance $x_0 + x_{10}$, have no existence at the higher frequencies. But as they are the values calculated by the usual formulas, they are given in Table I, for comparison with the true effective high-frequency values. The values r_2 and x_2, though given for all frequencies, have a meaning only for the very high frequencies, since at lower frequencies the condition of a conductor without return conductor can hardly be realized, as any conductor within a quarter wave length would act as a more or less effective return conductor.

TABLE I.

CIRCUIT CONSTANTS OF COPPER WIRE NO. 00 B. & S. GAGE PER METER LENGTH OF CONDUCTOR

a. return conductor at 6 ft. = 182 cm. distance.
b. return conductor at 2 × 30 ft. = 1820 cm. distance.
c. no return conductor
conductivity: $\gamma = 6.2 \times 10^5$

		Cycles f =	1	10	10^2	10^3	10^4	10^5	10^6	10^7	10^8
Low Frequency		r_0 =	.000240	.000240	.000240	.000240	.000240	.000240	.000240	.000240	.000240
		x_{10} =	.0000003	.000003	.00003	.0003	.003	.03	.3	3	30
	a.	x_0 =	.0000075	.000075	.00075	.0075	.075	.75	7.5	75	750
	b.		.0000104	.000104	.00104	.0104	.104	1.04	10.4	104	1,040
	a.	z_0 =	.000240	.000252	.000816	.0078	.078	.78	7.8	78	780
	b.		.000240	.000263	.001097	.0107	.107	1.07	10.7	107	1,070
	a.	cos ω_0 =	1.000	.952	.293	.0308	.0031	.00031	.000031	.000003	.0000003
	b.		1.000	.912	.218	.0225	.0022	.00022	.000022	.000002	.0000002
High Frequency		r_1 =	.000240	.000240	.000240	.000273	.000865	.00273	.00865	.0273	.0865
		x_1 =	.0000003	.000003	.000030	.000273	.000865	.00273	.00865	.0273	.0865
	a.	r_2 =	....	....	....	....	.0000048	.00048	.048	4.8	225
	b.		....	....	....	....	.000048	.0048	.48	22.5	194
	c.	r_2 =	.000002	.000020	.000197	.00197	.0197	.197	1.97	19.7	197
	a.	x_2 =	.0000075	.000075	.00075	0.075	.075	.75	7.5	75	490
	b.		.0000104	.000104	.00104	.0104	.104	1.04	10.4	78	505
	c.	x_2 =	.0000285	.000256	.00227	.0198	.169	1.40	11.2	83	515
	a.	z =	.000240	.000252	.000816	.00778	.0759	.753	7.5	75.2	539
	b.		.000240	.000263	.001097	.01068	.1049	1.043	10.4	81.2	541
	c.		.000242	.000367	.00234	.0201	.170	1.42	11.4	85.3	551
	a.	cos ω =	1.000	.952	.293	.0351	.0115	.0043	.0076	.0642	.418
	b.		1.000	.912	.218	.0257	.0087	.0072	.047	.277	.358
	c.		1.000	.710	.186	.112	.121	.141	.173	.232	.358
	a.	$\frac{r_1 + r_2}{r_0}$ =	1.000	1.000	1.000	1.14	3.62	13.4	236	20,100	937,000
	b.		1.000	1.000	1.000	1.14	3.80	31.3	2,030	94,000	808,000
	c.	$\frac{r_1 + r_2}{r_0}$ =	1.008	1.082	1.82	9.35	85.5	832	8,250	82,000	820,000

As seen from the equations, and illustrated in Table I, the thermal resistance of the conductor, r_1, that is, the resistance which converts electrical energy into heat in the conductor, is the true ohmic resistance at low frequencies, but with increasing frequency begins to rise due to unequal current distribution in the conductor (at about 1000 cycles in copper wire No. 00 B. & S. gage) and approaches proportionality with the square root of the frequency, hence reaches values many times the ohmic resistance, at very high frequencies.

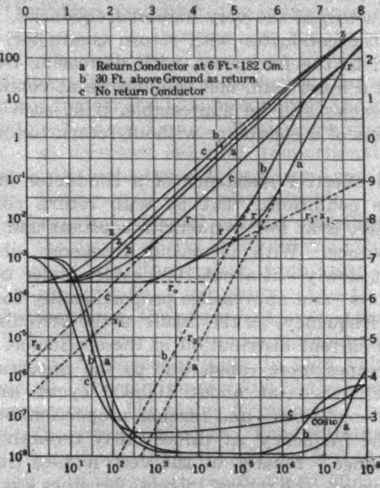

FIG. 1—HIGH-FREQUENCY CONSTANTS OF COPPER WIRE NO. 00 B. &. S GAGE

The radiation resistance of the conductor without return conductor, r_2, is proportional to the frequency, but the radiation resistance of the conductor with return conductor, r_3, is proportional to the square of the frequency, hence very small until high frequencies are reached – 10,000 to 100,000 cycles. The radiation resistance r_3 of the conductor with return conductor then increases very rapidly and reaches values many thousand times the ohmic resistance. At the very highest frequencies, many millions of cycles, its rate of increase becomes less again, and it approaches proportionality with the first power of the frequency, and approaches the value of radiation resistance r_2, of the conductor without return conductor, at frequencies of a wave length comparable with the distance of the return conductor. The radiation resistance r_3 of the conductor with return conductor is the larger, the greater the distance of the return conductor, and is proportional to this distance, within the range of which it is proportional to the square of the frequency. The radiation resistance of the conductor without return conductor, at the very highest frequencies, is the same as that of the conductor with return conductor, but, being proportional to the frequency, with decreasing frequency, it decreases at a lesser rate, and would even at commercial machine frequencies still be appreciable, if at such frequencies the conditions of a conductor without return conductor could be realized.

The total effective resistance of a conductor under transmission line conditions, that is, with return conductor at finite distance, is at low frequencies constant and is the true ohmic resistance. With increasing frequency, it begins to increase first slowly—at about 1000 cycles under transmission line conditions—and approaches proportionality to the square root of the frequency, as the result of the screening effect of the unequal current distribution in the conductor. Then the increase becomes more rapid, due to the appearance of the radiation resistance—at about 100,000 cycles under transmission line conditions—and reaches proportionality with the square of the frequency, at values many thousand times the ohmic resistance. Finally, at the very highest frequencies—10 million cycles—the rate of increase becomes less again, and approaches proportionality with the frequency.

It is interesting to note that the external reactance of the conductor with return conductor, or radiation reactance x_3, has up to very high frequencies, millions of cycles, the same value x_0 as calculated by the low-frequency formula, that is, by neglecting the finite velocity of the field, hence is proportional to the frequency. The internal reactance $x_1 = x_{10}$, and is proportional to the frequency at low frequencies, but drops behind x_{10} due to unequal current distribution in the conductor, and approaches proportionality with the square root of the frequency. As, however, the internal reactance in a small part of the total reactance, it follows, that the total reactance of the conductor and thus also the absolute value of the impedance (for all higher frequencies, where the reactance preponderates) can be calculated by the usual low frequency reactance formula, which neglects the finite velocity of the field. Hence, the inductance L of the conductor can be assumed as approximately constant for all frequencies up to millions of cycles; it decreases only very slowly by the decreasing internal reactance of unequal current distribution. Only at the very highest frequencies, where the wave length is comparable with the distance of the return conductor, the inductance L decreases, and the reactance $x_1 + x_3$ increases less than proportional to the frequency.

In a conductor without return conductor, the reactance at the very highest frequencies is approximately the same as in a conductor with return conductor. With decreasing frequency, however, $x_1 + x_2$ decreases less than proportional to the frequency, that is, the inductance L increases—and becomes infinity for zero frequency, if such were possible.

Without considering unequal current distribution in the conductor and the finite velocity of the electric field, the power factor cos ω would steadily decrease, from unity at very low frequencies, to zero at infinite frequency. Due to the increase of the effective resistance, the power factor cos ω first decreases, from unity at low frequency, down to a minimum at some high frequency, and then increases again to high values at very high frequencies. The minimum value of the power factor is the lower and occurs at the higher frequencies, the shorter is the distance of the return conductor. Thus with the return conductor at 6 ft. distance, the power factor is 0.43 per cent at 100,000 cycles; with the return conductor at 60 ft. distance, it is 0.72 per cent; in the conductor without return conductor the power factor is 11.2 per cent at 1000 cycles.

It is of interest to determine the effect of size, shape and material on the high-frequency constants of a conductor.

These high-frequency constants are, per unit length of conductor:

Internal constants:

Thermal resistance and internal reactance:

$$r_1 = x_1 = \frac{\pi}{l_1} \sqrt{\frac{0.4 \mu f}{\gamma}}\, 10^{-4} \text{ ohms per cm.} \qquad (8) \quad (9)$$

External constants:

Radiation resistance:

$$r_3 = \frac{8\pi^2 f^2 l'}{S} 10^{-9} \text{ ohms per cm.} \qquad (16)$$

External reactance:

$$x_3 = 4\pi f \log \frac{2\pi l'}{l_2} 10^{-9} \text{ ohms per cm.} \qquad (19)$$

These approximations hold for all but the very highest, and very low frequencies, that is, are correct within the frequency range with lower limit of about 1000 cycles, and upper limit of about 10 million cycles. Thus they apply for all those high frequencies which are of importance in the disturbances occuring in industrial circuits with the exception of the lowest harmonics of low-frequency surges.

The constants of the conductor material enter the equations only as the ratio $\frac{\mu}{\gamma}$, permeability to conductivity, in the internal constants r_1 and x_1. Thus higher permeability has the same effect in increasing the thermal resistance as lower conductivity, and for instance, a cast silicon rod of permeability $\mu = 1$, and conductivity $\gamma = 55$, has the same high-frequency resistance and reactance, as an iron rod of the same size, of wrought iron, of permeability $\mu = 2{,}000$ and conductivity $\gamma = 1.1 \times 10^5$, that is, of the same $\frac{\mu}{\gamma} = 0.0182$, though the latter has 2000 times the conductivity of the former.

Provided however, that size of conductor and frequency are such as to fulfill the conditions under which equations (**8**) and (**9**) are applicable, which is, that the conductor is large compared with the depth of penetration of the current into the conductor:

$$l_p = \frac{10^4}{\pi \sqrt{0.4 \ \gamma \mu f}}$$

Thus an iron rod of two inches (five cm.) diameter has at one million cycles the same thermal resistance as a silicon rod of the same size: 0.17 ohms per meter, since the depth of penetration is $l_p = 0.00034$ cm. for iron, 0.68 cm. for silicon, thus in either case small compared with radius of the conductor $l_r = 2.5$ cm.

At 10,000 cycles, however, the iron rod has the thermal resistance and internal reactance $r_1 = x_1 = 0.017$ ohms per meter, the penetration being $l_p = 0.0034$ cm. thus small. For the silicon rod, however, at 10,000 cycles the penetration is $l_p = 6.8$ cm. thus at the radius $l_r = 2.5$ cm. formulas (**8**) and (**9**) do not apply any more, but it is approximately (that is neglecting unequal current distribution): $r_1 = 0.093$ ohms per meter, or 5.5 times the resistance of the iron rod, while the internal reactance is $x_1 = 0.031$ ohms per meter, hence 80 per cent higher than that of the iron rod of the same size.

In the equations of the external constants, the radiation resistance and reactance, the material constants of the conductor do not enter, and the radiation resistance and the external reactance thus are independent of the conductor material.

The dimensional constants of the conductor, size and shape, enter the equation only as the circumference of the conductor l_1, l_2, that is, only the circumference of the conductor counts in high-frequency conduction, and all conductors of the same material, regardless of size and shape, have the same high-frequency resistances and reactances as long as they have the same conductor circumference. Thus a solid copper rod or a thin copper cylinder of the same outer diameter as the rod, or a flat copper ribbon of a circumference equal to that of the rod, are equally good high-frequency conductors, though the hollow cylinder or the ribbon may contain only a small part of the material contained in the solid copper rod. Provided, however, that the thickness or depth of the conductor,(the thickness of wall of the hollow cylinder, half the thickness of the copper ribbon) is larger than the depth of penetration of current into the conductor, which is

$$l_p = \frac{10^4}{\pi \sqrt{0.4 \, \gamma \, \mu}}$$

In the expression of the radiation resistance, r_3, neither the material nor the dimensions of the conductor enter, that is,the radiation resistance of a conductor is independent of size, shape or material of the conductor and depends only on frequency and distance of the return conductor.

Thus a thin steel wire or a wet string have the same radiation resistance as a large copper bar. Obviously, the thermal resistance of the former is much larger and the total effective resistance thus would be larger except at those very high frequencies, at which the radiation resistance dominates.

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As illustration, may be calculated, for frequencies from 10 thousand to 10 million cycles and for 60 cycles, the resistances and reactances and thus the total impedance, the power factor, the voltage drop per meter at 100 amperes, of various conductors, for the three conditions:

a. High frequency between conductors 6 ft. apart:

$$l' = 182 \text{ cm.}$$

b. High frequency between conductor and ground 30 ft. below conductor:

$$l' = 1820 \text{ cm.}$$

c. High-frequency discharge through vertical conductor without return conductor:

$$l' = \infty$$

For the conductors:

(1) Copper wire No. 00 B. & S. gage

$$l_r = 0.463 \text{ cm.} \quad \gamma = 6.2 \times 10^5$$

(2) Iron wire of the same size as (1):

$$\mu = 2{,}000 \quad \gamma = 1.1 \times 10^5$$

(3) Copper ribbon of thickness equal to twice the depth of penetration at 10,000 cycles, and the same amount of material as (1), that is, 0.128 cm. by 5.52 cm.

(4) Iron ribbon of the same size as (3).

(5) Two-inch iron pipe, ⅛ inch thick,

This gives the depth of penetration at $f = 10^4$ cycles:

for copper; $l_p = \dfrac{6.4}{\sqrt{f}} = 0.064$ cm.

for iron: $l_p = \dfrac{0.34}{\sqrt{f}} = 0.0034$ cm.

Table II gives the values of r_0, x_0, r_1, r_3, r_2, x_1, x_3, x_2, $r_1 + r_3$, $r_1 + r_2$, $x_1 + x_3$, $x_1 + x_2$, z, $\cos \omega$, e and l_p, for $f = 10^6$ cycles.

Table III gives the values of $r_1 + r_3$, or $r_1 + r_2$, z, $\cos \omega$ and e, for $f = 60, 10^4, 10^5, 10^6, 10^7$ cycles for the five kinds of conductors.

It is interesting to compare in Table III, the constants of the first four conductors, as they have the same section, representing about average section of transmission conductors, but represent two shapes, round wire and thin flat ribbon, and two kinds of material, copper—high conductivity and non-magnetic,—and iron—magnetic material of medium conductivity.

As seen, the effect of conductor shape and conductor ma-

TABLE II.

CIRCUIT CONSTANTS FOR $f = 10^6$ = ONE MILLION CYCLES, PER METER LENGTH OF CONDUCTOR.

a. return conductor at 6 ft. = 182 cm. distance.
b. return conductor at 2 × 30 ft. = 1820 cm. distance.
c. no return conductor

		(1) Copper wire:	(2) Iron wire:	(3) Copper ribbon:	(4) Iron ribbon:	(5) Iron pipe:	
$r_0 = \frac{100}{A\ \gamma} =$		0.00024	0.00135	0.00024	0.00135	0.000197	
$x_0 = 0.4\ \pi f \left(\log \frac{2\ \pi f l'}{l_1} + \frac{l_0}{l_1} \frac{\mu}{4} \right) 10^{-6} =$	a.	7.8	635	3.05	173	81	ohms
	b.	10.8	638	5.95	176	84	"
$r_1 = \frac{100\ \pi}{l_1} \sqrt{\frac{0.4\ \mu f}{\gamma}}\ 10^{-4} =$		0.00865	0.92	0.00233	0.248	0.167	"
$r_2 = \frac{0.8\ \pi^2 f^2 l'}{S} 10^{-6} =$	a.	0.048	0.048	0.048	0.048	0.048	"
	b.	0.48	0.48	0.48	0.48	0.48.	"
$r_2 = 0.2\ \pi^2 f\ 10^{-6} =$	c.	1.97	1.97	1.97	1.97	1.97	"
$x_1 = \frac{100\ \pi}{l_1} \sqrt{\frac{0.4\ \mu f}{\gamma}}\ 10^{-4} =$		0.00865	0.92	0.00233	0.248	0.167	"
$x_2 = 0.4\pi f \log \frac{2\ \pi f l'}{l_1} 10^{-6} =$	a.	7.5	7.5	2.97	2.97	2.47	"
	b.	10.4	10.4	5.87	5.87	5.37	"
$x_2 = 0.4\ \pi f \left(\log \frac{S}{l_1 f} - 0.5772 \right) 10^{-6} =$	c.	11.2	11.2	9.5	9.5	8.9	"
$r = r_1 + r_2 =$	a.	0.0567	0.97	0.0503	0.296	0.215	ohms
	b.	0.489	1.40	0.482	0.728	0.647	"
$r = r_1 + r_2 =$	c.	1.98	2.89	1.97	2.22	2.14	"
$x = x_1 + x_2 =$	a.	7.5	8.42	2.97	3.22	2.64	"
	b.	10.4	11.32	5.87	6.12	5.54	"
$x = x_1 + x_2 =$	c.	11.2	12.12	9.50	9.75	9.07	"
$z = \sqrt{r^2 + x^2} =$	a.	7.5	8.46	2.97	3.23	2.65	"
	b.	10.4	11.4	5.89	6.16	5.58	"
	c.	11.4	12.45	9.7	10.0	9.3	"
$\cos \omega = \frac{r}{z} =$	a.	0.76	11.5	1.7	9.2	8.1	per cent
	b.	4.7	12.3	8.2	11.8	11.6	"
	c.	17.3	23.3	20.3	22.2	23.0	"
$e = 100\ z =$	a.	750	846	296	323	265	volts
	b.	1040	1140	589	616	558	"
	c.	1140	1245	970	1000	930	"
$l_p = \frac{10^4}{\pi \sqrt{0.4\ \gamma \mu f}} =$		0.0064	0.00034	0.0064	0.00034	0.00034	cm.

TABLE III.

	Conductors 6 ft. apart:					Conductor 30 ft. above Ground:					Conductor without return conductor:				
Frequency, cycles:	60	10^4	10^5	10^6	10^7	60	10^4	10^5	10^6	10^7	60	10^4	10^5	10^6	10^7
Resistance, r_1 in ohms per meter:															
1. Copper wire	0.00024	0.00087	0.0052	0.0567	4.827	0.00024	0.00091	0.0075	0.489	22.5	....	0.0206	.200	1.98	19.7
2. Iron wire	0.0072	0.092	0.292	0.97	7.72	0.0072	0.092	0.297	1.40	25.4	....	0.112	.489	2.89	20.0
3. Copper ribbon	0.00024	0.00024	0.00122	0.0503	4.81	0.00024	0.00028	0.0055	0.482	22.5	....	0.0199	.198	1.97	19.7
4. Iron ribbon	0.0019	0.0248	0.079	0.296	5.58	0.0019	0.0248	0.083	0.728	23.3	....	0.0435	.265	2.22	20.5
5. Iron pipe	0.0013	0.0167	0.053	0.215	5.33	0.0013	0.0167	0.058	0.647	23.0	....	0.0364	.250	2.14	20.2
Impedance, z, in ohms per meter.															
1. Copper wire	0.00053	0.076	0.753	7.51	75	0.00069	0.105	1.043	10.41	81	....	0.115	1.14	11.4	85
2. Iron wire	0.0105	0.191	1.083	8.46	78	0.0106	0.218	1.37	11.4	85	....	0.233	1.49	12.45	88
3. Copper ribbon	0.00030	0.030	0.298	2.97	30	0.00043	0.059	0.588	5.89	49	....	0.097	0.97	9.7	72
4. Iron ribbon	0.0028	0.060	0.315	3.23	31	0.0030	0.087	0.67	6.16	51	....	0.127	1.06	10.0	74
5. Iron pipe	0.00195	0.0446	0.305	2.65	26	0.0021	0.072	0.592	5.58	47	....	0.112	0.97	9.3	70
Power Factor cos ω, in per cent:															
1. Copper wire	45.3	1.14	0.69	0.76	6.4	34.8	0.87	0.72	4.7	27.8	....	17.9	17.5	17.3	23.2
2. Iron wire	69.5	48.2	27.0	11.5	9.9	68.0	42.2	21.6	12.3	30.0	....	48.0	32.7	23.3	22.7
3. Copper ribbon	80.0	0.8	0.41	1.7	16.0	53.5	0.48	0.93	8.2	46.0	....	20.5	20.4	20.3	27.3
4. Iron ribbon	68.0	41.3	25.1	9.2	18.0	63.3	28.6	12.4	11.8	45.8	....	34.3	25.0	22.2	27.7
5. Iron pipe	66.7	37.5	17.4	8.1	20.4	62.0	23.2	9.8	11.6	49.0	....	32.5	25.8	23.0	29.0
Voltage, e, in volts per meter															
1. Copper wire	0.053	7.6	75	751	7500	0.069	10.5	104	1041	8100	....	11.5	114	1140	8500
2. Iron wire	1.05	19.1	108	846	7800	1.06	21.8	137	1140	8500	....	23.3	149	1245	8800
3. Copper ribbon	0.03	3.0	30	297	3000	0.043	5.9	59	589	4900	....	9.7	97	970	7200
4. Iron ribbon	0.28	6	36	323	3100	0.30	8.7	67	616	5100	....	12.7	106	1000	7400
5. Iron pipe	0.195	4.5	30	265	2600	0.21	7.2	59	558	4700	....	11.2	97	930	7000

terial is very great at machine frequencies, 60 cycles, but becomes small and almost negligible at extremely high frequencies. This is rather against the usual assumption.

The reason is that at machine frequencies, the unequal current distribution, or the screening effect of the internal magnetic field, is still practically absent in copper conductors, even in round wires of medium size, while it is practically complete in iron conductors, even in ribbon of $^1/_{20}$-inch thickness; while at very high frequencies the effect of radiation preponderates, which is independent of the material, and the radiation resistance even independent of the shape of the conductor.

Thus under transmission line conditions, first and second section of Table III, at 60 cycles the impedance, and hence the voltage drop in the iron conductor is from 7 to 20 times that of the copper conductor; at 10,000 cycles the voltage drop in the iron conductor is only from 1.5 to 2.5 times that in the copper conductor; the difference has decreased to from 14 per cent to 44 per cent at 100,000 cycles, 5 per cent to 12.5 per cent at one million cycles, while at 10 million cycles the voltage drop in the iron conductor is only 3 to 5 per cent higher, thus practically the same and the only difference is that due to the conductor shape.

The effective resistance, and thus the power consumption in the iron conductor at 60 cycles is from 8 to 30 times that of the copper conductor, but with increasing frequency the difference in the effective resistance increases to from 88 to 106 times at 10,000 cycles, reaches a maximum, and then decreases again, and is from 15 to 65 times that of the copper conductor at 100,000 cycles, only 1½ to 17 times at a million cycles, while at 10 million cycles and above, all the differences in the effective resistance practically disappear.

As the result, the power factor of the conductor,—being the same, 100 per cent, at extremely low frequency and not much different, and fairly high at machine frequency—decreases with increasing frequency, reaches a minimum and then increases again to considerable values at extremely high frequency,—where the high radiation resistance comes into play. The difference between iron and copper, however, is that the minimum value of the power factor, at medium high frequencies, is very low in copper, a fraction of one per cent, while in the iron conductor the power factor always retains considerable values, the minimum being 10 or more times that of the copper conductor. Thus an oscillation in an iron conductor must die out at a much faster rate then in a copper conductor and the liability of the formation of a continual or cumulative oscillation may exist in copper conductors but hardly in iron conductors.

The effect of the shape of the conductor on the impedance or voltage drop is fairly uniform throughout the entire frequency range, the voltage drop being the smaller, the larger the circumference.

With regards to the effective resistance, however, the effect of the conductor shape is considerable at very low frequencies in iron conductors, but absent with copper conductors, due to the absence of the screening effect in copper at low frequency. With increasing frequency, the difference appears in the effective resistance of the copper conductor also, with the appearance of unequal current distribution, and the ratio of the resistance of the round conductor to that of the flat conductor approaches the same value in copper as in iron. With the approach of very high frequency, however, the difference decreases again, with the appearance of radiation effect, and finally vanishes.

Thus to convey currents of extremely high frequency, an iron conductor is almost as good as a copper conductor of the same shape and cross section. As iron is very much cheaper than copper, it follows that in high-frequency conduction an iron conductor under the conditions of Table III should be better than a copper conductor of the same cost and the same general shape, due to the larger size or rather circumference of the iron conductor.

There is, however, a material advantage at extremely high frequencies as well as at moderately high frequencies, in lower voltage drop at the same current resulting from such a shape conductor as gives maximum circumference, such as ribbon or hollow conductor. This advantage of ribbon or hollow tube, over the solid round conductor, exists also in the resistance and thus power consumption at medium high frequencies, but not at extremely high frequencies, but at the latter, in power consumption all conductors, regardless of size, shape or material, are practically equal.

With the thickness of ribbon conductor considered in Table III, of about $^1/_{20}$ inch, which is about the smallest mechanically permissible under usual conditions, the screening effect even in copper conductors is practically complete at 10,000 cycles, that is, the depth of penetration less than one half the thickness of the conductor. It follows, that in the design of high-frequency conductor, the thickness of the ribbon or hollow cylinder is essentially determined by mechanical and not by electrical considerations; in other words, the thinnest mechanically permissible conductor usually is thicker than necessary for carrying the current. As iron usually cannot be employed in as thin ribbon as copper, due to its rusting, an iron conductor would have a larger section than a copper conductor of the same voltage drop and power consumption. Thereby a part of the advantage gained by the employment of the cheaper material would be lost.

The last section of Table III gives the constants of the conductor without return conductor, such as would be represented by the discharge circuit of a lightning arrester, by a wireless telegraph antenna, etc., while the first two sections correspond to transmission line conditions; high-frequency currents between line conductors and between line and ground.

In the third section of Table III, the 60-cycle values are not given, and the values given for the lower high frequencies, 10^4 and even 10^5 cycles, usually have little meaning, are rarely realizable; they would correspond to a vertical conductor, as lightning arrester ground circuit, under conditions where no other conductor is within quarter wave distance. Even at 10^5 cycles, however, the quarter wave length is still 750 m. Thus there will practically always be other conductors within the field of the discharge conductor, acting as a partial return conductor, and the actual values of impedance and resistance, that is, of voltage drop and power consumption in the conductor, thus will be intermediate between those given in the third section, for a conductor without return conductor, and those given in the first sections, for conductors with return conductors. Except at extremely high frequencies, at which the wave length gets so short that the condition of a conductor without return conductor becomes realizable. It is interesting to note, therefore, that at extremely high frequencies, the constants of the conductor without return conductor approach those of the conductor with return conductor. At lower high frequencies, impedance and resistance, and thus voltage drop and power consumption, of the returnless conductor are much higher than those of the conductor with return, the more so the lower the frequency.

However, while the case of the conductor without return conductor is not realizable at low and medium high frequencies in industrial circuits, it probably is more or less realized by the lightning discharge between ground and cloud, and the constants given in the third section of Table III would probably approximately represent the conditions met in the conductors of lightning rods such as used for the protection of buildings against lightning.

It is interesting to note that with such a conductor without return conductor, the power factor is always fairly high, even with copper as conductor material. The impedance and thus the voltage drop does not differ much from those of the conductor with return conductor. The resistance, however, and thus the power consumption are much higher, sometimes, in copper conductors, more than a thousand times as large, showing the large amount of energy radiated by the conductor—which reappears more or less destructively as "induced lightning stroke" in objects in the neighborhood of the lightning stroke.

III.—Wave Decay in Transmission Lines

From the equations given in Appendix C, numerical values of the line constants are calculated and given in Table IV, for average transmission line conditions, that is, a copper wire, No. 00, with 6 ft. = 182 cm. between the conductors, and an average height of 30 ft. = 910 cm. above ground, for the two conditions:

a. a high-frequency oscillation between two line conductors;

b. a high-frequency oscillation between one line conductor and the ground.

The table gives:

The thermal resistance r_1, the radiation resistance r_3, and the total resistance $r = r_1 + r_3$;

The internal reactance x_1, the external reactance x_3 and the total reactance $x = x_1 + x_3$.

The magnetic attenuation $u_1 = \frac{r}{2L}$, the dielectric attenuation $u_2 = \frac{g}{2C}$, and the total attenuation $u = u_1 + u_2$.

The table also gives the duration of a transient in microseconds t, and in cycles N, that is, the time which a high-fre-

ATTENUATION CONSTANT

per

$f =$ $\underline{/f} =$	25 1.398	60 1.778	500 2.699	750 2.875	1,000 3	2,000 3.301	5,000 3.699	7,500 3.875	10,000 4	15,000 4.176	2 4
$r_1 =$	.24	.24	.24	.24	.273	.386	.612	.750	.866	1.066	
$x_1 =$	.008	.019	.157	.236	.273	.386	.612	.750	.866	1.060	
a. return conductor at 6 ft. = 182 cm. distance.											
$r_2 =$	......	......	......	......	......	......	.0012	.0027	.0048	.0108	
$r =$	.24	.24	.24	.24	.273	.386	.613	.753	.870	1.071	
$x_2 =$	.187	.45	3.75	5.625	7.5	15	37.5	56.25	75	112.5	1
$x =$	.195	.469	3.907	5.861	7.773	15.39	38.11	57.0	75.86	113.6	1
$L =$	1.245	1.245	1.245	1.245	1.24	1.23	1.22	1.215	1.21	1.205	
$u_1 = \frac{r}{2L} =$	96	96	96	96	110	157	251	310	360	445	5
$u_2 = \frac{g}{2C} =$	......	......	.005	.011	.020	.080	.5	1.125	2.0	4.5	
$u = u_1 + u_2 =$	96	96	96	96	110	157	252	311	362	450	5
$\underline{/u} =$	1.982	1.982	1.982	1.982	2.041	2.196	2.401	2.493	2.559	2.653	
$l =$	10,400	10,400	10,400	10,400	9,090	6,370	3,970	3,210	2,760	2,220	1,9
$N =$	.26	.62	5.2	7.8	9.09	12.74	19.85	24.1	27.6	33.3	
$\cos^{\omega} =$	......	......	6.16	4.10	3.51	2.52	1.61	1.32	1.15	.94	
$\cos^{\omega}{}_C =$	......	......	......	......	......	......	......	......	.01	.01	
b. conductor 30 ft. above ground as return: $l' = 1820$ cm.											
$r_2 =$	......	......	......	......	......	......	.012	.027	.048	.108	
$r =$	.24	.24	.24	.24	.273	.386	.624	.777	.913	1.168	
$x_2 =$	.26	.624	5.2	7.8	10.4	20.8	52	78	104	156	2
$x =$	.268	.643	5.357	8.036	10.67	21.16	52.61	78.75	104.9	157.1	2
$L =$	1.705	1.705	1.705	1.705	1.70	1.69	1.68	1.675	1.69	1.665	
$u_1 = \frac{r}{2L} =$	70	70	70	70	80	114	186	232	273	350	4
$u_2 = \frac{g}{2C} =$	......	......	.04	.08	.14	.58	3.6	8.1	14.4	32.5	
$u = u_1 + u_2 =$	70	70	70	70	80	115	190	240	287	383	4
$\underline{/u} =$	1.845	1.845	1.845	1.845	1.903	2.061	2.279	2.380	2.458	2.583	
$l =$	14,300	14,300	14,300	14,300	12,500	8,700	5,260	4,170	3,480	2,610	2,0
$N =$	.36	.86	7.15	10.7	12.5	17.4	26.3	31.3	34.8	39.1	
$\cos^{\omega} =$	......	......	4.48	2.98	2.56	1.83	1.19	.98	.87	.74	
$\cos^{\omega}{}_C =$	......	......	......	......	......	.01	.02	.03	.05	.07	

TABLE IV.

NTS OF COPPER WIRE NO. 00 B. & S. GAGE.

per kilometer of conductor

20,000 4.301	50,000 4.699	75,000 4.875	10^5 5	2×10^5 5.301	5 × 10 5.699	7.5×10^5 5.875	10^6 6	2×10^6 6.301	5×10^6 6.699	Cycles
1.225	1.93	2.37	2.73	3.86	6.12	7.50	8.65	12.25	10.3	ohms
1.225	1.93	2.37	2.73	3.86	6.12	7.50	8.65	12.25	10.3	"
.0192	.12	.27	.48	1.92	12	27	48	192	1,200	"
1.243	3.05	3.64	3.21	5.78	18.1	34.5	56.6	204	1,219	"
150	375	562.5	750	1,500	3,750	5,625	7,500	15,000	37,500	"
151.2	376.9	564.9	752.7	1,504	3,756 9	5,632	7,509	15,012	37,520	"
1.205	1.20	1.20	1.20	1.20	1.1 5	1.195	1.195	1.195	1.195	henrys
516	855	1,100	1,340	2,410	7,570	14,400	23,700	85,600	510,000	
8.0	50	112.5	200	800	5,000	11,200	20,000	80,000	500,000	
524	905	1,212	1,540	3,210	12,570	25,650	43,700	165,600	1,010,000	
2.719	2.957	3.084	3.187	3.506	4.099	4.400	4.640	5.219	6.004	
1,910	1,100	825	649	311	79.6	39.0	22.9	6.04	1.00	micro-seconds
38.2	55.0	61.9	64.9	62.2	39.8	29.2	22.9	12.1	6.0	cycles
.82	.54	.47	.43	.38	.48	.61	.75	1.36	3.25	per cent
.01	.03	.05	.06	.13	.32	.48	.64	1.27	3.18	per cent
.192	1.2	2.7	4.8	19.2	120	270	480	1,920	12,000	ohms
1.417	3.13	5.07	7.53	23.06	126.1	277.5	488.6	1,932	12,0[illegible]	"
208	520	780	1,040	2,080	5,200	7,800	10,400	20,800	52,000	"
209.2	252	782	1,043	2,084	5,206	7,807	10,409	20,812	52,030	"
1.665	1.66	1.66	1.66	1.66	1.655	1.655	1.655	1.655	1.655	henrys
424	942	1,520	2,270	6,930	38,200	83,800	147,500	583,000	3,630,000	
57.6	360	810	1,440	5,760	36,000	81,000	144,000	576,000	3,600,000	
482	1,302	2,330	3,710	12,690	74,200	164,800	291,500	1,159,000	7,230,000	
2.683	3.115	3.367	3.569	4.103	4.870	5.217	5.465	6.064	6.8[illegible]	
2,070	768	429	269	78.8	13.5	6.07	3.43	.86	.14	micro-seconds
41.4	38.4	31.2	26.9	15.8	6.75	4.55	3.43	1.72	.69	cycles
.68	.60	.65	.71	1.10	2.42	3.55	4.70	9.24	22.5	per cent
.09	.23	.35	.46	.92	2.31	3.46	4.62	9.22	22.5	per cent

[STEINMETZ]

quency oscillation of the frequency *f* would last, if continuing with its initial intensity, and the number of cycles, which it would perform. It also gives the power factor, in per cent, of the series circuit, as determined by resistance and inductance, and of the shunt circuit, as determined by shunted conductance and capacity.

As seen, the attenuation constant *u* is constant up to nearly one thousand cycles. Thus in this range, all the frequencies die out at the same rate. From about one thousand cycles up to about 100,000 cycles, the attenuation constant gradually increases, and thus oscillations die out the more rapidly, the higher the frequency, as seen by the gradual decrease of the duration *t*. However, as the increase of the attenuation constant and thus the increase of the rapidity of the decay of the disturbance, in this range, is smaller than the increase of frequency, the number of cycles performed by the oscillation increases. Thus, at 25 or 60 cycles, the stored energy, which supplies the oscillating power, would be expended in less than one cycle, that is, a real oscillation would hardly materialize (except by other sources of energy, as the stored magnetic energy of a transformer connected to the line). At 1000 cycles, the oscillation would last 9 to 12 cycles, and at still higher frequencies reach a maximum of 41.4 cycles at 20,000 cycles frequency, in the oscillation against ground; 64.9 cycles at 100,000 cycles, in the oscillation between line conductors. This represents a fairly well sustained oscillation, in which the cumulative effect of successive cycles may be considerable. Above 100,000 cycles the attenuation constant begins to rise rapidly, and reaches enormous values, due to the rapidly increasing energy dissipation by radiation. As the result, the duration of the oscillation very rapidly decreases, and the number of cycles performed by the oscillation decreases, until, beyond a million cycles, the energy dissipation is so rapid, that practically no oscillation can occur; the oscillation dying out in a cycle or less, thus being practically harmless.

The attenuation constant is plotted, up to 15,000 cycles, in Fig. 2, with the frequency as abscissas, and is plotted in Fig. 3 in logarithmic scale, as (3).

Noteworthy is the great difference between the oscillation against ground, and the oscillation between line conductors. The oscillation against ground is more persistent at low frequencies, due to the greater amount of stored energy in the

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electric field of the conductor, which reaches all the distance to the ground. When reaching into very high frequencies however, the energy dissipation by radiation becomes appreciable at lower frequencies in the oscillation against ground, than in the oscillation between line conductors, and reaches much higher values, with the result that the decay of an oscillation between line and ground is much more rapid at high frequencies than the decay of an oscillation between line conductors. For instance, at 100,000 cycles, the latter performs 65 cycles before dying out, while the former has dissipated its energy in 27 cycles, that is, in less than half the time.

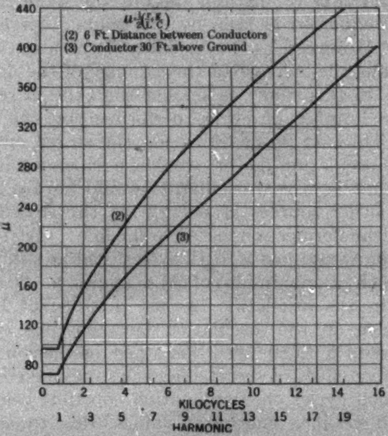

FIG. 2—ATTENUATION CONSTANT OF WIRE NO. 00 B. & S. GAGE COPPER
$u = 1/2\ (r/L + g/C)$

To further investigate this, in Tables V and VI the numerical values of effective resistance, power factor, attenuation constant and duration of a transient oscillation, in cycles, are given for six typical conductors and circuits, for frequencies from 10 cycles to five million cycles, and plotted in Fig. 3, 4, and 5, in logarithmic scale.

1, 2, and 3 are lines of high power: copper conductor No. 00 B. & S. gage, in 1 with 18 inches = 45.5 cm. between conductors, corresponding about to average distribution conductors, in 2 with 6 ft. = 182 cm., between conductors, corresponding to about average transmission line conductors with the oscillation between two lines, and in 3 with 60 ft. = 1830 cm. between conductor and return conductor, corresponding to an oscillation between line and ground, under average transmission line conditions, with the conductor 30 ft. above ground. 4, 5 and 6 give the same condition of an oscillation between line and ground, but in 4 an iron wire of the size of No. 00 B. & S. gage, such as has been proposed for the station end of transmission lines, to oppose the approach of high-frequency disturbances. In 5 a copper wire No. 4 B. & S. gage, that is, a low power transmission line is represented, and in 6 a stranded aluminum conductor of the same conductivity as copper wire No. 00 B. & S. gage.

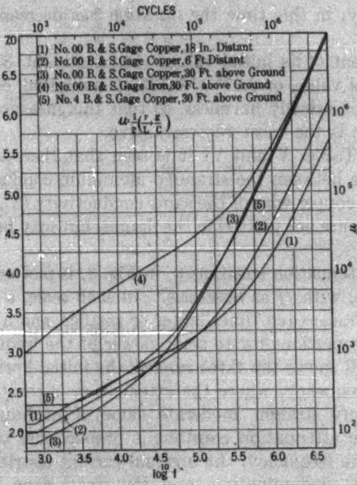

FIG. 3—ATTENUATION CONSTANT $u = 1/2\ (r/L + g/C)$ (TABLE VI)

The equations of the constants for these six circuits are given in Table V. This table also gives the limiting frequencies, between which the various formulas apply with sufficient accuracy for practical purposes, and the lower limits, where the effects become appreciable, in the various conductors.

In Fig. 3 the attenuation constants are plotted, in Fig. 4 the power factors and in Fig. 5 the duration, in cycles.

As such a transient oscillation dies out exponentially, theoretically it has no definite duration, but lasts forever, though practically it may have ceased in a few micro-seconds. Thus as duration is defined the time, or the number of cycles, which the oscillation would last if maintaining its initial intensity. In reality, in this time the duration has decreased to $\frac{1}{\epsilon}$, or 37 per cent of its initial value. Physically, at 37 per cent of its initial value, or $0.37^2 = 0.135$ of its initial energy, it has become practically harmless, so that this measure of duration probably is the most representative.

From Tables V and VI it is seen that there is no marked difference between the stranded aluminum conductor (6), and the solid copper wire of the same conductivity (3), and the values of (6) are not plotted in the figures, but may be represented by (3).

The attenuation constant *u*, in Fig. 3, is plotted in logarithmic scale, with $/\underline{f}$ as abscissas. In such scale, a difference of one unit means ten times larger or smaller, and a straight line means proportionality to some power of the frequency. This figure well shows the three ranges; the initial horizontal range at low frequency, where the attenuation is constant; the approximately straight moderate slope of medium frequency, where the attenuation constant is proportional to the square root of the frequency, the unequal current distribution in the conductor predominating, and the steep slope at high frequencies, where the radiation resistance predominates, which is proportional to the square of the frequency.

It is interesting to note that at high frequencies the distance of the return conductor is the dominating factor, while the effect of conductor size and material vanishes; in copper wire No. 00 the rate of decay is practically the same as in copper wire No. 4, though the latter has more than three times the resistance, or in the iron wire, which has nearly six times the resistance and 200 times the permeability. The permeability of the iron wire has been assumed as $\mu = 200$, representing load conditions, where by the passage of the low-frequency

LINE C

= frequency, in cycles per second.
S = 3 × 10^{10} cm. sec. = propagation velocity in
K = 1 = specific capacity.
l_0 = length of conductor, in kilometers = 10^5 cm.

	Size of round conductor, B. & S. gage: Radius of conductor, cm.: Distance of return conductor, cm.: Material of conductor: Conductivity: Permeability:	 $l_r =$ $l' =$ $\gamma = 10^5 +$ $\mu =$
Low	Ohmic resistance	$r_0 = \frac{10^8 l_0}{\pi l_r{}^2 \gamma}$ ohms =
frequency.	Internal reactance	$x_0^1 = 0.1\,\pi\mu f l_0 10^{-8}$ ohms $= f \times 10^{-3} \times$
	Limit, cycles:	$f =$
High	Thermal resistance, Internal reactance	$r_1 = x_1 = \frac{5 l_0}{l_r}\sqrt{\frac{0.4\mu f}{\gamma}}$ ohms $= 10^{-3} \times$
	Lower limit, cycles:	$f =$
frequency.	Radiation resistance	$r_2 = \frac{0.8\pi^2 f^2 l' l_0}{S} 10^{-3}$ ohms $= f^2 \times 10$
	Upper limit, cycles:	$f =$
	External reactance:	$x_2 = 0.4\pi f l_0 \log\frac{l'}{l_r} 10^{-3}$ ohms $= f \times 10$
	Total resistance:	$r = r_1\,(\text{or } r_0) + r_2$
	Total reactance:	$x = x_1\,(\text{or } x'_0) + x_2$
	Inductance:	$L = \frac{x}{2\pi f}$ henrys.
	Magnetic attenuation:	$\frac{r}{2L}$
	Dielectric radiation resistance:	$r_c = \frac{2Sl'}{K l_0} 10^{-14}$ ohms =
	Capacity reactance:	$x_c = \frac{S^2}{\pi f K l_0}\log\frac{l'}{l_r} 10^{-14}$ ohms $= \frac{10^9}{f} \times$
	Dielectric attenuation:	$\frac{g}{2C} = \pi f \frac{r_c}{x_c} = \frac{2\pi^2 f^2 l'}{S\log\frac{l'}{l_r}} = f^2 \times 10^{-3} \times$
	Attenuation Constant:	$\alpha = \frac{1}{2}\left(\frac{r}{L} + \frac{g}{C}\right)$

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	per kilometer of conductor:					
	(1) 00 0.463 45.5 copper 6.2 1	(2) 00 0.463 182 copper 6.2 1	(3) 00 0.463 1820 copper 6.2 1	(4) 00 0.45 1820 iron 1.1 200	(5) 4 0.258 1820 copper 6.2 1	(6) (00) 0.65 1820 aluminum 3.8 1
	0.240	0.240	0.240	1.35	0.770	0.240
	0.314	0.314	0.314	62.83	0.314	0.314
	10^3	10^3	10^3	10	2×10^3	10^3
×	8.65	8.65	8.65	290	15.5	7.86
	2×10^5	10^6	2×10^4	2×10^5	2×10^4	2×10^4
	0.012	0.048	0.48	0.48	0.48	0.48
	10	3×10^7	3×10^8	3×10^8	3×10^8	3×10^8
	5.76	7.5	10.4	10.4	11.1	9.95
	0.0273	0.1092	1.092	1.092	1.092	1.092
	13.2	17.3	23.7	23.7	25.4	22.8
	0.0065	0.020	0.144	0.144	0.135	0.151

TABLE VI.

ATTENUATION CONS

		f =	10c	60	500	1,000	2,000	5,000c	10 kc	20	50
1	No. 00 B. & S. G. l_r = 0.463 cm. l' = 45.5 cm. copper	r =	0.24	0.24	0.24	0.273	0.386	0.612	0.866	1.230	1.
		cos ω =	96.9	54.9	7.86	4.53	3.26	2.07	1.48	1.055	0.6
		s =	124	124	124	142	204	327	466	666	1,081
		N =	0.08	0.43	4.04	7.04	9.80	15.3	21.5	30.2	46.3
2	l' = 182 cm copper	r =	0.24	0.24	0.24	0.273	0.386	0.613	0.870	1.249	2.0
		cos ω =	95.1	45.4	6.16	3.51	2.52	1.61	1.15	0.822	0.5
		s =	96	96	96	110	157	252	362	524	905
		N =	0.10	0.62	5.20	9.09	12.74	19.85	27.6	38.2	55.0
3	l' = 1820 cm. copper	r =	0.24	0.24	0.24	0.273	0.386	0.624	0.913	1.417	3.1
		cos ω =	91.3	34.9	4.48	2.56	1.83	1.19	0.870	0.677	0.6
		s =	70	70	70	80	115	190	287	482	1,302
		N =	0.14	0.86	7.15	12.5	17.4	26.3	34.8	41.4	38.4
4	iron	r =	1.35	2.25	6.48	9.16	12.95	20.51	29.05	41.29	66.0
		cos ω =	(80)	61.7	48.5	42.4	35.8	27.1	21.5	16.0	11.3
		s =	(25)	148	873	1,470	2,410	4,444	6,914	10,210	18,060
		N =	(0.40)	0.40	0.57	0.68	0.83	1.13	1.45	1.96	2.7
5	No.4 B.&S.G. l_r = 0.258 cm. l' = 1820 cm. copper	r =	0.77	0.77	0.77	0.77	0.772	1.112	1.598	2.382	4.6
		c osω =	98.9	74.5	13.4	6.75	3.38	1.96	1.42	1.06	0.8
		s =	212	212	212	212	213	311	458	721	1,647
		N =	0.047	0.28	2.36	4.72	9.38	16.05	21.8	27.8	30.4
6	7 strand Al. equivalent to No.00 copper l_r = 0.65 cm. l' = 1820 cm.	r =	0.24	0.24	0.24	0.248	0.351	0.568	0.834	1.302	2.9
		cos ω =	91.9	36.0	4.64	2.42	1.72	1.12	0.83	0.65	0.5
		s =	73	73	73	76	109	181	275	467	1,298
		N =	0.14	0.82	6.85	13.2	18.3	27.6	36.4	42.8	38.5

100	200	500	1,000	2,000	5,000 kc
2.85	4.34	9.12	20.65	60.25	319.3
0.494	0.376	0.316	0.358	0.524	1.105%
1,815	2,620	6,595	17,740	58,800	336,500
64.5	76.4	76.0	56.4	34.0	14.9c
3.21	5.78	18.1	56.6	204.2	1,219
0.427	0.384	0.483	0.753	1.36	3.25 %
1,540	3,210	12,570	43,700	165,600	1,010,000
64.9	62.2	30.8	22.9	12.1	5.0 c
7.53	23.06	126.1	488.6	1,932	12,020
0.706	1.105	2.42	4.70	9.24	32.5 %
3,710	12,690	74,200	291,500	1,159,000	7,230,000
26.9	15.8	6.75	3.43	1.72	0.69 c
96.4	148.7	325	770	2,331	12,650
8.5	6.7	6.0	7.2	11.0	23.3
8,200	48,200	130,400	371,000	1,266,000	7,370,000
3.55	4.17	3.83	2.70	1.58	0.68 c
9.70	26.1	131	495	1,940	12,030
0.87	1.17	2.34	4.44	8.7	21.1 %
4,080	12,750	70,550	274,500	1,087,000	6,775,000
24.5	15.7	7.1	3.65	1.84	0.75
7.28	22.71	125.6	487.9	1,931	12,018
0.73	1.14	2.5	4.9	9.8	23.7
37.80	13,130	77,000	303,000	1,207,000	7,580,000
26.5	15.2	6.5	3.3	1.66	6.65

[STEINMETZ]

power current the iron is magnetically near saturation, and its permeability thus lowered. However, the decay of the oscillation between conductor and ground is six to seven times more rapid than that between conductors 6 ft. apart, and that between conductors 18 inches apart about three times less.

This shows, that to produce quicker damping of high-frequency waves, such as are instrumental in steep wave fronts, the most effective way is to separate the conductors as far as possible, perhaps even lead them to the station by separate single-conductor lines; but the use of high-resistance conductors, or of magnetic material, as iron, offers little or practically no advantage in damping very high frequencies or flattening steep wave fronts.

At medium and low frequencies however, the relation reverses, and the decay of the wave is the smaller the greater the distance of the return conductor. The reason is, that in this range the effective resistance is still independent of the conductor distance, while the inductance increases with increasing distance. At medium and low frequencies, the iron conductor offers an enormously increased attenuation—from 10 to 20 times that of non-magnetic conductors.

The power factor of the conductor is plotted in Fig. 4. As seen, it decreases from unity, at very low frequencies, to a minimum, at medium high frequencies, and then increases again to very high values at very high frequencies. The minimum value is a fraction of one per cent except with the iron conductor, where the minimum is very much higher. The power factor is of importance as it indicates the percentage of the oscillating energy, which is dissipated per wave of oscillation. This is represented still better by Fig. 5 the duration of the oscillation in cycles, that is, the number of cycles which an oscillation lasts before dissipating the stored energy which causes it.

At medium high frequencies, the oscillation is the more persistent the lower the ohmic resistance of the conductor and the further away the return conductor, while at very high frequencies the reverse is the case, and the oscillation is the more persistent, the shorter the distance of the return conductor, while the size and material of the conductor ceases to have any effect.

The maximum number of cycles is reached at medium high frequencies, in the range between 20,000 and 100,000 cycles—

depending on conductor size and distance of return conductor. It thus is in this range of frequencies, where an oscillation caused by some disturbance lasts the greatest number of cycles, that the possibility, by some energy supply by means of an arc, etc., to form a stationary oscillation or even a cumulative oscillation, thus to become continuous or "undamped", is greatest.

It would thus appear, that this range of frequencies, of 20,000 to 100,000, represents what may be called the "danger frequencies" of transmission systems. It is interesting to note, that experimental investigations have shown that the natural frequency of oscillation of the high-voltage windings of large power transformers usually is within this range of danger frequencies. The possibility of the formation of destructive cumulative oscillations or stationary waves in the high-voltage windings of large power transformers is therefore greater than probably with any other class of circuits, so that such high-

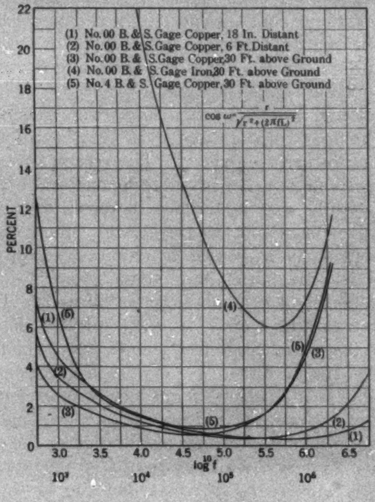

FIG. 4—POWER FACTOR OF SERIES CONSTANT $\cos \omega = \frac{r}{\sqrt{r^2 + (2\pi f L)^2}}$

(TABLE VI)

potential transformer windings require specially high disruptive strength and protection. This accounts for the not infrequent disastrous experience with such transformers, before this matter was realized.

Fig. 5 also shows, that the duration of an oscillation in iron wire, in cycles, is very low at all frequencies. Thus the formation of a stationary oscillation in an iron conductor is practically excluded, but such conductors would act as a dead resistance, damping any oscillation by rapid energy dissipation.

The duration of high-frequency oscillations, in cycles, increases with increasing frequency, to a maximum at medium frequencies. This obviously does not mean that the time during which the oscillation lasts increases; the time, in micro-seconds, naturally decreases with increasing frequency, due to the increasing attenuation constant. Thus in conductor (2) for instance, the oscillation lasts 65 cycles at a frequency of 100,000 cycles, but only 9 cycles at 1000-cycle frequency. However, the 65 cycles are traversed in 650 micro-seconds, while the 9 cycles last 9000 micro-seconds, or 14 times as long. It is not the total time of oscillation, but the cumulative effect due to the numerous and only slowly decreasing successive waves, which increases as represented in Fig. 5.

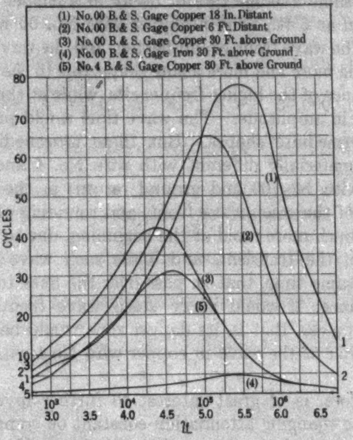

FIG. 5—DURATION OF OSCILLATION IN CYCLES (TABLE VI)

An oscillation between copper wires No. 4 B. & S. gage 6 ft. apart, would give a duration curve, which at moderate, frequencies follows (5) of Fig. 5, but at high frequencies follows (2). Thus the average duration and average rate of decay would be about the same as (3), an oscillation between copper wire No. 00 and ground. However, the oscillation would be more persistent in such a conductor at high, and less persistent at low frequencies. A complex wave, containing all the harmonics from low to very high ones, such as a steep wave front impulse or an approximately rectangular wave, as may be produced by a spark discharge, etc, would have about the same average rate of decay in a copper wire No. 4 with return at 6 ft., as in a copper wire No. 00 with ground return. The wave front would flatten, and the wave round off, approaching more and more a sine shape, due to the more rapid disappearance of the higher frequencies, while at the same time decreasing in amplitude. The wave thus would pass through many intermediate shapes. But these intermediate shapes would be materially different with wire No. 4 and return at 6 ft., as with No. 00 and ground return; in the latter, the flattening of the steep wave front, and rounding of the wave, would be much more rapid at the beginning, due to the shorter duration of the transient, and while such wave would last about the same time, that is, pass over the lines to about the same distance, it would carry steep wave fronts to much shorter distances, that is, its danger zone would be materially less than that of the wave in copper wire No.4 with return at 6 ft.

It therefore, is of great interest to further investigate the effect of the changing attenuation constant on complex waves, and more particularly those with steep wave fronts, as the rectangular waves of starting or disconnecting lines, etc.

IV.—Attenuation of Rectangular Waves

The destructiveness of high frequencies or steep wave fronts in industrial circuits is rarely due to over-voltage between the circuit conductors or between conductor and ground, but is due to the piling up of the voltage locally, in inductive parts of the circuit, such as end turns of transformers or generators, current transformers, potential regulators, etc., or inside of inductive windings as the high-potential coils of power transformers, by the formation of nodes and wave crests.

Such effect may be produced by high-frequency oscillations sustained over a number of cycles, as discussed in III, as oscillations lasting only a very few cycles or a fraction of a cycle, or due to non-oscillating transients, as single impulses, etc. As the high rate of change of voltage with the time, and the correspondingly high voltage gradients along the conductor are the source of danger, to calculate and compare oscillatory and non-oscillatory effects in this respect, it has become customary in the last years to speak of an "equivalent frequency" of impulses, wave fronts or other non-oscillatory transients.

As "effective" or "equivalent" frequency of an impulse, wave front etc., is understood the frequency of an oscillation, which has the same maximum amplitude, e or i, and the same maximum gradient $\frac{de}{dt}$ or $\frac{di}{dt}$. Thus assuming an impulse which reaches a maximum voltage $e = 60{,}000$, and has a maximum rate of increase of voltage of $\frac{de}{dt} = 10^{10}$, that is, a maximum voltage rise at the rate of 10^{10} volts per second, or 10,000 volts per micro-second. As the average voltage rise of a sine wave is $\frac{2}{\pi}$ times the maximum, the average rise of an oscillation of the same maximum gradient as the impulse would be

$$\frac{2}{\pi}\,\frac{de}{dt} = \frac{20{,}000}{\pi}$$

volts per microsecond. The total voltage rise of $e = 60{,}000$ thus would occur in

$$\frac{e}{\frac{2}{\pi}\,\frac{de}{dt}} = \frac{60{,}000\pi}{20{,}000} = 9.4 \text{ micro-seconds.}$$

A complete cycle of this oscillation thus would last $4 \times 9.4 = 37.6$ micro-seconds, and the equivalent frequency of the impulse would be

$$f = \frac{10^6}{37.6} = 26{,}600 \text{ cycles or } 26.6 \text{ kilo cycles.}$$

The equivalent frequency of a perfectly rectangular wave front if such could exist, obviously would be infinity.

A. QUARTER WAVE CHARGING OR DISCHARGING OSCILLATION OF LINE

Considering first the theoretically rectangular wave of connecting a transmission line to a circuit, or disconnecting it from the circuit.

Suppose a transmission line, open at the distant end, is connected to a voltage E. At this moment, the voltage of the line is zero. It should be, in permanent conditions, E. Thus the circuit voltage consists of a permanent voltage E (which is the instantaneous value of the alternating supply voltage at this moment $E_0 \sin \varphi$ and the transient voltage $-E$. We thus have a transient voltage, which uniformly $= -E$ all along the line, except at the switching point $l = 0$, where the transient voltage is zero.

Or, suppose a transmission line, open at the far end, is connected to a source of voltage, and at the moment where this voltage is E, the line short circuits at some point, by a spark discharge, flash-over, etc. Thus at this moment, the voltage $= 0$ at the point of short circuit, and is $= E$ everywhere between this point and the end of the line. Thus we get a line discharge leading to the same transient, a theoretically rectangular wave. In the part of the line between generator and short circuit, we have a different transient, a circuit of voltage $e = 0$ at one end, $e = E$ throughout the entire length at time $t = 0$, and $e = E$ continuously at the other end, where the generator maintains the voltage. However, this again leads to the same transient, of a theoretically rectangular wave.

Assuming thus, as an instance, a transmission line of 100-km. length, of copper wire No. 00 B. & S. gage, 30 ft. above ground, open circuited at the other end $l = 100$ km., and connected to a source of voltage E at the beginning, $l = 0$.

Then the beginning of the line, $l = 0$, is grounded, at the time $t = 0$, thus giving a quarter wave oscillation, with the terminal conditions:

Voltage along the line constant $= E$, at time $t = 0$, except at the beginning of the line, $l = 0$, where the voltage is 0.

Current along the line $= 0$ at $t = 0$, except at $l = 0$, where the current is indefinite.

The equation of the quarter wave oscillation of the line conductor against ground, as usually given[1], then is:

---

[1] See for instance: "Theory and Calculation of Transient Electric Phenomena," Section IV, Chapter VII, equation (57).

$$e = \frac{4\,E}{\pi}\,\epsilon^{-ut} \sum_{0}^{\infty}{}_{n} \frac{\sin(2\,n+1)\,\tau \cos(2\,n+1)\,\theta}{n+1} \tag{1}$$

where:
θ is the time angle of the fundamental wave of oscillation, of frequency:

$$f_0 = \frac{S}{4\,l_0} = \frac{3 \times 10^{10}}{4 \times 100 \times 10^5} = 750 \text{ cycles.} \tag{2}$$

τ is the distance angle, for $l_0 = 100$ km. = 90° = $\pi/2$, that is:

$$\left.\begin{aligned} \theta &= 2\,\pi\,f_0\,t \\ \tau &= \frac{\pi\,l}{2\,l_0} \end{aligned}\right\} \tag{3}$$

Equation (**1**) however assumes that u, and thus r, L, C and g are constant for all frequencies. As this is not the case, but u is a function of the frequency, and thus of n: ϵ^{-ut} cannot be taken out of the summation sign. Equation (**1**) thus must be .written:

$$e = \frac{4\,E}{\pi} \sum_{0}^{\infty}{}_{n}\, \epsilon^{u_n t} \frac{\sin(2\,n+1)\,\tau \cos(2\,n+1)\,\theta}{2\,n+1} \tag{4}$$

where u_n is the value of u for the frequency: $f = (2\,n+1)\,f_0$.

From (**4**) follows, as the voltage gradient along the line:

$$\frac{d\,e}{d\,\tau} = \frac{4\,E}{\pi} \sum_{0}^{\infty}{}_{n}\, \epsilon^{-u_n t} \cos(2\,n+1)\,\tau \cos(2\,n+1)\,\theta \tag{5}$$

$$= \frac{2\,E}{\pi} \left\{ \sum_{0}^{\infty}{}_{n}\, \epsilon^{-u_n t} \cos(2\,n+1)\,(\tau+\theta) + \sum_{0}^{\infty}{}_{n}\, \epsilon^{-u_n t} \cos(2\,n-1)\,(\tau-\theta) \right\} \tag{6}$$

The maximum voltage gradient occurs at the wave front, that is, for $\theta = \tau$. Substituting this, and substituting further, from (**3**):

$$d\,\tau = \frac{\pi}{2\,l_0}\,d\,l \tag{7}$$

gives, as the maximum voltage gradient:

$$G = \frac{d\,e}{d\,l} = \frac{E}{l_0} \left\{ \sum_0^{\infty}{}_n \epsilon^{-u_n t} \cos (2\,n + 1)\,2\,\tau + \sum_0^{\infty}{}_n \epsilon^{-\mu_n t} \right\} \tag{8}$$

It is however:

$$\sum_0^{\infty}{}_n \cos (2\,n + 1)\,2\,\tau = 0 \tag{9}$$

for all values of τ except $\tau = 0$ and $\tau = \pi$, that is, the beginning of the line, $l = 0$.

Thus, approximately, as u_n varies gradually:

$$\sum_0^{\infty}{}_n \epsilon^{-u_n t} \cos (2\,\pi + 1)\,2\,\tau = 0 \tag{10}$$

except for values of $\tau = 0$ or very near thereto.

Substituting (**10**) into (**8**) gives:

$$G = \frac{E}{l_0} \sum_0^{\infty}{}_n \epsilon^{-\mu_n t} \tag{11}$$

as the approximate expression of the maximum voltage gradient, that is, the steepness of the wave front, at time t, that is, at distance from the origin of the wave:

$$l = S\,t = 3 \times 10^{10}\,t \tag{12}$$

If θ differs materially from τ, the term with $(\tau - \theta)$ in equation (**6**) also vanishes, and $\frac{d\,e}{d\,\tau} = 0$, that is, there is no voltage gradient except at and near the wave front.

From (**11**) are now calculated numerical values of the steepness of the wavefront G, for various times t after its origin, and thus by (**12**), various distances l from the origin. These numerical values are given in Table VII, for $E = 60{,}000$ volts.

At a fundamental frequency of $f_0 = 750$ cycles, successive harmonics differ from each other by 1500 cycles, and for every value of t, values of $\epsilon^{-u_n t}$ thus have to be calculated for the frequencies:

$n =$	0	1	2	3	4	5	6	etc.
$f =$	750	2250	3750	5250	6750	8250	9750	cycles etc.,

until the further terms add no further appreciable amount to $\Sigma \epsilon^{-u_n l}$. In calculating, it is found that for instance for $t = 5$ microseconds, or $l = 1.5$ km., this occurs at $f = 5 \times 10^6$ cycle, thus beyond the $\frac{5 \times 10^6}{750} = 6670$th harmonic. Thus 6670 terms of the series would have to be calculated to get this one point of the wave gradient: more terms for shorter, less terms for longer distances l of wave travel. This obviously is impossible, and some simpler approximation, of sufficient accuracy, thus is required.

TABLE VII

ATTENUATION OF WAVE FRONT OF QUARTERWAVE OSCILLATION, OF 100 KM. LINE, 60,000 VOLTS.

Time t, micro-seconds.	Distance l km:	Distance l miles:	f, where $\Sigma \epsilon^{-ul}$ vanishes	Voltage $\frac{1}{E} \frac{de}{dt}$	Gradient G, volts per meter.	Wave Front: Equivalent kilo-cycles.	Wave Front: Length, meters.
from origin of wave.							
(3) Copper wire No. 00 B. & S. gage 30 ft. = 910 cm. above ground.							
0	0	0	—	∞	60,000	∞	0
.03	.01	.0062	....	5,800	5,500	8,800	6
.575	.1725	.107	10×10^6	1,370	1,290	2,060	73
1.15	.345	.215	8×10^6	1,025	967	1,540	98
2.3	.69	.43	6×10^6	670	630	1,000	150
11.5	3.45	2.15	4×10^6	300	280	450	330
23	6.9	4.3	2×10^6	218	205	330	460
92	27.6	17.2	10^6	104	98	156	960
230	69	43	$.7 \times 10^6$	62	58	92	1,630
2,300	690	430	$.2 \times 10^6$	13	12	19	7,900
23,000	6,900	4,300	50,000	1.3	1.2	1.9	79,000
(1) Copper wire No. 00 B. & S. gage, return conductor at 18 ins. = 45.5 cm distance.							
2.3	.69	.43	20×10^6	3,450	3,240	5,160	29
23	6.9	4.3	10×10^6	1,020	960	1,530	98
230	69	43	4×10^6	250	234	370	400
(5) Copper wire No. 4 B. & S gage, 30 ft. = 910 cm. above ground.							
23	6.9	4.3	2×10^6	227	213	340	440
230	69	43	$.7 \times 10^6$	64	60	94	1,570
(4) Iron wire No. 00 B. & S. gage, 30 ft. = 910 cm. above ground.							
23	6.9	4.3	2×10^6	143	135	215	700
230	69	43	$.3 \times 10^6$	11	10	16	9,400

This may be done as follows:

In the range from 5×10^5 to 10^6 cycles for instance, there are $\frac{10^6 - 5 \times 10^5}{1500} = 333$ harmonics. Instead of calculating u and ϵ^{-ul} for each of these harmonics, calculate ϵ^{-ul} for the average value of these 333 harmonics, and multiply by 333.

Thus, dividing the entire frequency range (beyond the lowest harmonics, which are calculated separately), into groups, and calculating one average value for each group, the calculation of $\Sigma\, \epsilon^{-ut}$ becomes feasible.

Since ϵ^{-ut} is calculated through $\underline{/\epsilon^{-ut}} = - u\, t\, \underline{/\epsilon}$, as values of t, multiples of $\frac{1}{\underline{/\epsilon}}$ have been chosen, to still further simplify the calculation, in deriving a curve of gradients G.*

Table VII gives the values of the maximum voltage gradient of the wave front, in volts per meter at 60,000 volts maximum initial line voltage, the equivalent frequency of the wave front, in kilo-cycles, and the length of the wave front, in meters, for various times of wave travel, from 0.03 micro-seconds up, and corresponding distances of wave travel, from 10 meters from the origin as rectangular wave, up to thousands of kilometers, for copper wire No. 00 B. & S. gage, 30 ft. = 910 cm. above ground, with the ground as return.

TABLE VIII.

ATTENUATION OF WAVE FRONT OF QUARTER-WAVE OSCILLATION, OF 100-KM. LINE, 60,000 VOLTS.

Conductor:	(3)	(1)	(5)	(4)
Size No.:	00	00	4	00
Material:	Copper	Copper	Copper	Iron
Distance of return conductor, cm.: . . .	1820	45.5	1820	1820
After 23 microseconds, 6.9 km.:				
Gradient, volts per meter:	205	960	213	135
Length of wave front, meters:	460	98	440	700
Equivalent kilocycles:	330	1530	340	215
After 230 microseconds, 69 km.:				
Gradient, volts per meter:	58	234	60	10
Length of wave front, meters:	1630	400	1570	9400
Equivalent kilocycles:	92	370	94	16

For comparison are given some data of the same conductor, with the return conductor at 18 inches = 45.5 cm., and also for a copper wire No. 4, and an iron wire No. 00, with the ground as return.

These data are plotted in Fig. 6, showing the wave front, as it gradually flattens out in its travel over the line, from the very steep wave at 172 meters from the origin, to the wave with a front of 1630 meters, 69 km. away.

A comparison of the data of the four circuit conditions is given in Table VIII, and plotted in Fig. 7.

It is interesting to note, that there is practically no difference in the flattening of the wave front on a low resistance conductor, No. 00, and a high resistance conductor, No. 4. There is, however, an enormous difference due to the effect of the closeness of the return conductor; with the return conductor at 18 inches distance, the wave front is still materially steeper at 6.9 km. distance, than it is in the conductor with ground return at 0.69 km. distance. That is, the flattening of the wave front in the conductor with ground return, is more than ten times as rapid, than in the same conductor with the return conductor

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*Here and in the following, the symbol $\underline{/}$ has been chosen for the common logarithm.

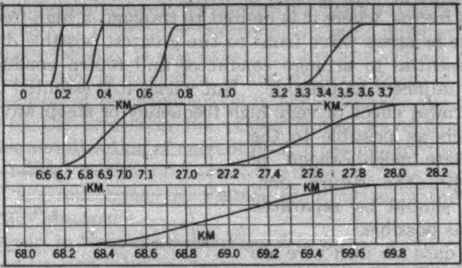


Fig. 6—Flattening of Steep Wave Front in Transmission Line No. 00 B. &. S. Gage Copper 30 ft. Above Ground

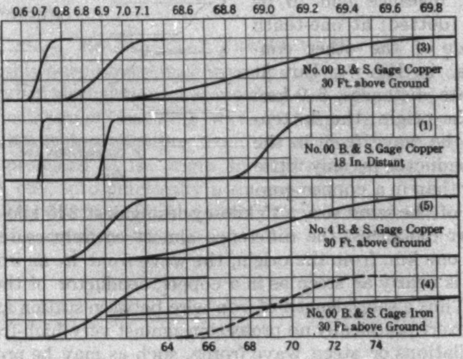
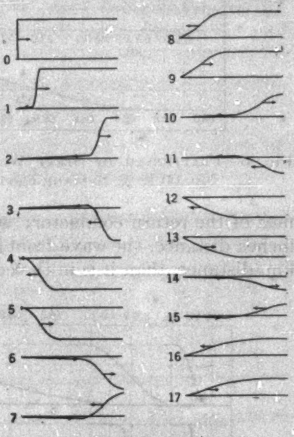


Fig. 7

closely adjacent. Or in other words, the danger zone of steep wave front, extends in a conductor with the return conductor closely adjacent, to more than ten times the distance than in the conductor with ground return.

This means, where it is desired to transmit a high-frequency impulse or steep wave front to the greatest possible distance, it is essential to arrange conductor and return conductor as closely adjacent as possible. But where it is essential to limit the harmful effect of very high frequency or steep wave front as much as possible to the immediate neighborhood of its origin, the return conductor should be separated as far as possible.



QUARTERWAVE CHARGING OSCILLATION OF TRANSMISSION LINE

FIG. 8

The data on iron wire are very disappointing; there is an enormous increase in the flattening of the wave front at great distances, by the use of iron as conductor material, so much so that the wave front of the iron conductor at 69 km. distance had to be shown (dotted) at one-tenth the scale of the other conductors, in Fig. 7. But at moderate distances, 6.9 km. from the origin, the flattening of the wave front in the iron conductor is only little greater than in a copper conductor of the same size: 215 kilo-cycles against 330 kilo-cycles. At short distances, the difference almost entirely ceases, and within one km. from the origin, the wave front in an iron conductor is nearly as steep as in a copper conductor of the same size. Thus a short length of iron wire between station and line would exert practically no protection against very high-frequency oscillations or steep wave fronts, such as may be produced by lightning strokes in the neighborhood of the station.

This was to be expected from the shape of the curve of the attenuation constant, shown in Fig. 3.

From the data in Table VII then are constructed and shown in Fig. 8, the successive curves of voltage distribution in the line, as it is discharging (or charging), by the originally rectangular wave running over the line, reflecting at the end of the line and running back, then reflecting again at the beginning of the line and once more traversing it, etc., until gradually the transient energy is dissipated and the line voltage reaches its average, zero in discharge, or the supply voltage in charge. The direction of the wave travel in the successive positions is shown by the arrows in the center of the wave front; the existence and direction of current flow in the line by the arrows in the (nearly) horizontal part of the diagram. As seen, after four to five reflections, the voltage distribution in the line is practically sinusoidal.

However, in these diagrams Figs. 6 to 8, the wave front has been constructed from the maximum voltage gradient derived by the calculation, assuming as approximation the shape of the wave front as a sinusoid. This usually is a sufficient approximation, since the important feature is the maximum gradient, that is, the steepest part of the wave front, which was given by the calculation; but it is not strictly correct, and the wave front differs from sine shape. It thus is of interest to investigate the exact shape of the wave front, in its successive stages of flattening.

RECTANGULAR TRAVELING WAVE

For this purpose we may investigate, as a further example, the gradual destruction of wave shape and decay of a 60,000-cycle rectangular traveling wave, during its passage over a transmission line, changing from the original rectangular wave shape produced at its origin by lightning stroke, spark discharge, etc., into practically a sine wave.

For this purpose, for the elementary symmetrical traveling wave, the equation is:

$$e = \frac{4\,E}{\pi} \sum_{0}^{\infty}{}_{n}\; \epsilon^{-u_n t} \sin{(2\,n + 1)}\, \varphi \qquad (13)$$

where $\varphi = \theta - \tau$ is the running time coordinate, in angular expression.

Values of e are calculated, for various times t, from one micro-second to 360 micro-seconds, for all the angles φ, where e has not yet become constant and equal to E.

In Table IX are given numerical values, plotted in Fig. 9, of

$$e \frac{\pi}{4E} = \sum_{0}^{\infty}{}_{n} \epsilon^{-u_n t} \sin (2n + 1) \varphi,$$

from 0 to 90 deg., and from one to 360 micro-seconds, and derived therefrom, the values of the length of wave front, in degrees and in meters, and the equivalent frequency, in kilo-cycles. Two values are given, the maximum kilo-cycles, derived from the steepest part of the wave front, and the average kilo-cycles, derived from the total wave front. The difference indicates the deviation of the shape of the wave front from a sinusoid.

TABLE IX.

ATTENUATION OF 60,000-CYCLE RECTANGULAR WAVE, IN LINE OF NO. 00 B. & S. GAGE COPPER, 30 FT. ABOVE GROUND.

Time: t =	0	1	2	5	10	20	40	100	360	ms.
Wave travel; l =	0	.3	.6	1.5	3	6	12	30	108	km.
φ = 0 degrees	0.785	0	0	0	0	0	0	0	0	0
1 "	....	0.250	0.170	0.106	0.074	0.052	0.347	0.022	0.010	
5 "	....	0.785	0.645	0.475						
10 "	....	....	0.785	....	0.606	0.468	0.582	0.217	0.100	
20 "	....	....	....	0.783	0.770	0.707	0.709	0.405	0.193	
30 "	....	....	....	....	0.780	0.762	0.740	0.547	0.282	
40 "	....	....	....	....	....	0.775	0.758	0.639	0.360	
50 "	....	....	....	....	....	....	0.765	0.691	0.426	
60 "	....	....	....	....	....	....	....	0.722	0.476	
70 "	....	....	....	....	....	....	....	0.730	0.513	
80 "	....	....	....	....	....	....	....	....	0.535	
90 "	0.785	0.785	0.785	0.783	0.780	0.775	0.765	0.730	0.540	
Wave Front degrees:	0	10	16	26	44	70	90	140	180	
Wave Front meters:	0	140	220	340	620	980	1250	1940	2500	
Wave Front kilo-cycles: max:	∞	1110	750	470	325	230	165	103	63	
Wave Front kilo-cycles: avg:	∞	1080	670	420	268	155	120	77	60	
Highest apprec. harmonic	∞	61	45	33	15	9	7	3	1	
Decrease of wave max:	0	....	....	....	0.6	1.3	2.6	7.0	31.2	%
Maximum gradient volts per meter:	∞	690	470	295	205	140	100	65	40	

As seen, due to the high fundamental frequency, 60,000 cycles, the number of significant harmonics is very greatly reduced, until frequencies are reached where the attenuation is so enormous, that the destruction of the wave by its energy dissipation occurs in a few meters.

Thus after one micro-second, or 300 meters wave travel, the calculation needs to extend only to the 61st harmonic; after two micro-seconds, to the 45th harmonic, etc., while after 100 micro-seconds, or 30 km. travel, only the third harmonic is still appreciable, and after 360 micro-seconds, or 108 km. travel, even this has disappeared, and the wave is essentially a sine wave.

In 30 km. travel, the wave maximum has decreased 7 per cent; in 108 km. it has decreased 31.2 per cent.

It is important to note however, that waves of relatively high frequency, within the range of the danger frequencies between 20 and 100 kilo-cycles, can travel considerable distances, if no other causes of rapid attenuation are at work but those considered here, and still retain a large part of their energy. Thus the 60,000-cycle wave, after traversing 100 km. of line, still retains about 70 per cent of its amplitude, that is, about 1/2 of its energy. Thus the danger from resonance of power transformer windings with such frequencies is not local but rather extends over a large part of the system.

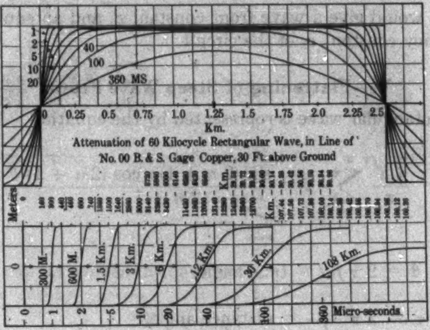


Fig. 9

The lower part of Fig. 9 shows the shape of the wave front at various distances from the origin; the upper part shows the gradual change of the wave from rectangular to flat top to sine wave.

Plotting for the 750-cycle quarter wave oscillation and the 60,000-cycle traveling wave, the logarithm of the length of the wave front, and of the equivalent frequency of the wave front, against the logarithm of the distance of wave travel, gives practically straight lines, (except for very great distances of wave travel, where the lower harmonics predominate), and from the slope of these lines it follows that: The length of wave front is approximately proportional and the equivalent frequency of the wave front approximately inversely proportional to the square root of the distance of wave travel:

$$l_f = c_1 \sqrt{l} \tag{14}$$

$$f_f = \frac{c_2}{\sqrt{l}} \tag{15}$$

This would give a wave front constant c_1 and an equivalent frequency constant c_2 of the circuit.

V.—Flattening of Steep Wave Fronts

A rectangular wave is represented by the equation

$$e = \frac{4\,E}{\pi} \sum_{0}^{\infty}{}_{n}\ \epsilon^{-ut}\ \frac{\sin\,(2\,n+1)\,\tau \cos\,(2\,n+1)\,\theta}{2\,n+1} \tag{1}$$

where

$\theta = 2\,\pi\,f_0\,t =$ time angle, (2)

$\tau = 2\,\pi\,\frac{l}{l_0} =$ distance angle, (3)

$f_0 =$ fundamental frequency,

$l_0 = \frac{S}{f_0} =$ wave length, (4)

$S = 3 \times 10^{10} =$ velocity of propagation,
$E =$ maximum voltage of wave.

The voltage gradient is

$$\frac{d\,e}{d\,l} = \frac{d\,e}{d\,\tau}\,\frac{d\,\tau}{d\,l}$$

$$= \frac{4\,E}{\pi}\,\frac{d\,\tau}{d\,l} \sum_{0}^{\infty}{}_{n}\ \epsilon^{-ut} \cos\,(2\,n+1)\,\tau \cos\,(2\,n+1)\,\theta \tag{5}$$

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Substituting for $\frac{d\tau}{dl}$ from (3), and resolving the cos-product, gives:

$$\frac{de}{dl} = \frac{4E}{l_0} \left\{ \sum_{0}^{\infty}{}_{n} \epsilon^{-ut} \cos (2n+1)(\tau - \theta) \right.$$

$$\left. + \sum_{0}^{\infty}{}_{n} \epsilon^{-ut} \cos (2n+1)(\tau + \theta) \right\} \qquad (6)$$

$$\sum_{0}^{\infty}{}_{n} \epsilon^{-ut} \cos (2n+1)(\tau + \theta) \text{ approaches zero for } \tau + \theta \neq 0. \qquad (7)$$

thus:

$$\frac{de}{dl} = \frac{4E}{l_0} \sum_{0}^{\infty}{}_{n} \epsilon^{-ut} \cos (2n+1)(\tau - \theta) \qquad (8)$$

The maximum voltage gradient occurs for $\theta = \tau$, and thus is:

$$G = \frac{de}{dl} = \frac{4E}{l_0} \sum_{0}^{\infty}{}_{n} \epsilon^{-ut} \qquad (9)$$

This can be written in the form:

$$G = \frac{2E}{f_0 l_0} \sum_{0}^{\infty}{}_{n} 2 f_0 \epsilon^{-ut} \qquad (10)$$

It is however, by (4):

$$f_0 l_0 = S \qquad (11)$$

thus:

$$G = \frac{2E}{S} \sum_{0}^{\infty}{}_{n} 2 f_0 \epsilon^{-ut} \qquad (12)$$

The values of ϵ^{-ut} are taken for all values of frequency differing from each other by $2 f_0$, and, at and near the wave front, the value of $\sum_{0}^{\infty}{}_{n} 2 f_0 \epsilon^{-ut}$ thus approaches the value of

$$\int_0^\infty \epsilon^{-ut} \, df$$

Substituting this into (**10**), gives as the equation of the maximum steepness of wave front:

$$G = \frac{d\,e}{d\,l} = \frac{2\,E}{S} \int_0^\infty \epsilon^{-ut}\,d\,f \tag{12}$$

As seen, in this expression (**12**), wave length and frequency have disappeared. Equation (**12**) thus applies to any wave, whether of finite length or not. That is, it represents broadly (though only approximately) the maximum gradient of any steep wave front or impulse, at the time t after its origin as rectangular wave front or impulse.

TABLE X.

ATTENUATION OF WAVE FRONT OF RECTANGULAR IMPULSE.

Copper Wire No. 00 B. & S. gage, 30 ft. above Ground, Ground Return.

f:	u:	$f\,\epsilon^{-ut}$; $t =$				
		10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
1	70	1	1			
2	"	2	2			
5	"	5	5			
10	"	9	10	88		
20	"	19	20			
50	"	47	50			
10^2	"	93	99	100		
2×10^2	"	186	198	200	8,888	
5×10^2	"	468	496	500		
10^3	80	923	992	1,000		
2×10^3	115	1,760	1,970	2,000		
5×10^3	190	4,150	4,920	5,000		
10^4	287	7,500	9,720	9,999	10,000	88,888
2×10^4	482	10,200	19,000	19,900	20,000	
5×10^4	1,300	10,800	44,000	49,400	50,000	
10^5	3,710	2,450	69,000	96,400	99,500	100,000
2×10^5	12,690	1	56,100	176,000	197,000	200,000
5×10^5	74,200	0	240	190,000	466,000	498,000
10^6	291,500	......	σ	54,300	748,000	970,000
2×10^6	1,160,000	......	......	18	625,000	1,780,000
5×10^6	7,230,000	......	......	σ	3,660	1,860,000
10^7	28,800,000	......	......	......	1	562,000
2×10^7	115,000,000	......	......	......	σ	200
$\Sigma =$		38,660	206,800	605,000	2,230,000	6,060,000
$+ 3 \times .4343 =$		29,700	158,700	464,000	1,710,000	4,650,000
$G = \frac{2\,E}{S} \times ; =$		.119	.635	1.86	6.84	18.6
[$l =$		300	30	3	.3	.03 km.]

As u is not a simple function of f, the integration of $\int_0^\infty \epsilon^{-ut}$

df in its general form meets with difficulties, and the integral may be evaluated as follows:

Equation (**12**) can be written in the form

$$G = \frac{2E}{S} \int_{-\infty}^{+\infty} f \epsilon^{-ut} \, d \log f \qquad (\mathbf{13})$$

Plotting then $f \epsilon^{-ut}$ as ordinates, with $\log f$ as abscissas, gives a curve, and the area of this curve gives the integral.

Instead of using $\log f$ as abscissas, it is more convenient to use the common logarithm, $\underline{/f}$, and divide the measured area by $\underline{/\epsilon} = 0.4343$.

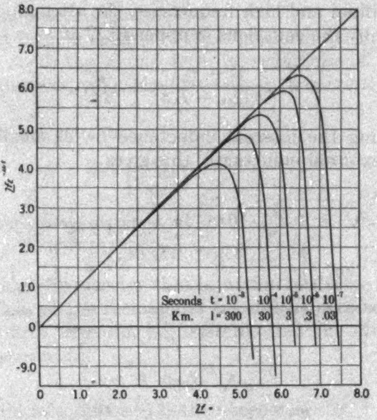

FIG. 10—ATTENUATION OF RECTANGULAR WAVE FRONT OF IMPULSE

Numerically, the integration is done by calculating $f \epsilon^{-ut}$ for (approximately) constant intervals of $\underline{/f}$, adding all the values, and multiplying the sum by the (average) difference between successive $\underline{/f}$.

As an instance is given, in Table X, the calculation for circuit (**3**) of the preceding, that is, copper wire No. 00 B. & S. gage, 30 ft. above ground, with the ground as return conductor, for the times $t = 0.1$, 1, 10, 100 and 1000 micro-seconds, corresponding to the distance of travel of the wave front of 0.03, 0.3, 3, 30 and 300 km. As frequency intervals are selected: 1, 2, 5, 10, 20, 50, 100, 200 etc., giving the logarithms: 0, 0.3, 0.6, 1, 1.3 etc.

These curves are plotted, in logarithmic scale, in Fig. 10.

As seen, all these curves rise as straight lines under 45 degrees, and then very abruptly drop to negligible values.

In Table X, the values of $f\,\epsilon^{-ut}$ are added, then divided by three, since there are three intervals for each unit of $\underline{/f}$, and multiplied by $\underline{/\epsilon}$, to reduce to natural logarithms. Multiplying by E/S then gives the gradient G.

For medium and high frequencies, the attenuation constant u is given by the equations of appendix C as:

$$u = \frac{r_1 + r_3}{2\,(L_1 + L_0)} + \frac{g}{2\,C} \tag{14}$$

Neglecting the internal inductance L_1, as small compared with the external inductance, this gives

$$\left.\begin{aligned} u &= \frac{\frac{\pi}{l_1}\sqrt{2.5\,\frac{\mu}{\gamma}}\,10^4}{\log\frac{l'}{l_r}}\sqrt{f} + \frac{4\,\pi^2\,l'}{S\log\frac{l'}{l_r}}\,f^2 \\ &= m_1\,\sqrt{f} + m_2\,f^2 \end{aligned}\right\} \tag{15}$$

where

$$m_1 = \frac{\pi\sqrt{2.5\,\frac{\mu}{\gamma}}}{l_1\log\frac{l'}{l_r}}\,10^4 \tag{16}$$

$$m_2 = \frac{4\,\pi^2\,l'}{S\log\frac{l'}{l_r}} \tag{17}$$

For the conductor (3) in Table X, it is:

$$\left.\begin{aligned} m_1 &= 2.6 \\ m_2 &= .29 \times 10^{-6} \end{aligned}\right\} \tag{18}$$

This expression (15) of u holds for all frequencies except very low frequencies—below 1000 cycles— and extremely high frequencies—many millions of cycles. The latter are of little importance, as they are wiped out in the immediate neighborhood of the origin of the rectangular impulse. At the low frequencies, the attenuation is so small, within the distances which come into consideration in the wave travel, and these low frequencies give such a small part of the wave front gradient, that the error made by the use of (**15**) is negligible. For instance, in the case of Table X, even at $t = 100$ microseconds, or 10 km. wave travel, the error made in the voltage gradient by altogether neglecting the attenuation of the frequencies up to 1000 cycles, would be only 0.01 per cent.

Thus the equation (**15**) of the attenuation constant can safely be used for all practical purposes.

As the first term in equation (**15**) is proportional to $\sqrt{f}$, the second term to f^2, the second term is negligible at low and medium frequencies, while the first term is negligible at high frequencies.

Both terms are equal at:

$$f = \left(\frac{m_1}{m_2}\right)^{2/3} \tag{19}$$

That is, in the above instance, at

$$f = 43{,}000 \text{ cycles.}$$

Thus, for high frequencies, that is, within moderate distances from the origin of the rectangular impulse—up to some kilometers—the first term can be neglected and the attenuation constant expressed by

$$u = m_2 f^2 \tag{20}$$

The integral in equation (**12**) then becomes,

$$F = \int_0^\infty \epsilon^{-m_2 t f^2} d f \tag{21}$$

Substituting:

$$x^2 = m_2 t f^2$$

gives:

$$d f = \frac{d x}{\sqrt{m_2 t}}$$

and:

$$F = \frac{1}{\sqrt{m_2 t}} \int_0^\infty \epsilon^{-x^2} d x$$

It is however:

$$\int_0^\infty \epsilon^{-x^2}\,dx = 1/2\,\sqrt{\pi} \tag{22}$$

thus:

$$F = \frac{1}{2}\sqrt{\frac{\pi}{m_2\,t}} \tag{23}$$

and:

$$G = \frac{E\sqrt{\pi}}{S\sqrt{m_2\,t}} \tag{24}$$

or, since:

$$l = S\,t \tag{25}$$

$$G = \frac{E\sqrt{\pi}}{\sqrt{m_2\,S\,l}} \tag{26}$$

Substituting (**17**) into (**26**), gives; in cm. and volts per cm.:

$$G = \frac{E}{2}\sqrt{\frac{\log\frac{l'}{l_r}}{\pi\,l'\,l}} \tag{27}$$

$$= 0.282\,E\sqrt{\frac{1}{l'\,l}\log\frac{l'}{l_r}}$$

Thus; approximately, the maximum gradient, or steepness of the wave front of a rectangular impulse, in the neighborhood and at moderate distances from its origin, decreases inversely proportional with the square root of the distance or time of wave travel.

It decreases with increasing distance l' of the return conductor, nearly in inverse proportion to the square root of l'

For the six types of circuits considered in the previous instances, it is:

	$G =$	$/G\sqrt{l}$ =
(1) Copper wire 00 B. & S, gage, 18 in. = 45.5 cm. from return con.:	$\frac{1}{\sqrt{l}} \times 1700$	3.230
(2) Copper wire " " " 6 ft. = 182 " " " "	968	2.986
(3) Copper wire " " " 60 ft. = 1820 " " " "	360	2.556
(4) Iron wire " " " 60 " = 1820 " " " "	360	2.556
(5) Copper wire 4 " " 60 " = 1820 " " " "	373	2.572
(6) Aluminum, stranded, same conductivity and arrangement as (3):	353	2.548

where G is given in volts per meter, at $E = 60{,}000$ volts, and l in kilo-meters:

$$\left.\begin{aligned} G &= 89 \times 10^{-3} E \sqrt{\frac{1}{l'l} \log \frac{l'}{l_r}} \\ &= 135 \times 10^{-3} E \sqrt{\frac{1}{l'l} \underline{/ \frac{l'}{l_r}}} \\ &= 8100 \sqrt{\frac{1}{l'l} \underline{/ \frac{l'}{l_r}}} \end{aligned}\right\} \qquad (\mathbf{28})$$

From Tables X, IX, and VII are collected the values of wave gradients G, and given in Table XI, together with their logarithms, and the $\underline{/ G_0}$, calculated from equation (**28**), for comparison of the different methods of calculation. Table XI then gives the difference Δ, and its value in per cent.

The values of G are plotted in Fig. 11. The drawn line gives the values calculated by equation (**28**); the three cornered stars the values from Table X, the crosses the values from Table IX, and the circles the values from Table VII.

As seen from Table XI and Fig. 11, the agreement of the equations (**27**) and (**28**) is satisfactory with the values of the wave front gradient taken from Tables X and IX.

The values of G from Table X differ erratically. This table was calculated by graphical integration, and it is probable that the intervals have been chosen too large, in that range where the curve drops very abruptly, as seen in Fig. 10.

The agreement with the values from Table IX is very close. In this table, representing the course of a 60,000-cycle rectangular traveling wave, the individual harmonics have been calculated, as they were relatively few, due to the high frequency of the fundamental. This agreement is important as it justifies equation (**27**). In deriving (**27**), we have substituted integration for summation, that is, have replaced the discontinuous values of the individual harmonics by a continuous curve. Any error resulting from this should be greatest where the number of discontinuous harmonics is the least. Table XI and Fig. 11 however, show that the agreement of the gradient of the 60,000 cycle traveling wave with equation (**27**), is good even at $l = 30$ km., where only two significant harmonics are left, the fundamental and the third harmonic.

Thus the method of deriving an equation for G by integration is justified.

Unsatisfactory however, is the agreement of equation (**27**) with the values of the gradient of the quarter wave oscillation, taken from Table VII. These values lay on a straight line in Fig. 11, given as dotted line, showing proportionality with the square root of the distance, but there is a constant error, and the gradients given in Table VII are about 50 per cent greater than those given by equation (**27**).

TABLE XI.

CALCULATION OF WAVE FRONT.

Copper Wire No. 00 B. & S. gage, 30 ft. above Ground.

Dist. l km.	Gradient, volts per meter at E = 60,000 V.				
	G:	/ G:	/ G_0*	Δ	= %
		Impulse, $f_0 = 0$: (Table X)			
.03	1860	3.269	3.317	+ .048	+ 11.7
.3	684	2.835	2.817	− .018	− 4.3
3	186	2.269	2.317	+ .048	+ 11.7
30	63.5	1.803	1.817	+ .014	+ 3.3
300	11.9	1.075	1.317	(+ .242)	
		Traveling Wave, $f_0 = 60,000$: (Table IX)			
.3	690	2.839	2.817	− .022	− 5.2
.6	470	2.672	2.667	− .005	− 1.1
1.5	295	2.470	2.468	− .002	− 0.5
3	205	2.312	2.317	+ .005	+ 1.1
6	140	2.146	2.167	+ .021	+ 5.0
12	100	2.000	2.016	+ .016	+ 3.7
30	65	1.813	1.817	+ .004	+ 0.9
108	40	1.602	1.539	(− .063)	
		Quarterwave, $f_0 = 750$: (Table VII)			
.01	5500	3.740	3.556	− .184	
.1725	1290	3.111	2.938	− .173	
.345	967	2.985	2.787	− .198	
.69	630	2.799	2.637	− .162	
3.45	280	2.447	2.287	− .160	
6.9	205†	2.312	2.137	− .175	
27.6	98	1.991	1.836	− .155	
69	58	1.763	1.637	(− .125)	
690	12	1.079	1.137	(+ .058)	
6900	1.2	.079	.637	(+ .558)	
		Same, 18 ins. = 45.5 cm. between conductors.			
.69	3240	3.510	3.311	− .199	
6.9	960	2.982	2.811	− .171	
69	234	2.369	2.311	(− .058)	

*Calculated by equation (28).

†131 to 293

The cause of the discrepancy probably is in the method used in calculating the gradients of the quarter wave oscillation given in Table VII. Due to the impossibility of calculating individually thousands of harmonics, the number of harmonics in successive intervals of frequency, has been multiplied with the average attenuation ϵ^{-ut} in this frequency range, and as the average has been used the mean value of the ϵ^{-ut} for the two extremes of the frequency range. However, ϵ^{-ut} drops exponentially and with great rapidity, so that the true average value is much lower than the average of maximum and minimum. Thus for instance in the range from 5×10^5 to 10×10^5 cycles, containing 334 harmonics, ϵ^{-ut} for $f = 5 \times 10^5$ (at $t = 23 \times 10^{-6}$ seconds), is 1.22×10^{-3} and for $f = 10 \times 10^5$, it is 181×10^{-3}, giving an average of 91.1×10^{-3}. However, for $f = 7.5 \times 10^5$, ϵ^{-ut} is 22.5×10^{-3}, thus less than a quarter of the average.

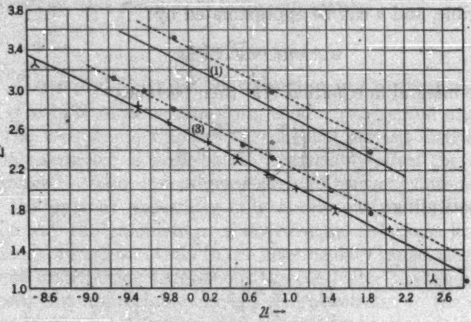

FIG. 11—WAVE FRONT GRADIENT

To check this, one value, at $t = 23 \times 10^{-6}$ or $l = 6.9$ km., has been re-calculated by using not the average, but the maximum and the minimum of ϵ^{-ut} in each interval, and the two gradients derived therefrom: $G = 293$ and 131, are marked with dotted circles in Fig. 11. As seen, the use of the minimum value of ϵ^{-ut} agrees nearer with equation (**27**), as was to be expected.

It appears probable that the equation (**27**) gives more reliable values of wave front gradient, within the range of its applicability, than the method of calculation used in Table VII. and as it is much simpler, it is preferable.

As seen from Table XI and Fig. 11, the parabolic law of wave front flattening, given by equations (**27**) and (**28**), holds good up to about 30 km. distance of wave travel, and with fair approximation even to 100 km. In the range beyond this—which is of lesser importance, as the flattening of the wave front has greatly reduced its danger at these distances—the values of the gradient G decrease with increasing rapidity, with distance and time, due to the medium high harmonics showing in the attenuation.

At great distances from the origin of the rectangular wave, when the very high harmonics have practically died out and the wave attenuation is determined by the medium-frequency harmonics, the second term of u in equation (**15**) becomes negligible, and u can be approximated by:

$$u = m_1 \sqrt{f} \tag{37}$$

The integral in equation (**12**) then becomes

$$F = \int_0^\infty \epsilon^{-m_1 t \sqrt{f}} \, d f \tag{38}$$

Substituting:

$$m_1 t \sqrt{f} = x$$

gives

$$d f = \frac{2 \, x \, d \, x}{m_1^2 \, t^2}$$

and

$$F = \frac{2}{m_1^2 \, t^2} \int_0^\infty x \, \epsilon^{-x} \, d \, x$$

It is however:

$$\int_0^\infty x \, \epsilon^{-x} \, d \, x = 1 \tag{39}$$

thus

$$F = \frac{2}{m_1^2 \, t^2} \tag{40}$$

and

$$G = \frac{4 \, E}{S \, m_1^2 \, t^2} \tag{41}$$

or:

$$G = \frac{4\,S\,E}{m_1^2\,l^2} \qquad \textbf{(42)}$$

Substituting (**16**) gives

$$G = \frac{1.6\,S\,\gamma\,l_1^2\left(\log\frac{l'}{l_r}\right)^2}{\pi^2\,\mu\,l^2}\,E \qquad \textbf{(43)}$$

That is, at great distances (or considerable time) from the origin, the flattening of the wave front approaches inverse proportionality with the square of the distance (or time) of the wave travel.

However, this range is of little importance.

APPENDIX A

Unequal Current Distribution in Conductor

It is simple to determine from conductor size and material, and the frequency, whether the unequal current distribution in the conductor is sufficient to appreciably effect the ohmic resistance.[1] Also, if conductor size and frequency are so high that the current penetrates only a thin surface layer of the conductor, the calculation of its effective resistance leads to simple expressions. Only in the intermediary range, where the current distribution throughout the conductor is already very uneven, but appreciable current still penetrates to the center of the conductor, the numerical calculation becomes laborious, involving a slowly convergent series or special functions of which tables are rarely available. However, this range is relatively narrow and can be approximated between the two extremes, with sufficient accuracy for the calculation of transients, while for low-frequency power transmission, conductors of a size reaching into this range would not be used, being uneconomical in conductor material.

As the case of such high frequencies or large conductor sizes, that the current penetrates only a surface layer, has been frequently investigated,[2] the derivation of the equations is unnecessary here. The most satisfactory way of representing it is to calculate the "depth of penetration" of the current into the conductor. The actual or effective resistance of the conductor then is given by circumference of the conductor, times the depth of penetration. This method, of calculating the depth of penetration, has the advantage that it applies to all sizes and shapes of conductors, solid round conductors or hollow tubes, flat ribbon and even such complex shapes as the railway rail, provided only that the depth of penetration is materially less than the depth of the conductor material.

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[1] "Theory and Calculation of Alternating-Current Phenomena, 5th edition, page 144.

[2] See for instance "Theory and Calculation of TransiebntPhenomena," Section III, Chapter VII.

The depth of the penetration into the conductor is

$$\left.\begin{aligned} l_p &= \frac{10^4}{\pi \sqrt{0.4\,\gamma\,\mu f}} \\ &= \frac{5030}{\sqrt{\gamma\,\mu f}} \end{aligned}\right\} \qquad (1)$$

where

γ = electrical conductivity of the conductor,
μ = magnetic permeability of the conductor,
f = frequency.

The effective resistance of the conductor then is, per unit length,

$$\left.\begin{aligned} r &= \frac{1}{\gamma\, l_p\, l_1} \\ &= \frac{\pi}{l_1}\sqrt{\frac{0.4\,\mu f}{\gamma}}\;10^{-4} \\ &= \frac{\sqrt{\dfrac{\mu f}{\gamma}}}{5030\, l_1} \text{ ohms per cm.} \end{aligned}\right\} \qquad (2)$$

where

l_1 = circumference of conductor (actual, that is, following all indentations, etc.)

As the true ohmic resistance of the conductor is,

$$r_0 = \frac{1}{\gamma\, s} \text{ ohms per cm.} \qquad (3)$$

where

s = conductor section,

the "resistance ratio", or ratio of the effective resistance of unequal current distribution, to the true ohmic resistance, is:

$$\left.\begin{aligned}\frac{r}{r_0} &= \frac{s\,\pi}{l_1}\sqrt{0.4\,\gamma\,\mu\,f}\\ &= \frac{s}{5030\,l_1}\sqrt{\gamma\,\mu\,f}\end{aligned}\right\} \qquad (4)$$

The internal reactance of unequal current distribution equals the resistance, in the range considered:

$$x_1 = r_1 \qquad (5)$$

while at low frequency, where the current distribution is still uniform, the internal reactance is, per unit length of conductor:

$$\left.\begin{aligned}x_{10} &= \pi\,f\,\mu\,10^{-9} \text{ henry per cm.}\\ &= \pi\,f\,10^{-9} \text{ for air } (\mu = 1)\end{aligned}\right\} \qquad (6)$$

The external inductance of the conductor, that is, the inductance due to the magnetic field outside of the conductor, calculated in the usual manner, that is neglecting the finite velocity of the magnetic field, is, per unit length of conductor,

$$L_0 = 2\log\frac{l'}{l_r}\,10^{-9} \text{ henry per cm.} \qquad (7)$$

where

l_r = radius of conductor

l' = distance of return conductor

or, for conductors of other shape, approximately:

$$L_0 = 2\log\frac{2\,\pi\,l'}{l_2}\,10^{-9} \text{ henry per cm.} \qquad (8)$$

where:

l_2 = shortest circumference of conductor (that is, not including indentations etc.)

Thus, the external reactance, per unit length of conductor is:

$$x_0 = 4\,\pi\,f\log\frac{l'}{l_r}\,10^{-9} \qquad (9)$$

$$= 4\,\pi\,f\log\frac{2\,\pi\,f'}{l_2}\,10^{-9} \text{ ohms per cm.} \qquad (10)$$

As the unequal current distribution in the conductor, and the increase of effective resistance and decrease of internal reactance caused by it, are the result of the magnetic field of the current inside of the conductor, it follows that stranding the conductor does not eliminate this effect, unless the stranding is done in such manner that every strand has the same distance from the conductor surface, that is, interior strands interchange periodically with exterior ones and inversely, and the individual strands are insulated from each other.

Otherwise the stranding has an indirect effect only, in reducing γ and μ in equation (**1**) etc.

Thus, if in the stranded conductor the part p is filled by the conductor material of conductivity γ_0,—the rest $(1 - p)$ being the space between the strands—, then the average conductivity of the conductor is

$$\gamma = p\,\gamma_0$$

In conductors of magnetic material stranding may materially reduce the effective resistance, as the spaces between the strands would greatly reduce the resultant permeability of the conductor, and thereby greatly increase the penetration of the current into the conductor.

APPENDIX B

Radiation Resistance, Inductance and Capacity

Consider a finite section of an infinitely long conductor of radius l_r, with the return conductor at distance l', and assume that l' is small compared with a quarter wave length of the electric field of the conductor.

$$\left.\begin{aligned} \text{Let } i &= I\cos(\theta - \alpha) = \text{current in the conductor} \\ e &= E\cos(\theta - \beta) = \text{voltage of the conductor} \end{aligned}\right\} \quad (1)$$

At a distance l from the conductor, the magnetic field intensity is $0.2/l$ times the current, and lags behind the current by the time required for the magnetic field to travel the distance l, that is, by the angle $a\,l$, where:

$$a = \frac{2\pi f}{S} \quad (2)$$

f = frequency,

$$S = 3 \times 10^{10} = \text{velocity of radiation} \quad (3)$$

The magnetic field intensity thus is given by:

$$H = \frac{0.2\, I \cos(\theta - \alpha - a\, l)}{l} \tag{4}$$

The magnetic component of field, per unit length of conductor (cm.), thus is:

$$\Phi = \int_{l_r}^{l'} H\, d\, l$$

$$= 0.2\, I \int_{l_r}^{l'} \frac{\cos(\theta - \alpha - a\, l)}{l}\, d\, l$$

$$= 0.2\, I \left\{ \cos(\theta - \alpha) \int_{l_r}^{l'} \frac{\cos a\, l}{l}\, d\, l \right.$$

$$\left. + \sin(\theta - \alpha) \int_{l_r}^{l'} \frac{\sin a\, l}{l}\, d\, l \right\} \tag{5}$$

It is however, for values of l' up to about one-half of a quarter wave length of the electric field, with sufficient approximation*:

$$\int_{l_r}^{l'} \frac{\sin a\, l}{l}\, d\, l = a\, l' \tag{6}$$

$$\int_{l_r}^{l'} \frac{\cos a\, l}{l}\, d\, l = \log \frac{l'}{l_r} \tag{7}$$

Thus $\Phi = 0.2\, I \left\{ \log \frac{l'}{l_r} \cos(\theta - \alpha) \right.$

$$\left. + a\, l' \sin(\theta - \alpha) \right\} \tag{8}$$

Similar considerations apply to the dielectric field of the conductor, in relation to its voltage.

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*See "Theory and Calculation of Transient Electric Phenomena," Section III, Chapter VIII: "Velocity of Propagation of Electric Field."

Thus, the two components of the electric field, the magnetic field and the dielectric field, are not in phase with current and voltage, respectively, but lag behind, due to the appearance of the sine component, resulting from the finite velocity of the electric field.

The magnetic field Φ induces a voltage in quadrature with the field Φ:

$$\begin{aligned} e' &= 2\,\pi f\,\frac{d\,\Phi}{d\,\theta}\,10^{-8} \\ &= 4\,\pi f I \left\{ \log \frac{l'}{l_r} \sin\,(\theta - \alpha) + a\,l' \cos\,(\theta - \alpha) \right\} 10^{-9} \end{aligned} \tag{9}$$

where the 10^{-8} reduces from absolute units to volts.

The reactive or sine component of this voltage e' is the e.m.f. of self induction, and gives rise to the reactance,

$$x = 4\,\pi f \log \frac{l'}{l_r}\,10^{-9} \text{ ohms} \tag{10}$$

and the inductance,

$$\begin{aligned} L &= \frac{x}{2\,\pi f} \\ &= 2 \log \frac{l'}{l_r}\,10^{-9} \text{ henrys} \end{aligned} \tag{11}$$

The cosine component of E' is in phase with the current, thus an energy voltage, and gives rise to an effective resistance, the magnetic radiation resistance:

$$r = 4\,\pi f\,a l'\,10^{-9} \text{ ohms.} \tag{12}$$

Substituting (**2**) into (**12**) gives:

$$r = \frac{8\,\pi^2 f^2\,l'}{S}\,10^{-9} \text{ ohms.} \tag{13}$$

That is, the radiation resistance is proportional to the distance from the return conductor, and to the square of the frequency, but is independent of size, shape or material of the conductor.

From (**11**) and (**13**) follow as the attenuation constant of magnetic radiation,

$$u_1 = \frac{r}{2\,L} = \frac{4\,\pi f^2\,l'}{2\,S \log \dfrac{l'}{l_r}} \tag{14}$$

At a distance l from the conductor, the dielectric or electrostatic field lags behind the voltage (1) by angle $a\,l$, thus, per unit length of conductor, is given by,

$$\psi = \Psi \cos (\theta - \beta - \alpha\, l) \tag{15}$$

The voltage gradient is $2/l$ times the dielectric flux, and the total voltage from conductor to return conductor thus is given by,

$$e = \int_{l_r}^{l'} \frac{2\,\psi}{l}\, d\,l$$

or:

$$\begin{aligned} E \cos (\theta - \beta) &= 2\,\Psi \int_{l_r}^{l'} \frac{\cos (\theta - \beta - a\,l)}{l}\, d\,l \\ &= 2\,\Psi \left\{ \cos (\theta - \beta) \int_{l_r}^{l'} \frac{\cos a\,l}{l}\, d\,l \right. \\ &\qquad \left. + \sin (\theta - \beta) \int_{l_r}^{l'} \frac{\sin a\,l}{l}\, d\,l \right\} \end{aligned} \tag{16}$$

hence, by (6) and (7):

$$\begin{aligned} E \cos (\theta - \beta) = 2\,\Psi \left\{ \log \frac{l'}{l_r} \cos (\theta - \beta) \right. \\ \left. + a\,l' \sin (\theta - \beta) \right\} \end{aligned} \tag{17}$$

The dielectric field gives rise to a current in quadrature with the field, the capacity or charging current of the conductor:

$$I_1 = 2\,\pi f\, \Psi \tag{18}$$

and, substituting (18) into (17), gives,

$$\begin{aligned} E \cos (\theta - \beta) = \frac{I'}{\pi f} \left\{ \log \frac{l'}{l_r} \sin (\theta - \beta) \right. \\ \left. + a\,l' \cos (\theta - \beta) \right\} \end{aligned} \tag{19}$$

where the change of the trigonometric functions represents the quadrature relation.

The reactive or sine component of this current I' gives rise to the capacity reactance,

$$x_c = \frac{S^2 \log \dfrac{l'}{l_r}}{\pi f} 10^{-9} \text{ ohms} \tag{20}$$

and the capacity:

$$C = \frac{1}{2 \pi f x_c}$$

$$= \frac{10^{-9}}{2 S^2 \log \dfrac{l'}{l_r}} \text{ farads} \tag{21}$$

where the factor S^2 reduces from electrostatic to electromagnetic units.

The cosine component of I' is in phase with the voltage, thus an energy current, and gives rise to an effective resistance, the dielectric radiation resistance.

$$r_c = \frac{a l' S^2}{\pi f} 10^{-9} \text{ ohms,} \tag{22}$$

or, by (**2**):

$$r_c = 2l' \; S \, 10^{-9} \text{ ohms} \tag{23}$$

If the specific capacity of the medium differs from unity, and is κ, then κ enters as factor in the numerator of C and in the denominator of x_c and r_c.

From (**21**) and (**23**) follow as the attenuation constant of dielectric radiation:

$$u_2 = \frac{g}{2C} = \frac{\pi f r_c}{x_c}$$

$$= \frac{2 \pi^2 f^2 l'}{S^2 \log \dfrac{l'}{l_r}} \tag{24}$$

$$= u_1$$

that is, the dielectric and the magnetic attenuation constant of magnetic radiation are equal; in other words, by electric radiation, the magnetic and the dielectric energy decay at the same rate, and their ratio thus remains constant.

It is interesting to note, that the expressions of inductance L and capacity C, (**11**) and (**21**), are the same as the values of

L and C, calculated without considering the finite velocity of the electric field, that is, are the "low-frequency values" of external inductance and capacity. Thus:

As long as the distance of the return conductor is small compared with a quarter wave length, electric radiation due to the finite velocity of the electric field, does not affect or change the values of external inductance L and of capacity C, but causes an energy dissipation by electromagnetic and by dielectric or electrostatic radiation, which is represented by effective resistances. The electromagnetic radiation resistance is a series resistance, proportional to the square of the frequency; the electrostatic resistance is a shunted resistance in series with the capacity. It is independent of the frequency and the power consumed by either resistance thus is proportional to the square of the frequency.

Equations (**6**) and (**7**), and thus the equations (**10**), (**11**), (**13**); (**20**), (**21**), (**23**), of the circuit constants x, L, r; x_c, C, r_c, apply only as long as the distance l' of the return conductor is small compared with a quarter wave length, and with sufficient approximation up to l' = one half of a quarter wave length.

With the return conductor at 6 ft. = 182 cm., the minimum wave length would be 8 × 182 = 1456 cm., and the maximum frequency thus:

$$f_0 = \frac{3 \times 10^{10}}{1456} = 20 \text{ million cycles, approx.}$$

With the ground as return, and the conductor 30 ft. above ground, thus the image conductor as equivalent return conductor at 2 × 30 ft. = 1820 cm. distance, the maximum frequency is:

$$f_0 = \frac{3 \times 10^{10}}{8 \times 1820} = 2 \text{ million cycles, approx.}$$

As seen, the approximations (**6**) and (**7**) apply to practically all industrially important transients, with rare exceptions.

For such high frequencies however, where the distance of the return conductor, l', approaches or exceeds a quarter wave length, or in the case of a conductor without return conductor ($l' = \infty$), such as is approximated by the wireless antenna, or by a lightning discharge to ground, the integrals (**6**) and (**7**) have to be evaluated.

They define new functions:*

$$\int_{l_r}^{l'} \frac{\sin a\, l}{l}\, d\, l = \text{col}\, a\, l_r - \text{col}\, a\, l' \qquad (25)$$

$$\int_{l_r}^{l'} \frac{\cos a\, l}{l}\, d\, l = \text{sil}\, a\, l_r - \text{sil}\, a\, l' \qquad (26)$$

and, since l_r is always small compared with a quarter wave length, it is:

$$\text{col}\, a\, l_r = \frac{\pi}{2} \qquad (27)$$

$$\text{sil}\ a\, l_r = \log \frac{1}{a\, l_r} - 0.5772 \qquad (28)$$

Substituting (**25**), (**26**), (**27**) and (**28**) instead of (**6**) and (**7**), into the previous discussion, gives the final complete equations:

$$(10) \quad x = 4\, \pi f \left\{ \log \frac{1}{a\, l_r} - 0.5772 - \text{sil}\, a\, l' \right\} 10^{-9}$$

$$= 4\, \pi f \left\{ \log \frac{S}{2\, \pi f\, l_r} - 0.5772 \right.$$

$$\left. - \,\text{sil}\, \frac{2\, \pi f\, l'}{S} \right\} 10^{-9} \text{ ohms} \qquad (29)$$

$$(11) \quad L = 2 \left\{ \log \frac{1}{a\, l_r} - 0.5772 - \text{sil}\, a\, l' \right\} 10^{-9}$$

$$= 2 \left\{ \log \frac{S}{2\, \pi f\, l_r} - 0.5772 \right.$$

$$\left. - \,\text{sil}\, \frac{2\, \pi f\, l'}{S} \right\} 10^{-9} \text{ henrys} \qquad (30)$$

$$(13) \quad r = 4\, \pi f \left\{ \frac{\pi}{2} - \text{col}\, a\, l' \right\} 10^{-9}$$

$$= 4\, \pi f \left\{ \frac{\pi}{2} - \text{col}\, \frac{2\, \pi f\, l'}{S} \right\} 10^{-9} \text{ ohms} \qquad (31)$$

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*Tables of the numerical values of these functions are given in the Appendix to the later editions of "Theory and Calculation of Transient Electric Phenomena."

$$(20)\quad x_e = \frac{S^2}{\pi f}\left\{\log\frac{1}{a\,l_r} - 0.5772 - \operatorname{sil} a\,l'\right\}10^{-9}$$

$$= \frac{S^2}{\pi f}\left\{\log\frac{S}{2\,\pi f\,l_r} - 0.5772 - \operatorname{sil}\frac{2\,\pi f\,l'}{S}\right\}10^{-9}\text{ ohms} \qquad \textbf{(32)}$$

$$(21)\quad C = \frac{10^{-9}}{2\,S^2\left\{\log\frac{1}{a\,l_r} - 0.5772 - \operatorname{sil} a\,l'\right\}}$$

$$= \frac{10^{-9}}{2\,S^2\left\{\log\frac{S}{2\,\pi f\,l_r} - 0.5772 - \operatorname{sil}\frac{2\,\pi f\,l'}{S}\right\}}\text{ farads} \qquad \textbf{(33)}$$

$$(23)\quad r_e = \frac{S^2}{\pi f}\left\{\frac{\pi}{2} - \operatorname{col} a\,l'\right\}10^{-9}$$

$$= \frac{S^2}{\pi f}\left\{\frac{\pi}{2} - \operatorname{col}\frac{2\,\pi f\,l'}{S}\right\}10^{-9}\text{ ohms} \qquad \textbf{(34)}$$

As seen from the tables of col x, the radiation resistances, and thus the energy dissipation by magnetic and dielectric radiation, increase with increasing distance of the return conductor, reach a maximum when the distance of the return conductor is one-half wave length, and then decrease again.

The maximum radiation resistances, at l' = one-half wave length, are:

$$r = 7.41\,\pi f\,10^{-9}$$

$$= 23.27 f\,10^{-9}\text{ ohms.} \qquad \textbf{(35)}$$

$$r_e = \frac{1.852\,S^2}{\pi f}\,10^{-9}$$

$$= \frac{0.531}{f}\,10^{12}\text{ ohms.} \qquad \textbf{(36)}$$

Substituting $l' = \infty$ into equations (**29**) to (**34**) gives the radiation constants of the conductor without return conductor.

$l = \infty$ gives:

$$\operatorname{sil} a\,l' = 0 = \operatorname{col} a\,l' \qquad \textbf{(37)}$$

thus:

$$(29)\qquad x = 4\,\pi f \left\{ \log \frac{S}{2\,\pi f\, l_r} - 0.5772 \right\} 10^{-9} \text{ ohms} \qquad (38)$$

$$(30)\qquad L = 2 \left\{ \log \frac{S}{2\,\pi f\, l_r} - 0.5772 \right\} 10^{-9} \text{ henrys} \qquad (39)$$

$$(31)\qquad r = 2\,\pi^2 f\, 10^{-9} \text{ ohms} \qquad (40)$$

$$(32)\qquad x_c = \frac{S^2}{\pi f} \left\{ \log \frac{S}{2\,\pi f\, l_r} - 0.5772 \right\} 10^{-9} \text{ ohms.} \qquad (41)$$

$$(33)\qquad C = \frac{10^{-9}}{2\,S^2 \left\{ \log \dfrac{S}{2\,\pi f\, l_r} - 0.5772 \right\}} \text{ farads} \qquad (42)$$

$$(34)\qquad r_c = \frac{S^2}{2 f}\, 10^{-9} \text{ ohms} \qquad (43)$$

APPENDIX C

Equations of Electrical Constants

Let: l = length of conductor, cm.

l_r = radius of conductor, cm.

l_1 = circumference of conductor, cm.

l_2 = shortest circumference of conductor, cm.

l' = distance of return conductor

f = frequency, cycles per second

γ = electrical conductivity, mhos per cm^3.

μ = magnetic permeability

$S = 3 \times 10^{10}$ = velocity of radiation in empty space,

it is then, log denoting the natural logarithm:

resistances:

true ohmic resistance (thermal):

$$r_0 = \frac{l}{\gamma\, \pi\, l^2{}_r} \text{ ohms.}$$

effective resistance of unequal current distribution (thermal):

$$r_1 = \frac{\pi\, l}{l_1} \sqrt{\frac{0.4\, \mu f'}{\gamma}}\, 10^{-4} \text{ ohms.}$$

effective magnetic radiation resistance:

(a) return conductor at distance l':

$$r_3 = \frac{8\,\pi^2 f^2 l'\, l}{S}\, 10^{-9} \text{ ohms}$$

at extremely high frequencies:

$$r_4 = 4\,\pi f l \left\{ \frac{\pi}{2} - \operatorname{col} \frac{2\,\pi f l'}{S} \right\} 10^{-9} \text{ ohms.}$$

(b) conductor without return conductor:

$$r_2 = 2\,\pi^2 f\, l\, 10^{-9} \text{ ohms.}$$

effective dielectric radiation (shunted) resistance:[1]

(a) return conductor at distance l':

$$r_c = \frac{2\,l' S}{l}\, 10^{-9} \text{ ohms}$$

at extremely high frequencies:

$$r_c = \frac{S^2}{\pi f l} \left\{ \frac{\pi}{2} - \operatorname{col} \frac{2\,\pi f l'}{S} \right\} 10^{-9} \text{ ohms}$$

(b) conductor without return conductor:

$$r_c = \frac{S}{2 f l}\, 10^{-9} \text{ ohms}$$

reactances:

low-frequency internal reactance:

$$x_{10} = \pi f l\, \mu\, 10^{-9} \text{ ohms.}$$

internal reactance of unequal current distribution:

$$x_1 = \frac{\pi l}{l_1} \sqrt{\frac{0.4\,\mu f}{\gamma}}\, 10^{-4} \text{ ohms, } = r_1$$

external reactance:

(a) return conductor at distance l':

$$x_0 = 4\,\pi f l \log \frac{l'}{l_r}\, 10^{-9} = 4\,\pi f l \log \frac{2\,\pi l'}{l_2}\, 10^{-9} \text{ ohms}$$

at extremely high frequencies:

$$x_4 = 4\,\pi f l \left\{ \log \frac{S}{2\,\pi f l_r} - 0.5772 - \operatorname{sil} \frac{2\,\pi f l'}{S} \right\} 10^{-9} \text{ ohms}$$

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*As shunted resistance and reactance, r_c and x_c are inverse proportional to the length of the conductor l that is, the longer the conductor, the more current is shunted across, and the lower therefor are r_c and x_c. For this reason, the shunt constants usually are given as conductance g and susceptance b. In the present case however, r and x give simpler expressions.

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(b) conductor without return conductor:

$$x_2 = 4\pi f l \left\{ \log \frac{S}{2\pi f l_r} - 0.5772 \right\} 10^{-9} \text{ ohms}$$

shunted capacity reactance:

(a) return conductor at distance l':

$$x_c = \frac{S^2 \log \dfrac{l'}{l_r}}{\pi f l} 10^{-9} = \frac{S^2 \log \dfrac{2\pi l'}{l_2}}{\pi f l} 10^{-9} \text{ ohms}$$

at extremely high frequencies:

$$x_c = \frac{S^2}{\pi f l} \left\{ \log \frac{S}{2\pi f l_r} - 0.5772 - \text{sil} \frac{2\pi f l'}{S} \right\} 10^{-9} \text{ ohms.}$$

(b) conductor without return conductor:

$$x_c = \frac{S^2}{\pi f l} \left\{ \log \frac{S}{2\pi f l_r} - 0.5772 \right\} 10^{-9} \text{ ohms}$$

inductances:

low frequency internal inductance:

$$L_{10} = \frac{l\mu}{2} 10^{-9} \text{ henrys}$$

internal inductance of unequal current distribution:

$$L_1 = \frac{l}{l_1} \sqrt{\frac{0.1\mu}{\gamma f}} 10^{-4} \text{ henrys}$$

external inductance:

(a) return conductor at distance l':

$$L_0 = 2\, l \log \frac{l'}{l_r} 10^{-9} = 2\, l \log \frac{2\pi l'}{l_2} 10^{-9} \text{ henrys.}$$

at extremely high frequencies:

$$L_4 = 2\, l \left\{ \log \frac{S}{2\pi f l_r} - 0.5772 - \text{sil} \frac{2\pi f' l'}{S} \right\} 10^{-9} \text{ henrys.}$$

(b) conductor without return conductor:

$$L_2 = 2\, l \left\{ \log \frac{S}{2\pi f l_r} - 0.5772 \right\} 10^{-9} \text{ henrys.}$$

capacity:

(a) return conductor at distance l':

$$C_0 = \frac{10^9}{2\, S^2 \log \dfrac{l'}{l_r}} \text{ farads}$$

at extremely high frequencies:

$$C_4 = \frac{10^9}{2\,S^2 \left\{ \log \dfrac{S}{2\,\pi f l_r} - 0.5772 - \operatorname{sil} \dfrac{2\,\pi f l'}{S} \right\}} \text{ farads}$$

(b) conductor without return conductor:

$$C_2 = \frac{10^9}{2\,S^2 \left\{ \log \dfrac{S}{2\,\pi f l_r} - 0.5772 \right\}} \text{ farads}$$

Herefrom then follow the Circuit Constants:

At Low Frequencies: (Machine Frequencies up to 10^3 cycles, approx.)

$$\begin{aligned} r &= r_0 \\ x &= x_{10} + x_0 \\ L &= L_{10} + L_0 \\ C &= C_0 \\ g &= 0 \end{aligned}$$

At Medium Frequencies: (10^3 to 10^5 cycles, approx.)

$$\begin{aligned} r &= r_1 \\ x &= x_1 + x_0 \\ L &= L_1 + L_0 \\ C &= C_0 \\ g &= 0 \end{aligned}$$

At High Frequencies: (10^5 to 10^7 cycles, approx.)

$r = r_1 + r_3$ (with return conductor)
$\;\;= r_1 + r_2$ (without return conductor)
$x = x_1 + x_0$ (with return conductor)
$\;\;= x_1 + x_2$ (without return conductor)
$L = L_1 + L_0$ (with return conductor)
$\;\;= L_1 + L_2$ (without return conductor)
$C = C_0$ (with return conductor)
$\;\;= C_2$ (without return conductor)
(approximately, or represented by x_e and r_e)
g represented by r_e and x_e:

$$g = \frac{r_e}{r^2_e + x^2_e}\ ;\ b = \frac{x_e}{r^2_e + x^2_e} = 2\,\pi f C$$

At Extremely High Frequencies: (above 10^7 cycles, approx.)

$r = r_1 + r_4$ (with return conducter)
$\;\;= r_1 + r_2$ (without return conductor)
$x = x_1 + x_4$ (with return conductor)
$\;\;= x_1 + x_2$ (without return conductor)

C and g represented by r_e and x_e, thus:

$$g = \frac{r_e}{r_e^2 + x_e^2}$$

$$C = \frac{x_e}{2\pi f (r_e^2 + x_e^2)}$$

From these follow the Derived Circuit Constants:

Magnetic Attenuation:

$$u_1 = \frac{r}{2L}$$

Dielectric Attenuation:

Usually zero at low and medium frequencies,

$$u_2 = \frac{g}{2C} = \frac{\pi f r_e}{x_e}$$

at high and very high frequencies.

Attenuation Constant:

$$u = u_1 + u_2 = \frac{1}{2}\left(\frac{r}{L} + \frac{g}{C}\right)$$

Series Power Factor:

$$\cos \omega = \frac{r}{\sqrt{r^2 + x^2}}$$

Shunt Power Factor:

Zero at low and medium frequencies,

$$\cos \omega_e = \frac{r_e}{\sqrt{r_e^2 + x_e^2}}$$

at high and very high frequencies.

Duration of Oscillation:[1]

$$t_0 = \frac{1}{u} \text{ seconds}$$

$$N_0 = \frac{f}{u} \text{ cycles}$$

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*Under "Duration" of a transient is understood the time (or the number of cycles), which the transient would last, that is, the time in which it would expend its energy, if continuing at its initial intensity. With a simple exponential transient, this is the time during which it decreases to $\frac{1}{\epsilon}$ or 36.8 per cent of its initial value. It decreases to one-tenth of its initial value in 2.3 times this time.

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REVIEW OF WORK OF SUB-COMMITTEE ON WAVE SHAPE STANDARD OF THE STANDARDS COMMITTEE*

BY HAROLD S. OSBORNE

ABSTRACT OF PAPER

The paper gives a review of the work done during the past three years by a sub-committee of the Standards Committee of the Institute appointed to make recommendations for changes in the Institute's rules regarding the wave shape of alternators. After a very careful consideration of the question and a large amount of experimental work the Sub-Committee last spring made recommendations that for the present the ten per cent deviation rule should be retained without change (except in wording), and that trial use should be made of a supplementary wave shape factor. The new factor, based on the relation between voltage wave shape and interfering effect in telephone circuits, when power and telephone lines parallel each other, is called the "telephone interference factor." The committee invites the broadest discussion and trial use of the new factor. No attempt will be made to decide upon limiting values of the factor until after wide experience shall have been obtained in its use, and the committee will welcome assistance on the part of any interested in collecting information to this end.

THIS paper is presented at the request of the sub-committee on Wave Shape Standard to encourage "the trial use of telephone interference factor and the collection of information by any who are interested."

Introduction. Until the past year the only standard of wave-shape included in standardization rules of the American Institute of Electrical Engineers has been the rule that the maximum deviation from the equivalent sine wave should not exceed 10 per cent of the amplitude of the equivalent sine wave. This provision has been in the standardization rules for many years and has done excellent service in preventing the use of machines having grossly distorted wave-shapes.

On a number of occasions papers and discussions have been presented to the Institute criticising the 10 per cent deviation rule. In two papers presented to the Institute in 1913 (*Wave-Form Distortions and Their Effects on Electrical Apparatus,* by P. M. Lincoln, and *Proposed Wave-Shape Standard,* by C. M. Davis) it was pointed out that the deviation standard has two marked disadvantages in that it sets approximately the same limit for harmonics of all frequencies and is relatively difficult of application.

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*The author acknowledges the assistance of the other members of the Sub-Committee on Wave Shape Standard. The members of this Sub-committee are: C. A. Adams, Frederick Bedell, L. W. Chubb, A. W. Copley, F. M. Farmer, H. S. Osborne, L. T. Robinson and A. E. Silver.

The discussion of these papers has led to the consideration of wave-shape factors other than the deviation and of more convenient means for determining them. The simplest method so far suggested of determining a wave factor is by devising an electrical network such that the current in one branch resulting from a complex voltage wave at the terminals of the network is an indication of the distortion of the voltage wave in question. The harmonics of the wave may be given a desired relative weighting by using a suitable network.

The wave-shape standard proposed by Mr. Davis in the paper cited above employed for its determination the simplest form of such a network. He suggested that the weighting of the harmonics be proportional to their frequency, and that this weighting be obtained by using as a measure of wave shape the ratio of the admittance of a condenser to the complex wave to the admittance of the condenser to a sine wave of the same frequency. The factor so measured is called the differential distortion factor. This and several other means of measuring wave shape were discussed again before the Institute in 1915 (*Distortion Factors,* by Professor Bedell). The attention which had been directed to the subject led in December, 1915, to the appointment by the Standards Committee of a sub-committee to consider this subject and make recommendations.

Outline of the Work of the Sub-committee. The form of wave-shape standard which was suggested by Mr. Davis has a number of characteristics which recommend it. In the form suggested, however, it would be exceedingly difficult of practical application, largely because of errors in measurement caused by the inductance and resistance of the machines under test and of the measuring instrument. The committee first turned its attention to the possibility of developing a satisfactory standard by simple modifications of that proposed by Mr. Davis formed by adding a series resistance or a resistance and inductance to the condenser which is the measuring circuit in that standard. It was hoped that a satisfactory standard of this sort could be developed that would be so simple that it could be set up from standard coils, condensers and meters available in any well equipped laboratory.

The different methods of measuring wave shape, which under these limitations could be given serious consideration, are discussed in a paper presented to the Institute in 1916 by Professor Bedell reporting the progress of the sub-committee's work (TRANSACTIONS, 1916, page 1154). However, it was the conclusion of the sub-committee, based on a large amount of experimental work,—that whatever form of network might be adopted it would need to be set up as a special piece of apparatus with the constants carefully specified. It was also found necessary to use with the apparatus a highly sensitive indicating instrument, such as a vacuum thermocouple with a direct-current milliammeter or a very sensitive alternating-current dynamometer milliammeter. Moreover, a network of any of the forms discussed in the progress report was felt to be not entirely satisfactory because the readings obtained would not vary rapidly with changes in wave shape and also because the relative penalizing of different harmonics was arbitrarily determined by the constants of the network.

These conclusions led the committee to consider the development of a rating of wave shape or wave-shape factor, unrestricted by the requirement of a simple series arrangement of measuring network, which would give as accurately as practicable a measure of the detrimental effect of any wave, taking into account effects on power systems and inductive effects on communication circuits. To such a factor the harmonics of different frequencies would contribute both in proportion to their magnitude and in proportion to the amount of the disturbance which they would individually cause if of the same magnitude.

The committee has found that but very little definite information is available regarding the relative effect of harmonics of different frequencies on the operation of power systems. For the present any modification of the 10 per cent deviation standard for the purpose of weighting the harmonics with reference to their effect on the operation of power systems would have to be based on general judgment and does not seem to be necessary.

On the other hand when telephone systems come within the inductive influence of power systems more trouble has been experienced, and this trouble bears a definite relation to the frequencies of the harmonics in the wave form. Consequently the committee has collected definite experimental data regarding the relative effect of harmonics of different frequencies in producing interference in telephone systems.

In view of these facts, the committee has proposed as a first step in a revision of the rules regarding wave shape and pending the obtaining of further information, that the present deviation factor be retained for all alternators and that an additional factor be adopted to be applied to alternators which will be used on open-wire * transmission and distribution systems. This additional factor is based entirely on the effect of the harmonics in producing interference in telephone circuits when the power and telephone circuits are parallel. In accordance with this conclusion the sub-committee has developed, and the Standards Committee has adopted for trial use, a new wave-shape factor which is called the telephone-interference factor. No attempt has been made to set values of the telephone-interference factor which should be used as limits, it being considered important that a large amount of data and experience be obtained before values are set. The purpose of this paper is to call attention to the new factor, to tell how it was obtained and to describe in detail the apparatus by which it may be measured so that any one interested can assist the committee in obtaining experience with the use of the new factor and in determining what would be the best limiting values of the factor to be adopted as standard.

New Standardization Rules. In the new standardization rules regarding wave-shape the old rule for maximum deviation remains unchanged except in the wording. The new rules are as follows:

Section 17 (new)

The deviation factor of a wave is the ratio of the maximum difference between corresponding ordinates of the wave and of the equivalent sine wave of equal length to the maximum ordinate of the equivalent sine wave when the waves are superposed in such a way as to make this maximum difference as small as possible. (See Section 406)

Section 406 (new)

A deviation factor, as defined in Section 17, of the terminal voltage wave on open-circuit of all synchronous machinery not exceeding 10 per cent is permissible except when otherwise specified.

The new factor, called a telephone-interference factor, is covered by new sections in the Standards' rules, reading as follows:

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*By open-wire system is meant one which is not in cable.

Section 17 A (new)

The interference factor of a wave is a measure of its departure from the sinusoidal shape based upon its disturbing effects when acting by induction on other circuits. (See Sections 407 and 408).

Section 407 (new)

The telephone interference factor is the ratio of the square root of the sum of the squares of the weighted values of all the sine-wave components (including in alternating waves both fundamental and harmonics) to the r.m.s. value of the wave.

Section 408 (new) (*For trial only*)

The weighting of the sine-wave components of different frequencies is as given in the following curve. (Fig. 1). The telephone interference factor of a voltage wave, corresponding to this weighting, may be measured by the use of the network shown in the following diagram (Fig. 2). With this network the telephone interference factor is the ratio of the current I in micro-amperes in the meter branch of the network to the voltage E applied to the external terminals of the network. The measurement may be made on the low-tension side of a potential transformer. A sensitive vacuum thermo-couple provided with a shunt, and a direct-current milliammeter have been found convenient for measuring the current.

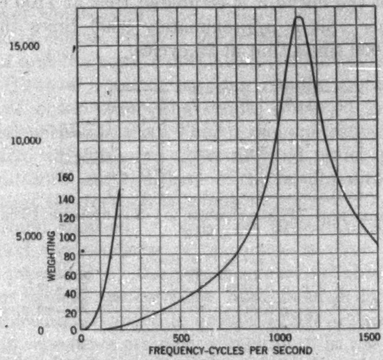

Fig. 1—Frequency-Weighting Curve for Telephone Interference Factor

Section 409 (new)

A telephone interference factor (See Sections 17A and 407) of the open-circuit terminal voltage wave not exceeding * . . . is permissible except when otherwise specified. This rule shall apply in cases where synchronous machinery is required for use in connection with open-wire transmission or distribution circuits and shall be supplementary to 406. The purchaser shall in each case indicate whether or not the conditions are such that this section applies.

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*The appropriate limiting value of this factor has not yet been determined, but will be recommended after further experience shall have been obtained.

Relation Between Wave Shape and Telephone Interference. The great importance of the harmonics in voltage and current waves of the power system in determining how much interference the power circuit will cause in parallel telephone circuits is well illustrated by the curve given in Rule 408. This curve represents the relative amount of interference caused by voltages of the same magnitude but different frequencies in the power circuit. From the curve it is estimated that at 1100 cycles the interference per volt is approximately 9000 times that at 60 cycles and 60,000 times that at 25 cycles.

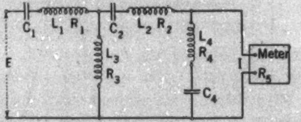

FIG. 2—NETWORK FOR MEASUREMENT OF TELEPHONE INTERFERENCE FACTOR

NETWORK CONSTANTS.

$C_1 = 0.9$ mf.	± 0.5%	$L_1 = 0.023$ henry	± 0.5%	$R_1 = 5$ ohms ± 2%
$C_2 = 0.9$ "		$L_2 = 0.0205$ "		$R_2 = 12$ " ± 2%
		$L_3 = 0.068$ "		$R_3 = 73$ " ± 1%
$C_4 = 7.5$ "		$L_4 = 0.019$ "		$R_4 = 22.5$ " ± 2%
				$R_5 = 43$ " ± 1%

This means that if the harmonics were entirely absent from a 25-or 60-cycle power system, the problem of inductive interference from the system with telephone service would practically vanish. Although this condition cannot, of course, be obtained, anything which causes an approach to it causes a reduction in the amount of interference under conditions otherwise the same and therefore makes possible a reduction in the expense which is necessary in the plants of the power and telephone systems in order to avoid excessive interference: There is therefore a direct economic advantage from the standpoint of present day conditions in designing and building alternators with as low a telephone interference factor as practicable; provided no material increase in cost is involved.

Determination of the Relative Importance of Frequencies in Telephone Interference. The curve given in new Rule 408 has been developed to represent the variation with frequency of the amount of interference in parallel telephone circuits from a given voltage applied to the power circuit, other conditions being the same. It is based on experimental work carried out under the direction of the chairman of the Sub-committee and which is described in Appendix A. The methods by which these results were obtained may be briefly summarized as follows:

Two methods have been used. In each case known amounts of current of different frequencies have been inserted in the telephone receiver of an ordinary telephone circuit, using for convenience artificial lines. With one method the observers listened to the reading of ordinary news matter over the noisy circuit and then over a quiet circuit of greater length and indicated over which circuit they could understand better. This method, is of course, crude if the results of a single observer be considered and the results obtained by different observers vary widely. However, averaging the results obtained in large numbers of observations the results obtained in two distinct sets of tests check closely. The two sets of tests were made four years apart, with different observers and under somewhat different conditions, and the check of the results obtained indicates strongly that the method is sound.

The second method is based on articulation tests. With this method the goodness of transmission over the telephone circuit is measured by the percentage of syllables spoken over the circuit which are correctly apprehended by the listener. The syllables are arranged in groups which are so selected that each group represents ordinary speech both in the relative number and in the arrangement of the fundamental sounds of which speech is composed. Therefore, the percentage of correctly received syllables is a measure of the degree of intelligibility of the conversations held over the circuit. The percentage of words correctly understood in an actual conversation is, of course, very much higher than that obtained in the articulation tests because the syllables used in the test are without context and, in general, without meaning. The articulation tests made in connection with this experimental work involve 60,000 observations, considering each syllable as an observation.

The articulation tests do not necessarily give a measure of the complete detrimental effect of the noise caused by extraneous currents because in addition to the reduction of intelligibility in the telephone circuit, there is a factor of annoyance to the person using the telephone. It is interesting to note that the relative results obtained by the articulation tests check those obtained by the reading tests for low frequencies, and then with frequencies above 1000 cycles, show progressively less effect. This indicated that the reading tests include in part the factor of annoyance which becomes relatively more important with the high frequencies.

The experimental work which is briefly discussed above gives a curve representing in relative terms the combined effect of varying sensitiveness of the telephone receiver and of the human ear. The shape of the curve depends much, of course, on the constants of the receiver. The receiver used in this work, however, was typical in this respect of all receivers in general use in the country and the curve represents a relation between sensitiveness and frequency of current which will probably be approximately maintained for a number of years.

In developing from this experimental work a curve to be made the basis of a new wave-shape factor, consideration has been given to other elements which help to determine the relative amount of noise caused by generator voltages and currents of different frequencies. The more important of these are outlined below:

a. The current induced in telephone circuits by a given voltage or current in the power circuit is approximately proportional to the frequency. This effect was taken into account in determining the weighting curve.

b. The attenuation of the harmonic components of the current as they are propagated along the power and telephone circuits varies with the frequency. Except in rare instances, this effect is small compared with the others which are taken into account. It was therefore omitted.

c. In some forms of telephone apparatus, there may be appreciable selective action tending to change the relative amount of currents of different frequencies which reach the receiver. Loaded telephone lines generally do not transmit currents above about 2500 cycles, but the effect of these currents is normally inappreciable in the production of noise because of their almost complete absence from the power system. Some forms of telephone apparatus tend to eliminate the currents of low frequencies. These currents have a very small effect on the amount of noise, however, except when present in very large volume. In general, the condition for a good telephone apparatus requires that it transmit with approximately equal efficiency all frequencies important in the reproduction of speech and it is just these frequencies which are important in the production of noise from extraneous sources. The effect of the apparatus is therefore in general small compared with the other factors which have been included and this effect has been neglected in determining the weighting curve.

In accordance with (a) the curve which is presented in Rule 408 was obtained by multiplying the experimental curve of relative interference of currents of different frequencies by a factor proportional to the frequency. As a result of the experimental work which has been done and of the above considerations the committee believes that the curve selected is the best curve for use at this time.

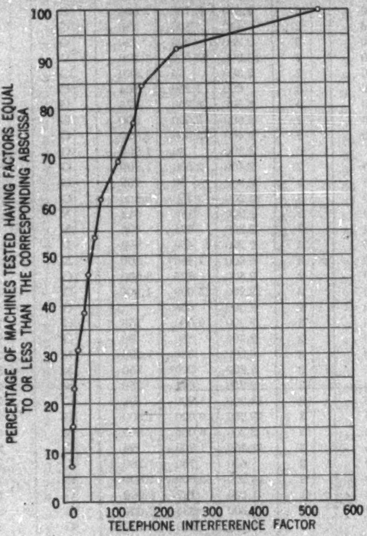

Fig. 3

Description of Apparatus and Method of Use. In order to make the use of the new telephone interference factor a simple matter, the committee has developed apparatus in which the ratio of current through one branch of a network to the voltage applied at its terminals gives a measure of the telephone inter-

TABLE I
RESULTS OF MEASUREMENTS WITH NETWORK

Number	Frequency	Number phase	Volts	Kv-a.	Rev. per min.	Type	Telephone interference factor determined by network	Oscillogram
1	25	3	2,200	6,000	1,500	G	5	
2	60	3	13,200	12,500	1,800	G	11	
3	60	3	600	450	138.5	G	11	
4	60	2	2,300	2,500	3,600	G	11	
5	60	3	13,200	12,500	1,800	G	12	
6	60	3	2,300	3,750	3,600	G	13	
7	25	3	440	800	136.3	G	14	
8	60	3	2,200	925	120	G	14	
9	60	3	13,200	25,000	1,800	G	16	
10	60	2	2,300	35	1,800	G	16	
11	60	3	600	1,250	164	G	16	
12	60	3	6,500	6,250	375	G	16	38
13	60	2	2,600	3,000	600	G	17	
14	60	2	2,400	2,500	1,800	G	18	
15	....	3	2,300	30	1,800	G	18	
16	60	2	2,400	2,500	1,800	G	18	
17	25	3	2,300	50	750	G	18	
18	60	3	13,200	2,350	3,600	G	18	
19	60	2	2,200	180	514	M	18	
20	25	3	2,200	625	500	M	19	
21	30	3	2,200	450	300	M	19	
22	25	3	6,600	1,440*	500	M	19	9
23	60	2	2,400	625	3,600	G	19	
24	60	3	13,200	25,000	1,800	G	20	
25	60	2	2,300	850	....	G	20	
26	25	3	13,200	9,000	750	G	20	
27	60	2	2,300	850	90	G	21	
28	60	3	2,300	300	1,200	M	22	
29	25	3	2,200	290*	750	M	23	
30	60	3	2,200	625	250	M	23	
31	25	3	13,200	1,800	94	G	24	
32	60	3	13,200	9,000	1,800	G	25	
33	60	3	2,200	6,000	300	M	25	
34	25	3	13,200	1,800	94	G	26	
35	60	3	600	750	1,200	C	26	
36	60	3	13,200	5,000	1,800	G	27	
37	60	1	1,100	187	1,200	G	27	
38	60	3	2,300	935	3,600	G	29	
39	60	2	2,500	500	400	G	30	
40	60	2	2,200	6,000	....	G	30	30
41	60	3	2,300	3,750	300	M	32	
42	60	3	250	1,000	600	C	34	35
43	60	2	2,600	750	90	G	34	
44	60	2	2,400	1,500	1,800	G	35	
45	60	3	7,000	500	100	G	35	
46	60	3	250	1,000	600	C	36	30
47	60	2	2,300	850	90	G	37	
48	60	3	600	450	150	G	38	
49	25	3	390	3,500	300	M	38	
50	50	3	480	400	1,000	C	42	31
51	60	3	6,600	2,500	3,600	G	42	
52	60	3	2,300	100	900	G	44	
53	60	2	2,300	1,000	1,800	G	46	
54	60	2	2,300	1,000	1,800	G	46	
55	60	3	2,200	625	250	M	46	
56	60	3	2,300	150	600	G	48	

TABLE I (*Continued*)

RESULTS OF MEASUREMENTS WITH NETWORK

Number	Frequency	Number phase	Volts	Kv-a.	Rev. per min.	Type	Telephone interference factor determined by network	Oscillogram
57	60	3	240	94	600	G	50	72
58	60	2	2,300	1,000	1,800	G	50	
59	25	3	13,200	5,000	500	G	50	
60	25	3	250	750	750	C	50	
61	60	3	250	300	1,200	C	51	68
62	60	3	250	500	1,200	C	51	23
63	60	3	250	500	1,200	C	51	
64	60	3	250	500	1,200	C	54	54
65	60	3	250	500	1,200	C	54	
66	60	3	250	500	1,200	C	60	
67	60	3	2,300	540	150	G	60	
68	60	3	2,300	50	1,200	G	61	
69	60	3	250	500	1,200	C	61	55
70	60	3	240	500	1,200	C	64	83
71	60	3	250	500	1,200	C	64	
72	60	3	2,300	300	450	G	66	
73	60	3	2,300	500	400	G	66	
74	60	3	3,810	500	720	G	67	54
75	60	3	2,300	150	225	G	67	
76	60	3	240	94	600	G	69	75
77	60	3	1,200	500	1,200	C	70	86
78	60	3	2,400	437	300	G	72	
79	60	1	1,150	30	1,500	G	75	
80	60	3	1,000	50	1,200	G	77	
81	60	....	13,200	20,000	720	G	78	
82	60	3	250	300	1,200	C	80	75
83	60	3	2,200	445*	900	M	83	
84	60	3	250	200	1,200	C	85	106
85	50	3	220	320	187.5	G	92	
86	50	3	2,200	580	750	G	105	
87	60	3	2,300	75	600	G	110	
88	60	3	2,200	225	900	M	111	
89	60	....	13,200	10,000	720	G	113	
90	60	3	600	800	180	G	113	100
91	50	3	5,250	625	....	G	122	139
92	60	3	220	75	77	G	132	140
93	60	3	2,300	1,000	300	G	136	
94	60	3	2,200	4,000	....	G	....	138
95	60	3	13,200	3,000	600	G	140	
96	62.5	3	2,200	2,000	....	G	....	142
97	60	3	250	500	1,200	C	145	57
98	60	3	2,200	225*	900	M	146	146
99	60	3	2,300	325	720	M	147	
100	60	3	2,200	4,000	....	G	....	147
101	60	3	6,600	1,080*	514	M	148	
102	60	3	2,200	225*	900	M	149	177
103	60	3	2,200	225*	900	M	152	
104	60	3	2,200	290*	900	M	153	195
105	60	3	2,200	225*	900	M	155	137
106	60	3	2,200	225*	900	M	156	175
107	60	3	2,200	290*	900	M	160	
108	60	3	2,200	435*	900	M	163	164
109	60	3	2,200	225*	900	M	164	151
110	60	3	2,200	225*	900	M	165	166
111	60	3	2,200	250	300	M	180	

TABLE I (*Continued*)
RESULTS OF MEASUREMENTS WITH NETWORK

Number	Frequency	Number phase	Volts	Kv-a.	Rev. per min.	Type	Telephone interference factor determined by network	Oscillogram
112	60	3	2,200	290*	900	M	181	189
113	60	3	13,200	3,000	720	G	190	
114	60	3	2,200	290*	900	M	191	171
115	60	3	2,200	750	300	M	193	
116	60	3	2,300	1,000	277	G	200	
117	60	3	2,200	225*	900	M	201	139
118	60	3	2,200	225*	900	M	207	221
119	60	3	2,300	300	450	G	230	
120	60	3	240	150	225	G	239	228
121	60	3	480	400	200	G	262	
122	60	3	240	33	1,200	G	299	
123	60	1	1,150	50	1,200	G	305	
124	60	3	600	75	900	G	352	
125	60	3	13,200	1,000	1,200	G	360	
126	60	3	2,200	210*	257	M	375	375
127	60	3	13,200	1,000	1,200	G	430	
128	25	3	11,000	500	1,500	G	450	
129	60	....	13,200	9,000	720	G	490	
130	60	2	1,150	50	900	G	543	

Types of Machine:
G = Generator
M = Synchronous Motor
C = Synchronous Converter
*Rating given in Horse Power.

ference factor. The essential electrical constants of the network are given in the diagram in Rule 406. A detailed description and specification of the apparatus is given in Appendix B to this paper.

The measuring instrument of the apparatus consists of a vacuum thermocouple connected to a direct-current milliammeter. When there are a number of harmonics in the voltage wave applied to the apparatus, this instrument gives a reading proportional to the square root of the sum of the squares of the effects of the individual harmonics. The data which have been obtained regarding the law of combination of the detrimental effects of a number of sources of noise disturbance in a telephone circuit, indicate that this law varies under different extreme conditions. It seems probable that the root-sum-square law best represents the average condition and it is far more practicable for use than any other law because of the characteristics of measuring instruments. It has therefore been adopted.

Summary of Results Obtained with Network. The sub-committee has used this network for measurements of the wave-factor of the open-circuit voltage of a considerable number of machines. Some of the results which have been obtained are given in Table I—Results of Measurements with Network. These results are summarized in the curve of Fig. 3.

In the last column of Table I are given the results obtained in the case of the machines in checking the reading of the network by computing the telephone interference factor from the analysis of oscillograms. In order to obtain this check it is necessary to carry the analysis out to the 25th harmonic, and in some cases higher, in order to include the tooth harmonics. These analyses have shown that the use of the network involves not only much less work but also gives greater accuracy than the computation of the factors from the analysis of the oscillograms.

Approximate analyses which were made in a number of cases show very clearly that in general the machines with high telephone interference factors are machines with large slot harmonics. It is evident that this should be so from the shape of the admittance curve of the network. This is illustrated by the following table which gives the relative amounts of harmonics of different orders which would give the same value of telephone interference factor assuming that one harmonic only in addition to the fundamental is present in the wave.

Harmonic	Approximate relative amount
3	100
5	26.7
7	10.4
9	6.6
11	4.4
13	3.0
15	1.86
17	.96
19	.73
21	1.25
23	1.90
25	2.57

Determination of Limiting Values. The question of what limiting values of telephone-interference factor should be adopted for the construction of new machinery will, of course, be an economic one. There will be a point beyond which further reduction in the telephone-interference factor of machines, would involve an increase in cost greater than would be warranted by the damage caused or greater than the cost of outside-plant methods of obtaining equally effective results. It is proposed by the sub-committee not to recommend any limiting values for use until adequate data shall have been obtained. In the meanwhile, the sub-committee wishes to enlist the cooperation of any who are interested in the work of gathering as much information as possible which can contribute to the result Practicability in application seems to require that the measurements of telephone-interference factor be made with the machines operating on open-circuit. Load on a generator modifies the wave-form and results obtained indicate that in general, the telephone-interference factor of the voltage wave of the power system is less than the no-load factor of the worst machine connected to it.

The sub-committee is particularly anxious to get from the operating engineers data regarding the relation between the telephone-interference factor of the system voltage under the usual load conditions and the factors of the individual generators and motors connected to it, as measured on open circuit. Valuable information can thus be obtained by measuring the wave factors of the generators and other important pieces of rotating machinery on open-circuit and at rated speed, and recording also the measurements of resultant interference factor on the system at regular intervals, and as far as it can be determined, the load conditions of all generators at the time the measurements are made and other conditions on the system which would affect the results. The data submitted should, when possible, include the following information regarding individual machines:

1. Telephone interference factor.
2. Complete name plate data and name of manufacturer.
3. Terminal voltage at time of measurement.
4. Speed at time of measurement.
5. Load condition at time of measurement.

Attention is particularly called to the fact that in many cases the telephone-interference factor varies rapidly with the speed. The speed should therefore be carefully controlled and should be as accurately measured as possible.

The members of the sub-committee will be pleased to hear from anyone who is willing to cooperate in this work, and to supply any desired information regarding apparatus and methods.

Future Work. Future work will involve the collection of the information necessary for recommendations regarding the limiting values of telephone-interference factor to be applied to new alternating-current rotating apparatus. Such values should not be recommended until a large amount of information shall have been obtained and full opportunity given all interested to acquaint themselves with the new factor and to offer comments. Furthermore, the application of the new rules should be so timed that manufacturers will not be inconvenienced in adjusting themselves to them. Future work should include also the collection of any available information regarding the effect of different harmonics on the operation of power systems in order that the desirability of modifying the present deviation factor may be reviewed.

In the present rules, the deviation factor and the telephone interference factor are applied only to synchronous machinery. A complete consideration of the subject would require a study of the effect of other classes of electrical machinery upon the wave forms of the currents and voltages of the systems to which they are connected.

APPENDIX A

RELATIVE INTERFERING EFFECTS OF SINGLE-FREQUENCY CURRENTS IN THE TELEPHONE RECEIVER

The frequency-weighting curve for the telephone-interference factor (Fig. 1) is based upon several experimental investigations to determine the relative interfering effect of single-frequency currents in a telephone receiver. These investigations were conducted at different times, and by different methods, by the Engineering Department of the American Telephone and Telegraph Company. The general principles of the different methods are given in the body of the paper. Details of procedure and data are given in this appendix.

"Reading" Tests of 1914. In the first series of tests* a typical telephone circuit was set up, using artificial cable, and arranged so that different single-frequency currents could be separately introduced in the telephone receiver at the listening end of the circuit, simultaneously with the normal telephone current. By throwing a switch the listener could remove the disturbing current from the circuit and at the same time reduce the volume of sound output from the receiver, the amount of this reduction and the amount of the disturbing current being set by the reader or caller and being unknown to the listener.

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*Made in 1914 at the request of the (California) Joint Committee on Inductive Interference.

The procedure was for the caller to read over the circuit while the listener switched from the noisy condition to the quiet condition and the reverse, until the latter made his judgment as to which of the two circuits would be preferable for carrying on a conversation. This judgment was signaled to the caller, who then changed the amount of volume-reduction or the noise and the procedure was repeated. By this means, and after an average of about five different settings of reduction in volume, the listener's opinion as to what reduction equaled in effect the given amount of single-frequency disturbing current was indicated. In these tests frequencies of 60, 180, 400, 800, and 1500 cycles were used. Four strengths of disturbing current of each frequency were employed, with an average of eight observers. This involved a total of 160 determinations, each of which was based on an average of five separate judgments, making a total of 800 judgments. The results obtained in these tests indicated a linear relationship between the amount of disturbing current and the volume reduction expressed in terms of miles of cable added to the standard testing circuit. The relative interfering effects of equal currents of the different frequencies employed in these tests are shown by the points marked "X" in Fig. 4.

"Reading" Tests of 1918. The tests made in 1914 were not considered final and in 1918 they were supplemented by further studies. In these latter tests it was felt that particular regard should be paid to 1100 cycles because the resonance point of telephone receivers is in the neighborhood of this frequency. The circuit was arranged, by using an amplifier, so that receivers for a number of observers were connected to the same circuit simultaneously while someone at the calling end switched from the noisy condition to the quiet condition with reduced volume, or the reverse, every 15 seconds. After some experimenting, it was found satisfactory to have the caller read for a total of 2 minutes, thus giving four periods of 15 seconds each for each condition. At the end of this time each observer recorded his judgment as to the relative desirability of the two conditions. For each frequency of interfering current, similar comparisons were made with two callers and for five reductions in volume on the quiet circuit. In this series of tests the comparisons were made for frequencies of 180, 400, 800, 1100, and 1500 cycles with from 8 to 11 observers, involving in all over 1000 judgments. For three of these

frequencies more than one value of current was used but for the other two only one value was used. As the previous tests established a linear relationship between current strength and reduction in volume expressed in miles of standard cable, the data could all be reduced to loss per unit current. The results obtained in this series of tests, together with those obtained in 1914, are represented by curve *A* of Fig. 4. It should be noted that for the frequencies common to the two series of tests, namely 180, 400, 800, and 1500 cyles, the results check very closely.*

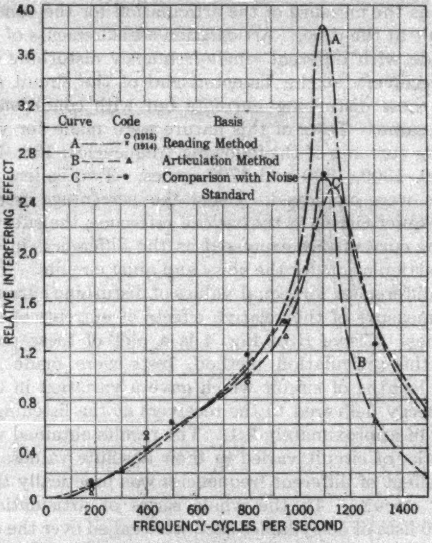

FIG. 4—RELATIVE INTERFERING EFFECT OF SINGLE FREQUENCY CURRENTS IN A TELEPHONE RECEIVER

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*The preliminary results obtained in 1914 have been presented to the Institute in the form of a curve (TRANSACTIONS, Vol. 34, page 1182, Fig. 4, Curve *C*). Within the precision of the data, this tentative curve agrees with the curve given here below 800 cycles, but is quite different in the range, between 800 and 1500 cycles, in which no observations had been made in the 1914 tests.

"Articulation" Tests. These tests were made by having a caller at the transmitting end of a typical telephone circuit speak into the transmitter lists of syllables which were phonetically balanced in groups of 50, while a number of observers, with receivers connected to the receiving end of the circuit through an amplifier, recorded the sounds which they heard. A comparison of these records with the lists of sounds called shows the percentage of syllables correctly received, and this is taken as the measure of the articulation for the condition of the circuit at the time. Articulation measurements of this sort were made with different single-frequency disturbing currents in the receivers at the listening end of the circuit and also without these disturbing currents but with conditions otherwise the same. Tests of this nature were made for values of disturbing currents of the following frequencies: 180, 400, 800, 950, 1100, 1300, 1500 and 1700 cycles. Two callers and five observers were employed. Using the percentage articulation over the quiet circuit as the basis of reference, the effects of the disturbing currents are expressed as the differences in the percentage articulation for the noisy and quiet circuits. The ratio of these differences for equal values of disturbing currents then gives a measure of the relative effects of currents of different frequencies. Curve *B* of Fig. 4 is a plot of these results.

With the articulation method, tests were made for two different lengths of circuit which gave a variation in the telephone energy delivered to the receivers at the listening end in the ratio of approximately 9:1. The results obtained with the two lengths of circuit varied in their absolute values, but the relative effect of different frequencies was practically the same for both circuits. In the whole series of articulation tests about 270 lists of 50 syllables each were called over the circuits, and about 60,000 syllables were recorded.

"Noise" Comparisons. Since the adoption of a frequency-weighting curve by the sub-committee, some tests have been made to compare the relative noise-producing powers of equal single-frequency currents in the telephone receiver with their relative interfering effects as determined by the methods just described.

In brief, these tests comprised measurements of the "noise" caused by the different single-frequency currents in a telephone receiver using the apparatus ordinarily employed by the telephone companies for measuring the magnitude of the noise on circuits subject to induction. In measuring the noise on a given circuit the observer listens with the receiver connected alternately to the noisy circuit and to a source of standard noise and adjusts the amount of the standard noise until in his opinion the noise from the actual circuit and from the standard would produce equal interference with a conversation if carried on over the circuit. By this method much is left to the judgment of the observer and the results obtained by any one observation can be only approximate. For convenience in this series of tests the method was slightly modified in that the setting of the noise standard shunt was fixed and the observer allowed to vary the magnitude of the single-frequency current until he judged the two of equal interfering effect. Nine observers were used, each making the comparison for

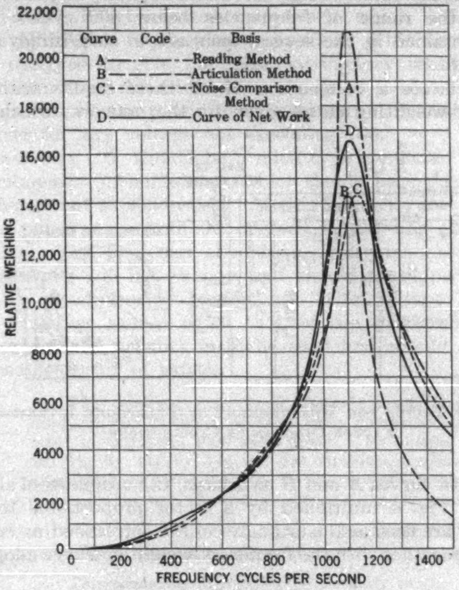

Fig. 5—Comparison of Frequency Weighting Curves

three different values of noise and for single-frequency currents of 180, 300, 500, 800, 950, 1100, 1300 and 1500 cycles per second. The results are shown by curve *C* of Fig. 4.

Comparison of Results. The different methods above described afford relative measures of the effects of different frequencies. In plotting the results thus obtained to a common scale, as shown by Fig. 4, the reduction factors were chosen so as to make the different curves coincide as nearly as practicable through the range of frequencies below 800 cycles. The results obtained in the several tests are in surprisingly good agreement.

To facilitate a comparison of all these results with the frequency-weighting curve given by the network, which was chosen with curves *A* and *B* as a basis, the ordinates of all the curves of Fig. 4 multiplied by a factor proportional to the frequency are used as the ordinates of the corresponding curves of Fig. 5, together with the frequency-weighting curve adopted.

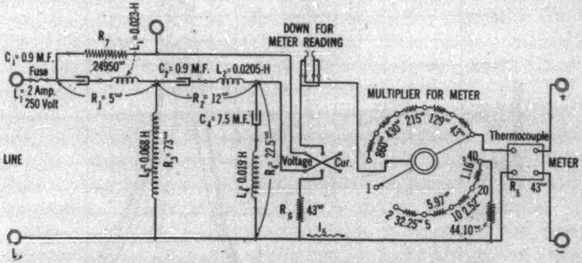
R₇
L₁=0.023-H
C₁=0.9 M.F.
Fuse
24950ω
C₂=0.9 M.F.
L₂=0.0205-H
L₁=2 Amp. 250 Volt
R₁=5ω
R₂=12ω
C₄=7.5 M.F.
DOWN FOR METER READING
MULTIPLIER FOR METER
860ω
430ω
215ω
129ω
43ω
Thermocouple
LINE
L₃=0.068 H
R₃=73ω
L₄=0.019 H
R₄=22.5ω
Voltage
Cur.
1
40
1.16ω
20
5.97ω
10 2.52ω
2 32.25ω 5
44.10ω
R₆ 43ω
I₅
R₅ 43ω
METER
+
−
L

FIG. 6—APPARATUS FOR MEASUREMENT OF TELEPHONE INTERFERENCE FACTOR-DIAGRAM OF CONNECTIONS

APPENDIX B

SPECIFICATIONS OF APPARATUS FOR MEASURING TELEPHONE INTERFERENCE FACTOR

These specifications describe in greater detail the electrical requirements for the network designed to measure the telephone-interference factor which is described in Rule 408 and illustrated in Fig. 2. In addition to the electrical requirements of the essential parts of the network, certain auxiliaries are also described which have been found to be very useful. The network including auxiliaries consists of the following parts: three condensers, four inductance coils, two resistances, one multiplier composed of a six-position dial switch and ten resistances, one vacuum thermocouple, one double-pole double throw switch, one voltmeter contact key and one fuse. These parts are connected electrically as shown in the wiring diagram of Fig. 6.

In a series of fine print notes are given data regarding a network which has been constructed and which satisfactorily meets all of the requirements specified. This network has been designed to give the most satisfactory results at 110 volts as most of the measurements will be made on the low-voltage side of the potential transformers. It may be used however for any voltage not exceeding 700.

Condensers. C_1 should be a mica type condenser having a capacitance of 0.9 microfarad, which should be correct to within 0.5 per cent at 900 cycles. At this frequency the deviation of its phase angle from 90 degrees should not exceed two minutes. The condenser should be rigidly mounted so that its constants will not change, and should be dipped in some suitable moisture-proof compound. The voltage rating of this condenser should equal that of the maximum voltage with which the network is to be used and should include a reasonable factor of safety.

This condenser consists of alternate sheets of tin-foil and mica securely fastened between brass mounting plates. It is rated to withstand 1000 volts direct. Its outside dimensions are 3¼ by 3⅝ by 3⅞ in. Its capacitance measures 0.901 microfarads at 900 cycles, and its effective resistance 0.09 ohms corresponding to a phase angle deviation of 1.7 minutes.

C_2 is a mica type condenser having the same requirements as C_1 except that the voltage rating need not exceed 500.

This condenser is constructed similarly to C_1. It is rated to withstand 500 volts direct. Its outside dimensions are 1⅛ by 3⅝ by 3⅞ in. Its capacitance measures 0.898 microfarads at 900 cycles and its effective resistance 0.09 ohms corresponding to a phase angle deviation of 1.7 minutes.

C_4 may be a paper type condenser having a capacity of 7.5 microfarads, and should be correct to within 0.5 per cent at 900 cycles and at a temperature of 25 deg. cent. The deviation of its phase angle from 90 degrees should not exceed 30 minutes at 900 cycles. The condenser should be suitably mounted in a container which will protect it from mechanical injury and which should be sealed with a suitable moisture-proof compound.

This condenser is made up of selected standard rolled paper units of nominal 2 microfarads and 0.5 microfarads sizes. Each unit is mounted in standard sheet metal containers of the following over-all dimensions: 2 microfarads: 1 25/32 by 1 21/32 by 5 in.

0.5 microfarad: 1¾ by 1 7/32 by 4⅞ in. and sealed with suitable moisture proof compound. The total capacitance measures 7.49 microfarads at 900 cycles and the effective resistance 0.12 ohms at 900 cycles and a temperature of 25 deg. cent. This effective resistance corresponds to a phase angle deviation of 18 minutes.

Inductance Coils. All of the four inductance coils should have non-metallic cores. It is quite desirable that one at least should be of the toroidal type, preferably L_3. In specifying the resistance of the coils allowance is made for the effective resistance of the associated condensers.

L_1 should have an inductance of 0.023 henry which should be correct to within 0.5 per cent at 1100 cycles, and an effective resistance of 4.9 ± 0.1 ohms at 1100 cycles. If this coil is constructed of stranded wire it may be adjusted at 900 cycles.

This coil as constructed is of the solenoid type wound with approximately 550 turns of single silk-covered stranded conductor, composed of 25 strands of No. 32 B. & S. gage black enameled copper wire, and has the following approximate dimensions:

Inside diameter ....2 7/12 in.
Outside " ....5 in.
Winding length ....1 ½ in.

The measured value of the inductance is 0.0230 henry and the effective resistance 4.9 ohms at 900 cycles.

L_2 should have an inductance of 0.0205 henry which should be correct to within 0.5 per cent and an effective resistance of 11.9 ± 0.2 ohms at 900 cycles.

This coil as constructed is of the solenoid type wound with approximately 700 turns of No. 24 B. & S. gage double cotton-covered copper wire and has the following approximate dimensions:

Inside diameter ....1 7/16 in.
Outside " ....3 in.
Winding length ....1 in.

The measured value of the inductance is 0.0204 henry and the effective resistance 11.8 ohms at 900 cycles.

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L_3 should have an inductance of 0.068 henry which should be correct to within 0.5 per cent and an effective resistance of 73.0±0.5 ohms at 900 cycles.

This coil as constructed is of the toroidal type. It is wound with approximately 4400 turns of No. 27 B. & S. gage double cotton-covered copper wire on a core of rectangular cross-section, having external and internal diameters of 2¾ in. and 1¾ in. respectively and a height of 1 in. The over-all dimensions of the coil are approximately as follows:

Outside diameter	3 5/16 in.
Axial height	1 11/16 in.

The measured value of the inductance is 0.0680 henry and the effective resistance 73.5 ohms at 900 cycles.

L_4 should have an inductance of 0.019 henry which should be correct to within 0.5 per cent and an effective resistance of 22.0±0.5 ohms at 900 cycles.

This coil as constructed is of the solenoid type wound with approximately 700 turns of No. 27 B. & S. gage double cotton-covered copper wire and has the following approximate dimensions.

Inside diameter	1 7/16 in.
Outside "	3 in.
Winding length	1 in.

The measured value of the inductance is 0.0190 henry and the effective resistance 21.9 ohms at 900 cycles.

Resistances. R_6 is provided so that for either position of the double-pole double-throw switch, the impedance of all branches of the network will remain the same. It should be non-inductively wound and have a resistance of 43 ohms which should be correct to within 0.5 ohm. It should have a continuous current carrying capacity of 0.1 ampere.

R_7 is provided as a multiplier for measuring the terminal voltage. It should have a d-c. resistance of 24,950 ohms which should be correct to within 0.5 per cent. Its inductance should not exceed 0.5 henry at 60 cycles. The unit should be designed to withstand the maximum voltage with which the network is to be used, and to dissipate 20 watts intermittently.

This resistance as constructed consists of approximately 2360 feet of No. 37 B. & S. gage double silk covered manganin wire wound in two sections on a wooden spool. The over-all dimensions of the completed coil are as follows:

Outside diameter	1½ in.
Length	4⅛ in.
Winding depth	½ in.

The resistance and inductance of this unit as measured are 24,960 ohms and 0.5 henry respectively.

Multiplier for Meter. This multiplier should consist of suitable shunt and series resistances so arranged that the input resistance will be 43.0±0.2 ohms for all settings and the ratio of its input to output current variable by definite steps so as to give satisfactory meter readings for any current from 0.001 to 0.1 ampere. The actual ratios of input to output currents should not differ by more than 1 per cent from the nominal values marked on the multiplier.

> The multiplier as constructed consists of a six-position dial switch, together with 10 non-inductive units having the following resistances; 860, 430, 215, 129, 43, 32.25, 5.97, 2.52, 1.16, and 44.1 ohms. These should be connected as shown in the wiring diagram of Fig. 6.

Meter. The measuring instrument used with this network should consist of a non-inductive meter having a resistance of 43.0±0.2 ohms at all frequencies and all current values. It should measure the effective value of currents having any wave-shape and should, together with the multiplier give satisfactory readings for all currents from 0.001 to 0.1 ampere.

> The measuring instrument used consists of a vacuum thermocouple whose heater is composed of material having a very low temperature coefficient, and which has a resistance of 43.0 ohms. The thermocouple is quite substantial in construction and should be able to withstand without a material change in calibration a heater current three times as large as that which gives a full scale deflection on the meter. The meter used with the thermocouple is a d-c. milliammeter having a sensitiveness which will give, with the thermocouple the current range specified.

Switches. The double-pole double-throw switch shown in Fig. 6 should be of such a design that there will be very little possibility of appreciable contact resistance developing as a result of extensive use. The voltmeter contact key should be inserted as shown in Fig. 6. This key will add materially to the protection of the thermocouple in reducing the possibility of overheating due to excessive currents.

Fuse. The fuse should be designed to operate on a voltage equal to the maximum for which the meter is to be used and with a sustained current of not over 2 amperes.

> The fuse as used with the instrument has the following rating:
>
> 250 volts
> 2 amperes.
>
> It is of the cartridge type and is mounted inside the box. When the network is used for a voltage higher than that of the

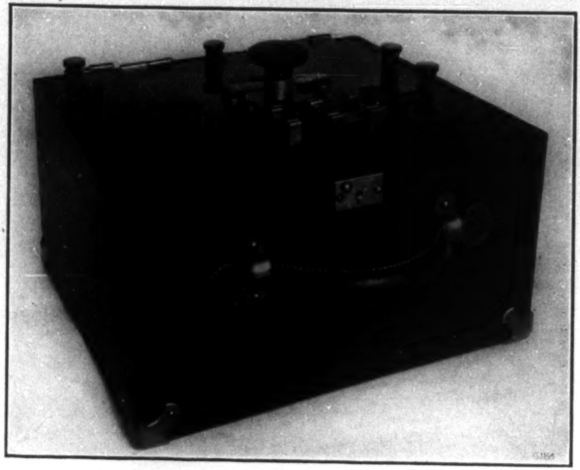

FIG. 8

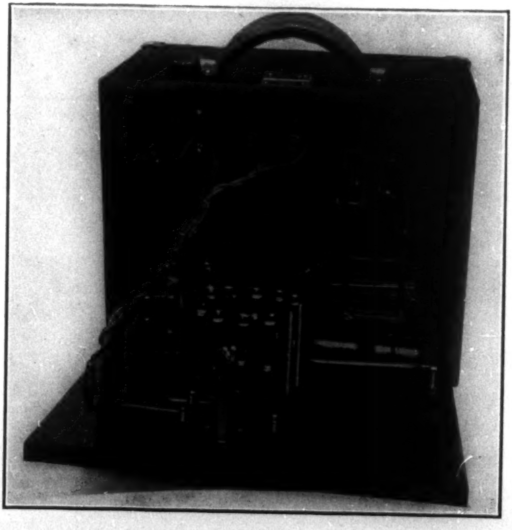

FIG. 9

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fuse rating, this fuse should be short-circuited and an external fuse of the proper size used.

Binding Post R. This terminal is provided so that R_7 may be shunted by a resistance connected between R and L_1. This permits the measurement of a lower terminal voltage than would otherwise be possible, which is particularly desirable in calibrating the network. In addition, with the switch in the voltage position, the posts R and L_G provide terminals directly connected to the multiplier. This arrangement is very useful in calibrating the multiplier and the thermocouple.

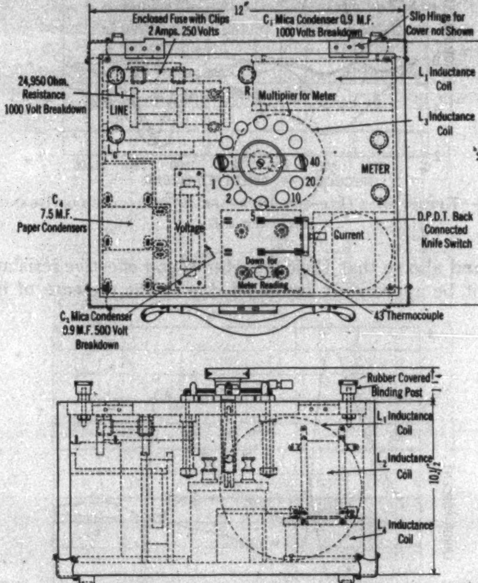
12"
Enclosed Fuse with Clips 2 Amps. 250 Volts
C1 Mica Condenser 0.9 M.F. 1000 Volts Breakdown
Slip Hinge for Cover not Shown
24,950 Ohm. Resistance 1000 Volt Breakdown
L1 Inductance Coil
R
Multiplier for Meter
LINE
L3 Inductance Coil
LG
40
1
20
2
10
5
METER
10"
C4 7.5 M.F. Paper Condensers
Voltage
Current
D.P.D.T. Back Connected Knife Switch
Down for Meter Reading
C2 Mica Condenser 0.9 M.F. 500 Volt Breakdown
43 Thermocouple
Rubber Covered Binding Post
L1 Inductance Coil
L2 Inductance Coil
10 1/2"
L4 Inductance Coil

FIG. 7—PORTABLE APPARATUS FOR MEASUREMENT OF TELEPHONE INTERFERENCE FACTOR

Assembly. The parts should be securely mounted in a suitable box which should be as small as possible to make the instrument convenient for field use. All parts should be as accessible as possible. The coils should be arranged so that there is practically no mutual inductance between any two of them and also so that their inductance and effective resistance will not be appreciably increased due to the presence of iron or other metal in their magnetic fields. The wiring should be well insulated and arranged as simply as possible.

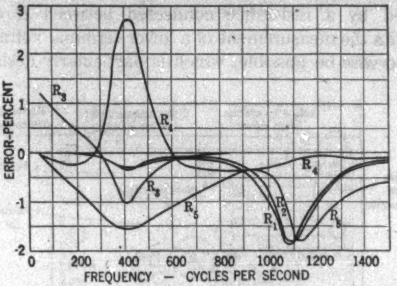

FIG. 10—EFFECT OF A ONE-OHM INCREASE IN RESISTANCE OF BRANCHES OF NETWORK

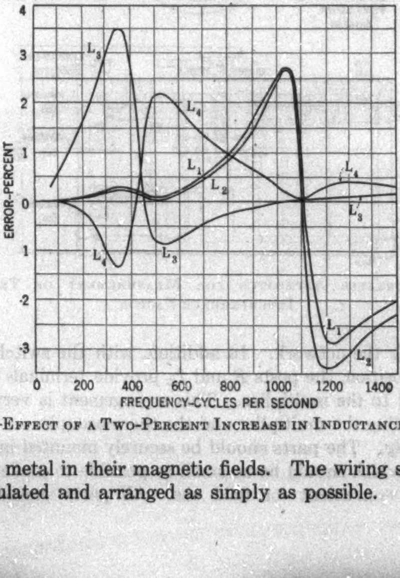

FIG. 11—EFFECT OF A TWO-PERCENT INCREASE IN INDUCTANCE OF COILS

In Fig. 7 are given working drawings of the apparatus as constructed showing the general arrangement of the parts. All switches are mounted on a hinged top which may be easily lifted to facilitate the inspection of the various parts. The solenoid-type coils are mounted with their axes mutually perpendicular so as to reduce mutual inductance between them. The fourth coil is of the toroidal type which has a negligible stray field. All coils are kept as far as possible from the condensers to reduce any effect which might be produced by their mounting plates or containers. Figs. 8 and 9 are from photographs of the completed apparatus. Its weight is 27 pounds.

Calibration. The network should be calibrated by connecting it to a source of single-frequency alternating current of known and variable frequency. The calibration of the network should include a curve showing the ratio of the current input to the multiplier to the terminal voltage for all frequencies from 0 to 1500 cycles which should agree with the theoretical curve shown in Fig. 1 and described in Section 408 of the Standardization Rules to within 5 per cent at all frequencies. In addition to this frequency-weighting curve the calibration curve of the thermocouple and meter should be given in case the combination is not direct reading. Care should be taken that all higher harmonics are practically eliminated from the testing voltage by the use of filters.

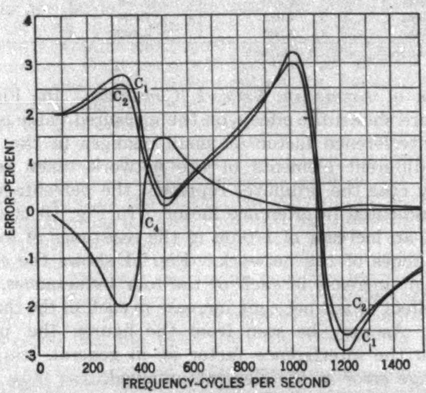

Fig. 12—Effect of a Two-Per Cent Increase in Capacitance of Condensers

The following table shows the calibration of the network constructed as described above, together with a comparison with thoretical values.

RATIO OF CURRENT TO TERMINAL VOLTAGE.
MICROAMPERES PER VOLT

Frequency	Theoretical	Actual*	Error per cent.
200	147.8	142.2	3.6
400	998.1	1,010	0.3
600	2,109	2,117	0.4
800	4,143	4,160	0.4
900	6,264	6,340	1.2
1,000	10,530	10,630	0.9
1,100	16,490	16,670	1.1
1,200	12,400	12,350	0.4
1,300	7,983	7,960	0.2
1,500	4,526	4,440	1.8

*At temperature of 21.0 deg. cent.

Effects of Errors in Network Constants. In Fig. 10, 11 and 12 are shown the effects on the measured value of the telephone-interference factor of small changes in the constants of the different elements of the network taken separately. In every case the ordinates represent the percentage increase in the indicated interference factor. In Fig. 10 is shown the effect of an increase of 1 ohm in the resistance of each of the five branches of the network. Fig. 11 shows the effect of a 2 per cent increase in each of the four inductances, and Fig. 12 the effect of a 2 per cent increase in each of the three capacitances. As may be seen from the figures the percentage effect on the interference factor is, in general, less than the percentage error in the element involved.

DISCUSSION ON "THE GENERAL EQUATION OF THE ELECTRIC CIRCUIT-III" (STEINMETZ) AND "REVIEW OF WORK OF SUB-COMMITTEE ON WAVE SHAPE STANDARD OF THE STANDARDS COMMITTEE" (OSBORNE), NEW YORK, N. Y., FEBRUARY 20, 1919.

C. F. Scott: There are certain things in circuits that we want, and as long as we get them and are satisfied with them, well and good, but we often get things we do not want. At first, they are very troublesome. They interfere with operation. Presently, theoretical study as it advances takes up these indefinite and unknown things, and finally it utilizes them and in many cases, a new order of electrical application arises. From the direct current we passed to the alternating current, and from ordinary frequencies to the higher frequencies, each opening up new fields. And the man who gives the advanced theory deals with things which are away beyond present practise and which explains the difficulties and the things not understood, announces what presently becomes the ground work of future text books and the foundation for further advance. It is a paper of this type which Dr. Steinmetz has given us.

Charles R. Underhill: I wish to supplement Prof. Scott's remarks. A few years ago I heard Dr. Milliken state that there was a gap between physics and electrical engineering which needed to be bridged. The physicists give us the so-called fundamental laws, or the theoretical information upon which electrical engineering is based, but this information often has to be interpreted or simplified so that it can be expressed in terms which the electrical engineer can apply to practical design.

For instance, Maxwell gave us an equation for the mechanical force of an electromagnet, but it could not be used by engineers until Dr. S. P. Thompson interpreted it in a form which engineers could use, but even then the results only were approximate.

About eight years ago Dr. Steinmetz gave us a general equation for the electromagnet which is more comprehensive than Maxwell's and which holds in every possible case.

Here, at least, is a case where the engineer has explained a physical phenomenon in a more comprehensive manner than the physicist, and the oftener that physical phenomena are studied by engineers and brought down to engineering standards, the greater will be the benefit to engineers.

A. E. Kennelly: Dr. Steinmetz's paper largely employs logarithmic scales in the illustrations, this is often a great advantage. The method should be aided by standardizing its nomenclature. If, in dealing with any physical magnitude expressed in numbers, we call the common logarithms of those numbers the orders of their magnitude, it would be very helpful in description. Thus a frequency of one million cycles per second would be a frequency of the sixth order. A frequency of 1000 would be of the third order. A frequency of 60 would be of the first order, or more precisely, of the 1.78th order. Again, a precision of the fourth order in measurements of any kind would be to one part in ten thousand.

The paper is of great value in clearly indicating the physical nature and essential relations of radiation resistance. It is perhaps the first paper to set the matter clearly from an engineering point of view. In particular, the paper brings the subject into the fields of electric lighting, power distribution and communication; whereas hitherto, radiation resistance has been regarded as the exclusive property of radio engineering. The paper shows that every pair of wires, such as the going and returning conductor of a single-phase alternating-current circuit inherently possesses radiation resistance; but that in the steady state, this radiation resistance is negligible at ordinary separating distances between the wires, until the frequency is very high. From this point of view, the paper should be widely read and studied.

There is a discrepancy on page 207 which, if its meaning is not misunderstood, should be corrected. Reference is there made to the skin-effect extra resistance of copper conductors of different shapes of cross section. It is intimated that the high-frequency linear resistance of all copper conductors having the same periphery will be the same. This we know is far from being the case. The linear resistance of a copper strap, say 2 mm. thick and 30 mm. wide, is very much greater than that of a copper tube or hollow cylinder of 1 mm. wall thickness and 60 mm. in periphery. This great difference is due to edge effect. The edge effect in many forms of copper conductors gives them much more linear resistance at high frequencies, owing to distorted current-density distribution, than does the simple skin effect.

The paper throws great light upon the rapidly increasing attenuation of very high-frequency or steep-front waves. This difficult subject badly needed elucidation, and we are all indebted to the paper for a relatively easy explanation of the shearing backward of a steep slope, as a vertically fronted wave runs along a conductor.

Dr. C. P. Steinmetz: I am very much gratified to have Dr. Kennelly come out so strongly in favor of the geometric scale, the scale of powers of 10, or logarithmic scale. I have urged this scale for a number of years as the most rational.

It is the scale of one of the most important and oldest human activities,—music. The musical scale is the geometric scale, for the reason that the perception of our senses follows this scale, that is, equal differences in the intensity of perception of any of our senses correspond to equal intervals on the geometric scale.

For this reason the most successful industrial scales have been geometric, as, the Brown & Sharp wire gauge, in which each successive number differs from the preceding one by a constant proportion. This is difficult to do in a merely numerical scale and herein lies the weakness which has always handicapped the merely numerical scales in gages.

As to the question of Dr. Kennelly regarding the skin effect of flat ribbon and round conductor, I refer to this in the beginning of Appendix A, where I say that there is an intermediate range between no skin effect and complete skin effect in which it is difficult to calculate it, but compared with the total range of frequencies of importance in our investigation, this range where there is still complete penetration but a decrease of current density towards the interior, is so small (less than a unit in the logarithmic scale), as to be negligible in its effect on the result, and we can therefore change abruptly from the equation of constant current density, at low frequency, to that of complete skin effect, at high frequency.

J. B. Whitehead: The first question on the Osborne paper which I would like to ask relates to the matter of noise. Mr. Osborne suggests that by the use of telephone receivers noise may be measured quantitatively. Can noise be measured quantitatively, and independently of the frequency, by means of a telephone receiver?

The other question that arises in my mind is with reference to the lower frequencies. For example, the third harmonic of a 25-cycle circuit (75 cycles), is it possible by means of the telephone receiver to measure a frequency as low as that, so that it may be applied in the form of factor that is suggested? And if not, does it not mean that the method will not take account of wave shape variations in the range mentioned?

W. J. Foster: The part of Mr. Osborne's paper that is of special interest to me is Table I. Would the author be willing to tell me whether Nos. 9, 24, 31 and 34 belong to the same company,—a large public utility company. I suspect they do, from the ratings of the machines. If so, it may be interesting to you to know that Nos. 31 and 34 which have a very low telephone interference factor, as determined by this instrument, are machines of very poor potential waves. They are machines which would have an interference factor higher than any given in the table, providing that the weighting was in the neighborhood of about 300 instead of 1140. Those machines would not come within a ten per cent deviation from a sine wave. On the other hand, at the lower end, of the table where the interference factor is 400 and higher I notice a large machine or two that to the eye have good potential waves as they appear on the oscillogram. The trouble, is, that these particular machines were built before any agitation had come up regarding telephone interference, before we knew what periodicities to avoid. The next to the last machine in the table, 9000 kv-a., 720 rev. per min., is a generator that has three slots per phase per pole, which, is a number we must steer clear of in the future in sixty-cycle generators unless we can guard against the slot frequency and neutralize its effect by introducing corrective features into the design.

I am curious to know, and I should like Mr. Osborne to tell us in closing, whether he has come to any conclusion in his own mind, as to what will be satisfactory as far as interference with the telephone is concerned, in the matter of the interference factor, whether it should be placed at 100 or 200. What would he suggest?

In looking at the matter from the standpoint of the designer, I see no reason why in any particular case in the design of a new generator it is not possible to attain a very low factor for the telephone interference factor. At the same time there have been many machines, such as those that appear here at the beginning of the table, with the low telephone interference factors, that have very poor potential waves, and have been in service for many years, and so far as I know they have never caused trouble of any kind. Consequently we want to be a little bit careful, when we establish our telephone interference factors, that some other interests will not come forward and prove a case of interference from some other periodicities than those which interest the telephone people.

John B. Taylor: Mr. Osborne's paper is a fair and open attempt to solve the problem in which we have all been interested for some time, and it is one of the few occasions on which a practical telephone man has stated these technical matters in a way to give information and to invite further discussion.

More specifically, as relates to the Standardization Rules themselves, as they appear in the new edition, I feel that there are points here which are not as they should be. In the first place we have an Interference Factor defined in Sec. 17a, as quoted in the first part of the paper, which states virtually that a sine wave will have an interference factor of zero, and then under that definition of interference factor, we have a Telephone Interference Factor, Sec. 407, which is something altogether different. It is not properly "Wave Shape" though under that heading. The essence of the T. I. F. is not one of wave shape, it is a matter of frequency, pure and simple, and if you read the paper carefully you will find where the matter is explained, not on the basis of shape, but on the basis of frequency.

To make this clear, under the rule referred to, a sine wave of 500 cycles would have a zero interference factor. This wave according to the table would have a telephone interference factor of, in round figures, about 1500. A 1000-cycle perfectly good sine wave would be listed as having no interference factor at all, because it is a sine wave, and yet its telephone interference factor would be 10,000.

Furthermore, in the presentation of the paper there is, I think, confusion all the way through between harmonics and what should be properly frequency. I will not attempt to point out all these spots, but merely cite an instance which occurs at the middle of page 273. The data are tabulated with reference to harmonics, but with nothing to tie them to any frequency. I presume we will find it is based on 60-cycles. The rules should be either given different names or worked differently, because sooner or later there will come up confusion over what is not wave shape but frequency.

From my talks with power transmission engineers, I know they are glad to do what they can to reduce interference, but most of them are not trained telephone men and they feel that the situation is advancing over their heads, so to speak. A better feeling can be brought about and a better working program arranged if in presenting a paper like this or similar papers the telephone data are presented a little more completely and clearly.

For example, the curves of Fig. 4 present test results on the sensibility of telephone receiver to currents of different frequencies and these curves are the basis of the proposed T. I. F. But though the strength of the current was doubtless recorded and the observations made in terms of miles of standard câble, the curves show relative values only and the strength of current employed is not given. If these results are offered as foundation for formal Standardization Rules, numerical values should be given so that the observations may be verified or compared with results of similar investigations.

Attention may be called to the fact that the proposed telephone interference factor rests not on the observed relation between disturbing current of a given value in the telephone circuit and corresponding noise, but on an assumption that induced currents in the telephone apparatus will be greater in direct proportion as the frequency of the harmonics in the the pressure wave at the terminals of the generator are higher.

On this assumption there is left out of consideration the effect of reactance, as the high-frequency ripple must pass through generator, protective reactance, step-up transformer and line; it will be soaked up to some extent by the capacity of the cables and lines; then it must be transferred by induction to the telephone line; the highest frequencies will again be soaked up by capacity of telephone line, and damped by reactance of telephone line and apparatus. Apparently the telephone interference factor curve is not based directly on the observations, but frequency values are all increased in proportion to their frequency, on top of what the tests indicated. Is there justification for increasing the observed values in direct proportion to the frequency employed?

Jesse J. Linebaugh: Mr. Osborne's paper is the most concise expression thus far in the attempt to obtain the quantities known as interference factor and telephone interference factor, but I believe if Mr. Osborne would elaborate on one or two points it will lead to a clearer appreciation of just what the telephone interference factor is supposed to cover. In Appendix A under *Noise Comparisons* Mr. Osborne refers to the "standard noise." Standard noise must have some definition of value as concerns the telephone receiver, and I think if Mr. Osborne would kindly define the standard noise, or give more information as to what makes this a standard noise, that it would help the ordinary engineer to gain a clearer appreciation of what the telephone interference factor is supposed to cover.

In Fig. 5 certain weightings also have been given. I would like to ask Mr. Osborne if he cannot in a few words show how these weightings can be transferred back or referred to noises in the telephone

E. E. F. Creighton: Since this is a report from a subcommittee of the Standards Committee, I wish to refer to the use of the term "open wire" or "open wiring," meaning the wires placed apart. I have always found it difficult to express that condition by the use of a single word. I attempted the use at one time of "proximate conductors" and "nonproximate conductors," but it seems to me if we are obliged to use a compound word in describing this condition, that if we say "near-spaced" and "widely spaced" wires we will avoid the confusion and misunderstanding that will naturally come from calling it "open wire" or "open wiring."

Looking at this problem from the standpoint of electrical protection, I think there are certain parts of the problem that must be solved by each side. In the matter of continuous interference between the power circuit and the telephone circuit it would seem that sooner or later some sort of a shunting condition could be created on the power circuit which would short-circuit the high frequencies near the station and absorb them to a sufficient degree to bring the interference to a workable value in the telephone circuits which parallel the power circuits.

On the other hand there is the problem that seems to have no direct solution at the present time, the question of interference during transient conditions in a lightning storm. It is one thing to have a few little harmless harmonics in the circuit, so far as the power circuit is concerned, a very harmless quantity in the circuit, and it is quite a different thing to have hundreds of thousands of kilovolt-amperes shunted to ground and returned by the earth. That must necessarily throw out the electro-magnetic waves to a very great distance and the telephone circuit must be affected during that period.

Now, on the power side I think it is going to be possible to limit the duration of these disturbances to about one-tenth of a second.

I should like to ask Mr. Osborne what has been done on the telephone side to develop some sort of a protective device which will absorb this relatively great amount of energy during the period of transient troubles on the line? It seems to me that something can be done to produce a protective device for the telephone circuits which for the brief period of one tenth of a second, would neutralize this condition. It seems to me that would be a fairly definite straightforward problem.

Dr. C. P. Steinmetz: This paper gives a very interesting and valuable investigation of the effect of the wave shape, or rather its constituent harmonics, in telephone interferences, but, as pointed out by the author, it is really not a wave-shape standard, and has practically nothing to do with wave-shape standards.

To begin with, it refers only to one feature, and one which, however important it may be, is really an extraneous feature. It has nothing to do with the purposes for which the alternating wave was created, or with its usefulness, or the troubles which it may create,—in short, it is not a wave-shape standard at all, to be considered by the electrical power engineer who desires to accomplish certain results with the greatest efficiency and the least trouble in his circuits.

It was stated that the old standard of less than 10 per cent deviation from sine shape has proven satisfactory. I beg to disagree with that, and I believe that the reason why no more trouble occurs is rather due to the development of the industry, which required very much larger machines, where designing conditions have led to a more complete sine shape by the use of distributed and fractional pitch windings, and so have eliminated inherently, without intention and without specification, any very great deviation from sine shape.

This I believe has saved the situation and in power engineering there seems no immediate need in most cases of wave-shape standards. It almost begins to look as if power engineering is not yet in the position to consider a wave-shape standard. Such a standard must deal, not with one particular feature, but with all features which come into consideration.

We might very quickly choose a standard by considering one feature only, specifying for example, that the maximum value of the actual wave must not exceed that of the equivalent sine wave by more than x per cent. This would mean that the insulation stresses produced by such wave would not be greater than those of the sine wave by more than an increase of voltage of x per cent.

Or, we may specify that the mean value of the actual wave must not exceed that of the equivalent sine wave by more than x per cent. This means that the energy losses in the magnetic circuit, excited by waves of this standard, would not exceed those of the sine wave by more than 1.6 x per cent.

Or, we may specify that the amplitude, in per cent, of the third harmonic and all its multiples must not be more than $3x$

times the percentage of internal reactance of the circuit, which would mean that the triple frequency circulating current in the system would not be more than $1/x$ of the full-load current.

Or, we may make specifications that the generator circuit must not be capable of resonating with frequencies above 10 kilo-cycles, and various other specifications which would only be of importance with regard to one particular application or use of the alternating circuit, but none of these is a complete wave-shape standard.

It may be that instead of considering a general wave-shape standard, for the next few years anyhow, the sub-committee of the Standards Committee which is dealing with these matters might be of more utility to the industry by studying, individually, the various conditions imposed by insulation, by energy losses in alternating circuits, by resonance, by triple frequency, and clear up each of these features in the same way and with the same completeness as they have cleared up that one feature of the telephone interference in the paper presented at this time, and so, instead of dealing further with the general wave-shape specification, give us information on the liability to troubles owing to various conditions, and the method of avoiding them, and the limitations which might be imposed in the individual conditions to overcome them.

Now, after all, when you come back to the problem, we must not forget that while we consider the sine wave as the standard, this standardization of the sine wave is, to a very much larger extent than we usually appreciate, based on sentiment and not on actual experience. There is almost no feature in which the sine wave is the preferred wave. It seems, and at least sentiment tells us so, that all around, it is the best wave, but there is more sentiment in that than we usually concede to ourselves.

A. E. Kennelly: This paper seems to proclaim two things. First it calls for non-interference with the frequency of 1100 cycles per second. A glance at Figs. 4 and 5 shows that the observed and experimental telephonic disturbance curve appears to have a sharp maximum near to this frequency. For this reason the telephone engineer requests complete relinquishment of that frequency in his favor, and that if any harmonics of this frequency are accidentally generated in neighboring power systems, that they shall be ruthlessly suppressed. Although there is a sharp peak in the interference curve near that frequency; yet the curve is not a mere vertical belt at this region. It descends in each direction to frequencies of the first or second order on the low side, and to frequencies of the 3.6th order on the high side; so that, in asking for free right of way on that peak frequency, the telephone engineers make a plea for operating as far away from the peak in both directions as may be practicable.

Secondly, and in accordance with the same suggestion, the paper seems to indicate that the prevalent notion of telephony being evenly carried on over a wide spectrum or range of electric frequencies is inaccurate. On the contrary, Figs. 4 and 5 suggest that electric telephony is mainly carried on at one frequency—the 1100-cycle frequency here so prominently displayed,—which is near to the resonant frequency of the diaphragm in the standard telephone receiver. I should like to inquire whether the experimental curve or mean curve of Fig. 4, is not approximately a simple resonance curve of the telephone receiver diaphragm to an alternating current of constant strength, but of adjustably varied frequency. It would seem as if telephony were mainly carried on at the resonant frequency of the diaphragms. Of course, all of the frequencies in vocal tones must be reproduced, within recognizable limits, by a receiver diaphragm; but these may be due to mere overtones or changes in nodal lines. The electric frequencies giving rise to these modulations of diaphragm vibration, may be very feeble relative to the dominant frequency of mechanical resonance, so that they may be swamped by the amplitude of resonant frequency, in the amplitude frequency curve.

If these ideas are even only roughly correct, they mean that the telephone engineers are justified in coming to the general body of electrical engineers and in bespeaking reservation for their special third-order frequency with as wide a margin of reservation as may be practicable. On the other hand, their brother electrical engineers are justified in saying that they will try to keep off this reservation, but that the telephone engineers in return should try to keep within the prescribed limits, lest after some years, when the alternating-current power generators may have been carefully designed to avoid reservation, the telephone receivers may be found to have preempted some new frequency.

L. W. Chubb: I will add a word to Mr. Foster's remarks. The designing engineer today in working up larger machines has practically a new design for each large machine. Now, if he knows what frequencies give trouble, he can hold the voltage wave shapes, with very little extra trouble, to quite reasonable limits of telephone interference factor.

There is one thing to be remembered, however, that is, the manufacturing company in order to give the customer quick and good service and to use tools, dies, etc. economically, must combine standard rotor dies and pole dies in different combinations for different ratings. This necessarily puts the small and intermediate machines in a class where the combination of existing parts may cause rather high telephone interference factors. If each new rating had to have a new pole, a new armature die, corresponding windings and dampers, it would be quite difficult to manufacture the various ratings economically.

I was interested in what Dr. Steinmetz just said about the other considerations and limitations for wave-shape standard.

Mr. Taylor's criticism of the general rule of interference factor is well founded, as frequency and not shape is considered in the specific definition of Telephone Interference Factor. The other considerations and limitations of shape mentioned by Dr. Steinmetz have a definite bearing on the work of the subcommittee. He spoke of the effects of wave shape on insulation stresses and iron losses. Such things have been considered by the committee and the only practicable thing seems to be to base tests on the pure sine wave and later if necessary place limits on crest factor and form factor. A distorted wave in a grounded system especially hard on the insulation or causing low core losses sent through a star-delta transformer changes from a peaked to a flat wave, giving the opposite conditions. Similar but more pronounced effects are produced by transforming by the Scott connection and it therefore seems impracticable to specify limits of crest factor and form factor unless all attending circumstances are given.

C. A. Adams: I understand that this is merely a discussion of telephone interference factor and not broadly a discussion of wave shape. The basis of the fears expressed or implied by Mr. Foster is not such as to give any real cause for uneasiness. Open-circuit voltages only are being discussed. Fortunately, under most operating conditions, most of these higher harmonics are largely absorbed by the reactance of the machines and transformers.

The committee intends later to extend its work into the field of load conditions; but that is a task of some magnitude and will take a long time to accomplish.

L. T. Robinson: When you have a certain condition to meet, the only thing to do, in a practical way, is to do what this sub-committee did, to find out what the trouble is, what you can do to find a cure for it, and leave the more complicated questions to be taken care of later on.

Apparently there is only one earth for us both to use and power must be transmitted and communication must be successfully undertaken. Therefore, we have to find some way to get along, and I think what has been started is a very definite step which will lead to the right end.

As President Adams said, there must necessarily be a good deal more to do, there is interference with telephone systems from direct current and other things in connection with power systems, aside from the machines, so that ultimately we must come to some basis where we can determine in advance what the disturbing factors of the system, as a whole, will be. That is, we can specify the machinery, and perhaps we can specify other things to avoid difficulties, and we finally, I hope can get into a position where we can predetermine what the disturbing effect of the whole system will be and overcome it more or less completely.

During the progress of this work, and as it nears completion, it will be undoubtedly possible to expand the general subject until we have more complete standards; that is, I feel that before we get through we should reach out and take in the telephone side more, that is, take into account the thing that is disturbed as well as the thing that disturbs. Without knowing what form it will finally take, I can only outline at this time what seems to be an ideal condition, and that is that we have some fairly definite idea of what the disturbing effect of a power line is, expressed in terms of this telephone interference factor, as applying to the power system as a whole, rather than to the machines alone, and to correspond with that, some form of standard for the telephone system, as well.

We are becoming more familiar with the difficulties involved in the general question of inductive interference and when we have progressed further with the work and thought enough about the general solution in the light of results obtained I feel that there should be no difficulty in agreeing on reasonable requests that should be imposed on both the power and communication circuits.

With regard to the wave shape standard for alternating-current generators I think the speakers have justified the position that the sub-committee took, that we should not do anything with this standard just now, and that consideration of the subject should be deferred to a later date. There is little to indicate that any change is needed.

C. F. Harding: (communicated after adjournment.) Although the measurement proposed in Mr. Osborne's paper may leave much to be desired as a general standard of wave shape, because of the fact that it confines the standardization to but one of the several effects of distorted wave form operating under no-load conditions only, the data which might be derived from the interference tests made throughout the country with apparatus similar to that proposed might prove very enlightening.

It occurred to the writer that the various University Engineering Experiment Stations, or even the A. I. E. E. Branches might be interested in making the desired tests in their respective localities, thereby accumulating a vast amount of valuable data upon the interference problem. In the case of Institute Branches, the student members, under the direction of their instructors for thesis work or as a laboratory test, might investigate conditions in several neighboring central stations. If this were done however, it would be advisable to accumulate a larger amount of data than that indicated in the paper in order that interference factors under various conditions of load might be determined. Little advance can be made if standards are confined to no-load conditions, when the disturbing wave is ordinarily that resulting from a loaded generator.

Referring now to the design of the equivalent branched circuit proposed in the paper, it is difficult to understand why the accuracy of the apparatus, Fig. 6, is limited by the use of a paper condenser at C_4. Some recent tests carried on at Purdue University upon commercial paper condensers indicate not only a variation in capacity in different condensers of the same make and rating, but a variation of from 10 to 15 per cent with different methods of measurement. The capacity was also found to vary with change of frequency and temperature. Reference to Circular No. 36 of the Bureau of Standards indicates paper condenser variation in capacity of 5 per cent, phase angle of one degree and power factor over a ratio of six to one with frequencies between 100 and 2900 cycles.

The paper further points out that the error in the final factors varies approximately with the error in the capacity C_4, at critical frequencies. To substitute a paper condenser for one of mica under these conditions for variable frequency work would seem to be unduly sacrificing accuracy for a slight gain in the economy and compactness of the instrument.

Harold S. Osborne: Professor Whitehead suggests that the proposed method is at fault because currents of low frequency, such as 75 cycles, cannot well be measured with a telephone receiver. However, with this method the current is measured by a vacuum thermocouple, not by a receiver. With the proposed network very little current of 75-cycle frequency will be caused to flow through the measuring instrument, unless the impressed voltage is relatively large. This is because 75-cycle voltage has very little effect in making telephone circuits noisy, compared with higher frequencies. The deflection of the meter is, for each frequency, proportional to the noise effect.

Professor Whitehead asks whether noise can be measured quantitatively by means of a telephone receiver. If he means measurement of the energy content and wave shape of acoustic waves, rather than their psychological or physiological effect, the answer is, "Yes." The procedure consists in using the receiver as a transmitter and measuring the electrical energy delivered by it. In an ordinary telephone receiver, the response varies greatly with frequency. The response at each frequency can be accurately determined, however, and the instrument is, therefore, satisfactory for measuring single-frequency sustained tones. With the complex wave shapes which correspond to what are commonly called "noises," it is much more satisfactory to use telephone apparatus having an approximately uniform response for the range of frequency under investigation. One instrument which is very valuable for work of this sort is described in a paper published in the "*Physical Review,*" July, 1917, by E. C. Wente, entitled, A" Condenser Transmitter as a Uniformly Sensitive Instrument for the Absolute Measurement of Sound Intensity."

Mr. Foster asks whether in Table I machines numbered 9, 24, 31 and 34 do not belong to the same company. I can check the fact that machines Nos. 9, 31 and 34 do belong to the same

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company. Machine No. 24, however, was tested in the factory, and I have no record of its destination.

Mr. Foster asks whether the author has come to any conclusion regarding satisfactory values of telephone interference factor. He has not. There does not seem any probability that it will prove economical to establish limiting figures low enough to eliminate all interference with telephone circuits, even in severe cases of parallelism. In so far as the telephone interference factor can be reduced, it will reduce the number of cases of interference and the amount of outside plant expense necessary to avoid interference in cases where telephone and power lines must parallel each other. The limiting values should be set at the point giving an economical balance between cost of generators and expense in outside plant of power and telephone companies, insofar as it is practicable to determine the balance point. It is the effort of the sub-committee now to gather information with this in view.

Mr. Taylor is correct in assuming that the table on page 273 is based on a 60-cycle frequency. He is mistaken, however, in stating that the data of Fig. 4 are expressed in miles of cable. As marked on the figure, these data are expressed in relative amount of interfering effect of the same amount of current of different frequencies. The method by which the interfering effect was measured is described for each curve. Mr. Taylor suggests that in determining the shape of the weighted curve factors should have been used other than those selected by the Committee. This matter was carefully considered by the Committee, and a discussion of various factors included and omitted is given on pages 268 and 269 of the paper. It is, of course, erroneous to think of 1100-cycle current as being soaked up by the capacity of the telephone line and damped by the reactance of the telephone apparatus. Eleven hundred cycles is in the most important part of the band of frequencies which must be transmitted by telephone circuits and apparatus, and they are designed to be as efficient as possible for this frequency and for the entire band between 300 and 2000 or more.

Mr. Linebaugh asks for a definition of standard noise. It is pointed out in the paper that this term is used to refer simply to an arbitrary unit of noise which is in use by the telephone companies. This unit is the noise caused in a telephone receiver by a given amount of current of a given wave shape. The wave shape is made very complex so that the noise may as nearly as possible resemble that generally heard over telephone circuits. Its complete description would require details of wave shape in which I suppose Mr. Linebaugh is not here interested. Mr. Linebaugh's question about the relation between weightings and noise is answered above in connection with Mr. Taylor's discussion.

It would seem that Mr. Creighton has misinterpreted the meaning intended to be given to the term, "open wire" in this paper. It does not refer to proximity or lack of proximity between the wires, but is a telephone term for non-insulated wires strung on insulators, as contrasted with insulated wires in a cable.

Mr. Creighton asks what has been done to develop a protective device for the telephone lines, which will absorb the relatively great amount of energy transferred to the telephone lines during the period of transient trouble from the power service. A device of this sort must, of course, differentiate between the induced currents and the telephone voice currents, which are themselves transient and very rapidly varying and which must not be absorbed by the device. The basis for differentiating is voltage; in other words, the device must absorb high voltages and not affect low voltages. Very careful study has been given to this problem, and there have long been in use on telephone lines various forms of "open space cutout" which perform this function very satisfactorily. When high voltages are produced these devices ground the line. The difficulty remains that in so doing, they necessarily throw the line out of service for the time being. Furthermore, at the instant of grounding the line there is often produced a sharp rush of current through the telephone receiver, which produces very severe acoustic shocks to the operator or subscriber using the telephone. There are numerous cases on record of very serious injuries to people by these acoustic shocks. The problem of avoiding them under such circumstances is still unsolved.

Dr. Steinmetz gives an interesting list of a number of important effects dependent upon wave shape. The fact that the telephone interference factor is concerned with a single effect of wave shape rather than all effects of wave shapes does not in the opinion of the Committee invalidate its use as a wave shape standard. Of course, it is not a complete standard. Even in the present rules one other standard, namely the deviation factor, is provided and I think that a complete consideration of the matter would require the establishment of a series of standards, as suggested by Dr. Steinmetz. The telephone interference factor does, of course, depend upon both wave shape and frequency, but this would be true of many wave shape standards set up on the basis of particular effects of wave shapes.

I believe there is something much stronger than sentiment which leads to the adoption of the sine wave as the standard of wave shape. In a circuit containing appreciable inductance and capacity, the sine wave is the only alternating current wave which does not change in wave shape. A periodic wave of complex form could be analyzed into a series of triangular waves, of flat topped waves, or waves of practically any form desired. In studying the propagation of a complex wave over an electric circuit, however, it is practically necessary to resolve it into sine waves because of their property of distortionless transmission. The propagation of each of these sine wave components can be obtained by relatively very simple mathematical treatment, and their recombination at the end of the circuit gives the form, magnitude, and phase of the transmitted wave.

Professor Kennelly asks whether the curve of Fig. 4 is not approximately a simple resonance curve of the telephone diaphragm. It is not. The form of this curve is very considerably different from a curve showing the response of the telephone receiver to currents of different frequencies. This is naturally the case, because the interfering effect on telephone conversations of a disturbing alternating current is not a simple linear function of the intensity of the sound which it produces, but varies in a complicated way with the intensity and frequency of the disturbing noise. Professor Kennelly is mistaken in the impression that the form of the curves of Fig. 4 indicates that telephone conversation is carried on mainly at one frequency. If only a narrow band on each side of the 1100 cycle frequency were transmitted, telephone conversations would be absolutely unintelligible. There is no close relation between the form of this curve and the curve showing the relative importance for the clearness of speech of the transmission of currents of different frequencies.

Professor Harding's discussion of paper condensers indicates that it is important, if paper condensers are to be used in the network designed for measuring telephone interference factor, to be very careful in the selection of these condensers. The condensers used in the models which have been made up have no such variations in capacity as were noted by Professor Harding. The variation of capacity with frequency between 300 and 3000 cycles is in these condensers less than 1 per cent. The variation in capacity with temperature is less than 0.2 of 1 per cent below 75 deg. fahr. At temperatures of 100 degrees or higher, the variation of capacity becomes relatively large, so that it is important in using these condensers to avoid excessive temperatures. The purpose of using a paper condenser is, of course, simply economy and increased portability, and condensers of the type discussed above would seem to give satisfactory performance for most conditions.

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RADIO TELEPHONY

BY E. B. CRAFT AND E. H. COLPITTS

ABSTRACT OF PAPER

This paper is divided into two parts. The first part describes the development of the art of radio telephony by the American Telephone and Telegraph Company and the Western Electric Company to the accomplishment of Trans-Atlantic telephony, followed by demonstrations of the use of radio telephony between ships and of methods of connecting radio and wire telephone systems. This first part covers work principally of an engineering research nature, the most important problems being the development of systems of generation, modulation, transmission and reception of radio telephone signals.

The second part is concerned almost entirely with the work of producing radio telephone and allied apparatus for the Army and Navy in the late war. The major problem of the second part was the design of light and compact sets which could be produced rapidly and in large numbers, for the main features of the generating, modulating and receiving systems were already well understood.

Historical

RADIO telegraphy has aroused so much popular interest that the main facts of its development to a commercial stage are known to every engineer and to many laymen. A detailed history of the growth of the art is therefore unnecessary in this paper and it is only for the sake of securing the proper point of view for a description of some recent developments in radio telephony that a short sketch of the fundamental principles involved will be presented. The work of Lodge, Marconi, Braun, Fessenden, Fleming, De Forest and many others is described in books written with this professed purpose; in fact a book of considerable dimensions is required to do justice to the historical side of the subject. These remarks are made because the present paper is concerned almost entirely with the work of the Bell Telephone System in radio telephony and little attempt is made to trace to their ultimate origins all the ideas which have entered into the building of the structure as it is at present.

Electromagnetic ether waves of "radio" frequency were first intentionally produced and studied by Hertz, who was guided by the electromagnetic equations of Maxwell, constructed twenty years earlier, which predicted electromagnetic wave propagation and the identity of light and electromagnetic radiation. Hertz succeeded in reflecting, refracting and polarizing his waves and in producing interference. The waves were generated by successive disruptive discharges of a condenser formed by two halves of an extended system, or oscillator, and were therefore damped waves occurring in trains. At each discharge a damped oscillatory current flowed to and fro along the oscillator and consequently set up outside the oscillator variable electric and magnetic states which, according to Maxwell's theory, were propagated as waves to remote regions. They were detected by means of a secondary circuit consisting of a nearly complete conducting ring broken by a minute spark gap.

This was the first radio telegraph system, and although Hertz was not concerned with any immediately practical consequences of his method of producing electrical waves, others were quick to see the commercial possibilities. Passing over the very extensive body of scientific work which was started by the researches of Hertz, the next notable achievements were the discovery by Braun of the coherer for making the waves evident, and the invention by Marconi of the present form of antenna which allows the radiation of large power by long waves. These two steps undoubtedly made radio telegraphy a commercial possibility.

Because of the light which is thereby thrown upon subsequent efforts, it is worth while to examine the situation as it existed at that time. Trains of damped waves were produced by the charging of the oscillator system to a high voltage and allowing an oscillatory discharge to take place after each charge. The charges occurred with a frequency which produced audible sounds while the radiated waves were damped out in a rather small number of oscillations. As a result a graph of the wave amplitude would show a succession of violent and rapidly damped disturbances separated by many periods of no disturbance. This method of generation had two effects: first the average power radiated could be made large only by greatly increasing the amplitude of each separate train. This was limited by the voltage which could be allowed on the antenna and associated apparatus. Secondly, a damped wave differs so much from a simple harmonic wave that sharp tuning was impossible and the advantages known to attend its use could not be realized. These difficulties could both be overcome if a sustained wave generator of sufficient power were available; consequently this was the most important technical problem at this time and resulted finally in the development of the high-frequency alternator and the arc generator.

In the meantime it had been found that bodies, such as crystals, having partial unilateral conductivity, could be used in connection with a telephone receiver to make the signals audible. In this case what was heard was the note produced by the succession of wave trains, the effect of the crystals being to rectify the received current. The rectified high-frequency current varied sufficiently slowly to produce in the telephone an audible sound which continued as long as the key at the sending station was depressed. It is not necessary that the detector be completely unilateral, but only that an applied voltage shall produce a greater current when acting in one sense than when acting in the other. It was soon found that this method of reception employing a telephone receiver and rectifier or detector was far better than that using a coherer, partly because of the greater sensitiveness, but more fundamentally because reception of an audible note by ear allows greater opportunities for discriminating against disturbances.

But with the advent of "continuous" or sustained waves, such a method could not be used without modification, because the rectified current in the telephone receiver was either zero (when the sending key was open) or constant and equal to the average value of the rectified current when the key was closed. Such a flat-topped wave of telegraph frequency produces no audible sound in a telephone receiver except at the make and break. Two solutions of the problem thus presented were given. The first consists in breaking up the received wave into groups at such a rate that the group frequency is in the audible range. This was done by means of a commutator. The other solution, which is extremely elegant and useful, consists in superposing upon the received signal at the receiving station a locally generated wave of frequency slightly different from that of the received signal. Beats are thereby produced which are of audible frequency, and by rectifying this composite wave the current in the telephone is made to vary at any frequency desired. Since the local oscillator is not required to furnish an appreciable amount of power it may be small and may, in fact, be combined with the tuning circuit of the receiver. A more important advantage is that the signals are amplified by this "heterodyne" method of receiving.

This brief sketch brings us to the early attempts at radio telephony. It was realized at an early date that a crystal rectifier, or any similar detector, in combination with a telephone, would produce in the telephone currents whose amplitudes varied approximately as the amplitude of the received high-frequency wave; consequently if the amplitude of this wave could be varied at the transmitting station in accordance with speech, radio telephony would be possible. The receiving apparatus was therefore in existence and all that was needed was a means of varying the amplitude of the transmitted wave, that is, the amplitude of the current in the transmitting antenna. Now, in wire telephony, the amplitude of a direct or alternating current is so varied by means of a microphone, and this device was tried for radio telephony, the method being to insert a microphone, or a combination of them, in some part of the antenna circuit traversed by high-frequency currents. This attempt was handicapped because of the fact that microphones, even when specially designed, cannot easily be made to carry large currents and still function properly. Their use, however, suggested that it might be possible to devise somewhat similar apparatus, and as a result there appeared numerous voice-operated resistance-varying devices. Using methods of this kind considerable success was attained; thus in 1912 Vanni succeeded in telephoning 1000 kilometers.

FUNDAMENTAL PRINCIPLES

In order, however, to understand the nature of the telephone problem, a short discussion is necessary to make clear what are some of the limiting factors in the transmission and reception of speech. Consider first the simplest case of the familiar wire telephone system, which will be taken as a speech-actuated microphone, a battery and an electromagnet in series. A certain current, I, flows in this series circuit and produces a pull, proportional to the square of the magnetic field strength, and consequently proportional to I^2, upon the armature. Now let the microphone be actuated and suppose its design is so good that a sinusoidal variation in air pressure produces a sinusoidal variation in the *carrier current* I. Then the result of singing a pure note of frequency $n/2\pi$ into the microphone will be to produce a current *variation* of amplitude $k\,I$, say, so that the current in the circuit is now

$$(1 + k \sin nt)\, I$$

The force on the armature will be proportional to

$$I^2 + 2\,k\,I^2 \sin n\,t + 1/2\,k^2\,I^2 + 1/2\,k^2\,I^2 \cos 2\,n\,t$$

This is made up of a constant part, which does not sound the telephone, a part, $2k\,I^2 \sin n\,t$, which is a perfect copy of the voice, and a harmonic, $1/2\,k^2\,I^2 \cos 2\,n\,t$. The distortion is relatively less the smaller the quantity k, which is called the fraction of complete modulation because if $k = 1$, the amplitude of the carrier current is periodically reduced to zero. It is clear that to completely modulate the carrier current the resistance of the microphone would need to vary from zero to infinity. This is done in telegraph sending, since the key is opened, but it can only be approximated with a microphone. There are two bad results of this incomplete modulation: The full power of the system is not used since it is only the variable part which transmits signals, and any irregularities or ripples in the carrier current have a relatively greater effect the smaller the modulation, since they are themselves modulations. There was therefore an imperative need of a resistance or current-varying device which would produce variations comparable with those produced by a telegraph key.

In our first circuit it is not necessary that the carrier current be constant; thus suppose it is represented by $I \cos r\,t$, in which $r/2\pi$ is a high (radio) frequency and hence inaudible. Upon modulation the current becomes

$$(1 + \sin nt)\, I \cos rt = I \cos rt + 1/2\,k\,I \sin (r + n)\, t - 1/2\,kI \sin (r - n)\, t$$

which defines a frequency band from $r - n$ to $r+n$. Thus if speech is to be transmitted (which requires all frequencies to about 3000) the high-frequency system must be capable of transmitting, not only the frequency $r/2\pi$ but a band extending 3000 cycles on each side of this frequency. This fact imposes a limit upon the sharpness of tuning allowable in carrier-wave telephony.

The above modulated current produces in the receiver (or an e.m.f. of this type produces in a rectifying detector) an effect roughly proportional to its square. Thus if there is impressed at the receiver another current, $A \cos r t$, in phase with $I \cos r t$, or equivalently, if the polarization of the receiver in the telephone case is made stronger, the audible frequency terms of the desired frequency will be reinforced while those of undesirable frequency will not play so prominent a part. Consequently, the received signal will be amplified with a minimum of distortion.

It is not intended to give here the theory of the modulation and detection of carrier waves, which should properly be treated in a separate paper, but this very brief sketch has shown the importance of complete modulation, the fact that radio telephony is concerned with the transmission of a high-frequency band instead of a single high frequency, and has indicated the possibility of new methods of transmission and reception.

INTRODUCTION OF THE AUDION

In 1906 Dr. Lee De Forest brought out a vacuum tube device which he called the " audion " and which has been described in the A. I. E. E. TRANSACTIONS. This was a very sensitive detector of electric oscillations and consisted of three electrodes in an exhausted envelope. One of the electrodes could be heated to incandescence with the result that electrons were emitted by it (the Edison effect). A battery connected between this electrode, as cathode, and another as anode resulted in a convection current of electrons from one to the other. Since negative electricity only was present, current could flow in but one direction. This is so far the action of the Fleming valve which also makes use of the Edison effect, but in the audion an epoch making advance was made in that the third electrode allows us to completely *control* the strength of the electron current *without consuming appreciable energy at that electrode or in its circuit.* In other words an inappreciable amount of power applied to the third electrode, or grid, will result in large changes in power in the anode circuit. Moreover, since the electrons have no appreciable inertia, the response in the anode circuit to stimuli in the grid circuit is practically instantaneous.

Now this kind of device is exactly what is needed as a repeater in telephony. It was therefore considered by the Bell

System and it was found that, provided the device could be so improved as to be able to handle the power required for wire telephony, the audion would make an ideal amplifier. In 1912 our audion development work was started with this in view, and the new device soon became a competitor of the so-called "mechanical" repeater, which it is displacing. This development of the telephone amplifier, fortunately, required a considerable amount of research in the field of high vacua and electron emission and also a rather extensive study of the electrical characteristics of thermoionic tubes.

This had two results immediately applicable to radio telephony. First, an application of the early studies of amplifying systems and circuits and particularly a study of "singing" or freely oscillating repeater systems by Dr. G. A. Campbell, resulted in a successful form of thermoionic oscillator, suitable for all frequencies, and second, through the work of Mr. E. H. Colpitts early in 1914, there was produced a thermionic modulation system giving approximately undistorted speech. Several related forms of modulating systems were soon after developed by van der Bijl and others.

Before describing the modulator it should be remarked that when one has a perfect amplifier and a perfect modulator, the problem of radio telephony is solved, at least theoretically, for all that is necessary is to modulate the output of an oscillator circuit and then, if necessary, amplify the modulated current. Or one may first generate oscillations of large power and modulate them by means of the amplified output of the telephone transmitter. It should also be remarked that a "perfect" modulator, *i.e.* one which may produce completely modulated undistorted carrier waves, is also a perfect detector of these waves.

Modulator. In order to understand the modulator, consider Fig. 1 in which is plotted a characteristic curve, C, of the audion. This curve shows the variation of anode, plate or output current with grid-filament or input voltage. Now let the tube be connected as shown in the circuit diagram in which two voltages are impressed simultaneously upon the grid. The first of these is due to a telephone transmitter, T, and is sufficient to vary the grid voltage from zero to the value $-E$. The second is of radio frequency and smaller than that due to the transmitter. The total voltage on the grid will then be shown, as a function of time, by the corrugated sine curve plotted vertically. The characteristic curve shows us that the resulting current in the output circuit will be represented by the variably corrugated curve extending to the right. The rapidly variable part of this curve represents a completely modulated high-frequency current which may be led to an antenna or amplifier by the transformer at the right. It will be seen that the modulator acts essentially as an amplifier whose amplification is not constant but varies from zero (at the point $-E$) to a maximum at the extreme right. The object of introducing the battery e in the grid circuit is to insure that the grid will not become positive with respect to the filament, for as long as it is negative no electrons can flow to it and there will be practically no consumption of power in the grid circuit. It is, however, sometimes desirable to allow current in the grid circuit, in which case the battery may be adjusted.

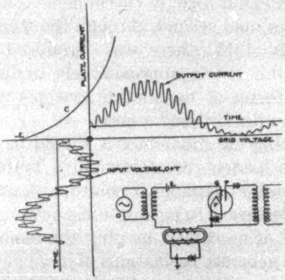

Fig. 1—Modulator Schematic

During the years 1912, 1913, 1914 a great deal of research work was carried on in connection with the development of the audion for telephony. There were many different problems involved in the manufacture of practical tubes which could handle the larger amounts of power necessary for amplifiers of telephone and radio currents. In the way of applications, laboratory apparatus was set up for amplifying telephone currents under long line conditions, a study of the detecting and rectifying properties of the tubes was made, circuits were designed for inter-connecting radio and wire telephone systems, oscillator circuits for both high and low frequencies were built, the audion was applied as a power-limiting device to protect circuits, and a number of other systems of modulation were devised.

FIRST FIELD TRIALS OF RADIO TELEPHONY

In 1914 it was decided that the apparatus and methods developed in the laboratory were sufficiently promising to warrant an attempt at Trans-Atlantic telephony.

In order that a practical demonstration of the method might be obtained an experimental station was constructed at Montauk, L. I., and a receiving station was constructed on the roof of the Dupont Building, in Wilmington, Del. A trial was made on April 4, 1915 in the presence of Professor Millikan and Colonel Reber, as well as Messrs. Carty, Gherardi, Jewett and others of the executives of the Engineering Departments of the American Telephone and Telegraph and the Western Electric Companies. The transmission was, of course, one way only. These gentlemen, therefore, witnessed the operation of the transmitting equipment at Montauk on one day and then on the following day visited the receiving station at Wilmington and listened to the incoming transmission. Wire connections between Montauk and Wilmington permitted the listeners at Wilmington to report immediately what words they had heard, or, as on the second day, it permitted the speaker at Montauk to report what he had said. The wire connection thus admitted of the verification of the experiment.

Following this demonstration the range was extended from Montauk to St. Simons Island off the coast of Georgia where a receiving antenna had been erected. For the purpose of this experiment the radio receiving circuit at St. Simons was connected to a telephone circuit leading to New York. Similarly, the radio transmitting equipment at Montauk was connected to a telephone line leading from New York. The speaker at New York talked over a wire circuit to Montauk, by wireless to St. Simons and then by wire to a listener in New York.

The method which was used in transmission was essentially that which was followed in later experiments when the Navy station at Arlington was used as a transmitting station. Prior to this time the inability of inventors to modulate large amounts of power in such a manner that the modulated current, when detected at the receiving station, could be obtained essentially free from distortion, had proven a serious limitation on the increase of range of transmission. It had been shown, however, that this limitation could be overcome by the use of the audion amplifier and modulator. The general method proposed, therefore, and the method finally developed was as follows. A small current of high-frequency was to be generated by means of a vacuum tube oscillator. This high-frequency current was to be modulated, essentially completely, by the voice current from the telephone transmitter. The resulting modulated current was to be amplified, or successively amplified, by distortionless amplifiers, of the vacuum tube type, until the energy which it represented was sufficient for transmission from an antenna over the desired distance.

For the production of a sustained high-frequency current a vacuum tube was used. With such a tube, provided its input and output circuits are coupled together and one of them contains a tuned circuit, the operation is one of successively amplifying its own output. This successive amplification will result after a moment in the development of a steady state of oscillation in which there is a definite maximum of output current. This maximum amplitude of the generated alternating current depends for its value upon the characteristics of the tube itself. An oscillation generator for producing alternating current is shown in Fig. 2. Power may be obtained from it by coupling loosely to the inductance of its circuit another inductance which in turn is connected to the point where this power is to be utilized.

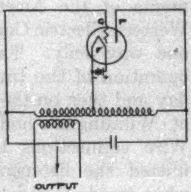

Fig. 2—Oscillation Generator Schematic

As noted above, the study of the vacuum tube had also indicated that although it was possible to use the tube as a distortionless amplifier, it was also possible to adjust its voltages and the impedances to which it was connected so as to produce a distortion of the input current in a manner suitable for the purposes of radio telephony. Under such conditions the output of the tube contains a component which is proportional to the product of such voltages as may be simultaneously impressed upon its input or grid-filament circuit. If one of these inputs is obtained from the oscillation generator just described and the other from a telephone transmitter, it is then possible to obtain an output which varies harmonically with the frequency of the oscillation generator, and also varies harmonically in amplitude with the frequency of the voice current impressed by the transmitter.

A vacuum tube, properly designed for the impedances between which it was to be connected and properly adjusted so as to emphasize the modulator characteristics, was used as a modulator in the system of transmission which was formally demonstrated at Montauk. To its input was coupled inductively the tuned circuit of the oscillation generator and also telephone transmitter circuit containing a local battery.

In order to obtain a sufficient amount of energy for transmission over appreciable distances, it is necessary to amplify the energy output of the modulator before impressing it upon the antenna. In case considerable amplification is desired it may be obtained by impressing the voltage from the modulator upon an amplifying system formed by two amplifiers in tandem. It was also realized that amplifiers might be worked in parallel, that is, with their input circuits connected in parallel and with their output circuits connected in parallel, without introducing disturbing interactions, provided that certain precautions are taken. The method of amplification adopted for the Montauk tests and later used for the Arlington experiment was therefore a system of two stages of amplification. At Montauk the first stage was obtained by a single tube and the second stage by a number of tubes in parallel. In Fig. 3 is shown a view of the experimental apparatus used at Montauk. On the left in the foreground is a rack containing the amplifiers of the second stage. On the right, immediately above the battery racks, are four boxes containing the other radio apparatus. The upper right-hand box contains the oscillation generator. The box immediately below this holds the modulator and its associated apparatus; the upper box on the left contains the vacuum tube of the first stage of amplification; and the box below it has a tuned circuit and certain controlling resistances which enter into the transmission system. Another view of this apparatus is shown in Fig. 4. On the left of this is seen again the rack of amplifiers, but now in such a view as to show the vacuum tubes themselves. These tubes, which were similar to those later used in the Arlington demonstration, were especially designed for this purpose and are much larger than the vacuum tube amplifiers of telephone practise.

THE ARLINGTON-DARIEN EXPERIMENT

The tests from Montauk to Wilmington and to St. Simons Island were so successful that it appeared practicable to carry the method to its logical conclusion and to extend the range of radio telephony to much greater distances. For this purpose experiments were undertaken in conjunction with the Navy Department. For the initial steps in making such arrangements and for assistance throughout the experiments, we are indebted and distinctly grateful to Admiral R. S. Griffin, Engineer in Chief, U. S. Navy, Captain now Admiral W. H. G. Bullard, Superintendent of the Radio Service, and to Commander now Captain Hepburn and Lieutenant now Commander S. C. Hooper of the Bureau of Steam Engineering, U. S. N.

For the purpose of these experiments a small operating house was constructed beside the main operating building of the Navy Wireless Station at Arlington, Va. An antenna switch was provided so that the antenna might be connected to the Western Electric equipment in this experimental station. Captain Bullard also arranged for observers from the Western Electric Company to be present at the Navy stations at Darien, San Diego and Mare Island. The necessary apparatus was installed in the early summer of 1915 and preliminary experiments were started at once. On August 27th successful transmission was obtained to Darien on the Isthmus and was received by Mr. R. H. Wilson of the Western Electric Company and by Lieutenant R. S. Crenshaw of the Navy.

On this day Colonel Reber, Lieutenant Bryant of Captain Bullard's office and Mr. G. H. Clark, Radio Expert of the Bureau of Steam Engineering, had been asked to speak from Arlington in the hope that communication would be established with Darien. Immediately prior to the time these gentlemen spoke, two selections were played on a phonograph placed in front of the telephone transmitter. These were correctly recognized by Wilson and Lieutenant Crenshaw. Each of the visitors then spoke for a minute or two. The voices of the speakers were not familiar to the observers at Darien. Wilson, however, recognized the change in voice caused by a change in speaker and received correctly several phrases and some scattered words. These words were also verified by Lieutenant Crenshaw. A report of these results was sent immediately from Darien by Navy code to Arlington and was compared by the speakers with their records of their spoken words.

PLATE XX.
A. I. E. E.
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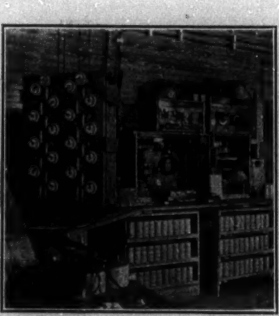


FIG. 3—MONTAUK EXPERIMENTAL APPARATUS

FIG. 4—MONTAUK EXPERIMENTAL APPARATUS

[CRAFT & COLPITTS]

FIG. 5—RADIO EXPERIMENTAL STATION, ARLINGTON, VA.

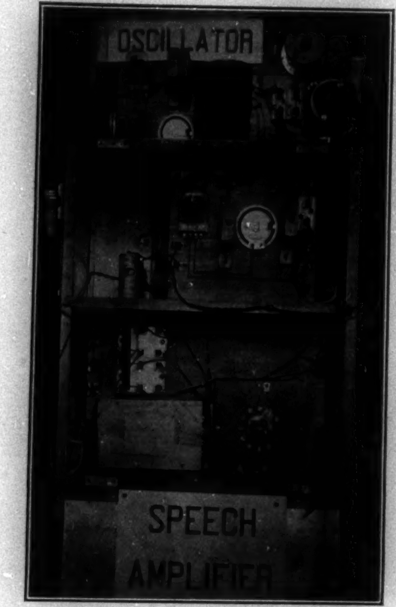
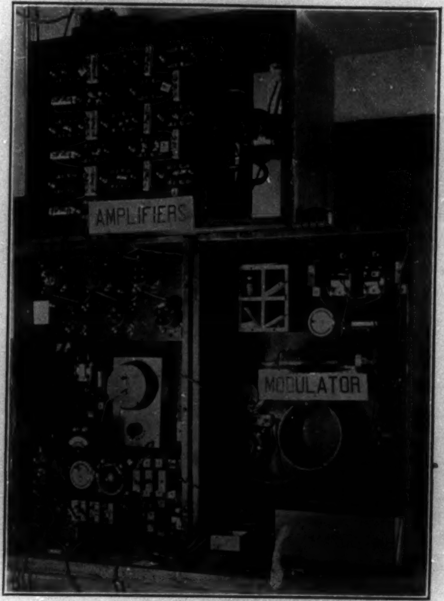

OSCILLATOR
SPEECH
AMPLIFIER

FIG. 6—SPEECH AMPLIFIER—RADIO EXPERIMENTAL STATION, ARLINGTON, VA.


AMPLIFIERS
MODULATOR

[CRAFT & COLPITTS]

FIG. 7—MODULATOR—RADIO EXPERIMENTAL STATION, ARLINGTON, VA.

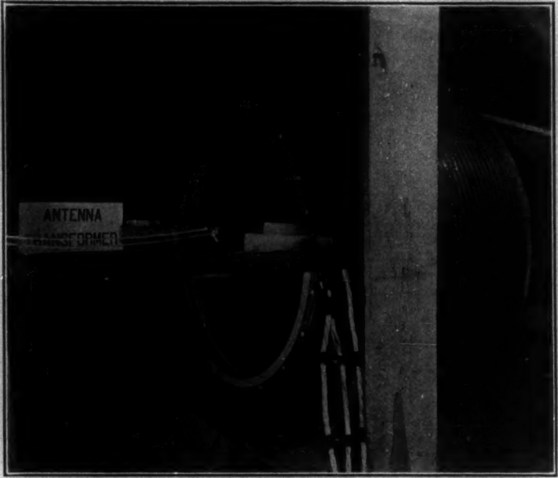

ANTENNA
TRANSFORMER

FIG. 8—ANTENNA TRANSFORMER—RADIO EXPERIMENTAL STATION, ARLINGTON, VA.

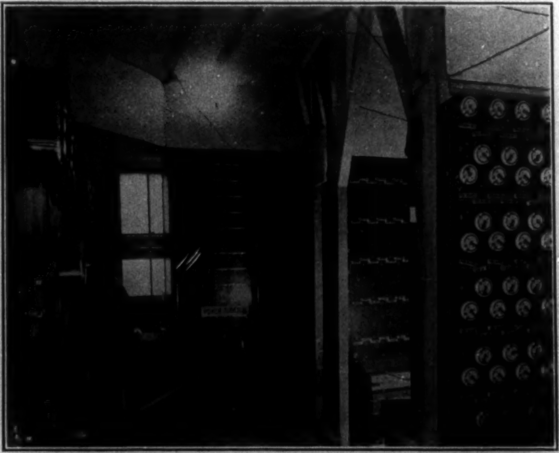


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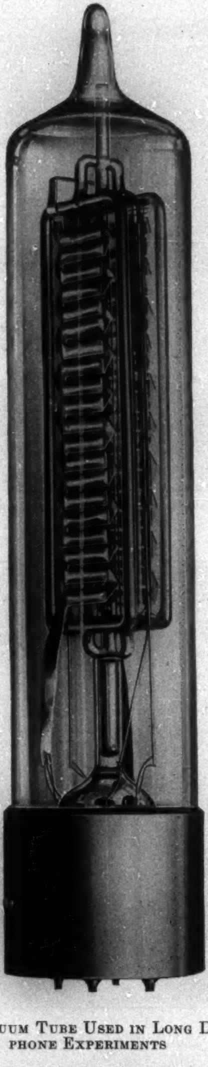
FIG. 10—POWER TUBE RACK—RADIO EXPERIMENTAL STATION, ARLINGTON, VA.



[CRAFT & COLPITTS]

FIG. 9—TYPE OF VACUUM TUBE USED IN LONG DISTANCE RADIO TELEPHONE EXPERIMENTS

The method of transmission was essentially that described above in connection with the Montauk experiment. The experimental station at Arlington is shown in Fig. 5. The transmitters used in these tests were of the ordinary commercial type, or of a type which was then in a stage of development and which it was thought might reproduce more exactly some of the higher harmonics of the voice. A phonograph was also used for the transmission of music. The output of the transmitter was amplified in a speech amplifier of the audion type, which is shown in Fig. 6. The high-frequency oscillator is shown in this figure above the amplifier where the tuning condensers and the coil for coupling to the inductance are plainly visible. The output of this speech amplifier and of the high-frequency oscillator were both impressed upon a modulator of the vacuum tube type, shown in Fig. 7. The voltage of this amplifier was not, however, high enough for the main amplyfying tubes, and it was stepped up by an intermediate group of amplifiers, of which six as a rule were used, which are shown at the left of the figure. The modulator output thus amplified was impressed upon a number of power tubes in parallel and then upon the antenna transformer shown in Fig. 8. The type of power tube is illustrated in Fig. 9. The amplifier tubes preceding the power tube and the modulator were also of the same form, but were of different constants, properly adapted to their purposes.

The current by which the filaments were heated was supplied by a local power company and brought in by underground cable to the operating station. Alternating current was used for heating the filament and any possibility of the superposition of a 60- or 120-cycle note, due to the frequency of this current, was eliminated by the use of a special scheme of connections. The transformers which were inserted in this circuit are shown at the left of the door in Fig. 5. Between the plate and the filament of the power tube a constant voltage of about 500 volts was impressed. It was obtained from one of the motor generators used by the Navy in operating the large Poulsen arc, with which they were then transmitting to Darien and other remote points. A large amount of energy, of course had to be dissipated in such an equipment because of the energy supplied to the filaments, and a blower was installed for this purpose as shown in Fig. 10 where the racks containing the power tubes are also visible. At the left of this figure is the switchboard for operating remotely the motor generator set in the main station of the Navy, for controlling the alternating current supplied to the filament, for operating the blower, and for metering the currents used for lighting the plant and for similar purposes.

The method of reception was varied from time to time as it was still in process of development. In some cases an amplifier for the high-frequency currents was introduced between the detector and the tuned circuit which was coupled to the antenna. In some cases a low-frequency amplifier was inserted between the detector and the receivers. In some of the later experiments a feed back circuit was used for the detector so that the detector tube itself acted as an oscillation generator as well as a detector. In that case the frequency of the oscillations was adjusted to be the same as those of the oscillation generator at the transmitting station.

THE NEW YORK-ARLINGTON-MARE ISLAND DEMONSTRATION

After obtaining successful communication with Darien, a distance of 2100 miles, attention was directed towards reaching the Pacific Coast stations of Mare Island and San Diego. As a test of the transmission efficiency, records were made at the W. E. Co's. laboratory in New York of the quality and intensity of the speech from Arlington, each time transmission was attempted. Records were also made occasionally at Darien, but no attempt was made to fit the time of transmission into the schedule of operation of that station. An order wire was set up between Arlington and the two Pacific Coast Stations, so that instructions as to the schedule of tests and reports as to its results might be conveniently interchanged.

Radio telephone transmission between Arlington and Mare Island was formally demonstrated on September 29th. On this date, Mr. J. J. Carty was inspecting the station at Mare Island. A demonstration for Mr. T. N. Vail was therefore arranged for this date.

The station at Mare Island consisted of a small house near enough to the operating house of the Navy so that the experimental apparatus might be connected to the antenna. The observers at the Mare Island station were Commander G. C. Sweet and Lieutenant Kenshit of the Navy, Mr. A. H. Babcock, Electrical Engineer, Southern Pacific Railroad, Mr. J. J.

Carty and others. Messrs. Arnold and Hartley of the Western Electric Company were the experimenters.

Arrangements had been made whereby Mr. Vail's apartment in New York could be connected with the radio apparatus at Washington. A telephone line extended from the transmitter assigned to Mr. Vail's use to the radio telephone apparatus at Arlington. This telephone circuit was connected to the radio apparatus in place of the telephone transmitter which had formerly been used at Arlington. From Arlington transmission was through the ether to Mare Island. The telephone set which Mr. Carty used at Mare Island consisted of a transmitter connected to a telephone line leading, via the New York test board, to a receiver in Mr. Vail's apartment. The receiver of Mr. Carty's set at Mare Island was one of several which were supplied in parallel by the detector of the radio receiving set. Arrangements were thus made whereby Mr. Vail and Mr. Carty might carry on two-way conversation, but what Mr. Carty and the other observers heard at Mare Island was transmitted from Arlington by wireless.

At the New York toll test board a telephone line was bridged onto the circuit leading from Mr. Carty to Mr. Vail. This line extended to Arlington and contained one-way repeaters so that transmission could take place only from New York to Arlington. At the latter place it supplied a group of telephone receivers which were also connected through a one-way telephone repeater with the line which bore Mr. Vail's speech to the radio apparatus. In this way some dozen observers at Arlington could hear both sides of this unique conversation. The use of one-way repeaters prevented any interaction between the oppositely directed speeches.

At the Arlington station the conversation was listened to by Captain Bullard, Colonel Reber, Commander Hooper, Lt. R. B. Coffman, Lt. Bastedo, Commander Bryant, and members of the telephone system. Messrs. T. N. Vail, U. N. Bethel, J. I. Waterbury, Bancroft Gherardi, and H. P. Charlesworth all conversed with Mr. Carty through the ether.

THE ARLINGTON-HONOLULU EXPERIMENT

Having successfully transmitted over water to Darien, a distance of 2100 miles and over land to Mare Island, a distance of 2500 miles, attention was now directed towards reaching the remoter points of Honolulu and Paris. Schedules were therefore arranged for experiments with these stations. With Paris, however, the arrangements as to schedules were necessarily delayed because of the difficulties in cable transmission occasioned by the war.

With Honolulu, on the other hand, arrangements for a transmitting schedule had been made shortly before the demonstration of the New York-Arlington-Mare Island transmission. The first deliberate attempt to communicate with this station occurred on the night following the demonstration to Mare Island. Mr. Lloyd Espenschied of the American Telephone and Telegraph Company, who was the experimenter at this point, reported by cable the following morning, giving a record of the words which he had heard spoken, and also the name of the speaker, whose voice he had recognized.

THE ARLINGTON-PARIS EXPERIMENT

At Paris were Mr. Shreeve and Mr. Curtis of the Western Electric Company, in which connection it may be mentioned that they are again in France, the former as Lieutenant Colonel and the latter as Lieutenant in the Signal Corps. These experimenters had installed their apparatus in the Eiffel Tower station of the French Army. This courtesy had been accorded our representatives by Lieutenant Colonel Ferrié, Director of the Military system of Radio Telegraphy, to whom they had been introduced by Commander W. R. Sayles, Naval Attaché of the American Embassy. The magnificent spirit which France has shown throughout this bitter struggle is well illustrated by the ability of that nation to consider scientific developments, which apparently were not of immediate military value, and to assist in them despite the demands of the war. The amount of time during which our experimenters could use the antenna was, of course, very limited, and was small as compared to their necessities. The permission to use the antenna at a time when France was wholly engaged in this bitter struggle represents a contribution to science altogether disproportionate to the time which the antenna was available to our men. Unfortunately, however, much of the time when Mr. Shreeve and Mr. Curtis had access to the Eiffel Tower antenna there was interference from other high-power stations on the Continent, such as Nauen, Eilvese, Clifden, Norddeich, Lyons and Vienna which were transmitting upon the same range of wave lengths as it was necessary for us to employ. Instructions from Paris to Arlington, to alter the wave length of the transmission, could be forwarded only by cable, and were, of course, subject to days of delay, so that a satisfactory and flexible working schedule was not to be hoped for. Further, it was possible to transmit in any one day only for a brief period. In spite of these difficulties, promising results in the way of clearly received speech at Paris were secured during short periods that it was possible to arrange for working, between October 12th and October 21st. On October 23d as a final demonstration a number of observers, including Commander Sayles and representatives of the French Army, listened to the telephone transmission from the Arlington station 3600 miles away. The wave length used was about 6000 meters and the antenna current at Arlington about 50 amperes.

On certain of the occasions when we had worked to Paris, Mr. Espenschied at Honolulu had been informed as to the schedule of tests. He had therefore listened in and copied the words spoken at Arlington. The conditions at his stations were, of course, much better that at Paris because of the absence of interference from high-power stations in the neighborhood. His observations were upon transmission over a distance some 25 per cent greater than that from Arlington to Paris, and in part over land as well as sea water.

Practical Developments

The impending danger of war put an end to the experiments as far as they were concerned with increase in range of transmission. The possibility of applying the principles which had been successfully developed in connection with these experiments in trans Atlantic and trans Pacific radio telephony to problems of immediate and patriotic value was of course evident. Subsequent developments and experimental tests were therefore directed solely with an aim toward their military value to the Army and the Navy. Some of these experimental developments which we made in this connection and the success with which they were accompanied may be of interest.

The first opportunity to demonstrate the use of the wireless telephone in war was presented early in 1916. Secretary Daniels desired a demonstration of what could be accomplished in the way of mobilizing the telegraph and telephone facilities of the country in case of war-time need. This was carried out by the American Telephone and Telegraph Company in May, 1916, and included as part of it wireless telephone communication with a battleship at sea.

The part played by the wireless telephone was to show the possibilities of quick communication by word of mouth between vessels at sea and Headquarters on land. A complete set was built and installed on the battleship New Hampshire. The transmitting to the battleship was done from our station at Arlington. The signals from the battleship were received at the Navy Yard at Norfolk with a set built for that purpose, and were automatically transferred to land telephone lines to Washington. The connections were brought directly to the mobilization switchboard at the Navy Department Annex. When it was desired to talk to the battleship, the regular telephone circuit from the Navy Department was connected up with the lines to Arlington and Norfolk, thus allowing communication by telephone to be held directly with the Captain of the vessel. It was found entirely practicable to hold two-way radio communication as effectively as over wire lines and to secure information or to give orders. Captain Chandler received his orders for manoeuvering in this experiment by wireless telephone from Secretary Daniels and Admiral Benson, and he reported to them each hour his position by the same means. The fact that the lines to the transmitting and receiving land stations came to Washington where these gentlemen were located did not mean that such an arrangement only was possible. This radio system was attached to the long distance telephone line and Capt. Chandler talked with Lt. Snyder of the Great Lakes Naval Station. It was then connected with the Transcontinental Telephone Line and he talked directly with Capt. F. M. Bennett, Commandant of the Mare Island Navy Yard. The ship during these conversations was 50 miles from Norfolk and outside the Capes.

The set installed on the New Hampshire differed from the set at Arlington only in size. The general circuit arrangement and modulation system was the same. The set was installed on the lower bridge deck and the telephone transmitter and receiver were installed on the bridge itself. This allowed the Captain, while on the bridge and directing the movement of the ship, to converse without leaving his post.

Battleship Equipment

The possibilities of the wireless telephone in naval maneuvering were quickly recognized by the Navy Department.

Commander Hooper of the Bureau of Steam Engineering asked us to cooperate in making the wireless telephone a useful piece of equipment in naval service. It was decided to build two experimental sets, install them on battleships and give the operating staff an opportunity to investigate and criticize them from the operating point of view. The construction of two sets was begun in December 1915 and they were completed about the middle of January. A short description of these sets will not be out of place at this time. Each set consisted of complete transmitting and receiving sets with a motor-generator. The receiving set was built to be set on top of the transmitting set. A photograph of the two thus placed is shown in Fig. 11. An extension circuit was provided so as to allow placing the hand set either on the bridge or in the Admiral's or Captain's cabin so that it would not be necessary for them to go to the radio room to talk. A duplicate hand set was provided for the operator in the radio room as well as a head telephone for listening-in. The normal condition of the circuit was with the antenna connected to the receiving set. Any call thus coming in would be received. When a call was received and it was desired to talk instead of to listen, a button on the hand set was pressed which caused the relay operated antenna switch to disconnect the antenna from the receiving set and connect it to the transmitting set and at the same time start the transmitting set operating. This operation required only a fraction of a second. It was thus possible to transmit and receive suc-

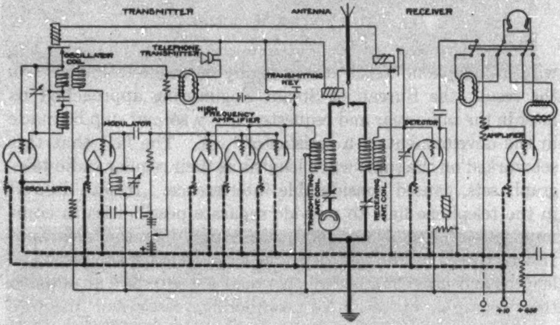

TRANSMITTER
ANTENNA
RECEIVER
OSCILLATOR COIL
TELEPHONE TRANSMITTER
TRANSMITTING KEY
HIGH FREQUENCY AMPLIFIER
MODULATOR
OSCILLATOR
TRANSMITTING ANT. COIL
RECEIVING ANT. COIL
DETECTOR
AMPLIFIER
−
+10
+60V

FIG. 12—CIRCUIT SCHEMATIC—BATTLESHIP SET

cessively on the same antenna by pressing the button whenever it was desired to talk. The wave length range of the set was 600 to 1200 meters. The circuit is shown in Fig. 12. This system was likewise similar to that at Arlington, but was, of course, quite small and produced only one and one-quarter amperes in the antenna. These sets were installed on the battleships Arkansas and Florida at Guantanamo, Cuba, the first part of February, 1916. It was found entirely practicable to hold a two-way conversation between vessels over 30 miles apart. There is no reason to think that was the limit of communication as some of these conversations were overheard in Jamaica, a distance of 175 miles. No attempt was made for range as that was not the object in building these sets.

MULTIPLEX WORKING

This experiment pointed out many things in the use of the wireless telephone which it was highly desirable to know. As the result, the Bureau of Steam Engineering approached us later in the same year and requested that a second step be made in the development of a satisfactory set. The fact that this set worked on the same wave lengths as their regular radio telegraph sets, caused considerable interference. It was desired in the telephone field to provide separate possibilities in communication for the Navy and not encroach upon the telegraph. It was, therefore, thought desirable to go to much shorter wave lengths and provide a set which would not interfere at all with the telegraphic operation of neighboring vessels and interfere very little with the telegraphic operations on the same vessel on which the telephone was being used. A few field experiments were made with short-wave antennas on two ships in July 1916 to provide data necessary for building these sets and their design and construction was started in October. The new set differed from the old one in that it operated on three wave lengths, 150, 189, and 238 metres and also in that it was multiplex and allowed nine conversations to be carried on three wave lengths. The basis of the multiplex system which was originated by Mr. R. A. Heising is as follows:

Suppose a wave of 25,000 cycles is modulated according to a speech signal. This wave when received in the receiving set and rectified reproduces the desired speech. If, however, instead of radiating this 25,000-cycle wave directly, we modulate a 150-meter wave with it and radiate the 150-meter wave, on

[CRAFT & COLPITTS]

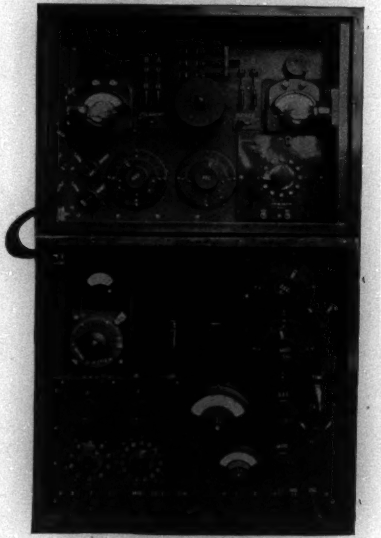


FIG. 11—BATTLESHIP RADIO TELEPHONE TRANSMITTING AND RECEIVING SET

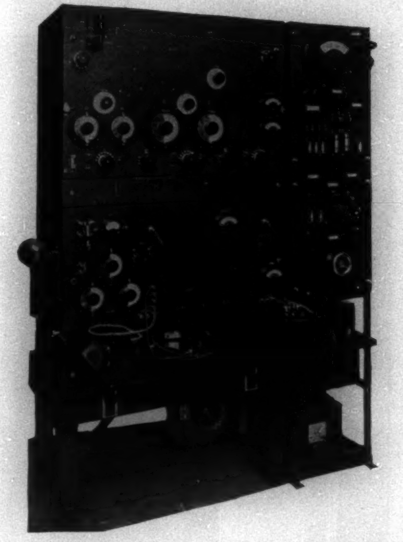


FIG. 15—MULTIPLEX TRANSMITTING AND RECEIVING SET

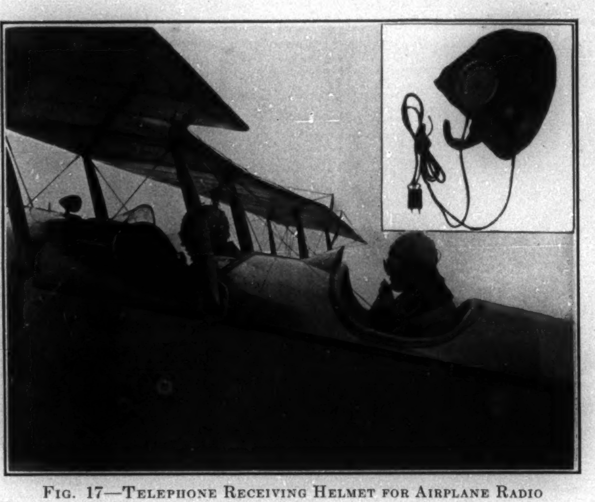


FIG. 17—TELEPHONE RECEIVING HELMET FOR AIRPLANE RADIO

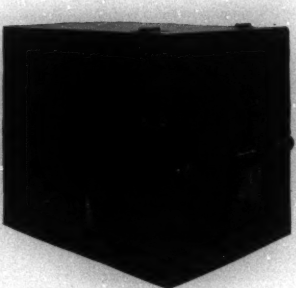
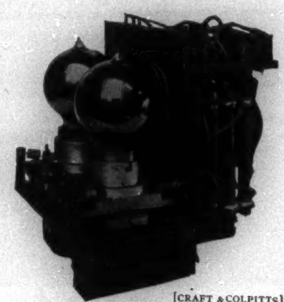


FIG. 19—EARLY FORM OF AIRPLANE TRANSMITTING SET



[CRAFT & COLPITTS]

FIG. 20—TRANSMITTING SET—OPEN VIEW

receiving and detecting at the receiving station the 150-meter wave there will be produced the wave of modulation, or the 25,000 cycle wave. This second wave, is, therefore, produced in the receiving station not by its direct radiation and reception, but is produced there by being carried by the shorter carrier wave. If it is then selected by tuned circuits and detected, the speech signal becomes audible. Therefore, with these sets provided, it was possible to have three ships transmit simultaneously on the same wave length, 150 meters for instance, one ship using an intermediate frequency of 25,000 cycles, the second an intermediate frequency of 35,000 cycles and the fourth, an intermediate frequency of 45,000 cycles. A receiving station tuned to 150 meters would receive all three double modulated waves. By using a second tuned circuit, one of the stations could pick out any one of these three conversations by tuning the second tuned circuit to 25,000, 35,000 or 45,000 cycles and detecting and would not be interfered with by the other two stations on the same wave length. A schematic diagram of the system is given in Fig. 13 and a circuit diagram of the receiving system is given in Fig. 14.

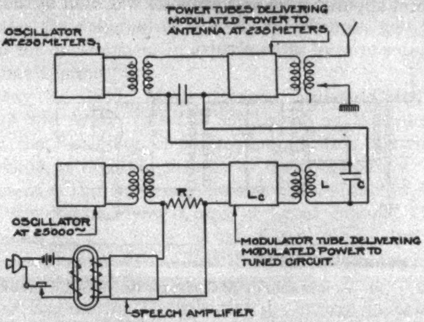

POWER TUBES DELIVERING MODULATED POWER TO ANTENNA AT 230 METERS
OSCILLATOR AT 230 METERS
R
Lc
L
C
OSCILLATOR AT 25000~
MODULATOR TUBE DELIVERING MODULATED POWER TO TUNED CIRCUIT.
SPEECH AMPLIFIER

FIG. 13—SCHEMATIC OF MULTIPLEX ARRANGEMENT

These sets were built upon the same plan as the previous ones in that transmitting and receiving sets were built to be placed one upon the other. An illustration is shown in Fig. 15. These sets were installed on the U. S. S. Pennsylvania, Wyoming and Seattle in January 1917, but scarcely had they been installed than diplomatic relations with Germany were severed and the assistance in the development of apparatus by the Operating Staff of the Navy was necessarily curtailed.

RADIO TELEPHONE APPARATUS FOR MILITARY USE

Beginning with the entry of this country into the war, the demands for radio apparatus for military purposes have resulted in the vary rapid commercial development of this type of equipment. While nothing fundamentally new has resulted from this work, there has been a very practical and valuable result in that the commercial development and manufacture of radio telephone apparatus in general has been put on a basis which otherwise could not have been reached for many years to come.

The first apparatus for war pruposes was built at the request of the Navy Department in March, 1917, and consisted of fifteen experimental sets which were proposed for use on submarine chasers. These sets were of the continuous wave type and were intended primarily for telegraph communication, but were also equipped with a telephone modulating attachment. The transmitting element employed four tubes of the type used in the trans-Atlantic experiments, and delivered approximately 1.5 amperes into the small antenna of the submarine chaser. In view of the fact that these sets were immediately replaced by an improved type which will be described later, the detailed construction of these sets need not be discussed, and they are mentioned merely to indicate the Navy Department's practical interest in this type of apparatus at a comparatively early date.

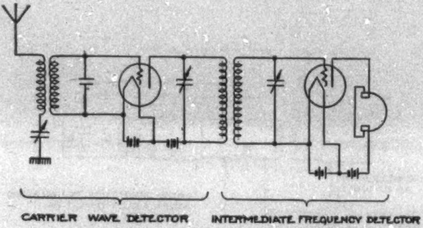


FIG. 14—SCHEMATIC OF MULTIPLEX RECEIVING CIRCUIT

AIRCRAFT TELEPHONY

The most prominent and probably most spectacular development of radio telephony during the war was in connection with its use on military aircraft. While radio telegraphy had been used before for fire control and scouting purposes, its field had not gone beyond that of one-way communication, and was confined practically to the use of simple spark gap types of telegraph apparatus.

Anticipating possible employment of radio telephony in this field, the Western Electric Company Engineering organization had done a considerable amount of experimental work in the way of applying the principles demonstrated in connection with earlier experiments in long distance radio telephony to short range work. This work had been carried on for some time under the direction of Mr. H. W. Nichols, and had resulted in the development in experimental form of satisfactory short range apparatus.

On May 22, 1917, Major General Squier, Chief Signal Officer of the Army, called a conference at Washington to consider the feasibility of intercommunication between airplanes while in flight by means of radio telephony. There were present at this conference besides General Squier, Colonel Rees of the Royal Flying Corps of Great Britain, Captain, now Colonel C. C. Culver of the Signal Corps and Major later Lieut. Colonel F. B. Jewett and Captain later Major E. B. Craft who had been assigned from the Western Electric Company Engineering Organization to engage in this work. At this time plans were in the making for the tremendous aircraft program which was later undertaken by the Army, and it was clear to all that a successful means of communication between battle planes when flying in squadrons would be of inestimable value and would greatly increase the efficiency with which these squadrons could be maneuvered. The early work had shown such promise that there was justification for assuring the Signal Corps that this means of communication could be worked out successfully and applied to aircraft of various types. As a result of this conference, orders were issued by the Signal Corps for the Western Electric Company to undertake the development of a wireless telephone system for the purpose. The experimental equipment available as a laboratory proposition at this time, served as a basis for the first experimental trials in the air. This transmitting set did not use the Colpitts system of modulation but the "constant current" system of R. A. Heising which was found to be more compact and easily operated. This system comprises a modulator and an oscillator tube with their plate circuits essentially in parallel for audio frequency currents. They are supplied from a generator through a choke coil whose function is to maintain a practically constant current to the two tubes no matter how the plate currents may vary individually. A circuit diagram is shown in Fig. 16 and the operation is as follows.

When the transmitter is not actuated there is a certain normal value of voltage impressed upon the grid of the modulator tube, this value being adjusted until the plate current of the modulator is about the same as that of the oscillator. Now the characteristic curve of Fig. 1 shows that as the grid becomes more negative the plate current decreases at constant plate voltage, and with positive grid voltage it increases. This may be described by saying that the resistance of the plate circuit may be varied by varying the grid voltage, and inspection of the curve shows that this variation may be from a very high value (point $-E$) to a small value for positive grid voltages. In the modulating system of Heising the transmitter voltage acts upon the grid of the modulator and causes the resistance of the plate circuit to vary through a wide range in accordance with the speech voltage. Since this circuit shunts the plate circuit of the oscillator tube (at audio but not at radio frequencies) the oscillator will be robbed of current or have additional plate current forced through it in accordance with

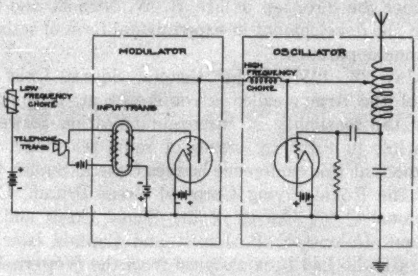


Fig. 16—Schematic of Heising Modulation System

the speech voltage. Its output will thus be varied and it has been found that speech is modulated in this way with very good quality. The efficiency of this system is high and greater than that of an "absorption" system in which power is variably diverted from an oscillating circuit and wasted in a modulating device. In the constant-current system, the voltage and current in the oscillator rise periodically to nearly twice their values when the transmitter is not acting, which fact accounts for the increased efficiency.

EXPERIMENTAL WORK

This fundamental plan seemed to be the most feasible one to pursue and on June 5, 1917, another conference was held at Washington to agree upon the various technical features and make plans for carrying out field tests.

It was early realized that the principal difficulty in airplane telephony would be the noises due to the motor and the wind, so that attention was concentrated on the problem of providing against this difficulty both in the transmitting and receiving ends of the combination. Laboratory work was directed particularly toward producing a form of telephone transmitter or microphone which would be as insensitive as possible to these extraneous noises and at the same time be responsive to the voice frequencies. In order that this work might be carried on at high speed, a sound proof room was constructed and a device provided which reproduced very accurately the noises of the engine exhaust. By the first of July experiments looked sufficiently promising to warrant tests in the field. On July 2nd a complete radio telephone transmitting equipment was taken into the air and speech of good volume and quality was received on the ground, with the transmitting plane two miles away. The development of a suitable receiving head set was carried on simultaneously and it was found possible to devise a leather helmet with the receiving elements so disposed and screened from external noises that the weak radio signals could be readily observed. On July 4th experimental receiving equipment was taken into the air and Mr. L. M. Clement of the Western Electric Company successfully received speech from the ground at a distance of several miles.

As stated before, the apparatus used in these tests was of an experimental form built in the laboratory. The transmitting set had an output of 0.7 amperes at a wave length range of

200 to 400 meters, the antenna being a trailing wire about 100 meters along.

Development of Apparatus

With the information resulting from these tests, the development of a practical airplane set began at once. During the previous month, comparative tests had been made on a number of proposed modulating systems, with the result that the constant-current system previously described seemed best for the purpose. The problem now was to produce sets of minimum size and weight, physical structures which would withstand the extreme vibrations and jars encountered in flying, especially in landing, the most convenient disposition of control elements, suitable sources of power for both high and low voltage, and a form of antenna which would not interfere with the evolutions of a plane in squadron formation.

It was realized that the solution of the last mentioned problem would consume more time than was at our disposal, and while work was immediately started in the field on antenna measurements and study, the design of the sets proceeded on the basis of using a trailing wire antenna.

The working out of a practical helmet design proved to be more difficult than the success of the earlier experiments indicated. It was found that the degree of interference experienced changed very rapidly with slight imperfections in fit. The problem was also complicated by the necessity of providing means for using oxygen at high altitudes, and of providing for the comfort of the wearer over a period of several hours. It was found that a very slight amount of pressure on certain portions of the ear caused excessive pains and headaches after a very short period, and the final design was a compromise between comfort and efficiency as to sound insulation. Fig. 17 illustrates the form of helmet that was eventually developed and adopted.

The problem of power supply was an interesting one. It was required that the weight should be reduced to a minimum, which precluded the possibility of employing storage batteries. There are obvious objections, also, to attaching any form of generating device to the propelling engine. Consideration of all the factors led to the adoption of a wind-driven generator for the purpose. This generator of about 100 watts direct-current output was required to produce a potential of 300 volts for the plate circuits of the vacuum tubes, and a potential of 25 volts for the filament circuits. It is necessary, to insure most efficient operation, that the filament current be kept constant, and the fact that the specifications called for operation with airplane speeds varying from 40 to 160 miles per hour made the problem of voltage regulation loom large. Ordinary forms of electro-mechanical regulating devices did not prove to be successful and while it was probable that something of this type would eventually have been developed, the problem was solved by a very ingenious arrangement proposed by Mr. H. M. Stoller, in which the vacuum tube is the essential element. Fig. 18 shows, schematically, the generator and regulator circuit. The voltage of the generator is held approximately constant by means of a vacuum tube regulator, which controls the field flux. Two field windings are provided, the main field, which is in series with the filament of the regulator tube, and a differential field which is in series with the plate circuit. At minimum speed the differential field is inactive and the generator behaves like an ordinary shunt machine, except that the main field has a small resistance in series due to the regulator tube filament. This filament is so designed

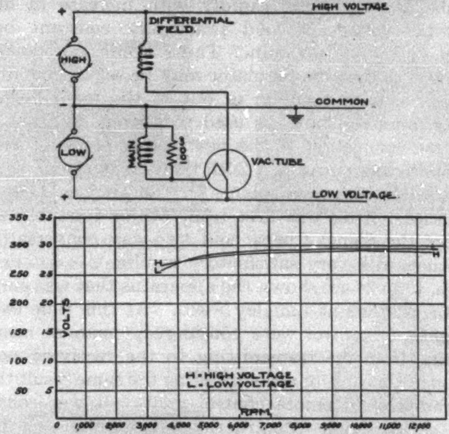

HIGH VOLTAGE
DIFFERENTIAL FIELD
HIGH
COMMON
LOW
MAIN
100
VAC. TUBE
LOW VOLTAGE
+
-
+
350 35
300 30
250 25
200 20
150 15
100 10
50 5
VOLTS
H
L
H - HIGH VOLTAGE
L - LOW VOLTAGE
RPM
0 1,000 2,000 3,000 4,000 5,000 6,000 7,000 8,000 9,000 10,000 11,000 12,000

FIG. 18—VOLTAGE REGULATION OF WIND-DRIVEN GENERATOR

that the main field current heats it to a temperature which gives a small electron current. This current flows through the differential field and reduces the resultant flux. At minimum speed, the differential current is small, but as the speed increases, the main field current tends to rise (as in any shunt generator) and this increases the temperature of the regulator tube filament. The electron current is, therefore, considerably increased, which current, flowing through the differential field, reduces the generator flux and thus restricts the rise in voltage. Due to the fact that the electron current of the regulator tube increases very rapidly with increase in filament current, the voltage is held practically constant between 4000 and 12,000 rev. per min. The 1.5-ohm resistance units are provided so that the regulator may be set to give different voltages by cutting them in or out of the main field. The 100 ohms shunt resistance is used to prevent hunting.

Upon the completion of the next set of models, field tests were resumed and on August 20th the first two-way telephone conversation between two planes in the air was successfully accomplished. After this first trial, Major Bartholf and Lt. Stevens of the Signal Corps held two-way conversation between planes with very satisfactory results.

In Figs. 19 to 24 are shown the apparatus that was employed in the August tests at Langley Field. At this time two general schemes of control were considered,—manual means for transferring from the transmitting to the receiving position, and automatic means for accomplishing the same result through the operation of a remote control relay. It was concluded, because of its simple construction, to employ the manual means of control, the idea being to locate the set in the observers position in the plane.

On August 22nd an informal demonstration of talk from airplane to ground was given for Secretary of War Baker, General Scott, and Colonel Baker. These experiments conclusively demonstrated the practicability of the system and apparatus, and we felt justified in proceeding with its commercial development.

Commercial Development

VACUUM TUBES

While the vacuum tubes that had been developed in the 1915 and 1916 experiments were satisfactory in operation under normal conditions, it was found that the mechanical vibration

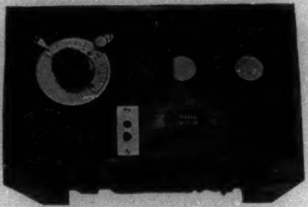


FIG. 21—EARLY FORM OF AIRPLANE RECEIVING SET

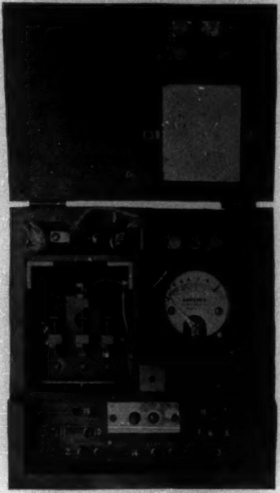


FIG. 24—REMOTE CONTROL BOX—AIRPLANE RADIO

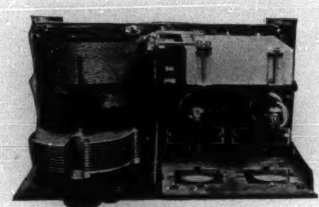


FIG. 22—AIRPLANE RECEIVING SET—OPEN VIEW

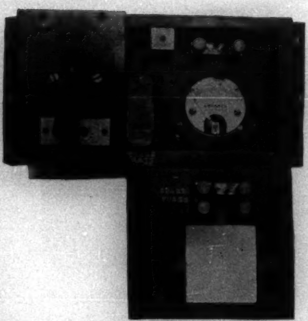
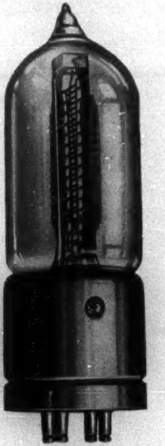


FIG. 23—MANUAL CONTROL BOX—AIRPLANE RADIO



[CRAFT & COLPITTS]

FIG. 25—WESTERN ELECTRIC RECEIVING TYPE VACUUM TUBE—ARMY AND NAVY STANDARD

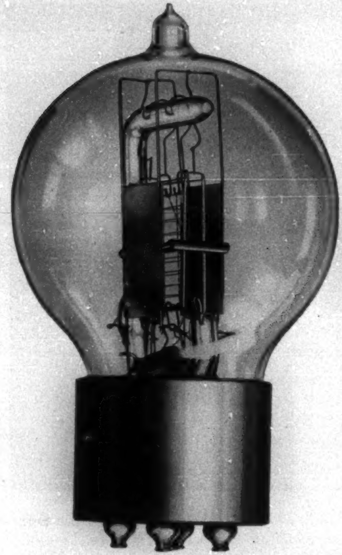


FIG. 26—EARLY FORM OF WESTERN ELECTRIC TRANSMITTING TYPE VACUUM TUBE

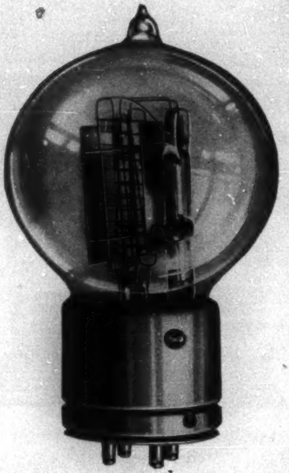
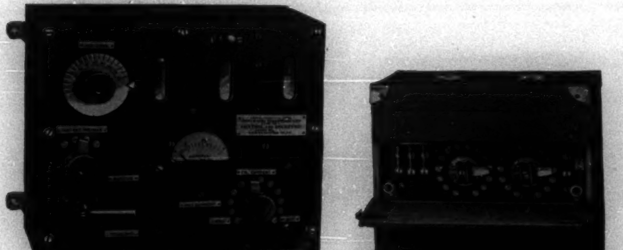


FIG. 27—WESTERN ELECTRIC TRANSMITTING TYPE VACUUM TUBE —ARMY AND NAVY STANDARD



[CRAFT & COLPITTS]

FIG. 30—WESTERN ELECTRIC AIRPLANE TRANSMITTING AND RECEIVING SET—FIRST STANDARD TYPE ADOPTED BY THE SIGNAL CORPS

encountered in the air was such as to necessitate special structures to withstand them. New forms of vacuum tubes, which successfully met these requirements were developed. Fig. 25 shows the receiving tube in which the elements are supported in a metallic framework, thus eliminating a great deal of breakage which occurred in the earlier structures. Fig. 26 shows the first form of power tube which was developed, later followed by the type shown in Fig. 27.

The vacuum tubes used in all this work were of the so-called "Wehnelt Cathode" type; that is, the electron-emitting cathode consisting of a metal filament—usually platinum—coated with a mixture of oxides which, when heated to a moderate temperature, gives off electrons in great numbers. When properly constructed and treated, such a type has several great advantages:

1. For a given electron emission, the temperature is not so near the point of destruction due to evaporation, etc., as in the case of a pure metal—for instance, tungsten. The result is longer life in operation.
2. For a given change in filament current at working ranges the change in electron emission is smaller in the case of the Wehnelt cathode, permitting of closer regulation in power output.
3. Evaporation of the filament does not go on so rapidly, consequently its electrical behavior is more uniform throughout its life.
4. The electron emission for a given amount of energy supplied to the cathode is larger, resulting in more economical operation of the sets.

It was found that for transmitting and receiving purposes, two types of tube were sufficient. The tube for receiving purposes was designated as VT1 by the Signal Corps and as the CW933 by the Navy. A transmitting tube capable of delivering a moderate amount of power—say from 3 to 5 watts high-frequency output—was known as the VT2 by the Signal Corps and as the CW931 by the Navy. Vacuum tubes are defined, as far as electrical characteristics in normal operation are concerned, by plate and filament voltages, plate current with normal plate voltage and zero grid voltage, normal filament current, filament life at this current, and by what is known as the amplification constant, which is approximately the ratio, μ, of that change in plate voltage which produces a given change in plate current to the change in grid voltage to produce the same change in plate current.

Some of these relations are conveniently shown in characteristic curves. The plate current-grid voltage characteristics of the *VT*1 and *VT*2 tubes are shown in Figs. 28 and 29. On these illustrations are given also the other data mentioned above.

As indicating some of the problems involved in the commercial production of this entirely new type of equipment, it is interesting to note that prior to August, 1917, the total output of commercial vacuum tubes of this general type was approximately 200 per week, their use being practically confined to long distance wire telephony and to radio detection purposes. On November 11th, 1918, deliveries were being made at the rate of 25,000 per week. This involved the organization of equipment and personnel to do a class of work for which there was practically no experienced talent available and the situation in many features was analogous to that in connection with airplane production.

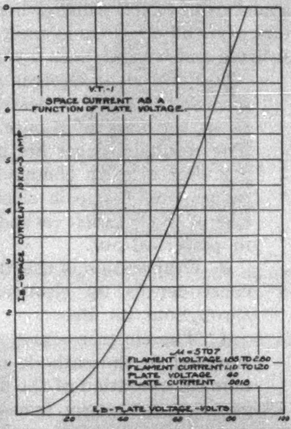


Fig. 28—Characteristics of Receiving Tube *V T*-1

APPARATUS

Tests of the first standardized sets for Signal Corps use were made at Langley Field on October 6th. These sets operated very successfully and official demonstrations of two-way communication were made. This set was made up of a combined control panel and receiver with two stages of amplification and a separate transmitting set, the electrical connections being made by flexible cords. These units are illustrated in Fig. 30. By means of the multi-contact manual switch located on the control panel, the operator can receive or transmit as desired. The receiver consists of a single tuned circuit of the very simplest type and the amplifier is made adjustable for the convenience of the observer.

While the original requirement was that communication should be maintained at a distance of 2000 yards, all of the tests indicated that successful communication could be obtained at much greater distances. On October 16th an official distance test was made, and communication between planes was maintained at a distance of 23 miles, and from plane to ground, 45 miles. The conditions under which these tests were made were extremely favorable, and are noted merely to show the possibilities of this particular equipment. The figure that was finally established as being representative of what would be required in service, was three miles.

A number of sets were immediately constructed, and complete equipment sent overseas with Signal Corps Officers, where they were submitted for the criticism of our military forces.

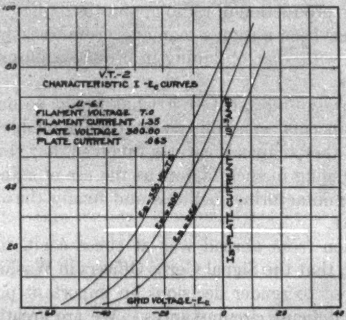


Fig. 29—Characteristics of Transmitting Tube *V T*-2

Several more demonstrations of this apparatus were made, culminating in the official trials at Dayton, Ohio, on December 2nd, 1917. There were present members of the Aircraft Production Board and the joint Army and Navy Technical Board, and various Signal Corps Officers. The demonstration consisted of a three-cornered conversation between two planes in the air and a ground station. At the ground station a loud speaking receiver was connected to the radio set in such a way that the entire party of about thirty could overhear the conversation between the fliers and also the speech between the planes and the ground. Under orders transmitted from the ground station, the fliers performed various evolutions in the air, and the observers could see these orders carried out and hear the acknowledgments. In these tests the pilot and observer in each plane were also connected, so that there were five people in constant communication. The maximum distance of the planes from the ground station was about eight miles, at which distance they were invisible. These tests so conclusively demonstrated the possibilities of the successful use of this apparatus, that quantity orders were immediately placed by the Signal Corps.

The time elements were very short and the problem of adapting the designs to commercial quantity manufacture were many and intricate. This probably is the first instance on record in which the production of radio apparatus, either telephone or telegraph, has been put on a manufacturing basis, in quantities comparable with that obtaining in ordinary lines of electrical manufacture.

The apparatus represents not what would be designed and built under normal conditions with time available for proper study of all the technical features involved, but the best compromise, bringing in such factors as the use of standard parts, available manufacturing facilities and finally the imperative need of haste.

Information from abroad was at times conflicting and inadequate, so that the Signal Corps Officers in Washington were often required to render decisions on important points under exceedingly difficult circumstances. We are greatly indebted to Lt. Col. N. H. Slaughter and other officers of the Signal Corps for their effective cooperation in the solution of difficult problems arising from all these causes.

The final standardized form of two-way airplane set is known by the Signal Corps as the SCR-68 set. Fig. 31 shows the complete equipment for a two-place airplane. The elements are so arranged that they can be mounted in the space available in various parts of the machine. The main set, however, necessitates being accessible to the operator.

The switch box, shown in the center, is to provide for communication between pilot and observer by means of the same helmets and microphones as are used for radio communication. This is one of the interesting by-products resulting from the radio development. Heretofore the only practicable means of communication between pilot and observer was by means

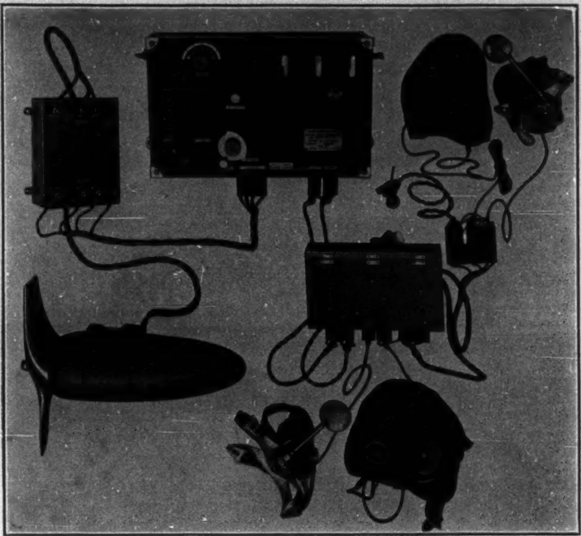


FIG. 31—COMPLETE WESTERN ELECTRIC RADIO TELEPHONE EQUIPMENT FOR TWO-PLACE AIRPLANE—SIGNAL CORPS STANDARD SCR-68

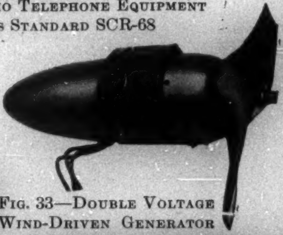


FIG. 33—DOUBLE VOLTAGE WIND-DRIVEN GENERATOR

FIG. 32—TRANSMITTING AND RECEIVING SET BOX —SIGNAL CORPS SCR-68

[CRAFT & COLPITTS]

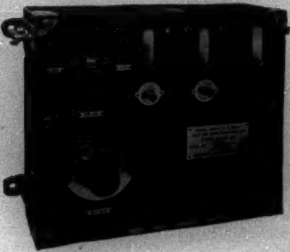


FIG. 37—WESTERN ELECTRIC RADIO TELEPHONE RECEIVING SET—SIGNAL CORPS S C R-59

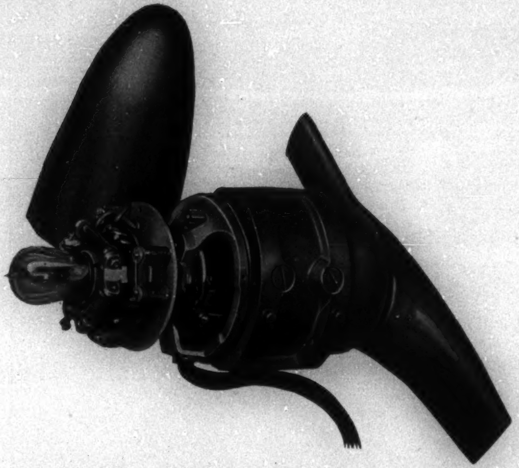


FIG. 34—WIND DRIVEN GENERATOR SHOWING VOLTAGE REGULATING TUBE

[CRAFT & COLPITTS]

FIG. 35—METHOD OF MOUNTING WIND DRIVEN GENERATOR

of manual signals. With the telephone equipment, satisfactory intercommunication could be provided at all times. This has been found very useful in connection with the training of fliers as well as in military operations.

The transmitting and receiving set proper is shown in Fig. 32. This set is approximately 17 by 10 by 7 in. in dimensions, and weighs 21 lb.

The wind-driven generator is shown in Fig. 33, and in Fig. 34 the tail cover has been removed showing the vacuum tube regulating device.

The generator is usually mounted in the slip stream of the propeller on one of the struts of the landing gear, as shown in Fig. 35.

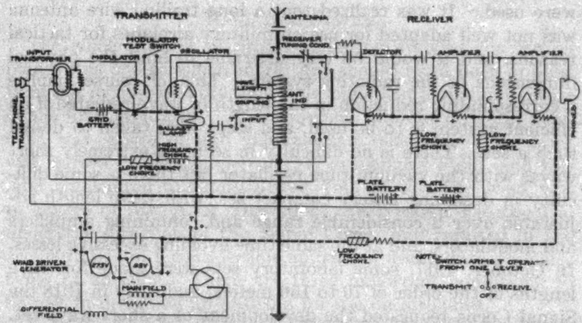

TRANSMITTER
ANTENNA
RECEIVER
MODULATOR TEST SWITCH
INPUT TRANSFORMER
MODULATOR
OSCILLATOR
RECEIVER TUNING COND.
DETECTOR
AMPLIFIER
AMPLIFIER
TELEPHONE TRANSMITTER
WAVE LENGTH
COUPLING
ANT IND
GRID BATTERY
BALLAST LAMP
INPUT
HIGH FREQUENCY CHOKE
LOW FREQUENCY CHOKE
LOW FREQUENCY CHOKE
LOW FREQUENCY CHOKE
PHONES
PLATE BATTERY
PLATE BATTERY
T
275V
25V
WIND DRIVEN GENERATOR
MAIN FIELD
DIFFERENTIAL FIELD
LOW FREQUENCY CHOKE
NOTE:- SWITCH ARMS T OPERA— FROM ONE LEVER
TRANSMIT
RECEIVE
OFF

FIG. 36—SCHEMATIC CIRCUIT DIAGRAM—RADIO TELEPHONE TRANSMITTING AND RECEIVING SET—SIGNAL CORPS SCR-68

The weight of the complete equipment including generator and two operators sets is approximately 58 pounds.

Fig. 36 shows, in simplified form, the electrical circuits.

When receiving only is required, the transmitting portion is omitted, and power is supplied to the filaments by a small storage battery. The receiving set is shown in Fig. 37.

Modifications of the SCR-68 sets adapted to different classes of service were developed and manufactured, one of the principal adaptations being a combination transmitting and receiving set for use on the ground. This is illustrated in Fig. 38. This set was employed principally for training purposes. With this means of communication at hand, the performance of the student could be observed in the air and his faults could be corrected by the instructor on the ground, thus greatly accelerating this training work. Energy for this set was provided by storage batteries, a dynamotor being operated by the storage battery to supply the plate potential.

Up to the time of the signing of the Armistice, many thousands of these sets had been manufactured and delivered.

SHORT WAVE TYPE

Since the first experiments the trailing wire antenna had been employed and wave lengths of from 200 to 500 meters were used. It was realized that a long trailing wire antenna was not well adapted for use on military airplanes for tactical reasons, and it would be very desirable to limit the physical dimensions of the radiating system. This, of course, implies a corresponding reduction in the length of wave radiated if an efficient system is to be used, and steps were taken to design such a set. There is no difficulty in securing extremely short waves with the vacuum tube oscillator but there is some difficulty in designing a very compact set, with wave length adjustable over a considerable range and containing amplifiers and modulators, and at the same time avoiding excessive losses. In October, 1917, some laboratory sets were made for wave lengths of the order of 70 to 150 meters, and early in 1918 the Signal Corps requested the development of a short wave set. This was built in April and was electrically essentially the same as the longer wave sets except for minor changes due to the use of higher frequencies of the order of 4,000,000 cycles.

Trials of this system were made at Camp Alfred Vail at wave lengths of 60 meters and above. The antenna first used was a very short structure on the top of the plane with two wires extending to the tail. This antenna had a natural wave length of 32 meters and a resistance of only one ohm. To increase the radiating qualities the rear portion of the antenna was raised and the resistance thereby brought up to nearly 3 ohms at 75 meters. This structure did not materially increase the head resistance of the plane and did not interfere with its operation. As described later, this form of antenna was soon replaced by another.

The use of short wave lengths brings froward prominently the problem of location of component parts of the set in the plane, for a few unnecessary feet of connecting wire carrying these high-frequency currents may cause the failure of the set. For this reason considerable thought was given to the question of location of transmitter and control box. Fig. 39 shows the final form of these short wave sets and it will be observed that the multiple unit plan of assembly has been carried further than heretofore, and this was due to the fact that difficulty was encountered in mounting the SCR-68 type set in the single place combat planes.

Fig. 40 shows the interior construction of the short wave transmitting and receiving units.

RADIATING SYSTEMS

The whole subject of proper radiating systems for use on planes was one to which more time should have been applied than was available at the time of development and manufacture of the first apparatus. A considerable amount of work, however, was carried on and valuable data obtained. This work was started in the summer of 1917 at Langley Field, and was later transferred to Camp Alfred Vail, N. J. The work was in charge of Mr. A. A. Oswald of the Western Electric Co. under the direction of the Signal Corps, and resulted in the accumulation of a considerable amount of useful data on many types of antenna. There are three important conditions to be met in designing an antenna for use on airplanes.

1. It must be an efficient radiator.
2. It must not be directive.
3. It must not interfere with complicated evolutions of the plane.

The third condition practically prohibits the use of a long trailing wire with a weight, because of the danger of fouling the propellor. This type of antenna is also quite directive.

The details of this antenna investigation should properly be covered in a separate paper. It may be well to state, however, that the best system, all things considered, was found to consist of two short unweighted wires, one from each wing tip. The two wires were joined in parallel above the fuselage, and worked against the conducting portions of the plane as a counterpoise. It was found that the use of one such wire as antenna and the other as counterpoise was not good.

In Fig. 41 the distribution of intensity of radiation is shown with the single long trailing wire and the two-wire type of antenna.

Ignition Interference

Another problem arose when sets began to be installed. This was that of noise in the receiving sets due to the engine ignition, and was very serious in some cases. Its solution was complicated by the fact that it was not allowable to alter the plane equipment in any way, for instance by changing the position of one magneto to secure greater shielding, but it was found that by covering the ignition wires with a flexible conducting tube grounded at intervals this trouble could be practically eliminated.

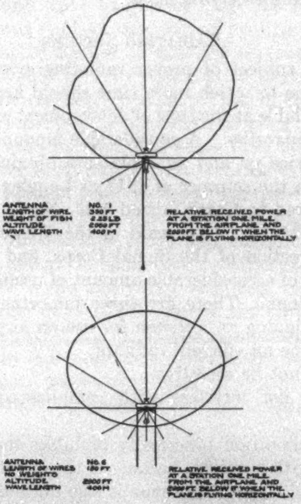


Fig. 41—Directional Characteristics of Airplane Antenna

It was found that in some cases the ignition spark started in the ignition system high-frequency oscillations which were of proper frequency to affect the receiver. This happened on the submarine chasers, which are driven by gasoline engines, and the trouble was remedied by inserting small iron core choke coils in the ignition leads to change the frequency of the oscillations. It is obvious that the "radio signals" sent out by the ignition spark are many millions of times more intense than those to be received and it is remarkable that they do not

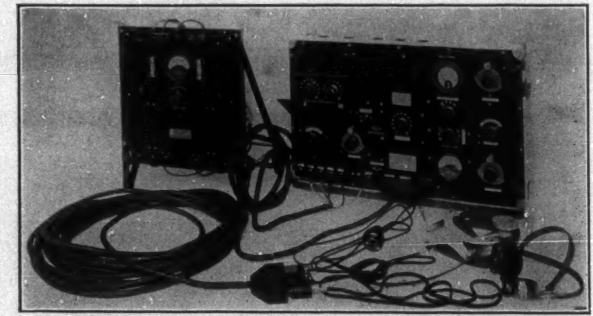


FIG. 38—RADIO TELEPHONE TRANSMITTING AND RECEIVING SET FOR GROUND USE—SIGNAL CORPS S C R-67

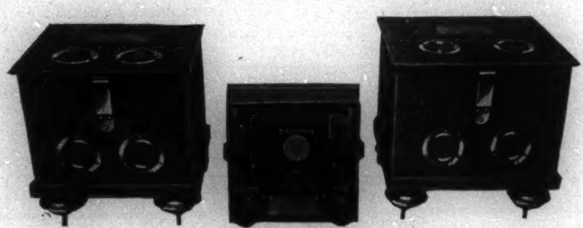
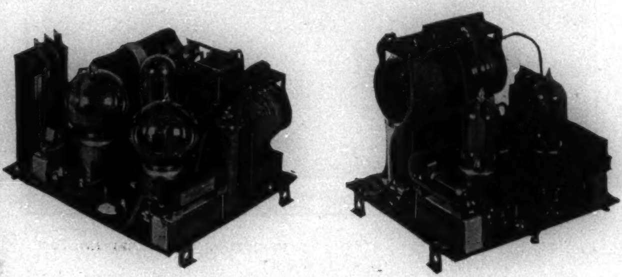


FIG. 39—SHORT WAVE TYPE AIRPLANE RADIO TELEPHONE SET



[CRAFTS & COLPITTS]

FIG. 40—INTERIOR CONSTRUCTION OF SHORT WAVE TRANSMITTING AND RECEIVING UNITS

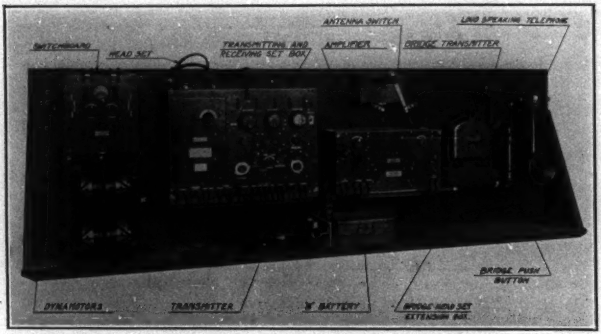
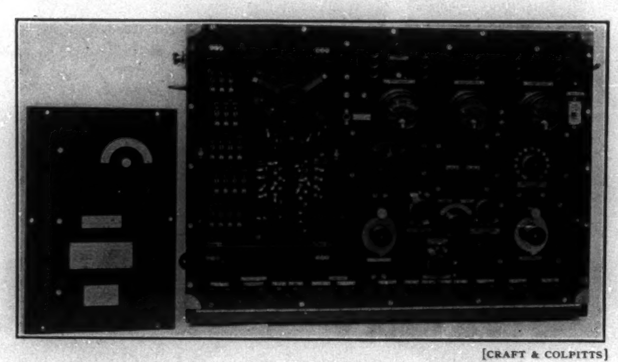

SWITCHBOARD
HEAD SET
TRANSMITTING AND RECEIVING SET BOX
ANTENNA SWITCH
AMPLIFIER
BRIDGE TRANSMITTER
LOUD SPEAKING TELEPHONE
BRIDGE PUSH BUTTON
DYNAMOTORS
TRANSMITTER
"B" BATTERY
BRIDGE HEAD SET EXTENSION BOX.

FIG. 42—COMPLETE WESTERN ELECTRIC TWO-STATION RADIO TELEPHONE EQUIPMENT FOR SUBMARINE CHASERS—BUREAU OF STEAM ENGINEERING STANDARD C W-936



[CRAFT & COLPITTS]

FIG. 43—SUBMARINE CHASER TYPE TRANSMITTING AND RECEIVING SET BOX—C W-936

entirely prevent reception. As a matter of fact, successful airplane telegraphy has been carried on in an experimental way by using the ignition magneto as a radio transmitter.

Submarine Chaser Equipment

During the war important use has been made of the wireless telephone in connection with naval work. Small 110-ft. craft had been largely employed in connection with anti-submarine operations. In order to make the most effective use of the various listening devices, and to co-ordinate the operations of the various units, it is very necessary that instantaneous and direct communication be established at all times. Under the direction of the Special Submarine Board of the Navy Department, modified forms of radio telephone equipment were developed for this purpose. Early in November 1917, the first practical trials were made, and satisfactory operation between chasers approximately five miles apart was obtained. These trials demonstrated the extreme value of this means of communication and sample equipments were immediately dispatched overseas for further trials under actual war conditions.

The circuits of this set are practically the same as the airplane equipment, except that power is obtained from small dynamotors operated from the 30-volt storage batteries with which the chasers are equipped. The set itself is located in the radio room, where it can be attended to by the regular radio operator. A telephone transmitter and receiver is, however, located in the pilot house so that the commanding officer can hold direct conversation with other vessels. The radio operator monitors the conversation and performs all tuning operations, thus leaving the commanding officer free to use the radio telephone as an ordinary wire line, except that he must press a button located at the side of the telephone when talking. The ordinary telephone head set is supplemented by a loud-speaking telephone receiver, which makes it possible for incoming signals to be heard without the use of head gear. Fig. 42 shows an assembled view of the various elements that go to make up a standard submarine chaser equipment. To provide sufficient energy for operating the loud speaking receiver a three-stage amplifier connected to the output side of the radio set is required. It is possible to connect the receiving portion of the radio set to either the ordinary head receiver or to the amplifier with its loud speaker.

Fig. 43 shows a close-up view of the radio transmitting and receiving set proper.

This equipment has an effective operating range of about ten miles when used on the 110-ft. submarine chasers. One feature not incorporated in the airplane sets consists of a wave length control gear by means of which the set can be operated on any one of five different wave lengths, ranging from 250 to 600 meters. This permits of a certain amount of selective operation within a chaser squadron.

A number of problems were met with in connection with this particular application not the least of which was the suppression of electrical disturbances in the antenna system due to the ignition system of the three propelling engines. Several thousand of these sets have been produced, and practically all submarine chasers sent overseas have been equipped with this apparatus. This apparatus is also being used for short range work on other types of naval vessels, with eminently satisfactory results. Modifications of the airplane type of equipment have also been made for use on naval seaplanes. For some classes of service the demand for longer range work has made necessary the employment of more powerful types of vacuum tubes.

The foregoing covers the most extensive application of vacuum tube radio telephony, the same general type of circuit and apparatus being used for short range telephony and telegraphy on land and water and in the air. For reasons already mentioned, quantity production has been limited to the earlier models, in spite of the fact that investigations carried on during production have indicated many changes which would improve the electrical and mechanical efficiency of the apparatus.

THE FUTURE OF RADIO-TELEPHONY

The possibility of communication by speech between any two individuals in the civilized world is one of the most desirable ends for which engineering can strive. For this reason it is particularly desirable to form some opinion of the part which radio telephony may play in securing this universal service. It is clear that the elimination of the Morse operator, which is accomplished by the use of radio telephony rather than radio telegraphy, is necessary for universal and direct communication.

Radio telephony and wire telephony offer several sharp contrasts. The latter requires fixed channels of communication whose construction and maintenance necessitates an accessible path between stations, but the results obtained include secrecy, power efficiency, selection of a desired station and freedom from interference. A large item of expense is the line. On the other hand radio telephony requires neither fixed nor accessible channels and no cost at all for line construction and maintenance, but it is non-secret in the practical sense of the word, its power efficiency is low, selection is at present not practicable except in a limited way by wave length, and freedom from interference is not at present an attained fact. Thus while the two systems may be contrasted they are not comparable but each is useful in its own field. It is easy to see that radio telephony can never compete with wire telephony in densely populated districts, while wire telephony is a physical impossibility at sea and in the air. Fortunately, however, the connection of a wire system to a radio system is no more complicated than connecting two wire lines by means of a repeater and therefore these two fields, although distinct, are adjacent. Leaving aside for the moment the particular methods by which radio telephone communication is to be carried on, it is clear that the establishment of communication between two given individuals will be most efficiently realized through the use of a combination of wire transmission on a network extending over perhaps 99 per cent of the stations and radio transmission to those relatively few stations to which it is either impossible or impracticable to build lines. These stations will be of two kinds:

1. *Moving*, such as ships, airplanes, trains, trucks.
2. *Fixed but inaccessible*, such as on islands, in deserts and in very sparsely settled regions.

A third class of service is that which is concerned, not with single individuals, but with groups; such service as the broadcasting of news, time and weather signals, and warnings. In some cases one objection to radio telephony would be an advantage in this class of service.

The choice of the particular kind of transmitting apparatus which will ultimately be used in thus extending the range of communication will involve careful consideration of costs and other engineering factors; its feasibility has, however, been conclusively demonstrated.

Presented at the 7th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 21, 1919.

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THEORY OF THE TRANSIENT OSCILLATIONS OF ELECTRICAL NETWORKS AND TRANSMISSION SYSTEMS

BY JOHN R. CARSON

Abstract of Paper

The purpose of this research was to make a broad theoretical study of transient phenomena with a view to developing methods of calculation directly applicable to engineering problems. The investigation starts with the problem of formulating the current in an electrical network or transmission system in response to a suddenly applied e.m.f. of arbitrary form. A simple formula is derived which expresses this current in terms of two independent functions: one, the applied e.m.f. expressed as a time function, and the other a characteristic function of the constants and connections of the system, this latter being termed the "indicial admittance" of the system. A knowledge of the indicial admittance, therefore, completely determines the behavior of the system to all types of applied voltages including both transient and steady states. A systematic investigation of methods for solving and computing the indicial admittance follows, in the course of which original solutions for transmission and artificial lines are derived and a new method involving integral equations is developed. This latter is particularly well adapted to handle the difficult problem of taking into account the effects of terminal impedances in transmission systems.

Introduction and Statement of Problem

WHEN an electromotive force is suddenly applied to an electrical network or system the current flowing in each and every part of the system is, at every instant of time, a determinate function of the form of the applied e. m. f. and of the connections and constants of the system. If the applied voltage is periodic (including, as a limiting case, the uniform steady voltage of zero periodicity) the system ultimately executes periodic oscillations of the same frequency as that of the applied voltage. These "steady-state" or periodic oscillations are the subject of the theory of alternating currents which takes up such a large part of the literature of electrical engineering. A more or less appreciable time after the application of the voltage elapses, however, before the proximate steady state is arrived at, and the oscillations which the system executes in this interval have been roughly termed "transient oscillations," this term implying, of course, that this "unsteady" state dies away or rather merges into the final steady state.

From a practical standpoint a knowledge of the steady state phenomena is sufficient in the great majority of problems. Theoretically, however, the so-called "transients" are of fundamental importance, and in certain problems, notably those of long distance telephony and telegraphy, the steady state theory is quite inadequate and the investigation of transient conditions becomes a matter of practical importance as well. The present paper is the outgrowth of an investigation, undertaken in connection with the writer's work in the Engineering Department of the American Telephone and Telegraph Company, of speech-excited transients in telephone systems, with particular reference to long, loaded cable circuits.

Stated in its broadest terms, our problem is to formulate, as a function of time, the current which flows in any part of an electrical system in response to a voltage of any specified form or type which is suddenly impressed on the system. After the development of the general theory and formulas, a considerable and by no means the least important part of the paper is devoted to deriving fundamental solutions, which admit of easy interpretation and computation, for the transmission line having continuously distributed inductance, capacity, resistance and leakage and the corresponding artificial line in which these constants are lumped. Formulas (**32**) to (**36**) and (**52**) to (**64**), which give the solutions for the current and voltage waves in all the important types of transmission system, are all the original results of the present research with the exception of formulas (**57**) and (**62**) which are the solutions for the current and voltage waves in the non-inductive, leakageless cable, first derived by Kelvin and Heaviside and formula (**53**) for the current wave in the leakageless line, which was independently derived by Heaviside and Poincare.

The investigation naturally falls into five parts:

1. The derivation of a general formula expressing the current as a function of the form of the applied voltage and the connections of the particular system under consideration. It will be shown that the current is simply expressed in terms of two functions, one the applied voltage, considered as a time

function, and the other a characteristic function of the system which is independent of the form of the applied voltage. This characteristic function will be termed the "indicial admittance" of the system, a terminology whose oppositeness will be justified by the physical and mathematical significance of the function. The word "indicial" indicates that the function is really an index of the system, and completely determines the behavior of the system to all types of applied voltages. On the other hand, the word "admittance" indicates that the function is of the nature of an admittance in a generalized sense, and that it has very real physical significance.

2. In practical problems, the applied voltage of the greatest importance is the mono-periodic or sinusoidal. For this particular type of e. m. f. special formulas are derived and a mechanical method of numerical solution is developed and described.

3. In (1) and (2) a knowledge of the "indicial admittance" is implied. A systematic investigation of the possible methods of solving for this function is taken up, and appropriate methods for particular types of systems, notably transmission systems, are developed.

4. The indicial admittance functions for all the important types of transmission systems, including the periodically loaded line are derived, and a large number of graphs of these functions is shown.

5. The appropriate treatment to take care of the effects of terminal apparatus in transmission systems is developed and applied to a number of specific problems.

I. General Theory and Formulas. The mathematical basis of the following investigation is furnished in a paper published by the writer in the *Physical Review* (Sept., 1917, pp. 217). A brief resume of the theory and formulas there developed will now be given as a necessary preliminary to the subsequent investigation; for a more detailed treatment the reader is referred to the original paper.

The current in any part of an electrical system or network is related to the impressed voltage (or voltages) by a set of linear differential equations in the variable t (time). If the applied voltage is taken as $E\,\epsilon^{pt}$, and if the differential operator $\frac{d^n}{d\,t^n}$ is replaced by p^n, the differential equations reduce to a set of simultaneous algebraic equations, and by purely algebraic processes we get the symbolic formula or solution

$$I = \frac{E\,\epsilon^{pt}}{Z\,(p)} \tag{1}$$

In the symbolic formula (**1**), $Z\,(p)$ is the impedance function; it is the ratio of two determinants (or the limit of such a ratio), and the equation $Z\,(p) = 0$, which is of frequent occurrence, will be termed the determinantal equation. If p is taken as a pure imaginary, formula (**1**) is simply the usual complex expression for the steady state current, and the real part of (**1**) the periodic current, corresponding to an impressed voltage of frequency $\frac{p}{2\,\pi}$. At present, however, no restriction is imposed on the operator p, and (**1**) is to be regarded as purely symbolic.

Starting with formula (**1**), the writer showed, in the paper referred to above, that, if the voltage is impressed on the system at time $t = 0$, the resultant current is expressed by the formula:

$$I = E\left[\frac{\epsilon^{pt}}{Z\,(p)} + \Sigma^{m}\,\frac{\epsilon^{p_m t}}{(p_m - p)\,Z'\,(p_m)}\right] \tag{2}$$

where p_m is the mth root of $Z\,(p)$, the summation is extended over all the roots of $Z\,(p)$ and

$$Z'\,(p_m) = \left[\frac{d}{d\,p}\,Z\,(p)\right]_{p\,=\,p_m}$$

In formula (**2**), p is no longer a mere operator and is assigned the value, real, imaginary or complex, which characterizes the impressed voltage. Thus if the impressed voltage is a unit d-c. voltage (zero before, unity after $t = 0$), $E = 1$, $p = 0$, (**2**) and becomes:

$$I = A\,(t) = \frac{1}{Z\,(0)} + \Sigma^{m}\,\frac{\epsilon^{p_m t}}{p_m\,Z'\,(p_m)} \tag{3}$$

which is the Heaviside expansion formula.

The function formulated by (**3**) is fundamental to our whole problem and will be defined as follows:

The *indicial admittance* of an electrical network or system is equal to the current, expressed as a time function, which flows in the system in response to a unit d-c. voltage (zero before, unity after $t = 0$) applied to the system at time $t = 0$. It is mathematically defined and formulated by (**3**) and will be denoted by $A\ (t)$ with appropriate subscripts. Thus if the applied voltage is inserted in the j^{th} mesh or branch of the network, the current in the k^{th} mesh or branch (assuming a unit d-c. voltage) will be denoted by $A_{jk}\ (t)$. The indicial admittance $A\ (t)$ is of the dimensions and character of an admittance in a generalized sense and is subject to the reciprocal relation $A_{jk}\ (t) = A_{kj}\ (t)$. When the two subscripts are the same the function may conveniently be referred to as the *driving point indicial admittance*, while when the subscripts are different, it will be termed the *transfer indicial admittance*, thus correlating our conceptions and terminology with those commonly employed in the theory of steady state or periodic phenomena. An important distinction, however, exists between the indicial admittance, as defined above, and the steady state admittance in that the former is a function of time while the latter is a function of frequency.

We shall now show that the "indicial admittance" completely determines the behavior of the system to all types of impressed voltages; in other words, that a knowledge of the indicial admittance alone is all that is required to completely specify the system, and predict its performance under all circumstances. The proof is based on a well known partial expansion which is employed and fully discussed in my paper referred to above. By a slight extension of the theory there developed it may be shown that all the impedance functions with which we are concerned admit of the partial fraction expansion:

$$\frac{1}{Z\ (p)} = \frac{1}{Z\ (\infty)} + \Sigma^m \frac{1}{(p - p_m)\ Z'\ (p_m)} \tag{4}$$

whence:

$$\frac{1}{Z\ (0)} = \frac{1}{Z\ (\infty)} - \Sigma^m \frac{1}{p_m\ Z'\ (p_m)} \tag{5}$$

Comparing (**3**) and (**5**) it is at once evident that

$$\frac{1}{Z\ (\infty)} = A\ (0)$$

$\frac{1}{Z\ (\infty)}$, and hence $A\ (0)$, is usually zero, except when, due to the absence of finite inductance, the current corresponding to $A\ (t)$ has an initial jump to a finite or even infinite value.

Returning now to equation (**2**) and replacing $\frac{1}{Z\ (p)}$ by its value as given by (**4**) we get:

$$I = E\left[\frac{\epsilon^{pt}}{Z\ (\infty)} + \epsilon^{pt}\ \Sigma_m\ \frac{\epsilon^{(p_m - p)t} - 1}{(p_m - p)\ Z'\ (p_m)}\right]$$

which is clearly equal to:

$$I = E\left[\frac{\epsilon^{pt}}{Z\ (\infty)} + \epsilon^{pt} \int_0^t \epsilon^{-p\lambda}\ \Sigma_m\ \frac{\epsilon^{p_m\lambda}}{Z'\ (p_m)}\ d\ \lambda\right] \tag{6}$$

But by (**3**)

$$\Sigma_m\ \frac{\epsilon^{p_m t}}{Z'\ (p_m)} = \frac{d}{d\,t}\ A\ (t) = A'\ (t)$$

$$\frac{1}{Z\ (\infty)} = A\ (0)$$

Substitution of these identities in (**6**)
gives

$$I = A\ (0)\ E\ \epsilon^{pt} + \int_0^t E\ \epsilon^{p\ (t-\lambda)}\ A'\ (\lambda)\ d\ \lambda \tag{7}$$

By obvious transformations (**7**) may also be written in either of the equivalent forms,

$$I = \frac{d}{dt}\int_0^t E\ \epsilon^{p\ (t-\lambda)}\ A\ (\lambda)\ d\ \lambda$$

$$= \frac{d}{dt}\int_0^t E\ \epsilon^{p\lambda}\ A\ (t-\lambda)\ d\ \lambda \tag{8}$$

which are sometimes more convenient to handle than is (**7**).

If the impressed voltage is the mono-periodic or sinusoidal function

$$E \sin\ (p\,t + \theta)$$

it is easy to show (by writing the sine function is its complex exponential form) that (**7**) becomes,

$$I = A(0)\, E \sin(p\,t + \theta) + \int_0^t E \sin(\,p\,(t-\lambda) + \theta\,)\cdot A'(\lambda)\, d\lambda \tag{9}$$

More generally if the impressed voltage $f(t)$ is not a pure sinusoid but is expressible (for $t > 0$) as a Fourier series:

$$f(t) = \Sigma^k\ a_k\ \sin(p_k\, t + \theta_k)$$

$$f(t-\lambda) = \Sigma^k\ a_k\ \sin(\,p_k\,(t-\lambda) + \theta_k\,)$$

equation (**9**) becomes, since the relations are linear,

$$I = A(0)\, f(t) + \int_0^t f(t-\lambda)\cdot A'(\lambda)\cdot d\lambda \tag{10}$$

while (**8**) becomes

$$I = \frac{d}{dt}\int_0^t f(t-\lambda)\cdot A(\lambda)\, d\lambda$$

$$= \frac{d}{dt}\int_0^t f(\lambda)\, A(t-\lambda)\, d\lambda \tag{11}$$

Equations (**11**) are the fundamental formulas which mathematically relate the current to the type of applied voltage and the connections of the system, and constitute the solution of the first part of our problem. The most important deductions from these formulas are expressed in the following theorems:

1. *The indicial admittance of an electrical network or system completely determines, within a single quadrature, the behavior of the system to all types of applied voltages.* As a corollary, a knowledge of the indicial admittance is the sole information necessary to predict completely the performance and characteristics of the system.

2. *The applied voltage and the indicial admittance are similarly and coequally related to the instantaneous current flowing in the system.* As a corollary, the form of the current may be modified either by changing the connections and constants of the system (and thus modifying $A(t)$) or by modifying the form of the applied voltage.

Since the applied voltage may be discontinuous formulas (**11**) not only determine the building up of the current in response to an applied voltage, but also its subsidence to equilibrium when an applied voltage is removed and the system is left to itself. In brief, *formulas* (**11**) *reduce the whole problem to a determination of the indicial admittances of the system,* a problem which forms the subject matter of succeeding sections of this paper.

The type of applied voltage of the greatest practical importance is the mono-periodic, or sinusoidal. In this case denoting the voltage applied at time $t = 0$, by $E \sin (p t + \theta)$, (**9**) is the appropriate formula. This may be written:

$$\begin{aligned} I = {} & A(0) E \sin (p t + \theta) \\ & + E \sin (p t + \theta) \int_0^t \cos p \lambda \cdot A'(\lambda)\, d\lambda \\ & - E \cos (p t + \theta) \int_0^t \sin p \lambda\, A'(\lambda)\, d\lambda \qquad (\mathbf{12}) \end{aligned}$$

which is the formula most convenient in practise. Another formulation which is sometimes preferable is derivable from the first of (**11**)

$$\begin{aligned} I = {} & E \sin \theta\, A(t) \\ & + E p \cdot \cos (p t + \theta) \int_0^t \cos p \lambda \cdot A(\lambda)\, d\lambda \\ & + E p \sin (p t + \theta) \int_0^t \sin p \lambda \cdot A(\lambda)\, d\lambda \qquad (\mathbf{12a}) \end{aligned}$$

Except in rare cases, however, formula (**12**) is preferable and further consideration will be limited to it. From formula (**12**) the final *steady state* current is given by

$$\begin{aligned} & A(0) E \sin (p t + \theta) \\ & + E \sin (p t + \theta) \int_0^\infty \cos p \lambda\, A'(\lambda)\, d\lambda \\ & - E \cos (p t + \theta) \int_0^\infty \sin p \lambda\, A'(\lambda)\, d\lambda \qquad (\mathbf{13}) \end{aligned}$$

While the transient distortion (*i. e.*, the difference between the actual and the steady state current) is given by

$$- E \sin (p t + \theta) \int_t^\infty \cos p \lambda A'(\lambda)\, d\lambda$$

$$+ E \cos (p t + \theta) \int_t^\infty \sin p \lambda A'(\lambda)\, d\lambda \qquad (\mathbf{14})$$

The resolution of the total current into two terms, first the steady state current as given by (**13**), and second, the transient distortion, as formulated by (**14**), is often of considerable practical advantage. In fact in many practical problems we are primarily concerned with the character and duration of the transient distortion. It should be observed that the "transient distortion" is formally defined by (**14**); it is the difference between the actual total current and the final steady state current.

Formula (**13**) gives the steady state current; this, of course, is derivable by other and far simpler methods so that (**13**) would, in practise, never be used to evaluate the steady state current. However, when equated with the usual steady state formulas, it furnishes an excellent check on the correctness of the theory and formulas, and also a means of evaluating some very complicated infinite integrals, the latter property being of very considerable mathematical interest.

Formula (**13**) also enables us to express the indicial admittance of the system in terms of its steady state admittance as follows. If the applied voltage is $E \sin (p t + \theta)$ and if

$$\frac{1}{Z(ip)} = \alpha(p) + i\,\beta(p)$$

the expression for the steady state current is by the usual methods

$$I = \alpha(p) \,.\, \sin (p t + \theta) + \beta(p) \cdot \cos (p t + \theta)$$

But this is equivalent to (**13**); hence equating the sine and cosine terms,

$$\alpha(p) = A(0) + \int_0^\infty \cos p \lambda A'(\lambda)\, d\lambda$$

$$\beta(p) = - \int_0^\infty \sin p \lambda A'(\lambda)\, d\lambda$$

$\alpha(p)$ and $\beta(p)$ are, of course, the real and imaginary components of the steady state admittances. The first of these equations can be solved giving

$$A(t) = A(0) + \frac{2}{\pi}\int_0^\infty (\alpha(p) - A(0))\,\frac{\sin(t\,p)}{p}\,d\,p$$

A very simple example must suffice for the present to illustrate the foregoing theory. Let the system consist of an inductance L in series with a resistance R. Then, by the expansion theorem,

$$A\,(t) = \frac{1}{R}\left(1 - \epsilon^{-\frac{R}{L}t}\right)$$

$$A'(t) = \frac{1}{L}\,\epsilon^{-\frac{R}{L}t}$$

Substituting in (**12**), the current is given by

$$I = \frac{E}{L}\sin(p\,t + \theta)\int_0^t \epsilon^{-\frac{R}{L}\lambda}\cos p\,\lambda\,d\,\lambda$$

$$-\frac{E}{L}\cos(p\,t + \theta)\cdot\int_0^t \epsilon^{-\frac{R}{L}\lambda}\sin p\,\lambda\,d\,\lambda$$

which integrates to

$$I = \frac{E}{(p\,L)^2 + R^2}\,\{R\sin(p\,t + \theta) - p\,L\cos(p\,t + \theta)\}$$

$$+\frac{E\,\epsilon^{-\frac{R}{L}t}}{(p\,L)^2 + R^2}\,\{p\,L\cos\theta - R\sin\theta\}$$

Here the first term is the steady state and the second term the transient distortion which is exponentially damped, so that the two components—steady state and transient distortion—are automatically separated by the process of integration.

Still limiting our consideration to sinusoidal voltages, the reader may have remarked that we can get the formal solution for the current directly from (**2**) by making p a pure imaginary and thus dispense with the definite integrals of formula (**12**) altogether. When the number of degrees of freedom of the system is small, and when the roots of the determinantal equation $Z(p) = 0$ are easily located, the direct evaluation of formula (2) is indicated; in the great majority of problems, however, we shall find that the employment of the definite integral formula (**12**) effects a great saving of labor, and that, in fact, the solution obtained directly from (**2**) does not admit, practically speaking, of either interpretation or computation. Without going into details, the chief reasons for this are briefly as follows:

1. The direct evaluation of the current from formula (**2**) requires that the indicial admittance be expressed as a sum of characteristic oscillations or vibrations, a form of solution which is not only usually to be avoided for practical reasons but which in a great many problems cannot be obtained.

2. *The definite integrals of formula* (**12**) *admit of mechanical integration when the indicial admittance is known.* On this fact rests the practical, as distinguished from the purely theoretical, value of the formula.

In order to explain the mechanical evaluation or integration of the integrals appearing in (**12**), we remark that they are of precisely the same form as the harmonic components of a Fourier series, differing therefrom only in that no periodicity on the part of $A\ (t)$ is either assumed or implied and in that the upper limit of integration is variable. The suggestion to employ a harmonic analyser is immediate and obvious, and upon investigation it was found that a Coradi harmonic analyser of the rolling sphere type was adapted to evaluate directly the integral terms of (**12**) when $A\ (t)$ is known.

The analyser is applied to the problem of mechanical integration and operates as follows: Having plotted $A\ (t)$ to a suitable time scale (depending on the frequency $p/2\pi$ which is specified, and the geometrical dimensions of the machine), the curve is traced with the pointer of the analyser, precisely as we trace a curve with an integraph or planimeter. As we trace this curve the recording mechanism of the analyser gives directly the progressive values, corresponding to any particular value of t, of the integrals

$$\int_0^t \cos p\,\lambda\,A'\,(\lambda)\,d\,\lambda$$

$$\int_0^t \sin p\,\lambda\,A'\,(\lambda)\,d\,\lambda$$

the value of p depending on the time scale to which $A(t)$ is plotted. But these are precisely the integrals we require to evaluate formula (**12**), and the analyser therefore furnishes the means for the mechanical solution of our problem in a simple direct manner with a minimum of labor. The analyser employed by the writer has three separate harmonic rollers so that formula (**12**) can be evaluated for three different frequencies simultaneously.

I do not wish to give the impression that the labor of computation is reduced to a negligible quantity, for it is still considerable by reason of the very nature of the problem. This follows from the fact that we require, not a single numerical value of I for a particular value of t, as is the case for steady state conditions, but a complete history of the current, as a function of time, from its start up to its steady state value. What I do claim, however, is that the labor is by no means prohibitive and that mechanical integration as described above, furnishes a thoroughly practicable, if not the only practicable means of numerical solution.*

II. The Determination of the Indicial Admittance $A(t)$

By virtue of formulas (**11**) and (**12**), derived and discussed in the preceding section, our problem is reduced to a determination of the indicial admittance $A(t)$. In other words, our problem is now restricted to the formulation of the current flowing in any part of the electrical system or network in response to a unit d-c. voltage (zero before, unity $t = 0$) applied to the system. To the solution of this problem Heaviside devoted the greater part of the first two volumes of his "Electromagnetic Theory" and no student of this subject can fail to express the greatest admiration for his researches, an acquaintance with which is indeed essential to any investigation of the problem. While new problems have presented themselves, the study of which has resulted in new methods and original solutions, Heaviside was the first to treat the problem comprehensively and to insist on solutions which admitted of physical and numerical interpretation, as distinguished from merely formal mathematical expressions which conceal the real nature of the solution as effectually as do the original differential equations of the problem. Furthermore, his expansion theorem alone is a powerful instrument and forms the basis, not only for the special methods of solution developed in this section but also for the transmission line formulas derived in the succeeding sections:

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*See Note 1 at end of paper.

The formal solution of our problem, the determination of the indicial admittance $A(t)$ is furnished by the expansion theorem (**3**):

$$A(t) = \frac{1}{Z(0)} + \sum^m \frac{\epsilon^{p_m t}}{p_m Z'(p_m)}$$

For systems or networks which have a relatively small number of degrees of freedom, and the roots of whose determinantal equation $Z(p) = 0$ admit of evaluation, formula (**3**) furnishes the working or practical as well as the formal solution. In the great majority of problems, however, the expansion theorem, as it stands, is of restricted utility as a working solution for several reasons, the chief of which are that it requires the location of the roots of $Z(p)$ and that it involves a prohibitive amount of labor for computation when the number of degrees of freedom is large. On the basis of the expansion theorem we therefore proceed to develop derived methods of solution.

1. *The Power Series Solution.* The solution for $A(t)$ can always be written down in the form of a power series in the variable t (time) without evaluating the roots of the determinantal equation $Z(p) = 0$, provided the function $\dfrac{1}{Z(p)}$ admits of asymptotic expansion* in the variable p. To prove this theorem and derive the power series solution we proceed as follows:

Return to formula (**3**) and expand the exponential functions in the usual power series. Since this expansion is absolutely convergent, the resultant expansion is absolutely convergent. Making use of the relation (**5**), formula (**3**) then becomes,

$$A(t) = \frac{1}{Z(\infty)} + \frac{t}{1\,!}\sum \frac{1}{Z'(p_m)} + \frac{t^2}{2\,!}\sum \frac{p_m}{Z'(p_m)} + \dots + \frac{t^n}{n\,!}\sum \frac{p_m{}^{n-1}}{Z'(p_m)} + \dots$$

Now take equation (**4**) and write it as

$$\frac{1}{Z(p)} = \frac{1}{Z(\infty)} + \frac{1}{p}\sum \frac{1}{Z'(p_m)}\left(1 - \frac{p_m}{p}\right)^{-1}$$

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*See Note 2.

Expanding the second factor of the summation by the binomial theorem, and arranging in inverse powers of p, we get,

$$\frac{1}{Z\ (p)} = \frac{1}{Z\ (\infty)} + \frac{1}{p}\ \Sigma\ \frac{1}{Z'\ (p_m)} + \frac{1}{p^2}\ \Sigma\ \frac{p_m}{Z'\ (p_m)}$$

$$+ \ldots . + \frac{1}{p'}\ \Sigma\ \frac{p_m{}^{n-1}}{Z'\ (p_m)} + \ldots . .$$

Comparing these two expansions we see at once that the coefficient of $\frac{t^n}{n\,!}$ in the former is identical with the coefficient of $\frac{1}{p^n}$ in the latter. Furthermore, the latter is nothing more than the asymptotic expansion of $\frac{1}{Z\ (p)}$. We therefore arrive at the following rule:

Expand the function $\frac{1}{Z\ (p)}$ *asymptotically in the form*

$$\frac{1}{Z\ (p)} = \sum_{0}^{\infty}{}^{k}\ \frac{a_k}{p^k},$$

Then the indicial admittance is given by

$$A\ (t) = \sum_{0}^{\infty}{}^{k}\ a_k \frac{t^k}{k\,!}$$

which is obtained from the asymptotic expansion by replacing $\frac{1}{p^n}$ *by* $\frac{t^n}{n\,!}$.

Those familiar with Heaviside's Electromagnetic Theory Vol. II, will recognize in the foregoing expansion the equivalent of his operational solution. Heaviside himself refrained from giving any justification of the solution beyond the pragmatic one that "it works," and so far as I am aware, the above furnishes the first formal proof of its correctness.

As regards the practical value of the power series expansion as a working solution, its utility is rather restricted (except when the expansion can be identified with known functions) by reason of the slow convergence of the series. A second disadvantage lies in the fact that the asymptotic expansions are usually difficult to obtain and of complicated form. Finally, it should be remarked that while $\frac{1}{Z(p)}$ always admits of asymptotic expansion for networks having a finite number of degrees of freedom, the asymptotic expansion of $\frac{1}{Z(p)}$ for transmission lines, where the number of degrees of freedom is infinite and trancendental functions are involved, is impossible. The method is, however, of very great mathematical interest, and is useful in computing $A(t)$ for small values of t.

We cannot at present take up the more developed series solutions of Heaviside where fractional powers of t appear. These solutions are of great mathematical interest and will be referred to later in connection with the treatment of transmission line problems. It may be remarked, however, in passing, that their chief application is in connection with the non-inductive cable which is of minor practical importance and whose solution is otherwise derived in Section IV.

2. *The Solution from Fourier's Integral.** It is always possible to express the current as a Fourier integral of the following form:

$$I = \frac{1}{2\pi i}\int_{-\infty}^{\infty} f(\lambda)\, d\lambda \int_{-\infty i}^{\infty i} \frac{\epsilon^{p(t-\lambda)}}{Z(p)}\, dp$$

where $f(t)$ is the impressed voltage and $Z(p)$ is the impedance function. It can be shown that this double integral is capable of reduction to the expansion theorem solution or the power series solution (for either integral or fractional powers of t), and it thus furnishes a check on these fundamental formulas.

3. *The Synthetic Method.* In many problems which do not admit of a working solution by direct methods of attack, the solution can be built up from the solutions for the component parts of the system under consideration by a method recently developed and employed by the writer. A notable example of the type of problem to which the method is applicable is furnished by the transmission line. Here, as we shall show later, the solution can be obtained for a line of any length if the voltage is applied directly to one end of the line and the other

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*See Note 3.

end is short-circuited. When, however, the voltage is applied through terminal apparatus, and the other end is closed through terminal apparatus, or when apparatus is inserted at any point along the line, the solution by direct frontal attack is impossible. The effects of such terminal or intermediate apparatus can be solved for, both formally and numerically, by the synthetic method which promises to be an instrument of value in the solution of practically important problems. Again the system may be a "compound" system or network which is made up of two separate networks, coupled or connected in any specified manner. This method furnishes us a means for determining the indicial admittances of the compound system in terms of the indicial admittances (assumed known) of the component networks, when direct methods are incapable of yielding a solution. A particular problem of this character is the determination of the modifying effects of inserting additional impedance elements in a network.

The method, as the name implies, is synthetic and physical in character, and the formulas are indeed based on physical reasoning rather than on mathematical analysis. From the mathematical standpoint, however, the method is of interest in that it applies the theory of integral equations* to the transient problem.

Briefly stated, the principle underlying the method consists in building up the solution for a network or system from the known solutions for its component parts. We now proceed to a discussion of the theory and a derivation of the formulas.

To make the application as general as possible we shall assume that we have two networks or systems, and we shall further assume that the indicial admittances of each are known. Our problem is to determine the indicial admittances (and hence the complete behavior and characteristics) of the compound network or system which is formed by connecting the two given systems in any specified manner. A sufficiently general manner of connecting the two networks is to open any mesh or branch of one network and any mesh or branch of the other network, and to connect the corresponding open terminals of the two networks, thus forming a compound network. This manner of coupling is illustrated schematically in the accompanying sketches.

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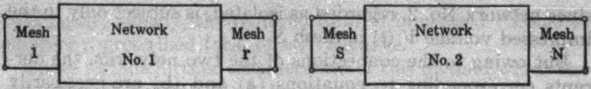
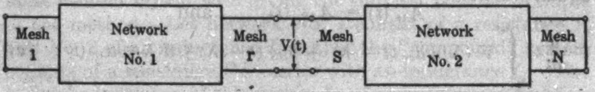
*See Note 4.





To proceed with the theory, let network No. 1 consist of m meshes or branches, numbered from 1 to m, and let network No. 2 consist of n meshes or branches, numbered from I to N. Let the indicial admittances of No. 1 be denoted by $a_{jk}(t)$ (the subscripts depending, of course, on the meshes with which we are concerned) and those of No. 2 by $b_{JK}(t)$. Now to couple these networks open any mesh, say the r^{th} of No. 1, and any mesh, say the S^{th} of No. 2, and connect the open terminals of mesh r to open terminals of mesh S. The two networks are thus connected together through their r^{th} and S^{th} meshes respectively and form a compound system. We require the indicial admittances of the compound system in terms of the known indicial admittances of the two component networks.

To formulate the required admittances, assume, as usual, a unit e.m.f. inserted at time $t = 0$ in any mesh of the compound system, say the 1st mesh of network No. 1; then by definition the current flowing in mesh j of network No. 1 is $A_{1j}(t)$ and the corresponding current flowing in mesh J of network No. 2 is $A_{1J}(t)$.

Now an unknown voltage $V(t)$ exists across the common terminals which connect the two networks; hence the current flowing in any mesh of network No. 1 is the same as that which would flow if the network were isolated and actuated by a unit e.m.f. inserted in mesh 1 and an e.m.f. $-V(t)$ in mesh r. Hence by reference to formula (**11**)

$$A_{1r}(t) = a_{1r}(t) - \frac{d}{dt}\int_0^t V(\lambda)\, a_{rr}(t-\lambda)\, d\lambda \qquad \textbf{(a)}$$

By precisely the same physical reasoning we get

$$A_{1S}(t) = \frac{d}{dt}\int_0^t V(\lambda)\cdot b_{SS}(t-\lambda)\, d\lambda \qquad \textbf{(b)}$$

since network No. 2, regarded as isolated, is subject only to the impressed voltage $V(t)$ in mesh S.

But owing to the connections of the two networks, the currents corresponding to equations (**a**) and (**b**) are necessarily equal, whence:

$$A_{1r}(t) = A_{1S}(t) \quad \text{, and}$$

$$\frac{d}{dt}\int_0^t V(\lambda)\left[a_{rr}(t-\lambda) + b_{SS}(t-\lambda)\right] d\lambda = a_{1r}(t) \qquad \mathbf{(c)}$$

Now this is an integral equation in the unknown voltage $V(t)$ and is reducible to the canonical form

$$V(t) = f(t) + \int_0^t V(\lambda)\, u(t-\lambda)\, d\lambda \qquad \mathbf{(d)}$$

where $f(t)$ and $u(t)$ are known functions.

The formal mathematical solution of this integral equation can be written down, by the method of successive substitutions, in the form of an absolutely convergent series. Furthermore, without solving the integral equations, we can immediately derive the power series solution (in case a power series solution in integral powers of t exists) by repeated differentiations of equation (**d**). When the power series solution involves fractional powers of t, this fact is at once evident from the form of the known functions appearing in (**d**), and the correct series solution can be derived from the integral equation as it stands. Having solved in some such manner for $V(t)$ the required indicial admittances are obtained from equations (**a**) and (**b**) by a single quadrature.

The practical value of this method of solution, as distinguished from its purely theoretical and mathematical interest, lies in the possibility of the numerical integration of the integral equations in cases where the formal and power series solutions are impracticable from the standpoint of interpretation or computation. The numerical integration works out well in practise, it may be made as accurate as we please, and gives the required numerical solution directly and with a minimum of labor. In practise, furthermore, the method is not so complicated as might appear from the sketch of the theory and often various short cuts are suggested by physical considerations. It is also applicable to problems other than that explicitly considered above; for example it yields a relatively easy numerical solution of an important problem in telegraphy, namely, the required form of impressed voltage to produce a prescribed form of arrival curve.

Space forbids further consideration of this method which the writer hopes to treat in greater detail in a future paper, and I must content myself with giving the simplest possible example of the method. Let network No. 1 consist of a resistance R and No. 2 of an inductance L so that the "compound" system consists of a resistance R in series with an inductance L. The indicial admittance of this system is, of course, known to be

$$\frac{1}{R}\left(1-\epsilon^{-\frac{R}{L}t}\right).$$

This solution will now be verified by the method just discussed. We have

$$a\,(t)=\frac{1}{R}$$

$$b\,(t)=\frac{1}{L}\,t$$

Substitution in (c) gives

$$\frac{d}{dt}\int_0^t V\,(\lambda)\left[\frac{1}{R}+\frac{1}{L}\,(t-\lambda)\right]d\,\lambda=\frac{1}{R}$$

Differentiating and rearranging

$$V\,(t)=1-\frac{R}{L}\cdot\int_0^t V\,(\lambda)\,d\,\lambda$$

By the method of successive substitutions the solution of this integral equation is:

$$V\,(t)=1-\frac{R}{L}\cdot\int_0^t d\,\lambda+\left(\frac{R}{L}\right)^2\cdot\int_0^t\int_0^\lambda d\,\lambda\,d\,\lambda_1$$

$$-\left(\frac{R}{L}\right)^3\cdot\int_0^t\int_0^\lambda\int_0^\lambda d\,\lambda\;d\,\lambda_1\;d\,\lambda_2+\ldots$$

$$=1-\left(\frac{R}{L}\right)\frac{t}{1\,!}+\left(\frac{R}{L}\right)^2\frac{t^2}{2\,!}$$

$$-\left(\frac{R}{L}\right)^3\frac{t^3}{3\,!}\;+$$

$$=\epsilon^{-\frac{R}{L}t}$$

Hence

$$A(t) = \frac{1}{R}(1 - V(t))$$

$$= \frac{1}{R}\left(1 - \epsilon^{-\frac{R}{L}t}\right)$$

which is the correct solution.

4. *Special Methods for the Solution of Transmission Systems.* In treating the problem presented by the transmission system, embracing, on the one hand, the transmission line having continuously distributed constants, and on the other, the artificial line whose constants are lumped or localized, and including as intermediate between these two the periodically loaded cable, we shall derive the basic solutions for the simplest possible configuration, namely when the voltage is applied directly to the line and the distant end is short-circuited. It will then be shown that the effect of terminal and intermediate apparatus can be formulated and numerically computed from the basic solution by aid of the synthetic method of treatment just discussed. The solution for the line itself is therefore fundamental.

Now limiting for the present our consideration to the line which is short-circuited at one end, and excited by the direct application of a voltage at the other end, the symbolic formula is readily written down; its general form is well known to be

$$\frac{1}{K}\frac{\cosh\gamma(s-x)}{\sinh(\gamma s)}\cdot\epsilon^{pt}$$

This is the symbolic formula for the current at point x of a line which is short-circuited at $x = s$ and is excited by a voltage applied at $x = 0$. K and γ are functions of the operator p, and the line constants. (If we are dealing with an artificial line s and x are to be interpreted as number of sections instead of distance; the form of function is, however, the same in both cases.)

Now for the smooth line (having continuously distributed constants) and for the artificial line, the expansion theorem solution can be written down without any trouble. As a working or practical solution this is, however, of no utility as it stands, owing to the fact that the number of terms in the summation is infinite for the smooth line (and very large for long artificial lines) and the convergence very slow. Physically this difficulty arises from the fact that the expansion theorem solution formulates the current or indicial admittance, as a sum of characteristic oscillations or vibrations, a formulation which is mathematically correct but physically artificial. In reality the current travels out in the form of a wave which is reflected at the distant end, and which is independent of the distant termination for a definite time. Each one of the modes of characteristic oscillation involves, however, the character of the distant termination. Investigation shows that if we deal with the *infinitely* long line the sum of characteristic oscillations is converted into the direct wave which alone exists in the infinitely long line, and that this wave is expressible in terms of known functions.

To handle the case of the infinitely long line we set up the expansion theorem solution for a line of length s and then let s approach the limit infinity. As s approaches infinity, the summation of the characteristic oscillations is converted into a definite integral, which, for the practically important types of lines considered in succeeding sections, admits of evaluation in terms of known functions.

To now apply the solution for the infinitely long line to the case of the line of finite length we remark that the current, say $\phi_x(t)$, which exists at point x and time t of a line of infinite length with unit d-c. voltage (zero before, unity after $t = 0$) applied at $x = 0$, is the solution of the problem whose symbolic formula or solution is

$$\frac{1}{K}\cdot\epsilon^{-x\gamma}$$

Now, take the formula given above, for the line of finite length s, express the hyperbolic functions in their equivalent exponential form, and expand; we get

$$\frac{1}{K}\epsilon^{-\gamma x}+\Sigma\frac{1}{K}\epsilon^{-\gamma(2ms+x)}+\Sigma\frac{1}{K}\epsilon^{-\gamma(2ms-x)}$$

Now, by definition, $\phi_x(t)$ is the solution for the first term; hence the solution for the term, $1/K\ \epsilon^{-(2ms+x)}$, is

$$\phi_{2ms+x}(t)$$

which is gotten from $\phi_x(t)$ (a function of x and t) by replacing x with $2ms + x$. Hence the solution for the short-circuited line of length s is simply

$$A_x(t) = \phi(t) + \Sigma\,\phi_{2ms+x}(t) + \Sigma\,\phi_{2ms-x}(t) \tag{15}$$

The first term in the above is the direct wave (the only wave which would exist if the line were infinitely long) the first summation represents the series of waves reflected from the sending end, and the second summation the series of waves reflected from the other end of the line. The summations are rapidly convergent, and the above furnishes the working solution which we shall employ.

If, now a voltage $V_1(t)$ is applied at $x = 0$, and a voltage $V_2(t)$ at $x = s$, the current $I_x(t)$ at point x is the aid of formula (**11**):

$$I_x(t) = \frac{d}{dt}\int_0^t V_1(t-\lambda)\left\{\begin{array}{l}\phi_x(\lambda) + \Sigma m\ \phi_{2ms+x}(\lambda) \\ \quad + \Sigma m\ \phi_{2ms-x}(\lambda)\end{array}\right\} d\lambda \tag{16}$$

$$- \frac{d}{dt}\int_0^t V_2(t-\lambda)\left\{\begin{array}{l}\phi_{s-x}(\lambda) + \Sigma m\ \phi_{(2m+1)s-x}(\lambda) \\ \quad + \phi_{(2m-1)s+x}(\lambda)\end{array}\right\} d\lambda$$

This is a formula of fundamental importance to the whole theory of transmission systems. If the line is of great length (large attenuation) and the applied voltages are alternating, the reflected waves are negligible and we have the approximate formula,

$$I_x(t) = \frac{d}{dt}\int_0^t V_1(t-\lambda)\,\phi_x(\lambda)\,d\lambda$$

$$- \frac{d}{dt}\int_0^t V_2(t-\lambda)\,\phi_{s-x}(\lambda)\,d\lambda$$

The *driving point* indicial admittance of the line is obtained from (**15**) by setting $s = 0$; denoting it by $A_1(t)$ it is

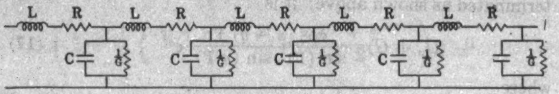
$$A_1(t) = \phi_0(t) + 2\sum_1^\infty m\ \phi_{2ms}(t) \tag{16}$$

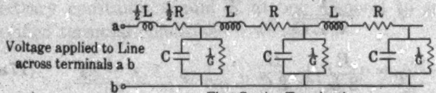
The *transfer* indicial admittance; *i. e.* the current at $x = s$ with line short-circuited at $x = s$, is obtained from (**15**) by setting $x = s$. Denoting this function by $A_2(t)$ it is:

$$A_2(t) = 2\sum_1^\infty m\ \phi_{(2m-1)s}(t) \tag{16a}$$

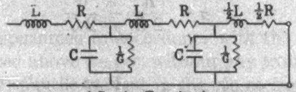
III. Solution for the Artificial Line

The artificial line is a periodic structure of the character shown in the sketch herewith.





First Section Termination



n^{th} Section Termination

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Each section of the structure consists of an inductance L and a resistance R in series with the line, and the parallel combination of a capacity C and a leakage conductance G in shunt across the line. The structure which we shall consider is of n sections in length, and the voltage is applied at "mid-series" position, and the nth section is short-circuited at mid-series as shown in the sketches herewith. These terminations are chosen not only because they simplify the analysis somewhat but because they are of practical importance.

A study of the transient properties of the artificial line is important for several reasons:

1. The artificial line is often used as a model of an actual transmission line and it is therefore of importance to determine theoretically the degree of correspondence between the two.
2. The solution for the corresponding transmission line with continuously distributed constants is derivable from the solution for the artificial line by keeping the total inductance, resistance, capacity and leakage constant or finite, and letting the number of sections approach infinity.
3. The artificial line is very closely related, in its properties and performance, to the periodically loaded cable, and its solution is, to a first approximation, a working solution for the loaded cable.

4. The structure is of great importance in its own right and is one form of "wave filter" employed in telephone systems.*

The starting point of the solution is the symbolic formula for the current in the qth section of a line of n sections in length, terminated as shown above; it is

$$-\,(C_p + G)\cdot\frac{\cos\,(n-q)\;\Gamma}{\sin\,(\Gamma)\cdot\sin\,(n\;\Gamma)}\;\epsilon^{pt} \tag{17}$$

where

$$v = \frac{1}{\sqrt{L\,C}}$$

$$\rho = \frac{R}{2\,L} + \frac{G}{2\,C} \tag{17a}$$

$$\sigma = \frac{R}{2\,L} - \frac{G}{2\,C}$$

and

$$\cos\Gamma = 1 + \frac{1}{2}\,\frac{(p+\rho)^2 - \sigma^2}{v^2} \tag{17b}$$

This formula may be deduced in a number of ways; perhaps the simplest is a follows: Considering the qth section, write down the circuital equation for the voltage drop around the closed section; after replacing $\frac{d}{dt}$ by p we get (denoting the circuital currents in the $(q-1)st$, qth and $(q+1)st$ sections by I_{q-1}, I_q and I_{q+1} respectively);

$$I_q\left(L\,p + R + \frac{2}{C\,p+G}\right) - \frac{1}{C\,p+G}\,I_{q-1}$$

$$-\frac{1}{C\,p+G}\cdot I_{q+1} = 0$$

or

$$I_q\left(\frac{(p+\rho)^2-\sigma^2}{v^2} + 2\right) - I_{q-1} - I_{q+1} = 0 \tag{18}$$

Equation (**18**) is a difference equation relating the currents in the three adjacent sections; in accordance, therefore, with usual methods we assume a solution of the form

$$I_q = A\;\epsilon^{iq\Gamma} + B\;\epsilon^{-iq\Gamma} \tag{19}$$

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*See Note 5.

where A and B are arbitrary constants and $i\,\Gamma$ is the exponential propagation factor which is to be determined. Substitution of (**21**) in (**20**) gives after simplification:

$$\left(A\,\epsilon^{iq\Gamma} + B\,\epsilon^{-iq\Gamma}\right)\cdot$$
$$\cdot\left(1 + \frac{1}{2}\left(\frac{(p+\rho)^2 - \sigma^2}{v^2}\right) - \frac{\epsilon^{i\Gamma} + \epsilon^{-i\Gamma}}{2}\right) = 0$$

Clearly this equation is satisfied and furthermore leaves two arbitrary constants A and B at our disposal to satisfy any specified terminal conditions if we set

$$\frac{\epsilon^{i\Gamma} + \epsilon^{-i\Gamma}}{2} = \cos\Gamma = 1 + \frac{1}{2}\,\frac{(p+\rho)^2 - \sigma^2}{v^2}$$

which agrees with (**17b**)

If A and B are now determined in accordance with the terminal conditions illustrated above we get, without any trouble, the symbolic formula (**17**) for the current in the qth section of the structure.

Assuming now a unit d-c. voltage applied to the line at time $t = 0$, the current in the qth section is by definition the indicial admittance of the line and will be denoted by $A_q\,(t)$, the double subscript being omitted for convenience; $A_q\,(t)$ is formulated by applying the expansion theorem to the symbolic formula (**17**) with the result that we get without any trouble

$$\begin{aligned} A_q\,(t) = &- \sum \frac{C\,p_m \cos\,(q\,\Gamma_m)}{n\cdot p_m\,.\left(\dfrac{d\,\Gamma_m}{d\,p_m}\right)\sin\,\Gamma_m}\,\epsilon^{p_m t} \\ &- \frac{G}{C}\sum \frac{C\cdot\cos\,(q\,\Gamma_m)}{n\cdot p_m\cdot\left(\dfrac{d\,\Gamma_m}{d\,p_m}\right)\sin\,\Gamma_m}\,\epsilon^{p_m t} \\ &+ \frac{G}{C}\sum \frac{C\cdot\cos\,(g\,\Gamma_m)}{n\cdot p_m\,\left(\dfrac{d\,\Gamma_m}{d\,p_m}\right)\sin\,\Gamma_m} \end{aligned} \qquad (\mathbf{20})$$

In equation (**20**) p_m and Γ_m are determined by the determinantal equation

$$\sin\,(n\,\Gamma) = 0$$

whence

$$\Gamma_m = m\,\pi/n \qquad\qquad m = 0, 1, 2, \ldots\ldots n$$

and

$$\frac{(p_m+\rho)^2-\sigma^2}{2\,v^2} = -\,(1-\cos\Gamma_m)$$

It will be observed that (**20**) may be written

$$\begin{aligned} A_q\,(t) = &-\sum \frac{C\cdot\cos\,(q\,\Gamma_m)}{n\cdot\sin\,\Gamma_m\cdot\left(\dfrac{d\,\Gamma_m}{d\,p_m}\right)}\,\epsilon^{p_m t} \\ &-\frac{G}{C}\int_0^t d\,t_1\cdot\sum \frac{C\cdot\cos\,(q\,\Gamma_m)}{n\cdot\sin\,\Gamma_m\cdot\left(\dfrac{d\,\Gamma_m}{d\,p_m}\right)}\cdot\epsilon^{p_m t_1} \end{aligned} \qquad \textbf{(21)}$$

since the last two terms are, within a constant, the integral of the first term. This effects a great simplification and reduces our problem to a solution of the first summation of (**20**) or (**21**).

After some straightforward simplification and rearrangement we get

$$\begin{aligned} F\,(t) &= -\,C\sum \frac{\cos\,(q\,\Gamma_m)}{n\cdot\sin\,\Gamma_m\cdot\left(\dfrac{d\,\Gamma_m}{d\,p_m}\right)}\,\epsilon^{p_m t} \\ &= \frac{\sqrt{2}}{\pi}\sqrt{\frac{C}{L}}\;\epsilon^{-\rho t}\sum\frac{\pi}{n}\,. \\ &\quad\cdot\frac{\cos\left(q\,m\,\dfrac{\pi}{n}\right)}{\sqrt{1-\cos\left(m\,\dfrac{\pi}{n}\right)-\dfrac{\sigma^2}{2\,v^2}}}\,. \\ &\quad.\sin\left(\sqrt{2}\,v\,t\sqrt{1-\cos\left(m\,\dfrac{\pi}{n}\right)-\dfrac{\sigma^2}{2\,v^2}}\right) \end{aligned}$$

Now in accordance with our previously discussed mode of treatment let n approach infinity and set

$$\frac{\pi}{n} = d\,\lambda$$

$$m\,\frac{\pi}{n} = \lambda$$

In the limit the summation passes into the definite integral and we get:

$$F(t) = \frac{\sqrt{2}}{\pi} \sqrt{\frac{C}{L}}\, \epsilon^{-\rho t} \int_0^{\pi} \frac{\cos(q\,\lambda)}{\sqrt{1 - \cos\lambda - \dfrac{\sigma^2}{2\,v^2}}} \cdot$$

$$\cdot \sin\left(\sqrt{2}\,v\,t\sqrt{1 - \cos\lambda - \frac{\sigma^2}{2\,v^2}}\right) \cdot d\,\lambda$$

If we now write for convenience

$$\begin{aligned} \rho/2\,v &= \eta \\ \sigma/2\,v &= \zeta \\ 2\,v\,t &= p_c\,t = z \end{aligned} \qquad \textbf{(22)}$$

and observe that

$$1 - \cos\lambda = 2\sin^2\left(\frac{\lambda}{2}\right)$$

we get finally, after a suitable change in the variable and limits of integration,

$$F(z) = \sqrt{\frac{C}{L}}\, \epsilon^{-\eta z} \cdot \frac{2}{\pi} \cdot \int_0^{\pi/2} \frac{\cos(2\,q\,\lambda)}{\sqrt{\sin^2\lambda - \zeta^2}}$$

$$\sin\left(z \cdot \sqrt{\sin^2\lambda - \zeta^2}\right) \cdot d\,\lambda \qquad \textbf{(23)}$$

and

$$A_q(t) = F(z) + (\eta - \zeta) \int_0^z F(z_1) \cdot d\,z_1 \qquad \textbf{(24)}$$

where now $A_q(t)$ is equal to the current in the qth section of an *infinitely* long line instead of the line of finite length with which we started. It will be observed that the integral term of (**24**) vanishes if the leakage conductance G is put equal to zero and that within a constant it is merely the progressive integral of the first term. For both the artificial line and the smooth line the effect of leakage is to introduce an integral term of this character. This integral term is readily evaluated by mechanical integration as soon as the first term of the formula is derived.

Returning to equation (**23**) we observe that

$$\int_0^{\pi/2} \frac{\cos(2\,q\,\lambda)}{\sqrt{\sin^2\lambda - \zeta^2}} \sin\left(z \cdot \sqrt{\sin^2\lambda - \zeta^2}\right) \cdot d\,\lambda =$$

$$\int_0^z d\,z_1 \int_0^{\pi/2} \cos(2\,q\,\lambda) \cdot \cos\left(z \cdot \sqrt{\sin^2\lambda - \zeta^2}\right) \cdot d\,\lambda$$

whence $$F(z) = \sqrt{\frac{C}{L}}\, \epsilon^{-\eta z} \int_0^z \Phi(z_1) \cdot d\, z_1 \tag{25}$$

where $$\Phi(z_1) = \frac{2}{\pi} \int_0^{\pi/2} \cos(2\, q\, \lambda) \cdot \cos\left(z_1 \sqrt{\sin^2 \lambda - \zeta^2}\right) \cdot d\, \lambda \tag{26}$$

Our problem is thus reduced to the solution of the definite integral of (**26**)

The key to the solution of this integral is furnished by the known value of the definite integral:

$$\frac{2}{\pi} \int_0^{\pi/2} (\cos(2\, q\, \lambda) \cdot \cos(z \cdot \sin \lambda) \cdot d\, \lambda = J_{2q}(z) \tag{27}$$

where $J_{2q}(z)$ is the Bessel function* of the first kind of order $2\, q$ and the argument z. To make use of this known function in solving (**26**) we proceed as follows: Expand the second cosine function appearing in (**26**) in the usual power series. We get

$$\Phi(z_1) = \frac{2}{\pi} \int_0^{\pi/2} d\, \lambda \cdot \cos(2\, q\, \lambda) \cdot$$

$$\Sigma_m \, (-1)^m \frac{z_1^{\,2m}}{(2\, m)\,!} (\sin^2 \lambda - \zeta^2)^m$$

expanding $(\sin^2 \lambda - \zeta^2)^m$ by the binomial theorem and arranging in powers of ζ^2 this becomes:

$$\Phi(z_1) = \frac{2}{\pi} \int_0^{\pi/2} d\, \lambda \cdot \cos(2\, q\, \lambda)$$

$$\left\{ \begin{array}{l} \Sigma\, (-1)^m \dfrac{z_1^{\,2m}}{(2\, m)\,!} \sin^{2m} \lambda \\ + \dfrac{\zeta^2}{1\,!} \Sigma\, (-1)^m (m+1) \dfrac{z_1^{\,2m+2}}{(2\, m + 2)\,!} \sin^{2m} \lambda \\ + \dfrac{\zeta^4}{2\,!} \Sigma\, (-1)^m (m+1)(m+2) \dfrac{z_1^{\,2m+4}}{(2\, m + 4)\,!} \sin^{2m} \lambda \\ + \ldots\ldots \end{array} \right. \tag{28}$$

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*See Note 6.

$$= \varphi_0(z_1) + \frac{\zeta^2}{1!}\varphi_1(z_1) + \frac{\zeta^4}{2!}\varphi_2(z_1) + \ldots$$

$$+ \frac{\zeta^{2n}}{n!}\varphi_n(z_1) \qquad (29)$$

where

$$\varphi_n(z_1) = \frac{2}{\pi}\int_0^{\pi/2} d\lambda \cdot \cos(2q\lambda)\cdot$$

$$\Sigma_m (-1)_m (m+1)\ldots(m+n)\frac{{z_1}^{2m+2n}}{(2m+2n)!}\sin^{2m}\lambda$$

We observe that

$$\varphi_1(z_1) = \frac{z_1}{2}\int_0^{z_1}\varphi_0(z_2)\cdot d\,z_2$$

$$\cdots\cdots\cdots$$

$$\varphi_n(z_1) = \frac{z_1}{2}\int_0^{z_1}\varphi_{n-1}(z_2)\cdot d\,z_2$$

and

$$\varphi_0(z_1) = \frac{2}{\pi}\int_0^{\pi/2}\cos(2q\lambda)\,.\cos(z_1\,.\sin\lambda)\,.\,d\lambda$$

$$= J_{2q}(z_1)$$

Hence (28) may be written:

$$\begin{aligned}\Phi(z_1) = {} & \varphi_0(z_1)\\ & + \frac{\zeta^2}{1!}\cdot\frac{z_1}{2}\int_0^{z_1}\varphi_0(z_2)\,.\,d\,z_2\\ & + \frac{\zeta^4}{2!}\cdot\frac{z_1}{2}\int_0^{z_1}\frac{z_2}{2}\,d\,z_2\int_0^{z_2}\varphi_0(z_3)\,.\,d\,z_3\\ & + \frac{\zeta^6}{3!}\cdot\frac{z_1}{2}\int_0^{z_1}\frac{z_2}{2}\,d\,z_2\,.\int_0^{z_2}\frac{z_3}{2}\,d\,z_3\,.\\ & \int_0^{z_3}\varphi_0(z_4)\cdot d\,z_4 + \ldots\end{aligned} \qquad (30)$$

Reversing the order of the multiple integrals appearing in (30) we get, after repeated integration and simplification, as the expression for $F(z)$ of equation (25)

$$F(z) = \sqrt{\frac{C}{L}}\,\epsilon^{-\eta z}\int_0^z d\,z_1 \cdot J_{2q}(z_1)\cdot \left\{ 1 + \frac{\zeta^2 (z^2 - z_1^2)}{2^2} + \frac{\zeta^4 (z^2 - z_1^2)^2}{2^2 \cdot 4^2} + \frac{\zeta^6 (z^2 - z_1^2)}{2^2 \cdot 4^2 \cdot 6^2} + \ldots\ldots \right. \tag{31}$$

The series appearing in the above is the expansion of the known function $I_0(\zeta\sqrt{z^2 - z_1^2})$ which is the Bessel function of order zero and pure imaginary argument $i\,\zeta\sqrt{z^2 - z_1^2}$ (the function itself is entirely real since even powers alone are present). Hence,

$$F(z) = \sqrt{\frac{C}{L}}\cdot\epsilon^{-\eta z}\int_0^z J_{2q}(z_1)\cdot I_0(\zeta\sqrt{z^2 - z_1^2})\cdot d\,z_1$$

and finally, by reference to (**24**), the final solution for $A_q(t)$ is

$$A_q(t) = \sqrt{\frac{C}{L}}\,\epsilon^{-\eta z}\int_0^z J_{2q}(z_1)\cdot I_0(\zeta\sqrt{z^2 - z_1^2})\cdot d\,z_1 + (\eta - \zeta)\cdot\int_0^z d\,z_1\cdot\sqrt{\frac{C}{L}}\cdot\epsilon^{-\eta z_1}\cdot \int_0^{z_1} J_{2q}(z_2)\cdot I_0(\zeta\sqrt{z_1^2 - z_2^2})\cdot d\,z_2 \tag{32}$$

This is the complete solution for the current in the qth section of an artificial line of infinite length when a unit voltage is applied to the line at time $t = 0$. The second term of (**32**) is merely the integral of the first term multiplied by $(\eta - \zeta)$, and for that reason offers no difficulties in the way of computation. The first term is expressed in terms of Bessel functions whose properties are known and which can be computed. Plots of $A_q(t)$ from equation (**32**) are shown later in this paper when a discussion of the formulas and their physical significance is taken up.

Formula (**32**) is quite general in that all four line constants (*R. L. C. G.*) are finite and unrestricted in value. Certain particular cases corresponding to particular values of or relations among the line constants are of interest and importance.

1. If the leakage conductance $G = 0$, then, by reference to formulas (**17**) and (**22**), $\eta = \zeta$ and (**32**) reduces to

$$A_q(t) = \sqrt{\frac{C}{L}}\,\epsilon^{-\eta z} \int_0^z J_{2q}(z_1) \cdot I_0(\zeta\sqrt{z^2 - z_1^2}) \cdot d\,z_1 \tag{33}$$

which is the solution for the *leakageless line* ($G = 0$)

2. If $\frac{R}{L} = \frac{G}{C}$, $\zeta = 0$, (32) reduces to

$$A_q(t) = \sqrt{\frac{C}{L}} \int_0^z \epsilon^{-\eta z_1} \cdot J_{2q}(z_1) \cdot d\,z_1 \tag{34}$$

which is the solution for the *quasi-distortionless line*. This name is, of course, suggested and justified by the fact that the corresponding smooth line (constants continuously distributed), when so proportioned that $\frac{R}{L} = \frac{G}{C}$, is exactly distortionless. We shall see later that the quasi-distortionless line, while not strictly distortionless, is characterized by a minimum of distorting action.

(3) If the line is non-dissipative $R = G = 0$, and (**32**) reduces to:

$$A_q(t) = \sqrt{\frac{C}{L}} \int_0^z J_{2q}(z_1) \cdot d\,z_1 \tag{35}$$

which is the solution for the *non-dissipative* artificial line made up of pure inductance and capacity only. This solution applies also to the non-dissipative loaded cable*

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*See Note 7.

(4) If $L = G = 0$, the solution reduces to:

$$A_q(t) = \frac{2}{R}\,\epsilon^{-\frac{2t}{RC}} \cdot I_q\left(\frac{2t}{RC}\right) \qquad (36)$$

which is the solution for the *artificial cable*. $I_q\left(\frac{2t}{RC}\right)$ is the Bessels function of order q and pure imaginary argument $\frac{2t}{RC}$. This solution is derivable from (**33**) by letting L approach zero as a limit; as a matter of fact, however, it was derived directly from the expansion theorem solution. The detailed solution is omitted, however, as involving nothing of special interest.

Before entering into a discussion of these formulas and an exposition of their nature and properties we shall first take up the problem of the smooth (continuously loaded) line and derive the solutions for both current and voltage.

IV. Solution for the Smooth or Continuously Loaded Line

The artificial line whose solution was derived in the preceding section becomes, in the limit, the corresponding smooth or continuously loaded line, if we keep the total value of the line constants finite and constant and let the number of sections approach infinity as a limit. Hence the solution for the smooth line is derivable by purely mathematical processes from the solution given by formula (**32**) for the artificial line. In this section we shall, however, derive the solution directly by aid of the expansion theorem. We shall find that the process of evaluation is precisely analagous, step by step, to that employed in the preceding section.

The symbolic formula for the current I_x at point x of a line of length s, short-circuited at $x = s$, is well known to be

$$I_x = \sqrt{\frac{Cp+G}{Lp+R}} \cdot \frac{\cosh \gamma (s-x)}{\sinh (\gamma s)} \cdot \epsilon^{pt}$$

$$= (Cp+G) \cdot \frac{\cosh \gamma (s-x)}{\gamma \cdot \sinh (\gamma s)}\,\epsilon^{pt} \qquad (37)$$

where R, L, C, and G are the values per unit length of the continuously distributed line constants and

$$\gamma = \sqrt{\{(Lp+R)(Cp+G)\}}$$

we write

$$\frac{1}{\sqrt{LC}} = v$$

$$\frac{R}{2L} + \frac{G}{2C} = \rho \tag{38}$$

$$\frac{R}{2L} - \frac{G}{2C} = \sigma$$

whence

$$\frac{G}{C} = \rho - \sigma$$

$$\gamma = \frac{1}{v} \sqrt{(p + \rho)^2 - \sigma^2} \tag{39}$$

Writing down the expansion theorem solution of (**36**) in accordance with formula (**3**) we get, after some easy simplifications,

$$A_x(t) = C \sum \frac{\cosh(\gamma_m x)}{s \cdot \gamma_m \cdot \left(\frac{d\,\gamma_m}{d\,p_m}\right)} \cdot \epsilon^{p_m t}$$
$$+ (\rho - \sigma) \int_0^t dt_1 \cdot C \sum \frac{\cosh(\gamma_m x)}{s \cdot \gamma_m \cdot \left(\frac{d\,\gamma_m}{d\,p_m}\right)} \epsilon^{p_m t_1} \tag{40}$$

$A_x(t)$ is, of course, the value of the current at point x and time t when a unit d-c. voltage is applied to the line at $x = 0$ and $t = 0$. We observe that the second term of (**40**) (which vanishes with the leakage), is merely the progressive integral of the first term, so that our problem is to evaluate the expansion formulated by the first term. p_m and γ_m are given by the determinantal equation

$$\sinh(\gamma s) = 0$$

$$\gamma_m = i m \frac{\pi}{s} \qquad m = 0, 1, 2, \ldots \infty$$

$$p_m = -\rho \pm i v \sqrt{\left(m \frac{\pi}{s}\right)^2 - \frac{\sigma^2}{v^2}}$$

For convenience we write

$$\begin{aligned} v\,t &= y \\ \rho/v &= a \\ \sigma/v &= b \end{aligned} \qquad \textbf{(41)}$$

and let the length of the line s approach infinity. By processes identical with those developed in the preceding section, the summations in (**40**) are thereby converted into definite integrals and we get without difficulty:

$$\begin{aligned} A_x(t) = {} & \sqrt{\frac{C}{L}}\,\epsilon^{-ay}\cdot\frac{2}{\pi}\int_0^\infty \frac{\cos(x\lambda)\cdot\sin\left(y\sqrt{\lambda^2-b^2}\right)}{\sqrt{\lambda^2-b^2}}\,d\lambda \\ & + (a-b)\int_0^y d\,y_1\cdot\sqrt{\frac{C}{L}}\,\epsilon^{-ay_1}\cdot\frac{2}{\pi} \\ & \int_0^\infty \frac{\cos(x\lambda)\cdot\sin\left(y_1\sqrt{\lambda^2-b^2}\right)}{\sqrt{\lambda^2-b^2}}\,d\lambda \end{aligned} \qquad \textbf{(42)}$$

where now $A_x(t)$ is equal to the current at point x and time t of the *infinitely long* line. Observe that the second term is the progressive integral of the first.

We set

$$F(y) = \sqrt{\frac{C}{L}}\,\epsilon^{-ay}\cdot\frac{2}{\pi}\int_0^\infty \frac{\cos(x\lambda)\cdot\sin\left(y\sqrt{\lambda^2-b^2}\right)}{\sqrt{\lambda^2-b^2}}\,d\lambda \qquad \textbf{(43)}$$

whence

$$A_x(t) = F(y) + (a-b)\int_0^y F(y_1)\cdot d\,y_1 \qquad \textbf{(44)}$$

and proceed with the evaluation of infinite integral factor of $F(y)$. We observe that

$$F(y) = \sqrt{\frac{C}{L}}\,\epsilon^{-ay}\int_0^y \Phi(y_1)\cdot d\,y_1 \qquad \textbf{(45)}$$

where

$$\Phi(y_1) = \frac{2}{\pi}\int_0^\infty \cos(x\,\lambda)\cdot\cos\left(y_1\sqrt{\lambda^2-b^2}\right)\cdot d\,\lambda \tag{46}$$

The key to the solution of this infinite integral is furnished by the known value of the infinite integral

$$\frac{2}{\pi}\int_0^\infty \frac{\sin(y\,\lambda)}{\lambda}\,d\,\lambda$$

which is a discontinuous function of value -1 for $y < o$ and $+1$ for $y > o$.

Returning to (**46**) expand the second cosine function in the usual power series, and precisely as in the preceding section, subsequently expand $(\lambda^2 - b^2)^m$ by the binomial theorem. We get, after arranging in powers of b,

$$\Phi(y_1) = \frac{2}{\pi}\int_0^\infty d\,\lambda\cdot\cos(x\,\lambda) \left\{ \begin{aligned} & \Sigma(-1)^m \frac{y_1^{2m}\lambda^{2m}}{(2\,m)\,!} \\ & + \frac{b^2}{1\,!}\,\Sigma(-1)^m \frac{y_1^{2m+2}}{(2\,m+2)\,!}(m+1)\cdot\lambda^{2m} \\ & + \frac{b^4}{2\,!}\,\Sigma(-1)^m \frac{y_1^{2m+4}}{(2\,m+4)\,!}(m+1)(m+2)\,\lambda^{2m} \\ & + \ldots \end{aligned} \right. \tag{47}$$

which corresponds exactly with formula (**28**) of the preceding section. By analysis precisely the same as that employed in deriving (**30**), equation (**47**) is reducible to the form,

$$\begin{aligned} \Phi(y_1) = {} & \varphi(y_1) \\ & + \frac{b^2}{1\,!}\cdot\frac{y_1}{2}\int_0^{y_1} \varphi(y_2)\cdot d\,y_2 \\ & + \frac{b^4}{2\,!}\cdot\frac{y_1}{2}\int_0^{y_1} \frac{y_2}{2}\,d\,y_2 \int_0^{y_2} \varphi(y_3)\cdot d\,y_3 \\ & + \frac{b^6}{3\,!}\cdot\frac{y_1}{2}\int_0^{y_1} \frac{y_2}{2}\,d\,y_2\cdot\int_0^{y_2} \frac{y_3}{2}\,d\,y_3 \int_0^{y_3} \varphi(y_4)\cdot d\,y_4 \\ & + \ldots \end{aligned} \tag{48}$$

where

$$\varphi(y_1) = \frac{2}{\pi}\int_0^\infty \cos(x\lambda)\cdot\cos(y_1\lambda)\cdot d\lambda$$

$$= \frac{d}{d\,y_1}\cdot\frac{2}{\pi}\int_0^\infty \frac{\cos(x\lambda)\cdot\sin(y_1\lambda)}{\lambda}\,d\lambda \qquad (49)$$

$$= \frac{d}{d\,y_1}\,\psi(y_1)$$

Now the integral of (**46**) is by (**48**)

$$\int_0^y \Phi(y_1)\cdot d\,y_1$$

$$= \int_0^y \varphi(y_1)\cdot d\,y_1$$

$$+ \frac{b^2}{1\,!}\int_0^y \frac{y_1}{2}\,d\,y_1\int_0^{y_1}\varphi(y_2)\cdot d\,y_2$$

$$+ \frac{b^4}{2\,!}\int_0^y \frac{y_1}{2}\,d\,y_1\int_0^{y_1}\frac{y_2}{2}\,d\,y_2\cdot\int_0^{y_2}\varphi(y_3)\cdot d\,y_3$$

$$+ \ldots$$

which sum up to

$$\int_0^y \Phi(y_1)\cdot d\,y_1 = \int_0^y \varphi(y_1)\cdot I_0\left(b\sqrt{y^2-y_1^2}\right)d\,y_1$$

$$= \int_0^y \frac{d}{d\,y_1}\,\psi(y_1)\cdot I_0\left(b\sqrt{y^2-y_1^2}\right)\cdot d\,y_1 \qquad (50)$$

Now $\psi(y_1)$ is a discontinuous function which is zero before, unity after $y_1 = x$. Hence the definite integral of (**50**) is identically zero for values of the upper limit $y < x$, and its total contribution is entirely concentrated at $y_1 = x$.
Hence

$$\int_0^y \Phi(y_1)\cdot d\,y_1 = 0 \quad \text{for} \quad y = v\,t < x$$

$$= I_0\left(b\sqrt{y^2-x^2}\right) \quad \text{for} \quad y \geq x \qquad (51)$$

Hence by (**44**) and (**45**) our solution is:

$$
\begin{aligned}
A_x(t) &= 0 \qquad \text{for} \qquad y = vt < x \\
&= \sqrt{\frac{C}{L}} \cdot \epsilon^{-ay} \cdot I_0\left(b\sqrt{y^2 - x^2}\right) \\
&\quad + (a-b)\int_x^y \sqrt{\frac{C}{L}}\, \epsilon^{-ay_1} \cdot I_0\left(b\sqrt{y_1^2 - x^2}\right) \cdot d\,y_1 \\
&\qquad\qquad\qquad\qquad \text{for} \quad y \geq x \qquad (\mathbf{52})
\end{aligned}
$$

which is the final solution.

In the foregoing solution, we have dealt with discontinuous functions and doubtless some of the processes involved require further examination and justification from a purely mathematical standpoint. The final solution is, however, quite correct, and may be verified as the limit of formula (**32**); a solution to whose derivation no exception can be taken.

Special cases of the general solution will now be considered. If leakage is absent $G = 0$ and $a = b$ whence

$$
\begin{aligned}
A_x(t) &= 0 \qquad \text{for} \qquad vt = y < x \\
&= \sqrt{\frac{C}{L}}\, \epsilon^{-ay} \cdot I_0\left(b\sqrt{y^2 - x^2}\right) \qquad \text{for } y \geq x \qquad (\mathbf{53}) \\
&= \sqrt{\frac{C}{L}}\, \epsilon^{-\rho t} \cdot I_0\left(\rho\sqrt{t^2 - \frac{x^2}{v^2}}\right) \quad \text{for} \quad t \geq \frac{x}{v}
\end{aligned}
$$

which is the solution for the *leakageless line*.* While the general solution (**52**) is original, the special solution (**53**) was previously derived by Heaviside by his operational method.

If $\dfrac{R}{L} = \dfrac{G}{C}$, $b = 0$ and the solution reduces to

$$
\begin{aligned}
A_x(t) &= 0 \qquad \text{for } y < x \\
&= \sqrt{\frac{C}{L}}\, \epsilon^{-ax} \qquad \text{for } y \geq x \qquad (\mathbf{54})
\end{aligned}
$$

which is the solution for the *distortionless* line.

If $R = G = 0,\ \ a = b = 0$ and

$$
\begin{aligned}
A_x(t) &= 0 \qquad \text{for } y < x \\
&= \sqrt{\frac{C}{L}} \quad \text{for } y > x \qquad (\mathbf{55})
\end{aligned}
$$

---

*See Note 8.

which is the solution for the non-dissipative line.

In all cases the current is propagated with a true finite velocity $v = \frac{1}{\sqrt{LC}}$ and is identically zero until $t = \frac{x}{v}$. When we come to examine the solution for the artificial line we shall see that there is no true velocity of propagation in an exact sense, but that there is a *virtual* velocity $\frac{1}{\sqrt{LC}}$ just as in the corresponding smooth line.

If $L = 0$, the solution is derivable from (**52**) by the method of limits. It is

$$A_x(t) = \sqrt{\frac{C}{\pi R}} \cdot \frac{\epsilon^{-\frac{RCx^2}{4t}}}{\sqrt{t}} \cdot \epsilon^{-\frac{G}{C}t}$$

$$+ \frac{G}{C}\sqrt{\frac{C}{\pi R}} \int_0^t \epsilon^{-\frac{G}{C}t_1} \frac{\epsilon^{-\frac{RCx^2}{4t_1}}}{\sqrt{t_1}}\, dt_1 \tag{56}$$

which is the solution for the *leaky cable*. Observe that here we have no finite velocity of propagation owing to the absence of inductance.

If the leakage G as well as the inductance L is zero, we get

$$A_x(t) = \sqrt{\frac{C}{\pi R}} \; \frac{\epsilon^{-\frac{RCx^2}{4t}}}{\sqrt{t}} \tag{57}$$

which is the Kelvin-Heaviside solution.

IV A. Solution for the Voltage Formulas for Smooth Line

Of almost equal importance and interest are the formulas for the voltage existing at any point x and time t on a smooth line of infinite length when a unit voltage is applied to the line at $x = 0$ and $t = 0$. The corresponding formulas for the artificial line are deducible from the current or indicial admittance formulas, but they are of complicated form and their consideration is omitted for the present.

We denote by $V_x(t)$ the voltage at point x and time t when a unit voltage is applied to the line at $x = t = 0$, and start with the differential equation of the line,

$$C \frac{d}{dt} V + G V = - \frac{d}{d x} I$$

where V and I are perfectly general. This equation may be integrated in the form:

$$V = - \frac{1}{C} \epsilon^{-(\rho-\sigma) t} \int_0^t \epsilon^{(\rho-\sigma)\lambda} \cdot \frac{d}{d x} I(\lambda) \cdot d\lambda$$

If $I(t)$ is the current due to unit d-c. voltage applied at $x = t = 0$ it is of course $A_x(t)$ as given by formula (**52**) and the corresponding voltage is $V_x(t)$. Substituting $A_x(\lambda)$ as given by (**52**) for $I(\lambda)$ in the integral and paying due regard to the discontinuous character of $A_x(\lambda)$ we get:

$$\begin{aligned} V_x(t) &= 0 \qquad \text{for } v t < x. \\ &= \epsilon^{-ax} \\ &\qquad + b^2 x \int_x^y \frac{\epsilon^{-a y_1} I_1(b\sqrt{y_1^2 - x^2})}{b\sqrt{y_1^2 - x^2}} d y_1 \end{aligned} \tag{58}$$

which is the required solution. $I_1(b\sqrt{y^2 - x^2})$ is the Bessel function of order unity and pure imaginary argument $i b \sqrt{y^2 - x^2}$ tables of which exist and whose computation is quite simple.

If leakage is absent, $G = 0$ and $b = a$ in the above formula. In this case the solution may be conveniently written,

$$\begin{aligned} V_x(t) &= 0 \qquad \text{for } T < A \\ &= \epsilon^{-A} + A \int_A^T \frac{\epsilon^{-T_1} \cdot I_1(\sqrt{T_1^2 - A^2})}{\sqrt{T_1^2 - A^2}} d T_1 \end{aligned} \tag{59}$$

where
$$A = \frac{R}{2} \sqrt{\frac{C}{L}}\, x$$

$$T = \frac{R}{2 L} t = \rho t$$

If $L = 0$ but the other three line constants are finite, the formula becomes, by approaching the limit $L = 0$,

$$V_x(t) = x \sqrt{\frac{R C}{4 \pi}} \int_0^t \frac{\epsilon^{-\frac{G}{C} t_1} \cdot \epsilon^{-\frac{R C x^2}{4 t_1}}}{t_1^{3/2}} d t_1 \tag{60}$$

which is reducible to the form

$$V_x(t) = \epsilon^{-\alpha x} - \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{\frac{RCx^2}{4t}}} \epsilon^{-\left(\lambda^2 + \frac{\alpha^2}{4\lambda^2}\right)} \cdot d\lambda \qquad (61)$$

where $\alpha = \sqrt{RG}$

These are the formulas for the *leaky cable.*

If finally both G and L are zero

$$V_x(t) = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{\frac{RCx^2}{4t}}} \epsilon^{-\lambda^2} \cdot d\lambda \qquad (62)$$

a formula which was first derived by Heaviside.

V. Approximate Solution for Loaded Cable

The loaded cable is defined as a non-inductive, leakageless line ($L = G = 0$) with load coils of pure inductance L inserted in series with the lines at periodic intervals. The exact solution for this structure has not yet been worked out, as it presents a problem of enormous complexity incapable of treatment by the direct methods employed in dealing with the smooth and artificial lines in the preceding section of this paper. The writer has applied a functional method of treatment, involving integral equations, to the problem which gives good promise of yielding a formal solution. This investigation has, however, not been completed, and as yet gives little hope of resulting in a practical working solution.

Fortunately, however, the close relationships obtaining between the loaded cable and the corresponding smooth and artificial lines has enabled me to arrive at a quasi-empirical solution, which very closely represents the facts, although it is not functionally exact.

To arrive at this solution, we remark that physically the loaded cable, as defined above, is intermediate between the corresponding smooth and artificial lines, and its solution must to a certain extent share the properties of the known solutions of the corresponding lines. More exactly, the required solution must satisfy the following conditions:

1. It must be asymptotic to the solution of the correspond-

ing smooth line as t approaches infinity. That is, it must approach formula (**53**) and ultimately (**57**) as t approaches infinity.

2. It must degenerate into formula (**35**) for the non-dissipative artificial line as the distributed resistance of the cable approaches zero.

3. It must degenerate into formula (**57**) for the smooth cable as the load coil inductance L approaches zero.

4. It must degenerate into formula (**53**) for the smooth line as the load coil inductance L and the distributed resistance R and capacity C per section all approach zero while the number of sections approach infinity.

5. It must give the correct virtual velocity of propagation of the structure and correctly formulate the steady state characteristics in accordance with formula (**13**).

Such a function is the following:

$$\sqrt{\frac{C}{L}}\,\epsilon^{-\rho t}\cdot I_0\left(\sqrt{(\rho\,t)^2-A^2}\right)\cdot\frac{\epsilon^{hA}}{\operatorname{sech}\left(\frac{h\,A}{2\,q}\right)}\int_0^{2vt}\epsilon^{-\lambda\,\cdot\,\sinh\left(\frac{h\,A}{2\,q}\right)}\cdot J_{2q}(\lambda)\cdot d\,\lambda \qquad \textbf{(63)}$$

Where in accordance with previous notation

$$\rho=\frac{R}{2\,L}\text{ and }\quad v=\frac{1}{\sqrt{L\,C}}\quad\text{and}\quad A=\frac{R\,q}{2}\sqrt{\frac{C}{L}}$$

This is the required quasi-empirical formula for the current in the q^{th} section or load coil of a loaded cable in response to a unit d-c. voltage applied at mid-load position at time $t=0$. R, C, and L, are the constants *per section* of the structure. The first two being distributed and the last concentrated in the load coil. For values of R. L. and C. encountered in practise: formula (**62**) may be replaced by

$$\sqrt{\frac{C}{L}}\,\epsilon^{-\rho t}\cdot I_0\left(\sqrt{(\rho\,t)^2-A^2}\right)\cdot\epsilon^{hA}\cdot\int_0^{2vt}\epsilon^{-\frac{h\,A}{2\,q}\lambda}\,J_{2q}(\lambda)\cdot d\,\lambda \qquad \textbf{(64)}$$

The symbol h which disappears in the first four limiting cases listed above, is a numeric. When assigned the value ½, formula (**62**) gives the steady state characteristics with a good degree of approximation.

The foregoing formulas, while not functionally exact, and which are indeed, empirical, represent the phenomena with sufficient exactness for engineering purposes and have been verified as closely as could be expected by oscillographic tests. The physical character of the solution will be discussed and illustrated by curves in the succeeding section.

VI. Numerical Solution of Formulas and Discussion of Curves

By virtue of the mathematical analysis of the preceding sections we are now in possession of a fairly systematic formulation of the general problem, and in particular of the formulas which determine the progress and distortion of electric waves in transmission systems. While the phenomena are reduced to their simplest possible formulation they are still so complex and the line constants are involved in such a complicated manner that an understanding of the inner meaning of the solutions requires that analysis be superseded at this stage by a systematic and comprehensive program of computation and study of the resulting numerical solutions, when plotted in the form of curves. Such a systematic study is beyond the scope of the present paper and would indeed occupy a whole treatise in itself. We shall, however, exhibit a representative series of numerical solutions in the form of curves, and briefly discuss the physical character of the phenomena which they represent.

Restricting attention to the line formulas (32)-(64,) their evaluation depends on the numerical computation of Bessel functions and the integration of definite integrals. The latter operation presents no difficulties and the integrations are easily and accurately effected, once the integrands are computed and plotted as time functions, by means of a Coradi integraph which traces the progressive values of the integrals. Fortunately, also, the Bessel functions, which are of such great importance in so many branches of mathematical physics, have been the subject of exhaustive study and tabulation. While no tabulations exist for the high orders of functions which we require, their computation is effected without any serious difficulty from the asymptotic formulas of Nicholson.

We shall first consider the artificial line formulas (32)-(36) and since one of the prime objects of our investigation is an examination of the correspondence existing between the artificial and smooth line, the corresponding functions for the smooth line are plotted to the same scale on the same sheet. We shall have frequent occasion in discussing the artificial and the periodically loaded lines to refer to the "corresponding smooth line," which we define as the line having continuously distributed constants of the same aggregate value as the lumped constants of the artificial line. Thus if R, L, C, G, denote the constants *per section* of the artificial or periodically loaded line and the current in the qth section is under consideration, the corresponding smooth line has constants R, L, C, G per *unit length* and the current at point $x = q$ is the corresponding current.

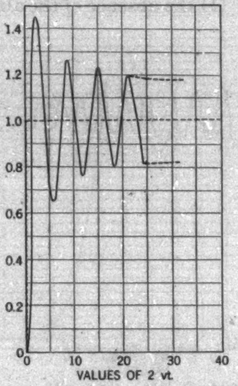

FIG. 1—[COMPUTED FROM FORMULA (35)] CURRENT ENTERING LOADED CABLE

Non-dissipative: $R = G = O$. Multiply ordinates by $\sqrt{C/L}$

Figs. 1, 2, 3 and 4 are graphs of formula (**35**) for $q = 0$, 200, 1000 and 2000 respectively and thus represent the current in the initial, the 100th, the 500th and the 1000th sections of the non-dissipative artificial line or periodically loaded cable. It will be understood that the line is infinitely long and that a unit d-c. voltage applied to the line at mid-load position and at time $t = 0$ is assumed. A study of the non-dissipative line, which of course is ideal and unrealizable, is nevertheless of practical importance because in this type of line the effect of the discontinuous loading of the periodically loaded cable is isolated and thereby exhibited in the clearest possible manner.

The dotted curves represent the current in the corresponding smooth line as computed from formula (**55**). For the smooth line, the current, as we see, is discontinuous, being identically zero for a time $v\,t = q$ and having an instantaneous jump to its final value $\sqrt{\frac{C}{L}}$ at $v\,t = q$. The current in the artificial line differs from that in the corresponding smooth line in three important respects; (1) the absence of the abrupt discontinuous wave front; (2) the presence of superposed oscillations and (**3**) *the absence of a true finite velocity of propagation.* It will be observed, however, that the current in any section is negligibly small or even sensibly zero until $v\,t = q$, so that the current is propagated with a *virtual* velocity $\frac{1}{\sqrt{L\,C}}$ per section. The presence of a well marked wave front is also evident although this is not abrupt, as in the smooth line. The effective slope of the wave front becomes smaller as the current wave travels out on the line decreasing noticeably as the number of sections is increased. When the number of sections becomes large,

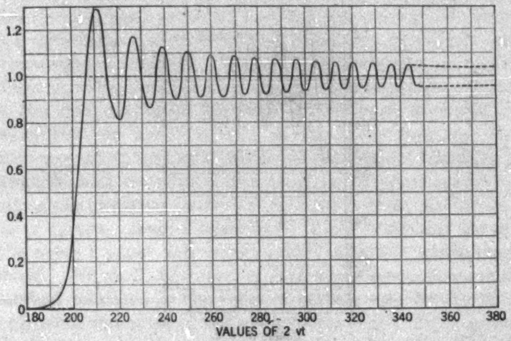

FIG. 2—[COMPUTED FROM FORMULA (35)] CURRENT IN 100TH SECTION OF ARTIFICIAL LINE

Non-dissipative: $R = G = O$. Multiply ordinates by $\sqrt{C/L}$

however, the decrease in the slope is not rapid, being in the 500th section about 60 per cent of that in the 100th section.

The superposed oscillations are of interest. These are initially of a frequency depending upon and decreasing with the number of sections, q, but in all sections ultimately attaining the frequency $\frac{1}{\pi\sqrt{LC}} = \frac{v}{\pi}$ which is the critical or cut-off frequency of the line, above which steady-state currents are attenuated during transmission and below which they are unattenuated. When $v\,t$ is large compared with q the amplitude of these oscillations becomes $\sqrt{\frac{1}{\pi v t}}$ so that they ultimately die away and the current approaches the value $\sqrt{\frac{C}{L}}$ for all sections. The current in the loaded cable is thus asymptotic to the current in the corresponding smooth line and oscillates about it with diminishing amplitude and increasing frequency.

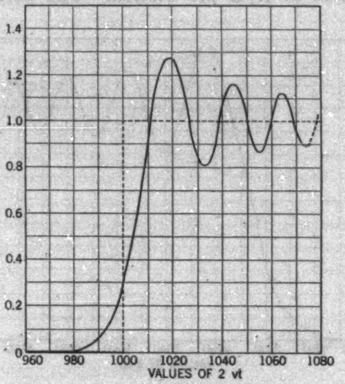

FIG. 3—[COMPUTED FROM FORMULA (35)] CURRENT IN 500TH SECTION OF ARTIFICIAL LINE

Non-dissipative: $R=G=0$. Multiply ordinates by $\sqrt{C/L}$

Since the abscissas of these curves represent values of

$2vt = \frac{2}{\sqrt{LC}}\,t$, and the ordinates are to be multiplied by $\sqrt{\frac{C}{L}}$ to translate into actual values, the curves are of universal application for all values of the constants L and C.

The additional effect of resistance in the artificial line is exhibited in Figs. 5, 6 and 7 which are computed from formula (**33**) for $q = 100$. They therefore represent the current in the 100th section of an artificial line without leakage for three different values of R/L per section. The current functions (or indicial admittances) for the corresponding smooth lines, computed from formula (**53**) are plotted on the same sheet. Curves 5, 6 and 7 correspond to values of $\frac{R}{2}\sqrt{\frac{C}{L}}\,q$ of 2.65, 5.29 and 14.75 respectively; $\frac{R}{2}\sqrt{\frac{C}{L}}\,q$ is the exponential attenuation factor and an examination of the curves shows that, for the smooth line, the discontinuous wave front is attenuated by this factor during transmission. For values of $\frac{R}{2}\sqrt{\frac{C}{L}}\,q$ greater than two, the tail of the wave exceeds in

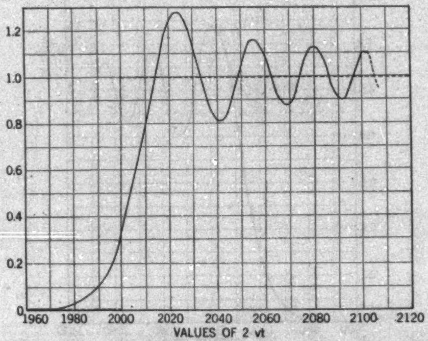

FIG. 4—[COMPUTED FROM FORMULA (35)] CURRENT IN 1000TH SECTION OF ARTIFICIAL LINE

Non-dissipative: $R = G = 0$. Multiply ordinates by $\sqrt{C/L}$

amplitude the head. For large values of $\frac{R}{2}\sqrt{\frac{C}{L}}\,q$ (see curve 7) the head of the wave is insignificant compared with the tail. It should be observed that curves 5, 6 and 7 are plotted in a scale to exhibit the character of the wave in the neighborhood of the head and do not show the way the tail builds up to its maximum and ultimately dies away. The tail of the wave builds up more and more slowly as $\frac{R}{2}\sqrt{\frac{C}{L}}\,q$ is increased and ultimately dies away in accordance with the law $\sqrt{\frac{C}{\pi R t}}\cdot\epsilon^{-\frac{RC}{4t}q^2}$. This will be recognized as the non-inductive cable formula (57) so that the line ultimately behaves like the cable to d-c. impressed voltages. Its behavior to alternating voltages is entirely different, however, as may be inferred from the difference in the curves in the neighborhood of the wave head.

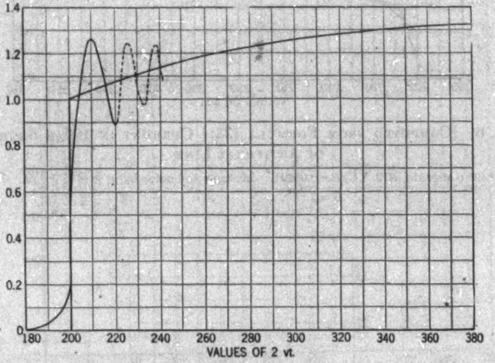

FIG. 5—[COMPUTED FROM FORMULA (33)] CURRENT IN 100TH SECTION OF ARTIFICIAL LINE

Line constants: $R/2\ \sqrt{C/L} = 0.02646$. Multiply ordinates by $\epsilon^{-2.646\sqrt{C/L}}$

$G = 0$

As regards the artificial line curves, we see that, precisely as in curves 1, 2, 3 and 4, they oscillate about the corresponding

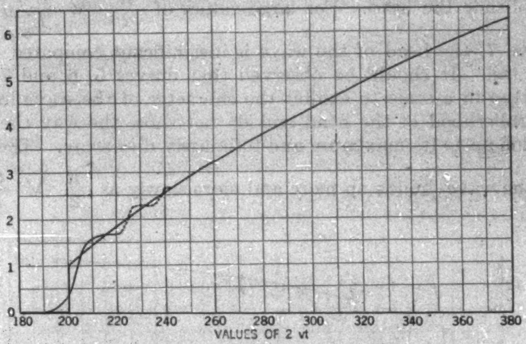

FIG. 6—[COMPUTED FROM FORMULA (33)] CURRENT IN 100TH SECTION OF ARTIFICIAL LINE

Line constants: $R/2\ \sqrt{C/L} = 0.05291$. Multiply ordinates by $\epsilon^{-5.291}\ \sqrt{C/L}$

$G = 0$

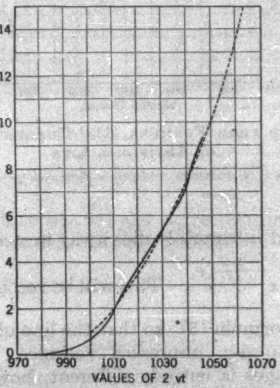

FIG. 7—[COMPUTED FROM FORMULA (33)] CURRENT IN 500TH SECTION OF ARTIFICIAL LINE

Line constants: $R/2\ \sqrt{C/L} = 0.0295$. Multiply ordinates by $\epsilon^{-14.75}\ \sqrt{C/L}$

$G = 0$

smooth line curves and ultimately coalesce therewith. They are distinguished, also, by the absence of the abrupt wave head and the presence of the superposed oscillations. The same virtual velocity per section $\frac{1}{\sqrt{LC}}$ exists independently of the resistance but the superposed oscillations diminish in relative amplitude as $\frac{R}{2}\sqrt{\frac{C}{L}}\,q$ is increased, as is clear from a comparison of curves 5, 6 and 7.

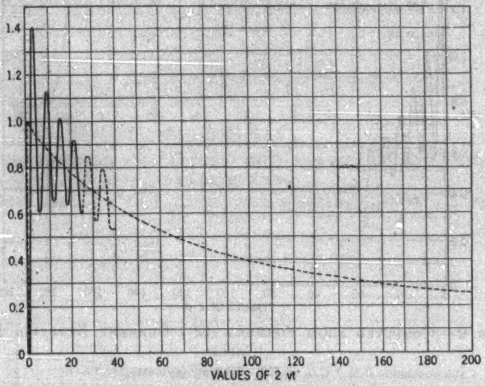

FIG. 8—[COMPUTED FROM FORMULA (62)] CURRENT ENTERING LOADED CABLE

Cable constants: $R/2\sqrt{C/L}=0.02646$. Multiply ordinates by $\sqrt{C/L}$

$G=0$

Some typical loaded cable curves computed from formula (**63**) are shown in Figs. 8, 9, 10 and 11. Curves 8 and 9 are plots of the current entering the cable (corresponding to a unit d-c. voltage impressed at mid-load position) for two different values of $\frac{R}{2}\sqrt{\frac{C}{L}}$ (0.01323 and 0.02646 respectively) while curves 10 and 11 show the current in the 100th load coil for two different values of $\frac{R}{2}\sqrt{\frac{C}{L}}\,q$ (2.65 and 5.291 respectively). The corresponding smooth line current curves are shown in dotted lines. The difference between the curves for the loaded cable and the corresponding artificial line is quite small except for fairly large values of $\frac{R}{2}\sqrt{\frac{C}{L}}q$ (greater than three) and consists in a slower damping out of the superposed oscillations. This is precisely the kind of effect we should expect when we recall the fact that the attenuation of alternating currents for large values of $\frac{R}{2}\sqrt{\frac{C}{L}}q$ is much smaller in loaded cables than in artificial lines. At the same time it must be remembered that the loaded cable under consideration is ideal in that the loading coils are assumed as pure inductances without resistance. In practise the resistance of the loading coils would modify the current curves, which are actually (as has been shown by oscillographic tests) intermediate in character between the theoretical curves for the artificial line and the ideal periodically loaded cable.

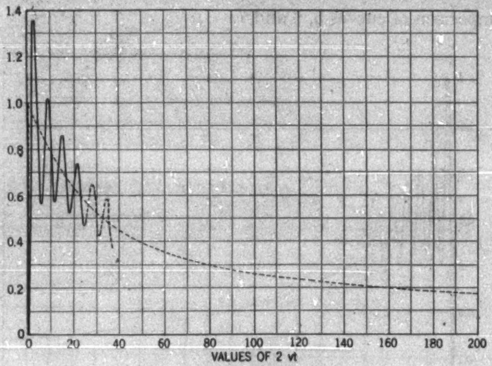

FIG. 9—[COMPUTED FROM FORMULA (62)] CURRENT ENTERING LOAD CABLE

Cable constants: $R/2\sqrt{C/L}=0.529$. Multiply ordinates by $\sqrt{C/L}$

$G=0$

The effect of leakage in the artificial line (when so proportioned as to make $\frac{G}{C}=\frac{R}{L}$) is shown in Figs. 12 and 13

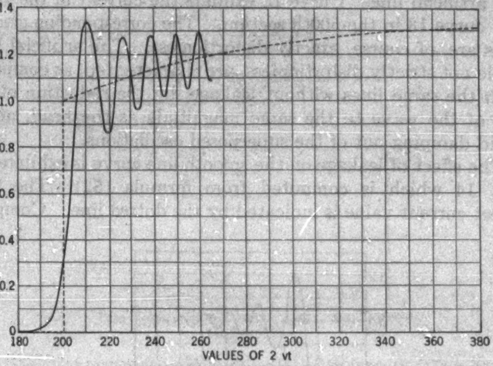

FIG. 10—[COMPUTED FROM FORMULA (162)] CURRENT IN 100TH SECTION OF LOADED CABLE

Cable constants: $R/2\sqrt{C/L}=0.02646$. Multiply ordinates by $\epsilon^{-2.646\sqrt{C/L}}$
$G=0$

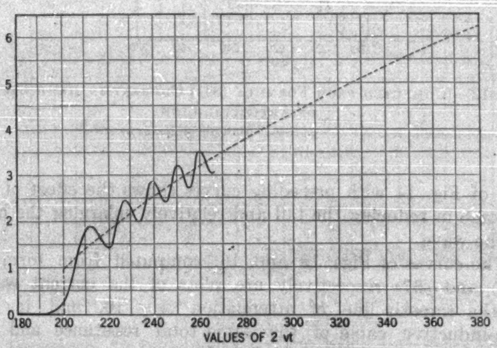

FIG. 11—[COMPUTED FROM FORMULA (62)] CURRENT IN 100TH COIL OF LOADED CABLE

Cable constants: $R/2\sqrt{C/L}=0.05291$. Multiply ordinates by $\epsilon^{-5.291\sqrt{C/L}}$
$G=0$.

which are computed from formula (**34**) for the quasi-distortionless artificial line. Curve 12 exhibits the current in the 100th and curve 13 in the 500th section. The corresponding smooth lines are, of course, exactly distortionless. The artificial lines while not strictly distortionless, are characterized, as compared with the same lines without leakage, by the reduction of the tail of the wave to the same magnitude as the head, and a rapid damping out of the superposed oscillations.

The effect of leakage on the smooth line curve is exhibited in Fig. 14 which is computed from formula (**52**). The final direct-current value is indicated by the dotted line. Comparison of Fig. 14 with preceding curves shows the effect of the leakage in reducing the tail and relatively enlarging the head of the wave.

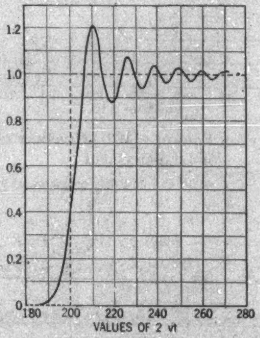

FIG. 12—[COMPUTED FROM FORMULA (34)] CURRENT IN 100TH SECTION OF ARTIFICIAL LINE

Line constants: $R/2\sqrt{C/L}=0.02646$. Multiply ordinates by $\epsilon^{-5.291\sqrt{C/L}}$

$R/L=G/C$

The curves of Figs. 15 and 16, computed from formulas (**53**) and (**57**) respectively are plots of the current waves in the smooth line of attenuation $A = 10$, and in the non-inductive cable of the same total resistance and capacity. The curves are plotted to the same argument $\dfrac{4\,t}{R\,C \times 2}$ in order to facilitate comparison. Referring to Fig. 15, the large value of the attenuation makes the discontinuous head of the wave quite negligible in comparison with the tail, and it will be seen that the head of the wave is not distinguishable on the scale to which the ordinates are drawn. In fact for lines of this or greater order of attenuation the inductance plays a quite minor role when the applied voltage is uniform except that its presence results in a finite velocity of propaga-

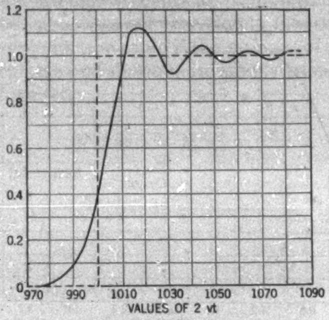

FIG. 13—[COMPUTED FROM FORMULA (34)] CURRENT IN 500TH SECTION OF ARTIFICIAL LINE

Line constants: $R/2\sqrt{C/L}=0.02646$. Multiply ordinates by $\epsilon^{-26.46\sqrt{C/L}}$,
$R/L=G/C$

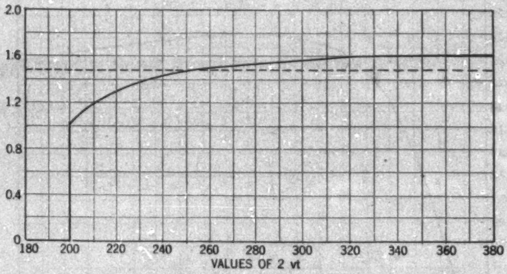

FIG. 14—[COMPUTED FROM FORMULA (52)] CURRENT AT POINT $X=100$ OF CONTINUOUSLY LOADED LINE

Line constants: $R/2\sqrt{C/L}+G/2\sqrt{L/C}=0.0706$. Multiply ordinates by $\epsilon^{-7.06\sqrt{C/L}}$
$R/2\sqrt{C/L}-G/2\sqrt{L/C}=0.0353$

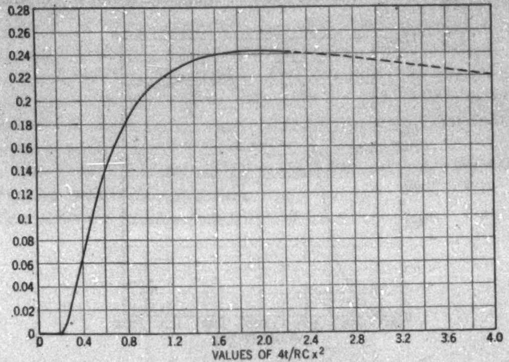

FIG. 15—[COMPUTED FROM FORMULA (53)] CURRENT AT POINT X OF SMOOTH LINE

Line constants: $R/2\sqrt{C/L}\,X = 10$ Multiply ordinates by $2/RX$
$G = 0$

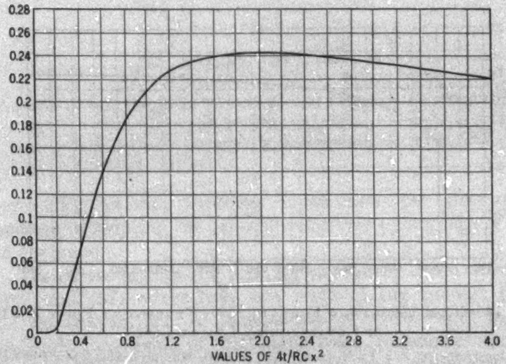

FIG. 16—[COMPUTED FROM FORMULA (57)] CURRENT AT POINT X O SMOOTH CABLE (R, C)

Multiply ordinates by $2/RX$

tion, whereas in the non-inductive cable velocity is infinite. Otherwise the current waves, as may be seen by comparison of Figs. 15 and 16, are almost identical.

The artificial cable behaves very closely like the corresponding smooth non-inductive cable if the number of sections is not too small. This may be shown quite simply from formulas (**36**) and (**57**). If we set

$$\frac{4\,t}{R\,C\,x^2} = \frac{4\,t}{R_0\,C_0} = u$$

formula (**57**) becomes

$$A_x(t) = \frac{2}{\sqrt{\pi}\,R_0}\,\frac{1}{\sqrt{u}}\,\epsilon^{-1/u}$$

$$= \frac{2}{\sqrt{\pi}\,R_0}\cdot\frac{1}{\sqrt{u}}$$

$$\left\{1 - \frac{1}{1\,!\,u} + \frac{1}{2\,!\,u^2} - \frac{1}{3\,!\,u^3}\;\ldots\ldots\right\} \qquad \textbf{(a)}$$

Here, of course, R_0 and C_0 denote the total value of the line constants in the length x of the line.

If the corresponding artificial line is made up of q sections, the constants per section are R_0/q and C_0/q and formula (**36**) becomes,

$$A_q(t) = \frac{2\,q}{R_0}\,\epsilon^{-\frac{q^2 u}{2}}\cdot I_q\left(\frac{q^2\,u}{2}\right)$$

Expanding the Bessel function in its asymptotic form:

$$A_q(t) = \frac{2}{\sqrt{\pi}\,R_0}\,\frac{1}{\sqrt{u}}$$

$$\left\{\begin{aligned} &1 - \frac{1}{1\,!\,u}\left(1 - \left[\frac{1}{2\,q}\right]^2\right) \\ &\quad + \frac{1}{2\,!\,u^2}\left(1 - \left[\frac{1}{2\,q}\right]^2\right)\left(1 - \left[\frac{3}{2\,q}\right]^2\right) \\ &\quad - \frac{1}{3\,!\,u^3}\left(1 - \left[\frac{1}{2\,q}\right]^2\right)\left(1 - \left[\frac{3}{2\,q}\right]^2\right) \\ &\qquad\qquad \left(1 - \left[\frac{5}{2\,q}\right]^2\right)\ldots \\ &\quad + \;\ldots\ldots \end{aligned}\right. \qquad \textbf{(b)}$$

Comparison of (**a**) and (**b**) shows that, except for very small values of the argument u the artificial cable simulates the corresponding smooth cable quite closely if q is not too small. The departure between the two formulas decreases very rapidly as either u or q is increased in value. We observe, therefore, that the non-inductive cable admits of mush closer simulation by an artificial structure than does the transmission line which includes inductance as well as resistance and capacity.

The curves of Figs. 17, 18 and 19 exhibit the voltage waves on smooth lines, curves 17 and 18 being computed from formula (**59**) while curve (19) is plotted from formula (**58**) and exhibits the additional effect of leakage. Referring to Figs. 17 and 18 the voltage is identically zero until $T = A$ or $v\,t = x$, and has an instantaneous jump to the value ϵ^{-A}, after which it builds up slowly to its final value, unity. When leakage is present the wave head has likewise an instantaneous jump at $v\,t = x$, and thereafter builds up to its final value (shown by the dotted line in Fig. 19) $\epsilon^{-x\sqrt{RG}}$. The constants corresponding to curve 19 are so chosen as to make the line almost but not quite distortionless, the ratio ρ/σ being 5.

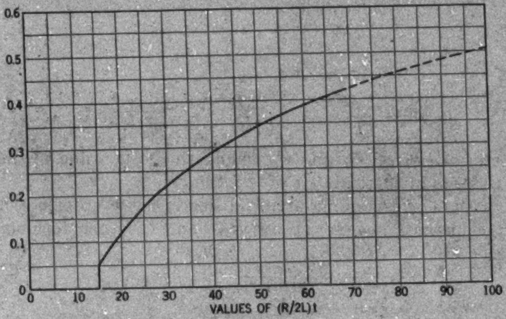

FIG. 17—[COMPUTED FROM FORMULA (59)] VOLTAGE ON CONTINUOUSLY LOADED LINE

Line constants: $R/2\sqrt{C/L}\,X = 3$

$G = 0$

The preceding curves all represent the current or voltage waves in response to a unit d-c. voltage applied to the line. Figs. 20 and 21 show how the current starts to build up when the line is excited by alternating voltages of two different frequencies. The line is the non-dissipative periodically loaded cable and the current is that existing in the 1000th section. Observe that in Fig. 21 the current has not yet attained ten per cent of its final steady state value, so that in this case the building up is very slow. These curves have been computed by special methods applicable to the non-dissipative loaded cable, which have proved considerably more rapid than the

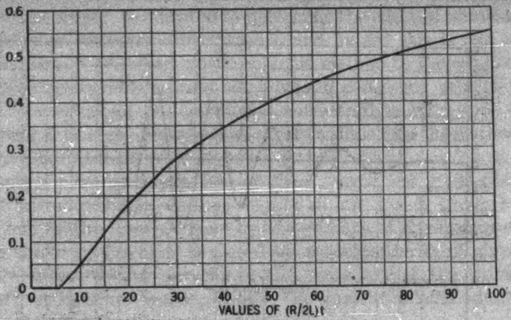

FIG. 18—[COMPUTED FROM FORMULA (59)] VOLTAGE ON CONTINUOUSLY LOADED LINE

Line constants: $R/2\sqrt{C/L}\,X = 6$

$G = 0$

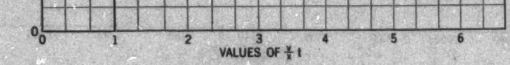

FIG. 19—[COMPUTED FROM FORMULA (58)] VOLTAGE ON CONTINUOUSLY LOADED LINE

Line constants: $R/2\sqrt{C/L}\,X = 3$

$G/2\sqrt{L/C}\,X = 2$

mechanical evaluation by the Coradi harmonic analyser as described in Section I. These special methods, however, are restricted to the non-dissipative loaded cable, and are therefore

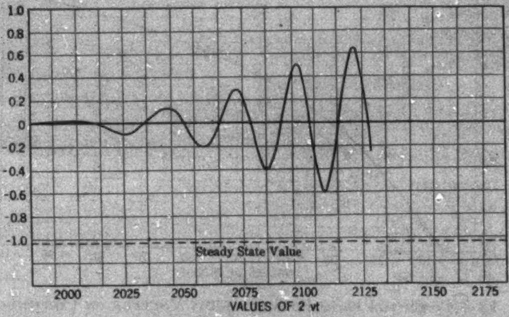

FIG. 20—CURRENT IN 100TH SECTION OF LOADED CABLE IN RESPONSE TO VOLTAGE cos pt

Non-dissipative: $R=G=0$. $f/fc=0.25$. Multiply ordinates by $\sqrt{C/L}$

not of sufficient general interest to warrant their discussion here.

The foregoing discussion is merely suggestive and makes no pretense to more than point out briefly the prominent outstanding facts. It is hoped, however, that the discussion is full enough to show that a real interpretation and understanding of the complex phenomena of wave propagation is deducible from the formulas derived in this paper.

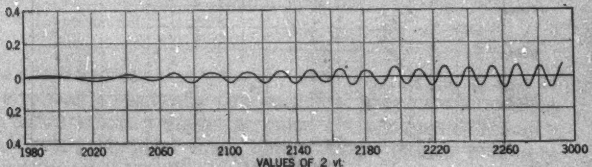

FIG. 21—CURRENT IN 1000TH SECTION OF LOADED CABLE IN RESPONSE TO VOLTAGE cos pt

Non-dissipative: $R=G=0$. $f/f_c=0.6283$. Steady state amplitude $=1.286$

VII. Application of Synthetic Method to Transmission Lines with Terminal Impedances

The synthetic method, developed and discussed in a previous part of this paper, is adapted to determine the effect of terminal apparatus in transmission systems, and indeed this is one of its chief uses. We shall now apply the method to the solution of a practical problem, and since we are concerned primarily with the method of treatment and not with detailed results, a relatively simple problem will be chosen to illustrate the method and its possibilities. A statement of the problem is as follows:

A smooth transmission line of given length and given constants has a unit d-c. voltage applied at $x = t = 0$, while the other end of the line is closed by the primary of a transformer the secondary of which is closed by a resistance R_2 of given value. What is the current in the secondary of the transformer; or in other words, what is the transfer indicial admittance of the system?

We denote the length of the line by s; its constants per unit length by R, L, and C (leakage is assumed absent); the primary and secondary inductances of the transformer (assumed as unity ratio and without magnetic leakage) by L_1; the unknown line current and voltage at $x = s$ by $I_s\,(t)$ and $V_s\,(t)$ respectively. We also denote by $A_1\,(t)$ and $A_2\,(t)$ the driving point and transfer indicial admittances of the line per se. These are then equal to the currents existing at $x = 0$ and $x = s$ respectively when a unit d-c. voltage is applied to the line at $x = 0$ and $t = 0$. By aid of formulas (**15**) and (**53**), $A_1\,(t)$ and $A_2\,(t)$ are given by

$$A_1\,(t) = \sqrt{\frac{C}{L}}\,\epsilon^{-\rho t}\left\{ I_0\,(\rho\, t) + 2\,I_0\left(\rho\sqrt{t^2 - \left[\frac{2\,s}{v}\right]^2}\right) + 2\,I_0\left(\rho\sqrt{t^2 - \left[\frac{4\,s}{v}\right]^2}\right) + \dots \right\} \qquad \textbf{(a)}$$

$$A_2\,(t) = 2\sqrt{\frac{C}{L}}\,\epsilon^{-\rho t}\left\{ I_0\left(\rho\sqrt{t^2 - \left[\frac{s}{v}\right]^2}\right) + I_0\left(\rho\sqrt{t^2 - \left[\frac{3\,s}{v}\right]^2}\right) + \dots \right\} \qquad \textbf{(b)}$$

In these formulas, I_0 denotes the Bessel function of order zero, and the successive terms of the series come into existence at the time when their arguments pass from imaginary to real values. For smaller values of the time they are to be regarded as identically zero since they represent successive reflected waves.

Now the line, regarded by itself, is actuated by a unit d-c. voltage applied at $x = 0$ and a voltage $V_s(t)$ applied at $x = s$ by formula (**16**) the current $I_s(t)$ existing at $x = s$ is given by

$$A_2(t) - \frac{d}{dt} \int_0^t V_s(\lambda) \, . \, A_1(t - \lambda) \cdot d\lambda \qquad (\mathbf{c})$$

Now this same current is that which enters the primary of the transformer which is connected across the line at $x = s$. We can therefore express this current as that entering the terminal apparatus in response to the voltage $V_s(t)$; it is then

$$\frac{1}{R_2} V_s(t) + \frac{1}{L_1} \int_0^t V_s(\lambda) \cdot d\lambda \qquad (\mathbf{d})$$

Equating (**c**) and (**d**) we get after simplification

$$V_s(t) = \left[\frac{R_2}{1 + R_2 A_1(0)} \right] \cdot$$

$$\left[A_2(t) - \int_0^t V_s(\lambda) \left\{ A'_1(t - \lambda) + \frac{1}{L_1} \right\} \cdot d\lambda \right] \qquad (\mathbf{e})$$

which is the integral equation in $V_s(t)$. Since the voltage across the secondary of the transformer is $V_s(t)$ also, the secondary current is simply

$$\frac{1}{R_2} V_s(t)$$

The numerical integration of this equation can, of course, be effected only when the numerical values of the constants of the problem are assigned. We take the following values.

$R_2 = A_1(0) = \sqrt{\frac{C}{L}} = 1935$ ohms

$L_1 = L_2 = 0.4$ henrys

$\frac{R}{2L} = \quad = 292$

$v = \quad = 1.105 \times 10^4$

$s = 100$

The results of the numerical integration of the equation are shown in Fig. 22 which is a plot of the voltage existing across the end of the line and hence is directly proportional to the current in the secondary of the terminal transformer. In this particular case the effect of the secondary reflections is insignificant and the voltage sensibly zero before the secondary reflected wave has arrived.

As a second example of the synthetic method we shall develop the solution for the voltage existing at the terminals of an infinitely long non-inductive cable when a unit d-c. voltage is applied at time $t = 0$ to the cable through a condenser C_0. This problem is not only of some practical importance but is of very great theoretical interest as well in that we are led not only to the formal solution but also to an asymptotic solution identical with that gotten by Heaviside by his remarkable "fractional differentiation." We shall have occasion, therefore, to discuss briefly Heaviside's mode of solution in this connection.

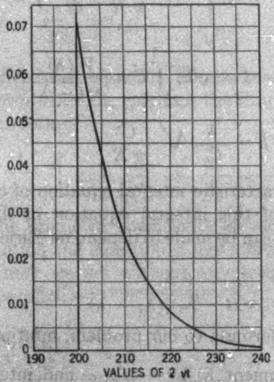

FIG. 22—[COMPUTED FROM FORMULA (63)] VOLTAGE ACROSS TERMINAL IMPEDANCE ON SMOOTH LINE. FOR DATA SEE SECTION VII

To proceed with the problem let R and C be the constants per unit length of the cable and C_0 the terminal condenser capacity. By formula (**57**) the indicial admittance of the cable is $\sqrt{\frac{C}{\pi R t}}$ so that, if $V(t)$ denote the required terminal voltage, the current entering the cable is, by formula (**11**),

$$\sqrt{\frac{C}{\pi R}} \cdot \frac{d}{dt} \int_0^t d\lambda \cdot \frac{V(\lambda)}{\sqrt{t-\lambda}}$$

But since this current is likewise equal to C_0 times the time rate of increase of the voltage $1 - V(t)$ which exists across the condenser, we have

$$C_0 \frac{d}{dt} \left(1 - V(t)\right) = \sqrt{\frac{C}{\pi R}} \frac{d}{dt} \int_0^t d\lambda \frac{V(\lambda)}{\sqrt{t-\lambda}}$$

whence

$$V(t) = 1 - \sqrt{h} \int_0^t d\lambda \frac{V(\lambda)}{\sqrt{t-\lambda}} \qquad (\mathbf{a}_1)$$

where $\sqrt{h} = \frac{1}{C_0} \sqrt{\frac{C}{\pi R}}$

which is the characteristic integral equation of our problem.

The solution of this integral equation in the form of an asymptotic series can be made to depend on Dirichlet's formula,

$$\int_\lambda^t \frac{d\lambda_1}{\sqrt{(t-\lambda_1)(\lambda_1-\lambda)}} = \pi$$

To apply this formula to our problem, multiply the integral equation (to argument λ) by $\frac{1}{\sqrt{t-\lambda}}$ and integrate between the limits 0 and t; this gives

$$\int_0^t d\lambda \frac{V(\lambda)}{\sqrt{t-\lambda}}$$

$$= 2\sqrt{t} - \sqrt{h} \int_0^t d\lambda \cdot V(\lambda) \int_\lambda^t \frac{d\lambda_1}{\sqrt{(t-\lambda_1)(\lambda_1-\lambda)}}$$

$$= 2\sqrt{t} - \pi\sqrt{h} \int_0^t V(\lambda)\, d\lambda \qquad (\mathbf{b}_1)$$

by Dirichlet's formula.

By aid of this equation, the integral equation is reducible to the form,

$$V(t) - \pi h \int_0^t V(\lambda) \cdot d\lambda = 1 - 2\sqrt{h t} \qquad (\mathbf{c}_1)$$

the formal solution of which is

$$V(t) = \epsilon^{\pi h t} - \sqrt{h}\int_0^t d\lambda \cdot \frac{\epsilon^{\pi h \lambda}}{\sqrt{t-\lambda}}$$

This is the formal solution of the problem and might have been gotten directly from the original integral equation by the method of successive substitutions. The advantage of the method followed here is, however, that we have succeeded in reducing the original integral equation to the following differential equation which results from differentiating c_1 with respect to t:

$$V(t) - \frac{1}{\pi h}\, V'(t) = \frac{1}{\pi\sqrt{h\,t}}$$

Now this differential equation admits of easy asymptotic solution by known methods; perhaps the simplest is to assume a solution of the form

$$V(t) \sim \frac{1}{\sqrt{t}}\left[a_0 + \frac{a_1}{t} + \frac{a_2}{t^2} + \frac{a_3}{t^3} + \ldots \right]$$

and identify the unknown coefficients by substitution in the differential equation and equation of like powers of t. Proceeding in this manner we get without any trouble the required asymptotic expansion:—

$$V(t) \sim \frac{1}{\pi\sqrt{h\,t}} \cdot$$

$$\left\{ 1 - \left(\frac{1}{2\pi h t}\right) + 1.3\left(\frac{1}{2\pi h t}\right)^2 - 1.3.5\left(\frac{1}{2\pi h t}\right)^3 + \ldots \right\} \qquad (\mathbf{d}_1)$$

This expansion, although it does not formally satisfy the original integral equation of the problem is a true asymptotic solution, and may be used for numerical computation with an error less than the last term included. Its convenience for large values of the argument is obvious, and it shows by mere inspection that $V(t)$ ultimately approaches zero in accordance with the law $\frac{1}{\pi\sqrt{h\,t}}$.

Heavisides operational solution is arrived at as follows: If we go back to the original differential equations of the problem and replace $\frac{d}{dt}$ by p, we get the symbolic formula

$$V = \frac{\sqrt{\frac{R}{C\,p}}}{\sqrt{\frac{R}{C\,p} + \frac{1}{C_0\,p}}}$$

Rearranging and introducing the symbol

$$\sqrt{h} = \frac{1}{C_0}\sqrt{\frac{C}{\pi R}}$$

this may be written

$$V = \frac{\sqrt{\frac{p}{\pi h}}}{1 + \sqrt{\frac{p}{\pi h}}} = \sqrt{\frac{p}{\pi h}}\left[1 + \sqrt{\frac{p}{\pi h}}\right]^{-1}$$

Expanding by the Binomial theorem and rearranging we have symbolically

$$V = \left\{1 + \frac{p}{\pi h} + \left[\frac{p}{\pi h}\right]^2 + \ldots\right\}\sqrt{\frac{p}{\pi h}} - \left\{\frac{p}{\pi h} + \left[\frac{p}{\pi h}\right]^2 + \ldots\right\}$$

This equation, as it stands is quite without meaning. Nevertheless Heaviside deduces the correct asymptotic solution of the problem from the symbolic formula by the following empirical rule: Discard all integral powers of the operator p, identify $\sqrt{p}$ as $\frac{1}{\sqrt{\pi t}}$ and replace any p^k in the first bracketted series by $\frac{d^k}{dt^k}$; then,

$$V(t) = \left\{1 + \frac{1}{\pi h}\frac{d}{dt} + \frac{1}{(\pi h)^2}\frac{d^2}{dt^2} + \ldots\right\}\frac{1}{\pi\sqrt{h}\,t}$$

$$= \frac{1}{\pi\sqrt{h\,t}}\left\{1 - \left(\frac{1}{2\,\pi\,h\,t}\right) + 1.3\left(\frac{1}{2\,\pi\,h\,t}\right)^2 - \ldots\right\}$$

which agrees with the asymptotic expansion derived above.

Compared with the foregoing operational solution, the asymptotic expansion from the integral equation of the problem may appear cumbersome and involved, and it must be admitted that it entirely lacks the extraordinary directness and simplicity of the Heaviside solution. On the other hand there is this to be said for the solution from the integral equation; it is rigorous and demonstrably correct, whereas Heaviside's remarkable method was as he professes, the outcome of experiment and still lacks the logical demonstration which we are accustomed to require of our mathematical solutions. Again it must be remembered that the chief difficulty lies not so much in deriving as in subsequently computing and interpreting the solution. From this standpoint a series solution leaves much to be desired since its behavior and significance can be arrived at only by direct numerical computation. When, however, the solution is defined by an integral equation, we can not only always derive the formal series solution, and the asymptotic expansion (where such exists), but a study of the integral equation as it stands in many cases gives an insight into the behavior of the function without computation. Furthermore its numerical computation is usually far less laborious than is the computation of the equivalent series solution.

As a final example of the derivation of the asymptotic expansion from the integral equation of the problem we shall require the current at distance x from the sending end of the cable considered above. By formulas (**11**) and (**57**) the required current $I(t)$ is given by

$$\sqrt{\frac{C}{\pi R}}\,\frac{d}{dt}\int_0^t d\,\lambda \cdot V(t-\lambda)\cdot\frac{\epsilon^{-a/\lambda}}{\sqrt{\lambda}}$$

where $V(t)$ is defined by equation ($\mathbf{a}_1$) and $a = \dfrac{R\,C\,x^2}{4}$

Consequently, as may be shown without any trouble, the integral equation which defines the required current is

$$I(t) = C_0 \sqrt{h} \frac{\epsilon^{-a/t}}{\sqrt{t}} - \sqrt{h} \int_0^t d\lambda \frac{I(\lambda)}{\sqrt{t-\lambda}} \qquad (\mathbf{a}_2)$$

where, as above, $\sqrt{h} = \frac{1}{C_0} \sqrt{\frac{C}{\pi R}}$

Multiplying this equation (to argument λ) by $\frac{1}{\sqrt{t-\lambda}}$, integrating with respect to λ from o to t, and applying Dirichlet's formula, we get:—

$$\int_0^t d\lambda \cdot \frac{I(\lambda)}{\sqrt{t-\lambda}} = C_0 \sqrt{h} \int_0^t d\lambda \frac{\epsilon^{-a/\lambda}}{\sqrt{t\lambda - \lambda^2}}$$

$$- \pi \sqrt{h} \int_0^t I(\lambda) \cdot d\lambda \qquad (\mathbf{b}_2)$$

and finally, from $(\mathbf{a}_2)$ and $(\mathbf{b}_2)$,

$$I(t) - \pi h \int_0^t I(\lambda) \cdot d\lambda$$

$$= C_0 \sqrt{h} \frac{\epsilon^{-a/t}}{\sqrt{t}} - C_0 h \int_0^t d\lambda \frac{\epsilon^{-a/\lambda}}{\sqrt{t\lambda - \lambda^2}} \qquad (\mathbf{c}_2)$$

But

$$\int_0^t d\lambda \frac{\epsilon^{-a/\lambda}}{\sqrt{t\lambda - \lambda^2}} = \sqrt{a\pi} \int_0^t d\lambda \cdot \frac{\epsilon^{-a/\lambda}}{\lambda^{3/2}}$$

$$= \pi - 2\sqrt{\frac{a\pi}{t}} \left\{ 1 - \frac{1}{3 \cdot 1!} \left(\frac{a}{t} \right) + \frac{1}{5.2!} \left(\frac{a}{t} \right)^2 + \ldots \right\} \qquad (\mathbf{d}_2)$$

and

$$\frac{\epsilon^{-a/t}}{\sqrt{t}} = \frac{1}{\sqrt{t}} \left\{ 1 - \frac{1}{1!} \left(\frac{a}{t} \right) + \frac{1}{2!} \left(\frac{a}{t} \right)^2 \ldots \right\} \qquad (\mathbf{e}_2)$$

Substitution of $(\mathbf{d}_2)$ and $(\mathbf{e}_2)$ in $(\mathbf{c}_2)$ and rearrangement gives

$$\pi h \int_0^t I(\lambda) \cdot d\lambda - I(t) = \pi h C_0$$

$$- C_0 \sqrt{\frac{h}{t}} \left\{ \begin{array}{l} \left[1 + \frac{2}{1} \sqrt{a h \pi} \right] \\ - \frac{1}{1!} \left[\frac{a}{t} \right] \left[1 + \frac{2}{3} \sqrt{a h \pi} \right] \\ + \frac{1}{2!} \left[\frac{a}{t} \right]^2 \left[1 + \frac{2}{5} \sqrt{a h \pi} \right] \\ - \frac{1}{3!} \left[\frac{a}{t} \right]^3 \left[1 + \frac{2}{7} \sqrt{a h \pi} \right] \\ \quad \cdots \end{array} \right. \qquad (\mathbf{f}_2)$$

The asymptotic expansion of this equation can now be derived without any difficulty; as the expression becomes somewhat complicated we shall not however, carry out the solution in detail. From the equation, as it stands, it is clear that the total charge passing any point x is simply C_0 and that $I(t)$ ultimately becomes

$$\frac{1}{2\pi} \frac{C_0}{t\sqrt{h\,t}} \left(1 + 2\sqrt{a h \pi}\right)$$

To enter upon more advanced applications of the synthetic method is beyond the scope of the present paper. The simple examples dealt with above, will, it is hoped, serve to indicate the appropriate treatment and the wide range of problems to which the method is applicable.

NOTE 1

Owing to the very great importance of mono-periodic currents, the formulas appropriate to the sinusoidal e.m.f. $\sin(p\,t + \theta)$ have been the subject of particular study. While formula (**12**) is fundamental and its evaluation by aid of the harmonic analyzer is usually indicated, the derived formulas given below are of value in particular cases, especially when a knowledge of the behavior of the system to very low or very high frequencies is required.

If the cosine and sine functions in the intergrand of formula (**12**) are expanded in the usual convergent power series, it is easy to show that we get by repeated partial integrations and some easy simplifications

$$I = \sin\theta .\quad u(t) + \cos\theta \cdot v(t) \qquad \textbf{(a)}$$

where

$$u(t) = A(t) - p^2 \int_0^t \int_0^\lambda A(\lambda_1) \cdot d\lambda\, d\lambda_1$$

$$+ p^4 \int_0^t \int_0^\lambda \int_0^{\lambda_1} \int_0^{\lambda_2} A(\lambda_3)\, d\lambda\, d\lambda_1\, d\lambda_2\, d\lambda_3 \qquad \textbf{(b)}$$

$$- p^6 \int_0^t \ldots \int_0^{\lambda_4} A(\lambda_5) d\lambda \ldots d\lambda_5$$

$$\ldots\ldots$$

and

$$v(t) = p \int_0^t A(\lambda)\, d\lambda - p^3 \int_0^t \int_0^\lambda \int_0^{\lambda_1} A(\lambda_2) d\lambda\, d\lambda_1 d\lambda_2$$

$$+ p^5 \int_0^t \ldots \int_0^{\lambda_3} A(\lambda_4)\, d\lambda \ldots d\lambda_4 \qquad \textbf{(c)}$$

$$- p^7 \int_0^t \ldots \int_0^{\lambda_5} A(\lambda_6)\, d\lambda \ldots . d\lambda_6$$

$$\ldots\ldots$$

Both of these series are absolutely convergent for all values of p, but their direct computation is practicable only for relatively small values of p. In the limit $p = 0$, $u(t)$ reduces to $A(t)$ and $v(t)$ to zero as we should expect.

The functions $u(t)$ and $v(t)$ are obviously connected by the relation,

$$v(t) = p \int_0^t u(\lambda) . d\lambda \qquad \textbf{(d)}$$

so that $v(t)$ is calculable from $u(t)$ by mechanical integration.

Furthermore it is easy to show from formula (**b**) that $u(t)$ satisfies the integral equation

$$u(t) = A(t) - p^2 \int_0^t \int_0^\lambda u(\lambda_1) . d\lambda\, d\lambda_1$$

which can be written (by reversing the order of integration of the multiple integral)

$$u(t) = A(t) - p^2 \int_0^t (t - \lambda) . u(\lambda) . d\lambda \qquad \textbf{(e)}$$

The integral equation is now in canonical form, and in some problems its numerical integration is simpler than the mechanical evaluation of formula (**12**). For a brief account of integral equations and the methods for their formal and numerical solution, see Note 3.

The foregoing formulas are often useful when we wish to investigate the behavior of the system to applied voltages of low frequency; when, on the other hand, the applied frequency is high the following formula is applicable and is of considerable value in interpreting the ultimate character of the *transient distortion:*

If the applied voltage be $\sin\,(p\,t+\theta)$ *and if* p *be greater than* (p_m) *where* p_m *is the greatest root of the determinantal equation* $Z\,(p)=0$, *then the transient distortion, as defined by formula* (**14**), *is formulated by the convergent series:*

$$\cos\theta\,.\,g\,(t)+\sin\theta\,.\,h\,(t)$$

where

$$g\,(t)=\frac{1}{p}A^{(1)}(t)-\frac{1}{p^3}A^{(3)}(t)+\frac{1}{p^5}A^{(5)}(t)\,\ldots.$$

$$h\,(t)=\frac{1}{p^2}A^{(2)}(t)-\frac{1}{p^4}A^{(4)}(t)+\frac{1}{p^6}A^{(6)}(t)\,\ldots.$$

and

$$A^{(n)}\,(t)=\frac{d^n}{dt^n}A\,(t)$$

The proof of this last formula and the sufficient condition for its convergence are easily established. If the frequency is very high it is clear that the transient distortion becomes ultimately,

$$\frac{1}{p}A'\,(t)$$

and therefore, if the frequency be high enough the *transient distortion in all systems is inversely proportional to the applied frequency,* provided that the characteristic oscillation frequencies have a finite upper limit. Furthermore, even when divergent, series of the above form, in many cases, constitute asymptotic expansions which are valid for numerical computation.

NOTE 2

From our restricted standpoint an asymptotic expansion is defined as an expansion of a function in inverse powers of the argument. The theory of asymptotic expansions occupies an important place in modern analysis. With the questions of divergence, convergence and summability to which pure analysis is largely restricted, we are not concerned, but rather with methods of asymptotic expansions, regarding which knowledge is quite fragmentary. For a good brief discussion of asymptotic expansions from the point of view of analysis, the reader is referred to Whittaker and Watson—"A Course in Modern Analysis," Chap. VIII.

When the system under consideration consists of a finite number of branches or meshes, the asymptotic expansion of the function of $\frac{1}{Z(p)}$ is always possible. A general rule for the expansion is as follows: Replace p by $\frac{1}{y}$ in $\frac{1}{Z(p)}$ and write

$$\frac{1}{Z(p)} = \frac{1}{Z\left(\frac{1}{y}\right)} = \varphi(y)$$

Then expand $\varphi(y)$ in the usual Taylor's series

$$\varphi(y) = \varphi(0) + \varphi'(0) \cdot \frac{y}{1\,!} + \varphi''(0) \cdot \frac{y^2}{2\,!}$$

$$= \Sigma\, \varphi^{m}(0) \cdot \frac{y^m}{m\,!}$$

Then the required asymptotic expansion is given by

$$\frac{1}{Z(p)} \sim \Sigma\, \varphi^{m}(0) \cdot \frac{1}{m\,!\; p^m}$$

In many problems, however, much shorter ways of expanding suggest themselves, and often the binomial expansion can be directly applied.

NOTE 3

In this connection the writer takes the opportunity of calling attention to a formula, as yet unpublished, which was derived by Mr. T. C. Fry of the Western Electric Company in Nov. 1916. Starting with the Fourier integral which formulates the current in response to an applied voltage $f(t)$ and applying a theorem of Hermites' thereto, Mr. Fry arrived at the original formula (adopting the notation of the present paper)

$$I = \int_0^t f(\lambda)\, d\lambda \,.\, \Sigma \frac{\epsilon^{p_m(t-\lambda)}}{Z'(p_m)}$$

Mr. Fry restricted consideration to systems for which $\frac{1}{Z(\infty)} = A(o) = o$; for this case it is equivalent to formula (**10**) of this paper.

The writer has derived the power series solution from the Fourier integral formulation by asymptotically expanding $\frac{1}{Z(p)}$ in the integrand and then evaluating the infinite integrals by the calculus of residues.

Since this paper was written the author has been privileged to see some more recent work, as yet unpublished, of Mr. Fry's, in which he has secured results of great originality and value by attacking the Fourier integral by the methods of advanced analysis.

NOTE 4

The integral equation, known as Fredholm's equation is of the following form

$$v(t) = f(t) + \int_b^a v(\lambda) \cdot u(t, \lambda) \cdot d\lambda$$

where $f(t)$ and $u(t, \lambda)$ are known functions. The type of integral equation with which we are concerned is less general in form:

$$v(t) = f(t) + \int_0^t v(\lambda) \cdot u(t - \lambda) \cdot d\lambda \qquad \textbf{(a)}$$

differing from the general equation in that the upper limit of integration is variable and in that the kernel or nucleus of the equation is a function of $t - \lambda$ instead of being a perfectly arbitrary function of t and λ. When the upper limit of integration is variable the equation is usually called Voltenra's equation in honor of the Italian mathematician who has studied it extensively.

The formal solution of this equation is gotten by successive or repeated substitution of the right hand side of the equation for $v\ (t)$ in the integral. It is

$$
\begin{aligned}
v\ (t) = f\ (t) &+ \int_0^t u\ (t-\lambda) \cdot f\ (\lambda) \cdot d\ \lambda \\
&+ \int_0^t u\ (t-\lambda) \cdot \int_0^\lambda u\ (\lambda-\lambda_1) \cdot f\ (\lambda_1) \cdot d\ \lambda\ d\ \lambda_1 \\
&+ \int_0^t u\ (t-\lambda) \int_0^\lambda u\ (\lambda-\lambda_1) \int_0^{\lambda_1} u\ (\lambda_1-\lambda_2) \\
&\qquad\qquad f\ (\lambda_2)\ d\ \lambda\ d\ \lambda_1\ d\ \lambda_2 \\
&+ \cdots
\end{aligned}
$$

Pure analysis is concerned, of course, not with the numerical interpretation of this solution but with questions as to the uniqueness, convergence, etc. and it has been shown that the solution is not only unique but absolutely convergent.

Except, however, for very simple forms of the functions $f\ (t)$ and $u\ (t)$ the formal solution given above is quite useless for our purpose, which must always be to arrive at interpretable and computable results. Indeed our conception of a solution is quite distinct from that of pure analysis, and our tests must always be pragmatic.

If neither $f\ (t)$ or $u\ (t)$ involve fractional powers of t the *power series solution* can be gotten from (a) by repeated differentiations as follows:

From (a) we have:

$$v\ (o) = f\ (o)$$

$$v'\ (t) = \frac{d}{d\,t}\ v\ (t) = f'\ (t) + u\ (o) \cdot f\ (t) + \int_0^t v\ (\lambda) \cdot u'\ (t-\lambda) \cdot d\ \lambda$$

$$v'\ (o) = f'\ (o) + u\ (o) \cdot v\ (o)$$

Proceeding with repeated differentiations we get

$$v''\ (o) = f''\ (o) + u\ (o) \cdot v'\ (o) + u'\ (o) \cdot v\ (o)$$
$$v'''\ (o) = f'''\ (o) + u\ (o) \cdot v''\ (o) + u'\ (o) \cdot v'\ (o) + u''\ (o) \cdot v\ (o)$$

Hence $v\ (o)$, $v'\ (o)$, $v''\ (o) \ldots v^{(m)}\ (o)$ are all expressible in terms of $f\ (t)$ and $u\ (t)$ and their derivatives. Hence the solution is

$$v\,(t) = v\,(o) + v'\,(o) \cdot \frac{t}{1\,!} + v''\,(o) \cdot \frac{t^2}{2\,!} + -$$

$$= \Sigma\, v^{(m)}\,(o) \cdot \frac{t^m}{m\,!}$$

where $v^m\,(o)$ is gotten from (*b*).

If either $u\,(t)$ or $f\,(t)$ contains fractional powers of t, the solution can be gotten by *assuming* a series solution of the form

$$v\,(t) = \Sigma\, a_m\, t^m + \sqrt{t}\, \Sigma\, b_m\, t^m$$

substituting in the integral equation, integrating and then determining the unknown coefficients of the series solution by direct equation of like powers of t.

Corresponding to every series solution of the form

$$\Sigma\, a_m\, t^m + \sqrt{t} \cdot \Sigma\, b_m\, t^m$$

Heaviside, by his operational treatment, arrives at an asymptotic solution of the form

$$\Sigma\, \frac{c_m}{t^m} + \frac{1}{\sqrt{t}}\, \Sigma\, \frac{d_m}{t^m}$$

which, although divergent, may be used for computation, in spite of the fact, which may be easily proved, that the latter series does not satisfy the integral equation and is therefore not a formal solution of the problem. We shall show later that the asymptotic solutions are derivable from the integral equations of the problem, and thus verify Heaviside's remarkable results.

The numerical integration of the integral equation (**a**) is carried out as follows: We remark that the integral appearing in (**a**) is the limit of the summation:

$$\begin{aligned}
&[v\,(o) \cdot u\,(t) + v\,(\Delta\, t) \cdot u\,(t - \Delta\, t)]\,\Delta\, t/2 \\
&+ [v\,(\Delta\, t) \cdot u\,(t - \Delta\, t) + v\,(2\Delta\, t) \cdot u\,(t - 2\Delta\, t)] \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \Delta\, t/2 \\
&+ \cdots \\
&+ [v\,(t - \Delta\, t) \cdot u\,(\Delta\, t) + v\,(t) \cdot u\,(o)] \cdot \Delta\, t/2 \\
&= v\,(o) \cdot u\,(t) \cdot + v\,(t) \cdot u\,(o) \Delta\, t/2 \\
&+ \Delta\, t \sum_{1}^{n-1}{}_{m}\; v\,(m\Delta\, t) \cdot u\,(t - m\Delta\, t)
\end{aligned}$$

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When $t = n\triangle t$

Equation (**a**) then is the *limit* of the expression

$$\left[1 - u\,(o) \cdot \frac{\triangle t}{2} \right] v\,(t) = f\,(t) + v\,(o) \cdot u\,(t) \cdot \frac{\triangle t}{2} \cdot$$

$$+ \triangle t \sum_{1}^{n-1}{}_{m} \; v\,(m\triangle t) \cdot u\,(t - m\triangle t)$$

When $n\triangle t = t$.

or

$$v\,(t) = \frac{1}{1 - u\,(o) \cdot \frac{\triangle t}{2}} \left\{ \begin{array}{l} f\,(t) + f\,(o) \cdot \frac{\triangle t}{2} \cdot u\,(t) \\ + \triangle (t) \sum_{1}^{n-1}{}_{m} v\,(m\triangle t) \cdot u\,(t - m\triangle t) \end{array} \right.$$

If we now call $t = t_1;\ 2t = t_2; \ldots$
$n\triangle t = t_n$ we have:—

$$v\,(o) = f\,(o)$$

$$v\,(t_1) = \frac{1}{1 - u\,(o) \cdot \frac{\triangle t}{2}} \left\{ f\,(t_1) + f\,(o) \cdot u\,(t_1) \cdot \frac{\triangle t}{2} \right.$$

$$v\,(t_2) = \frac{1}{1 - u\,(o) \cdot \frac{\triangle t}{2}} \left\{ \begin{array}{l} f\,(t_2) + f\,(o) \cdot u\,(t_2) \cdot \frac{\triangle t}{2} \\ \qquad + v\,(t_1) \cdot u\,(t_1) \cdot \triangle t \end{array} \right.$$

$$v\,(t_3) = \frac{1}{1 - u\,(o) \cdot \frac{\triangle t}{2}} \left\{ \begin{array}{l} f\,(t_3) + f\,(o) \cdot u\,(t_3) \cdot \frac{\triangle t}{2} \\ + \triangle t \,\{ v\,(t_1) \cdot u(t_2) + v\,(t_2) \cdot u\,(t_1) \} \end{array} \right\}$$

. . . . .

In each of these equations the factors appearing on the right hand side are known from the preceding equations, and we thus can compute $v\,(t)$ for any required value of t.

The time element $\triangle t$ should be chosen with reference to the character of the known function involved in the integral equation; that is, it should be so small as to make the curvature, or second differences of these functions small within the interval $\triangle t$. The precision of this method increases, of course, as the time interval $\triangle t$ is diminished and can be made as exact as we please by taking $\triangle t$ sufficiently small.

The method has been tried out pretty thoroughly in a series of problems and works well with a precision quite sufficient for practical purposes. The labor is very much less than that involved in a computation of a power series solution; and the accuracy of the results is readily checked.

NOTE 5

The steady-state properties and characteristics of periodic structures (of which the artificial line is a particular case) were investigated by Dr. G. A. Campbell, Research Engineer of the American Telephone and Telegraph Company, who discovered their extremely interesting and important action as "wave-filters," due to their property of transmitting with small attenuation steady-state currents of all frequencies lying within a continuous range or ranges of frequency (which may be termed transmission bands) while sharply attenuating all currents of frequencies outside these transmission bands. For a fairly full account of the wave filter and its theory, see U. S. patent No. 1,227,113 to G. A. Campbell.

NOTE 6

The Bessel function of the first kind, $J_n(Z)$ and $I_n(Z)$ are defined, when n is a positive integer, by the absolutely convergent series:

$$J_n(Z) = \left(\frac{Z^n}{2^n \cdot n!}\right) \cdot \left\{ \begin{aligned} &1 - \frac{Z^2}{2(2n+2)} + \frac{Z^4}{2.4(2n+2)(2n+4)} + \dots \\ &- \frac{Z^6}{2.4.6(2n+2)(2n+4)(2n+6)} - + \dots \end{aligned} \right.$$

$$\begin{aligned} I_n(Z) &= i^{-n} J_n(iZ) \\ &= \left(\frac{Z^n}{2^n \cdot n!}\right) \cdot \end{aligned}$$

$$\left\{ \begin{aligned} &1 + \frac{Z^2}{2(2n+2)} + \frac{Z^4}{2.4(2n+2)(2n+4)} \\ &+ \frac{Z^6}{2.4.6(2n+2)(2n+4)(2n+6)} + \dots \end{aligned} \right.$$

The terms of $J_n(Z)$ are alternating in sign while those of $I_n(Z)$ are all positive.

For values of Z very large compared with n, the functions can be computed from the asymptotic formulas:

$$J_n(Z) = \sqrt{\frac{2}{\pi Z}} \left\{ \begin{array}{l} P_n \,.\, \cos\left(\frac{2n+1}{4}\pi - Z\right) \\ \quad + Q_n \,.\, \sin\left(\frac{2n+1}{4}\pi - Z\right) \end{array} \right\}$$

where

$$P_n = 1 - \frac{(4n^2-1)(4n^2-9)}{2\,!\,(8Z)^2}$$

$$+ \frac{(4n^2-1)(4n^2-9)(4n^2-25)(4n^2-49)}{4\,!\,(8Z)^4} \ldots$$

$$Q_n = \frac{4n^2-1}{8Z} - \frac{(4n^2-1)(4n^2-9)(4n^2-25)}{3\,!\,(8Z)^3}$$

and

$$I_n(Z) = \frac{\epsilon^Z}{\sqrt{2\pi Z}} \left\{ 1 - \frac{4n^2-1}{1\,!\,.\,2^3 Z} + \frac{(4n^2-1)(4n^2-9)}{2\,!\,.\,2^6\,.\,Z^2} \ldots \right.$$

For the large order of functions with which we are mainly concerned these asymptotic expansions are useless for values of argument of practical importance, since we are concerned with the behavior of the functions in the neighborhood of $Z = n$. The computation of the functions is carried out without much difficulty from the valuable asymptotic formulas given by J. W. Nicholson in a series of papers appearing in the *Philosophical Magazine* (Dec. 1907, August 1908, July 1909, Feb. and Dec. 1910). These formulas are too long to reproduce here and the original papers should be consulted.

The properties of the integral function $\int_0^Z J_n(Z_1)\, d\, z_1$, are of course determined by the corresponding properties of the Bessel function appearing in the integrand; it does not appear, however, that the integral function has been the subject of any extensive study in spite of its extremely interesting properties. A few of these properties which are of importance for our purposes are:

(1) $\int_0^\infty J_n(z)\, dz = 1$ for all values of n.

(2) For values of z very large compared with n,

$$\int_0^Z J_n(z_1)\, dz_1 \sim 1 - \sqrt{\frac{2}{\pi}} \int_i^\infty \cos\left(z_1 - \frac{2n+1}{4}\pi\right) \frac{dz_1}{\sqrt{z_1}}$$

the integral term of which can be resolved into Fresnel integrals.

(3) For large values of n, the function $\int_0^Z J_n(z_1)\, dz_1$ is insignificant for values of Z less than n. From a mathematical standpoint, this property explains the virtual velocity of propagation of the artificial line.

(4) $$\text{Limit } n \to \infty \left[\int_0^{\lambda n} J_n(z_1)\, dz_1\right] \begin{array}{l} = o, \quad \lambda < n \\ = 1, \quad \lambda > n \end{array}$$

This relation enables us to derive the solution for the smooth line as the limiting case of the artificial line, as the aggregate value of the constant is kept finite and the number of sections approaches infinity.

NOTE 7

The correctness of the solution formulated by (**35**) may be verified by deriving the corresponding steady-state formula by aid of equation (**13**). If a voltage $\sin(pt + \theta)$ is applied at midload position to an infinitely long periodically loaded cable the steady-state or periodic current in the *qth* load-coil is given by substitution of (**35**) in (**13**); it is

$$\sqrt{\frac{C}{L}} \left\{ \begin{array}{l} \sin(pt + \theta) \int_0^\infty \cos(w\lambda) \cdot J_{2q}(\lambda) \cdot d\lambda \\ - \cos(pt + \theta) \int_0^\infty \sin(w\lambda) \cdot J_{2q}(\lambda) \cdot d\lambda \end{array} \right.$$

When $w = \frac{p}{p_c} = \frac{p}{2v} = \frac{p}{2}\sqrt{LC}$

p_c indicates $2\pi f_c$ where f_c is the "critical or cut-off frequency of the line.

The infinite integrals have been evaluated, they are (see Nielsen—*Handbuch der Cylinderfunctionen*, p. 195.)

$$\int_0^\infty \cos(w\lambda)\cdot J_{2q}(\lambda)\cdot d\lambda = \frac{\cos(2q\sin^{-1}w)}{\sqrt{1-w^2}} \qquad w<1$$

$$= -\frac{\sin q\pi}{\sqrt{w^2-1}}\left(w-\sqrt{w^2-1}\right)^{2q} \qquad w>1$$

$$\int_0^\infty \sin(w\lambda)\cdot J_{2q}(\lambda)\cdot d\lambda = \frac{\sin(2q\sin^{-1}w)}{\sqrt{1-w^2}} \qquad w<1$$

$$= \frac{\cos q\pi}{\sqrt{w^2-1}}\left(w-\sqrt{w^2-1}\right)^{2q} \qquad w>1$$

Hence, if $w<1$, $p<p_c$, the frequency is less than the critical frequency and

$$|I_q| = \sqrt{\frac{C}{L}}\,\frac{1}{\sqrt{1-w^2}}$$

If $w>1$ the frequency is above the critical frequency and

$$|I_q| = \sqrt{\frac{C}{L}}\,\frac{1}{\sqrt{w^2-1}}\left(\frac{1}{w+\sqrt{w^2-1}}\right)^{2q}$$

which agree exactly with the results gotten by the usual steady state or periodic theory, and show the critical character of the frequency $\frac{p_c}{2\pi}$.

NOTE 8

From formulas (**12**) and (**53**) we can arrive at some important deductions regarding the relations obtaining between the frequency of the applied voltage and the character of the transient distortion on the smooth line.

If the applied e.m.f. is taken as $\sin(pt+\theta)$ (applied at

$t = o$ and $x = o$), the current at point x on the line is by (**12**) and (**53**)

$$I_x = 0 \qquad \text{for } v\,t < x$$

$$I_x = \sqrt{\frac{C}{L}}\ \epsilon^{-ax} \sin\left(p \left[t - \frac{x}{v} \right] + \theta \right)$$

$$+ \sqrt{\frac{C}{L}}\,\epsilon^{-ax} \left\{ \begin{array}{l} \sin\,(p\,t + \theta) \displaystyle\int_x^{vt} \cos\left[\frac{p}{v}\,\lambda \right] \frac{d}{d\,\lambda}\,\varphi\,(\lambda)\,d\,\lambda \\ - \cos\,(p\,t + \theta) \displaystyle\int_x^{vt} \sin\left[\frac{p}{v}\,\lambda \right] \frac{d}{d\,\lambda}\,\varphi\,(\lambda)\,d\,\lambda \end{array} \right.$$

where

$$\varphi\,(\lambda) = \epsilon^{-a(\lambda - x)}\, I_0\,(a\,\sqrt{\lambda^2 - x^2})$$

Inspection of this formula shows that the first term is the final steady state provided the frequency of the applied voltage is high enough to justify the employment of the usual approximate formulas for the steady-state characteristic impedance and propagation factor. The integral terms represent transient distortion. To a first order of approximation the integral terms are of an order of magnitude

$$\frac{1}{p} \cdot \frac{R}{2\,L} \cdot (a\,x - 2)$$

provided $a\,x > 2$.

The building up of the current wave to its final steady state value may therefore be described as follows:

If a voltage of frequency f is applied to a smooth transmission line the current at distance x out on the line is identically zero for a time interval x/v after the application of the voltage; at the end of this time interval it has an instantaneous jump to its final steady state value, but is thereafter subjected to a transient distortion the magnitude of which is roughly *inversely proportional to the applied frequency*, and the sensible duration of which diminishes with the applied frequency. The transient distortion is negligibly small for $a\,x < 2$, while for large values of $a\,x$ it increases rapidly at a more than linear rate with the product $R/L\,a\,x$. If the applied frequency is made indefinitely great the transient distortion approaches zero both in magnitude and duration.

From formulas (**15**) and (**53**) the complete wave solution for the leakageless line of length s, short-circuited at $x = s$, and actuated by a unit voltage applied at $x = o$ and $t = o$, is

$$A_x(t) = \sqrt{\frac{C}{L}}\,\epsilon^{-\rho t}\cdot \left\{ \begin{aligned} & I_0\left(\rho\sqrt{t^2 - \left[\frac{x}{v}\right]^2}\right) \\ & \quad + 2 \sum_1^{\infty}{}_m\, I_0\left(\rho\sqrt{t^2 - \left[\frac{2\,m\,s + x}{v}\right]^2}\right) \\ & \quad + 2 \sum_1^{\infty}{}_m\, I_0\left(\rho\sqrt{t^2 - \left[\frac{2\,m\,s - x}{v}\right]^2}\right) \end{aligned} \right. \qquad (\mathbf{a})$$

This solution will be discussed with particular reference to the velocity of transmission regarding which there seems to exist considerable confusion of thought or at any rate looseness of statement. The case of the leakageless line is chosen purely for the sake of simplicity and the following remarks apply also to the general case when leakage is present.

Now in the light of the derivation of formula (**53**) it is to be understood that the terms of formula (**a**) come into existence not at time $t = o$ but *only as the arguments of the* I_0 *functions pass from imaginary to real values.* That is, while all the terms are written down in formula (**a**), they come into existence at successive intervals of time.

Thus for $o < t < x/v$, $A_x(t)$ is identically zero, and at $t = x/v$ the direct wave (which alone exists on the infinitely long line) arrives at x, and it alone exists there until $t = \dfrac{2\,s - x}{v}$ when the first reflected wave arrives. At a subsequent time $t = \dfrac{2\,s + x}{v}$ the second reflected wave arrives and so the final steady state is built up. Analytically,

$$A_x(t) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 0 < t < x/v$$

$$A_x(t) = \sqrt{\frac{C'}{L}}\,\epsilon^{-\rho t}\, I_0\left\{\rho\sqrt{t^2 - \left(\frac{x}{v}\right)^2}\right\} \ldots\ldots\ldots\ldots$$

$$x/v < t < \frac{2s - x}{v}$$

$$A_x(t) = \sqrt{\frac{C}{L}}\,\epsilon^{-\rho t}\left\{ I_0\left(\rho\sqrt{t^2 - \left[\frac{x}{v}\right]^2}\right)\right.$$

$$\left. + 2\, I_0\left(\rho\sqrt{t^2 - \left[\frac{2s - x}{v}\right]}\right) \quad \frac{2s - x}{v} < t < \frac{2s + x}{v}\right.$$

. . . . . .

Not until $t = \infty$ do all the terms finally come into existence.

Now the outstanding deduction from this interpretation of formula (**a**) is that it represents a series of waves which are propagated with finite velocity $x = \dfrac{1}{\sqrt{LC}}$ which is independent of the dissipation occurring during transmission whether due to line resistance or leakage. Consequently formula (**a**) is the wave solution; the first term represents the direct wave, which alone exists on the infinitely long line; the second summation represents a series of secondary waves reflected at $x = s$ and the first summation a series of secondary waves reflected at $x = o$. Each wave is represented by the product of an exponential term and a Bessel function, the latter formulating the distortion which the wave suffers during transmission, and which is absent on the non-dissipative or distortionless line. The foregoing discussion of the wave solution holds for all types of applied voltages except that the character of the distortion of the wave during transmission is modified in accordance with formula (**11**). It is also applicable to all types of terminal arrangements, due regard being had to the fact that the form of the reflected waves is modified by the character of the terminal apparatus. In no case, however, is the velocity of transmission of the series of waves modified by the type of applied voltage or character of the end terminations.

Now in accordance with the method of derivation of formula (**a**) it must be equivalent to the expansion (**40**) (setting $G = o$) and this expansion will accordingly be investigated. After easy simplification formula (**40**) with G put equal to zero may be written,

$$A_x(t) = 2\sqrt{\frac{C}{L}}\,\epsilon^{-\rho t}\cdot\sum_{1}^{\infty}{}_{m}\frac{\cos\left[m\cdot\frac{x\,\pi}{s}\right]}{s\sqrt{\left[m\,\frac{\pi}{s}\right]^2-\left[\frac{\rho}{v}\right]^2}}$$

$$\cdot\sin\left(v\sqrt{\left[m\,\frac{\pi}{S}\right]^2-\left[\frac{\rho}{v}\right]^2}\,t\right)+\frac{1}{R\,s}\left(1-\epsilon^{-\frac{R}{L}t}\right) \qquad (\mathbf{b})$$

$$=\sqrt{\frac{C}{L}}\,\epsilon^{-\rho t}\,\Sigma\,m$$

$$\left\{\begin{array}{l}\sin\left(m\,\frac{v\,\pi}{s}\sqrt{1-h^2_m}\left[t-\frac{x}{v\sqrt{1-h^2_m}}\right]\right)\\[2ex] +\sin\left(m\,\frac{v\,\pi}{s}\sqrt{1-h^2_m}\left[t+\frac{x}{v\sqrt{1-h^2_m}}\right]\right)\end{array}\right\}$$

$$\frac{1}{\pi\,m\sqrt{1-h^2_m}}+\frac{1}{R\,s}\left(1-\epsilon^{-\frac{R}{L}t}\right) \qquad (\mathbf{c})$$

where $\quad h_m=\frac{1}{m}\,\frac{\rho\,s}{\pi\,v}$

This type of solution is often referred to or regarded as a wave solution; in fact, however, it is a formulation in terms of normal or characteristic *vibrations* and in a more precise terminology is a normal function or vibrational solution. Each term of (**b**) is the product of a cosine function of space and a sine function of time which may be thrown into the form of (**c**) in which the individual terms appear to represent propagated waves. The propagation is only apparent, however, since each term of (**c**) is instantaneously produced on the spot at $t = o$ and is not *propagated* as are the component waves of the wave solution (**a**). The instantaneously produced terms of (**c**), however, combine for a definite time x/v to annul each other so that the summation is identically zero until $t = \frac{x}{v}$

the solution is therefore mathematically correct but physically artificial and deceptive. The most serious confusion of thought results from a tendency to interpret the summation (**c**) from a consideration of a single term and from the prevalent error of regarding the individual characteristic *vibrations* as *propagated waves*. Thus by inspection of (**c**) each term has an *apparent* velocity of phase propagation $v\sqrt{1-h^2_m}$ whereas the actual velocity of propagation of the wave is identically v. From this viewpoint the velocity of propagation of the vibrations *appears* to increase with the frequency of the vibrations up to the limiting value v. $v\sqrt{1-h^2_m}$ is merely the *apparent velocity of phase propagation* of the characteristic vibration and only in the case of the non-dissipative and distortionless line (where $h_m = o$) is the apparent velocity of phase propagation equal to v which is the true "speed of propagation of the electric phenomenon in the circuit." The increase of $v\sqrt{1-h^2_m}$ with the frequency of the characteristic vibration does not indicate an increased speed of propagation with frequency but merely a *change of form* or *distortion* of the wave during transmission.

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TELEPHONE CIRCUITS WITH ZERO MUTUAL INDUCTION

BY WILLIAM W. CRAWFORD

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Abstract of Paper

The paper deals with the reduction of inductive interference in telephone circuits.

Various relative positions of two or more circuits, in which the mutual inductance is zero, and the mutual capacitance unbalance is approximately zero, are discussed. The most important case is that when the two wires of one circuit occupy opposite ends of one diagonal of a square, and the other circuit, the ends of the other diagonal. A phantom formed on circuits so related is nearly immune from external induction.

Several forms of construction embodying this arrangement, and built largely with standard parts, are illustrated. In these forms, about one third of the inductive relations of each physical circuit are with circuits in a position for approximately zero mutual inductance. The form of phantom transposition used automatically balances the circuits not so related.

Calculations and tentative designs are presented to show that the use of these forms of construction will give greatest refinement of balance against induction from power circuits, and possibly also against cross-talk, increased flexibility in coordinating with the variations in exposure to power circuits, a simplification of the transpositions system, fewer transposition poles and transpositions, and when desired, the realization of a part of these advantages with the lead compressed into less than the normal space. The improvement is greatest in the phantoms.

Adverse factors are the increased sensibility to cross-talk due to variations in sag, and the slightly increased capacitance of the phantoms.

The cost of initial construction of the proposed forms should not be materially higher than for the present standard form. The cost of conversion of an existing lead to one of the proposed forms will be considerably greater than that of re-transposing the lead according to present practises.

I. General

THE method of transposition as applied at present in telephone practise involves balancing against each other fairly large inductive effects in adjacent sections of the circuits. Mr. Harold S. Osborne has in a recent paper[1] named various influences which render it impossible to make the opposing effects completely annul each other. By proper design the residual unbalances are ordinarily reduced to a point which gives satisfactory operation. The design of the transposition system is, however, subject to severe limitations in the case where the telephone line is to parallel electric power circuits. Particularly, it is difficult to obtain proper co-ordination between the transpositions in the telephone circuits and the points where changes take place in the degree of exposure to power circuits.

There are certain relative positions of two or more circuits composed of parallel wires, in which the electrostatic and electromagnetic co-efficients of mutual induction are continuously zero. Between circuits so positioned not only should the residual effects disappear, but there should be no need for transposition. It seems worth while to inquire how far the employment of such arrangements will go toward giving greater refinement of balance, increasing the flexibility of design, and rendering usable for telephone leads locations which are not permissible in present practise. The idea involved is a very old one[2], and the aim of this paper is to show that it has certain merits in connection with modern practise which it did not have before the advent of the phantom circuit.

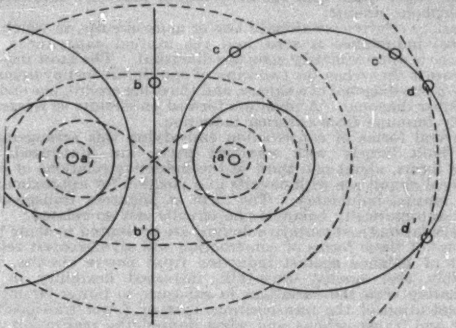

Fig. 1

In Fig. 1 the solid lines represent the electrostatic equipotential surfaces due to equal and opposite charges on the

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[1] Trans. A. I. E. E., 1918, p. 897.

[2] See A. V. Abbott, Electrical Transmission of Energy, 1898, Art. 52.

parallel wires $a\,a'$. Placing the two wires of the circuit $c\,c'$ in the same equipotential surface as shown, subjects both wires to the influence of the same potential, there is no tendency to induce a current around the circuit $c\,c'$. If, however, a and a' have like charges, the equipotentials are as shown by the dotted lines, c and c' are unequally affected.

A like situation arises with respect to electromagnetic induction. The lines of magnetic force due to a current out a and back a', or vice versa, follow the same curves as shown by the solid lines in Fig. 1. For such currents there is no mutual inductance between $a\,a'$ and $c\,c'$. But for currents travelling in the same direction along a and a', the lines of force follow the dotted curves, they link between c and c', and induce a current around that circuit.

The voice currents in the physical circuits travel around the circuits, but the phantom voice currents, and some classes of stray currents due to induction from outside sources, travel along both wires in the same direction. Therefore the arrangement of Fig. 1 would tend to eliminate cross-talk between the physical circuits $a\,a'$ and $c\,c'$, but not cross-talk between these circuits and a phantom set up on them, nor the tendency of stray lengthwise currents in one physical circuit to render the other physical circuit noisy. The latter effects are removed by transposing both circuits alike.

The same is more or less true of every mutually non-inductive relative position of the two circuits except that shown by $a\,a'$ and $b\,b'$, Fig. 1. Wires $b\,b'$ lie in the same electrostatic equipotential, or link zero flux, for either like or unlike charges or currents on $a\,a'$. The circuit $a\,a'$ is also balanced for currents or charges in $b\,b'$. A circuit in the position $d\,d'$ is balanced with respect to $a\,a'$, but $a\,a'$ is only partially balanced with respect to $d\,d'$.

If a phantom is set up on the circuits $a\,a'$ and $b\,b'$ the, phantom is very nearly immune from external fields. The average effect of an external field on wires a and a' will approximately balance that on wires b and b', unless the field is very non-uniform. There is, moreover, no mutual induction between the phantom and its own side circuits—a marked contrast to present standard practise.

The above statements are inaccurate to the extent that the affected wires may have unequal capacities to ground, so that equal potentials induce unequal charges, and that the electrostatic field is distorted by the presence of other wires on the lead, which behave as if grounded[3]. Consequently, there is no single relative position which will in all cases give perfect balance for both electrostatic and electromagnetic effects. Such transpositions as are necessary for other reasons, are sufficient to eliminate also these effects.

II. APPLICATION TO COMPLETE LEADS

The practicability of applying the proposed group arrangement in practise will be discussed in terms of the forms of construction shown in Figs. 2 to 9 inclusive.

In form *A*, Fig. 2, the cross-arm spacing is made equal to the pin spacing. Wires 1 and 12 form a physcial circuit, as do the wires 2 and 11, all the other wires are arranged in similar groups. There is a considerable mutual induction between wires 1-12 and 4-13, also between 2-11 and 3-14. Between 1-12 and 3-14, 5-16, etc., there is an approach to the same relation as between $a\,a'$ and $c\,c'$, Fig. 1.

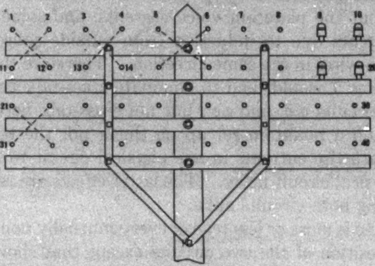

FIG. 2—FORM *A*

There being no necessity of transposing the two physical circuits of a group relatively to each other, each group is treated as a unit in transposing relatively to the other groups. Special insulators or pins are not necessary to make a transposition. The wires are merely rotated around the quadrangle, for example, 1 replacing 2, 2 replacing 12, 12 replacing 11, and 11 replacing 1. For a physical transposition this rotation is continued in two spans, whereupon the two wires of each circuit are interchanged, see Fig. 3.

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[3] Loc. cit. page 925.

As shown in Table III, there is some mutual inductance between phantoms, though this is smaller than it is in the present system, and falls off much more rapidly with distance. The amount is sufficient to require transposition of the nearest phantoms. For a phantom transposition, the wires are rotated in only one span. This method transposition is the same as that now used in polyphase power circuits.

At the center of the span, the distance between the wires is reduced to $1/\sqrt{2}$ of its value at the poles, see dotted square in Fig. 3.

If existing crossarms are used to construct form *A* or *C*, the central group is not quite equilateral, there being a greater space between the pole pins. The chief effect of this distortion is to introduce into this one group a mutual inductance between the side circuits, about equal to that between a phantom and each of its side circuits in present practise, see Tables I and III. The phantom transpositions, by interchanging the pin positions of the side circuits, balance out this mutual inductance. Considerations of type unbalance between the side circuits may require transposing the phantom of this group to a pattern having more frequent transpositions than would be required for the benefit of the phantom alone.

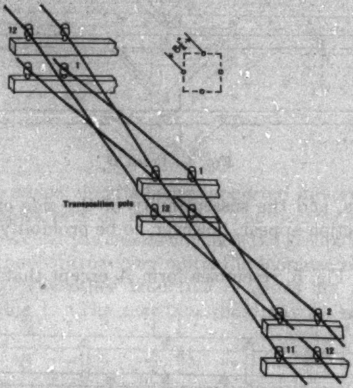

FIG. 3—PHYSICAL TRANSPOSITION

In form *B*, Fig. 4, the use of present standard crossarm spacing, as well as standard pin spacing, is contemplated. The only alteration required to convert a present type lead into form *B* would be to re-transpose, and re-connect the jumpers at the ends of the line. The mutual inductance between the side circuits, and the susceptibility of the side circuits to external induction appear, however, to be prohibitive, see Tables II and VII.

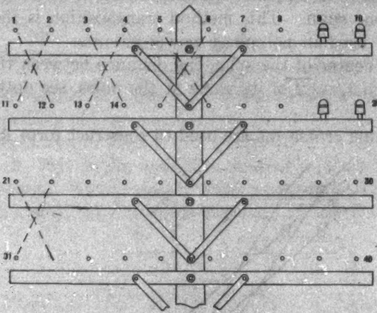

FIG. 4—FORM *B*

Form *C*, Fig. 5, resembles form *A* except that the distance between the various horizontal rows of groups is increased, giving electrical characteristics somewhat better than form *A*, and better accessibility of the wires.

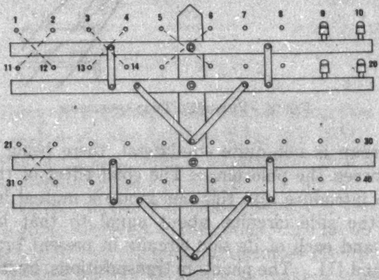

FIG. 5—FORM *C*

In form *D*, Fig. 6, the aim is to provide still better conditions as to freedom from induction. The unsymmetrical center group is removed, and the spacing between the other groups is increased.

Form *E*, Fig. 7, is obtained by adding to a lead, constructed in accordance with present practise, four pairs of wires per crossarm, the added circuits being supported in vertical planes on special duplex pins or brackets. The necessity of having the lower wires clear the crossarm makes the groups unsymmetrical, but without introducing mutual inductance between the side circuits. The other inductive relations are affected as shown in Table I. The pole-pin wires may be reserved for non-phantomed circuits, or phantomed by the method now in use.

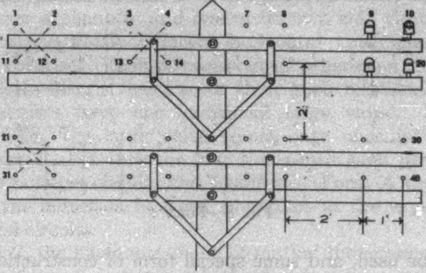

Fig. 6—Form *D*

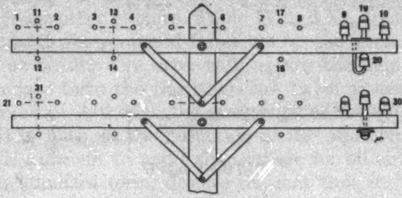

Fig. 7—Form *E*

One or more form *E* groups may be incorporated in a lead which is otherwise standard, giving proper attention to the co-ordination of the transpositions of the two different styles of construction. The form *A* group may also be constructed with duplex pins or brackets, in which case it becomes feasible to use this form in the same way as the form *E*.

Form *F*, Fig. 8, is an idealised form *E*, in which the groups are symmetrical. Form *G*, Fig. 9, bears the same relation to form *F* that form *D* does to form *A*. On account of the small clearance between wires in form *F*, and *G*, standard crossarms cannot be used, and some special form of construction would have to be devised.

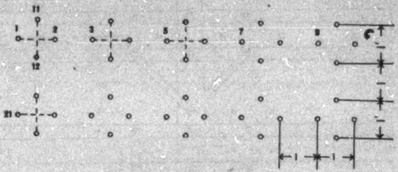

Fig. 8—Form *F*

Certain other designs, in which the form *A* group is alternated with the form *E* group, or in which the different horizontal rows of groups have different spacings, give zero mutual inductance between some of the adjacent phantoms. The possibilities presented by these designs have not yet been fully worked out.

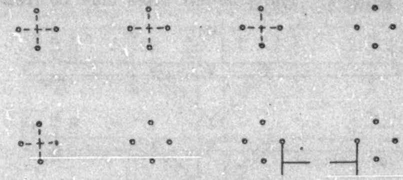

Fig. 9—Form *G*

III. Comparisons

In comparing the susceptibility to induction of the various forms it has unfortunately been possible to take account only of the mutual inductance, and not of the capacitance unbalances, because of the great labor required to compute the latter[4]. Probably the two effects will vary nearly enough in the same proportion, as between different forms, so that the comparisons on a basis of inductance alone, if used with due caution, will not give false conclusions. Values of mutual inductance are given in Tables I, II, and III.

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[4] Loc. cit. page 902.

In form *A*, the maximum value of mutual inductance (that between circuits related like 1-12 and 4-13) is about equal to that between a phantom and one of its own side circuits, or between two vertically adjacent horizontal type phantoms, in present practise. Each physical circuit other than the pole-pin circuits in form *A* has two or three such relations. The pole-pin circuits have one additional large value. In the standard form, the large value comes only once into each physical circuit, but three or four times into each phantom, except the pole-pin circuits and phantom. Form *A* therefore decreases the induction between phantoms at the expense of the physical circuits.

In form *C*, the maximum value of mutual inductance is the same as in form *A*, but this value occurs less frequently.

Forms *D*, *F*, and *G* give on the whole a very great reduction in mutual inductance. Forms *F* and *G* are interesting from the large number of values of zero mutual inductance which they afford. Most of these are due to the relation $a\,a'$ to $d\,d'$, Fig. 1.

Form *E* has a large value of mutual inductance, about equal to that in the standard form, which comes once into each phantom, and once each into half the side circuits.

A summation was obtained of the inductive effects on each circuit of all the other circuits (Table IV and V), showing that the phantoms in the proposed forms are on the average about one tenth as sensitive to cross-talk as are the side circuits. In the standard form the phantoms are twice as sensitive as the side circuits. Although the sensitivity of the side circuits is increased in form *A*, the reduction of the sensitivity of the phantoms is sufficient to make the average for all circuits less than in the standard form, despite the fact that in form *A* a given number of circuits are compressed into half the normal space. Such a compression of the standard form would result in considerable increases in the mutual inductance values, with consequent increase of the sensitivity to cross-talk. In form *C* the sensitivity of the side circuits is about the same as in the standard form, the improvement of the phantoms is clear again. In the other forms, the sensitivities of side circuits as well as phantoms, are decreased.

Corresponding figures (Table VII) for the sensitivity to induction from power circuits show a similar result. In forms *A*, *C*, and *D*, the sensitivity of the side circuits is increased because in these forms the distance between the two wires of a side circuit is $\sqrt{2}$ times what it is in present practise. The sensitivity of the phantoms is, however, negligible, except when the disturbing circuit is very close to the telephone circuit.

On account of the shorter average distance between the wires in a group in the proposed forms, it is to be expected that the effect of accidental displacements of the wires from their correct positions will be greater than in standard practise. The unbalance produced will in each case be greatest between the circuits forming one group, and the comparison is made only on the basis of such circuits. (Table VI).

For horizontal displacements, the various forms have nearly the same sensitivity as standard construction. Horizontal displacements should be rare, because the horizontal spacing is fixed by the pin locations.

The effective vertical spacing depends on the sag, and this is likely to vary. For displacements as small as the value assumed for comparison (root-of-mean-square displacement, 0.1 foot, or 0.03 meter) the effect is practically nil in the standard form, but in all the proposed forms, it is appreciable. This is a distinct disadvantage, the importance of which can be determined only by the aid of data from actual construction. Comparison with the values in Table V appears to show that unless the r. m. s. value of displacement is considerably greater than the value assumed, the total unbalance due to these displacements will not be more serious than the unbalance which occurs in standard practise due to the transpositions being located at the nearest poles, instead of at the theoretically correct points. The tendency of the proposed forms being to reduce the effect of inexact location of the transpositions, the total effect of all irregularities should not be much greater than in present practise.

Keeping the crossarms carrying wires in the same groups accurately parallel is evidently essential in forms *A*, *C*, and *D*, the method of bracing illustrated in these forms is intended to provide for this.

The effect of displacements of the wires on the induction from power circuits is the same in the proposed forms as in the standard form.

Number of Transpositions and Number of Transposition Patterns. Certain comparisons are fairly obvious between the number of transposition patterns (types) and the total number of transpositions required in the standard and proposed forms.

The condition of zero mutual induction between a phantom and its side circuits does not mean that the three circuits can be transposed after the same pattern. If that were done, each time a side circuits reversed, it would move to the other pin position, for a given pin position it would always have the same polarity, it would not be balanced with respect to external induction. This fact is exemplified in the standard *E*-section[4] in circuits 5-6 and 15-16, which have zero mutual inductance with the phantom, nevertheless, they are transposed to different patterns from the phantom (Table IX).

If each circuit is to have its own transposition pattern, thirty patterns are required for the commonly used physical and phantom circuits on the first four arms in the standard form. In the proposed forms, each pair of side circuits being transposed as a unit, only twenty patterns would be required.

In the *E*-section, 16 types of transposition pattern are used, and mutual inductance and capacitance unbalances as large as 5.44 microhenries per kilometer, and 3.0 micro-microfarads per kilometer, respectively, are allowed. Adopting these limits for the proposed forms permits a still further reduction in the number of patterns, and in the number of transpositions, required.

The effect of simultaneous phantom transpositions in two groups is to reverse the mutual inductance between the circuits whose planes are at right angles, like 1-12 and 4-13, Figs. 2, 5, or 6. Only the circuits related like 1-12 and 3-14 are left unbalanced, and the mutual inductance between these is small. Whenever the mutual inductance of two phantoms is small enough so that they can be transposed to the same pattern, the same is true of the physical circuits.

Fig. 10 shows a transposition system which utilized this fact. Four types of group transposition are used, denoted by I, II, III, IV. There are only sixteen transposition poles per section. The procedure in design was as follows:

Physical transpositions were inserted in all the groups at the odd-numbered transposition poles, balancing the physical circuits for outside induction every eighth section. Next, phantom transpositions were inserted at the poles 2, 6, 10, and 14. This balances the phantoms every quarter section for sources of induction which are too near for the phantoms to be immune due to their form. As already stated, these phantom transpositions greatly reduce the unbalance between physical circuits. Poles 4, 8, and 12 remain for balancing out the induction between phantoms and the residual induction between physical circuits in the four groups. Only two phantom types are used, leaving the diagonally adjacent phantoms untransposed. It is then possible to obtain complete balance of the four adjacent pairs of physical circuits. There are several ways of doing this, one of which is shown. It is also possible to balance the four phantoms, leaving a slight residual unbalanced in the diagonally adjacent pairs of physical circuits.

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[4] Loc. cit. page 911, Fig. 5.

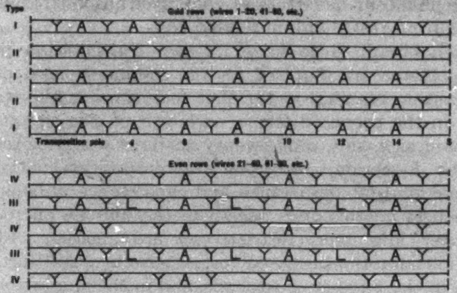
Type
Odd rows (wires 1-20, 41-60, etc.)
I
II
I
II
I
Transposition pole
4
6
8
10
12
14
S
Even rows (wires 21-40, 61-80, etc.)
IV
III
IV
III
IV

Fig. 10

Y—physical transposition
A—phantom transposition, right-handed
L—phantom transposition, left-handed

The largest values of unbalanced mutual inductance in Fig. 10 are as follows:

Form	*A*	*C*	*D*	*F*	*G*
Diagonally adjacent groups					
Phantoms	4.77	0.76	0.93	1.17	0.23
Alternate groups in the same horizontal row					
Phantoms	1.17	1.17	0.23	0.29	0.06
Physical circs.	1.55	1.55	0.31	0.40	0.08

Limits in the standard *E*-section
Inductance unbalance, 5.44, capacitance unbalance, 3.0

The inductance unbalance is within the limit even for form

A. The limiting value of capacitance balance in the *E*-section, however, occurs between circuits which are separated by a large number of intervening wires, the screening action of which has a large effect in reducing the capacitance unbalance. There will be very little screening action between the diagonally adjacent groups in the proposed forms, hence, in form *A* the capacitance unbalance between the diagonally adjacent phantoms will be much too large if Fig. 10 is used. In the other forms, however, the inductance unbalances are so far below the limit that it is probable that the capacitance unbalances are not excessive, and Fig. 10 can be used.

Fig. 11 shows a system in which the mutual inductance between the diagonally adjacent phantoms is eliminated. The method of design was the same as in Fig. 10, except that physical balance was obtained between horizontally and vertically adjacent groups at the outset, giving the necessary added flexibility to permit transpositions on poles 4, 8, and 12 to completely balance the four groups. In this case, the maximum unbalance is between alternate groups in the same horizontal row. Fig. 11 is apparently suitable for use with form *A*. By using a slightly larger number of transpositions, with the same number of transposition poles, non-adjacent groups can be completely balanced if desired.

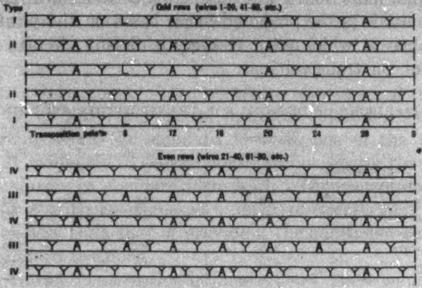
Type
Odd rows (wires 1-20, 41-60, etc.)
I
II
II
I
Transposition poles
8
12
16
20
24
28
Even rows (wires 21-40, 61-80, etc.)
IV
III
IV
III
IV

FIG. 11

Y—physical transposition
A—phantom transposition, right-handed
L—phantom transposition, left-handed

The extension of Figs. 10 and 11 to more than forty wires is a matter only of repeating the same system, additional transposition poles are not necessary.

These designs were not obtained by a systematic study of all the possibilities, but rather, by a logical process from the assumed conditions, to which method the proposed form of construction appears to be favorable. The designs are of a tentative nature, and are intended merely as illustrations of the possibilities of the proposed forms.

In standard practise, a distinction is drawn between the effect of distant and near disturbing circuits, on account of the effect of the phantom transpositions in shifting the pin positions of the physical circuits. In Figs. 10 and 11, this effect does not need to be considered, since each section of physical circuit between two phantom transpositions is completely balanced for external induction. The distinction between near and distant circuits is in the proposed forms based on the phantoms, the balance of which with respect to distant circuits is not dependent on transpositions.

COMPARISONS OF DESIGNS FOR TRANSPOSITION SECTIONS

	Standard	Proposed	
	E-section	Fig. 10	Fig. 11
No. of transposition poles per section			
First 40 wires	32	16	32
Additional wires	64	16	32
No of transpositions per section, 40 wires	288	141	179
Wires cut	876	564	716
Balanced to distant non-transposed circuits, every	Eighth-section. Eighth-section.	Eight-section. Eighth-section.	Eighth-section. Eighth-section.
Balanced to near non-transposed circuits, every	Half-section.	Quarter-section.	Quarter-section.
Balanced to distant 3-phase power circuits for	Two successive half-mile barrels. 3, 6, or 12-mile barrels.	Single half-mile barrels. 3, 6, or 12-mile barrels.	Two successive half-mile barrels. 3, 6, or 12-mile barrels.
Balanced to near 3-phase power circuits for	Quarter-mile barrels, 12-mile barrels.	Quarter or half-mile barrels, 6 or 12-mile barrels.	Quarter-mile barrels, 6 or 12-mile barrels.
Neutral points for discontinuities of telephone lead.	*S*–poles	*S*–poles	*S*–poles, half-section points.

Balance for a half-section, equal to Fig. 10 for a whole section.

The permissible locations for discontinuities of the telephone lead depend a good deal on the form of construction. For example, a form *D* lead transposed by Fig. 10 or 11 could be discontinued at a half-section, or even at a quarter-section point, with only a very slight resultant unbalance.

Provision for Non-Phantomed Circuits. Where non-phantomed circuits occur in pairs, they may be treated in the same manner as phantomed circuits, except, possibly, in the case of circuits which have transmitters of large output, in which the unbalance due to sag variations may make it undesirable to associate the circuits in groups of the proposed form. Each single non-phantomed circuit will have to occupy the space appropriate for a group. If there are only a few such circuits, the loss of space will be small, for a larger number, one or more crossarms of standard construction may be incorporated in the lead.

Effects of the Constants of the Circuits. The capacitance of the phantoms is about 30 per cent greater in the proposed forms than in present practise, and the inductance correspondingly less. The change in the constants of the side circuits is very slight.

Cost. On account of the necessity of re-spacing the crossarms, or of providing special insulator supports, the cost of converting from the standard form to one of the proposed forms is evidently in general considerably greater than that of re-transposing a standard form lead. The cost of converting to form *E* with an increased number of circuits may in some cases be less than that of providing the additional circuits by the standard method.

For initial construction of forms *A* or *C* there would apparently be no increased cost for materials, and a saving in the cost of transposing. Form *D* presents a slight increased cost of materials, due to the increased number of crossarms per circuit-mile.

IV. CONCLUSIONS

The anticipation that the use of the proposed forms of telephone line construction and transposition under exposed-line conditions will give greater refinement of balance and increase the flexibility in co-ordinating with transpositions and discontinuities of the disturbing circuits, particularly, when the telephone circuits are phantomed, appears to be justified.

The proposed forms appear to be especially well adapted for use on joint pole lines and in other cases of severe or irregular exposure. Obviously, they affect only the particular phases of inductive interference for which transposition is a remedy. The worst effect of severe exposure to power circuits is that of the high longitudinal electromotive forces and currents induced in the telephone circuits when an adjacent power circuit operates normally with a ground return, or is accidentally grounded[5]. The increased balance of the open-wire circuits afforded by the proposed forms should permit operating with larger values of longitudinal currents than in the standard form, except where the limiting value of the current or e. m. f. is determined by conditions of safety, or by the balance of the terminal apparatus.

Present-type phantoms are often used as parts of longer lines, despite the fact that they are considerably more susceptible to noise and cross-talk than are the physical circuits, and that a given amount of noise is of greater importance in the longer circuit. The proposed forms in which the phantoms are relatively immune from disturbances, bring the desirable qualities of noiselessness and good transmission together in the circuits where they are most needed.

Form *A* is for use in new construction where space limitations are extreme.

Form *C* permits an appreciable saving of space, with a good degree of balance, a saving in the number of transpositions and transposition poles, and increased flexibility in the location of discontinuities in power circuits, as compared to the standard *E*-section.

Form *D* gives a considerable reduction of type unbalance and unbalance due to unequal transposed lengths, with a slight increase of cost, as compared to form *C*.

Partial or complete conversion to form *E* recommends itself when it is desired to route one or more high-grade phantom circuits through locally adverse conditions, or where it is desired to add a few circuits to the lead under conditions of limited space. Partial conversion to form *A*, *C*, or *D*, may also be effected; in these forms, the phantoms have better balance than in form *E*. Such conversion is of course conditional on the ability of the crossarms to support the additional weight.

Forms *F* and *G* promise the highest degree of balance of all the forms shown, but would probably not be considered in the present state of the art on account of the special construction required.

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[5] "Transposition and Relative Location of Power and Telephone Wires," P. M. Lincoln, TRANS. A. I. E. E., 1903.

"Inductive Effects, etc.," H. S. Warren, TRANS. A. I. E. E., 1918, p. 503.

V. NUMERICAL VALUES AND METHODS OF CALCULATION

a. *Mutual Inductance.* The calculations were made on the basis of a spacing between the pole pins equal to that between the other pins, except in forms *D* and *G*, where the distance between groups was made uniform. The order of magnitude of the differences due to these assumptions is indicated in Table I. Where the differences are large enough to affect the comparisons appreciably, allowance has been made for them.

From the values given in Tables I, II and III, the mutual inductances of most of the other pairs of circuits can be found, by determing to which of the circuits given in the table, their relative location is similar.

TABLE I

EFFECT OF ASSUMPTIONS AS TO SPACING

Circuits	Mutual inductance, microhenries per kilometer	
	Actual	Assumed
Standard Form		
1–2 and 5–6	–16.00	–12.92
1–2 and 9–10	–2.92	–3.16
Forms *A* and *C*		
5–16 and 6–15	–115.08	0
Phants. 3–14–4–13 and 5–16–6–15	–19.38	–17.86
Phants. 3–14–4–13 and 7–18–8–17	–0.84	–1.17
Forms *E* and *F*	Form *E*	Form *F*
1–2 and 13–14	–15.58	0
Phant. and its own vertical side	122.38	0
Phants. 1–2–11–12 and 3–4–13–14	9.87	4.77
Phants. 1–2–11–12 and 21–22–31–32	–0.40	4.77

b. *Summated Effects of Unequal Transposed Lengths and Type Unbalance.* The summation of these effects was obtained as follows:

Assume a given circuit to be affected by equal values of current *I* and equal unbalanced lengths *l* in all the other circuits. The voltages induced will be

$$E_2 = -jM_2\,\omega_2\,I_2\,l$$

$$E_3 = -jM_3\,\omega_2\,I_2\,l$$

etc.

and if the frequencies are nearly equal, the energies induced in the first circuit by the various other circuits will be proportional to

$$E_2^2,\ E_3^2,\ \text{etc.}$$

The total energy will be proportional to ΣE^2, or, since $j I \omega l$ is assumed to be constant, to ΣM^2. ΣM^2 is therefore a measure of the total sensitivity of the circuits to electromagnetic induction from the other circuits. For a complete result, of course, a like value for electrostatic induction should be obtained.

TABLE II
MUTUAL INDUCTANCES BETWEEN PHYSICAL CIRCUITS
Microhenries per Kilometer.

Form of construction Present		Proposed *A*		*B*	*D*	*F*		*G*
Effect on physical circuit 1-2		1-12		1-12	1-12	1-2		1-2
Of Circ.	*M*	Of Circ.	*M*	*M*	*M*	Of Circ.	*M*	*M*
3-4	−57.54	2-11	0	277.3	0	11-12	0	0
5-66	−12.92	3-14	22.31	140.2	4.82	3-4	−57.54	−23.55
7-8	−5.64	4-13	102.17	−81.1	44.63	13-14	0	0
9-10	−3.16	5-16	1.55	38.7	....	5-6	−12.92	−5.63
11-12	44.63	6-15	25.03	57.5	....	15-16	0	....
13-14	1.55	7-18	0.31	17.1	0.31	7-8	−5.64	−5.64
15-16	−5.92	8-17	11.11	60.4	11.11	17-18	0	0
17-18	−4.02	9-20	0.09	9.5	0.07	9-10	−3.16	−2.47
19-20	−2.61	10-19	6.26	15.3	4.93	19-20	0	0
21-22	12.12	21-32	22.31	−32.5	4.82	21-22	44.63	21.08
23-24	6.07	22-31	−102.17	−69.7	−44.63	31-32	0	0
25-26	4.03	23-34	−57.54	−57.5	−23.55	23-24	1.55	0.30
27-28	−1.54	24-33	0	−28.4	0	33-34	−25.04	−11.11
29-30	−1.49	25-36	−16.25	−25.6	....	25-26	−5.92	......
31-32	5.48	26-35	11.99	1.4	....	35-36	−8.01	....
33-34	3.98	27-38	−6.92	−9.3	−6.17	27-28	−4.02	−2.66
35-36	1.52	28.37	8.00	7.8	5.33	37-38	−3.00	−3.56
37-38	0.01	29-40	−2 72	−3.2	−2.66	29-30	−2.61	−1.78
39-40	−0.56	30-39	5.19	7.6	3.55	39-40	−1.38	−1.25
Effect on		2-11		....	2-11	11-12		11-12
		Of	*M*		*M*	Of	*M*	*M*
		26-35	15.69	....	....	19-20	3.12	1.78
		28-37	6.06	....	7.05	25-26	6.06	....
		30-39	2.81	....	2.67	27-28	....	5.48
Form *C*. First row like Form *A*. Second row:								
Effect on		1-12		1-12		2-11		
		Of	*M*	Of	*M*	Of	*M*	
		21-32	4.82	26-35	4.46	24-33	26.83	
		22-31	−44.63	27-38	−7.17	28-37	7.05	
		23-34	−30.15	28-37	5.33	26-35	14.84	
		24-33	−11.78	29-40	−3.60	30-39	3.61	
		25-36	−15.93	30-39	4.13			

ΣM^2 may be taken for each pair of circuits on the lead, and the average per circuit or per group taken as representing the whole system.

The induced e. m. f. or current due to type unbalance is, for a given relative transposition type, also proportional to the co-efficient of mutual induction. ΣM^2 may therefore be considered to represent in a measure the total effect of type unbalance, although the comparison will be affected to some extent by the practise of reducing the effect of large values of

TABLE III

MUTUAL INDUCTANCE BETWEEN PHANTOMS AND BETWEEN PHANTOMS AND PHYSICAL CIRCUITS

Microhenries per Kilometer

Form of construction							
Present		Proposed					
		A	*D*	*F*	*G*		
Effect on phantom 1-2-3-4		1-12-2-11			1-2-11-12		
Of Phantom	*M*	Of Phantom	*M*	Of Phantom	Of Phantom	*M*	*M*
5–16	–60.02	3–13	–17.86	–3.64	3–14	4.77	0.93
7–10	–24.75	5–15	–1.17		5–16	0.29	
11–14	117.34	7–17	–0.23	–0.23	7–18	0.06	0.06
17–20	–18.88	9–19	–0.08	–0.04	9–20	0.03	0.01
21–24	41.35	21–31	–17.86	–3.64	21–32	4.77	0.93
25–36	–19.90	23–33	4.77	0.93	23–34	–1.17	–0.23
27–30	–5.34	25–35	0.21		25–36	–0.05	
31–34	20.38	27–37	–0.05	0.04	27–38	0.02	–0.01
37–40	2.28	29–39	–0.03	–0.01	29–40	0.01	0.003
Of Phy. Circ.		Of Phy. Circ.			Of Phy. Circ.		
1–22	109.86	1–12	0	0	1–2	0	0
3–4	109.86	2–11	0	0	11–12	0	0
5–6	–58.78	3–14	–29.39	–8.12	3–4	14.32	3.96
7–8	–28.77	4–13	–29.39	–8.12	13–14	0	0
9–10	–8.51	5–16	–3.31		5–6	1.62	
11–12	47.78	6–15	–3.31		15–16	0	
13–14	47.78	7–18	–0.95	–0.95	7–8	0.46	0.46
15–16	–7.66	8–17	–0.95	–0.95	17–18	0	0
17–18	–9.97	9–20	–0.40	–0.28	9–10	0.20	0.14
19–20	–6.54	10–19	–0.40	–0.28	19–20	0	0
21–22	19.28	21–32	–29.39	–8.12	21–22	0	0
23–24	19–28	22–31	29.39	8.12	31–32	–14.32	–3.96
25–26	5.45	23–34	14.32	3.96	23–24	–3.31	–0.95
27–28	–1.84	24–33	0	0	33–34	3.31	0.95
29–30	–3.02	25–36	1.78		25–26	0.18	
31–32	9.79	26–35	–2.45		35–36	–1.08	
33–34	9.79	27–38	0.18	0.54	27–28	0.22	0.06
35–36	5.42	28–37	–1.08	–0.75	37–38	0.32	0.32
37–38	1.38	28–40	–0.03	0.06	29–30	0.14	0.07
39–40	–0.62	30–39	–0.50	–0.32	39–40	0.12	0.10

M by more frequent transposition, which practise may be favored by one form of construction more than another.

TABLE IV
VALUES OF ΣM^2 BY CIRCUITS

$$\frac{\text{Microhenries}}{\text{Kilometer}} \div 10{,}000$$

Standard Form							
Wires	1–2		3–4	5–6	7–8		9–10
Sides	2.10		2.89	1.66	2.89		2.10
Phants		5.51		4.01		5.51	
Wires	11–12		13–14	15–16	17–18		19–20
Sides	2.54		3.39	1.87	3.39		2.54
Phants		7.51				7.51	

For the lower two cross-arms, use the same values in the reverse order.

Form *A*					
Wires	1–12	3–14	5–16	7–18	9–20
Sides	2.84	4.22	5.64	4.22	2.70
Wires	2–11	4–13	6–15	8–17	10–19
Sides	2.70	4.22	5.64	4.22	2.84
Phants	0.44	0.67	0.67	0.67	0.44

For the second row of groups, put 21–32 for 2–11, 22–31 for 1–12, etc.

Form *D*					
Wires	1–12	3–14		7–18	9–20
Sides	0.50	0.74		0.75	0.49
Wires	2–11	4–13		8–17	10–19
Sides	0.49	0.75		0.74	0.50
Phant	0.034	0.050		0.050	0.034

For the second row of groups, put 21–32 for 2–11, 22–31 for 1–12, etc.

Form *F*					
Wires	1–2	3–4	5–6	7–8	9–10
Sides	0.65	1.06	1.08	1.06	0.65
Wires	11–12	13–14	15–16	17–18	19–20
Sides	0.64	0.80	0.92	0.80	0.64
Phants	0.049	0.073	0.074	0.073	0.049

Second row of groups, same values.

Form *G*					
Wires	1–2	3–4		7–8	9–10
Sides	0.120	0.188		0.188	0.120
Wires	11–12	13–14		17–18	19–20
Sides	0.120	0.175		0.175	0.120
Phants	0.004	0.006		0.006	0.004

Second row of groups, same values.

The values of ΣM^2 in Tables IV and V are found for a 40-wire lead except in forms *D* and *G*, where 32 wires are assumed.

The values for form *C* in Table V were estimated by combining the values in Table IV for forms *A* and *D*.

The improvement in forms *F* and *G* over *A* and *D* respectively shown in Table V, is found to be due primarily to the reduced size of the square on which the wires are spaced. Like results could be obtained in forms *A* and *D* by a corresponding reduction in the minimum distance between wires.

c. *Effect of Accidental Variation in Spacing.* A given displacement of one wire in a group in general changes the mutual inductances between the side circuits and between each side circuit and the phantom. If M represents a change of mutual inductance, the sensitivity of the group to these effects may be expressed as $(\Sigma M)^2$, where the summation is taken over the three circuits and the two inductive relations for each.

TABLE V

Average $\Sigma M^2 + 10{,}000$

Form	Physical circuits	Phantom circuits	Per group
Standard	2.52	6.01	11.05
A	3.92	0.58	8.42
C	2.82	0.42	6.06
D	0.62	0.042	1.28
F	0.83	0.0644	1.72
G		0.005	0.307

The effects of opposite directions of displacement are averaged. In the standard form, the effect depends on whether wire 1 or 2 is displaced, the values given are averages for displacements of the two wires.

In forms *E*, *F*, *G*, for the effect of displacements of wires 11–12, interchange columns 2 and 3. The average for vertical displacements of any one of the four wires is 0.238 per group.

The reasoning in arriving at the relative importance of sag variation and other sources of unbalance was as follows:

On the assumption that the displacements follow the probability law for errors, the resultant value of M for a considerable number of spans will be the square root of the sum of the squares of the values for the individual spans. The value of $(\Sigma M)^2$ for n spans will then be n times the average value for one span. Dividing the value of ΣM^2 for a group in the standard form, viz, 11.05 by the values of $(\Sigma M)^2$ in Table VI therefore gives the number of spans in which the displacement effect must occur to be the equivalent of one span of unbalanced length in the standard form. This is 91 spans for forms *A*, *C*, and *D*, and 46 spans for forms *E*, *F*, and *G*. In the *E*-section, the frequently-used transposition patterns have only 14 to 40 spans per transposition. At each of these transpositions there is a liability to one span unbalance, assuming that the transposition have not been set exactly at the correct points. In a long line these unbalances will add by the same law as the displacement unbalances and will produce a greater total unbalance than that due to sag variation unless the r. m. s. displacement is considerably larger than the value assumed. The value of $(\Sigma M)^2$ due to displacements of all four wires appears to be about the same as that for dis-

TABLE VI
EFFECT OF DISPLACEMENTS

Standard of Comparison, effect of displacing one wire 3 cm. (0.1 foot)
Values of $(\Sigma M)^2 + 10{,}000$

Circuit of which one wire is displaced, marked		
Effect on circuit	Horizontal displacement	Vertical displacement
Standard Form		
1-2	0.043	Less than 0.001
3-4	0.015	"
Phant	0.034	"
Group	0.092	"
Forms *A*, *C*, *D*		
1-12	0.051	0.051
2-11	0.051	0.051
Phant	0.020	0.020
Group	0.122	0.122
Forms *E*, *F*, *G*		
1-2	0.040	0.158
11-12	0	0.198
Phant	0.040	0.040
Group	0.080	0.396

placements of only one wire, due to the large number of cases in which the displacements of two or more wires compensate each other.

The effect of turning corners is to reduce the horizontal spacing without affecting the vertical spacing. A 52 deg. turn (Fig. 12) is required to reduce the horizontal spacing by 10 per cent. in a whole span. The value of $(\Sigma M)^2$ due to this distortion is zero, within the group, for standard construction, and for forms *E*, *F*, and *G*. In forms *A*, *C*, and *D*, it is as follows:

Per physical circuits	0.177
Per phantom circuit	0
Per group	0.354

The effect is always in the same direction until a phantom transposition is reached, when it is reversed. Consequently the total effect varies as the square of the number of corners. There would have to be an unbalance equivalent to five or six 52 deg. turns of the style shown in Fig. 12, between successive phantom transpositions, in order to equal the unbalance of $\Sigma M^2 = 11.05$ which may occur in standard practise due to transposition poles not being located at the exactly correct points.

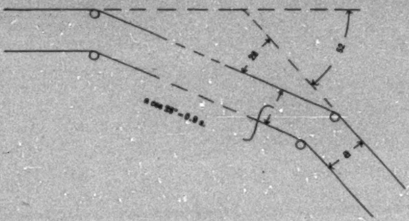

FIG. 12

d. *Induction from Power Circuits.* Following a reasoning analogous to that for cross-talk, the sensitivity of a given circuit to induction from a power circuit is taken as proportional to average E^2, where E is the voltage induced in the telephone circuit by a standard value of I in the power circuit, and the average is taken for every relative direction of the power circuit from the telephone circuit. Assuming a constant frequency, average E^2 is proportional to average ϕ^2, where ϕ is the flux through the telephone circuit due to unit current in the power circuit. For the effect of a single wire on a physical circuit,

$$\phi = 2 \frac{a^2}{d^2} \cos \phi \tag{1}$$

and $$\text{avg. } \phi^2 = 2 \frac{a^2}{d^2}, \text{ approximately} \tag{2}$$

where a is the distance between the wires of the telephone circuit and d is the distance of the power wire from the center of symmetry of the telephone circuit, and θ is the angle between a and d.

TABLE VII.

ELECTROMAGNETIC INDUCTION FROM POWER CIRCUITS, SINGLE WIRE AT DISTANCE OF 1.5 METERS (5 FEET)

Form	Circuit	a feet	Avg. ϕ^2	
			Per circ.	Per group
Standard	Physical	1	0.08	
	Phantom	2	0.32	0.48
A, C, D	Physical	V2	0.16	
	Phantom	V2	0.0008	0.321
F, G	Physical	1	0.08	
	Phantom	1	0.0002	0.160
B	Physical	V5	0.4	0.8

For a present type phantom, formulas (1) and (2) are used, assuming that the phantom is made up of two circuits in parallel For the proposed phantoms,

$$\phi = \frac{a^2}{2d^2} \cos 2\phi \tag{3}$$

$$\text{avg. } \phi^2 = \frac{a^4}{8d^4} \tag{4}$$

where a is the diagonal of the group, and d and θ are as before. Comparative values are given in Table VII.

In form *E*, the distortion of the group increases average ϕ^2 for the phantom by 0.01, making the total for the group 0.17.

The above values represent also a reduction in the amount of noise induced in a given phantom by stray longitudinal currents in the other wires of the telephone lead.

e. *Characteristics of Standard E-Section.*

In the following table, M_I is the mutual inductance, in microhenries per kilometer, computed from Table II, and M_C the mutual capacitance unbalance, in micro-microfarads per kilometer, computed from the table given by Osborne[6]. In computing both M_I and M_C account was taken of the two pin positions of each physical circuit.

f. *Computation of Unbalances in Proposed Forms.* In Fig. 2, let the mutual inductance (or capacitance unbalance) between the physical circuits of one group (say 1–12–2–11) and another group (say 3–14–4–13) be represented as follows:

Between 1–12 and 3–14, by w
" 1–12 " 4–13, by x
" 2–11 " 3–14, by y
" 2–11 " 4–13, by z

TABLE VIII.

Type	Transp. per Section	Circuits Physical	 Phant.	M_I	M_C
i	23	5–6	37–40	5.44	0.08
j	22	...	5–16	....	...
k	21	15–16	31–34	5.44	0.8
l	20	13–14, 27–28	.....	−4.14	2.5
m	19	29–30, 31–32	.....	−4.14	2.4
n	18	9–10, 23–24	.....	−0.14	0.6
o	17	1–2	27–30	−2.43	3.0
p	16	7–8	21–24	−2.43	3.0
A	15	11–12, 39–40	.....	−0.14	0.6
B	14	19–20, 21–22	.....	−4.14	2.6
C	13	... ...	25–36	....	...
D	12	3–4, 17–18	.....	−4.14	2.8
E	11	37–38	11–14	−2 43	2.3
F	10	35–36	7–10	5.44	0.8
G	9	25–26	1–4	5.44	0.8
H	8	33–34	17–20	−2 43	2.2

Table IX shows the changes in these coefficients due to successive phantom transpositions of one or both groups.

The use of this table permits working out readily the degree of balance between the physical circuits when the phantoms are transposed to any given patterns. If there are three equidistant phantom transpositions in one group between the phantom transpositions of the other, there is complete balance for all the inductive relations of the physical circuits. With a few exceptions, there is balance for the physical circuits whenever one phantom has gained three transpositions on the other one, whether this gain be made in one or several transposition sections of the line. Some patterns give balance of the physical circuits in two sections. The effect of phase displacement prevents depending on the phantom transpositions to balance the physical circuits except when a satisfactory degree of balance is obtained in one section.

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[6] Loc. cit. facing p. 902.

TABLE IX.

EFFECT OF PHANTOM TRANSPOSITIONS ON COEFFICIENTS OF INDUCTION BETWEEN CIRCUITS IN DIFFERENT GROUPS.

Circuits initially in the position	Number of Transposition:									
		1	2	3	4		1	2	3	4
	Number of Element:									
	1	2	3	4	5	1	2	3	4	5
	Group 1–12–2–11 transposed									
	To right					To left				
1–12 and 3–14	w	y	$-w$	$-y$	w	w	$-y$	$-w$	y	w
1–12 and 4–13	x	z	$-x$	$-z$	x	x	$-z$	$-x$	z	x
2–11 and 3–14	y	$-w$	$-y$	w	y	w	w	y	$-w$	$-y$
2–11 and 4–13	z	$-x$	$-z$	x	z	z	x	$-z$	$-x$	z
	Group 3–14–4–13 transposed									
	To right					To left				
1–12 and 3–14	w	x	$-w$	$-x$	w	w	$-x$	$-w$	x	w
1–12 and 4–13	x	$-w$	$-x$	w	x	x	w	$-x$	$-w$	x
2–11 and 3–14	y	z	$-y$	$-z$	y	y	$-z$	$-y$	z	y
2–11 and 4–13	z	$-y$	$-z$	y	z	z	y	$-z$	$-y$	z
	Both groups transposed									
	Same direction					Opposite direction				
1–12 and 3–14	w	z	w	z	w	w	$-z$	w	$-z$	w
1–12 and 4–13	x	$-y$	x	$-y$	x	x	y	x	y	x
2–11 and 3–14	y	$-x$	y	$-x$	y	y	x	y	x	y
2–11 and 4–13	z	w	z	w	z	z	$-w$	z	$-w$	z

For groups with their diagonals in parallel planes, as is the case in all the forms illustrated, x is equal to y, whatever the relative position of the groups, so that simultaneous phantom transpositions of both groups in the same direction balance the x and y values. Referring to Table II and the figures, shows that when the sides of the groups are in line, w and z are small compared to x and y. If the diagonals are in line, and in most other cases, w and z are opposite in sign and nearly equal. Hence in any case the simultaneous phantom transpositions leave only a small residual unbalance, equal to $(w + z)/2$ per unit length, between the circuits whose planes are parallel.

For example, let the following groups in Form *A* have the same pattern of phantom transposition:

1–12–2–11 and 3–14–4–13. $w = 22.31$, $x = 102.17$, $y = 102.17$,
$z = 22.31$, $(w + z)/2 = 22.31$, M between phantoms, –17.86.
1–12–2–11 and 23–34–24–33. $w = -57.54$, $x = 0$, y 0,
$z = 44.63$, $(w + z)/2 = -6.46$ M between phantoms, 4.77.
1–12–2–11 and 5–16–6–15. $w = 1.55$, $x = 25.03$, $y = 25.03$,
$z = 1.55$, $(w + z)/2 = 1.55$, M between phantoms, 1.17.
1–12–2–11 and 25–36–26–35. $w = -16.25$, $x = 11.99$, $y = 11.99$.
$z = 16.69$, $(w + z)/2 = -0.28$, M between phantoms, 0.21.

These figures illustrate the statement previously made that whenever the mutual unbalance between phantoms can be neglected, the same is true of the physical circuits.

Then following table of co-efficients between groups I and IV in Fig. 10 illustrates the use of Table IX.

Transp. pole		2	4	6	8	10	12	14
1–12 to 21–32	w	z	$-x$	y	$-w$	$-z$	x	$-y$
1–12 to 22–31	x	$-y$	w	z	$-x$	y	$-w$	$-z$
2–11 to 21–32	y	$-x$	$-z$	$-w$	$-y$	x	z	w
2–11 to 22–31	z	w	y	$-x$	$-z$	$-w$	$-y$	x

The sum of each horizontal line is zero, and inspection will show that since $x = y$, the larger co-efficients, are balanced in each quarter section.

The phantom transpositions with left and right handed rotation resemble the present types 2 and 3 phantom transpositions. By using transposition insulators and brackets, types analogous to 1 and 4 could be introduced, but apparently there would be no advantage in doing so.

g. *Formulas Used.* In Fig. 1, the solid lines are circles for which the radius is:

$$R = \frac{2\,a\,q}{q^2 - 1} \tag{5}$$

and the intercept on the X-axis is

$$x_1 = a\,\frac{q - 1}{q + 1} \tag{6}$$

where

$$q = \left(\frac{2\,a}{r}\right)^{1-2n} \tag{7}$$

n being the fraction of the flux, or potential difference, to be included within the given curve, and a, half the distance between wires. Curves are drawn for $n = 0.4, 0.45, 0.5, 0.55, 0.6$.

The dotted lines have the equation

$$y^2 = \pm \sqrt{(a^2 - x_1^2)^2 + 4\,a^2\,x^2} - a^2 - x^2 \tag{8}$$

When x_1 is less than a,

$$x_1^2 = a^2\left[1 - \left(\frac{a}{2\,r}\right)^{-1}\right] \tag{9}$$

When x_1 is greater than a, the sign of the second term within the brackets is reversed. The unit of flux ($n = 1$) is the amount of flux linking only one wire. For $n = 1$, the curve shaped like a Fig. 8 is obtained. Curves are drawn for $n = 0.8, 1.0, 1.2, 1.4, 1.6$.

Mutual Inductances. Physical circuits 1–2 and 3–4

$$M = \log_e \frac{d_{14}^2\,d_{23}^2}{d_{13}^2\,d_{24}^2} \tag{10}$$

where d_{14} = distance from wire 1 to wire 4
etc. etc.

Phantom 1–2–3–4 and its own side circuit 3–4.

$$M = \frac{1}{2}\log_e \frac{d_{14}^2\,d_{24}^2}{d_{13}^2\,d_{23}^2} \tag{11}$$

Phantom 1–2–3–4 and circuit 5–6

$$M = \frac{1}{2}\log_e \frac{d_{16}^2\,d_{26}^2\,d_{35}^2\,d_{45}^2}{d_{15}^2\,d_{25}^2\,d_{36}^2\,d_{46}^2} \tag{12}$$

Phantom 1–2–3–4 and Phantom 5–6–7–8

$$M = \frac{1}{4}\log_e \frac{d_{17}^2\,d_{18}^2\,d_{27}^2\,d_{28}^2\,d_{35}^2\,d_{36}^2\,d_{45}^2\,d_{46}^2}{d_{15}^2\,d_{16}^2\,d_{25}^2\,d_{26}^2\,d_{37}^2\,d_{38}^2\,d_{47}^2\,d_{48}^2} \tag{13}$$

Self-inductances. Physical circuit,

$$L = 1 + 4\log_e a/r \tag{14}$$

in which a is the distance between wires, r the radius of the wires.

Phantom circuit,

$$L = 1/4\,(L_1 + 2\,M + L_2) \tag{15}$$

in which L_1 and L_2 are the self-inductances of the component circuits 1–3 and 2–4 (or 1–4 and 2–3), and M is the mutual inductance between these circuits, the side circuits being 1–2 and 3–4.

The above formulas give mutual and self-inductances in c.g.s. units per centimeter length. For microhenries per kilometer, multiply by 100.

Capacitances. Physical circuit,

$$C = \frac{1}{4 \log_e a/r} \tag{16}$$

Phantom, standard form, horizontal type,

$$C = \frac{A' + B'}{A\,B' + B\,A'} \tag{17}$$

where $A = \log_e a/r,\qquad B = \log_e 2,$
$A' = \log_e a/r - \log_e 2,\qquad B' = \log_e a/r + \log_e 3/2$

Phantom, proposed form, non-distorted,

$$C = \frac{1}{2\,\log_e a/r - \log_e \sqrt{2}} \tag{18}$$

These formulas give capacitances in e.s.u. per cm. length.

DISCUSSION ON "RADIO TELEPHONY" (CRAFT AND COLPITTS), "THEORY OF THE TRANSIENT OSCILLATIONS OF ELECTRICAL NETWORKS AND TRANSMISSION SYSTEMS" (CARSON) AND "TELEPHONE CIRCUITS WITH ZERO MUTUAL INDUCTION" (CRAWFORD), NEW YORK, N. Y., FEBRUARY 21, 1919.

George O. Squier: To save time, I will refer only to the conclusion as presented by the first paper. That gives an outline of the marvellous state in which the inter-communicating systems for the transmission of intelligence have been linked up, and we are apt to forget the significance of that from a national and international standpoint, unless we pay special attention to it and try to see what it portends.

The author of the first paper described the wonderful and beautiful experiments whereby he spoke as far as Honolulu and Paris, and then linked these up with the great wire system of the country. Then, to cap the climax he gave you an account of the communication which was had with the New Hampshire at sea, which was a combination of the radio system at sea and the wire system on land. And only a few days ago from an airplane, we spoke through radio to the wire system, so that we are gradually linking up all the different systems of communication. I think we are within a fairly measurable distance of the goal when, on occasion any two persons shall be able to speak with each other from any two points. That, of course, is the ultimate object.

That brings out the point of correlating these systems which are of immediate value as independent links in this chain. In this connection there is urgent need for improvement in the engineering features of the present cable system as now operated throughout the world.

During the last few months the pressure on the cables in the Pacific and Atlantic Oceans has been unprecedented, and particularly so just now, with the necessity for cabling the doings of the Peace Conference. All departments of the Government, the State Department, the Navy Department, the War Department, the Signal Corps, which is part of the War Department, and other departments realize the necessity of bringing to bear the best engineering skill of the country to try to improve cable engineering practise.

An ocean cable is a most simple sort of circuit. It reaches across the Atlantic Ocean for instance, which represents a distance of some three thousand miles, and has a capacity of something like 1000 microfarads; very little inductance and practically no leakance at all. It was one of the first accomplishments of telegraphy and is over fifty years old now. This fact gives an idea of the engineering state of the cables which are so much older than the new art we have heard about this morning, which does all these wonderful things, I will have to say to you that today the ocean cable system is operated, so far as transmitting is concerned, and very largely so far as receiving is concerned, practically the same as it was in the beginning. The Duplex is the greatest advancement which has been made since the beginning.

The present method of signaling connects the end of the cable for a given length of time to a battery and then opens the circuit and connects the cable to earth.

But since the cable has this enormous capacity, when you close the key to make a dot or dash, there is a tremendous inrush of current charging up the cable. And when you open the key it takes a considerable time for the energy thus stored to run off to ground before you can send another dot or dash. Thus for each individual letter, an appreciable time is consumed in letting that charge run off, and during that interval, the cable is not in use for the purpose of sending messages. That amount of time, it is estimated, in the case of a long cable is 40 per cent of the total time of each element of a letter, if I am not mistaken. Therefore if both ends of an Atlantic cable were operating at maximum speed for twenty-four hours a day, we can say that every day the Atlantic cable system is practically shut down forty per cent of the twenty-four hours or nine and a half hours, during which time the cable is not performing any useful work. In the life of the cable, extending over a period of fifty years, the cable has been non-producing something like forty per cent of the whole time, or twenty years.

This is in contrast with the beautiful results we have heard described this morning, and is an indictment of present cable engineering practise. What I wish to draw your attention to, is the present attitude of the Government, in trying to remedy this state of affairs, because the cables are very important from an international standpoint. The League of Nations plan will be largely dependent on an international system of signalling, including what we have heard described this morning.

As Chief Signal Officer at the War Department I will furnish any assistance that is in my power to anyone who wishes to attack any of these questions, mathematically or practically, and will turn over the use of our cable to Alaska, as far as is consistent with the public traffic on that cable for testing any improvements.

M. I. Pupin: Just twenty years ago I heard General Squier say exactly the same things about cable communication. The art has not advanced one bit and it is fortunate it has not because engineers have been engaged in other fields, notably radio telephony.

People long ago realized that you cannot do much with the cable. There it is, just like a long amphibious animal stretching from one shore to the other. No matter what you do to it it moves slowly. There is no life in it.

Now, General Squier says if any one of you will make a new, fresh start in the direction of improving the cable the Government is at your service. That is a very attractive offer, but I do not think they will get many acceptances, for the simple reason that the engineer will choose radio telegraphy and radio telephony. This art is new, it is full of life, it is full of the brightest dreams. It is the beautiful science connected with this new art which attracts us and stirs up our imagination.

Now, let me add a few things, if I can, to the history of this beautiful art. As long ago as twenty years, in 1899, I was very much interested in Peter Cooper Hewitt's work, the mercury vapor lamp, as an outsider and as a looker-on. It was a beautiful piece of work. Then, after that I took an interest in the Wehnelt tube and thought it was a beautiful piece of work, and it is; and as an outsider, unprejudiced, I compared the merits of the two inventions. I found practically no difference. You are dealing with a hot cathode and a cold anode, you are dealing with practically, under certain conditions, a semi-thermyonic current, as we call it. The thermyonic current is not new, it is older than Wehnelt tubes or the Cooper Hewitt tube. It takes us back fifty years, it was Hittorf who first showed that if you heat the cathode you can pass an electric current through a vacuum tube very easily with a few volts whereas if you have both a cold anode and a cold cathode you cannot pass through any thermyonic current or practically none. Of course, with a little bit of exterpolation of the Hittorf theory you can infer from that that the current can pass easily through a hot cathode and a cold anode. But if you apply the electromotive force the other way, you will not get the current.

Then came the so-called Edison effect—that is specifically a case of the Hittorf tube, both the Hittorf tube and the ordinary vacuum tube,—I mean that the ordinary incandescent bulb can act as a rectifier. I must say that to me that was absolutely clear long before the tubes were employed for wireless telegraphy. I did not attach much importance to it, for reasons that I will not explain, but when the Fleming valve came out I was really surprised that a patent was granted for it, but as the Patent Office did grant the patent, I therefore assume that he had a right to it.

I think I was one of the first to realize the importance of a rectifier in wireless telegraphy. I made an electrolytic rectifier which was operated by myself and Professor Merritt of Cornell University for the purpose of rectifying Hertzian oscillations. The paper in which I described my experiments was read before the American Physical Society in November, 1899; twenty years ago. Three years after that it became known in the art as the Fessenden barreter. I asked Professor Fessenden why he called it a barreter. I told him I thought it was a rectifier. He said, No, it is not a rectifier, it does not act in accordance with the principles you mention." But it was a rectifier and it was mentioned here to day as the Fessenden rectifier by a good friend of mine, who ought to know better. If it is a rectifier then I say it is the Pupin rectifier, and if it is not, then it is the Fessenden barreter. The rectifier is very important, no doubt to day, the rectifier employed by the Western Electric Company, the American Telephone & Telegraph Company and others, the vacuum tube rectifier, is not a rectifier of that kind at all. It operates differently; it operates better, it is an entirely improved form. It needs three electrodes and the man who added the third electrode to the vacuum tube is the man who made this wonderful epoch-making invention referred to here today possible, and that was Lee de Forest. Lee de Forest read a paper before this body in 1906. I was asked to discuss that paper.

Well, I acknowledge, I did not understand at that time quite fully the importance of the paper. Perhaps the author did. However, whether he understood it quite fully or not, he did make an epoch-making invention, one which is perhaps more important than any other electrical invention made during the last twenty years. Mr. Peter Cooper Hewitt also used a third electrode in the vacuum tube, when he used mercury for his negative electrode. At that stage I said, Here is a thing which is full of the greatest promise. This is a thing which needs investigation, scientific reasearch of the most profound character. I called up J. J. Carty, of the American Telephone & Telegraph Company, knowing that the directors of that organization lay great weight upon his judgment. I said, "Carty, here is a thing that must be developed," and Carty was not slow in getting the thing installed in 1912. It is 1912 that marks the beginning of this great art, that is the time when the American Telephone & Telegraph Company took up the Peter Cooper Hewitt and Lee de Forest patents.

As the Chairman of the Committee on wireless communication between airplanes of the Committee of the National Advisory Board for Aeronautics, I was requested to find various means by which wireless communication between airplanes could be advanced. That was in 1917. Early in that year, or the latter part of 1916, I met Captain Culver, who had just returned from San Diego where he was flying and making experiments with wireless communication between airplanes and from airplanes to earth. It seemed that there was no difficulty whatever in receiving a wireless telegraphic message while in an airplane, in spite of the great noise made by the machine, provided the ears of the man in the airplane were suitably protected. His information was so encouraging that I said, "Here, Captain Culver, you ought to run over to the Western Electric Company and see if they cannot give you an outfit for wireless telephony". I told General Squier the same thing and hence the conference on May 22, 1917.

From that conference to the present time, a period of less than two years, I think that the developments of the wireless telephony art, as used in airplanes today, and as it has been used during the war, is perfectly marvellous. It shows the great usefulness of having a great organization maintaining a first class research laboratory. That is what accomplished it.

These great research laboratories are national assets, they are not merely local organizations belonging to a particular company. We are proud that the value of pure science has been recognized not only by the universities—for that is their business, they are teaching pure science—but by practical men, the men who are the directors of great industrial organizations. I am very glad from that point of view that we have these splendid results to convince these men and the directors of other industrial organizations that the best investment which a great industrial organization can make is to maintain a great research laboratory.

The idea which I have just expressed has been the guiding idea of the men who during the last three years have been working hard to organize the National Research Council, the object of which will be to co-ordinate and stimulate the scientific research of this country for the good of mankind, and particularly for the good scientific reputation and standing of this country. They will attempt to co-ordinate the work of such organizations as we have, like the great industrial research laboratories of the large manufacturers and to stimulate other industries to establish similar research laboratories. To establish points of contact between industrial laboratories and the universities and the scientific men in general, so that they will understand each other and work for the common good, that is, for the advancement of science, for the glory of mankind and for the glory of the scientific reputation of this country.

Alfred H. Cowles: I would like to criticize one point which appears in the historical part of the paper of Messrs. Craft and Colpitts. It will be noted, that it is stated on the second page of their paper in effect that Maxwell "*predicted* electromagnetic wave propagation and the identity of light and electromagnetic radiation". The point I wish to make is, that electromagnetic radiation was known to Maxwell as a phenomenon and he explained it instead of predicting it. His became an accepted theory which classified and reduced to order known facts and phenomena and not an unproven hypothesis.

In about 1879, there were but two telephone lines in the city of Cleveland, one between my home and my father's office, the other between the office of the Brush Electric Company and that of their President. I had occasion to talk over one wire and a voice came in, that was a strange voice. I was calling my father. . . . . and the voice said, "who is this." I said, "Cowles." It said, "ha ha ha, this is General Leggett, *our lines are crossed.*" The strange voice was that of General M. D. Leggett, then President of the Brush Electric Company. By the expression "our lines are crossed," he did not mean that the wires were in contact. Now there was a case where we had wireless telephony almost in the sense of today. There passed upon the wires successive alternating waves corresponding with the fundamental vibrations of the human voice, superimposed upon these waves were various harmonics due to the resonance, fixed by the form of one's organs of speech which with pitch of tone enables the distinguishing of one voice from another. All of these fundamental and supplemented concurring vibrations were in this instance transmitted from one wire to the other by Maxwell's electromagnetic waves of propagation, thus causing similar induced waves in the crossed wire that brought to me the recognizable voice of General Leggett.

Hertz did not forecast or even think of wireless telegraphy in connection with his published papers in 1888. Neither he nor his predecessors had mentioned such a thing excepting in the sense of my 1879 experience. In 1899 Nikola Tesla made his wonderful experiments at Colorado Springs. He lighted incandescent lamps in the yard about his laboratory by wireless. I had occasion several years later to read one of his papers that was presented to the Franklin Institute, at Philadelphia, February 24th 1893, and I was astounded to see in that paper a statement which I can almost remember literally. He said—"It behooves the electrical engineers of the world to give their closest attention to the proposition of wireless telegraphy to all parts of the world. It is possible and it is sure to come." That gave me an impression of Nikola Tesla's genius which I felt in later years that a great many other electrical engineers had failed to recognize, and I think it ought to be recognized today.

His work was several years before you began to hear about wireless. He had published the fact, that he had observed by aid of his delicate instruments the passage of an electric impulse from a flash of lightning and had detected its return electric echo from the far side of the earth. His instruments, to explain, had given two indications, one the flash and the other the echo. Upon measuring the difference of time between the two impulses and calculating what it should be, knowing the velocity of electromagnetic waves he found the difference of time to be that which he looked for. To be exact it was periodic echoes of stationary waves that he measured. From this he drew the conclusion that it was possible to have wireless communication to all parts of the world. All that was necessary was to set up two instruments in synchrony at any two points on the earth's surface. By the use of one give to the charge on the earth a sufficiently heavy set of impulses of a definite vibratory period in which case the other instrument would be bound to take up these impulses when in resonance with the first.

It seems to me that in this instance we had the most wonderful forecast of the real possibility of wireless. He had prior to July 1899 invented his oscillator and magnifying transmitter and had them long in use. He could take at will, according to design, the energy stored during a given length of time of flow along any wire and discharge that quantity of energy in a millionth or less part of such time. This enabled him to produce what were equivalent to flashes of lightning 100 feet long.

While Hertz in his experiments in 1888, accomplished the transmission of impulses through a stone wall, succeeded in refracting electromagnetic waves, producing interferences, etc., they were not far different from similar experiments being performed in other physical laboratories the world over. None of these experiments suggested or hinted at world wide wireless. They were insignificant when compared with that which Tesla forecasted and afterward did. Using Tesla's theory of explanation, his impulses would go through or around the earth which seemed to me perfectly logical when I first read his February 1893 paper. He viewed the earth as a charged sphere. The presence of Maxwell's electromagnetic waves were to be inferred as accompanying the phenomena that Tesla observed. His method of explaining these phenomena did not embody one point which puzzled many electricians, *viz,* the question as to how ordinary electromagnetic waves could pass around segments of the earth's circumference, yet fundamentally his theory was in harmony with the generalizations of Maxwell and could just as logically be applied to explain long distance wireless as the electric wave theory through either. The Craft and Colpitt's paper seems to me to unnecessarily differentiate as between Maxwell's electromagnetic waves and Hertzian waves which are truly one and the same thing inasmuch as what is now called the Hertzian wave in their paper was well understood before the date of Hertz's experiments.

It seems to me that from 1890 through to recent years Tesla's publications and patents were not generally understood, because of his starting from the static end, while others have begun their explanation of wireless from the dynamic portion of electrical science. I have no personal interest in this matter other than that I wish to see fair credit given to an American inventor.

Michael I. Pupin: I want to call your attention to the fact, as announced on the second page of the paper, that the author deals with electrical phenomena in which all the reactions can be expressed by first or second differential coefficients, with constant coefficients, and, therefore, it is not as general as you might suppose. For instance the equations will not apply to the theory of the induction motor, because in that theory you will find that one of the reactions is expressed in terms of a differential coefficient multiplied by a function of the time.

If you take an iron core and subject it to periodic magnetization there, too, you have the development of harmonics, because the permeability is not a constant quantity, so from that point of view the paper is limited to the so-called linear differential equations and constant coefficients. That, however, is not a serious limitation, because there are very few electrical phenomena that fall outside of that orbit, that is, so far as we know at the present time.

Mr. Carson discussed artificial lines. I gave the first discussion on artificial lines in March, 1899, and in another paper a year after that I discussed the connection of lines with periodic structures, taking special cases, but stated at that time that other combinations could be treated by the same method.

In that paper I showed how to analyze the connection of an artificial line. First, where the artificial line is capable of oscillation, and secondly where it is aperiodic, and so that in one case you have periods of oscillations from which you can compound your general solution, and in the other case you have a certain number of types of exponential functions and you can compound any complete solution by taking these types and putting them together in the proper way. As far as I know, no such study had been made prior to that time.

Afterward I wanted to show, and I did show in a paper—not published yet, although the paper is nearly twenty years old—the first case treated by Kirchhoff and also by Kelvin—what happens when we impress constant voltage on an overhead line or on a cable. I discussed that from the point of view considering the cable as an artificial line with resistance and capacity, and the overhead line as an artificial line consisting of sections of resistance, capacity and inductance. I found the general solution which, when the number of terms was raised to infinity, resulted in Kirchoff's and Kelvin's solutions, and I said, "This is a more general solution, because from this solution you can get others, whereas from others you cannot get this one." In fact, the problems of wave propagation of continuous lines can be handled by considering the problems, first, on artificial lines; but I must say that it is more difficult to handle the artificial line than it is to handle the continuous line in most cases. In some cases, however, it is easier, because for any solution in one you can get a corresponding solution in the other, whereas in the one case of the artificial line with a finite number of sections, you deal with the sum of the finite number of turns; and in the continuous line, if the limit is an artificial line of an infinitely small section, you deal with the integral.

I think in the papers of March, 1899 and June, 1900, that subject has been treated pretty exhaustively, except that I did not go into any arbitrary impressed forces. Mr. Carson does, and he does it very well, indeed. That, however, a man can do by studying the problems which have already been solved for continuous lines, and then he can guess pretty well, if he is skilled in the art, what will be the corresponding solution in the case of the artificial line.

The artificial lines, as Mr. Carson says, stand on their own legs—they are independent structures for filtering, they call them filterers in the Western Electric Company, I call them pilot conductors at Columbia University, but they are exactly the same thing. You can use these artificial lines, and the theory I published in 1899 shows that you can use them as filterers. They will let through certain frequencies and exclude very strongly and very perfectly other frequencies. You can use two structures like that so that they will transmit only certain intervals of frequencies. I have used them at Columbia University but here is the difficulty—I will tell you something that is not contained in the paper—two structures having inductance, capacity and resistance to be effective filters and to be used as such must have resistances very small in comparison with the reactances. What happens, then, is this. Any disturbance occurring in the vicinity of an impulsive nature will start oscillations in the circuit, oscillations of about the same frequency as the signaling frequency. Although you exclude other simple harmonic states from it, you are bothered by the local oscillations produced in the circuit itself, and for that reason the circuit is not as universally applicable as might be thought.

The best structure is, of course, aperiodic structure. In fact, all the work I have done in Columbia University in the line of excluding, for instance, the disturbance on a commutator employs aperiodic structures. I find that sometimes it is desirable to use a dynamo-electric machine in place of the storage battery and dry cells in wireless work because it is less expensive and needs less attention. However the commutator of the machine makes noises, and these noises have to be excluded which is done by aperiodic artifical lines. They are very much more effective on account of the absence of local oscillations. We should congratulate ourselves upon having in one little pamphlet a presentation of the mathematical apparatus that, when put together, is useful to every engineer, who essays, in particular, any investigation into high-frequency electrical work.

A. S. Dana: I should like to draw the attention of members of the Institute to two papers which may serve as an introduction to part of this paper and which were presented to the I. R. E. and A. I. E. E. by Dr. V. Bush in 1916 or 1917. The "Threshold Impedance" introduced in Dr. Bush's papers is equal to $p_m Z^1 (p_m)$ in Mr. Carson's "indicial admittance", equation (**3**).

A. Pinto: (communicated after adjournment): In the last two paragraphs of the section on "Aircraft Telephony" of the Craft-Colpitts paper an explanation is given of the principle of operation of the so-called "constant-current" system of modulation, devised by R. A. Heising. On the basis of this explanation the writers claim that the function of the "low-frequency choke" is to maintain the total current supplied by the main battery to both the modulator and the oscillator plate circuits practically constant; so that, if the telephone transmitter is spoken into, and the modulator plate current thereby increased or decreased (through the action of the modulator grid), the oscillator plate circuit will be either "robbed of current" or else "will have additional current forced through it", and thus the output of the oscillator will be changed in accordance with the speech currents.

The above seems to me both inexact and confusing. For, how could the plate circuit of the oscillator be "robbed of current" or "have additional current forced through it" unless the voltage applied to it were varied? And how could this voltage vary if the current through the "low-frequency choke" were practically constant, as claimed by the writers? On the contrary, if the amplitude of the high-frequency current generated by the oscillator is to vary, *i. e.*, if modulation is to take place, the modulator must cause to flow through the "low-frequency choke" a *varying*, and not a *constant*, current.

Thus, by speaking into the telephone transmitter a result is produced which is equivalent to varying the impedance of the modulator plate circuit. Since this is in multiple with the oscillator plate circuit, the joint impedance of the two plate circuits is varied, and hence the impedance of the entire battery circuit (which includes the "low-frequency choke") is varied. It follows that the total current supplied by the main battery (or the current flowing through the "low-frequency choke") is varied, and so is the potential drop taking place over the "low-frequency choke". This varying drop, subtracted from the constant voltage of the main battery, produces across the oscillator plate circuit a voltage which varies at audio-frequency, and which causes the amplitude of the antenna current to vary at audio-frequency and in a manner more or less corresponding to the vibrations of the telephone transmitter diaphragm.

It may be easily seen that the impedance of the "low-frequency choke" at audio-frequency must be comparable with the joint impedance of the two plate circuits. For, if it were very much higher, any variation of the joint impedance of the two plate circuits would produce such a small variation of the impedance of the entire main battery circuit that the current therein would remain practically constant, and thus produce an inappreciable change in the voltage across the oscillator plate circuit, with resultant ineffective modulation. On the other hand, if the impedance of the "low-frequency choke" were very low, a variation of the joint impedance of the two plate circuits would cause such a small change of voltage drop to take place over the "low-frequency choke" as to again produce ineffective modulation.

With reference to the names "low-frequency choke" and "high-frequency choke", applied to two coils appearing in the schematic diagram of the Heising circuit, I wish to say that it has been my experience, in connection with the training of radio officers at the Air Service School established at Columbia University, to find these two names a source of confusion and misunderstanding. I would, therefore, suggest that the "low-frequency choke" be called the "modulating coil", and the "high-frequency choke" the "feed-back coil", these two names being more nearly in accord with the functions which the two coils are called upon to perform.

V. Bush (communicated after adjournment): The very remarkable analysis presented in Heaviside's "Electromagnetic Theory" is rapidly being placed upon a more secure mathematical foundation, extended to meet present day needs, and placed in such form as to be readily available for the solution of constantly recurring, practical problems.

Heaviside obtained many results of great interest by applying the symbolic operator developed into a power series. This method is shown in the Carson paper in connection with the problem of the infinite cable with terminal condenser; where it indeed applies, as stated, with extraordinary simplicity and directness. As the author points out, however, the method has never been justified mathematically; and its use is hence attended with the same liability to missteps as are all empirical formulas. On account of the importance of this method I will attempt below to throw some light on its aspects mathematically.

In the problem stated, the operational solution is, in the notation of the paper:

$$V = \left[\frac{E}{\sqrt{\pi h}} \left\{ p^{1/2} + \frac{1}{\pi h} p^{3/2} + \frac{1}{(\pi h)^2} p^{5/2} + \dots \right\} - E \left\{ \frac{1}{\pi h} p + \frac{1}{(\pi h)^2} p^2 + \dots \right\} \right] \xi \qquad (1)$$

where p^n replaces $\frac{d^n}{d t^n}$, and where for convenience the discontinuous function, which is zero when $t < 0$ and the constant value E when $t > 0$, and which is operated upon, is represented here explicitly by ξ.

The interpretation of this expression, and of similar expressions where they occur in the application of this operational method, necessitates a determination of the value of

$$p^n \xi \qquad (2)$$

where n may have any value integral or fractional.

When n is a negative integer, the desired result follows by direct integrations. When n is a positive integer the matter is, as Heaviside states, not so simple as it looks. The trouble however is not mathematical, but lies in the fact that we have not defined ξ at the point $t = 0$, and hence cannot state anything in regard to its derivatives at this point. When $t > 0$ we know that expression (**2**) is zero for a positive integral n by simple inspection. The fact that we do not know its value at $t = 0$ need not trouble us. If we state that our solution applies only when $t > 0$ we may drop out all terms of this form. The solution at $t = 0$, that is the initial voltage or current, is usually known from physical considerations.

We need then the value of (**2**) when n is fractional, in particular when n is half an integer. This value Heaviside obtained by solving a special case "from both ends" and hence discovering what value must be assigned to give a correct result. The following analysis attempts to derive the value of this expression directly.

The discontinuous function ξ may be expressed by the Fourier line integral

$$\xi = \frac{E}{2\pi j} \int_{-j\infty}^{j\infty} \frac{\epsilon^{zt}}{z} dz \qquad (\mathbf{3})$$

the path of integration being the axis of imaginaries; avoiding the origin, which is a first order pole of the integrand, by a small semicircle on the positive side. It may readily be checked that the integral over this path properly describes our function.

Applying p^n to this expression

$$p^n \xi = \frac{E}{2\pi j} \frac{d^n}{dt^n} \int_{-j\infty}^{j\infty} \frac{\epsilon^{z}}{z} dz \qquad (\mathbf{4})$$

The variables z and t are entirely independent; hence we may differentiate under the sign of integration, and write

$$p^n \xi = \frac{E}{2\pi j} \int_{-j\infty}^{j\infty} \frac{\epsilon^{zt}}{z^{-n+1}} dz \qquad (\mathbf{5})$$

Consider now the case where $n < 1$. The integrand of expression (**5**) is regular along the above path, and also everywhere on the negative side of the axis of imaginaries; and it is zero at ∞ on this side of the axis. Hence the path of integration may be deformed to a circle taken once positively about the origin.

$$p^n \xi = \frac{E}{2\pi j} \int_0 \frac{\epsilon^{zt}}{z^{-+1}} dz \qquad (\mathbf{6})$$

Let us now consider Cauchy's Integral Formula

$$f^n(Z_0) = \frac{\Gamma(n+1)}{2\pi j} \int_0 \frac{f(z)}{(z - z_0)^{n+1}} dz \qquad (7)$$

which gives the *n*th derivative of a complex function at a point z_0 in terms of a line integral about a boundary surrounding this point. When written as above with the gamma function in place of the factorial, in accordance with

$$\Gamma(n+1) = n!$$

the formula (**7**) serves as an expression also for the fractional derivative.

If now Z_0 is taken at the origin, and if we consider the function

$$f(z) = \epsilon^{zt}$$

(**7**) reduces to

$$p^n(\epsilon^{zt})_{z=0} = \frac{\Gamma(n+1)}{2\pi j} \int_0 \frac{\epsilon^{zt}}{z^{n+1}} dz \qquad (8)$$

or

$$\frac{t^n}{\Gamma(n+1)} = \frac{1}{2\pi j} \int_0 \frac{\epsilon^{zt}}{z^{n+1}} dz \qquad (9)$$

Comparing now with (**6**) we have immediately

$$p^n \xi = \frac{E t^{-n}}{\Gamma(-n+1)} \qquad (10)$$

where $n < 1$

In particular, since

$$\Gamma(\tfrac{1}{2}) = \sqrt{\pi}$$

we may write

$$p^{1/2} \xi = \frac{E}{\sqrt{\pi t}} \qquad (11)$$

the expression noted in the paper, which was assumed by Heaviside.

By applying p successively we may then obtain

$$p^{3/2} \xi = -\frac{E}{2\sqrt{\pi t^3}} \text{ etc.} \qquad (12)$$

The above results, applied to the expansion of the operator in powers of p, justify this method as far as its interpretation is concerned, and allow us to avail ourselves of its directness.

Mr. Carson notes correctly that while the method of expansion in a power series often obtains solutions with remarkable ease, the resulting series are often inconvenient for computation. There are usually two such series, one excellent for very large values of t, and the other for very small values of t. In the intermediate region either series will often lead to prohibitive labor of computation.

Another method of applying the operator which works excellently for certain problems is to expand in partial fractions in $p^{1/2}$ instead of in series.

For the cable without inductance or leakage, but with terminal apparatus, this may often be done to advantage.

The value of

$$\frac{1}{1 + m\,p^{1/2}}\,\xi \tag{13}$$

may be worked out once for all, for real and complex values of m, and tabulated. A solution then consists of first splitting the operator up into partial fractions of this form, and then looking up in these tables.

Effects out on the line may be found similarly by tabulating

$$\epsilon^{-l p^{1/2}}\frac{1}{1 + m\,p^{1/2}}\,\xi \tag{14}$$

the operator

$$\epsilon^{-l p^{1/2}} \tag{15}$$

giving the effect at a distance

$$\frac{l}{\sqrt{R\,C}}$$

out on the line.

This tabulation requires a curve sheet for each assumed value of l.

I have not been able to bring this method of development in partial fractions to a point such as to warrant publication as yet, for my efforts have been otherwise directed during the war. It holds considerable promise. One result was however obtained incidentally by this method which should be of interest in connection with submarine cables. It has been found, if in addition to a terminal series condenser a coil be used between cable and ground, that very interesting results are obtained. The current entering the cable reverses after the initial pulse. The wave form at a certain distance out on the cable is practically without a "tail". The form may of course be controlled by proper choice of the terminal constants.

It seems to me probable that this might be used to advantage in rapid cable telegraphy, adjusting the constants such that the arrival wave would be substantially without tail.

In conclusion I wish to emphasize that the application of this kind of analysis is greatly aided if we keep in mind the analogy with the solution for the alternating-current circuit in the steady state. The function $Z(p)$ of the paper which has been previously called the *threshhold impedance,* may be immediately formed from the ordinary impedance of the circuit by replacing ω with p. This is an excellent guide in formulating the *indicial admittance,* which is derived from the Heaviside expansion theorem, and which forms the starting point of Mr. Carson's admirable paper.

J. H. Morecroft: (communicated after adjournment): The record of this accomplishment in the field of radio telephony adds one more victory to the credit of the commercial research laboratory. It is a record which illustrates how well and quickly an organized corps of research and construction engineers can solve a problem, which is definitely stated, the solution of which is possible.

It has been my opportunity to analyze in some detail, the construction and operation of the set known as the *S C R* 67, one of the sets described in the Craft-Colpitts paper. When the Signal Corps sent men to Columbia University to be trained as radio officers it fell to my lot to explain the principles and operation of the radio telephone outfits which were to be used. When I first opened the cover of the *S C R* 67 and saw the mass of tubes, coils, resistances, condensers, batteries, the compactness of the design, the spring suspension of the tubes, etc. I realized what a tremendous advance in radio telephony had been suddenly accomplished. Upon first seeing one of these sets one naturally anticipates all sorts of trouble in maintenance and adjustment but in this respect we were happily disappointed; the sets functioned very regularly and no special care was required.

In gathering sufficient data to properly explain the operation of the set I did considerable laboratory work in obtaining the principal characteristics; as these technical points are not brought out in the paper, and as there are undoubtedly many electrical engineers who are not acquainted with the functioning of vacuum tubes, it seems worth while to give with the paper a brief discussion, technical, but elementary in character. My remarks will deal only with the power generating tube; the modulator action and the amplifier action are much more evident than is that of the oscillating tube and its circuit.

As explained in the paper the grid potential of the three-electrode tube controls the plate current, this current increasing as the grid goes positive and decreasing as the grid becomes negative; the curves in Fig. 29 of the paper show this action. If the tube is connected to a closed circuit made up of L, R, and

C, in series, and the grid, filament, and plate are connected to the proper points on this circuit, the tube will act as a generator of alternating-current power, the frequency of the alternating current being fixed (nearly) by the well known equation for resonance,

$$f = \frac{1}{2\pi\sqrt{LC}}.$$

The simplest possible circuit is shown in Fig 1. The input to the tube comes from the plate battery and is equal to the product of the plate voltage E_p and the plate current I_p. In the tube used with the telephone set the normal plate current is about 50 milliamperes and the plate voltage 300, giving an input of 15 watts. In these low power tube circuits the efficiency is seldom as great as 30 per cent, we might expect then from the tube used with this set an output of about 5 watts; this much may be obtained when the proper adjustments are made.

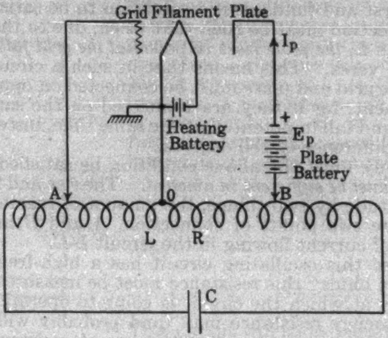

FIG. 1

Many, and apparently different, circuits have been devised for use with oscillating tubes, but they all function to produce the same result. Suppose the oscillating condition has been established in the circuit of Fig. 1. There is an alternating current flowing around the circuit *L*—*C*; a pulsating current flowing in the plate circuit, through the tube, and through that part of the inductance between *O* and *B*; and a comparatively small current in the grid circuit. This latter current flows only when the grid is positive with respect to the filament and so consists of a series of unidirectional pulses. For the *V T* 2 tubes (used in the *S C R* 67) the relative value of these currents might be as follows—the alternating current in the L-C circuit from 0.25 to one ampere, depending upon the resistance of the circuit; plate current of average value 40 milliamperes, fluctuating between 10 and 70 milliamperes; grid current having a maximum value of 5 to 10 miliamperes, its average value being about 3 milliamperes.

If one should connect a tube and circuit as shown in Fig. 1 it would be found that certain conditions must be satisfied before the tube will oscillate. The alternating current flowing in the closed circuit L-C will produce an alternating potential difference along the coil L; when one end of the coil rises in potential the other will fall. This rise and fall of potential are referred to the potential of the filament which, for the sake of discussion, is grounded and so held at zero potential.

The first and fundamental condition to be satisfied is this—the connection must be such that when, due to the alternating current in L, *the plate rises in potential the grid falls in potential* and vice versa. This means that in such a circuit as that in Fig. 1 the grid and plate must be connected on opposite sides of the filament, for if they are connected on the same side they will rise and fall in potential at the same time, instead of having a phase difference of 180 degrees.

Not only must the above condition be satisfied but *the rise and fall must be sufficient in amount.* The rise and fall are fixed in magnitude by two things, the amount of reactance between O and the two points of connection, A and B, and upon the amount of current flowing in the circuit L-C.

Suppose this oscillating circuit has a high-frequency resistance of 4 ohms; this resistance must be measured at the high frequency at which the circuit is going to operate because the high-frequency resistance may (and probably will) be several times as great as the continuous-current resistance. As the tube can furnish about 4 watts of power we may expect one ampere of alternating current, putting $I^2 R = 4$ watts. Then the reactance between O and B must be such as to give, with one ampere of current, a reactance drop the maximum value of which is equal to the plate voltage E_p. The amount of reactance between O and A determines the voltage impressed on the grid and this should be about one half as much (for the *V T* 2) as the drop between O and B. This ratio of grid volts to plate volts depends upon the type of tube used, the finer the mesh of the grid the less is the ratio.

The amount of voltage impressed on the grid as a result of the above consideration, will result in the flow of too large a current in the grid circuit if the average potential of the grid is zero; for this reason it is customary to introduce an extra voltage in the grid circuit to keep its average potential several volts (perhaps 20) negative with respect to the filament. If this is not done there will be excessive loss of power in the grid circuit and there is danger of burning out the tube.

To get maximum power output from a tube the impedance of the oscillating circuit, when measured between the two points to which the filament and plate are connected, must be resistance only, and this resistance must be equal to the resistance of the tube. The resistance of the tube is variable, depending upon the grid voltage, filament current, etc. and is moreover different for an alternating current than for a steady current. The alternating resistance is about one half the continuous-current resistance. This resistance (c. c.) is found by dividing the plate voltage by the plate current; for a plate voltage of 300 and plate current of 0.04 amperes it would be 7500 ohms. The a-c. resistance for this condition would be about 3750 ohms.

Now it would seem that the circuit of Fig. 1, containing L, R and C in series could not have a power factor of unity between the points B and O, but if this power factor is not unity the tube cannot deliver its maximum power. Here however an interesting property of resonant circuits is encountered. Suppose the circuit of Fig. 2, consisting of L, R, C_1 and C_2 in series, is connected to an a-c. line at the points A, B, and suppose further that the impressed frequency is such as to satisfy the relation

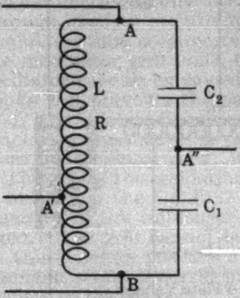

FIG. 2

$$f = \frac{1}{2\pi\sqrt{L\,C}}$$

where

$$C = \frac{(C_1 \times C_2)}{C_1 + C_2}$$

This circuit will show parallel resonance; the line power factor will be unity and the impedance between A and B will be a resistance only. This effective resistance will not be the actual resistance of the circuit but many times greater. In fact it will be equal to the actual resistance multiplied by the ratio $\left(\frac{\omega L}{R}\right)^2$. That is, the lower the actual resistance, the higher is the effective resistance between the points A and B.

Now if the power supply is connected to the points A'-B or A''-B or A'-A'', the power factor will still be unity; in fact the power supply may be connected at *any two points* in the circuit and the circuit will offer reisistance only. The value of this resistance will vary of course, as the points of application of the power are changed; it has a maximum value for the points A-B and for any other points of application it is equal to the value for the points *A*-*B* multiplied by the square of ratio of the reactance in either arm of the parallel path (the reactances of the two branches are equal in magnitude for all points of application) to ωL.

The circuit of Fig. 2 is used in the oscillating tube circuit of *S C R* 67; the manner in which the tube is connected in is shown in Fig. 3. The filament, instead of being connected to the inductance at point *O*, between *A* and *B*, as shown in Fig. 1, is connected at point *O′*, which is the electrical antipode of point *O*. In every closed electric circuit there must be two points which are continually at the same potential and such points are *O* and *O′*; if therefore *O* is a proper point for connecting the filament so is *O′* such a point.

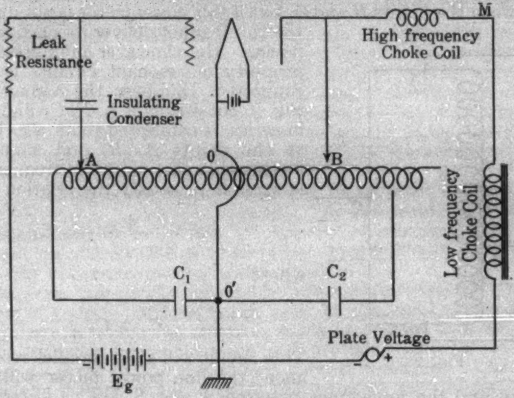

Fig. 3

The tube then oscillates and furnishes power to the circuit *L*, C_1 and C_2; the oscillating circuit is connected to the power supply (the tube) at points *O′* and *B*. The impedance offered between these points will be a resistance and the value of the resistance depends upon the location of the point *B*, which is a movable contact in the actual set. Although the actual resistance of the oscillating circuit is only a few ohms the resistance offered between points *O′* and *B* is perhaps 3000 ohms in normal operation.

The voltage impressed by the oscillating tube between points *O′* and *B* is alternating, with a maximum value about equal to the plate voltage E_p, say 300 volts. The alternating current which the tube supplies to these points varies from – 0.030

amperes to $+0.030$ amperes, its maximum value being somewhat smaller than the normal plate current of the tube. The power supply to the oscillating tube may now be calculated as it would be for any a-c. circuit having a power factor of unity, by getting the product of the effective voltage and current, it is equal to

$$\frac{E_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}},$$

or

$$\frac{300}{\sqrt{2}} \times \frac{0.030}{\sqrt{2}} = 4.5 \text{ watts},$$

which is about what we have said the tube would supply.

The function of the low-frequency choke coil is to maintain through itself an essentially constant current; the plate of the modulator tube is connected at M and so by the rise and fall of the modulator grid volts, takes more or less current from the oscillator tube as explained in the paper. The high-frequency choke coil prevents the partial short-circuiting of the points O'-B by the plate-filament circuit of the modulator tube. Its reactance must be of a proper value to get maximum power from the oscillator tube; in the set it is about 50,000 ohms.

The grid of the oscillating tube is excited by the drop between A and O'; in the *S C R* 67 this drop is varied by changing the capacity of the condenser C_1, the point A being permanently fixed. The grid cannot be directly connected to the point A as this point is positive about 300 volts with respect to the filament, while the grid must actually be held negative with respect to the filament. So a condenser is introduced between the grid and point A, permitting the action of the alternating potential difference but at the same time making it possible to maintain the average grid potential at any desired value by the battery E_g acting through the so called *leak resistance*. This resistance acts as a partial short circuit between the two points O' and A of the oscillating system and so consumes some power; it must be of suitable value for the satisfactory operation of the tube, for the *V T* 2 it is about 10,000 ohms. If too small the resistance consumes too much power and if too large the plate current is caused to decrease, unduly decreasing the power generated by the tube.

In the actual circuit of the *S C R* 67 the condenser C_1 is a group of mica condensers having a total capacity of about 3 millimicrofarads adjustable in about 12 equal steps. The condenser C_2 is really the antenna, but in parallel with the antenna there is a variable condenser occupying a position in the circuit the same as that of C_2 in Fig. 3; by means of this condenser and the taps provided on the coil L the oscillating circuit may be set at any wave length desired within the range of the set.

In Figs. 4, 5, and 6 are shown the more important operating curves of such a set. These curves are qualitative rather than quantitative as they will vary considerably according to the antenna employed.

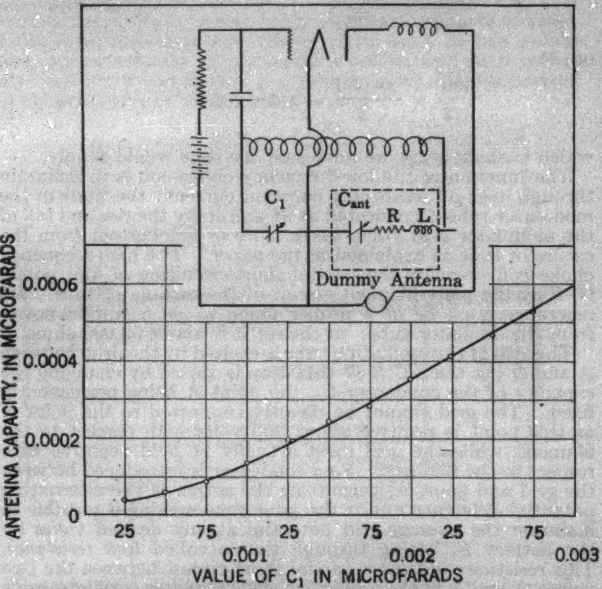

FIG. 4—CURVE SHOWING NECESSARY MINIMUM VALUE OF ANTENNA CAPACITY TO PRODUCE OSCILLATIONS, FOR A GIVEN SETTING OF C_1

With antenna capacity less than the value given by the curve (for a given setting of C_1) the grid does not get sufficient excitation to produce oscillations. Values depend upon the tube, varying to some extent for different tubes of the same type.

It is evident from the analysis of the action of the circuit, that if the grid is not sufficiently excited the set will not oscillate. As the proportion of the total voltage in the oscillating circuit impressed on the grid is determined by the ratio of the condensers C_1 and C_2, Fig. 3, it seems as though for a given setting of C_1, C_2 must have a certain minimum value to produce oscillations; if less than this critical value the grid will not get enough voltage to sustain oscillations. With the idea of bringing out this operating feature of the set a series of readings were taken to determine the maximum permissible value of C_1 for a given antenna capacity. C_1 was set at a certain value, the antenna capacity being zero. The antenna capacity was then increased until oscillations started and the value of C_2 noted. Then C_1 was set for its next larger adjustment and the new critical value of C_2 found. The results Fig. 4 show that for the conditions obtaining in the test the grid must be supplied with at least one fifth of the total voltage in the oscillating circuit, this ratio being nearly the inverse ratio of grid capacity to antenna capacity for the critical condition.

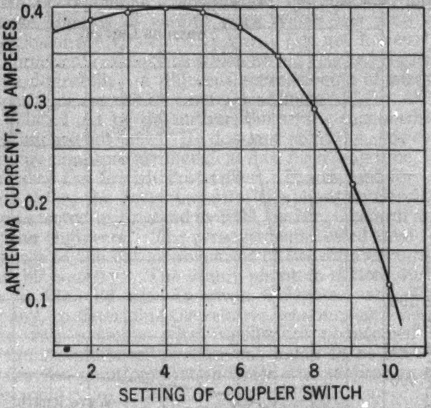

FIG. 5—EFFECT OF VARYING THE COUPLING BETWEEN THE TUBE AND OSCILLATING CIRCUIT. LARGER NUMBERS CORRESPOND TO WEAKER CURRENT.

Wave length = 406 meters. Dummy antenna used had constants: $C = 0.0004\ \mu f$ $R = 8$; $\lambda = 325$ meters; C_1 set at $0.001\ \mu f$.

Fig. 5 shows the effect of changing the position of contact B, Fig. 3; this is called the *coupling* adjustment of the set. The dummy antenna used had a resistance of about 8 ohms. The ωL of the antenna circuit for the conditions used was 400 ohms giving the oscillating circuit a maximum effective resistance of 20,000 ohms, much too large to match the impedance of the tube. The exact values of coupling used in the results of Fig. 5 were not known but it is evident that a certain coupling gives a maximum output which was equal to $0.4^2 \times 8 = 1.2$

watts. (The grid was not properly excited to give maximum tube output in this run, as shown by the next figure).

The effect of varying the grid excitation on the output and wave length of the set is shown in Fig. 6; in this run C_1 was the only thing varied. The dummy antenna had an actual resistance of 8 ohms so that the maximum output was

$$0.725^2 \times 8 = 4.2 \text{ watts.}$$

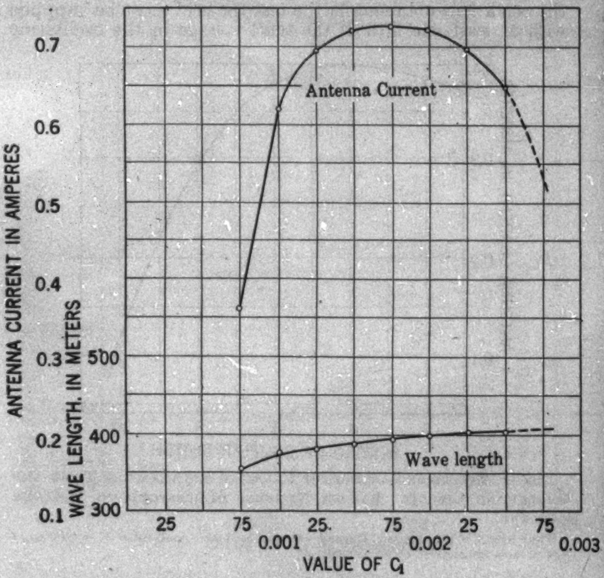

Fig. 6

The wave length increases as C_1 increases, more rapidly for the low than for the higher values as would be expected. When getting a maximum output in this run the continuous current input was 0.040 amperes at 375 volts, giving an efficiency of 28 per cent. In calculating the efficiency the tube was not charged with the power required to heat the filament; if this is taken into consideration the efficiency of the tube generator is only about 15 per cent.

The telephone sets furnished for instruction purposes at

Columbia operated excellently; they seldom gave any trouble and then it was generally some minor defect such as failure of battery, etc. The engineers who designed and constructed these sets have contributed immensely to the radio art.

A. G. Chapman: In checking over the late Mr. Crawford's paper an error was noted in formula (**71**) on the last page of the paper. In the interests of accuracy it should be noted that the values of capacitance given by this formula are too large by a factor of 4. This slight error in the otherwise excellent paper would, no doubt, have been corrected in a final revision by Mr. Crawford.

Harold S. Osborne: Only men who have worked with the problem of telephone transposition systems can appreciate the large amount of careful work which has gone into the production of Mr. Crawford's paper. I do not however concur in the practical conclusions which he has reached regarding the relative desirability of different arrangements of wires and of transpositions for use on commercial lines.

As pointed out by the author, the arrangements which he is suggesting are not new. He does not point out, however, that the square phantom arrangement has, for a long time, been the standard of the British Post Office. It was used for two-wire circuits before the development of the phantom circuit, and as such was tested in England in 1899, by Mr. Gherardi and other American engineers. The arrangements tested were found to be inferior to the flat arrangement of two-wire circuits then in use in this country. The square phantom system was studied again in 1913 by the American Telephone and Telegraph Company, to determine the relative economy and desirability of the two systems when applied to phantomed circuits. The square phantom system was found to be less desirable than the flat phantom system which was adopted as standard in this country.

In 1915 the results obtainable with the square phantom system were again carefully reviewed. In this investigation particular attention was paid to the possibilities which the square phantom system gives of providing for a transposition balance against external interferences. The matter came up in connection with the work of the Joint Committee on Inductive Interference of California, and the proposal that the American Telephone and Telegraph Company should develop a system of transpositions better adapted for coordination of transpositions with parallel 3-phase power circuits than those which were then standard. This study, therefore, took into special consideration the possibilities offered by the square phantom arrangement in obtaining coordinated arrangements of transpositions in parallel telephone and power circuits. This study, like its predecessors, led to the conclusion that the square phantom arrangement was not as desirable as the flat phantom arrangement for use in any of the conditions then prevailing.

A brief outline is given below of the factors of the problem which lead to this conclusion.

1. *Unbalance between telephone circuits.*

Most of the work which has been done in studying the relative tendency for crosstalk between telephone circuits in accordance with the two systems has been based on consideration of mutual inductances only. This is for the reason that the mutual inductances can be computed, whereas, elaborate experimental investigation is necessary to study the mutual capacity unbalances.

If perfect spacing be assumed the advantage in low mutual inductances is somewhat in favor of the square phantom.

The square phantom is much worse than the flat phantom arrangement in liability to large unbalances due to slight irregularities in construction. With the square phantom arrangement the two side circuits of a phantom are much more intimately related than with the flat phantom system, and therefore slight changes often introduce relatively very large mutual inductances. For example, a 2 or 3-degree change in the angle of a crossarm, such as is often caused by warping or other practical construction conditions, introduces a large mutual inductance between the pairs of a square phantom as exists normally between the pairs of a flat phantom. This unbalance is, of course, much more difficult to neutralize than the corresponding unbalance in the flat phantom system, because it enters as an irregularity, varying in an entirely unknown way from one span to the next, whereas in the flat phantom the unbalance is fixed in amount and therefore can largely be neutralized by opposing the effects in adjacent sections.

The electrostatic unbalances are not neutralized to as high a degree as the mutual inductances. This is because the mutual capacity unbalances between the circuits depend not only on the location of the circuits themselves, but also upon the location of other circuits on the pole line. Square phantoms having perfect spacing, therefore, and consequently having zero mutual inductances between the circuits of a given phantom group, generally do have mutual capacity unbalances, and under some conditions, these are large.

As a conclusion, it may be stated that the existing evidence is a little uncertain regarding whether, on the average, the flat phantom arrangement or the square phantom arrangement gives greater ease in reducing unbalances between telephone circuits. It seems probable, however, that because of the large effect in the square phantom system of slight irregularities of construction, the flat phantom system may be somewhat better.

2. *Induction from outside circuits.*

The coefficient of induction into the phantom circuit from outside circuits is very much lower with square phantoms than with flat phantoms. This characteristic is urged by the proponents of the square phantom system as a chief advantage of that system. However, in general, the coefficients of induction to the side circuits are worse with the square phantom system because the wires of the side circuits are more widely spaced for a given minimum separation of wires at crossarms. In some cases only one of the side circuits is worse, and one better. It is difficult to give a single figure which would represent the relative magnitudes of the coefficients of induction with the two systems of telephone transpositions because of the large difference in the comparison for different conditions and the difference in importance of different coefficients of induction. The average of computations on a phantom group for four different conditions with horizontal and triangular arrangement of the power circuit gives an induction between wires from balanced voltages on the power circuit which is somewhat greater with the square than with the flat phantom system. These coefficients are generally the most important in producing noise. The coefficients for the same conditions for induction from balanced currents are materially lower with the square phantom system. For both currents and voltages, however, the coefficients computed for two cases with a vertical arrangement of the power circuit are very much greater with the square phantom than with the corresponding flat phantom arrangement. The coefficients for residual voltage and current computed for two cases average somewhat lower for the square phantom arrangement.

It should be noted that the arrangement of the telephone conductors does not materially influence the magnitude of the voltages induced between telephone conductors in ground and these voltages are in many cases an important factor in determining the amount of interference.

It may be concluded, therefore, that from the standpoint of coefficients of induction from outside circuits, the investigations which have been made show no large difference between the square phantom and the flat phantom arrangements.

3. *Cost.*

With the square phantom arrangement, some of the transpositions are cheaper because they are obtained by a mere rotation of the wires and some must be made in ways similar to the present. The rotation of the wires, however, very materially increases the cost of stringing the wires, as compared with American practises.

In order to obtain as good a comparison as possible between the costs of systems built in accordance with these two plans, there was included in the 1915 investigation a complete design of a square phantom system for a forty-wire pole line intended to meet the same requirements as the "Exposed Line System" which uses flat phantoms. The best system which could be designed with the available information, according to the square phantom arrangement, gave a slightly greater cost than the flat phantom system for the same limits.

4. *Maintenance.*

When applied to 8 or 10-pin crossarms, the square phantom system brings in difficulties in maintenance because the crossarms are separated by only one foot instead of two. This leaves hardly sufficient space for a lineman to reach through to the outer pins. This difficulty is overcome in some cases in British construction by using 4-pin crossarms but limiting the crossarms in this way adds to the construction cost.

5. *Application to Existing Plant.*

The above factors relate to newly constructed lines. When one considers the application of the square phantom system to lines already built, additional difficulties are encountered. These difficulties are of a good deal of practical importance because most of the cases in which the use of the square phantom system would be considered are cases in which inductive interference arises from the building of power lines near existing telephone toll lines.

Modern telephone toll circuits for long distance service are very generally provided with loading coils and very frequently must work with telephone repeaters. For successful operation under these conditions it is necessary that the inductance of all loading coils on a given circuit be very exactly the same and that the capacity of the sections of circuit between adjacent loading coils also be very exactly the same. Furthermore, in order to avoid great difficulties in reducing the crosstalk between the circuits, all circuits must have their loading coils placed at the same points. The phantom circuits have a different capacity from the side circuits and correspondingly have loading coils of a different inductance.

With the square phantom arrangement the capacity of a phantom circuit per unit length is much greater and the capacity of the side circuits per unit length materially smaller than with the flat phantom arrangement. If, therefore, it is proposed to retranspose according to the square phantom arrangement a short section of existing line which is transposed by the flat system, difficulty is at once encountered. To meet the conditions outlined above for loaded lines, side circuit loading coils must be spaced farther apart in the square phantom section than in the flat phantom section and the phantom circuit loading coils must be spaced nearer together. However, this makes it no longer possible to place phantom coils and side circuit coils at the same points and constitutes a very serious difficulty.

This discussion may therefore be summed up by the statement that the possibility of obtaining advantage by the use of the square phantom or some similar arrangement has for a great many years been present in the minds of telephone engineers, that it has been very carefully studied at various times with a view to the possibility of its proving advantageous but that as yet no condition has arisen in which it seems preferable to the flat phantom system which is in use here.

J. B. Carson: In reply to Professor Pupin's interesting remarks I do not believe that any apology is needed for limiting the explicit treatment of the problem to systems which can be described by a set of simultaneous linear differential equations with constant coefficients. This limitation is, as Professor Pupin remarks, not serious and leaves the scope of treatment co-extensive with the problems in alternating-current theory which can be dealt with by the usual symbolic method involving the operator j. As an illustration of the limitation involved, however, Professor Pupin has instanced the induction motor as a problem outside the scope of the methods developed in my paper. Professor Pupin has himself, however, shown how to build up the steady-state theory of the induction motor from the usual periodic solution and by precisely analogous methods its transient states can be investigated by the present theory. In fact the method can be employed to deal with systems in which non-linear differential equations appear, such as the three-element vacuum tube.

I have re-read Professor Pupin's important 1899 and 1900 papers to which he calls attention in his discussion. Without in any sense minimizing their importance and originality, I cannot agree with him that they treated the subject exhaustively if by that Professor Pupin means the transient phenomena and oscillations of such structures. Pupin's treatment of this phase of the problem is limited to writing down the transients as a sum of characteristic damped vibrations, each of which is multiplied by an undetermined constant of integration. Now this is correct as far as it goes but it is in fact merely a formulation of the problem and not a solution. I feel sure that Professor Pupin will agree with me that the real problem lies in determining the integration constants in accordance with the initial configuration of the system and the type of applied forces, and in then throwing the solution into a manageable form which enables us to take account of terminal impedances.

Mr. Bush in his remarks raises some very interesting questions regarding the Heaviside operational methods, to which, however, I do not see that he has given satisfactory or conclusive answers. Consider the symbolic or operational expansion (equation (**1**) of Mr. Bush's discussion) of the problem discussed and solved on pages 405–409 of my paper. Now the problem involved in this operational expansion is two-fold; first to justify the expansion in the operator p itself, and secondly to determine (adopting Mr. Bush's notation) the significance of the symbols $p^n\zeta$. The first phase of the problem which can in no sense be neglected in a rigorous discussion, Mr. Bush ignores entirely and the second he begs by indentifying the symbol p^n with the differential operator d^n/dt^n at the start of the discussion. This is all the more unjustified because he subsequently assigns non-integral values to n which makes the operator d^n/dt^n meaningless. Now it is true that if we start with the solution expressed as the Fourier integral given on page 359 of the paper it is possible to evaluate the symbolic expression $p^n\zeta$ with some reservations involving the legitimacy of differentiating under the integral signs but this leaves the first phase of the problem still untouched. For we have merely shown that *assuming the legitimacy of the expansion in the operator p*, the series resulting is a *formal* solution. The real problem as I see it, still remains: to establish the legitimacy of the expansion in the operator p and then to investigate the significance of the resulting divergent series in inverse powers of t and its relation to the actual convergent solution. It is in the investigation of these important questions, without which the solution must remain in doubt, that I have found the application of integral equations, as developed in my paper, of the greatest value.

Owing to the importance of the foregoing I shall elucidate it in connection with the specific problem stated and solved on pages 405–409 and employed by Mr. Bush in his discussion: namely the evaluation of the voltage $V(t)$ existing across the cable terminals where a "unit e.m.f." is impressed on the cable through a condenser.

The solution from the integral equation of the problem, is, as given at the top of page 407:

$$V(t) = e^{\pi h t} - \sqrt{h}\int_0^t d\lambda \, . \, \frac{e^{\pi h \lambda}}{\sqrt{t-\lambda}}$$

and from the theory of integral equations we know that the solution is unique and absolutely convergent. Following this solution on page 407 it is shown that the integral equation is reducible to a differential equation which is *formally* satisfied by the series solution:—

$$V(t) \sim \frac{1}{\pi\sqrt{h\,t}} \left(1 - (1/2\,\pi\,h\,t) + 1.3\,(1/2\,\pi\,h\,t)^2 \ldots \right)$$

This is identical with that gotten by the Heaviside operational method. So far, however, we have merely shown that this is a *formal* solution and we have established no relation, numerical or otherwise between it and the actual solution gotten from the integral equation. It is just here that the essential weakness of the Heaviside operational solution appears; it is quite powerless to establish any such relation, except empirically, or to show that the divergent series is a true asymptotic expansion of the actual solution. The solution derived from the integral equation, however, enables us to answer this question at once. To show this we write the solution as

$$V(t) = \epsilon^{\pi h t} - \sqrt{h}\ \epsilon^{\pi h t} \int_0^t d\lambda\, \frac{\epsilon^{-\pi h \lambda}}{\sqrt{\lambda}}$$

$$= \sqrt{h} \int_t^\infty d\lambda\, \frac{\epsilon^{-\pi h \lambda}}{\sqrt{\lambda}}$$

If we now perform repeated integration by parts in this expression we get:—

$$V(t) = \frac{1}{\pi \sqrt{h\,t}}$$

$$\left\{ \begin{array}{l} 1 - (1/2\,\pi\,h\,t) + 1.3\,(1/2\,\pi\,h\,t)^2 \\ + (-1)^n\, 1\,.\,3\,.\,5\,\ldots\,(2\,n-1)\,.\,(1/2\,\pi\,h\,t)^n \end{array} \right\}$$

$$- R_n$$

Where the remainder R_n is

$$(-1)^n \frac{\epsilon^{\pi h t}}{\pi \sqrt{h}} \frac{1\,.\,3\,.\,5\,\ldots\,(2\,n-1)}{(2\,h)^n} \int_t^\infty \frac{\epsilon^{-\pi h \lambda}}{\lambda^{n+1}\ \sqrt{\lambda}}$$

Clearly the remainder R_n is less than the last term of the series, consequently the series is a true asymptotic expansion of the solution, and may be used for numerical calculation with an error less than the value of the last term included.

The investigation of the operational solution by rigorous methods is by no means of merely academic interest. For example the method breaks down in at least one specific case which is gotten by replacing the condenser of the present problem by an inductance coil. Here the divergent series obtainable by a straight forward application of the operational method is not a true asymptotic expansion. Heaviside noted this himself and added an extra term which he justified by reasoning which I have been unable to follow, involving as it does an appeal to his somewhat esoteric theory of "generalized differentiation."

The foregoing merely touches on questions of the greatest mathematical interest which I hope to deal with more fully in a future paper.

Another important property of the solution, when expressed as an integral equation, which is not mentioned in the paper, is that it enables us to determine the current and voltage surges where an energized system is suddenly opened. I have gotten solution of problems of this type with a very considerable saving of labor as compared with older methods.

A final word, in the interest of historical accuracy, concerning formula (**11**). At the time my paper was published I believed that this formula originated in the independent work of Mr.

Fry and myself. I have since learned that Vaschy, "Electricite et Magnetisme," Vol. II, page 54, in his discussion of telephony gives a formula for the arrival current which is in the notation of the present paper:—

$$\int_0^t f'(\lambda)\, A\,(t-\lambda)$$

Vaschy does not make a general application of this formula nor does he appear to have appreciated its fundamental importance, and it has gone unnoticed all these years. By a curious coincidence Colonel J. B. Pomey, Chief engineer of the French Telegraph, published in the issue of February 8, 1919 of the *Revue Generale de l'Electricite* a formula superficially the same as that given in note 3 of my paper. Pomey, however, bases his formula on a consideration of the differential equation

$$\left(a_n \frac{d^n}{dt^n} + a_{n-1} \frac{d^{n-1}}{dt^{n-1}} + a_0\right) I = f\,(t)$$

for which his formula is a correct formal solution. In the problem of electrical networks, however, we are not at liberty to start with such a differential equation but are under necessity of dealing with a set of simultaneous differential equations equal in number to the degrees of freedom of the system. For such a problem it is easy to show that Pomey's formula except for an extremely restricted type of applied force, does not and cannot apply.

Presented at the 7th Mid-winter Convention of the American Institute of Electrical Engineers, New York, February 21, 1919.

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IONIZATION OF OCCLUDED GASES IN HIGH-TENSION INSULATION

BY G. B. SHANKLIN AND J. J. MATSON

Abstract of Paper

This paper deals with the ionization of occluded gases in solid insulation from the standpoint that these gas spaces are the weakest part of an insulation design and should receive first consideration. The stress at which ionization starts in different types of built-up insulation, such as used in cables and coils, is measured and from these measurements a safe working stress determined. The paper brings out, more than anything else, the importance of reducing the gas spaces to a minimum size, and using materials of the lowest possible permittivity, since the higher the permittivity the greater the stress on the gas spaces

If an insulation design is worked at a stress so high that internal ionization occurs it should be considered as "over-stressed" and its life will certainly be shortened. Just how much it will be shortened depends upon such factors as; type of insulation used, degree of over-stressing, looseness and porosity, etc. Roughly, the life of an over-stressed design is anywhere from six months to six years, judging from numerous tests made. This shows that the damage is very gradual.

Introduction

THE selection of a safe voltage rating for high-tension equipment, such as cables and coils, or more properly speaking, the selection of a safe thickness of insulation, has not, like other factors in electrical engineering, been based on scientific grounds. The method of selection has been more a question of judgment and experience, than any thing else.

It is generally admitted that high-potential testing and dielectric strength data are very unsatisfactory criterions. They are of value only in testing for flaws and defects. But assuming the insulation in perfect condition physically and as having a dielectric energy loss sufficiently low at maximum operating temperature to assure against accumulative heating or undue rise in temperature, no satisfactory method of predetermining the proper thickness has been available.

In studying the service strength of any design the weakest point has first consideration. Occluded gas is the weakest point of insulation designs, such as considered above. The solid insulation alone, always assuming, of course, that it is a good grade of material and workmanship, will safely withstand much greater stress than when occluded gases are present. When the stress is high enough to ionize these gases, the insulation should be considered "over stressed." This has been generally recognized but the fact that occluded gas is, almost without exception, inherent in organic insulation has received little recognition.

It is the purpose of this paper to present the results of an extensive study of these gas spaces and the effects they introduce. In doing so we do not wish to over emphasize the importance of this one factor. What we do wish to emphasize is that this method of study or test fills a breach in insulation engineering. The method of "judgment and experience" has, as is usually the case, given correct results. It is interesting to note in studying the data given in this paper how closely the maximum allowable working stress agrees with that which is accepted as good practise.

OCCLUDED GAS AND ITS CAUSES

There are so many variables that enter into the formation and characteristics of gas pockets, from the most minute pocket to the largest, that it is impossible to give a complete analysis at the present stage of the investigation. A certain amount of speculation, although an effort will be made to reduce it to a minimum, must be indulged in.

Each detail in the process of manufacture and application of materials introduces some new factor into the formation of gas spaces. Quite often the methods of installation and operation introduce additional factors.

Impregnated Paper Cable, is usually made by first wrapping dry paper tape around the conductor. To simplify the illustration, single-conductor, lead-sheath cable only will be considered.

The dry paper cable is put into a vacuum tank and the greater part of the moisture and air extracted under heat. It is not a perfect vacuum and a certain amount of impure air remains. Melted filling compound is then let into the tank until the cable is immersed. The compound acts as a valve and stops up all the exposed passages through the paper wrappings. Pressure is then applied and the compound forced thoroughly into the cross section. The slight amount of impure air remaining in the cable is driven back into the cross section and entrapped in the spaces or voids between wrappings. The applied pressure compresses these gas spaces to a most minute size.

After the cable has been in the tank long enough to assure complete impregnation the pressure is reduced to atmospheric and the heat cut off. The gas spaces thereby automatically expand to atmospheric pressure and remain at this pressure during cooling. After the cable has cooled down to a temperature at which the compound is somewhat thickened but still thin enough to drip it is lifted out and the excess compound cleaned off. The cable is lead jacketed while still warm and is then immune from external pressure except through the ends. Every one is familiar with the exuding of compound from the exposed ends of cable when heated. This is due to a tendency to equalize the pressure inside and out.

When the cable has cooled to a temperature of about 55 deg. cent. or lower,depending upon the type of filling compound, the compound has thickened to such an extent that pressure is not transmitted to the gas spaces and their size remains constant at lower temperatures, except for the slight enlargement that occurs due to shrinkage of the compound.

At room temperature, then, the pressure in the spaces must be something less than atmospheric. Assuming the temperature at which the compound thickens as 55 deg. cent. the pressure at 25 deg. cent. should be approximately,

$$\frac{760\ (273 + 25)}{273 + 55} = 690 \text{ mm.},$$

or allowing 4 per cent for shrinkage of compound, about 650 mm.

In speculating upon where these spaces are distributed we are inclined to believe that the greater percentage of them are initially down towards the center of the cross section, near the conductor. They are forced down into the cross section by the entrance of the impregnating compound. Either this is the case or the whole cross section contains a maze of gas spaces, for the test results invariably show spaces in the neighborhood of the conductor, where maximum stress occurs.

The aim should be always to keep these spaces reduced to a minimum, for the larger spaces are, the less the stress required to ionize the gas. It is obvious that the paper wrappings should be tight and compact. This prevents the gas from accumulating at one point in the filling compound and forming a bubble of appreciable size between loose layers of paper, or in creases or crinkles.

Varnished Tape and Sheet Materials. The conditions under which these are applied and treated is somewhat different from cable paper. Usually they are wrapped on while exposed to the room air. Varnish or shellac is brushed on between wrappings to stick them together and fill up space. It is reasonable to expect that some air will be entrapped during this application and is originally at atmospheric pressure. When the material is baked or dried out the volatile solvent joins with the air to set up considerable internal pressure and this forces passages through the thickness so that the gases can escape. A cured material is consequently more or less porous and the gas pockets all have small capillary passages to the surrounding atmosphere. They are, then, normally at atmospheric pressure and it is possible to supply fresh air through the outlet passages as the material cools down. This is called "breathing."

If the tapings or sheets are applied tightly to conductors of round cross section the thickness of the gas spaces should be about in the same order as those experienced in well wrapped paper cables. If, however, the material is applied to conductors of rectangular cross section the thickness of the spaces might vary anywhere between those found in round cross sections and values much greater, depending upon the type of insulation considered, the process of application, conditions of use, etc. There is a tendency for the insulation to bulge away from the rectangular sides, which accounts for the wider variation in thickness of gas space.

IONIZATION CURVES

The loss introduced by ionization of occluded gases is combined with the inherent dielectric loss of the solid material and in obtaining data it is necessary to measure the total loss. Fortunately the ionization loss is an appreciable part of the total and is easily and accurately distinguished.

The dielectric energy loss measurements were made by the "Compensated Dynamometer Wattmeter Method." This was described in detail in an article published previously.[1]

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[1] "Compensated Dynamometer Wattmeter Method of Measuring Dielectric Energy Loss," G. B. Shanklin, *General Electric Review*, Oct. 1916.

A better conception of the method, its accuracy and the way the data are worked up and analyzed can be obtained from a paper read before the Institute in June, 1917.[2]

During the three years in which we have followed this work all possible precautions have been taken to eliminate external sources of loss, phase-angle errors and other forms of inaccuracy. Numerous checks have been made besides those described in the two publications referred to. Familiarity with the method and analysis of a mass of data make us feel that an accuracy better than 2 per cent is obtained at the lowest dielectric power factors ordinarily met with in solid materials.

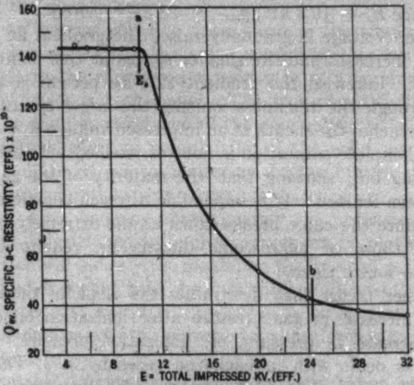

FIG. 1—TYPICAL IONIZATION CURVE

A typical ionization curve is given in Fig. 1. It is calculated from 60-cycle energy-loss data taken at room temperature on a 2/0 single-conductor cable with 9/32 in. thickness of impregnated paper and lead sheath. The watts and current were measured at the impressed voltages indicated. From these readings the effective a-c. restivity per cm³ ($\rho_{av.}$ average through the thickness) was calculated by formula (1) in the Appendix.

Let it be assumed that there are gas spaces sparcely scattered throughout the cross section of cable and that the thicknesses of these spaces vary from zero to that of the largest met with. Up to 10.3 kv. (a) the loss represented by $\rho_{av.}$ is the inherent dielectric loss of the solid material and varies practically as the square of the voltage. The largest gas spaces next to the conductor are the first to be ionized. The voltage gradient at the conductor (in the solid material, not in the gas space) will be

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[2] "Insulation Characteristics of High-Voltage Cables," Clark and Shanklin, A. I. E. E. TRANS., Vol. XXXVI, 1917, p. 447.

$$g_{max.} = \frac{E}{a \log_e b/a} = 2.218\, E,$$

and since $E_s = 10.3$ kv., $g_{max.} = 22.8$ kv. per cm.

As the voltage is gradually raised the gradient at the conductor increases and the smaller spaces at this point will be ionized. Likewise, the gradient 22.8 kv. per cm. is traveling out through the thickness ionizing the larger spaces until it finally reaches the sheath at an impressed voltage $E = 24.3$ kv. (b). Complete saturation is not yet reached but the curve is flattening out, showing that the majority of the gas spaces have been ionized. It is impossible to reach complete saturation before the cable breaks down as the extremely small gas spaces, those of microscopic dimensions, require excessive stress to ionize them.

Another factor that determines the slope of these curves is the increase in gas pressure after ionization once starts. This increase in pressure, or "ionization pressure" as it is called, is dealt with in books on gases by J. J. Thompson[3] and J. S. Townsend.[4] A paper by E. H. Warner[5] also covers the subject.

Since the gas pressure at the instant ionization starts is that with which we are dealing, the "ionization pressure" does not bear on our work, but it is mentioned merely as a matter of interest.

The relative permittivity ($K_{av.}$, average through thickness) is calculated from the energy-loss data by formula (**2**) in the Appendix. For a full description of these characteristics reference should be made to the Institute paper previously referred to.

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[3] "Conduction of Electricity through Gases," by J. J. Thompson, Cambridge Press, 1906.

[4] "Electricity in Gases" by J. S. Townsend, Oxford Press, 1915.

[5] "Ionization Pressure, by E. H. Warner, *Phys. Review*, Vol. 8, 1916, p. 285.

$K_{av.}$ is not the true relative permittivity. In calculating $K_{av.}$ it is assumed that the dielectric circuit is a parallel circuit, that is, the leakage path (a-c.) and capacity are assumed in parallel. Actually the circuit is a maze of series-parallel paths. The series component determines the change in $K_{av.}$ with frequency, depending upon the type of material considered. For instance, in varnished cambric or impregnated paper the difference in $K_{av.}$ at 25 and 60 cycles is small, especially at higher temperatures, because the series component is small. In pasted mica the difference is greater due to the "barrier effect" of the mica flakes.

$K_{av.}$ is an average effective value through the whole thickness and is consequently the value needed in this work. It corresponds to the effective a-c. resistance of a conductor in which skin effect is appreciable. The true permittivity corresponds to the d-c. resistance of the conductor.

Knowing the effective permittivity of the solid insulation and the voltage at which ionization begins, curves can be calculated showing the stress across gas spaces of any assumed thickness. Also, knowing approximately the pressure of the gas and its dielectric strength, cross curves can be plotted and a comparison will indicate the maximum thickness of gas space. This method of dealing with the data will be followed throughout the paper. It would be impossible to present all of the energy-loss data or even the ionization curves alone, as hundreds of them were taken.

In a number of cases measurements were taken at both 25 and 60 cycles. The voltages at which ionization started agreed almost exactly. In other words, the ionization starting point is practically independent of frequency over the usual commercial range.

During the initial stages of this work considerable difficulty was experienced in obtaining uniformly smooth and consistent curves, such as that shown in Fig. 1. In particular, the points taken at the lower voltages would not line up on a smooth curve but plotted in a haphazard manner. We at first came to the conclusion that the ionization was inherently unstable and followed no simple law, but further investigation showed this trouble to be due to two causes:

1. The errors in voltage reading are squared in the calculation of $\rho_{av.}$ and come in once more in the abscissas of the curves. The greatest possible accuracy in voltage reading over the whole range is therefore necessary.

2. The wave shape varied slightly over the voltage range. This not only changed the values of ρ_{av}. but also gave an ionization starting point at variance with that of a sine wave. This error was corrected by connecting a relatively large capacity in parallel with the test sample. A pure sine wave over the whole voltage range resulted.

The effects of a badly distorted wave are shown in Fig. 2 the data being taken at 60 cycles, room temperature, on a sample of mica-paper insulation 0.16 in. thick, wrapped on a conductor of rectangular cross section. Baking at high temperature had caused the insulation to loosen up and form numerous gas spaces. Curve (*a*) shows measurements with a distorted wave and curve (*b*) with a sine wave. Oscillographic records of the two waves are shown in Fig. 3. The higher harmonics in (*a*) were caused by an iron core reactance in the low voltage side. Wave shape (*b*) was taken after 0.05 microfarads capacity was connected in parallel with the test sample.

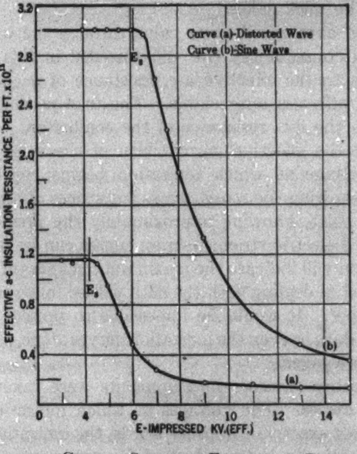
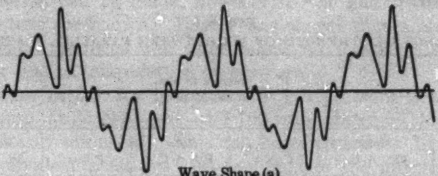


FIG. 2—IONIZATION CURVES SHOWING EFFECT OF DISTORTED WAVE SHAPE

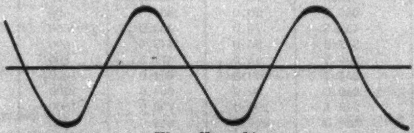
Ionization starts at 6.1 kv. effective with the sine wave, while with wave (*a*) it starts at 4.2 kv. The starting point with wave (*b*) is $6.1 \times \sqrt{2} = 8.62$ kv. maximum, with wave (*a*) it is $4.2 \times 2.03 = 8.53$ kv. maximum. These two values correspond closely.

STRENGTH OF AIR

Before the thickness of gas space in a test sample, can be determined the strength of gas under exactly the conditions, met with must be known. From the nature of these conditions the gas considered must be a mixture of air and the volatile organic contents of the insulation, the amount of impurity depending upon the vapor tension of the different components, the ability to supply fresh air to the gas spaces, etc.



Wave Shape (a)



Wave Shape (b)

FIG. 3—OSILLOGRAPH RECORDS OF WAVE SHAPES REFERRED TO IN FIG. 2

In those cases where components of low vapor tension, such as benzol, are present the nature of the conditions is such that passages are forced from the gas pockets to the outside air and the impurities mostly escape.

Impregnating compounds are heat or vacuum treated before using, and their components have a comparatively high vapor tension. When impregnating compound is admitted to the filling tank the vacuum decreases but little faster than the decrease in gas volume, which shows that the amount of impurity liberated is not great.

We can say roughly, that, the occluded gas in insulation is a mixture of air and from zero to about 50 per cent impurity, the impurity as a general rule being an organic vapor. The best that can be done is to consider the strength of air alone. The amount of impurity will vary in different cases and complete ionization data are available only for air.

On pages 466 to 471 of J. J. Thompson's book he discusses and analyzes data on various gases and vapors. The strength of CO_2 and water vapor is about equal to that of air over the range of pressure and gap spacing needed in our work. The strength of benzol vapor varies from about 1.0 to 2.5 times that of air.

TABLE I
DIELECTRIC STRENGTH OF AIR BETWEEN PARALLEL PLATES

P = pressure in mm.
t = air gap in mm.

$P \times t$	Volts Eff. A-C.	$P \times t$	Volts Eff. A-C.	$P \times t$	Volts Eff. A-C.
1.05	1371.5	9.0	260.0	160.	1025.
1.10	1226.5	10.0	270.0	200	1195
1.35	813.0	15.0	308.0	300	1602
1.50	686.0	20.0	336.5	400	1980
2.00	452.5	25.0	363.0	500	2350
2.50	346.5	35.0	417.5	600	2705
3.00	295.5	45.0	471.5	800	3402
4.00	254.5	60.0	554.0	1000	4075
5.00	249.0	80.0	664.0	1200	4720
6.00	249.5	100.0	756.0	1500	5655
7.00	250.0	120.0	848.0	1700	6260
8.00	252.0	140.0	937.0		

In discussing Bouty's tests on different gases between glass plates, Thompson concludes that the nature of the electrodes (whether metal or dielectric material) has no effect unless the spacing is comparable with the mean free path of the gas molecules. Tests in our laboratory at 60 cycles, atmospheric pressure, showed no difference in the strength of air between brass electrodes and glass plate electrodes down to the minimum spacing (0.05 cm.) that could be measured accurately.

Table No. 1 gives the strength of air in terms of Pachen's Law;

$$Pt = \text{a constant voltage.}$$

where P = mm. pressure, and
t = mm. gap spacing.

The data in this table represent an average of the values obtained by a number of investigators with both direct and alternating voltages. Most of the data were taken from the books by J. J. Thompson and J. S. Townsend. In some cases original sources were referred to.[6]

Pashen's Law as given above is based on the assumption that $P \times t =$ a constant mass of gas per unit area of gap. Also, the figures in Table I are based on an air density equivalent to a temperature of 25 deg. cent.

It is obvious that when increasing the temperature of a test sample above 25 deg. cent. this condition is not met with. A correction can be made, however, if the gas pressure and temperature are known. Harris[7] has shown that a change in air density produced the same effect on dielectric strength whether due to temperature or pressure.

In paper cables, for instance, the gas pressure is assumed as 650 mm. at 25 deg. cent. and 760 mm. above 55 deg. cent., as previously discussed. This can be corrected for by assuming "equivalent pressures" proportional to the absolute temperatures, instead of the actual pressure. The "equivalent pressures" are:

650 mm. at 25 deg. cent.
760 mm. at 55 deg. cent.

$$760 \times \frac{328}{373} = 668 \text{ mm. at } 100 \text{ deg. cent.}$$

In wrapped insulation that has been cured the gas pressure is assumed as 760 mm. over the whole temperature range. The "equivalent pressures" therefore are:

760 mm. at 25 deg. cent.

$$760 \times \frac{298}{373} = 607 \text{ mm. at } 100 \text{ deg. cent.}$$

$$760 \times \frac{298}{473} = 479 \text{ mm. at } 200 \text{ deg. cent.}$$

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[6] "Laws of Elec. Discharge in Gases," by W. R. Carr, Phil. Trans. of Royal Society A, Vol. 201, 1903.

"Comparison of A-C. and D-C. Spark Potentials, by J. C. Jansen, *Phys. Review*, Vol. 8, 1916.

"Dielectric Phenomena in High-Voltage Engineering," by F. W. Peek, McGraw-Hill Co., 1915.

[7] W. S. Harris, Phil. Trans. Royal Society, 1834, p. 230.

TABLE II.
SINGLE-CONDUCTOR PAPER CABLE:
FILLED WITH MINERAL HYDRO-CARBON BASE COMPOUND. LEAD SHEATH. NEVER IN SERVICE.

Sample No.	Type of conductor	Dia. cond. inches	Thick. insul. inches	G_{max} kv. per cm. at cond.	Temp. deg. cent.	Freq.	K_{av}	E_s Start of ionization kv.	$G_{max.}$ at start kv. per cm.	Remarks
No. 1	stranded	0.412	9/32	2.218*E*	25	60	3.28	10.2	22.65	Wrappings compact and well filled. Inherent loss higher than average.
					50	60	3.30	10.9	24.20	
					75	60	3.45	11.0	24.40	
No. 2	stranded	0.412	9/32	2.218*E*	21.5	60	2.93	10.1	22.40	Wrappings compact but not very well filled. Inherent loss less than average.
					51.5	60	2.96	11.2	24.85	
					73.0	60	2.90	12.0	26.60	
					100.0	60	2.93	13.2	29.30	
No. 3	stranded	1.346	14/32	1.17*E*.	25.0	60	2.85	20.0	23.40	Wrappings compact and well filled. Inherent loss less than average.
					47.5	60	2.87	22.0	25.75	
					72.5	60	2.92	24.0	28.07	
No. 4	stranded	1.186	26/32	0.770*E*	23.	60	3.17	30.0	23.10	Wrappings compact and fairly well filled. Average inherent loss.
					60	60	3.27	31.5	24.25	
					100	60	3.67	33.5	25.80	
					60	25	3.55	31.5	24.25	
					100	25	4.20	33.5	25.80	
No. 5	smooth	1.311	24/32	0.7875*E*	26	60	3.10	28.5	22.45	Conductor surface smoothed with 1/16 in. coating of lead. Wrappings compact and fairly well filled. Average inherent loss.
					60	60	3.23	32.0	25.20	
					100	60	3.70	32.75	25.80	
					28	25	3.30	29.0	22.85	
					60	25	3.50	32.5	25.60	
					100	25	4.20	33.0	26.00	
					SAMPLE WITH LARGE GAS SPACES.					
No. 6	stranded	1.186	22/32	863.*E*	23.5	60	2.91	15.0	12.95	Wrappings loose and badly wrinkled. Only partly filled. Inherent loss less than average.
					59.0	60	3.05	16.0	13.81	
					97.0	60	3.20	17.0	14.68	
					23.5	25	3.11	15.5	13.38	
					59.5	25	3.24	16.0	13.81	
					98.0	25.	3.68	16.5	14.25	

When determining the strength of air from Table I the "equivalent pressures" should be used instead of the actual pressures.

IONIZATION DATA

Single-Conductor Paper Cable: Filled with Mineral Hydrocarbon Base Compound. Lead Sheath. In Table No. 2 data are given for samples of single-conductor paper cable filled with mineral hydro-carbon base compound. The various cross sections cover a wide range. The voltage at which ionization starts (E_s) was taken from measured curves similar to that in Fig. 1. In every case it is the minimum value measured on two or more separate samples.

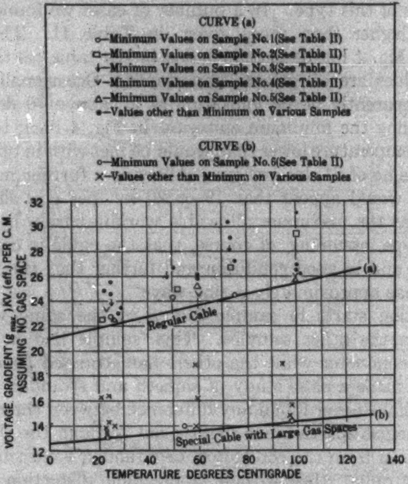

FIG. 4—MAXIMUM VOLTAGE GRADIENT BETWEEN CONDUCTOR AND SHEATH (ASSUMING NO GAS SPACE) AT START OF IONIZATION

Single-conductor paper cable—filled with mineral hydro-carbon base compound— new cable; never in service

In Fig. 4 the voltage gradient (effective kv. per cm.) in the solid insulation at the conductor, that is, assuming no gas spaces present, is given for the different cables at the instant ionization starts. The assumption of no gas spaces is allowable, for even when present they are so small as not greatly to effect the distribution of stress in the solid material. Also, it would be expected that the larger number of the stress lines between conductor and sheath pass only through solid material without encountering gas spaces.

The gradient (g_{max}) agrees surprisingly well for the different cross sections. All of the points above curve (a) in Fig. 4 are representative of the values met with in ordinary commercial cable of this type. In a number of cases we found values of g_{max} higher than those given in Table II. These are given in Fig. 4 but are not included in the table, as the minimum values are of greater importance. Occasionally there would apparently be no ionization at all above 70 deg. cent.

Extending the minimum curve (a) in Fig. 4 back to 0 deg. cent., a temperature lower than would be met with in operation, gives a value of 21. kv. per cm. Allowing a further margin of safety it would appear that 19 or 20 kv. per cm. should be selected as the maximum allowable working stress in a cable of this type, assuming, of course, that the cable is compactly made up and in good condition and further that the working voltage has approximately a sine wave.

Ionization starts in sample No. 5 at the same value of g_{max} as in the other samples. This sample had a smooth surfaced conductor while the others had stranded conductors. We have made a close study of smooth and stranded conductors and have never found any difference between their effects. Either the gas spaces occur slightly out through the thickness of insulation from the conductor, or stranded conductors give a uniform radial stress without appreciable distortion next to the conductor.

It now remains to determine approximately the maximum thickness of gas space met with in this type of cable. Taking sample No. 5 as a representative sample the stress across gas spaces of assumed thicknesses was calculated at 26 deg. and 100 deg. cent. using formula (3) in the Appendix. The stress (g_a) across the gas space is calculated as the average by making $x = a + 1/2\, t_a$ when the gas space is assumed next to the conductor, or $x = b_1 + 1/2\, t_a$ when the space is assumed out in the thickness of insulation, which is allowable because there is only one or two per cent difference between the maximum and minimum stress across the gas space. In other words, the lines of stress across the gas space are practically parallel. This is due to its extreme thinness when compared with radius a.

The calculated curves discussed above are given in Fig. 5. The gas space (t_a) is first assumed as next to the conductor and then 5 and 15 per cent out in the thickness of insulation from the conductor. The difference in the permittivities of gas and solid material makes the stress much higher in the gas space, as can be seen by comparing Figs. 4 and 5.

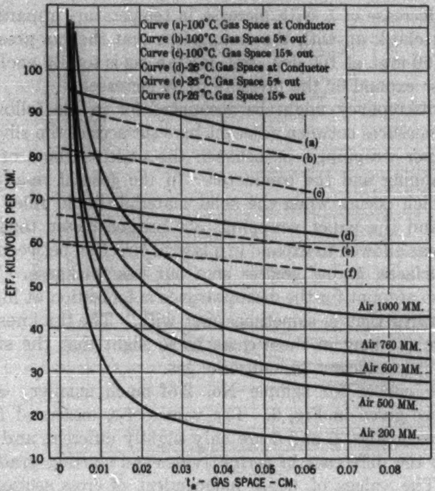

FIG. 5—CROSS CURVES FOR DETERMINING THICKNESS OF GAS SPACE—SAMPLE NO. 5 (SEE TABLE II)

The strength of air, taken from Table I, is cross plotted in Fig. 5 at various assumed pressures. It is questionable whether the strength of pure air is closely representative of the impure mixture in the gas spaces. However, it serves the purpose of approximation and comparison of the different materials considered. It is interesting to note that according to Thompson the strength of the impure mixture must be higher, not lower, than that of pure air.

Assuming a gas pressure of 650 mm. at 25 deg. cent. and 760 mm. at 100 deg. cent., as previously considered, the corresponding "equivalent pressures" are;

650 mm. at 25 deg. cent.
668 mm. at 100 deg. cent.

With the gas space assumed next to conductor the values of t_a, estimated from Fig. 5 at points where the curves cross are:

0.0068 cm. at 25 deg. cent.
0.0036 cm. at 100 deg. cent.

This decrease of t_a with increase in temperature apparently does not check up with the assumption that the gas pressure is only 760 mm. at 100 deg. cent. for the gas is totally enclosed and must expand or the pressure must increase.

This discrepancy might be accounted for in the following way; t_a measures between adjacent bare surfaces of the slightly loose paper wrappings, or between the under surface of the first wrapping and the conductor. In the general re-adjustment which occurs when the solid material, liquid filler and gas expand the paper wrappings are pressed closer together and the gas allowed to expand in a lateral direction between the paper surfaces, giving greater area but less thickness. This could also account for the disappearance of ionization at higher temperatures that is sometimes met with. The thickness of gas space (t_a) may in these cases be so slight that the stress across it is insufficient to ionize the gas.

Similar curves for sample No. 2 of much smaller cross section are given in Fig. 6. The values of t_a measured from the curves in Figs. 5 and 6 are only slightly different and this is due to the difference in permittivities and starting gradient (g_{max}). The values of t_a are independent of cross section as long as t_a is small compared with the thickness of insulation.

Cable With Large Gas Spaces. Sample No. 6 in Table II was a special length made up with comparatively large gas spaces for test. The first 5 per cent of the paper wrappings were applied loose and crinkled, the remaining wrappings being uniformly firm. The length was then vacuum treated in the usual way but was not left in the compound tank long enough to thoroughly fill it. Compound penetrated the whole cross section but not thoroughly, as was shown by examination after test, the gas spaces in the loose crinkled section being plainly visible when the wrappings were carefully removed.

Curve (*b*) in Fig. 4, extended back to 0 deg. cent. gives a voltage gradient of 12.5 kv. per cm., a dangerously low value. Curves for determining the maximum thickness of gas space are given in Fig. 7. The "equivalent pressures" are:

$$650 \times \frac{296.5}{298} = 647 \text{ mm. at } 23.5 \text{ deg. cent.}$$

$$760 \times \frac{328}{370} = 674 \text{ mm. at } 97 \text{ deg. cent.}$$

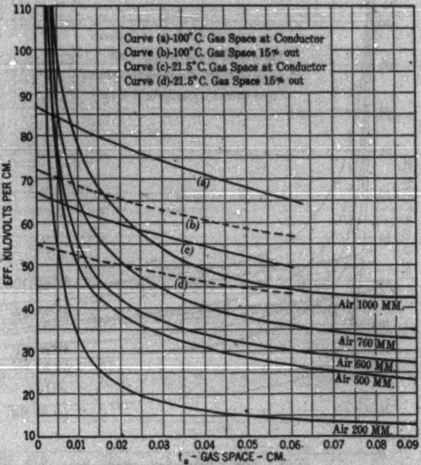

FIG. 6—CROSS CURVES FOR SAMPLE NO. 2 (SEE TABLE II)

With gas spaces assumed next to conductor the values of t_a are:

0.047 cm. at 23.5 deg. cent.
0.023 cm. at 97 deg. cent.

The spaces in this sample are therefore about seven times larger than those in samples Nos. 2 and 5 which are representative of average good cable of this type. It is obvious that large gas spaces are dangerous. All engineers have recognized this. The data given above merely verify a generally accepted view.

It will be noted that the 23.5 deg. cent. curve in Fig. 7 falls below the curve of the dielectric strength of air at 760 mm. This is very good proof that the pressure in the gas spaces is something less than atmospheric at room temperature.

Three-Conductor Paper Cable, Filled with Mineral Hydrocarbon Base Compound. We have had very poor success in attempting to investigate internal ionization and other characteristics of three-conductor cables with actual three-phase voltage. The degree of accuracy in the measurements has not been sufficient to obtain uniform and consistent ionization curves. Our work in this direction agrees with that of F. M. Farmer,[8] in that, as far as can be determined, there is no difference in the actual three-phase losses and those calculated from single-phase measurements. The investigation is being continued and we are hoping for better results later.

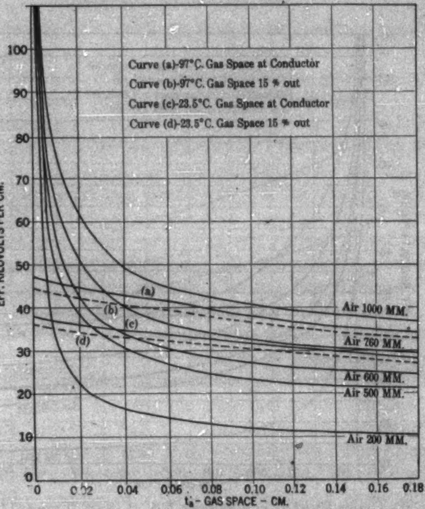

FIG. 7—CROSS CURVES FOR SAMPLE NO. 6 (SEE TABLE II)

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[8] Measurements of Power Loss in Voltage of Three-Conductor High-Tension Cables, by F. M. Farmer, TRANS. A. I. E. E., 1918, p. 221.

TABLE III.

THREE-CONDUCTOR PAPER CABLE:

FILLED WITH MINERAL HYDRO-CARBON BASE COMPOUND. LEAD SHEATH. STRANDED CONDUCTORS.

Sample No.	Dia. cond. inches	Thick. insul. inches	$G_{max.}$ kv. per cm. at cond.	Temp. deg. cent.	Freq.	K_{av}	E_s start of ionization	$G_{max.}$ at start kv. per cm.	Remarks
				NEW SAMPLES—NEVER IN SERVICE					
No. 7	0.412	$\frac{9}{32} \times \frac{7}{32}$	1.45*E*	25	60	3.26	15.5	22.48	Three conductors in.
				25	60	3.26	15.5	22.48	Two of conductors withdrawn.
No. 8	0.412	$\frac{9}{32} \times \frac{7}{32}$	1.45*E*	22.5	60	3.24	15.0	21.75	Wrappings compact but not very well filled. Loss less than average.
				60	60	3.33	16.0	23.20	
No. 9	0.412	$\frac{9}{32} \times \frac{7}{32}$	1.45*E*	48.	60	3.53	16.0	23.20	Average well filled. Loss about average.
				74.	60	3.60	16.4	23.79	
				94.	60	3.61	16.8	24.37	
No. 10	0.728	$\frac{7}{32} \times \frac{7}{32}$	1.396*E*	23	60	3.69	17.6	24.57	Well filled. Loss about average.
No. 11	Semi Sector	$\frac{9}{32} \times \frac{5}{32}$	1.121*E*	21	60		20.1	22.53	Assumed as concentric single cond. a = 2.104 cm.
				98	60		22.0	24.70	
				OLD SAMPLES—HAD BEEN IN SERVICE.					
No. 12	0.373	$\frac{9}{32} \times \frac{7}{32}$	1.51*E*	24	60	3.23	20.0	30.20	Well filled. Loss a little higher than average.
				51	60	3.40	22.0	33.20	
				77	60	3.44	22.6	34.15	
				101	60	3.53	23.4	35.30	
No. 13	0.412	$\frac{9}{32} \times \frac{7}{32}$	1.45*E*	26	60	3.35	20.0	29.00	Well filled. Loss a little higher than average.
				64	60	3.45	22.0	31.90	
				100	60	3.52	24.0	34.80	

The start of ionization between any two conductors cannot be studied with single-phase voltage, but only that between conductor and sheath. This is unfortunate, for in actual three-phase operation the stress is greatest between conductors, whereas with, single-phase voltage the stress is greatest between conductor and sheath, and ionization naturally starts here first, blanketing the results between conductors. A study of the stress between conductor and sheath is instructive, however, and will be given.

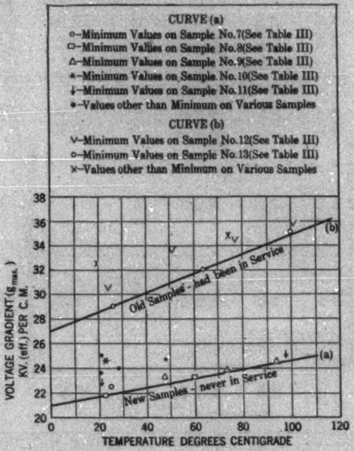

FIG. 8—MAXIMUM GRADIENT BETWEEN CONDUCTOR AND SHEATH (ASSUMING NO SPACE) AT START OF IONIZATION

Three-conductor paper cable filled with mineral hydro-carbon base compound

Calculation of the maximum stress between conductor and sheath (assuming no gas space present) is made by formula (**5**) in the Appendix. In this formula, it is assumed that the proximity of the other two conductors has no effect on g_{max}. The correctness of this assumption is verified by a comparison of the values of g_{max} for new cable in Table III and Fig. 8 with those of single-conductor cable in Fig. 4. A special test on sample No. 7 in Table III gives, perhaps, better verification.

Sample No. 7 was first tested by applying high voltage to all three conductors and grounding the sheath. The voltage at which ionization started (E_s) was found to be 15.5 kv. Two of the conductors were then carefully withdrawn from the sample, strand at a time, and the test repeated. E_s agreed exactly with that found in the first test.

The minimum values of $g_{max.}$ for new samples of three-conductor cable, in Fig. 8 (curve *a*), agree closely with the corresponding values for single-conductor cable, Fig. 4 (curve *a*), and it must be concluded that the conditions, such as thickness of gas space, gas pressure, etc. are the same in both types of cables. This would be expected, since they are made of the same materials and by the same process.

Sample No. 11 has semi-sector shaped conductors. The outer circumference of these conductors are arcs, concentric with the lead sheath. As far as the maximum stress between conductor and sheath is concerned this type can be considered as a single-conductor cable and the max. stress calculated in the usual way;

$$g_{max} = \frac{E}{a \log_e b/a}$$

where, a = the radius to the outer surface of conductor, and b = inner radius of sheath.

Tests on old samples that had been in service, samples No. 12 and 13, gave very interesting results. Curve (*b*) in Fig. 8 shows these to have much higher values of g_{max} when ionization starts. This same permanent increase in E_s has been noted several times in new samples after they had been heated above the melting point of the mineral compound filler. We can only account for it by suggesting some sort of "settling" or re-adjustment of the melted compound and gas spaces in such a way as to make the spaces permanently smaller, at least those nearest the conductor. This probable explanation has previously been offered for the increase of $g_{max.}$ with temperature, although in that case the increase considered was only temporary. The two effects are probably closely related, as will be brought out later.

The study of old samples has been limited and we do not know whether this settling effect is inherent or not. It would hardly be safe to rely upon improvement with aging in every case, and the minimum curve (*a*) in Figs. 4 and 8 should stand as a safer guide. However it is re-assuring to know that the change in ionization with aging is in the right direction.

TABLE IV.
THREE-CONDUCTOR PAPER CABLE:
FILLED WITH VEGETABLE HYDRO-CARBON BASE COMPOUND. LEAD SHEATH. STRANDED CONDUCTORS.

Sample No.	Dia. cond. inches	Thick. insul. inches	$G_{max.}$ kv. per cm. at cond.	Temp. deg. cent.	Freq.	K_{av}	E_s start of ionization	$G_{max.}$ at start kv. per cm.	Remarks.
				New Samples—Never in Service.					
No. 14	0.528	$\frac{7}{32} \times \frac{7}{32}$	1.624E	21	60	3.88	13.5	21.93	Well filled. Losses high.
				58	60	4.43	12.5	20.30	
				100	60	5.40	11.0	17.87	
No. 15	Semi sector	$\frac{6}{32} \times \frac{6}{32}$	1.243E	25.5	60		13.0	16.16	Assumed as concentric single cond. a = 2.44 cm. Losses high.
				110.5	60.		14.5	18.02	
				Old Samples—Had Been in Service.					
No. 16	0.528	$\frac{7}{32} \times \frac{7}{32}$	1.624E	29	60	5.03	10.0	16.24	Charred in center. Losses very high.
				55	60	5.52	10.2	16.56	
				75	60	6.10	10.4	16.90	
				111	60	9.37	11.6	18.85	
No. 17	0.528	$\frac{7}{32} \times \frac{7}{32}$	1.624E	25	60	4.25	8.8	14.30	Undamaged. Poorly filled. Losses high.
				75	60	5.29	10.0	16.24	
				100	60	7.19	10.1	16.40	
No. 18	0.528	$\frac{7}{32} \times \frac{7}{32}$	1.624E	27	60	4.74	9.5	15.45	Undamaged. Well filled. Losses very high.
				100	60	8.36	10.0	16.20	
No. 19	Semi sector	$\frac{6}{32} \times \frac{6}{32}$	1.243E	25.5	60		15.0	18.65	Assumed as single cond. $a = 2.44$ cm. Losses high.
				106	60		16.0	19.90	

The relative permittivity (K_{av}) of the insulation between conductor and sheath, as given in Table III, is calculated with a formula for three-conductor cables developed by Alex. Russell. It is given as formula (**6**) in the Appendix. The calculated K_{av}. agrees favorably with that for single-conductor cable.

Three-Conductor Paper Cable; Filled with Vegetable Hydrocarbon Base Compound. Before discussing the tests on paper cable filled with vegetable hydrocarbon base compound we wish to point out that our tests were limited to only two lots of this kind and, in particular, that a study of these was made because they had given trouble in operation.

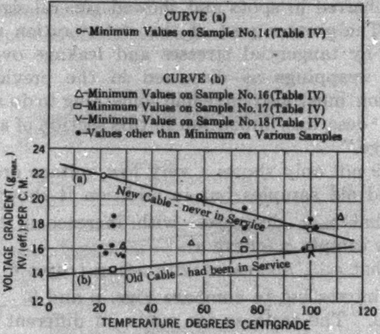

Fig. 9—Maximum Gradient Between Conductor and Sheath (assuming no gas space) at Start of Ionization

Three-conductor paper cable—filled with vegetable hydro-carbon base compound

Ionization data are given in Table IV. Samples Nos. 14, 16, 17, and 18 are from the same batch of cable described in the June, 1917, Institute paper previously referred to. Sample No. 14 had never been in service. The others were cut from the lines near points of failure after five years operation. The start of ionization is shown in Fig. 9. It is very low compared with that in Fig. 8, which shows that this lot of cable could not be safely operated at so high a voltage as the other type.

The dielectric losses and temperature coefficient were extremely high, so high in fact, that the temperature rise in operation was appreciable even without load on the cables. This factor alone would limit the voltage rating. With internal ionization and the deterioration it must cause probably included, there is no wonder that the cable gave trouble.

The stress between conductors at which ionization starts could not be determined with single-phase voltage. We cannot say definitely, then, that there was ionization during operation, but judging from the low values of g_{max} between conductor and sheath in Fig. 9 it seems very probable that there was. The paper wrappings between conductors were badly charred in spots and showed general signs of deterioration. The greater part of this deterioration may have been caused by tangential stresses and leakage over the surfaces of the wrappings as discussed in the previous paper, but ionization must also have had something to do with it. Hochstadter[9] describes ionization in this section of a cable as "glow discharge."

Fig. 9 not only shows a very low voltage gradient for both new and old samples but in addition it shows:

1. that $g_{max.}$ decreases with increase in temperature in the new sample. (curve *a*).
2. that $g_{max.}$ decreases with aging (curve *b*).

It is interesting to compare Figs. 8 and 9. The effects are reversed. The tests so far made on different types of cable insulation have shown that whenever there is an increase or decrease in $g_{max.}$ with temperature in new samples there is a corresponding increase or decrease with aging. This would tend to show that the two effects, whatever their cause are closely related.

Formula (3) in the Appendix does not apply to three-conductor cable and there is no way of approximating the thickness of gas space t_a. It is apparent, however, that t_a is greater than that found for paper cable filled with mineral hydrocarbon compound, although the higher value of permittivity is also a determining factor in making $g_{max.}$ lower.

Samples No. 15 and 19 in Table IV are from a different lot of cable than those dealt with above. The curves for these samples are in Fig. 10. Their characteristics are not the same as those of the first lot, more nearly resembling those in Fig. 8 although the values of $g_{max.}$ are considerably lower.

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[9] "Three-Core Cable," by M. Hochstadter, *Electrotechische Zeitschrift*, Heft 47, 1915.

This lot of cable did not give as much trouble in operation as the first, even though the dielectric losses were just as high. Allowing for possible differences in the operating conditions it would still seem that the lesser trouble experienced with this lot was due to its more favorable ionization characteristics, that is, improvement with aging.

Varnished Cambric Tape; Wrapped Half-lap on Conductors of Round Cross Section. The ionization data for 0.010 in. black varnished cambric tape, wrapped half-lap on round conductors is given in Table V.

Two samples of lead-sheathed single-conductor cable (samples No. 20 and 21) are also given in this table. Sample No. 20 is varnished cambric cable and sample No. 21 varnished paper. They both were well filled with cable compound, this being accomplished by running the tape through the compound.

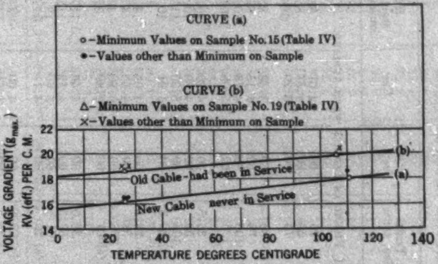

FIG. 10—MAXIMUM GRADIENT BETWEEN CONDUCTOR AND SHEATH (ASSUMING NO GAS SPACE) AT START OF IONIZATION

Three-conductor paper cable—filled with vegetable hydro-carbon base compound

The other samples under the heading, "Filled as Tapings were Applied." were wrapped on brass tubes, wet varnish being brushed on as the tapings were applied. These were first tested without heat treatment, and then after baking at 100 deg. cent., as designated in the table. Metal foil tape was wrapped over the surface and bound down tightly with adhesive tape.

The minimum curve of $g_{max.}$ *vs.* temperature for the filled samples is in Fig. 11 and the cross curves of stress in gas space

TABLE V.
0.010″ VARNISHED CAMBRIC TAPE:
WRAPPED HALF-LAP ON CONDUCTORS OF
ROUND CROSS SECTION.

Sample No.	Type of conductor	Dia. cond. inches	Thick. insul. inches.	$g_{max.}$ kv cm. at cond.	Temp. deg. cent.	Freq.	K_{av}	E_s start of ionization	$g_{max.}$ at start kv. cm.	Remarks.
					FILLED AS TAPINGS WERE APPLIED.					
No. 20	stranded	0.412	9/32	2.218E	25	60	4.07	8.5	18.85	Black varnished cambric cable. Lead sheath.
					50	60	4.55	8.4	18.64	
					75	60	5.20	none		
No. 21	stranded	0.412	9/32	2.218E	35	60	3.53	8.3	18.41	Black varnished paper cable. Lead sheath.
					75	60	4.15	8.5	18.85	
					100	60	4.76	10.1	22.40	
					125	60	5.08	10.6	23.50	
No. 22	smooth	0.538	.331	1.82E	23.5	60	4.20	10.2	18.56	Black varnished cambric wrapped on brass tubes. Tin foil wrapped tightly outside.
					58	60	4.80	10.1	18.39	
					104	60	5.95	11.2	20.40	
					(a) 20.5	60	4.01	10.0	18.20	
No. 23	smooth	0.528	.0975	4.74E	23.5	60	4.05	4.0	18.95	Same as Sample No. 22.
					58	60	4.55	4.2	19.90	
					103	60	6.20	4.5	21.30	
					(a) 21	60	3.80	3.8	18.02	
No. 24	smooth	1.527	.277	1.665E	22.5	60	4.46	11.5	19.16	Same as Sample No. 22.
					57.5	60	4.72	11.8	19.66	
					101	60	5.60	12.9	21.50	
					(a) 20	60	4.17	11.1	18.50	
No. 25	smooth	1.533	.0845	4.92E	23.5	60	4.00	3.8	18.70	Same as Sample No. 22.
					56.5	60	4.51	4.0	19.68	
					103.	60	6.40	4.3	21.15	
					(a) 21	60	3.70	3.7	18.21	

TABLE V.—(*Continued*)
0.010″ VARNISHED CAMBRIC TAPE
WRAPPED HALF-LAP ON CONDUCTORS OF
ROUN CROSS SECTION.

Sample No.	Type of conductor	Dia. cond. inches.	Thick. insul. inches.	δ_{max} kv. cm. at cond.	Temp. deg. cent.	Freq.	K_{av}	E_s Start of ionization	δ_{max} at start kv. cm.	Remarks.
					(b) NO FILLING TAPING APPLIED DRY.					
No. 26	smooth	0.530	0.294	1.988*E*	23.5	60	4.16	9.0	17.90	Black varnished cambric wrapped on brass tubes. Tin foil wrapped tightly outside.
					62	60	4.68	10.0	19.88	
					102.5	60	5.15	11.1	22.07	
					(c) 23.5	60	4.08	8.8	17.50	
					23.5	25	4.08	9.1	18.10	
					62.	25	4.71	10.5	20.87	
					103.	25	5.43	11.1	22.07	
					(c) 23.	25	4.08	8.8	17.50	
No. 27	smooth	0.534	0.133	3.63*E*	19.5	60	4.87	4.6	16.70	Same as Sample No. 26.
					61.5	60	5.62	5.2	18.90	
					101.	60	5.96	5.5	19.95	
					(c) 23.	60	4.79	4.6	16.70	
					23.5	25	5.00	4.6	16.70	
					62.5	25	5.96	5.2	18.90	
					102.	25	6.21	5.6	20.33	
					(c) 22.5	25	4.82	4.7	17.05	
No. 28	smooth	1.529	0.292	1.592*E*	23.5	60	3.90	11.0	17.50	Same as Sample No. 26.
					59.5	60	4.35	12.2	19.40	
					101.5	60	4.76	13.0	20.70	
					(c) 23.5	60	3.94	11.0	17.50	
					19.5	25	4.10	11.4	18.15	
					61.5	25	4.50	12.2	19.40	
					102.5	25	4.95	12.8	20.40	
					(c) 23.	25	3.98	11.0	17.50	
No. 29	smooth	1.510	0.0925	4.525*E*	21	60	3.75	4.0	18.10	Same as Sample No. 26
					57	60	4.35	4.2	19.00	
					102.5	60	4.86	4.7	21.27	
					(c) 24	60	3.88	3.9	17.64	
					24	25	4.00	4.2	19.00	
					62	25	4.99	4.2	19.00	
					102.5	25	5.22	4.7	21.27	
					(c) 23.	25	3.90	4.0	18.10	

(a) NOTE: These measurements were taken after samples had been baked in oven 24 hrs. at 50 deg. cent. and 5 hrs. at 100 deg. cent.
(b) All of the samples under this heading were baked 72 hrs. at 50 deg. cent. and 24 hrs. at 100 deg. cent. before testing.
(c) Baked an additional 24 hrs. at 50 deg. cent. and 5 hrs. at 100 deg. cent.

and strength of air in Figs. 12 and 13. The stress in the gas spaces (g_a) was calculated by determining values of E_s from the minimum curve in Fig. 11 and using an average low value of K_{av} from Table V.

The thickness of gas space (t_a) measured from the cross curves in Figs. 12 and 13 is therefore the maximum expected in this type of insulation.

Assuming the gas pressure as 760 mm. at both 25 deg. cent. and 100 deg. cent. the "equivalent pressures" are:

760 mm. at 25 deg. cent.
607 mm. at 100 deg. cent.

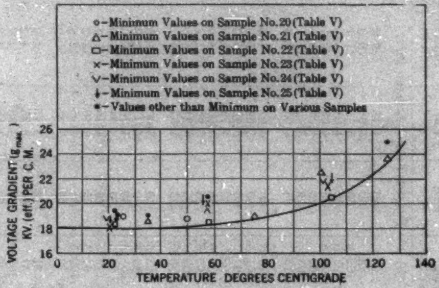

FIG. 11—MAXIMUM GRADIENT BETWEEN CONDUCTOR AND OUTER METALLIC WRAPPING (ASSUMING NO GAS SPACE) AT START OF IONIZATION

Varnished cambric tape—wrapped half lap on conductors of round cross section—filled as tapings were applied

With the gas space assumed next to conductor the values of t_a are:

0.008 cm. at 25 deg. cent.
0.003 cm. at 100 deg. cent.

The thickness of gas space given above is within the same order as that for paper cables filled with mineral hydrocarbon compound. The values of $g_{max.}$ however, are lower because of the higher permittivity of varnished cambric.

Ionization data are given under a separate heading in Table V for samples of 0.010-in. black varnished cambric tape wrapped on brass tubes without a filler, that is, no wet varnish brushings were applied. These samples were thoroughly dried out in an oven just before testing to make sure that no excess moisture was present. The minimum curve of g_{max}. is in Fig. 14. It is surprising to note that it is only slightly lower than the corresponding curve in Fig. 11. Larger gas spaces at the edges of the overlapped wrappings would naturally be expected.

The values of t_a corresponding to those given for the filled samples are:

0.011 cm. at 25 deg. cent.
0.0035 cm. at 100 deg. cent.

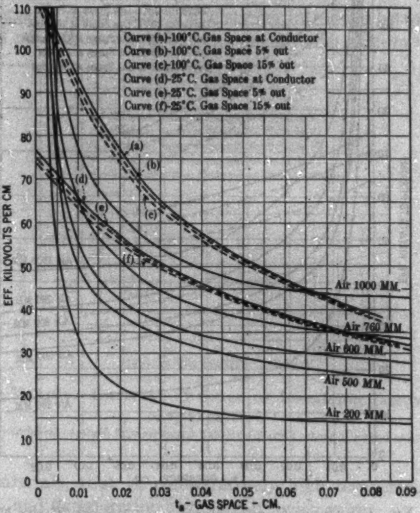

FIG. 12—CROSS CURVES FOR SAMPLE NO. 25 (SEE TABLE V)

These values were taken from curves similar to the curves in Figs 12 and 13.

The actual maximum thickness of the space at the edge of the wrappings must be greater than estimated above, as the wrappings were each 0.010 in. (0.0254 cm.) thick. We believe the discrepancy can be accounted for in the following way:

In calculating g_a it is assumed that the dielectric lines of flux are uniformly straight radial lines. In passing through the thin gas spaces of comparatively large area, usually met with between the surfaces of wrappings, they doubtless are, at least towards the center of the space where t_a is a maximum and ionization first occurs. But, the space at the edge of the wrappings is nothing but a narrow thread-like spiral and the stress lines are partly shunted around it through the solid material of higher permittivity. Consequently, either the ordinary spaces between surfaces are ionized first, in which case the calculations are correct, or the narrow edge spaces are actually ionized at a lower value of g_a than that wrongly estimated.

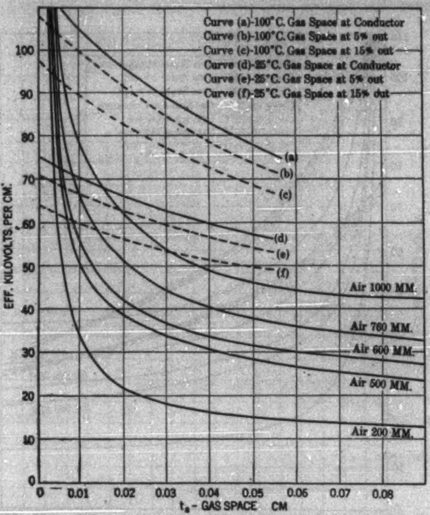

FIG. 13—CROSS CURVES FOR SAMPLE NO. 22 (TABLE V)

This same line of reasoning would apply to unfilled wrinkles or creases in wrappings, and also to the small capillary passages which form as the filler is dried out.

Flexible Mica-Paper Tape; Wrapped Half-lap on Conductors of Round Cross Section. Pasted flexible mica-paper has characteristics somewhat different from the usual run of flexible organic materials. This difference is due to the interruptions, or barrier effect, of the laminated inorganic mica flakes. Pasted mica-paper can be better understood if it is considered as having the combined characteristics of mica flakes and varnished sheet material such as paper or cambric.

Pure mica is more permanently stable than any other insulating material. Its electrical characteristics, such as dielectric strength, permittivity and energy loss are almost independent of length of voltage application and temperature between 0 deg. and 300 deg. cent.

The organic paper backing and sticker introduce between the flakes of mica those characteristics usually met with in organic solid materials, that is, comparatively high temperature coefficient, and deterioration and porosity with aging.

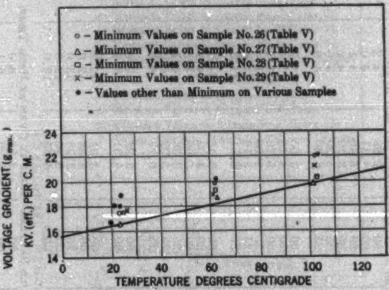

Fig. 14—Maximum Gradient at Start of Ionization (assuming no gas space)

Unfilled varnished cambric tape wrapped half-lap on conductors of round cross section

The a-c. leakage path (also porosity and deterioration when present) follows the path of the organic components, zig-zagging in and out around the mica flakes and wrappings searching out the shortest and easiest path through the thickness.

When mica-paper is wrapped on conductors of rectangular cross section extreme care must be taken to prevent the insulation from loosening up and bulging away from the sides. The natural spring of mica flakes and softening of the sticker under heat treatment tend to cause this bulging. On round cross sections there is no tendency to bulge and, hence, one of

TABLE VI.
FLEXIBLE MICA PAPER TAPE:
WRAPPED HALF-LAP WITH STICKING VARNISH ON CONDUCTORS OF ROUND CROSS SECTION. STICKER DRIED BY HEAT CURING.

Sample No.	Dia. Cond. inches	Thick. insul. inches	$\mathcal{E}_{max}$ kv. per cm. at cond.	Temp. deg. cent.	Freq.	K_{av}	E_s start of ionization	$\mathcal{E}_{max}$ at start kv. per cm.	Remarks.
				Fresh Samples—Cured but not Aged.					
No. 30	0.536	0.07075	6.27E	26.	60	2.98	4.2	26.30	Metal foil tape wrapped over outer surface and tightly bound down with adhesive tape.
				65.	60	4.11	4.0	25.10	
				105	60	5.46	4.1	25.70	
				25	25	3.10	4.2	26.30	
				67	25	5.20	4.0	25.10	
				102	25	6.40	4.0	25.10	
No. 31	0.535	0.2512	2.22E	26	60	3.05	11.7	26.00	
				68	60	4.16	11.4	25.30	
				105	60	5.70	11.4	25.30	
				26	25	3.28	11.4	25.30	
				67	25	5.16	11.6	25.75	
				104	25	6.51	11.8	26.20	
No. 32	1.51	0.073	5.66E	26	60	3.15	4.7	26.60	
				64	60	4.09	4.25	24.05	
				105	60	5.99	4.3	24.35	
				25	25	3.36	4.7	26.60	
				68	25	5.22	4.3	24.35	
				103	25	6.77	4.5	25.50	
No. 33	1.51	0.118	3.58E	22	60	3.22	7.0	25.06	
				61	60	4.31	6.9	24.71	
No. 34	1.53	0.211	2.155E	25	60	3.02	12.2	26.30	
				106	60	5.38	12.0	25.90	
				28	25	3.25	13.0	28.00	
				65	25	5.26	12.0	25.90	
				103	25	6.65	12.0	25.90	
				Old Samples—Aged in Oven at High Temp.					
No. 35	1.51	0.120	3.53E	24	60	2.95	6.0	21.18	Baked 24 hr at 50 deg. cent.
				62	60	3.20	6.1	21.53	" 24 hr at 75 deg. cent.
				103	60	3.99	6.0	21.18	" and 84 hr. at 100 deg. cent.
No. 36	1.51	0.116	3.66E	25	60	2.87	5.7	20.87	Same baking as sample No. 35 and in addition 84 hr. at 150 deg. cent.
				101	60	3.10	5.6	20.50	
				152	60	3.51	5.4	19.77	
No. 37	1.51	0.118	3.58E	28	60	2.65	5.8	20.75	Same baking as sample No. 36 and in addition 290 hr. at 200 deg. cent.
				106	60	2.85	5.6	20.05	
				200	60	3.12	5.5	19.70	

the most troublesome characteristics of pasted mica is eliminated when used in this way.

Ionization data for representative samples of mica paper insulation are given in Table VI and the minimum curves of g_{max} in Fig. 15. The wrappings were compactly applied half-lap with sticking varnish on round brass tubes, and heat cured to drive out the solvent.

The minimum curve of g_{max}. for fresh material in Fig. 15 is higher than that for any other material so far considered.

Samples No. 35, 36 and 37 in Table VI were aged in an oven respectively at 100 deg. cent., 150 deg. cent. and 200 deg. cent. They are minimum representatives of a number of samples aged for different periods of time at the above temperatures.

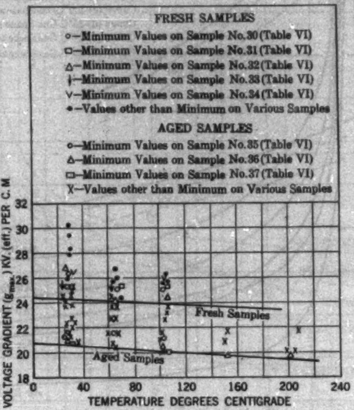

FIG. 15—MAXIMUM GRADIENT AT START OF IONIZATION (ASSUMING NO GAS SPACE)

Flexible mica-paper-wrapped half-lap on conductors of round cross section.

Baking at high temperature volatilizes a considerable part of the organic components and renders the insulation brittle and extremely porous. The degree of porosity was shown clearly in two ways:

1. The additional dielectric loss caused by ionization was much greater than the corresponding loss in fresh samples. This was shown by comparison of ionization curves similar to that in Fig. 1.

2. When aged samples were exposed to moisture the absorption by the capillary passages of the porous organic components increased the dielectric loss enormously. In fact it was almost impossible to reach voltage E_s without breakdown when the samples were well saturated.

The minimum curve of g_{max} for aged samples in Fig. 15 indicates that aging enlarges the gas spaces, as would be expected.

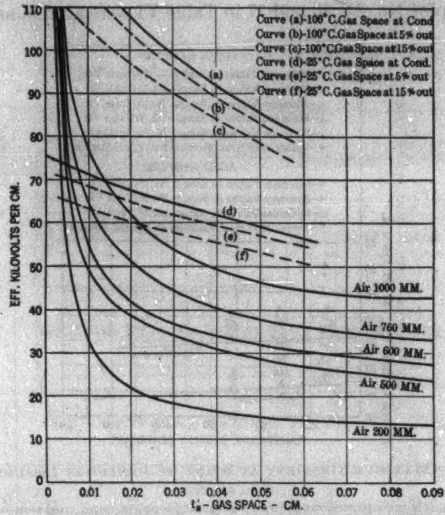

FIG. 16—CROSS CURVES FOR SAMPLE NO. 31 (SEE TABLE VI)

The cross curves for determining the maximum thickness of gas space for fresh samples are in Fig. 16. The curves of g_a were calculated by taking values of E_s from the minimum curve in Fig. 15 and minimum average values of K_{av}. The maximum ratio of b/a (sample No. 31) is considered.

Assuming the gas pressure as 760 mm. both at 25 deg. cent. and 100 deg. cent. the "equivalent pressures" are:

760 mm. at 25 deg. cent.
607 mm. at 100 deg. cent.

From the cross curves the values of t_a assumed next to conductor are:

0.0075 cm. at 25 deg. cent.
0.0025 cm. at 100 deg. cent.

The cross curves for sample No. 37, a representative aged sample, are shown in Fig. 17. Assuming a constant gas pressure of 760 mm. the "equivalent pressures" are:

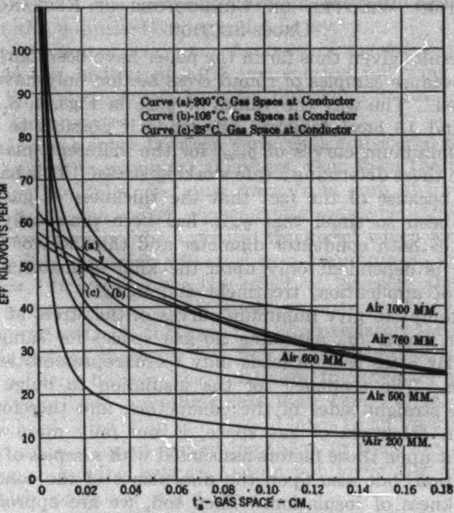

Fig. 17—Cross Curves for Sample No. 37 (See Table VI)

$$760 \times \frac{298}{304} = 752 \text{ mm. at 28 deg. cent.}$$

$$760 \times \frac{298}{379} = 598 \text{ mm. at 106 deg. cent.}$$

$$760 \times \frac{298}{473} = 479 \text{ mm. at 200 deg. cent.}$$

From the cross curves the values of t_a assumed next to conductor are.

0.023 cm. at 28 deg. cent.
0.012 cm. at 106 deg. cent.
0.0065 cm. at 200 deg. cent.

Aging has enlarged the gas spaces appreciably.

INSULATION WRAPPED ON CONDUCTORS OF RECTANGULAR CROSS SECTION

The results given thus far in the paper have been fairly consistent because samples of round cross section only have been considered. The way the values of $g_{max.}$ in Figs. 4, 8, 9, 10, 11, 14 and 15 have bunched has made it possible to assign definite minimum curves of $g_{max.}$ for the different materials, and from these determine a safe working stress. This has been possible because of the fact that the thickness of gas space (t_a) has been so small that $g_{max.}$ has been practically independent of both conductor diameter and thickness of insulation. It is dependent only upon the kind of material used, method of application, treatment and aging.

An attempt to give minimum curves of the stress at which ionization starts (g_s, assuming no gas space) for samples of rectangular cross section leads only to misrepresentation and confusion. The tendency for the insulation to bulge away from the straight sides of the conductors, and therefore the maximum thickness of gas space, is not only more vitally dependent upon those factors associated with samples of round cross section, but also upon the dimensions of the conductor and thickness of insulation. Then too, we are approaching values of t_a that are appreciable when compared with the thickness of insulation and for this reason g_s varies with thickness.

The most satisfactory way to deal with rectangular cross sections, such as armature coils, is to measure the voltage at which ionization starts in each individual case. The value of E_s will indicate whether the particular type of coil can be safely operated at its rated voltage or not. Also, the thickness of gas space determined from these measurements will indicate the degree of puffing or bulging by comparison with a similar sample of round cross section, which can be considered as a standard representing the best possible attainment, that is, as having the least values of t_a for the given conditions.

Before dealing with actual tests on coils consideration will again be given to round cross sections to illustrate, by indirection, a peculiar characteristic which has been met with only in rectangular cross sections.

Referring to Fig. 12, as representative of round cross sections, the 25 deg. cent. curve of stress in gas space at conductor crosses the 760 mm. strength of air curve at both 0.009 cm. and 0.075 cm. It is evident that for any thickness smaller than 0.009 cm. or larger than 0.075 cm. there will be no ionization at the impressed voltage of 3.64 kv.

Now, the actual maximum thickness of gas space is selected as 0.009 cm. instead of 0.075 cm. for the following reasons:

a. For any thickness between the above limits, ionization would have started at a lower value of E_s.

b. If a gas space between surfaces is considered as having 0.075 cm. as its maximum thickness, then from the nature of its shape the thickness must vary from zero at its edges to 0.075 cm. in the center, consequently somewhere through the gas space there will be a critical thickness, which we will call t_c, at which ionization will start at a minimum E_s lower than 3.64 kv. It would have the effect of lowering the 25 deg. cent. stress curve so that it would be just tangent to the 760-mm. curve at t_c, (in this case t_c is approximately 0.035 cm.)

This critical thickness has not been experienced in round cross sections because the maximum thickness of gas space has always been something less than this. It is, however, experienced quite often in coils and leads to an interesting and important conclusion concerning them, in that, any degree of puffing or bulging beyond t_c does not lower the voltage E_s at which ionization starts.

Ionization data for a few representative coils are given in Table VII. Sample No. 38 is one of a number made up and tested under conditions whereby the gas spaces would be reduced to the smallest possible size. The insulation was applied as compactly as possible and it was tightly clamped in moulds during the treating process and throughout test. Metal foil was wrapped over the outer surface and tightly bound down, serving as one electrode.

The voltage stress (g_s) at which ionization starts is found merely by dividing E_s by the thickness of insulation. This is a closer approximation than would at first appear, as the bulging is always a maximum at the center of the widest straight side where the lines of stress are practically parallel. Divergent

TABLE VII.
FLEXIBLE MICA-PAPER TAPE:
WRAPPED HALF-LAP WITH STICKING VARNISH ON CONDUCTORS OF RECTANGULAR CROSS SECTION. CURED DURING APPLICATION.

Sample No.	Cond. inches	Thick. insul. inches	g kv. per cm.	Temp. deg. cent.	Freq.	K_{as}	E_s start of ionization	g_s at start kv. per cm.	Remarks.
TESTED IN MECHANICAL CLAMPS.									
No. 38	1/2 × 3	0.0960	4.10E	24.7	60	4.00	5.6	22.97	Insulation compactly applied and well cured while clamped. No aging.
				100	60	5.20	5.5	22.55	
				151	60	6.20	5.6	22.97	
		0.0930	4.23E	25	60	3.10	5.5	23.27	Aged in an oven while clamped for 170 hr. at 200 deg. cent.
				100	60	3.30	5.3	22.42	
				198	60	3.60	5.6	23.70	
COMPACT COILS TESTED WITHOUT CLAMPS									
No. 39	1/2 × 3	0.112	3.51E	25	60	3.80	5.1	17.91	Insulation applied and cured compactly. Good grade. No aging.
				60	60	4.40	5.1	17.91	
				102	60	4.90	5.1	17.91	
No. 40	1/2 × 3	0.322	1.22E	26	60	3.80	12.4	15.13	Same as Sample No. 39.
				60	60	4.40	12.3	15.01	
				101	60	4.90	12.2	14.88	
TESTED WITHOUT CLAMPS—NOT VERY COMPACT.									
No. 41	1/2 × 3	0.110	3.58E	25	60	3.87	4.5	16.11	Insulation bulged but not badly.
				61	60	4.40	4.5	16.11	
				100	60	4.90	4.4	15.75	
No. 42	1/2 × 3	0.322	1.22E	25	60	3.80	10.0	12.20	Same as Sample No. 41.
				60	60	4.40	10.0	12.20	
				102	60	4.90	10.0	12.20	
COIL BADLY PUFFED—NO CLAMPS.									
No. 43	1/2 × 3	0.328	1.196E	25	60	3.85	10.0	11.96	Insulation caused to bulge badly by sudden application of high temperature.
				61	60	4.30	8.4	10.05	
				100	60	4.75	7.2	8.61	

stress lines occur only at the corners and the insulation is very compact here.

The values of g_s are comparable with corresponding values of $g_{max.}$ for mica samples of round cross sections in Table VI, showing how compact this coil was. The clamps prevented bulging when the coil was aged at 200 deg. cent. and consequently there was not much change in g_s.

The working stress of coils varies over a wider range than for other insulation designs. It is usually somewhere between 20 and 37 volts per mil (7.87 and 14. 57 kv. per cm.) depending upon such factors as mechanical stress and abrasion, working temperature, etc. Ionization started in sample No. 38 at 57 volts per mil (22.5 kv. per cm.) If it were possible to actually operate this coil under the same favorable conditions it is evident that it could be safely stressed much higher than the very highest stress used in ordinary operation.

The stress in gas spaces of assumed thicknesses cannot be calculated unless the effective permittivity (K_{av}) is known. K_{av} cannot be determined from measurements on rectangular cross sections because the dielectric field is not symmetrical. The best that can be done is to approximate g_a by selecting values of K_{av} by judgment, using those values found in round cross sections of similar material as a guide. The results are fairly good but K_{av} varies too much in different batches, treating processes, etc., to make this method more than a fair approximation. Whenever the maximum thickness of gas space is equal to or greater than the critical thickness (t_c), K_{av} can be found directly in the following way:

For a first trial a value of K_{av} is selected as above to correspond as closely as possible to the true value. The curve g_a vs. t_a is calculated and compared with the strength of air curve of the right "equivalent pressure." If K_{av} is properly chosen it will be almost tangent to the strength of air curve. Another curve is drawn in parallel to the first and exactly tangent. The point of tangency will indicate the value of t_c. Knowing this, the true value of K_{av} and the correct curve of g_a vs. t_a can be calculated.

The cross curves for sample No. 38 are in Fig. 18. The g_a vs. t_a curves all cross their corresponding "equivalent pressure" curves, showing that the gas space is less than t_c. In selecting the values of K_{av} weight was given to the following factors:

(a) K_{av} should be higher than for the round cross sections in Table VI as the insulation was applied differently in the two cases. For the round cross sections all of the insulation was wrapped on with wet sticker between wrappings. The whole was then cured at high temperatures until the varnish was well dried out. The escaping gases consequently caused porosity, and added to this, there must have been many very small spaces formed between surfaces from shrinkage of the sticker as it dried out. The percentage of solid material was reduced and thereby the effective permittivity. For the rectangular cross sections, the layers were carefully cured and moulded as applied, largely closing up capillary passages and other small spaces generally distributed through the thickness and increasing the effective permittivity.

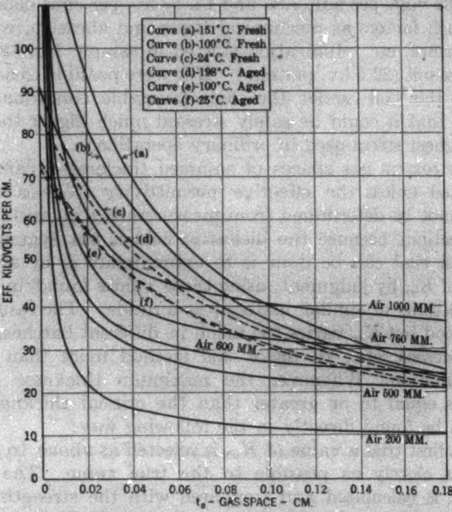

FIG. 18—CROSS CURVES FOR SAMPLE NO. 38 (TABLE VII)

(b) Energy loss measurements give the total capacity of the sample and show how this capacity varies with temperature. The actual permittivity should, of course, vary exactly the same. There remains, then, only to select an arbitrary value of K_{av} for one temperature.

(c) In samples of the same kind, the insulation can be caused to bulge, by sudden application of high temperature until t_a is equal to or greater than the critical thickness at room temperature. Tangent curves as previously discussed will give K_{av} direct. This K_{av} should, however, be a little less than that for sample No. 38, (see sample Nos. 41 and 42) which was tightly clamped and more compact.

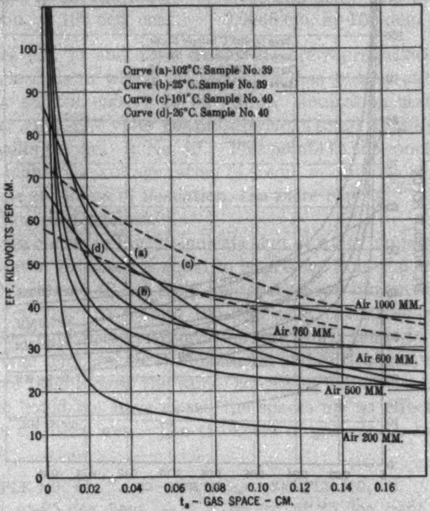

FIG. 19—CROSS CURVES FOR SAMPLES NOS. 39 AND 40 (SEE TABLE VII)

Using "equivalent pressures" in the usual way, the values of t_a in Fig. 18 are:

FRESH	AGED
0.005 cm. at 24 deg. cent.	0.010 cm. at 25 deg. cent.
0.003 cm. at 100 deg. cent.	0.007 cm. at 100 deg. cent.
0.0025 cm. at 150 deg. cent.	0.0045 cm. at 198 deg. cent.

These thicknesses are even smaller than those in samples of round cross section. We believe this is accounted for partly by the tight clamping and partly by the more compact nature of the material, that is, presence of more organic sticker.

Samples No. 39 and 40 are exactly similar to No. 38 but were not tested in clamps. They are representative of good grade compactly made coils in which every reasonable care has been taken to reduce the gas spaces to a minimum. No. 39 has 0.2845 cm. (0.112 in.) thickness of insulation and ionization starts at a stress of 17.91 kv. per cm. (45.5 volts per mil) No. 40 has 0.818 cm.(0.322 in.) thickness and ionization starts at only 15.0 kv. per cm. (38.2 volts per mil). In other words, the thicker the insulation the less the stress it will stand. This characteristic is inherent in rectangular cross sections and is due to the gas spaces being appreciable compared with thickness of insulation. It is evident that there will be no ionization in sample No. 39 at the very highest stress ordinarily used in operation, while No. 40 would be limited to a working stress of about 14.17 kv. per cm. (36 volts per mil), which is a little

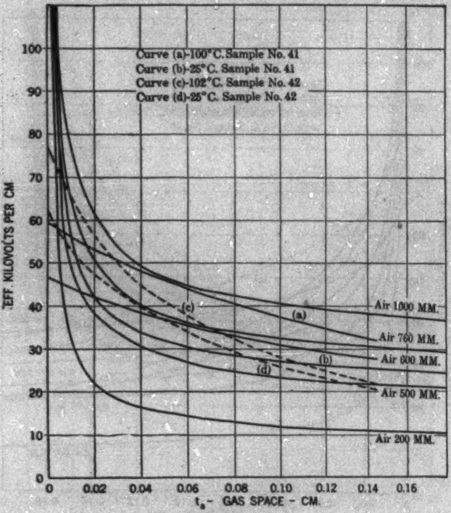

Fig. 20—Cross Curves for Samples Nos. 41 and 42 (Table VII)

less than the maximum limit but actually higher than would be required in operation for so great a thickness. A working stress of 36 volts per mil is based on the assumption that the operating conditions, such as maximum temperatures, etc., will be such that the gas spaces will not be enlarged with aging. The cross curves for No. 39 and 40 are in Fig. 19. The values of t_a are:

SAMPLE NO. 39	SAMPLE NO. 40.
0.015 cm. at 25 deg. cent.	0.018 cm. at 26 deg. cent.
0.005 cm. at 102 deg. cent.	0.0065 cm. at 100 deg. cent.

Samples No. 41 and 42 in Table VII are representative of poor grade coils in which the insulation has bulged out, not extremely so, but enough to be visible. Ionization starts at 16 kv. per cm. (40.7 volts per mil), in No. 41 and 12.2 kv. per cm. (31 volts per mil) in No. 42. This points to the conclusion that the higher the voltage rating of a coil, in other words, the greater the thickness of insulation, the more carefully bulging must be guarded against.

The cross curves in Fig. 20 indicate that at room temperature the maximum gas space is equal to or greater than the critical in both No. 41 and 42, for the g_a vs. t_a curves are tangent to the 760 mm. curves. At 60 and 100 deg. cent. the stress curves cross their "equivalent pressure" curves, which tends to show that at room temperature the maximum and critical gas spaces must have been almost the same, otherwise expansion of the insulation could not have closed the spaces up so effectively at higher temperatures. The thickness of gas spaces in Fig. 20 are:

SAMPLE NO. 41.	SAMPLE NO. 42.
0.035 cm. at 25 deg. cent. (critical)	.06 cm. at 25 deg. cent. (critical)
0.007 cm. at 100 deg cent.	.011 cm. at 102 deg. cent.

This apparent reduction in thickness of gas space with increase in temperature is based entirely on the assumption adhered to throughout the paper of atmospheric pressure at the higher temperatures as well as at room temperature. If the pressure is assumed to increase with temperature it is possible to even get an increase in gas space with temperature. We are confident that the assumption of atmospheric pressure at room temperature is correct but are free to confess that we have

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sometimes doubted the assumed pressure at higher temperatures because of the unreasonable decrease in gas space. Every bit of evidence we have, with the exception of this one contradiction, has forced us to the conclusion of atmospheric pressure at all temperatures. When the temperature is raised too rapidly there is certainly an increase in gas space, due to internal gas pressure, as bulging is plainly visible. In all of our tests, however, the temperature is brought up very slowly and everything allowed to stabilize before readings are taken.

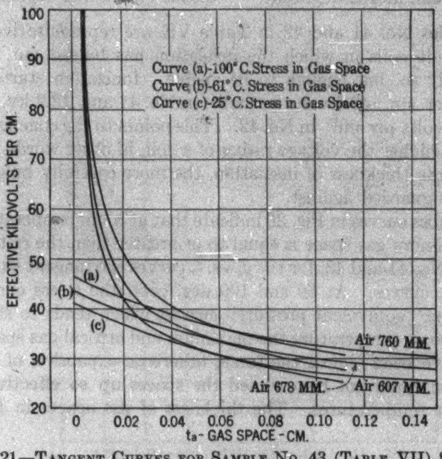

Fig. 21—Tangent Curves for Sample No. 43 (Table VII) Showing Critical Thickness of Gas Space

For this reason, it is possible that the pressure is held at atmospheric by the small capillary orifices.

The data on sample No. 43, a badly puffed coil, prove beyond any doubt that the gas pressure was practically atmospheric for all temperatures. Whether the internal gas pressure of a coil in this condition is representative of the more compact coils is problematical. The g_a vs. t_a curves in Fig. 21 are all tangent to their respective "equivalent pressure" curves and the permittivity K_{av}, found in this way is so nearly that which would be expected that there is little doubt of the assumed pressures being correct. Apparently the maximum thickness of gas space is considerably greater than the critical, at least at room temperature. The values of t_c are:

0.063 cm. at 25 deg. cent.
0.062 cm. at 61 deg. cent.
0.061 cm. at 100 deg. cent.

No. 43 had an insulation thickness of 0.833 cm. (0.328 in.) At 100 deg. cent. ionization starts at a minimum stress of 8.61 kv. per cm. (21.8 volts per mil). With lesser thickness of insulation, ionization would not have started at such low stress, but it is apparent that in any badly bulged coil there is danger of ionization under normal operating conditions.

EFFECT OF INTERNAL IONIZATION

The damage caused by internal ionization in samples of rectangular cross section having various types and thicknesses of insulation has been studied for the past four years. The samples are over stressed for long periods of time, some continuously, others intermittently to produce "breathing" effect.

The result can be summarized by saying that without exception a chemical analysis[10] of the deteriorated material deposited on the walls of the gas spaces showed it to be a nitro-product. Exactly the same effect is produced by the attack of nitric acid. This can only mean that some source of air supply was available and the conclusion is that air was supplied through the capillary passages.

In general the degree of damage depended upon the ability to supply fresh air. The damage in loose porous samples was much greater than in compact ones. Likewise it was greater in those compact samples operated intermittently, where "breathing" supplied additional air. Another feature was that the voltage stress at which ionization started seemed to be independent of the degree of damage. There was no decrease in the starting voltage as the damage progressed.

We have never tested impregnated paper cable in this way. There is no source of air supply in lead sheathed cable and such a test would be interesting.

ACKNOWLEDGMENT

In concluding this paper, we wish to express our appreciation and thanks to Dr. C. P. Steinmetz and Messrs. W. S. Clark,

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[10] Thanks are due to Dr. C. G. Van Brunt who analyzed these materials.

H. G. Reist, E. Knowlton, D. A. Ballard, H. L. Schermerhorn, and W. C. Hayman for their cooperation and helpful advice.

APPENDIX

For a concentric cross section, such as a single-conductor sheathed cable, the effective a-c. resistivity per cm³., (average through thickness of insulation) can be calculated by the following formula:

$$\rho_{av.} = \frac{2\,\pi\,l\,E^2}{W\ \log_e \dfrac{b}{a}} \qquad \mathbf{(1)}$$[11]

where, $\rho_{av.}$ = average effective a-c. resistivity per cm.³
l = length of sample considered in cm.
a = radius of conductor in cm.
b = radius of inner side of metal jacket in cm.
E = impressed volts (effective)
W = watts dielectric loss in length, l.

For the same conditions above the relative permittivity (average through thickness,) is:

$$K_{av.} = \frac{I \sin\theta \cdot \log_e \dfrac{b}{a}}{(2\,\pi)^2\,l\,f\,E\,K_a} \qquad \mathbf{(2)}$$

where, I = amperes, dielectric current for length, l.
θ = dielectric power factor angle.
f = frequency.
K_a = capacity per cm³ of air = 0.08842×10^{12} farads.

With the following nomenclature and that already given the formula for concentric cross sections having dielectric layers of different permittivities is derived below:

g = voltage gradient $\left(\dfrac{d\,e}{d\,x}\right)$ at radius x.
C = capacity in farads = $\dfrac{I \sin\theta}{2\,\pi\,f\,E}$
$b_1, b_2 \ldots\ldots b_n$ = radius in cm. to outer surface of layers.
K = relative permittivity at radius x.
$K_1, K_2 \ldots\ldots K_n$ = relative permittivity of layers.

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[11] For derivation of this formula see, "Insulation Characteristics of High-Voltage Cables," Clark and Shanklin, loc. cit.

$$\frac{1}{C} = S = S_1 + S_2 \ldots\ldots + S_n$$

$$S = \frac{1}{2\pi l K_a}\left(\frac{\log_e \frac{b_1}{a}}{K_1} + \frac{\log_e \frac{b_2}{b_1}}{K_2} \ldots\ldots + \frac{\log_e \frac{b_n}{b_{n-1}}}{K_n}\right)$$

$$d s = \frac{1}{K} \cdot \frac{1}{2\pi l K_a} \cdot \frac{d x}{x}$$

$$d e = \frac{I \sin\theta\, d s}{2\pi f} = \frac{E}{S} \cdot \frac{1}{K} \cdot \frac{1}{2\pi l K_a} \cdot \frac{d x}{x}$$

$$g = \frac{d e}{d x} = \frac{E}{S} \cdot \frac{1}{K} \cdot \frac{1}{2\pi l K_a} \cdot \frac{1}{x}$$

$$g = \frac{1}{x K}\left(\frac{E}{\frac{\log_e \frac{b_1}{a}}{K_1} + \frac{\log_e \frac{b_2}{b_1}}{K_2} \ldots\ldots + \frac{\log_e \frac{b_n}{b_{n-1}}}{K_n}}\right) \tag{3}$$

When $a = \alpha$ the dielectric lines are parallel between plane surfaces.

$$t_1 = b_1 - a \qquad t_2 = b_2 - b_1 \qquad t_n = b_n - b_{n-1}$$

$$g = \frac{E}{K\left(\frac{t_1}{K_1} + \frac{t_2}{K_2} \ldots + \frac{t_n}{K_n}\right)} \tag{4}$$

In calculating the maximum voltage gradient between conductor and sheath in a three-conductor cable it is assumed that g_{max} is not influenced by the proximity of the other two conductors. The problem then simplifies to the determination of g_{max} for two eccentric conductors.

a = radius in cm. of conductor.
b = radius in cm. to inside surface of sheath.
d = distance in cm. between axis of a and b.
g_{max} = voltage gradient (volts/cm.) at surface of cond.

From Russell's "Alt. Currents", Vol. 1, pp. 169-170.

$$g_{max} = \frac{E}{a\ \omega}\left[\frac{b^2 - (a - d)^2}{b^2 - (a + d)^2}\right]^{1/2} \qquad (5)$$

where,

$$\omega = 2 \log_e [\sinh (\omega/2) + \cosh (\omega/2)]$$

and,

$$\sinh (\omega/2) = \left[\frac{(S - a)\ (b - S)}{a\ b}\right]^{1/2}$$

$$\cosh (\omega/2) = \left[\frac{S\ (S - d)}{a\ b}\right]^{1/2}$$

where,

$$S = \frac{a + b + d}{2}$$

The relative permittivity (K_{av}) of the insulation between conductor and sheath of a three-conductor cable is determined as follows:

Let,

a = radius in cm. of conductor.

b = radius in cm. to inside surface of sheath.

d = distance in cm. between axis of a and b.

c_1 = 1/3 the total measured capacity between the three conductors in parallel and sheath, in farads per ft. length of cable.

From Alex. Russell's, "Alt. Currents" Vol. 1, p. 193.

$$K_{av} = 5.904 \times 10^{10}\ c_1 \log_e \frac{b^6 - d^6}{3\ (b^3\ d^2\ a)} \qquad (6)$$

Presented at the 7th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 21, 1919.

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THE DIELECTRIC STRENGTH OF AIR FILMS ENTRAPPED IN SOLID INSULATION AND A PRACTICAL APPLICATION OF THE PROBLEM FOR ALTERNATOR COILS AND CABLES

BY F. DUBSKY

ABSTRACT OF PAPER

It is of theoretical importance, as well as of practical importance in design, to be able to predetermine the dielectric strength of air films or bubbles entrapped in solid insulation.

If such films are over-stressed the solid insulation may be damaged by chemical action, local heating, mechanical action etc., due to corona.

An experimental investigation was made of the strength of air films of various thicknesses between glass plates. In the arrangement used, the breakdown of the air or the starting point of corona could be readily observed. Tests were also made of the dielectric strength of air films between other solid insulations.

It was found that the dielectric strength of air films between insulations was practically the same as the dielectric strength of air films between conductors.

The second part of the paper is devoted to the practical application of the data to the design of armature coils and cables, and several specific examples are given.

Part I

THE DIELECTRIC STRENGTH OF AIR FILMS

THIN air films or bubbles are often entrapped in the solid insulation used in coils, cables and other electrical apparatus. Air is weaker than solid insulations, and when placed in series with them in an electric field, as is usually the case for entrapped air, it takes a much greater unit stress. This follows because the same flux passes through the different dielectrics and the air has the lowest permittivity or offers the highest resistance to the flux.

It becomes of great practical importance to be able to predict the behavior of these films and the voltage at which their break-down occurs. The break-down of an air bubble or film usually will not cause immediate break-down of the solid insulation; it may result, however, in gradual deterioration for the following reasons:

1. Heating due to the losses in corona.
2. Chemical action due to the formation of ozone and nitric acid.
3. Mechanical action due to a rapid movement of the corona streamers.
4. The formation of conducting paths by chemical action and carbonization.

The dielectric strength of air in the form of very thin spacings or films *between conductors* has been studied by a number of investigators†; it has recently been treated very thoroughly by F. W. Peek, Jr. and the resulting laws and data are published in several papers and in his book‡.

It is the object of the present investigation to determine experimentally if the laws and data referred to apply when the air films are not between metallic conductors, but exist as pockets in the solid insulation. For this reason, in the tests which will be described later, the break down strength of air was measured on films between glass plates and between insulated conductors.

The data on air films between metal surfaces referred to above will first be discussed.

Air Films Between Metal Surfaces. The dielectric strength of air is the voltage gradient or the volts per unit distance which must be reached before the spark-over of an air space in a uniform electric field can occur. For the thin films, or small spacings under consideration here, the field may be considered as uniform.

The dielectric strength of air at large spacings is proportional to the air density, which varies directly with the pressure and inversely as the absolute temperature. If the air density at a temperature of 25 deg. cent. and a barometric pressure of 76 cm. is taken as unity, the relative air density δ at other temperatures and pressures may be expressed in terms of it, thus:

$$\delta = \frac{3.92\, b}{273 + t} \qquad (1)$$

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†Harris J. Ryan, TRANS. A. I. E. E., 1911, Vol. XXX, Part I, page 8 Fig. 6, giving the Baille-Paschen-Schuster results.

‡F. W. Peek, Jr., TRANS. A. I. E. E., 1913, Vol. XXXII, Part II, pages 1335 to 1355, *Law of Corona and Dielectric Strength of Air.*

F. W. Peek, Jr., In his book "Dielectric Phenomena in High-Voltage Engineering", Chap. III on Visual Corona, Chap. X on Design of Apparatus where Solid, Liquid and Gaseous Insulations enter in Combinations.

where b = barometric pressure in centimeters,
t = temperature in degrees centigrade.
If G_0 = dielectric strength of air in effective volts per mil in a uniform electric field at 25 deg. cent. and 76 cm. barometric pressure, and
G = dielectric strength of air in effective volts per mil in a uniform electric field at temperature of t deg. cent. and barometric pressure of b cm.

$$G = G_0\,\delta \tag{2}$$

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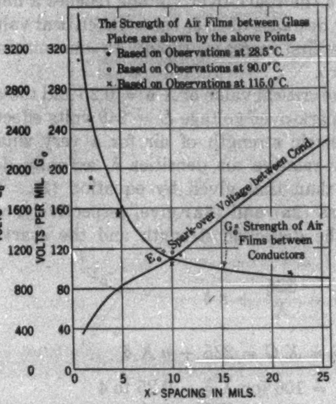

FIG. 1A—DIELECTRIC STRENGTH AND SPARK-OVER VOLTAGE OF AIR FILMS AT $t = 25$ DEG. CENT. AND $b = 76$ CM.

Further, if X = air space in mils, since the field is uniform,

$$E_0 = X\,G_0 \tag{3}$$

$$E = X\,G \tag{4}$$

where E_0 and E are respectively the effective sine-wave spark-over voltages of the air space.

The relation $G = G_0\,\delta$ does not hold for spark-over between parallel plates of small spacings at low air densities. It is used here for simplicity. It is of sufficient accuracy for the range of spacing and air density used in this paper. If greater accuracy is desired the following relation should be used:

According to Paschen's law the spark-over voltage is a function of the product of gas pressure and spark length; the spark-over voltage remains constant when both the gas pressure and the spark gap are altered in such a way that their product does not change.

From the investigations referred to in "Conduction of Electricity through Gases" by J. J. Thompson, *Cambridge Press*, 1906, pages 430 to 528, it follows that the spark-over voltage in gases diminishes as the product $X\,\delta$ of spark length X and relative air density δ decreases, until at a certain critical value of $X\,\delta$ the spark-over voltage reaches a minimum; with still further decrease of $X\,\delta$, below its critical value, the spark-over voltage first increases rapidly to some maximum and then drops to zero.

For air the critical value of $X\,\delta = 0.25$ and the corresponding minimum spark-over voltage $E = 249$ volts effective.

The dielectric strength of air for a very small spacing at various low relative air densities δ, actually decreases at a lesser rate than that given by equation (**2**). The following two formulas (**2a**) and (**4a**) give, respectively, with sufficient accuracy, the dielectric strength and the spark-over voltage for air:

$$G = \frac{225}{X} + n\,\delta \qquad \textbf{(2a)}$$

$$E = X\,G = 225 + n\,X\,\delta \qquad \textbf{(4a)}$$

Where $n = 106$ for $X\,\delta = 0.25$ to 4
$n = 100$ " $X\,\delta = 6$
$n = 96$ " $X\,\delta = 8$

$n = 93$ " $X\,\delta = 10$
$n = 89$ " $X\,\delta = 15$
$n = 82$ " $X\,\delta = 25$

In Figs. 1A and 1B are plotted two curves showing the strength G_0 and the spark-over voltage E_0 for different thicknesses of air films between metal surfaces at 25 deg. cent. and 76 cm. barometer. These data were obtained from the investigation referred to above*; the spacing is expressed in terms of mils to be more conveniently applied.

It would be expected that this strength of air would hold also whenever thin air spacings occur in combination with layers of various solid insulations. In order to apply this data, however, it is first necessary to calculate the stress or gradient on the entrapped air. That this may be simply done is shown below.

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*F. W. Peek, Jr., TRANS. A. I. E. E., Vol. XXX, Part II, 1913, p. 1348.

Calculation of the Stress or Gradient in Air Films in Solid Insulation. Taking a general case when n dielectrics of thicknesses $X_1, X_2 \ldots X_i \ldots X_n$ mils and of relative permittivities $k_1, k_2 \ldots k_i \ldots k_n$ respectively, are arranged in a uniform electric field in series so that the same dielectric flux density passes through each material, the expression for the gradient in any dielectric of the combination of the n insulations is

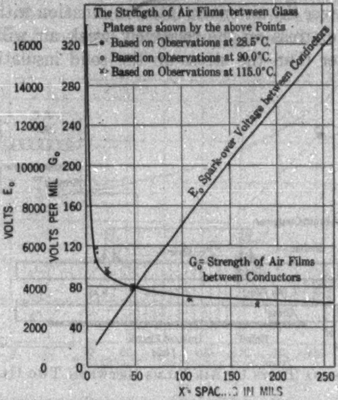

Fig. 1b—Dielectric Strength and Spark-over Voltage of Air Air Films at t = 25 deg. cent. and b = 76 cm.

$$g_i = \frac{E}{k_i\left(\frac{X_1}{k_1} + \frac{X_2}{k_2} + \ldots + \frac{X_i}{k_i} + \ldots + \frac{X_n}{k_n}\right)} \quad (5)$$

where E = voltage across the terminals in volts.

If X_1, g_1 and k_1 are respectively the thickness, gradient and permittivity of the air film, and hence $k_1 = 1$, it follows that

$$g_i = \frac{E}{k_i\left(X_1 + \dfrac{X_2}{k_2} + \ldots + \dfrac{X_i}{k_i} + \ldots + \dfrac{X_n}{k_n}\right)} \quad (6)$$

and $$g_1 = \frac{E}{\left(X_1 + \dfrac{X_2}{k_2} + \ldots + \dfrac{X_i}{k_i} + \ldots + \dfrac{X_n}{k_n}\right)} \quad (7)$$

The latter equation is used to determine the dielectric stress in air whenever the air enters into combination with solid insulation. The stress on the relatively weak air will always be much greater than the stress on the solid insulation. When the dielectric stress reaches the critical value, the air film breaks down with the appearance of visual corona, characterized by a pale violet glow, a hissing noise and the odor of ozone.

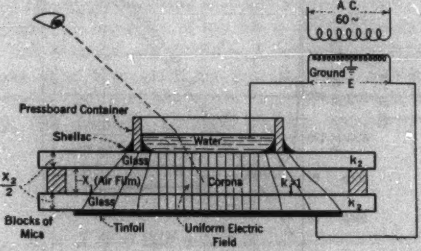

FIG. 2—CORONA TESTS ON AIR FILMS BETWEEN TWO GLASS PLATES

Description of Tests to Determine the Strength of Air Films between Solid Insulations. The corona tests were made with varying air space between two pieces of white French plate glass each 8 in. by 15 in. (see Fig. 2). Small blocks of mica were used to secure the desired space between the glass plates. The tests were made at practically sine-wave voltage, at a frequency of 60 cycles per second.

The lower terminal was a piece of tinfoil 6 in. by 12 in. pressed into intimate contact with the glass. The upper terminal consisted of water held in place by a container of pressboard fastened to the glass with shellac. Water was used as the upper electrode so that the start of corona could be readily observed through the water and the glass plate. The air film did not make contact with either electrode, but did with the glass plates.

The tests were made first at room temperature and then at higher temperatures by the use of an oven. In the latter case the same arrangement was used with the exception of the top terminal, to which calcium chloride was added in order to raise the boiling point of the water.

The voltage was very gradually increased and the start of corona was observed in a dark room by looking through the water terminal and the upper glass plate. The break-down point, as indicated by the pale violet glow in the air film, could be very accurately determined.

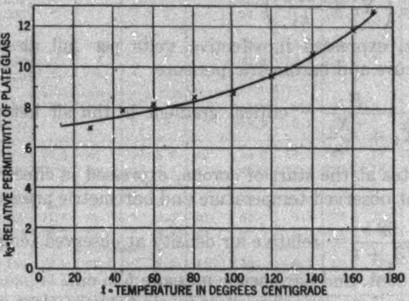

FIG. 3—VARIATION OF PERMITTIVITY OF PLATE GLASS WITH TEMPERATURE

This interesting experiment was suggested by Mr. F. W. Peek, Jr.

The observed data and the values calculated therefrom are given in Table I where the following designations are used:

X_1 = air space in mils,
X_2 = average thickness of both glass plates,
E = effective sine wave volts between terminals,
b = barometric pressure in cm.,
t = average temperature in deg. cent. of glass plates and air space,

k_2 = relative permittivity of the plate glass. In Fig. 3 is plotted a curve giving the relation between the relative permittivity of the plate glass (used in the above tests) and the temperature from 25 to 170 deg. cent. The data for this curve were obtained from capacitance measurements, on one of the 8 in. by 15 in. glass plates. A sheet of tinfoil 6.625 in. by 13.625 in. was pasted on each side of the glass plate with dry shellac and smoothed out with a hot iron. The maximum, minimum and average values of thickness in 20 measurements were 0.194 in., 0.160 in. and 0.176 in. Capacity measurements were made with a Hoxie capaciometer.

$$g_2 = \frac{E}{k_2\left(X_1 + \frac{X_2}{k_2}\right)}$$ = gradient in the glass at the start of corona, expressed in effective volts per mil at observed temperature and barometric pressure.

$$g_1 = \frac{E}{X_1 + \frac{X_2}{k_2}}$$ = critical gradient in the air between the glass plates at the start of corona, expressed in effective volts per mil at observed temperature and barometric pressure.

$$\delta = \frac{3.92\, b}{273 + t}$$ = relative air density at observed temperature t in deg. cent. and barometric pressure b in cm.

$g_{1,0} = g_1/\delta$ = critical gradient in air between glass plates at start of corona, expressed in effective volts per mil reduced to 25 deg. cent. temperature and 76 cm. barometric pressure = the dielectric strength of air between glass plates, expressed in effective volts per mil at 25 deg. cent. temperature and 76 cm. barometric pressure.

In the last column of Table I is given

G_0 = dielectric strength of air *between conductors* in effective volts per mil at 25 deg. cent. temperature and 76 cm. barometric pressure which values were taken for comparison from Figs. 1A and 1B for the air spacings X_1 referred to in Table I.

Finally it should be stated that the values $g_{1,0}$ obtained in Table I are plotted in Figs. 1A and 1B along the curve giving G_0. The three symbols represent points based on three different temperatures of observation, (all reduced to t = 25 deg. cent. and b = 76 cm.).

The points, (especially those based on observations at 28.5 deg. cent.), practically coincide with the curves if the relative accuracy due to small spacings between glass plates is taken in consideration.

The results from corona tests on air space between two insulated conductors are given in the lower part of Table I in which case X_2 designates the both sides thickness of the conductor insulation built up of tapings.

TABLE I

CORONA TESTS ON AIR FILMS BETWEEN TWO GLASS PLATES

X_1	E	b	t	X_2	k_2	$\frac{X_2}{k_2}$	$X_1 + \frac{X_2}{k_2}$	g_2	g_1	δ	$g_{1,\delta}$	G_0
1	14,900	73.6	28.5	357	7.2	49.6	50.6	41	294	.96	307	353
2.1	12,300	"	"	"	"	"	51.7	33	238	"	247	250
4.8	8,200	"	"	"	"	"	54.4	21	151	"	157	170
11.0	6,600	"	"	"	"	"	60.6	15.1	109	"	113	106
22.2	6,400	"	"	"	"	"	71.8	12.4	89	"	93	90
48.0	7,600	"	"	"	"	"	97.6	10.8	78	"	81	79
108	10,100	"	"	"	"	"	157.6	8.9	64	"	67	70
179	13,500	"	"	"	"	"	228.6	8.2	59	"	61.5	66
2.0	6,500	72.3	90	357	8.5	42	44.0	17.4	148	.78	190	256
4.8	5,750	71.5	86	"	8.4	42.5	47.3	14.5	122	.78	156	170
10.2	4,810	71.5	87	"	8.4	42.5	52.7	10.9	91	.78	117	107
22.0	4,750	71.5	89	"	8.5	42	64.0	8.7	74	.775	95	91
47.3	5,625	72.2	88	"	8.45	42.3	89.6	7.4	63	.785	80	79.5
107	7,750	72.5	89.5	"	8.5	42	149	6.1	52	.785	66	70.5
179	11,000	72.5	88	"	8.45	42.3	221.3	5.9	50	.785	64	66
2.0	5,370	72.9	114	357	9.3	38.4	40.4	14.3	133	.738	181	256
4.8	4,940	"	114	"	9.3	38.4	43.2	12.3	115	.738	155	170
10.2	3,630	"	120	"	9.5	37.6	47.8	8.0	76	.728	105	107
22	4,125	72.5	116	354	9.4	37.7	59.7	7.3	69	.730	95	91
47.3	4,680	"	117	"	9.4	37.7	85.0	5.9	55	.728	76	79.5
108	7,125	"	114	"	9.3	38.1	146.1	5.3	49	.733	67	70
179	10,000	"	115	"	9.3	38.1	217.1	5.0	46	.732	63	66
Corona Tests on Air Films between Insulated Conductors.												
250	19,000	72.8	25	254	3.2	79	329	18.1	58	.95	61	63
250	15,800	72.8	107	254	3.5	73	323	13.9	48.7	.75	65	63

Conclusions from above Corona Tests. A comparison of the dielectric strength of *air films between conductors,* and *air films entrapped in solid insulation* shows as good agreement as could be expected; in a uniform electric field the phenomenon of start of corona in thin air films between layers of solid insulation apparently follows the same law as between conductors. It is important to show how practically to apply this data. This is done in Part II.

Part II

EFFECT OF AIR FILMS, AIR BUBBLES OR POCKETS, ENTRAPPED IN SOLID INSULATION UPON THE DESIGN OF ARMATURE COILS OF AIR-COOLED HIGH-VOLTAGE ALTERNATING-CURRENT MACHINES AND UPON THE DESIGN OF CABLES.

General Description of Coil Arrangement and Insulation in Alternators, Motors and Induction Regulators. The armature winding of air-cooled high-voltage alternating-current machines is usually of the barrel type. Three-phase windings are preferably connected in Y with grounded or ungrounded neutral. The form-wound coils are embedded in open slots; there are two coil-sections per slot.

There may be one or more turns per coil as shown in Fig. 4. Occasionally, when the number of turns per coil is large, the coil is wound with several layers and with several turns per layer.

The turns are often wound with several copper strands in multiple. Single-cotton covering, compound treatment, or both are used to insulate the strands so as to eliminate eddy current loss between strands.

The first coil of each phase, being connected to the line terminal, has full line potential to ground (volts between lines $/\sqrt{3}$); the potential of other coils to ground is less and decreases down to zero at the common neutral.

The potential difference between two coils and between top and botton leg of coils depends upon the location of the coils under consideration, and varies between a maximum value equal to the voltage between two line terminals and a minimum value equal to the induced voltage of two coils.

Usually all coils are wound and insulated alike. The insulation of each coil to ground must be designed for the line potential to ground, E, and the total insulation from copper to copper between two coils must stand a potential difference $\sqrt{3}\,E$.

Since there are two thicknesses of insulation between coils it is possible to reduce the thickness of the coil insulation in the end portion of coils to about 87 per cent of the thickness of the coil insulation in the slot portion. The mechanical reenforcement rings supporting the end portion of the winding must then be sufficiently insulated and spaced from the insulated coils so as to give from copper to ground at least the same dielectric strength in the end portion as in the slot portion of the winding. The end portions of the insulated coils are properly spaced for ventilation of the winding, and this spacing helps to reduce the gradients in the coil insulation at these points.

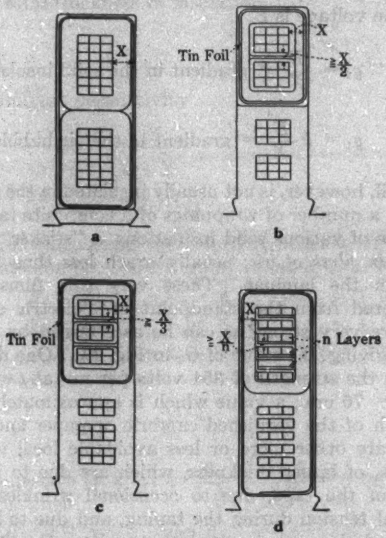

FIG. 4—ARMATURE COILS OF BARREL TYPE WINDING

The coil insulation, being of laminated form, is built up of suitable materials, such as paper and mica, varnished cambric, oiled linen, or micanite, cemented together with some binding sticker and built up to the proper thickness to meet the operating voltage.

Method of Calculating the Stress on the Entrapped Air in Coils with Practical Example. As a simple example, take a coil insulated with a uniform, homogeneous insulation of a thickness X and permittivity k. It is sufficiently accurate to assume the conductor of the coil as a flat plate and the iron core or tinfoil as the other plate. The thickness of the entrapped air bubbles will then be so extremely small compared to X that the capacity or total flux will not be appreciably changed thereby.

If the voltage is E,

$$g_i = \frac{E}{X} = \text{gradient in the solid insulation,}$$

$$g_1 = k\frac{E}{X} = \text{gradient in the air bubbles.}$$

A coil, however, is not usually insulated in the above manner, but by a number of wrappings of a composite tape made up of laminae of various solid insulations, a "sticker", and possibly *very thin films of air*, usually much less than one mil thick, between the laminae. These very thin films need not be considered from the standpoint of dielectric strength, since they are very strong as can be ascertained by examining the curve giving the value of G_0 in Fig. 1A. One mil thickness of air has the strength of 354 volts per mil at $t = 25$ deg. cent. and $b = 76$ cm., a value which is approximately equal to the strength of the varnished cambric or paper and mica taping. There are other more or less avoidable local air bubbles, or pockets, of larger thickness, which are due to imperfect butt joints of the tape, due to occasional wrinkles produced by unequal tension during the taping, and due to bulging of the coil insulation, whose strength requires attention. The very thin air films between laminae must be considered, however, in calculating the resultant permittivity, K, since they reduce the dielectric flux density in the coil insulation.

Calculation of the stresses in any of the insulation materials of the combination may then be made from equation (5),

$$g_i = \frac{E}{k_i\left(\frac{X_1}{k_1} + \frac{X_2}{k_2} + \ldots + \frac{X_i}{k_i} + \ldots + \frac{X_n}{k_n}\right)} \quad (5)$$

For convenience let us substitute for the expression in the parenthesis in the above denominator

$$\left(\frac{X_1}{k_1}+\frac{X_2}{k_2}+\ldots+\frac{X_i}{k_i}+\ldots+\frac{X_n}{k_n}\right)$$

$$=\sum_1^n\frac{X_i}{k_i}=\frac{X}{K} \qquad (8)$$

where X is the total thickness of insulation

$$X=X_1+X_2+\ldots+X_i+\ldots+X_n=\sum_1^n X_i \qquad (9)$$

and K is the resultant permittivity

$$K=\frac{X}{\sum_1^n\frac{X_i}{k_i}} \qquad (10)$$

The method of calculating the resultant permittivity, K is best shown by an example. Let a taping be made up as follows:

$X_1 = 1$ $\quad X_2 = 4$ $\quad X_3 = 6$ $\quad X_4 = 1.5$ mils

$k_1 = 1$ $\quad k_2 = 2.6$ $\quad k_3 = 5.5$ $\quad k_4 = 4.5$

$X = X_1 + X_2 + X_3 + X_4 = 1 + 4 + 6 + 1.5 = 12.5$

$$\sum_1^n\frac{X_i}{k_i}=\frac{X_1}{k_1}+\frac{X_2}{k_2}+\frac{X_3}{k_3}+\frac{X_4}{k_4}$$

$$=\frac{1}{1}+\frac{4}{2.6}+\frac{6}{5.5}+\frac{1.5}{4.5}=3.96$$

$$K=\frac{X}{\sum_1^n\frac{X_i}{k_i}}=\frac{12.5}{3.96}=3.15$$

The resultant permittivity is calculated in Table II for several insulation units. It will be seen that the resultant permittivity, K, of the coil insulation with reference to a temperature of 25 deg. cent. is about 3 to 3.5 for the various tapings and about 4 for micanite.

The resultant permittivities, K, in Table II apply for a

TABLE II

RESULTANT PERMITTIVITY $K = \dfrac{X}{\sum_{1}^{n} \frac{X_i}{k_i}}$ OF COIL INSULATION-UNITS BASED ON PERMITTIVITIES k_i OF DIELECTRICS AT A TEMPERATURE OF 25 DEG. CENT.

Insulation unit	Composed of materials	X_i Thickness in mils	k_i Permittivity at 25 deg. cent.	$\frac{X_i}{k_i}$ at 25 deg. cent.	$\frac{X}{\sum_{1}^{n} \frac{X_i}{k_i}} = K$ at 25 deg. cent.
One-half lap	Paper	4	2.6	1.54	$\frac{12.5}{3.96} =$
taping with	Pasted Mica	6	5.5	1.09	
(.005)" = .001 + .003 + .001	Sticker	1.5	4.5	.33	= 3.15
Paper-Mica-Paper Tape	Air	1	1.0	1.0	
	Total	12.5		3.96	
One-half lap	B. V. C.	24	3.8	6.30	$\frac{27}{7.66} =$
taping with	Varnish	2	5.6	.36	
(.012)" Black Varnished	Air	1	1.0	1.00	
Cambric Tape	Total	27		7.66	= 3.52
One-half lap	B. V. C.	16	3.5	4.57	$\frac{19}{5.93} =$
taping with	Varnish	2	5.6	.36	
(.008)" Black Varnished	Air	1	1.0	1.00	
Cambric Tape	Total	19		5.93	= 3.2
One-half lap	Cotton Tape	10	2.6	3.84	$\frac{15}{5.55} =$
taping with	Varnish	4	5.6	.71	
(.005)" Cotton Tape	Air	1	1.0	1.00	
+ 2 Coats of Varnish	Total	15		5.55	= 2.7
One butted taping	Cotton Tape	5	2.6	1.92	$\frac{9.5}{3.13} =$
with (.005)" Cotton	Varnish	4	5.6	.71	
Tape + 2 Coats of	Air	.5	1.0	.50	
Varnish	Total	9.5		3.13	= 3.03
Micanite			4.0		
Horn Fibre, varnished			2.8		
Press Board, varnished			3.7		

temperature of 25 deg. cent.; at higher temperatures these values become greater, because the permittivities of the solid dielectrics increase with temperature.

From equations (8) and (5) we obtain now

$$g_i = \frac{E}{k_i \frac{X}{K}} = \frac{K}{k_i} \frac{E}{X} \tag{11}$$

and for air $k_i = k_1 = 1$ and $g_i = g_1$

$$g_1 = K \frac{E}{X} \tag{12}$$

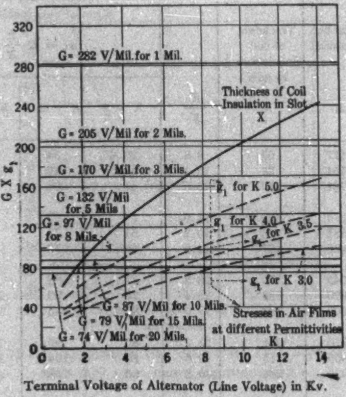

FIG. 5—INVESTIGATION OF START OF CORONA IN COIL INSULATION SHOWN BY A TYPICAL CURVE X

X = Thickness in mils of coil insulation in slot portion (from copper to ground).
g_1 = Dielectric stress in effective volts per mil of entrapped air pockets
G = Dielectric strength of air films at 100 deg. cent. expressed in effective volts per mil

where g_1 = dielectric stress or gradient in air bubbles or pockets, expressed in effective volts per mil;
K = resultant permittivity of the coil insulation;
E = effective sine wave voltage between line terminal and core or grounded tinfoil, expressed in volts = line voltage $/\sqrt{3}$;
X = thickness of coil insulation in slot portion measured from copper to core or grounded tinfoil and expressed in mils;

In case of composite coil insulation which is made up of several layers of different kind of insulation—units, as layer A next to the coil, then B, then C, having layer thicknesses X_A, X_B, and X_C and resultant permittivities K_A, K_B and K_C respectively, the resultant of the resultant permittivities is

$$K = \frac{X}{\frac{X_A}{K_A} + \frac{X_B}{K_B} + \frac{X_C}{K_C}} \tag{13}$$

TABLE III

DIELECTRIC STRESSES IN AIR FILMS IN COIL INSULATION SHOWN BY CURVE X IN FIG. 5.

Line Voltage	Potential to ground	Thickness of coil insulation in mils		Calculated gradients in volts per mil in air films of coil insulation $g_1 = K\frac{E}{X}$ for			
$\sqrt{3}E$	E	X	$\frac{E}{X}$	$K = 3$	$K = 3.5$	$K = 4$	$K = 5$
1,000	577	60	9.6	29.0	34	38	48
2,000	1,154	88	13.1	39	46	53	66
3,000	1,731	110	15.8	47	55	63	79
4,000	2,310	128	18.0	54	63	72	90
5,000	2,885	143	20.2	61	71	81	101
6,000	3,460	157	22.0	66	77	88	110
7,000	4,045	170	23.8	71	83	96	119
8,000	4,620	182	25.4	76	89	102	127
9,000	5,200	193	27.0	81	95	108	135
10,000	5,770	204	28.3	85	99	113	142
11,000	6,350	215	29.6	89	104	119	148
12,000	6,930	225	30.8	93	108	123	154
13,000	7,500	233	32.2	97	113	129	161
14,000	8,090	241	33.6	101	118	135	168

where K_A, K_B and K_C are obtained from equation (**10**) or from Table II for the corresponding insulation-unit in the layer A, B and C respectively.

In Fig. 5 is given a typical curve, X, showing the thickness of coil insulation in the slot portion of alternator windings—in relation to the terminal voltage (line voltage).*

In Table III this curve is analyzed with respect to the dielectric stresses in air pockets in the coil insulation, by applying equation (**12**) and using the resultant permittivities $K = 3$, 3.5, 4 and 5 respectively.

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*See Standard Handbook for Electrical Engineers, Fourth Edition, Section 7, Alternating-Current Generators and Motors by Comfort A. Adams and Henry M. Hobart, page 484, Fig 65.

The stresses g_1 in air bubbles for different resultant permittivities, K, are shown in the relation to the terminal voltage by the broken curves in Fig. 5.

In Fig. 6 is reproduced the curve of dielectric strength G_0 of air at $t = 25$ deg. cent. and $b = 76$ cm. in relation to spacing, (see also Fig. 1A), and therefrom is derived—by applying equations (**1**) and (**2**)—the curve of dielectric strength G of air films at $t = 100$ deg. cent. and $b = 76$ cm.

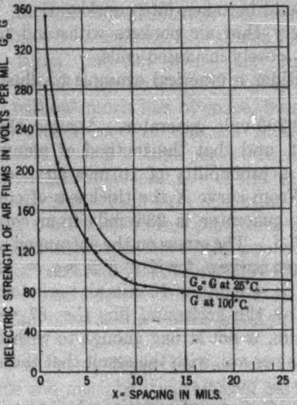

Fig. 6—Dielectric Strength of Air Films at $b = 76$ cm. and $t = 25$ and 100 deg. cent. Respectively

The following values of dielectric strength, G, of air at $t = 100$ deg. cent. and $b = 76$ cm. for spacings;
$X_1 =$ 1, 2, 3, 5, 8, 10, 15, 20 mils; namely $G =$ 282, 205, 170, 132, 97, 87, 79, 74 volts per mil are shown by lines parallel to the abscissa axis in Fig. 5 and *their intersections with the broken curves g_1 indicate the starting points of the corona* in the entrapped air bubbles or pockets.

As stated above we have to consider the strength of entrapped air bubbles or pockets of larger thickness only, namely of such air bubbles or pockets which are due to imperfect butt joints of the tape, due to occasional wrinkles produced by unequal tension during the taping, and due to bulging of the coil insulation.

Since air has the lowest value of both permittivity and dielectric strength among the insulating materials which enter into the combination of the coil insulation, we should keep the dielectric stress of those air bubbles or pockets below the critical values which are plotted in Fig. 6 in relation to the air spacing. Both temperature and barometric pressure (altitude) of the location affect the strength of air pockets entrapped in coils, and should be taken into consideration. Tightly wound coils with very thin air pockets withstand higher dielectric stresses than loosely insulated coils.

Let us consider a practical example to illustrate the use of Fig. 5.

Take a 13,200-volt generator. Assume that $K = 3.5$ at 100 deg. cent. and that the method of manufacture is such that there is a probability of 10 mils air pockets in the coil insulation. From curve X the thickness of coil insulation for a 13,200-volt generator is 235 mils from copper to core or grounded tinfoil. The stress on the entrapped air for $K = 3.5$, from the broken curve g_1 for $K = 3.5$, is $g_1 = 113$ volts per mil.

The dielectric strength of 10 mils air pocket at 100 deg. cent., as indicated by the horizontal line $G = 87$ volts per mil for 10 mils, air film, is not strong enough to withstand a stress of $g_1 = 113$ volts per mil, with the result that there will be corona in the 10 mils air pockets.

If the method of manufacture can be improved so that the air bubbles in the coil insulation are all reduced to about 5 mils or less, there will be no internal corona, since the strength of a 5 mils pocket is then 132 volts per mil, as indicated by the horizontal line $G = 132$ volts per mil for 5 mils air film.

The above can, of course, also be calculated for any coil insulation directly from equation (**12**) and the conclusion reached by inspection of Fig. 6.

It should be borne in mind that we have considered so far the dielectric stresses in air in a uniform electric field, which is nearly the case for the sides of the coils adjacent to the core. At the edges produced in the slots by the air ventilating ducts in stator iron and at the edges of the coils, layers and turns, there are some concentrations of the dielectric field which produce somewhat higher local stresses on the coil insulation and the occluded air pockets than at the location of the uniform field.

The edges of stator ducts and stator teeth may produce external corona and cause pitting of the coil insulation at the coil surface. This danger can be guarded against by providing a grounded tinfoil armor extending 1½ to 2 in. at each end beyond the slot portion of the coil. The coil insulation must be properly reenforced under and above the edges at the ends of the tinfoil armor.

The edges of layers and turns are locations of increased stress for the internal coil insulation between layers and turns. The problem is comparatively simple for coils which have but few turns per coil with only one turn per layer. Here the turns may be heavily taped, as shown in Figs, 4B and 4C without much loss of space, because the turn insulation constitutes a part of the coil insulation. It is recommended that coils with n turns per coil receive an insulation between turns equal to at least $1/n$ of the thickness of the coil insulation to ground. Such turn insulation will stand up indefinitely under normal operating conditions and will also withstand occasional line surges when they penetrate the winding. It is in most cases the insufficient turn insulation that, after comparatively short operation, ages and deteriorates due to action of internal corona, and causes break down usually in the line coil.

Coils wound with several layers and several turns per layer are more delicate and must be insulated yet more liberally to withstand safely a long time service. Such coils are usually filled with compound before the insulation is applied; the purpose of this treatment is to eliminate so far as possible all the air between turns and layers inside the coil.

In conclusion it is evident that the dielectric stress on the air bubbles in solid insulation of armature coils should be calculated and kept below the dielectric strength by using suitable thickness of insulation for the required operating voltage and by applying the insulation as tightly as possible. The internal insulation of coils between layers and turns should be adequate and selected in proper relation to the coil insulation to withstand the increased dielectric stresses at the edges of layers and turns and to withstand the line surges.

General Description of Insulation in Cables and Leads. The insulation of cables and leads is also of laminated form, being built up of paper or varnished cambric tape, and occludes also thin air bubbles or pockets. The latter are, of course, much thinner than those in the coil insulation, because the taping of the cylindrical conductor can be done with uniform tension; and in the case of cables the taping is applied with a machine.

The dielectric problems with which we will deal here are, therefore, similar to those encountered in the coil insulation, except that the electric field in the cable insulation is not uniform, but radial,—as shown in Fig. 7.

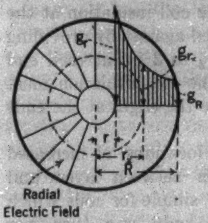

Fig. 7—Single-Conductor Cable with Lead Sheath

Let

r = radius of wire in mils,

r_x = any radius of the insulation around the conductor in mils,

R = inside radius of the lead sheath in mils,

E = effective sine-wave voltage between the conductor and the grounded lead sheath,

g_r = gradient in effective volts per mil on the cable insulation at radius r,

g_{r_x} = gradient in effective volts per mil on the cable insulation at radius r_x.

g_R = gradient in effective volts per mil on the cable insulation at radius R.

The following equations hold for the gradients at different points in the radial electric field of a homogeneous insulation:

$$g_r = \frac{E}{r} \frac{1}{\log_e \frac{R}{r}} \tag{14}$$

$$g_{r_x} = \frac{E}{r_x} \frac{1}{\log_e \frac{R}{r}} \tag{15}$$

$$g_R = \frac{E}{R} \frac{1}{\log_e \frac{R}{r}} \tag{16}$$

If K is the resultant permittivity of the cable insulation, we may determine the dielectric stresses $g_{1,r}$, g_{1,r_x}, $g_{1,R}$ on occluded air pockets at different points in the radial electric field as follows:

$$g_{1,r} = K g_r = K \frac{E}{r} \frac{1}{\log_e \frac{R}{r}} \tag{17}$$

$$g_{1,r_x} = K g_{r_x} = K \frac{E}{r_x} \frac{1}{\log_e \frac{R}{r}} \tag{18}$$

$$g_{1,R} = K g_R = K \frac{E}{R} \frac{1}{\log_e \frac{R}{r}} \tag{19}$$

The gradient g_i, r_x, in any dielectric of permittivity k_i at any radius r_x, can be calculated from the following equation:

$$g_{i,r_x} = \frac{K}{k_i} \frac{E}{r_x} \frac{1}{\log_e \frac{R}{r}} \tag{20}$$

In case that the cable insulation is made up of one insulation-unit the K for equations (**17**), (**18**), (**19**) and (**20**) can be obtained with sufficient approximation from equation (**10**) or from Table II for the corresponding insulation-unit.

In case that the cable insulation is graded by using several layers of different kind of insulation-units, as layer A next to the conductor of radius r, then B, then C, having outside radii R_A, R_B and R_C and resultant permittivities K_A, K_B and K_C respectively, the resultant of the resultant permittivities, K, for equations (**17**), (**18**), (**19**) and (**20**) is

$$K = \frac{\log_e \frac{R_C}{r}}{\frac{1}{K_A} \log_e \frac{R_A}{r} + \frac{1}{K_B} \log_e \frac{R_B}{R_A} + \frac{1}{K^C} \log_e \frac{R_C}{R_B}} \tag{21}$$

where K_A, K_B and K_C are obtained from equation (**10**) or from Table II for the corresponding insulation-unit in the layer A, B and C respectively.

The gradient $g_{1,r}$ on the air pockets at the conductor radius r has the maximum value because it occurs in the densest electric field.

From equation (**17**) it will be seen that the occluded air is subjected to a higher dielectric stress than the solid insulation. Because the dielectric strength of the air bubbles is lower than the dielectric strength of the solid insulating materials which enter into the combination of the cable insulation, we should, to avoid corona, keep the dielectric stress, $g_{1,r}$ on the air pockets, below the critical values which are plotted in Fig. 6 in relation to air spacing.

Conclusions from the Applications in Part II. To avoid corona in the insulation of armature coils, cables and other high-tension apparatus operating in air, it is necessary that under normal operating conditions the dielectric stress on air is below the dielectric strength of such air spaces, air bubbles or pockets which cannot be avoided in the process of manufacture.

A solid insulation which is not subjected to corona will show not an abnormal deterioration with time but will preserve practically its full dielectric strength and enable it to withstand exceptional high-voltage stresses of very short duration, caused by surges and lightning.

This discussion does not cover, of course, the phenomenon generally known as "aging", that is the fatigue of solid insulations with time due to slow oxidation in the atmospheric air, or from effects of heating and dielectric stress below corona starting point.

It is not within the scope of this paper to deal with the difference in action between air films or thin pockets which have a practically uniform dielectric field and air bubbles or air spaces of such a shape as to distort the field. The latter may introduce additional phenomena as lateral stresses at the surface of division of air and taping.

DISCUSSION ON "IONIZATION OF OCCLUDED GASES IN HIGH-TENSION INSULATION" (SHANKLIN AND MATSON) AND "THE DIELECTRIC STRENGTH OF AIR FILMS ENTRAPPED IN SOLID INSULATION AND A PRACTICAL APPLICATION OF THE PROBLEM FOR ALTERNATOR COILS AND CABLES" (DUBSKY), NEW YORK, N. Y., FEBRUARY 21, 1919.

Henry G. Reist: Injury from corona is an old trouble with us that is continually taking new forms. I remember many years ago the engineers in England and the continent, had very serious trouble from internal corona, about which we knew nothing in this country. It has been the practise of American manufacturers to use form-wound compound filled coils ever since large high-voltage generators were built. In Europe, on the contrary, the conductors were drawn through tubes of paper or micanite, and no filling was used, leaving a very considerable amount of air around the conductors. It was around these conductors that the internal corona was experienced, and the insulation on the individual turns of the conductors was soon destroyed and the machines were short-circuited.

As far as my experience goes, the first trouble experienced with corona was outside of the insulation on the coils. This appeared on machines usually in certain places, more particularly perhaps when they were located in high altitudes. But now we are coming to a new kind of corona trouble, that is, corona within the walls of the external insulation. Due to filling the coils, there is very little corona next to the conductors.

Corona in the external insulation, seems to have been increasing in recent years, probably due to the fact that we are making larger coils than formerly, the present coils being two, three or more inches in depth, with straight sides, on which it is difficult to apply the insulation so that it will lie flat on the surface. There is always a tendency for the insulation to bulge, due to the springing of the material. This tendency is probably greater with mica than when paper or cloth is used, because of the natural elasticity of the mica. If the section of the coils were round, there would be no difficulty in applying the insulation closely.

Insulated coils are usually moulded, and when new the external insulation is very compact, but most of the binding materials soften under heat and will release the insulation from the coil so that air pockets may be formed. These are largely prevented in the slots by the walls of the slot not allowing the insulation to go very far. At the ends it is not so easy to avoid this and prevent pockets from forming.

If we could keep the thickness of the spaces down to 0.003 in. or 0.004 in., we would apparently be free from troubles, as pointed out by the writers of these papers, but it is not always possible to do this.

As a practical remedy, we may use materials that are least affected by the corona, such as, mica and certain varnishes, some of which are much more resistant than others. In this way, paper and cloth may be protected from the acids which are formed by the corona.

L. W. Chubb: The formation of corona in the air pockets has, of course, been known for years, and the endeavor of the manufacturer has always been to reduce this trouble by either confining the insulating material to low specific capacitance, by mechanical means, by filling the coils so that they will not breathe and by using insulation that will not be injured.

Years ago, in considering the problem, it was felt that materials having low specific inductive capacitance and the necessary dielectric strength would be rather hard to find, especially those that would stand the higher temperature, so that the problem was attacked by improving the mechanical methods of insulating coils. The mica is very desirable because it can stand the high temperature. It is, of course, attacked by nitric acid, and there are cases where pitting has been produced.

The way that the problem was solved to a great extent was to put the mica on hot, and by a machine which has ironing action and puts the wrapper in circumferential tension. This operation eliminates all air pockets and by the heavy pressure and ironing action removes all bond except that necessary to fill the irregularities of the voids.

Mr. Reist has told about the great trouble with the flat sided-coil. This is usually very much greater with mica. The machine wrapped coil, however, is so solid and can be made so true to dimensions that when, properly gaged and pressed in, will be supported by the sides of the slot, the tension with which it was wound will keep it from bulging at the corners and edges, where there is no support and a chance to bulge.

Mr. Reist spoke of the end of the coil not being supported. That is true but the end of the coil has not as much potential gradient, because of no grounded material outside.

In Mr. Dubsky's paper (second paragraph), a list of troubles is given. The first one is: "Heating due to the losses in corona". This is not so great at commercial frequencies. At high frequencies it is quite important, and protection must be obtained so that local heating will not be produced with high-frequency troubles. The next item in the list of troubles is: "Chemical action due to the formation of ozone and nitric acid". Nitric acid is the more important. Its effects can be greatly reduced by the filling of coils. If a coil is not allowed to breathe, the troubles, even though occluded air is broken down, will not be very serious, and the corrosion will be very slight. The next trouble is indicated as being due to, "Mechanical action due to a rapid movement of the corona streamers". I do not understand just what is meant by this and would like the author to explain it. The fourth reason given is: "The formation of conducting paths by chemical action and carbonization." It seems to me, that fourth item is an advantage rather than a disadvantage. If we have an air film the surfaces of which becomes conducting, the space between will be subjected to no potential gradient. Such conducting surfaces will be an equipotential surface and remove all stress from the entrapped air.

The method of using the water for the upper electrode for observing corona was quite ingenious. It seems as though Dr. Whitehead's schemes of recording the breakdown of the air could be used to get very much closer indications of the start of corona.

John B. Whitehead: Both of the foregoing speakers have emphasized the importance of the subject of these papers, but it is noticeable that neither of them has made any comment on the material presented. While agreeing entirely as to the importance of the subject, I, too, find it very difficult to make specific comment. The explanation lies, in my opinion in the unfortunate manner in which the material is presented.

Both papers deal with the presence of air spaces in insulation and each discusses the subject as though these air spaces existed in the form of thin films. Neither paper presents satisfactory evidence of the existence of such air films and if, as seems probable, air is entrapped in insulation, it is quite certain that it does not exist in the shape of uniform films. On the contrary, it is probably in the form of bubbles of highly irregular shape and size. It appears to me therefore unwarrantable to reason as to the permissible electric intensity in insulation from the premises assumed by the authors.

The late Professor Rowland used to conduct a journal meeting, in which the graduate students reported on current articles in physics. The painstaking student would often undertake an exhaustive description of all the subject matter of a paper. Rowland was anxious only for the results presented and would exclaim with impatience "Stop all that, tell me what he did and not what he thinks." While the anecdote may over stress the point to be made, I think it may fairly be said that the authors of the present papers have extended their speculations, as based on the assumptions of uniform thin films, to a point where the actual results they have to present are considerably confused. There is an increasing tendency, in the presentation of papers to the Institute to reduce to an extreme minimum, sometimes only to the drawing of a curve, the statement of the conditions of observation and accuracy of experimental results. The criticism applies not only to the papers before us, but to numerous others. What is wanted in a paper describing experimental observations is a simple statement of the problem, the conditions of observation, a reasonable account of the observations themselves, showing their accuracy, and a description of conclusions that may fairly be drawn from the experiments.

Returning to the subject of the papers, there are two obvious methods of investigation which would appear to offer promise of useful result: first, insulate a number of armature conductors in as many different ways as can be thought of for the exclusion of air, then measure the losses in different types of insulation by the very interesting methods described in the Shanklin and Matson paper; second, if it can be found that gas and air are actually present in the insulation in the form of films, prepare insulated conductors in which films of known thickness are actually included. Measurements may then be made on this type of conductor also.

In the face of the painstaking and earnest work that has been put on these papers, it is perhaps ungracious to appear to criticize them. The spirit, however, is intended to be constructive and to make an earnest plea for a more scientific method in our experimental work and in the form of its presentation.

C. A. Adams: I think all of those present who have been concerned with research work in any thorough and careful way will appreciate the force of what Major Whitehead has said. In every experimental research we start out with some problem to solve, certain definite results we wish to obtain, and with some recognition of the variables involved. If we analyse the results carefully as the work progresses we will usually find some inconsistency between the results and our theory, which takes account of only those variables previously considered as significant. We are thus forced to take account of other variables, possibly of some not previously even recognized. In fact the chief value of a piece of research is dependent very largely upon the insistence on knowing the reason why; the digging down deeper and deeper, after each step is made, into the theory of the problem, introducing one by one additional factors until we have been able to explain completely all the observations that we have made.

I do not like to take up your time to discuss at any length one of my hobbies, one to which I have been devoting a large amount of time for the past year or two; but I cannot resist the opportunity to emphasize briefly the importance of co-operative research.

In almost any industrial field you will find several researches being carried on in several places by different men, all overlapping more or less, and frequently none of them comprehensive or thorough. It often happens in research work that a suggestion made by another mind, with a slightly different point of view, may throw important light on the subject which at a particular juncture will turn the direction of the research at right angles, or more, and finally result in something which is really worth while; while the one-man researches are in many cases incomplete, misleading and wasteful. If all those interested would only get together, divide up the work, meet periodically to discuss results and get the reaction not of one, but of several able minds, we might solve many of our problems in one tenth or less of the time now required and at much less expense. This field of insulation is one of the most important of them all, and because of its breadth and depth has been cultivated only in small patches. It is therefore one which promises the most from comprehensive cooperative treatment.

W. J. Foster: After Dr. Whitehead's remarks the thought that came to me, was the difficulty in determining in advance by any laboratory methods what is going to be the effect of usage on different insulations. That is the great difficulty that we are laboring under. We are now getting experience, more and more of it, and we can in the case of any new problem, where we have a difficult machine to construct, decide upon certain experiments; we may go so far as to apply mechanical force to our insulated coils so as to distort them somewhat. It is possible to imitate the distortions that we think the coils will be subjected to in assembling, and then apply our tests again, and find out how much the insulation has been weakened. but after all this is done, we may meet with a surprise in the course of two or three years from some actions going on that we did not anticipate.

Such experiences have emphasized the importance of using insulations that will stand high temperatures. We all see that that is a desideratum—that we must, if possible, have insulations that will stand temperatures that will be met with above what we foresee. At the same time we are faced with the effect of heat expansions, and contractions on the life of insulation. I think without any doubt insulations have proven insufficient due to the time action of heating and cooling.

There are a number of points in these two papers that I would like to discuss, but I will not take up the time to speak of many of them, but along the line of usage there is one point in Mr. Dubsky's paper *viz*: where he refers to the turn insulation. He lays down a rule which perhaps is a good guide and may be a good one to follow, but I should limit its application to certain branches of work. He makes the recommendation that the turn insulation be $1/n$ of the external, that is, in the case of a two-turn coil the insulation should be one-half of the external, in the case of three turns, one third, and in the case of twenty turns, one twentieth. Now, there are other factors than the number of turns, as those of us who are practical designers and builders of machinery know, such as the voltage between turns, which, in the case of a machine of any given capacity, say a 30,000 kilowatt turbo-generator is just as high in the case of a 2300-volt machine as in the case of a 13,000-volt machine, and in that case the rule that Mr. Dubsky suggests would prove inadequate, whereas in the case of a much smaller size machine, say 1000 kilowatts, if we had fifteen or twenty turns of small conductor, and an external insulation very thick, the turn insulation prescribed would probably be altogether too much; it would be too costly, and result in a machine of poorer characteristics in other respects.

Henry W. Fisher: I was very much interested in what Dr. Whitehead had to say because it is difficult for me to conceive how there could be air films of the magnitude described by the authors.

In the determination of costs, we have often measured the thickness of paper as we were applying a definite number of wraps and our data show that it would be almost inconceivable to see how air films having thickness of two or three mils could exist.

In applying high-voltage stress to cables insulated with certain compound, we have found that after the voltage reaches a certain point the resistivity which has previously been gradually decreasing may commence to rise again. This action corresponds to polarization with direct current, but why it should occur with alternating current is difficult to conceive.

It seems rather remarkable that the voltage stress at the point of ionization given by the authors should in no case be very far from the average result of 22 kilovolts per centimeter. Calculations which we made in connection with tests conducted some time ago indicate about the same value as above.

It is very important to use the right kind of compound to saturate paper-insulated cables. If the compound is thin, it may be quite easy to fairly fill the insulation, but if the cable happens to be in a slanting position when in operation, the compound may run out through the voids caused by stretching the lead when the cable was installed, thus leaving vacant spots where ionization may take place with disastrous results.

With fluid compound the draining effect may be such as to cause the disintegration or carbonization of the filler in the middle of the cable, or in other places as is pointed out by Mr. Shanklin in a former paper.

It is important to have cables with low dielectric losses but I would a great deal sooner have the cables filled with a compound of proper physical properties even if by doing so the dielectric loss would be somewhat increased.

R. E. Argersinger: We have been having serious trouble from corona on certain generator coils, resulting in breakdowns and service interruptions. In one case of two generators operating side by side, one was insulated entirely with mica and the other with a composite insulation. Both showed very marked corona under normal operating conditions, the mica, as one might expect, showing more than the other, but, as far as the damage went, the mica-insulated machine suffered very little.

On a number of occasions I have heard the statement made that mica was never attacked by corona, and I was therefore somewhat surprised when one of the speakers this afternoon said he had seen mica insulation badly pitted.

I took one of these mica coils where the slot wrapper had been practically destroyed by the action of the corona, and cut out a section of insulation. From this I sliced off layers from the back side so as to get down as close to the slot wrapper as possible to see how deeply the corona had penetrated. To the best of my ability I could only find that the very outside layer of mica had been affected. That had practically gone with the slot wrapper, but the action apparently had not penetrated into the real body of the mica insulation. The varnished cambric, of course, was severely attacked.

C. Francis Harding: Apropos of the suggestion of Dr. Whitehead and Prof. Adams it may be of interest to learn of some of the information obtained in the midst of a research investigation in which the speaker has been interested. This investigation, which we have been carrying on at Purdue University for about two years, is on the fixation of atmospheric nitrogen by the so-called silent discharge method, a method which was reported by Dr. Hugo Spiel, of Germany, preceding the war. The apparatus used is in principle like Dr. Whitehead's corona voltmeter, with the exception that it was so designed that the volume of air, temperature, pressure and the moisture could be controlled and all variables held constant in each instance. This investigation has led to some interesting results.

The work has not been completed, but there are a few things which may be reported, which may be of assistance to others carrying on research investigations along parallel lines.

It is very evident that in studying the yield of the nitrous-oxide gas, or the nitric acid when this gas has been dissolved in water, that the yield varies greatly with the volume of air supplied, the temperature, the pressure and the moisture. That is, if you hold all the variables constant, except one, and vary that, you get a peculiarly peaked curve not unlike the one which Mr. Osborne showed in his telephone paper; so that one critical velocity of the air, the other variables being held constant; one critical temperature, or very small range of temperature, and a very small range of pressure, determine to a very marked extent the maximum yield of the nitrous-oxide and therefore the nitric acid.

Of course, we have worked with very large volumes, but it is reasonable to suppose that in the small volumes of air included in the insulation similar laws would hold.

In attempting to find suitable material for the conduction of these gases to the absorption chambers, and for the absorption chambers themselves, tests on materials which were subjected to boiling nitric acid before they were used, seemed to indicate that certain materials were not readily attacked, while these same materials in this fixation apparatus were very quickly attacked. That led us to believe that the combination of ozone with the nitrous-oxide was a very much more serious attacking agent than the nitrous-oxide or the nitric acid alone. For instance, in making connections between glass or aluminum tubes, using the different insulating materials such as rubber, varnished cambric, etc., many of these, when subjected to the combination of ozone and nitrous fumes, would disintegrate in a very few minutes time, whereas many of them in boiling nitric acid would not be readily attacked.

We noted also that these critical conditions, combinations of temperature and pressure and volume of air confined in the apparatus, would change very suddenly the nature of the gases coming through the discharge chamber. One instant the brown fumes of the higher oxides of nitrogen, would appear, and the next moment one would get a white fog in their place, with apparently no change in the variables themselves. Just what those white fumes are, we do not know, but they are at least one of the indications of a particular combination of variables in the nitrogen apparatus.

It is hoped that during the next year we may be able to show some very definite results from this series of experiments. They have to do, not with the arc process at all, but with the silent corona discharge between conductors.

Ralph W. Atkinson: I am going to confine my remarks particularly to the paper of Messrs. Shanklin and Matson, and more especially to some differences we have found in our results as compared with theirs. In a great measure, our data are quite parallel to theirs.

In the first place, however, I wish to point to some things which I believe are errors in the paper. In Table IV, in column 4, the value given as 1.624 for several samples should be 1.43, with the corresponding changes in the last column. That actually will emphasize the conclusions that are already drawn. These are cables that were said to show ionization at an unusually low voltage gradient. With the above suggested changes, if the figures I have given are correct, the conclusion will be still more apparent; that is the samples already shown to be bad are even worse than indicated. In Table III near the middle of the fourth column, the value 1.396 should be 1.29, which will change the figures in the last column similarly, and which will bring the data on this cable more in line with those on the others in the same table.

The authors show an ionization curve, stating that it is typical. Reference to the paper published in 1917 by Mr. Shanklin in collaboration with Mr. W. S. Clark shows that the large number of ionization curves there given are strikingly different from this typical one. Part of the difference undoubtedly is due to the fact that fewer points were taken on the old curves and thus sharp turns might be shown as rounded.

However, in some cases, the actual points obtained are inconsistent with a curve of the nature now given as typical. Also though I have tested many specimens showing curves similar to this "typical curve," I have tested a large number of specimens showing curves of quite different characteristics.

Three-phase measurements cannot be made with the same facility or accuracy as can single-phase measurements, but important additional information is given by such tests. On account of the convenience single-phase will continue to constitute an important factor in the tests of three-conductor cable, but conclusions seriously in error may be drawn if the three-phase tests are neglected. For example, we have found that when three-phase voltage is applied to a three-conductor cable, in some instances the beginning of ionization may occur at a materially lower gradient than when single-phase voltage is applied between the three conductors and sheath.

I have had occasion to measure at different parts of the cross-section the voltage gradient in a three-conductor cable. The results are very interesting and instructive, and I hope to extend these tests and to make them available for presentation before the Institute: they have a particular application in the conclusions drawn by the authors from data of single-phase tests. Consider the sample No. 7. A cable of this make-up might normally be used on a commercial circuit of 24 to 26 kv. In the paper given in 1917 by Mr. W. S. Clark and Mr. Shanklin, such a cable was mentioned as having operated successfully at 24 kv. for five years. Now it can be shown that the maximum gradient in such a cable operating on a 24 kv. three-phase circuit, is much in excess of 22 kv. per cm. Therefore, accepting the above and the data supplied by the authors and unless the recently tested cables show ionization at a lower stress than the ones described in the older paper, cables have operated successfully at a gradient much in excess of the ionization point.

The data given by the authors are very interesting and instructive; the theory and conclusions drawn must be given weight and consideration and doubtless afford a partial explanation of the facts presented, though too inadequate fully to serve that purpose. In fact, I consider that the data presented when fully analyzed, as well as tests which I have conducted, show distinctly that the method given for determining the voltage rating of a cable does not give results which entirely accord with operating experience.

Delafield DuBois: What I have to say rather supplements the criticism of Mr. Whitehead. I wish to show why it is that the statement by Messrs. Shanklin and Matson on page 495 "Knowing the effective permittivity of the solid insulation and the voltage at which ionization begins, curves can be calculated showing the stress across gas spaces of an assumed thickness" is only true under very limited conditions.

What might be termed the Ohm's law of the dielectric circuit is that

$$D = k\,g$$

or the flux density is equal to the permittivity times the gradient.

If we have solid insulation intercepted by a layer of air of uniform thickness, of large area as compared with its thickness and running normal to the general direction of the flux, the flux lines, at least near the center of this air space, will be unchanged in direction when passing from the solid insulation and through the air, and the flux density in the air will be the same as in the solid insulation. In this case we may write $k_1\,g_1$ for the air $= k_2\,g_2$ for the solid and as $k_1 = 1$ $g_1 = k_2\,g_2$ which is the formula upon which both these papers are based. But this is only true under very ideal conditions. It is not true if the film of air is not normal to the general flux direction, as in this case the flux density in the air will be lower than in the solid material. It is not true if the air film is not of uniform thickness. If for instance the surfaces of the solid insulation are rough the flux in crossing the gap will concentrate between opposing projections, giving gradients in the air much greater than indicated by the formula. Finally it is not true if the air spaces are small in area. In this case so much of the flux is deviated around the edges of the air pocket that even in the center the density is reduced. As an extreme case the gradient, instead of being equal to $g_2\,k_2$ would approach g_2, the gradient in the solid dielectric.

In a cable there may be air spaces that comply with the conditions under which calculations as to thickness are possible, but there are surely also many that do not and these would give false results if measured by this method.

L. W. Chubb: I would like to emphasize what Prof. Harding just said with regard to the critical conditions necessary for the formation of corrosive gases. I recall the accidental formation of ozone, in a test, in such volume that it made several men sick. The structure of the corona member, the conditions of the circuit, etc. were noted. Later, we tried to duplicate the conditions but could get only a very small yield of ozone. We do not know just why but under similar conditions of operation and construction these gases will be formed and corrosive action take place in one case, while in other cases of the same kind no trouble will be experienced.

Prof. Harding also expressed the view that it was combination with the ozone that made the gases active. It is my opinion that it is not the presence of the ozone as much as it is the ionized or nascent condition of the gas formed. Platinum, for instance, will not be attacked by hydrochloric acid gas or chlorine. Platinum as corona electrode, however, in these gases will react and form chloride of platinum.

F. W. Peek, Jr.: Fortunately the universal medium in which we are immersed, air, is an insulation and not a conductor. Unfortunately, however, it has not the highest dielectric strength and has low permittivity. Experience has shown that it is almost impossible to rid certain insulations of air films and bubbles in one form or another. On account of its low permittivity, the unit stress, or gradient, on the air films is usually several times what it is on the solid insulation. The strength of air is constant at a given density for large spacings. For small spacings, however, the unit strength increases very rapidly with decreasing spacings. With small spacings the strength may be as great as that for solid insulations. It is still, however, under the disadvantage of taking more than its share of the stress due to its low permittivity.

The breakdown of the air films does not mean that the apparatus will break down immediately; it may take years. it is undesirable because it causes gradual deterioration of the solid insulation. This deterioration may be due to carbonization, mechanical action or chemical action, depending upon the nature of the insulation. Soft rubber, for instance, is almost immediately ruined by the ozone. It appears almost as if it had been cut. Varnished cambric attacked for several years by corona has a honey-combed appearance of mechanical action. The strength of the air films is thus generally the limiting feature. The stresses must be such that the films do not break down.

Mr. Dubsky measured the strength of air films of different thicknesses between solid insulation. These data are of great scientific interest because he has found that the strength of these films surrounded by insulation is the same as it is between metal electrodes. These data are practically applied by estimating the thickness of the largest films in a given type of insulation and so proportioning it that the stress on the air is below the measured breakdown value.

Messrs. Shanklin and Matson have investigated the problem in a different way. They have measured the losses in the insulation. They find that the losses and the alternating-current resistances increase rapidly at a certain point, apparently when the air films begin to break down. These are important data and the papers fit in well together.

I cannot be pessimistic about insulations. High-voltage cables have not been built principally because of lack of demand. There are other limiting features beside voltage. For instance, the leading wattless kv-a. of a high-voltage cable network would be enormous. Transformer bushings operating at several hundred thousand volts are really short high-voltage cables. Transformers have been built up to almost a million volts.

Comfort A. Adams: First, I want to say that I fear Mr. Peek has misinterpreted my statement, which I intended to be something like this—that in the case of high-voltage armature insulation the actual thickness of mica is such that if it were an absolutely solid, homogeneous air-free mass of mica insulation it would stand up indefinitely under voltages several times, those normally impressed.

Now, just a word in regard to the deterioration of insulation under corona attack. It is very difficult to say how much this is due to chemical action, how much to some kind of corona bombardment and how much to the heat resulting from corona. I wish now to express the opinion that the deterioration in mica at least, is due more to corona heat and corona bombardment, than to chemical action, although the latter doubtless plays some part in certain types of insulating materials.

This opinion is based upon both theoretical analysis and experimental evidence.

Speaking first from the theoretical point of view, there are two factors not at first sight obvious which tend to increase the heat produced by internal corona.

a. High frequency of corona discharge. Consider a thin flat air space perpendicular to the electric flux in an otherwise homogeneous wall of insulation. When the voltage gradient in the air space exceeds the critical point, corona is established, and the heat developed is approximately proportional to the current, other things being equal. But if the air space is very thin as compared with the thickness of the insulation wall, the corona current is the displacement current through this wall and is therefor proportional to the voltage gradient in the wall, the dielectric constant of the insulating material and the frequency. However the frequency is not that of the circuit fundamental but that of the discharge itself, which is of the order of hundreds of thousands of cycles per second, as both the inductance and the capacity of the discharge circuit are very low. Thus the corona displacement current and the heat developed is thousands of times greater than if at fundamental frequency.

Also the resulting high-local temperature lowers the initial voltage at which corona begins and causes the corona to last through a larger part of each half cycle. This accounts for the unexpectedly large losses disclosed by the experiments of Shanklin and Matson.

b. Corona Blast. In an air space of any appreciable thickness and reasonable length, the corona current is deflected by the magnetic field at right angles thereto, and is driven longitudinally in the air space against the end, impinging against any exposed edges of the very thin mica films actually fusing them in some cases. These films are peeled up and fused one at a time, gradually increasing the length and thickness of the space and the violence of the action.

Ample evidence backing up these conclusions has been discovered in the insulation of machines which have been in service less than two years.

Chemical action does certainly take place but as this depends upon the presence of air and moisture the access of either to the interior of impregnated masses of insulation must be so slow as to render it practically harmless. Moreover there is as yet no experimental evidence to indicate that this corona-chemical action affects the mica appreciably even after years of service, although it does affect impregnating compounds.

Finally the problem is very complicated and very important from the practical point of view. There is hardly any field of electrical engineering research where the results are so likely to extend the limits or boundaries of practise.

E. E. F. Creighton: Some time ago in making a study of insulation we arrived at the same conclusion as the authors in regard to air and its disagreeable effects. In making the investigation we found it impossible, in ordinary carbonaceous materials, to get rid of the effect of heat, and its deteriorating influences on carbonaceous material, and we were led to make tests on porcelain in which we could eliminate the factor of carbonization. One can get porcelain of all degrees of porosity by different degrees of firing. With porcelain there can be no carbonization to limit and change the effect of the bombardment of the air in the pockets.

Now, by choosing the different kinds of porcelain, the dry-process being one type, with very long spaces, and the wet-process procelain, with longer or shorter air spaces, according to the degree of firing, it is possible to study the air pockets more thoroughly than can be done with carbonaceous insulation.

Unfortunately, the exigencies of war work required us to discontinue our tests, and it has been impossible to put them into shape for publication. We used an Alexanderson alternator with a frequency of about 200,000 cycles. This high frequency of continuous alternating current heats the porcelain rapidly. There was a marked difference in heating that took place due to the difference in degrees of porosity.

J. J. Matson: Relative to Mr. Chubb's statement on the pitting of mica of which Mr. Argersinger also spoke, I will say, that we also in extreme cases found pitting; but as far as our cases go, and we have not had many, we have not found very much pitting. It has been more often, as President Adams said, a corona blast and fusing. We have found the same thing, the creepage of the corona, but we did not touch on that in the paper, due to the fact that it does not affect the beginning and it was the starting of the corona that we were interested in.

Prof. Harding spoke of the color of the material coming from the insulation. We have found different colors in the corona trouble. That is due to the different complex inorganic substances which you have in the coil or in any insulation. The nitric acid will affect the various complex substances in various ways and give different shades or hues, but we have usually found that holds for a given compound in the same spectrum, that is, it will either fade or brown with age.

W. I. Middleton and E. W. Davis: (communicated after adjournment.) Considerable work has been done both in this country and abroad on the effect of potential stress on insulating materials and all experimenters are convinced that it is possible to permanently injure them by overstressing. Just how much stress we can put upon an insulating material without injury is a matter of much discussion. It is possible to stress a material to a point of apparent hurt but if the stress is removed the material will recover its normal condition.

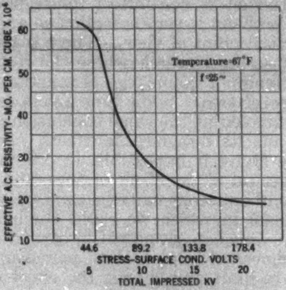

Fig. 1—A-C. Resistivity vs. Volts

6 str. B. & S.—7/32 in. wall paper insulated—lead covered.

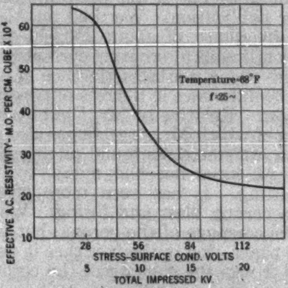

Fig. 2—A-C. Resistivity vs. Volts

2/0 str. B. & S.—9/32 in. wall paper insulated—lead covered

In many cases it is difficult to determine what has taken place as the physical properties of the material show no apparent change and it is only by electrical tests that we can detect the effect of stress. In many cases where heat has been generated the effect is plainly visible.

The authors state that ionization takes place at about 57 volts per mil in a well made cable. We are led to believe we should keep the working stress below this value. Good practise tells us this is not necessary as many paper cables are operating successfully at twice this stress. In fact, if we are to advance in the manufacture of high-tension cables, we must go well beyond twice this value.

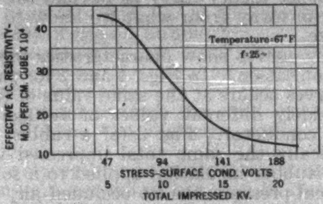

FIG. 3—A-C. RESISTIVITY VS. VOLTS

6 sol. B. & S.—7/32 in. wall rubber insulated—lead covered.

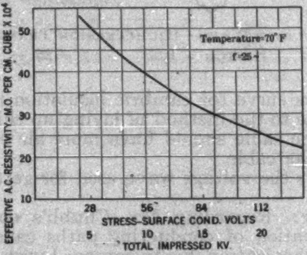

FIG. 4—A-C. RESISTIVITY VS. VOLTS

2/0 str. B. & S.—9/32 in. wall cambric insulated—lead covered.

When we begin to lay down theoretical laws for allowable stresses on cables we obtain results that practise condemns. We must therefore, be careful how far we allow ourselves to be influenced by such laws. For example—there has been in operation for a number of years a 3-conductor No. 2 stranded cable operating at 25,000 volts with a 9/32 in. wall of paper on each conductor and a 4/32 in. jacket. The stress at the surface of the conductor is 108 volts per mil. With these same walls of insulation to keep the stress down to 57 volts per mill, the conductors would have to be 2,000,000 cir. mils, or a 3-conductor No. 2 stranded operating at 25,000 volts would require 46/32 in. or nearly 1 ½ in. of insulating material on each conductor.

We must not let theory carry us beyond the limits of good common sense or away from what we know to be good commercial practise.

The following curves show characteristics inconsistent with the theory of ionization. In Figs. 1 and 2 for paper cables the knee of the curves occurs at the same value of total impressed voltage but widely different values of voltage stress at the surface of the conductor. Fig. 3 for a rubber cable shows a knee at practically the same total impressed voltage as the paper cable. Rubber insulation applied to a solid wire under great mechanical pressure has no occluded air, therefore, this knee could not be due to ionization of occluded gases.

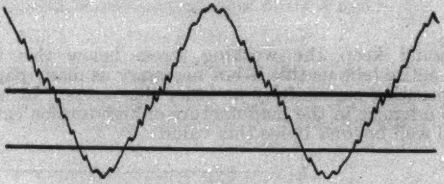

FIG. 5—VOLTAGE WAVE—HIGH-TENSION TRANSFORMER FORM FACTOR = 1.53 $f = 25 \sim$

Fig. 4 shows curve for cambric insulation in which there is no knee. Due to the method of laying up this type of insulation, a cambric cable should have more air entrapped than a well made paper cable.

Fig. 5 shows the voltage wave used for getting the data for Figs. 1 to 4.

F. Dubsky: Regarding Mr. Chubb's comments on the effect of formation of conducting paths caused by chemical action and carbonization due to corona, I wish to point out that when this effect takes place in air bubbles at the flat sides of coils, between the inside wall of the coil insulation and the conductors, it is liable to weaken there the layer and turn insulation.

With regard to Mr. Whitehead's comments I wish to state the following:

The most concerned air bubbles are of comparatively large area, that is of such a shape so as not to distort, practically, the straight line electric field. The thickness of such air bubbles varies, of course, from zero to some maximum thickness and the gradient on air at any point of the air bubble may be determined from the equation

$$g_1 = K \frac{E}{X + X_a K} \qquad \textbf{(12a)}$$

where g_1, K, E and X have the same signification as in equation (**12**) page 515, and X_a is the thickness of the air space at any point of the air bubble.

A lateral dielectric stress can occur only in the narrow air pockets, as in the thread-like spiral air spaces at the supposed butt joint edges of wrapping. In this case the stress lines are partly shunted around through the solid insulation of higher permittivity and thus the dielectric stress in the narrow air space is reduced.

The dielectric strength and the dielectric stress are expressed in this paper in volts per mil rather than in kilovolts per centimeter in order to be more convenient in application for the designing engineer, who is accustomed to express the dielectric strength of insulating materials in volts per mil.

With regard to Mr. Foster's comments on internal coil insulation I would like to emphasize my opinion, that the layer and turn insulation of coils embedded in slots should be determined rather in relation to the insulation to ground than with regard to the normal induced voltage existing between layers and turns. If this is done an internal coil insulation is obtained which will safely meet occasional high-voltage stresses due to surges and lightning.

Presented at the 7th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 21, 1919.

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ABNORMAL VOLTAGES WITHIN TRANSFORMERS

BY L. F. BLUME AND A. BOYAJIAN

Abstract of Paper

Mathematical analysis is made of a rectangular wave impinging upon a transformer winding and quantitative values deduced of the resulting internal voltage stresses in terms of transformer constants. It is shown that the conclusions also apply in part to abrupt impulses and approximate idea is given of the reaction of a transformer to high frequencies. The difference between operating transformer with isolated and grounded neutral is shown. Energy losses are not considered in the mathematics although the manner in which the results are affected is pointed out. Finally, theoretical results are compared with impulse and high-frequency tests made in the laboratory.

I.—Introduction

To avoid breakdown of transformers resulting from abnormal voltages, the present practise is to either safeguard the transformer winding by external protective devices which are designed to prevent line disturbances from reaching the transformer, or to insulate the windings to withstand them. A third but less obvious method is so to design the transformer that voltage disturbances entering the winding will not result in excessive voltage concentration.

By showing in what way the transformer constants influence the voltage concentration this paper indicates how a transformer may be so designed as to mitigate the stresses occasioned by a given voltage disturbance.

II.—Conclusions

Theory and experiment both of which are fully described later in the paper confirm the following conclusions.

1. Transformers differ from transmission lines on account of the presence of coil-to-coil capacitance and mutual inductance.

2. Three kinds of disturbances are recognized:

A—abrupt impulses: B—rectangular waves with long tails: C—high-frequency voltages sustained and damped.

A. Abrupt Impulses

3. The presence of coil capacitance causes the transformer to respond as a capacitance and not as an inductance to abrupt impulses and towards all frequencies* above its lower natural frequencies of oscillations.

4. The voltage distribution within the transformer windings for abrupt impulses and for high frequencies depends upon the factor,

$$\alpha = \sqrt{\frac{C_g}{C_s}}$$

where C_g is the total capacitance of winding to ground and C_s is the total coil capacitance from one end of the coil stack to the other.

5. The greater the value of alpha, the greater the concentration of voltage at the line end and the smaller the voltage in the interior of the winding. The maximum volts per unit length of coil stack at the line end is equal to alpha times the value of the voltage corresponding to uniform distribution. In high-voltage power transformers of disk high-voltage, and concentric-barrel low-voltage windings, the value of alpha varies from 5 to 30.

6. The more abrupt the impressed impulses, or the higher the impressed frequency, the greater is the concentration of voltage at the line ends. The maximum voltage expressed in the preceding paragraph corresponds to a perpendicular wave front or infinite frequency impressed.

7. If the turns are uniformly distributed throughout the windings, alpha is also the factor giving the concentration of the volts per turn at the line end; but on account of extra turn insulation, the usual practise is to considerably reduce the number of turns on the end coils.

As the voltage concentration in the end coils depends upon the value of series and ground capacitances and is independent of the number of turns, it follows that the volts per turn on the end turns is inversely as the number of turns per coil. For this reason it is evident that extra turn insulation will be effective only when it is so designed that the increase in dielectric strength is greater than the increase in voltage concentration.

8. For abrupt impulses and for very high-frequency

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*See paragraph 15.

oscillations it is immaterial whether the neutral is isolated or grounded.

B. Rectangular Waves with Long Tails

9. If the impulse has a long tail, the phenomena within the transformer will be initially as described in paragraphs 1 to 5 above, followed by a series of damped oscillations, the amplitudes of these oscillations depending not only on the amplitude of the impressed wave but also on the value of alpha, and on whether the neutral is isolated or grounded.

10. In addition to being proportional to the value of the impressed wave the amplitude of oscillation for a given harmonic is smaller the higher the harmonic and the smaller the value of alpha.

11. There is no simple relation between wave length and frequency and on that account waves cannot penetrate into a transformer winding without distortion.

12. As these oscillations occur simultaneously within the transformer windings, the resultant voltage distribution at any instant is obtained by re-combining the harmonics at that instant.

13. For isolated neutral and for alpha equal to 10, a maximum of 2.8 times impressed volts exists from neutral to ground and the volts per turn in the interior of the windings is approximately four times normal.

14. For grounded neutral and for alpha equal to 10, a maximum of 1.5 times impressed volts exists from the windings to ground, and the volts per turn in the interior of the windings are increased to four times normal and at the neutral to 7.5 times normal.

C. High Frequency

15. When the applied high frequency is greater than the natural frequency of the transformer, the reaction of the transformer is similar to its reaction towards abrupt impulses, and paragraphs 3 to 8 apply.

16. Theoretically, by impressing a sustained high-frequency voltage across transformer terminals at a natural oscillating frequency of the windings, the transformer can be made to resonate and indefinite voltages built up limited only by internal losses. However, if the transformer is excited by a transmission line, the voltage built up by resonance is also

limited by the relative values of surge impedance of transformer and line.

17. However in the high frequency tests here reported extreme high voltages were not observed. This is probably due to the following reasons:

(a) The impressed high-frequency was highly damped.
(b) The presence of internal capacitances shunting the windings decreases the rapidity with which the voltage is accumulated.
(c) Internal losses within the transformer windings.

18. The effects of (b) and (c) are greater the higher the frequency of oscillation. For this reason high internal voltages are built up, only at the lower frequencies.

III.—Description of Phenomena

The theory of electrical waves in transmission lines cannot be directly applied to transformers due to the fact that a transformer, unlike a transmission line, possesses capacitance between coils and mutual inductance.

Assuming that the inductance and capacitances are uniformally distributed along the winding as diagrammatically represented in Fig. 1, mathematical analysis can readily be applied, but if the constants are bunched as is the case for transformers in which the high and low voltage windings are interlaced, the problem becomes very complicated.

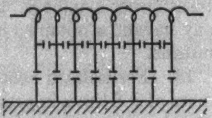

FIG. 1

A good example of a concentric winding transformer to which the conclusions of this report are applicable is given in Fig. 4. This is a three-phase 2500-kv-a. transformer, the 100,000-volt winding of which is Y connected and made up of three stacks of disk coils. It is necessary to assume that the capacitances between turns and layers are very large compared with capacitances between coils and ground to make Fig. 1 applicable.

The purpose of this paper is to show what takes place within the transformer windings under the impact of rectangular waves. Rectangular waves are chosen rather than periodic waves on account of the greater simplicity of the problem, and

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*See *Abnormal Voltages on Transformers*, by J. M. Weed, A. I. E. E. TRANS. 1915, p. 1621.

waves of long duration (long compared with the time of a complete oscillation within the transformer) are assumed because the resulting voltages are greater than with the very short waves or impulses.

In general there will be two cases requiring separate treatment depending upon whether the neutral is isolated or grounded.

Although the phenomena with which we are dealing involves in its entirety only a very small fraction of a second, it is nevertheless convenient to divide it into three distinct time periods, each requiring separate treatment. These will be designated; 1, initial period; 2, transient period; 3, final period.*

The initial period is the period from the beginning of the disturbance to the time of maximum applied voltage. The time involved is so small that there is no appreciable growth of magnetic flux and therefore the phenomena may be considered entirely electrostatic without appreciable error. This is followed by a transient period during which there is a continual interchange of energy between the electric and magnetic fields. The energy interchange usually takes the form of highly damped high-frequency voltage and current oscillations. When the oscillations become negligible the final period has been reached in which the winding has again assumed a permanent condition of equilibrium.

TRANSFORMER REACTING AS A CONDENSER

Due to the combination of coil and ground capacitances, the transformer, during the initial period, reacts as a concentrated capacitance. The condensers are rapidly charged and the voltage across them rises to nearly double the voltage of the incoming wave. The time required for charging is so very small that it is safe to assume that twice the voltage of the incoming wave is applied to the terminals of the transformer windings before an appreciable inductive current can be established in the windings. The equivalent or effective capacitance of the transformer, while being charged, is approximately given by equation (**21**),

$$C_{eff} = \sqrt{C_g \times C_s} \qquad \textbf{(21)}$$

where C_g is the total capacitance of the surface of transformer

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*See paper by Karl Willy Wagner entitled "Progress of Waves in Windings." *Elektrotechnik und Maschinenbau.*" Feb. 21, and 28th, 1915.

winding to ground, C_s is the capacitance between the two ends of the winding. The calculation of the latter capacitance is simplified by assuming that the ends of the winding are electrically connected to clamping plates or end rings. C_s is then defined as the capacitance between these plates.

INITIAL VOLTAGE DISTRIBUTION

The voltage distribution throughout the transformer winding when the condensers are charged is given by formula (**11b**) for isolated neutral, and formula (**13b**) for grounded neutral.

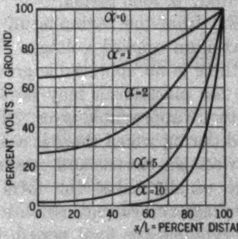


FIG. 2—NEUTRAL ISOLATED

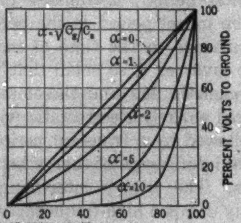


FIG. 3—NEUTRAL GROUNDED

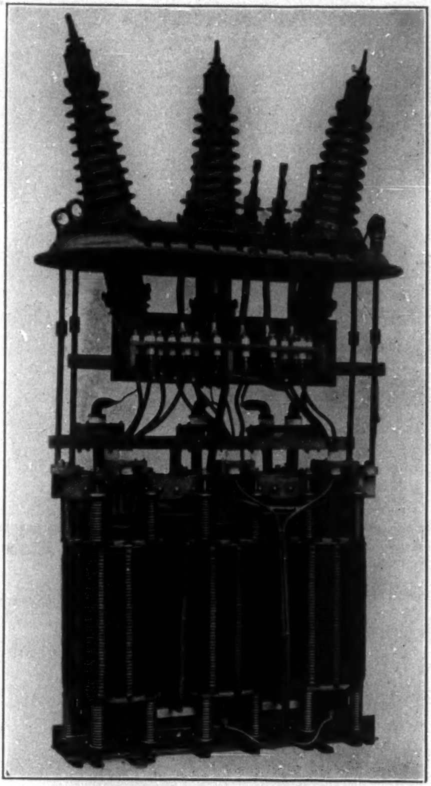
$$e = E_0 \frac{\cosh \dfrac{\alpha x}{l}}{\cosh \alpha} \qquad \textbf{(11b)}$$

$$e = E_0 \frac{\sinh \dfrac{\alpha x}{l}}{\sinh \alpha} \qquad \textbf{(13b)}$$

where E_0 is value of voltage to ground on the transformer terminals. This will be either twice the value of the incoming wave, or, if lightning arresters are assumed, the maximum voltage permitted by the protective device.

The value of alpha is the square root of the ratio of the ground capacitance (C_g) to coil capacitance (C_s).

Plotting the ratio e/E_0 as ordinate and the ratio x/l as abscissa gives the system of curves shown in Figs. 2 and 3. For alpha equals zero, that is, ground capacitance zero, the curve is a straight line (uniform voltage distribution). The



[BLUME AND BOYAJIAN]

FIG. 4—THREE-PHASE—60-CYCLE—2500 KV-A.—100,000-VOLT TRANSFORMER

greater the value of alpha the less uniform becomes the voltage distribution and the more is the voltage concentrated at the end of the winding. This is the reason for the modern practise of putting extra insulation on the coils and turns at the line ends.

It should be noted that these curves are based on windings in which the turns are uniformly distributed along the coil stack. On account of the space required by the extra end turn insulation it follows that there are fewer turns per unit length of coil stack at the ends of the windings. The effect of this is to cause a still greater concentration of voltage at the ends.

FINAL VALUE OF VOLTAGE DISTRIBUTION

(a) Eventually, in case of isolated neutral, the voltage of the winding will be brought to a potential of twice the original value of the wave with zero current flowing.

(b) For the case of grounded neutral the final condition will be a straight line with neutral at ground potential and line at twice the voltage of the original wave, with a current in the winding increasing at a constant rate, (assuming no resistance).

TRANSIENT VOLTAGE DISTRIBUTION

The transient by which the voltage distribution changes from the initial to the final value can be analyzed into a complex series of damped oscillations or standing waves at various frequencies, wave lengths and damping factors.

The maximum values of these standing waves are readily obtained by analyzing the initial voltage distribution as given in curves, Fig. 2, into space harmonics with respect to the final voltage distribution as axis of reference. Thus we have for $\alpha = 10$ the following initial values of the standing waves.

NEUTRAL ISOLATED		NEUTRAL GROUNDED.	
Wave length on winding n	Amplitude in per-cent of line to ground voltage E_n/E_0	Wave length on winding. n	Amplitude in per cent of line to ground voltage e/E_0
1/4	125%	1/2	58%
3/4	34%	3/2	23%
5/4	15%	5/2	6%
7/4	7%	7/2	4%
9/4	3%	9/2	3%

These oscillations at various frequencies occur simultaneously within the transformer windings and the complex wave at a given time is obtained by recombining the harmonics at that time (Fig. 5).

The equation of any standing wave in a transformer winding. is

$$e_x = E_n \operatorname{Sin}\left(2\,\pi\,n\,\frac{x}{l}\right)\cos w_n t$$

or **(33d)**

$$e_x = E_n \operatorname{Cos}\left(2\,\pi\,n\,\frac{x}{l}\right)\cos w_n t$$

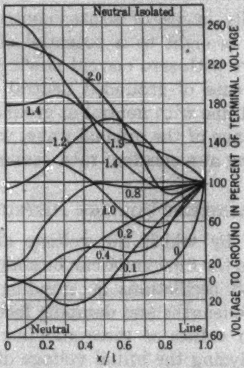


FIG. 5—VOLTAGE DISTRIBUTION WITHIN TRANSFORMER WINDINGS AT VARIOUS TIMES AFTER IMPACT OF RECTANGULAR WAVE

where n is the number of wave lengths per harmonic along the stack. For the fundamental, n is ½ when neutral is grounded, and ¼ when neutral is isolated. The frequency of oscillation is given by the equation:

$$f = \frac{2\,\pi\,n^2}{\sqrt{L\,(C_g + 4\,\pi^2\,n^2\,C_s)}} \tag{35e}$$

These equations are particular solutions of the general differential equations and are derived under the heading of "Mathematical Development."

An inspection of formula (35e) shows that the natural frequencies are a function of the wave lengths. For $\alpha = 10$ the variation of wave lengths with frequency is as follows:

NEUTRAL ISOLATED		NEUTRAL GROUNDED	
Wave length	Relative * frequencies	Wave length	Relative * frequencies
n		n	
1/4	1	1/2	4
3/4	8	2/2	14
5/4	20	3/2	28
7/4	33	4/2	42
9/3	47	5/2	56

*These figures are not accurate due to the variation of inductance with wave length as discussed on another page.

This peculiar relation between frequency and wave length is due to the presence of coil capacitance and mutual inductance, and causes a continual change in the form of the complex wave from instant to instant. This is in marked contrast to the phenomena which take place in the transmission line, where a complex wave of any shape will be propagated without distortion, except for the effect of resistance.

MAXIMUM VOLTAGE TO GROUND AND MAXIMUM VOLTS PER TURN

The considerations of greatest interest in connection with these curves are the maximum voltages to ground and the maximum volts per turn at various points within windings. These are plotted in Figs. 6 and 7. Of course the maximum values of voltages at the various points do not occur at the same instant.

An inspection of these curves shows that, for the case of isolated neutral, the maximum voltage to ground occurs at the neutral and is equal to 2.8 times the voltage from line to ground and for grounded neutral the maximum volts to ground is 1.5 times line to ground voltage. The volts per turn for isolated neutral is 10 times normal at line end, 4 times normal in the major portion of winding, and zero at the neutral. With neutral grounded it is 10 times normal at line end 5 times normal in the major portion of the winding and 7.5 times normal at the neutral.

SURGE IMPEDANCE

In order to have a convenient means of calculating the maximum value of current associated with each oscillation the ratio of terminal volts to maximum current is given by

$$Z = \frac{E}{I} = \frac{1}{2\,\pi\,n}\sqrt{\frac{L}{C_g + 4\,\pi^2\,n^2\,C_s}} \qquad \text{(39b)}$$

The value of surge impedance calculated for the transformer shown in Fig. 4 is 60,000 for the fundamental, 29,000 for the third harmonic, 17,000 for the fifth harmonic and 12,000 for the seventh harmonic.

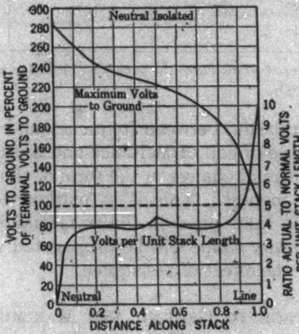


FIG. 6—VOLTAGE DISTRIBUTION WITHIN TRANSFORMER WINDINGS MAXIMUM VALUES AFTER IMPACT OF RECTANGULAR WAVES

In the analysis just given it has been tacitly assumed that the voltage E applied across the terminals of the transformer winding is maintained throughout the oscillations. Strictly speaking, this is not true, because during the time when the voltage to ground is increasing in the interior of the winding, the transmission line is delivering current to the transformer windings, and during the time when this voltage to ground is decreasing, the transmission line is receiving current from the transformer windings, or in other words, there is a continual interchange of energy between the transmission line and the winding throughout the oscillation, and as the ratio of voltage to current associated in the rectangular wave in the transmission line is a constant value, this interchange of energy cannot take place without modifying the line voltage. However, calculations of equivalent surge impedance show that in all practical transformers the amount of energy absorbed and delivered by the transformer windings is very small compared with the energy residing in the rectangular wave, and that, therefore, this fluctuation in voltage is negligible in amount.

Again it may at first appear that the axis of oscillation taken in the case of a grounded-neutral transformer, cannot be correct because the final condition for the case of grounded neutral is zero voltage throughout the windings owing to the fact that the windings become a dead short circuit. This is again theoretically true but it has no bearing on the problem because in all practical cases the time taken for the transformer windings to degenerate into a short circuit is so very long compared with the time involved in the oscillation of the fundamental frequency that it can be safely assumed that the transformer winding oscillates through a complete cycle of the fundamental before the voltage applied to the terminals of the transformer windings can be appreciably lowered.

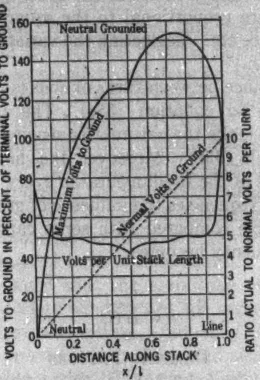


FIG. 7—VOLTAGE DISTRIBUTION WITHIN TRANSFORMER WINDINGS MAXIMUM VALUES AFTER IMPACT OF RECTANGULAR WAVE

NATURAL FREQUENCY OF OSCILLATION OF A WINDING

Formula (**35e**) is useful for the purpose of obtaining the natural frequency of a winding. For a solenoid consisting of a single one-layer coil, the capacitance between turns may be neglected and the formula becomes.

$$f = \frac{2\,\pi\,n^2}{\sqrt{L\,C_g}} \tag{35f}$$

From this equation we reach the very interesting conclusion that the natural frequency of oscillations are not simple multiples of the fundamentals in the order of 1, 2, 3, 4, etc., but that they are proportional to the square of these values that is 1, 4, 9, 16, etc., (neglecting the variation of inductance with wave length.)

If the capacitance between turns or coils is not negligible the formula reverts to equation (**35e**). The influence of coil capacitance is to make the natural frequencies lower than those given by formula (**35f**). Moreover, the harmonics are not simple multiples of the fundamental in every case.

At very high frequencies the influence of ground capacitance becomes negligible compared with series capacitance and the formula for frequency becomes,

$$f = \frac{n}{\sqrt{L\,G_s}} \tag{35g}$$

DAMPING FACTORS

Of course the voltage distribution as given in Fig. 5 departs considerably from what occurs in practise on account of dielectric losses, which introduce a damping factor, which is greater the higher the frequency.

It may be, therefore, especially where the order of frequencies progresses as the square of the wave lengths, that before the fundamental has completed one-half of a complete oscillation most of the higher frequencies have become negligible. A knowledge of the dielectric losses would be necessary to determine whether the voltage stresses are appreciably restricted by them.

HIGH-FREQUENCY TRANSIENTS

An idea of the transient phenomena occurring within a transformer winding when sustained high frequency is impressed can best be obtained by considering the applied high frequency to be made up of successive positive and negative rectangular waves, the frequency of which is the same as the fundamental frequency of the transformer winding. During the time of the first half cycle, phenomena are as described above. When the applied wave changes from positive to negative, the voltage to ground within the transformer winding due to the fundamental has reached its maximum value. The effect of this change in applied voltage is to increase the amplitude of the fundamental approximately by twice the amplitude of the applied voltage. In the example given the above fundamental has a value of 1.24 times the terminal applied voltage for the case of isolated neutral. At each cycle this value is increased by twice the value of the terminal voltage, causing the voltage to ground to increase regularly in steps so that the maximum potential to ground at the end of the second half cycle is 4.72, at the end of the third half cycle 7.20 etc.

The above assumes that the internal losses are insufficient to reduce its amplitude and neglects the amplitude of all other harmonics. Of course, if the applied frequency happens to be equal to the frequency of one of the harmonics the building up of voltage by resonance as described above would apply to that harmonic and not to the fundamental.

It should be noted that the amplitude of the oscillations within the transformer winding are not necessarily the amplitude of the impressed voltage, but they depend upon the amplitude of impressed wave voltage and also upon the values given in the table under "Transient Voltage Distribution." These values are dependent upon the value of "α"; the smaller the value of "α" the smaller will be the value of internal oscillations, and moreover, for a given value of "α" the amplitudes of the oscillations are smaller for the shorter wave lengths.

However, when a sustained high frequency, or a damped oscillation, having a frequency equal to a natural frequency of the transformer, is impressed, the resulting stress as previously shown may very greatly exceed those given by the curves, the values being limited by the value of "α", the internal losses, and the damping factor of the impressed wave. For a reliable quantitative analysis an accurate knowledge of all these factors is necessary.

IV.—Impulse Tests on Small High-Voltage Transformer Winding

A 30,000-volt transformer winding made up of a stack of 52 disk coils approximately 15 in. long and 10 in. in diameter was used for these tests. Grounded metal cylinders were placed inside and outside of the stack, to simulate low-voltage winding and tank.

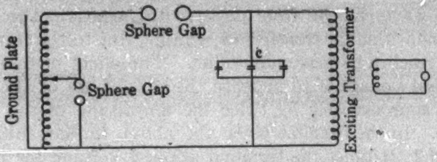


FIG. 8—IMPULSE TESTS ON SMALL HIGH-VOLTAGE TRANSFORMER-CONNECTION DIAGRAM

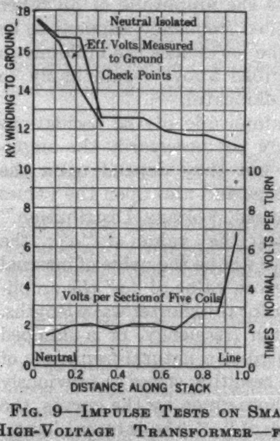


FIG. 9—IMPULSE TESTS ON SMALL HIGH-VOLTAGE TRANSFORMER—FOR CONNECTIONS SEE FIG. 8

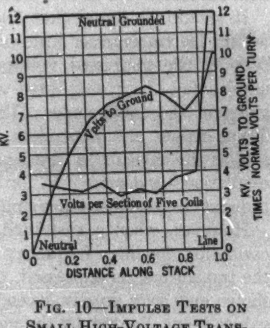


FIG. 10—IMPULSE TESTS ON SMALL HIGH-VOLTAGE TRANSFORMER

The connection used in testing is shown in Fig. 8. Voltage was applied from line to ground; in some tests the neutral (one end) of the coil was grounded to the metal cylinders and in the other test isolated. A spark gap *A* was connected in series with the coil, and a large condenser *C* in multiple with it. The operation is as follows: As the voltage rises, it charges the condenser *C*. This voltage also appears across the gap *A*, a very small part being consumed in the capacitance of the coil. When the gap arcs over, the entire voltage, abruptly applied across the winding, gives rise to a series of oscillations, within the winding. The use of the condenser *C* (which must be as large as practicable) is to maintain the applied voltage while the capacitance of the coil is being charged and also to allow an easy return path for the high-frequency oscillations set up

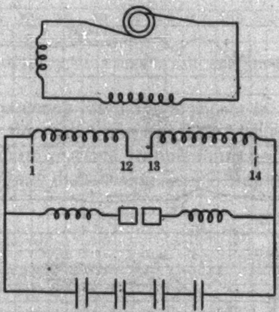


Fig. 11—High-Frequency Investigation—60 Cycles—3000 Kv-a.—140,000 Volts—Diagram of Connections

in the coil. As these oscillations die out before the 60-cycle voltage has appreciably changed, the impressed voltage is in effect a sustained rectangular wave.

The voltage to ground and the voltage gradients are plotted in Fig. 9 and Fig. 10.

Constants of the Coil

Ground capacitance $C_g = 12 \times 10^{-10}$ farads.
Series capacitance $C_s = 4.4 \times 10^{-12}$ farads.

$$\alpha = \sqrt{\frac{C_g}{C_s}} = 16.5$$

V.—High-Frequency Tests*

High-frequency tests on the high-voltage windings of 3000-kv-a. 140,000-volt transformer (Fig. 4) compared with theoretically calculated stresses resulting from impact of rectangular waves.

Transformer constants;

C_g = Ground capacitance = 9×10^{-10} farads.
C_s = Coil capacitance = 9×10^{-12} farads.

$$\alpha = \sqrt{\frac{C_g}{C_s}} = 10$$

Connection diagrams for these tests are given in Fig. 11, and

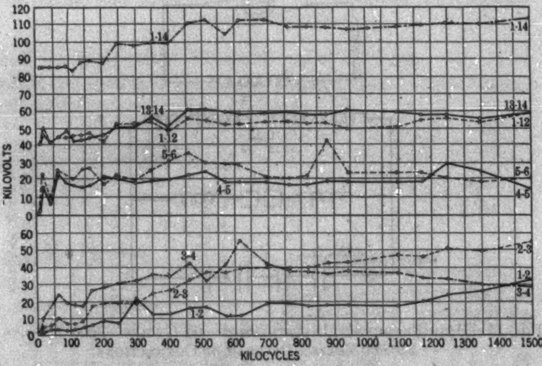


FIG. 12—HIGH-FREQUENCY VOLTAGES MEASURED BETWEEN TAPS

high-frequency voltages obtained across taps are plotted in Fig. 12 and Fig. 13. In Fig. 14 is plotted the approximate volts obtained by these tests at any frequency (curve marked by dots). For comparison the volts per turn as theoretically obtained for rectangular waves (See Fig. 6) and corrected for end turn insulation has been plotted on the same sheet (curve marked by crosses.) The theoretical values have been multiplied by two, for the following reason.

Discharge takes place every one-half wave at the maximum point of the wave. Condensers discharge into the inductive circuit, the maximum value of high-frequency voltage being equal to maximum value of the 60-cycle voltage. One-half wave of the high-frequency discharge is shown in Fig. 15. It is seen that during this half cycle the high-frequency voltage changes from maximum positive to maximum negative. The original 60-cycle voltage distribution, just before the arc-over, is the straight line shown in Fig. 16. Half a high-frequency cycle after the arc-over, the resultant of the 60-cycle and high-frequency voltage distribution is shown by curve "*a*" in Fig. 16, assuming that the frequency is high enough for the wind-

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*Tests made by J. M. Weed and J. E. Clem.

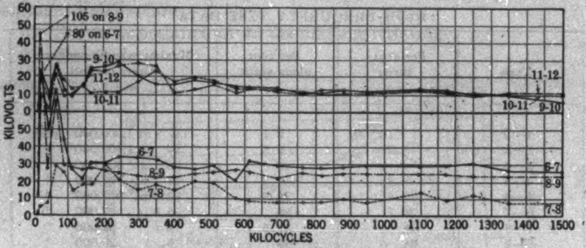

FIG. 13—HIGH-FREQUENCY VOLTAGES MEASURED BETWEEN TAPS

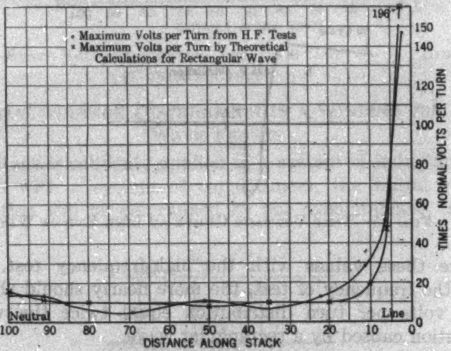

FIG. 14—3000-KV-A.—140,000-VOLT TRANSFORMER

ing to act as condenser for the first half cycle of the high-frequency wave. Thus it is evident that the combination of the high- and low-frequency waves doubles the voltage gradient.

HIGH-FREQUENCY TEST DATA

Taps	Per cent turns	Per cent along stack	Maximum volts per turn times normal
1—2	.19	2.1	147 at 1,500,000 Cycle
2—3	1.2	6.3	52 1,500,000 "
3—4	4.	11.4	28 at 600,000 "
4—5	8.2	16.7	11 at 62,000 "
5—6	13.5	22.5	13.5 at 900,000 "
6—7	30.7	36	5.6 62,000 "
7—8	48	51.5	11.5 62,000 "
8—9	68	69	6 20,000 "
9—10	89	86	9.9 62,000 "
10—11	94	91.5	13 62,000 "
11—12	98	97	14 62,000 "

The volts per turn occurring at ends of windings are the same as if a rectangular wave of voltage 2 *E* had been applied. For this reason the values for volts per turn as theoretically obtained for rectangular waves have been multiplied by two in order to compare these values with the high-frequency test. The higher the frequency of test, the more nearly should the condenser volts per turn distribution equal twice the voltage distribution caused by a rectangular wave.

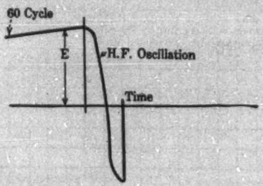

FIG. 15—TIME-VOLTAGE CURVE AT LINE END OF WINDING

V—Mathematical Development

1. Initial voltage distribution in a winding for a steep wave front.

2. Initial effective or equivalent capacitance of a winding for a steep wave front.

3. Time required to charge the initial effective capacitance of a winding.
4. Analysis of transient phenomena following the initial voltage distribution.
5. Surge impedance of an oscillating winding.
6. Example.

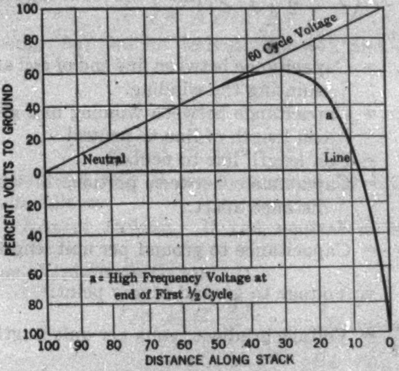

FIG. 16—SPACE-VOLTAGE CURVES FOR $t = 0$ AND $t = \frac{1}{2}$ HIGH-FREQUENCY CYCLE

1. INITIAL VOLTAGE DISTRIBUTION IN A WINDING FOR STEEP WAVE FRONTS

If voltage is suddenly applied to a system of inductances in multiple with a system of capacitances, the initial voltage distribution will be determined by the capacitances. At the instant of impact, the condensers short-circuit the steep wave front. If the wave is maintained, the condensers become charged, and the voltage rises. The time required for this charging is extremely small as given by equation (22).

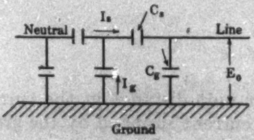

FIG. 17

The voltage distribution, when the condensers are charged, is the same as that when an alternating voltage is applied to the system of condensers with the inductance disconnected. Considering the system of internal and ground capacitances by themselves (Fig. 17), and assuming an alternating voltage of frequency f, $\left(f = \frac{\omega}{2\pi}\right)$, applied, the following relations will be evident.

Let C_s = Capacitance between line end of coil and neutral, shunting the winding.

C_g = Capacitance between winding and ground for a coil length of line to neutral.

l = Coil length line to neutral.

$l\,C_s$ = Capacitance between portions of winding unit distance apart.

$\frac{C_g}{l}$ = Capacitance to ground per unit length of coil

e = Voltage to ground at any point.

$\frac{d\,e}{d\,x}$ = Voltage gradient, volts per unit length of coil.

E_0 = Impressed voltage line to ground.

ω = $2\,\pi\,f$ = angular velocity.

I_s = Current in the coil capacitance C_s.

$$= \omega\, C_s\, l\; \frac{d\,e}{d\,x} \qquad (1)$$

I_g = Current in ground capacitance C_g (total.)

$\frac{d\,I_g}{d\,x}$ = Current in ground capacitance per inch of coil.

$$= \omega \frac{C_g}{l}\, e \qquad (2)$$

The increase per unit distance in the current of the shunt capacitance is the current supplied by the ground capacitance per unit length, that is,

$$\frac{d\,I_s}{d\,x} = \frac{d\,I_g}{d\,x} \qquad (3)$$

Substituting in equation (3) the value of $\frac{d\,I_s}{d\,x}$ from (1), and

$\frac{d\,I_g}{d\,x}$ from (**2**), and simplifying,

$$\frac{d^2\,e}{d\,x^2} - \frac{1}{l^2}\,\frac{C_g}{C_s}\,e = 0 \qquad \textbf{(4)}$$

The solution of this equation is of the form

$$e = A\,\epsilon^{px} \qquad \textbf{(5)}$$

Substituting (**5a**) and its derivative in (**4**), and solving for p,

$$p = \pm\frac{1}{l}\sqrt{\frac{C_g}{C_s}} \qquad \textbf{(6)}$$

and

$$e = A\,\epsilon^{px} + B\,\epsilon^{-px} \qquad \textbf{(7)}$$

The constants A and B are determined by the terminal conditions as follows.

Case I.—Neutral Isolated. If the neutral is isolated at

$x = 0 \quad I_s = 0$ **(8)**

The value of I_s is, from (**1**) and (**7**).

$$I_s = \omega\,l\,C_s\,\frac{d\,e}{d\,x} \qquad \textbf{(9a)}$$

$$= \omega\,l\,C_s\,(A\,\epsilon^{px} - B\epsilon^{-px}) \qquad \textbf{(9b)}$$

Substituting, $x = 0$, and $I_s = 0$

$$0 = \omega\,l\,C_s\,p\,(A - B) \qquad \textbf{(10a)}$$

$$A = B \qquad \textbf{(10b)}$$

Substituting, (**10b**) in (**7**),

$$e = A\,(\epsilon^{px} + \epsilon^{-px}) \qquad \textbf{(10c)}$$

$$= A'\cosh\,(p\,x) \qquad \textbf{(10d)}$$

Now, at $x = l \quad e = E_0$ **(10e)**

$$e = E_0\,\frac{\cosh\,(p\,x)}{\cosh\,(p\,l)} \qquad \textbf{(11a)}$$

$$= E_0\,\frac{\cosh\left(\frac{\alpha\,x}{l}\right)}{\cosh\,(\alpha)} \qquad \textbf{(11b)}$$

where $\alpha = p\,l = \sqrt{\frac{C_g}{C_s}}$ and the neutral is isolated

Case II.—Neutral Grounded. The terminal conditions are

at $x = 0 \qquad e = 0$ **(12a)**

and at $x = l \qquad e = E_0$ **(12b)**

Applying condition (**12a**) to equation (**7**).

$$0 = A + B$$

$$A = -B$$

$$e = A\,(\epsilon^{px} - \epsilon^{\;px}). \qquad \textbf{(12c)}$$

$$e = A' \sinh p\,x \qquad \textbf{(12d)}$$

Applying condition (**12b**) to equation (**12d**)

$$E_0 = A' \sinh p\,l \qquad \textbf{(12e)}$$

$$A' = \frac{E_0}{\sinh p\,l}$$

Therefore, $e = E_0 \dfrac{\sinh p\,x}{\sinh p\,l}$ **(13a)**

$$= E_0 \frac{\sinh\left(\frac{\alpha\,x}{l}\right)}{\sinh \alpha} \qquad \textbf{(13b)}$$

where, $\alpha = p\,l = \sqrt{\dfrac{C_g}{C_s}}$ and the neutral is grounded.

Initial voltage distribution curves for both the isolated neutral and the grounded neutral are given in Figs. 2 and 3.

GENERAL REMARKS ON THE INITIAL VOLTAGE DISTRIBUTION

When the capacitance to ground is negligible compared with the capacitance between coils, that is, $C_g = 0$, the initial voltage distribution is determined by the series capacitances and is a straight line if the capacitance between coils is uniform.

As the capacitance to ground more and more predominates, the curve becomes steeper near the line end, that is, the voltage is concentrated more and more near the line end.

The voltage gradient (volts per unit length) is $e_s = \dfrac{d\,e}{d\,x}$

$$= \frac{\alpha\,E_0 \sinh\left(\frac{\alpha\,x}{l}\right)}{l \cosh \alpha} \text{ neutral isolated} \qquad \textbf{(14a)}$$

$$= \alpha\,\frac{E_0}{l}\,\frac{\cosh\left(\frac{\alpha\,x}{l}\right)}{\sinh \alpha} \text{ neutral grounded} \qquad \textbf{(14b)}$$

The maximum voltage gradient is at the line end, $x = l$,

$$e_s = \frac{\alpha E_0 \sinh \alpha}{l \cosh \alpha} \text{ neutral isolated} \qquad \textbf{(15a)}$$

$$= \alpha \frac{E_0}{l} \tanh \alpha \text{ neutral isolated} \qquad \textbf{(15b)}$$

Similarly $e_s = \alpha \dfrac{E_0 \cosh \alpha}{l \sinh \alpha}$ neutral grounded **(16a)**

$$= \alpha \frac{E_0}{l} \coth \alpha \text{ neutral grounded} \qquad \textbf{(16b)}$$

For α greater than 3,
Tanh $\alpha = \coth \alpha = 1$

This is true for most transformers, and the voltage gradient at the line end becomes $\left(\dfrac{\alpha E_0}{l}\right)$ volts per unit length.

If the voltage were uniformally distributed throughout the winding, the gradient would have been $\left(\dfrac{E_0}{l}\right)$. Therefore, when the total capacitance to ground is very much larger than the coil capacitance between neutral and line, the voltage gradient at the line end is α times the value corresponding to uniform distribution.

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It will also be noticed that for values of α greater than 5, the initial voltage distribution for a steep wave front is practically the same for the case of isolated as for grounded neutral.

INITIAL EFFECTIVE CAPACITANCE OF A WINDING FOR A STEEP WAVE FRONT.

In calculating the time of charging the system of coil and ground capacitances by an incoming wave, it becomes necessary to know the effective capacitance of the combination, This is equal to the effective capacitance for alternating voltage.

Representing this effective capacitance by C_{eff}, line current by I, line voltage (a-c.) by E, frequency by "f".

$$I = 2 \pi f C_{eff} E \qquad \textbf{(17a)}$$

$$\text{or } C_{eff} = \frac{I}{2 \pi f E} \qquad \textbf{(17b)}$$

$$\text{But } I = I_s \text{ at } x = l \qquad \textbf{(17c)}$$

$$\text{and } I_s = 2 \pi f C_s l \left(\frac{d e}{d x}\right) \qquad \textbf{(17d)}$$

The value of $\left(\frac{d\,e}{d\,x}\right)$ must be taken from equation of initial voltage distribution.

Case I.— Neutral Isolated. By equation (**15b**) at $x = l$

$$\frac{d\,e}{d\,x} = \alpha\,\frac{E}{l}\,\tanh\alpha \qquad \textbf{(18a)}$$

$$\text{and } I = 2\,\pi\,f\,C_s \times \alpha\,\frac{E}{l}\,\tanh\alpha \qquad \textbf{(18b)}$$

Substituting (**18b**) in (**17b**).

$$C_{eff.} = \sqrt{C_g\,C_s} \times \tanh\alpha \text{, neutral isolated} \qquad \textbf{(19)}$$

Case II.—Neutral Grounded. Solving similarly for the above case, but using equation (**16b**) instead of (**15b**),

$$C_{eff} = \sqrt{C_g\,C_s}\ \coth\alpha \qquad \textbf{(20)}$$

As stated before, for values of α greater than 3, which is the case with most transformers, tanh α equals one, coth α equals one, and

$$C_{eff} = \sqrt{C_g\,C_s} \qquad \textbf{(21)}$$

whether the neutral is grounded or isolated.

TIME REQUIRED TO CHARGE THE CONDENSERS TO THE INITIAL VOLTAGE DISTRIBUTION

The time required to charge a condenser by an incoming wave is given by the following formula, modified from one given by G. Faccioli in *G. E. Review*, 1914, page 749.

$$t = 2.3\,C_{eff}\,Z\,\log_{10}\left(\frac{100}{100 - \%\,E_0}\right) \text{seconds.} \qquad \textbf{(22)}$$

$C_{eff.}$ = Equivalent massed capacitance of the winding from line to neutral, calculated by formula, (**19**), (**20**), or (**21**).

Z = Surge impedance of the transmission line, and may be taken between 300 and 400.

$\%\,E_0$ = The actual terminal voltage in per cent of the final terminal voltage.

As an example, we will calculate the time necessary to charge the capacitance of the transformer described above to 90 per cent of its final value.

$C_g = 9 \times 10^{-10}$ farads.
$C_s = 9 \times 10^{-12}$ farads
$C_{eff.} = 9 \times 10^{-11}$ by formula (**21**)

Substituting this value of C_{eff} in (**22**) we obtain

$$t = 2.3 \times 9 \times 10^{-11} \times 300 \times \log_{10}\left(\frac{100}{100-90}\right)$$

$= 6.2 \times 10^{-8}$ seconds, which is about one twenty millionth of a second. It is justifiable, then, to assume that the initial voltage distribution by the condensers takes place before the inductance can begin to draw any appreciable current.

ANALYSIS OF TRANSIENT PHENOMENA FOLLOWING THE INITIAL VOLTAGE DISTRIBUTION

Let I_L = Current in the winding.
I_s = Current in the coil capacitance.
I_g = Current in the ground capacitance (total).

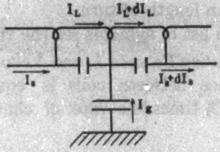

FIG. 18

The following relation will be evident from Fig. 18.

$$\frac{d\,I_L}{d\,x} + \frac{d\,I_s}{d\,x} - \frac{d\,I_g}{d\,x} = 0 \qquad \textbf{(23a)}$$

Equation (**23a**) states that the increase in the currents in the winding and the internal capacitance (as we go towards the line end) is equal to the current contributed by the ground capacitance, which is evident from Fig. 18.

For reasons that will be seen later, we differentiate (**23a**) with respect to t, and obtain

$$\frac{d^2 I_L}{dxdt} + \frac{d^2 I_s}{dxdt} - \frac{d^2 I_g}{dxdt} = 0 \qquad \textbf{(23b)}$$

It will be noticed that equations (**23a**) and (**23b**) are true generally and are independent of what the initial or final voltage distributions may be.

Equation (**23**) involves three variables, I_L, I_g and I_s, and, before it can be solved, it must be expressed in one variable.

Since we are interested mostly in the voltages, we will express the variables in terms of voltages.

Let e = voltage to ground.
e_s = voltage gradient (volts per inch of coil) in the winding.
$= \frac{d\,e}{d\,x}$
N = Total turns (line to neutral)
φ = Total flux linking any turn.
φ_L = Leakage flux linking any turn.
φ_M = Main flux linking the total coil.
B_L = Leakage flux density at any point.
h = Effective length of leakage path.
L = Leakage inductance of coil.
(MLT) = Mean length of turn.
(For a complete list of symbols see the end of this paper.)

To Find the Relation Between I_s and Voltage. The current I_s in the capacitance between coils is equal to the capacitances per inch $l\,C_s$ times the rate of change of voltage per inch $\left(\frac{d\,e_s}{dt}\right)$.

$$I_s = l\,C_s \frac{d\,e_s}{dt} = C_s\,l \;\frac{d^2 e}{dx dt} \tag{24}$$

$$\frac{d^2 I_s}{dx dt} = C_s\,l \;\frac{d^4 e}{dx^2\,dt^2} \tag{25}$$

Equation (**25**) can be directly substituted in equation (**23b**).

To Find the Relation Between I_g and Voltage. The current density in the capacitance to ground is equal to capacitance to ground per inch of coil times the rate of change of voltage to ground.

$$\frac{d\,I_g}{d\,x} = \frac{C_g}{l}\,\frac{d\,e}{dt} \tag{26}$$

and

$$\frac{d^2 I_g}{dx dt} = \frac{C_g}{l}\,\frac{d^2 e}{dt^2} \tag{27}$$

Equation (**27**) can be directly substituted in (**23b**).

To Find the Relation Between I_L and Voltage. The relation between the magnetizing current (I_L) and the voltage is somewhat more complicated than that of the capacitance currents and voltage, due to the mutual inductance between turns and the presence of leakage and main fluxes. The connecting links between current and voltage to ground at any point x are current at x, ampere turns or magnetomotive force effective in producing leakage flux, flux density at point x, flux producing voltage at point x, volts per turn and volts. See Fig. 19. The mathematical relation between these physical quantities are as follows:

The voltage gradient e_s, *i. e.*, volts per inch, equals number of turns per inch N/l times the rate of change of total flux φ (= main flux φ_m + leakage flux φ_L). Thus,

$$e_s = \frac{N}{l} \frac{d\varphi}{dt} 10^{-8} \tag{28a}$$

$$= \frac{N}{l} 10^{-8} \frac{d}{dt} \left\{ \varphi_L + \varphi_m \right\} \tag{28b}$$

The leakage flux (φ_L) linking x is obtained by integrating the leakage flux density (B_L). Thus,

$$e_s = \frac{N}{l} 10^{-8} \frac{d}{dt} \left\{ (MLT) \int_x^l B_L\, dx + \varphi_m \right\} \tag{28c}$$

If we differentiate (**28c**) with respect to x we get rid of the integral sign, and also of the last term because it is independent of x. Thus,

$$\frac{d e_s}{d x} = -\frac{N}{l} 10^{-8} (MLT) \frac{d B_L}{dt} \tag{29a}$$

The negative sign in (**29a**) is due to x being the negative limit of integration in (**28c**). Of course it could have been assumed of the opposite sign provided other signs were kept consistent with it.

The leakage flux density B_L in (**29a**) is produced by the ampere turns enclosed by the flux.

$$B_L = \frac{N\mu}{lh} \int_0^x I_L\, dx \tag{29b}$$

where $\mu = 4\frac{\pi}{10}$ when using dimensions in cm.

$= 3.2$, when using dimensions in inches.

h = effective length of leakage path.

It may be questioned why in **(28c)** we use the limits $\int_x^l$ and in **(29b)** $\int_0^x$. The reason may be stated thus; the flux linking a current is outside of it, and the ampere turns producing the flux are inside of it. Therefore, if the total flux linking a certain point be integrated on the right of that point, the ampere turns producing that flux must be integrated on the left of the point. As a concrete example consider Fig. 19.

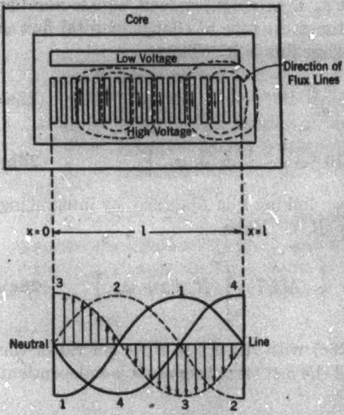

Fig. 19—Natural Oscillation Within Transformer Winding for ¾ Wave Length with Isolated Neutral and Line End Connected to Transmission Line

Curve 1, Volts to ground = E. Curve 2, Volts per turn = $D\,E/D\,X$. Curve 3, Flux density, or Magnetomotive force = $D^2\,E/D\,X^2$. Curve 4, Current = $D^3\,E/D\,X^3$.

The flux producing voltage in a turn at the point x is all the flux from x to l; but the ampere turns producing the flux at x are all the ampere turns from 0 to x.

Returning to the main problem, **(29b)** substituted in **(29a)** gives

$$\frac{d\,e_s}{d\,x} = -\frac{3.2\,N^2\,(MLT)}{l^2\,h\,10^8}\,\frac{d}{d\,t}\int_0^x I_L\,d\,x \qquad \textbf{(29c)}$$

$$= \frac{L}{l^3}\,\frac{d}{d\,t}\int_0^x I_L\,d\,x \qquad \textbf{(29d)}$$

where

$$L = \frac{3.2\,l\,N^2\,(MLT)}{h\,10^8} \text{ (See note opposite)} \qquad \textbf{(29e)}$$

Differentiating (**29d**) with respect to x eliminates the integral sign. Thus,

$$\frac{d^2 e_s}{d x^2} = - \frac{L}{l^3} \frac{d I_L}{dt} \tag{30a}$$

Remembering that e_s (voltage gradient) is equal to $\frac{d e}{d x}$ (derivative of voltage to ground), (**30a**) becomes

$$\frac{d^3 e}{d x^3} = \frac{L}{l^3} \frac{d I_L}{dt} \tag{30b}$$

(**30b**) gives the value of $\frac{d I_L}{dt}$, while, equation (**23a**) involves $\frac{d I_L}{d x}$. If we differentiate the former with respect to x and the latter with respect to t, substitution becomes possible. This is the reason why (**23a**) was differentiated with respect to time giving (**23b**).

Differentiating (**30b**) with respect to x and rearranging.

$$\frac{d^2 I_L}{dxdt} = \frac{- l^3}{L} \frac{d^4 e}{d x^4} \tag{31}$$

(**31**), (**25**) and (**27**) express in terms of voltage the three terms of the fundamental differential equation (**23b**). Making the substitution, (**23b**) becomes

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NOTE:—In discussing the leakage flux relations, we have assumed the effective length of the path as constant and independent of wave length. This is approximately true for only short waves on disk coils when the radial build of the coils is comparable to the wave length. Otherwise, the formula for L will not be accurate enough for numerical calculations. L, however, can be determined experimentally for various wave lengths as follows. Take one portion of the winding equal to one quarter of the wave length under consideration and short-circuit it. Take an equal portion adjacent to it, excite it as primary, and measure its inductance (= Reactance/377 for 60-cycle test). Multiply this value of inductance by 32 n^3 to get the effective value of L for that wave length.

If the winding consists of disk coils, for approximate calculations the effective length of leakage path may arbitrarily be assumed as equal to the radial coil-build plus one-sixth of wave length:

$$h = b + \frac{l}{6 n} \tag{29f}$$

$$\frac{d^4 e}{d x^4} - \frac{L C_s}{l^2} \frac{d^4 e}{dx^2 dt^2} + \frac{L C_g}{l^4} \frac{d^2 e}{dt^2} = 0 \qquad \textbf{(32)}$$

(**32**) is the general differential equation of electrical phenomena occurring in windings, except that it does not take into account resistance and dielectric losses.

A solution of this equation is

$$e = A \cos p\, x \cos \omega\, t. \qquad \textbf{(33)}$$

Substituting in equation (**32**) gives

$$\omega = \pm \frac{p^2 l^2}{\sqrt{L\,(C_g + l^2 p^2 C_s)}} \qquad \textbf{(34)}$$

These equations show that a transformer is capable of sinusoidal oscillations in space and time at various frequencies and wave lengths.

Determination of the Constants. The voltage distribution may be expressed as the sum of a transient and a permanent component. The permanent component is the final voltage distribution. For a d-c. voltage applied, the final voltage distribution is $N\,B$ (Fig. 20) for the case of neutral isolated, that is, the whole coil assumes the impressed voltage E_0 against ground. The actual initial voltage being given by the hyperbolic curve $O\,A\,B$, the transient component is the difference between $N\ B$ and $O\,A\,B$. Expressing the same relation in another way $O\,A\,B$ represents the actual voltage when referred to $O\,M$ as the axis of reference, but represents the transient component when referred to $N\,B$ (the line of equilibrium) as the axis of reference. For neutral grounded (Fig. 21) $O\,B$ represents the permanent or equilibrium line, and $O\,A\,B$ (referred to $O\,B$ as axis) represents the transient component of the voltage distribution.

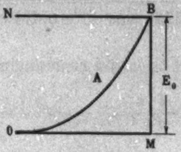

FIG. 20

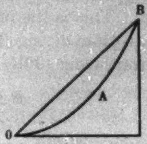

FIG. 21

To try to substitute the hyperbolic formula of initial voltage distribution in equation (**33**) leads to an indeterminate form, but if analyzed into space harmonics (with respect to the line of final voltage distribution), substitution is readily effected, and the actual voltage to ground is the sum of the harmonics and the voltage of the reference line to ground.

An inspection of Fig. 5 shows that for neutral isolated the initial voltage distribution curve *O A B* is a quarter wave length (and its odd harmonics) with respect to the equilibrium line *N B* with maximum voltage at the neutral. An inspection of Fig. 6 shows that for neutral grounded the initial voltage distribution curve *O A B* is a half wave length (and its even and odd harmonics). The numerical values of the harmonics are derived by harmonic analysis.

Representing wave length by λ, initially, for neutral isolated,

$$e' = E_\lambda \cos \frac{2\,\pi\,x}{\lambda} \tag{35}$$

and for neutral grounded

$$e' = E_\lambda \sin \frac{2\,\pi\,x}{\lambda} \tag{35a}$$

where $E\lambda$ is the maximum value of the wave.

The value of p in equation (**34b**) is therefore $\dfrac{2\,\pi\,x}{\lambda}$ and

$$\omega = \pm \frac{4\,\pi^2\,l^2}{\lambda^2 \sqrt{L\left(\dfrac{C_g + 4\,\pi^2\,l^2\,C_s}{\lambda^2}\right)}} \tag{35b}$$

$$= \pm \frac{4\,\pi^2\,(l/\lambda)^2}{\sqrt{L\,\{\,C_g + 4\,\pi^2\,C_s\,(l/\lambda)^2\,\}}} \tag{35c}$$

Now l/λ is the ratio of coil length to wave length and we shall designate it by small n. The longest wave to which the winding will oscillate is a quarter wave length, and $n = 1/4$. Thus n may be looked upon as the order of the space harmonic. If n equals one, the wave length equals the coil length; if n equals two, the wave length is one-half of the coil length, or, there are two complete wave lengths from line to neutral. Substituting n for l/λ,

$$\omega = \frac{4\,\pi^2\,n^2}{\sqrt{L\,(C_g + 4\,\pi^2\,n^2\,C_s)}} \tag{35d}$$

$$f = \frac{2\,\pi\,n^2}{\sqrt{L\,(C_g + 4\,\pi^2\,n^2\,C_s)}} \tag{35e}$$

For long wave lengths (low harmonics, n small) capacitance to ground predominates,

and
$$f = \frac{2\,\pi\,n^2}{\sqrt{L\,C_g}} \tag{35f}$$

For the very short wave lengths (high order of harmonics, n large) the coil capacitance C_s predominates.

$$f = \frac{n}{\sqrt{L\,C_s}} \tag{35g}$$

Since there are two values of ω, one positive and the other negative, and since $\cos(+\,\omega)$ equals $\cos(-\,\omega,)$ equation (**33**) becomes

$$e' = A\cos(2\,\pi\,n\,x/l)\cos(\omega_n t) + B\cos(2\,\pi\,n\,x/l)\cos(-\,\omega_n t) \tag{33a}$$

$$= A\cos(2\,\pi\,n\,x/l)\cos\omega_n t + B\cos(2\,\pi\,n\,x/l)\cos\omega_n t \tag{33b}$$

$$= (A + B)\cos(2\,\pi\,n\,x/l)\cos\omega_n t \tag{33c}$$

$$= E_n\cos(2\,\pi\,n\,x/l)\cos\omega_n t \tag{33d}$$

e' is the voltage to the axis of oscillation, and for every harmonic there is a term like (**33d**). The voltage to ground is, for neutral isolated,

$$e = E_0 - E_n\cos(2\,\pi\,n\,x/l)\cos\omega_n t \tag{33e}$$

and for neutral grounded

$$e = E_0\,(x/l) - E_n\sin(2\,\pi\,n\,x/l)\cos\omega_n t \tag{33f}$$

As the voltage oscillates above and below the line of equilibrium, and as the frequency is different for the different waves, we may assume that at some instant the maxima of all the harmonics at a point will add arithmetically, and that will represent the maximum voltage to ground at that point. That is, maximum voltage to ground equals, for neutral isolated.

$$\max e = E_0 + E_n\cos(2\,\pi\,n\,x/l) + \ldots \tag{36a}$$

and for neutral grounded

$$\max e = E_0\,(x/l) + E_n\sin(2\,\pi\,n\,x/l) + \ldots \tag{36b}$$

Of equal importance is the voltage gradient (volts per turn). The maximum voltage gradient at a point occurs when maximum voltage gradients of the various harmonics at that point add directly. This gives, for neutral isolated,

$$\max\frac{d\,e}{d\,x} = (2\,\pi\,n/l)\,E_n\sin(2\,\pi\,n\,x/l) + \ldots \tag{37a}$$

and for neutral grounded

$$\max \frac{d\,e}{d\,x} = E_0/l + (2\,\pi\,n/l)\,E_n \cos\,(2\,\pi\,n\,x/l) + \,.\,.\quad \text{(37b)}$$

It is to be noted that although the higher harmonics may be small in magnitude yet the volts per turn which they produce may not be negligible on account of the volts per turn varying directly with the order of the harmonic. For instance, a tenth harmonic having a magnitude of ten per cent of the fundamental will cause as much strain per turn as the fundamental.

The initial voltage gradient was discussed under equations (**14**,) (**15**) and (**16**).

SURGE IMPEDANCE OF A WINDING

The surge impedance of a transformer winding may be defined as the ratio of voltage impressed to current at line end of winding.

By equation (**23**)

$$\frac{d\,I_L}{d\,x} = \frac{d\,I_g}{d\,x} - \frac{d\,I_s}{d\,x}$$

$$= \frac{C_g\,d\,e}{l\,dt} - l\,C_s\,\frac{d^2\,e}{dxdt}$$

Substituting the general value of e, that is, $e = E\cos p\,x \cos \omega\,t$

$$\frac{d\,I_L}{d\,x} = -\,\omega\,E\,(C_g/l + l\,C_s\,p^2)\cos p\,x \sin \omega\,t$$

and integrating

$$I_L = \frac{\omega\,E}{p}\,(C_g/l + l\,C_s\,p^2)\sin p\,x \sin \omega\,t \quad \text{(38a)}$$

At the line end, $\sin p\,l$ equals one, and the maximum of the current occurs when $\sin w\,t$ equals one.

$$\max I_L = \frac{w\,E}{p}\,(C_g/l + l\,p^2\,C_s) \quad \text{(38b)}$$

$$Z = \frac{E}{I_L} = \frac{p\,l}{\omega\,(C_g + l^2\,p^2\,C_s)} \quad \text{(39a)}$$

$$Z = \frac{1}{2\,\pi\,n} \times \sqrt{\frac{L}{C_g + 4\,\pi^2\,n^2\,C_s}} \quad \text{(39b)}$$

It is, therefore, evident that the surge impedance of a winding is not a constant but a function of the wave length, the shorter the wave length the less the surge impedance and hence, for a given voltage the current associated with higher harmonics is greater.

The current surges back and forth into the line, and it is assumed that the surge impedance of the line is low enough not to impede this free oscillation as will be seen in the following example.

EXAMPLE

The constants of the high-voltage winding of the above mentioned transformer are as follows,

$$\begin{aligned} C_g &= 9 \times 10^{-10} \text{ farads} \\ C_s &= 9 \times 10^{-12} \text{ farads} \\ \alpha &= 10 \end{aligned}$$

By formula (**29e**)

$$\begin{aligned} L &= 8 \text{ for } n = 1/4 \\ &= 14 \text{ `` } n = 1/2 \\ &= 20 \text{ `` } n = 3/4 \\ &= 23 \text{ `` } n = 1 \\ &= 27 \text{ `` } n = 5/4 \\ &= 30 \text{ `` } n = 3/2 \\ &= 32 \text{ `` } n = 7/4 \\ &= 34 \text{ `` } n = 4/2 \end{aligned}$$

The initial voltage distribution is, by equations (**11b**) and (**13b**).

$$\% \, e = \frac{e}{E_0} = \frac{\cosh (10\, x/l)}{\cosh (10)} \text{ neutral isolated}$$

$$= \frac{\sinh (10\, x/l)}{\sinh (10)} \text{ neutral grounded}$$

These are plotted in Figs. 2 and 3 together with curves for other values of α.

Analyzing the curve for $\alpha = 10$ into its space harmonics with respect to the corresponding axis of oscillation, we find the following values:

Neutral Isolated.—

Order of harmonic	Number of wave lengths n	Voltage $\left(\frac{E_n}{E_0}\right)$
Fundamental	1/4	− 125 %
3rd	3/4	+ 34 %
5th	5/4	− 15 %
7th	7/4	+ 7 %
9th	9/4	− 3.5%
11th	11/4	+ 1 %

The frequencies corresponding to these harmonics, calculated by equation (**35e**), are

Order of harmonics	n	λ	ω	f	Ratio
Fundamental	1/4	168 in.	28,000	4,600	1
3rd	3/4	56 "	150,000	24,000	5.2
5th	5/4	34 "	310,000	49,000	10.6
7th	7/4	24 "	480,000	76,000	16.5

Expressing the voltage to ground in per cent of line voltage (line to ground) the complete equation is

$$\begin{aligned} \% e = \frac{e}{E_0} = 1 - 1.25 \cos\left(\frac{\pi}{2} \times \frac{x}{l}\right) \cos\ 28{,}000\, t \\ + 0.34 \cos\left(\frac{3\,\pi}{2} \times \frac{x}{l}\right) \cos 150{,}000\, t \\ - 0.15 \cos\left(\frac{5\,\pi}{2} \times \frac{x}{l}\right) \cos 310{,}000\, t \\ + 0.07 \cos\left(\frac{7\,\pi}{2} \times \frac{x}{l}\right) \cos 480{,}000\, t \text{ etc.} \end{aligned}$$

The maximum possible voltage to ground at any point at any time is equal to

$$\begin{aligned} \text{max. } \% e = 1 + 1.25 \cos\left(\frac{\pi}{2} \times \frac{x}{l}\right) \\ + 0.34 \cos\left(\frac{3\,\pi}{2} \times \frac{x}{l}\right) \end{aligned}$$

$$+ 0.15 \cos\left(\frac{5\,\pi}{2} \times \frac{x}{l}\right)$$

$$+ 0.07 \cos\left(\frac{7\,\pi}{2} \times \frac{x}{l}\right) \text{ etc.}$$

This is plotted in Fig. 6.

The maximum voltage gradient is the derivative of the above equation.

$$\text{max. } \%\, e_s = \frac{d\,e}{d\,x} = \frac{\pi}{2\,l} \times 1.25 \sin\left(\frac{\pi\,x}{2\,l}\right)$$

$$+ \frac{3\,\pi}{2\,l} \times 0.34 \sin\left(\frac{3\,\pi\,x}{2\,l}\right)$$

$$+ \frac{5\,\pi}{2\,l} \times 0.15 \sin\left(\frac{5\,\pi\,x}{2\,l}\right)$$

$$+ \frac{7\,\pi}{2\,l} \times 0.07 \sin\left(\frac{7\,\pi\,x}{2\,l}\right) \text{ etc.}$$

This is also plotted in Fig. 6. Calculating the voltage gradient at the line end by this formula we find that it is less than that obtained by equation (**15b**). This means that a sufficient number of harmonics are not included in the above formula. Neglecting the highest harmonics is permissible for the interior of the coil as they may be damped out before the highest possible maximum would be reached, but for the line end it is better to calculate the voltage gradient by the hyperbolic formula. This has been done in the calculation of the above curve.

Neutral Grounded. The initial voltage distribution for this case is practically the same as for isolated neutral (see Fig. 3, $\alpha = 10$). Analyzing this curve into space harmonics with respect to the slant line $\alpha = 0$, we find the following values:

No. of wave lengths on the stack n	$\frac{E_x}{E_0}$	ω	f	Ratio
$\frac{1}{2}$ −	.58	84,000	13,000	1
$2 \times \frac{1}{2}$ +	.23	230,000	37,000	2.8
$3 \times \frac{1}{2}$ −	.13	390,000	62,000	4.8
$4 \times \frac{1}{2}$ −	.06	580,000	92,000	7.1

The maximum voltage to ground is

max. $\%\, e$

$$= \frac{x}{l} + 0.58 \sin\left(\frac{\pi x}{l}\right) + 0.23 \sin\left(\frac{2\pi x}{l}\right)$$

$$+ 0.13 \sin\left(\frac{3\pi x}{l}\right) + 0.06 \sin\left(\frac{4\pi x}{l}\right) \text{ etc.}$$

This is plotted in Fig. 7.

The maximum voltage gradient is the derivative of the above.

$$\text{max. } \%\, e_s = \frac{d\,e}{d\,x} = \frac{1}{l} + \frac{0.58\,\pi}{l} \cos\left(\frac{\pi x}{l}\right)$$

$$+ 0.23 \times \frac{2\pi}{l} \cos\left(\frac{2\pi x}{l}\right)$$

$$+ 0.13 \times \frac{3\pi}{l} \cos\left(\frac{3\pi x}{l}\right)$$

$$+ 0.06 \times \frac{4\pi}{l} \cos\left(\frac{4\pi x}{l}\right) \text{ etc.}$$

This also is plotted in Fig. 7.

Surge Impedance. By equation (**39b**)

n =	1/4	1/2	3/4	1	1 1/4	1 1/2	1 3/4	2
Z =	60,000	38,000	29,000	22,000	17,000	14,000	11,500	9,600

SYMBOLS

α = $\sqrt{C_g/C_s}$

A = Constant of integration

B = Flux density, also constant of integration.

C = Capacitance.

C_g = Capacitance to ground (total.)

C_s = Coil capacitance (total), from line end to neutral.

E_0 = Terminal voltage, *i. e.*, line to ground.

E_n = Max. value of the "n" th harmonic to the axis of oscillation.

e = Voltage to ground at any point at any instant.

e_s = Voltage gradient in the winding, volts per unit length, proportional to volts per turn.

f = Frequency cycles per second, $\omega/2\,\pi$.

g = Subscript meaning ground.
I = Current.
l = Length of coil (or coil stack), line to neutral.
N = Number of turns total, line to neutral.
n = Number of wave lengths on the winding for a given harmonic.
ω = Angular velocity, radians per second.
$= 2\pi f$
Z = Surge impedance.

DISCUSSION ON "ABNORMAL VOLTAGES WITHIN TRANSFORMERS" (BLUME AND BOYAJIAN), NEW YORK, N. Y., FEBRUARY 21, 1919.

H. R. Summerhayes: I would like to have the author give some additional information on the value of alpha, it goes over one per cent; now, it has occurred to me that in some classes of apparatus, for instance, as induction regulators or in generator coils, that alpha might be less than one, I may be wrong,but it is even conceivable that if the iron is brought close to all of the coils as in a generator or as in a transformer which might have ground shields between the coils, that the value of alpha might be less than one. I would like to know how that would modify the conclusions.

I am interested to notice the difference between grounded neutral systems and those with isolated neutral. Apparently, in most cases the grounded neutral has some advantage in the effect on the apparatus, especially with rectangular waves.

G. Faccioli: I knew by experience and theory that transformer coils—like men—do not always cooperate, so that when an abnormal condition of operation arises some of the coils are left to take the brunt of the disturbance while other coils do not do their share. I also knew, as all the engineers interested in this class of phenomena knew, that the capacity of the transformer winding is the controlling factor of the distribution of voltage when a steep wave front impinges upon the winding but I did not know the definite law governing the relation between the distribution of voltage and the capacity. Now Lord Kelvin said that you never know a phenomenon until you can put it into figures. The authors of this paper have succeeded for the first time in putting this phenomenon into figures and therefore I feel that they have for the first time really explained it, rendering thereby a great service to the art.

C. A. Adams: I feel that we owe a debt of gratitude to those men who are willing to devote their valuable time to the preparation of papers of this kind. Such papers are not popular, and are frequently criticized by many who attend our meetings, but they are of the utmost fundamental and practical importance and constitute the foundation on which the practical designing engineer must build if he is to be sure of his superstructure.

Referring now to this particular subject, we all realize that the mathematical analysis is approximate. It is impossible to state in accurate mathematical terms the complete physical problems, as the conditions are too complex, but when the mathematical analyses have been made as carefully as we know how under the circumstances, and when the conclusions have been checked by careful research work, they become tremendously useful in rationalizing our attack on the practical problem in hand, and in dealing with new problems in the same field.

H. R. Summerhayes: It seems to me that the paper has a bearing possibly upon testing, I think the author mentioned that it might be possible to build transformers in accordance with these principles, which would be strong for the service and cheap, so to speak, but would not necessarily meet the present rules of testing, and I think that is where the paper has important bearing in its possible effect on the rules.

C. A. Adams: You refer to the Standardization Rules?

H. R. Summerhayes: Yes, the rules for insulation testing.

C. A. Adams: That is certainly a very important bearing.

J. F. Peters: (Abstracted by L. W. Chubb.) It has been known for many years that during disturbances high voltages are developed in transformer windings and especially in that part of the winding adjacent to the line. The paper is of considerable value and interest because it affords a method of approximately determining the magnitude and distribution of these abnormal voltages under certain conditions.

The authors have computed and plotted in Figs. 2 and 3 the voltage distribution in a transformer winding for various values of α when a voltage having a sheer front or rectangular wave is suddenly applied to the terminals. By an exactly similar method the voltage distribution in any coil of the stack can be determined. The authors have taken as a unit, one coil length. Now, if we consider one coil and take as a unit, one turn, and consider X along the plane of the coil (at right angles to the length of the stack) then the solution for the coil is the same as for the stack with grounded neutral. C_g becomes the capacitance between the coil under consideration and the next adjacent coil and C_s becomes the capacitance between the inner and outer turns of the coil. The voltage distribution within the coil

$$e' = E_0' \frac{\sinh \dfrac{a' x'}{l'}}{\sinh a'} \qquad a' = \sqrt{\frac{C_g'}{C_s'}}$$

E_0' = voltage across the coil considered.
e' = voltage distribution in coil.

a' is always greater than zero and in many cases may be greater than a.

Therefore, the maximum voltage per turn may be considerably greater than that given by the slope of the curves in Fig. 2 and 3. In insulating the windings for these abnormal voltages between turns and between coils one has, as in the design of most apparatus, the choice of two evils. The voltage for any coil of the stack is approximately constant irrespective of the number of turns it contains. Therefore, decreasing the number of turns in the line coil in order to get more space for insulation increases the voltage per turn of the coil and one must choose between volts per turn and insulation space between turns.

The information that the paper gives on the effect of grounding the neutral is very interesting. For the example that the authors have computed, the maximum voltage to ground for grounded neutral is 1.5 normal as compared with 2.8 times for isolated neutral. The voltage per turn for the major part of the winding is approximately the same for both. It is four times normal for the isolated connection and five times normal for the grounded connection. The voltage per turn for the line end of the stack is the same for both connections, while for the neutral end, the voltage per turn is greater for the grounded connection, but is less than the volts per turn at the line end. Both ends of the stack are generally insulated the same for reasons of symmetry; therefore there is no additional difficulty encountered in the end turn insulation for the grounded neutral and there is a decided gain in the maximum voltage to ground.

I feel that the conclusions drawn regarding the transient voltage distribution following the initial voltage are not entirely fair. Equations (**33e**) and (**33f**) give these transients only when the damping factors due to resistance and dielectric losses are neglected. When including the damping factors the parts of these equations that are multiplied by E_n are also multiplied by the damping factors. For the high frequencies in these transients the damping is undoubtedly large and by the time all the harmonics have simultaneous maximums the damping factors will have gotten in their good work on decreasing the amplitudes and it is questionable whether the assumptions leading to equations (**36a**) and (**36b**) are justified.

F. Dubsky: The factor alpha for alternator, motor and induction regulator windings, having coils embedded in slots, may be estimated with the following values:

For one single coil $\alpha = 0.7$ to 2 (the larger value for deep and narrow slots).

For one phase of the winding $\alpha =$ (0.7 to 2) times the number of coils connected in series per phase.

Since the factor alpha for one coil is comparatively small it follows that the voltage distribution within a coil can be considered as uniform.

The factor alpha per phase may attain, of course, a considerably large value and produce, therefore, a greater concentration of voltage across the line coil.

Since all coils of the alternator, motor and induction regulator windings are usually wound and insulated alike, their layer and turn insulation should be selected in proper relation to the coil insulation to ground.

F. W. Peek, Jr.: The effects of sustained high-frequency oscillations have been mentioned. Some time ago we made extensive experiments with sustained high-frequency oscillations impressed on transformers. Voltages were measured over a wide range of impressed frequencies by gaps placed across taps brought out from the various coils throughout the winding.

It was found that voltages equivalent to line voltage could be built up across a very small fraction of the winding. The point or points of maximum voltage occurred at different parts of the winding, depending upon the frequency of the applied oscillations. These high internal voltages existed without appreciable increase in voltage across the terminals. A gap arrester is thus not effective for protection against these oscillations. Protection may be obtained, however, by shunting the apparatus by a condenser and resistance in series.

These data are of great importance, in design, to determine the best methods of applying insulation and of building transformers as free from the probability of high internal voltages as possible. Mr. Blume's paper is of importance from both of these standpoints.

Fortunately, there need be but very little fear of conditions producing high internal voltages on a properly insulated, connected, and operated system.

A. E. Kennelly: (communicated after adjournment). The paper is of special interest from the standpoint of artificial electric lines. It brings to our notice a new type of artificial line, and one which deserves consideration from the operating engineer. This new type is an all-capacitance line, with its series elements shunted by inductances. The conditions, however, are such that at the moment of initial disturbance, the inductance shunts are virtually non-existent, by reason of their sluggishness of action; so that the artificial line presented in a high-tension transformer is at first, very nearly a pure all-capacitance line. After the lapse of a certain brief period, the inductance shunts begin to come into play. The artificial line thereafter changes to one of mixed character, with condensers and inductances in series, and condensers in shunt. It is this automatic change in the nature of the artificial line of the transformer with time, that makes the case presented in the paper so interesting.

The theory of all-capacitance artificial lines has been already outlined.* It has been shown that in any pure all-capacitance line, the potential and current distributions are independent of the frequency, and belong to the hyperbolic-sine and hyperbolic-cosine series of numbers, whether the distant end be grounded or freed. From this point of view, the quantity α, defined on the second page of the paper, is the angle subtended by the line, uncorrected for lumpiness, is a real angle expressible in hyperbolic radians, and is the same at all frequencies.

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* *Artificial Electric Lines,* by A. E. Kennelly, New York 1917, Chapter XVII.

The quantity described in (**21**) as the effective capacitance of the transformer, is the surge admittance of the line, when multipled by $j\omega$, where ω is the impressed angular velocity. Again the quantity defined as $\alpha X/l$, in (**11**) and (**13**), is the position angle of a point on the line. Equations (**13**) and (**14**) merely restate the well known propositions that the potential and current at any point on an artificial line are respectively proportional to the sine and cosine of the position angle at the point. It is to be noted that a value for α such as 10, given in the paper, is a very large real hyperbolic angle. It shows that such transformers may develop large angles, and that alternating-current artificial lines may present real angles, whereas it is very commonly supposed that all such lines develop complex angles varying with the frequency.

No oscillations are possible in a homologous line; of which an all-capacitance line is one type; because the energy is all of one type—in this case electric energy—and the energy must be capable of appearing in more than one type—such as either electric or magnetic energy—in order that oscillations may occur. When the inductance shunts are awakened, and come into action, the oscillations discussed in the paper begin to appear.

It will be of very great interest, and also of great practical importance, to ascertain how long it takes to wake up the inductance shunts, and how gradual the awakening is. It is to be hoped that the authors will attempt to answer these questions experimentally.

L. F. Blume: In reply to Mr. Summerhayes' question regarding the value of alpha (ratio of ground capacitance to series capacitance) this design constant in practise may vary from values approaching zero to very high values. It is possible to have values less than one where the ground capacitance is small compared with series capacitance. However, I doubt if this condition occurs very often in practise. In apparatus other than transformers in the majority of cases, the value of alpha is quite large. In the case of a transmission line the value of alpha is infinity because series capacitance is zero.

Mr. Peters raises the question concerning the initial distribution of potential within a coil. In the type of transformer winding analyzed in our paper, the value of internal capacitance (capacitance from layer to layer) is relatively large, that is, large compared with capacitance from edge of coil to ground, due to the small distance between layers. And since this capacitance is essentially in series with ground capacitance, it can have but little effect on the voltage distribution to ground, and between layers within the coil. However this is not necessarily true for all types of windings and for this reason, it is important that the distribution within the coil should be properly analyzed.

The influence of internal losses on oscillations is undoubtedly of very great practical importance because the insulation and other losses combined, very quickly absorb the energy of oscillation. It should be remembered in this connection that only a small fraction of energy of an impinging wave enters a transformer winding, the majority being returned to the transmission line by reflection. To take care of the energy losses in the theory would have made the mathematics very much more involved. We have only attempted to outline in a rough way the influence of losses in dampening out these oscillations.

Presented at the 348th Meeting of the American Institute of Electrical Engineers, Boston, Mass., March 14, 1919.

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SOME POSSIBILITIES OF STEAM RAILROAD ELECTRIFICATION AS AFFECTING FUTURE POLICIES

BY CALVERT TOWNLEY

ABSTRACT OF PAPER

Electricity fills every requirement of railroad service, but as it involves a large investment, electrification has proceeded slowly. Electrification has also been retarded because the problem has been largely considered one of replacing the steam locomotive by the electric locomotive whereas in reality the problem is much broader. It really offers a fundamentally different method of train propulsion because the limitations of the steam locomotive disappear and the strictly limited motive power is replaced by one that is practically unlimited, thereby opening up many possibilities in the methods of railroad operation. While there are a number of different systems of electric traction all of the systems have many features in common and the possibility of unlimited electric power is a characteristic of them all. A brief review is given of electrified sections of railways showing the advantages which have been realized in both the freight and passenger service. Existing electrifications have been operated for a sufficient length of time so that operating statistics are now available, and any proposed undertaking may therefore be predicated on established facts. While electrification will greatly increase track capacity, there is a large railroad mileage which already has more than sufficient capacity, in which case electrification would not be justified. On the other hand, there are so many cases where its advantages are clear and conclusive that when the railroads are able to finance their required electrification it will test the capacity of the electric factories of the country to serve them.

ELECTRICITY now performs every railroad service previously rendered exclusively by steam locomotives and in every case does it better than it was done before. But in order to use electricity a large investment in equipment and installation must be made and electrification has proceeded slowly because railroad executives were not convinced that the advantages to be gained are always worth the cost.

The progress of electrification has also been impeded, first, before the war by the difficulty in financing, due to conditions other than the merits of electrification, and second, since the war began, because every one has been too busy to consider any work that could be deferred and because the government's taking over the railroads has created an unsettled situation not conducive to the investment of new capital for future returns. Now, however, there seems to be ground for hoping that these bars to progress will be removed in the not distant future so that electrification can be again studied on its merits, therefore our consideration of the subject is timely.

In reviewing the past twenty years history of this question, I cannot escape the conclusion that we electrical men, and not our steam road colleagues are responsible for the slow progress made. We have not known enough about either the science or the art of railroading. Our belief in, and our zeal for our own profession has lead us, albeit with entire honesty of purpose, to make more or less extravagant claims as to what we could do and to underestimate the cost of doing it. The inevitable reaction of mind which followed an accurate determination of facts of course disturbed confidence in our judgment. But if at times we have injured the cause of electrification by claiming too much, strange as it may sound, we have injured it a great deal more by not claiming enough. Electrical engineers not having always been railroad men, have been unable to study railroad problems as they should have been studied, that is to say with only real and not with any arbitrary limitations before them. It has been natural for the electrical man to ask the railroad man for a statement of the conditions he was expected to meet. It was equally natural for the railroad man to prescribe the conditions upon which his steam service was predicated. Under these circumstances the problem became largely one of replacing one sort of locomotive with another, and of balancing hoped-for economies in operation and maintenance on the one hand, against fixed charges for the additional investment required, on the other. Right there comes the mistake. A perfectly natural but yet a fundamental mistake, for which no individual or class should be censured but for which the unusual development of the art is responsible. We cannot blame railroad men for not being electrical engineers nor electrical engineers because they are not railroad men but the progress of electrification has had to lag until both should be able to see, each with the eyes of both. It is only by combining the railroad's man knowledge of the fundamental requirements of his service with the electrical man's skill in applying electricity to perform that service that all the possibilities of any specific problem may be developed.

The electrification of a railroad is not simply the substitution of one kind of locomotive for another. It is far more than that. It is the adoption of a fundamentally different method of train propulsion. It is conservative to say that, within the bounds of ordinary practise, electricity can furnish every train with all the pulling power that can be used. The limitations of the steam locomotive in this respect disappear and ruling grades rule no longer. A strictly limited motive power is replaced by one that is practically unlimited.

There are a number of so-called "systems" of electric traction and heavy emphasis has been laid by the advocates of each upon its points of difference from every other. So much has been said about these differences and so little about the points of similarity as to create an entirely misleading impression. It is a fact that there are more kinds and types of steam locomotives in use many times over than there are electric systems. It is a fact that except for the storage battery locomotive, which has but a limited field of application, all electric systems have many more common features than differences. It is a fact that they agree on fundamentals and differ in detail only. Their costs may not be the same, their efficiencies may vary but they all do their work and do it successfully and well. The possibility of unlimited electric power is a characteristic not of any one system but of all. It is due to basic differences between steam and electric equipment. A steam locomotive is a complete independent unit which not only generates but also utilizes its power. The electric locomotive generates no power at all. It is only a translating device receiving energy from an outside and a remote source. The electric power house always having much greater capacity than any one locomotive, can supply ample power for the heaviest train on the steepest grade. The steam locomotive which carries its own power house with it, is limited to the capacity of its one boiler. By the multiple unit principle, as many electric locomotives as may be needed can be coupled together and operated in synchronism by one crew from any cab. Any required tractive effort can thus be exerted without slipping the wheels, without imposing undue strains on the rails or bridges and without increasing the number of engine crews.

The business of a railroad is to transport freight and passengers. I put freight first because on the average it produces 73 per cent of the revenue. Unlimited motive power permits longer trains and higher schedule speeds. On the Elkhorn grade of the Norfolk & Western the schedule speed was doubled. It cuts the operating cost by hauling more cars with the same or a smaller crew. The Norfolk & Western uses two electrics to do the work of three Mallets. These new opportunities at one fell swoop banish many of the railroad's time honored traditions. The traffic possibilities must be studied from a new angle and advantage taken of every facility. It is a new thought to realize that train length is limited not by motive power but by the yard tracks and length of sidings, or that all the trailing tonnage that the draw bars will stand can be hauled. Nor are these new limits fundamental. Sidings can be extended, draw bars can be made stronger, if it pays to do it. In a word electrification opens up tremendous possibilities of increasing the freight capacity of a road and without it being necessary to build additional tracks.

While not as important as freight, passenger traffic likewise comes in for its share in the widened horizon and the vanishing tradition. Unlimited power of course is available but the absence of combustion is another basic advantage. Smoke and cinders disappear. Tunnel operation loses its terrors. Unobscured signals permit normal speeds with undiminished safety. Projects like the Pennsylvania terminal in New York depending entirely on submarine tunnel operation and previously impracticable, become immediately possible. Railroads owning valuable realty in cities can erect buildings thereon, where before smoky locomotives made any structure above the ground level impracticable. The aerial rights are now valuable. Multiple unit operation has in fact made suburban traffic. The rapid acceleration made possible by electric traction has directed attention to the equal value of rapid retardation and has quickened the study of braking accordingly; also of modified coach design to bring about the more efficient loading and discharge of passengers. These combined possibilities secure increased schedule speeds and attract patronage. The people not only get over the line in a shorter time but as a corollary more people get over it in the same time. Again, it is seen therefore that in passenger as in freight traffic the ability to do something that could not be done before, rather than to do the same thing at a lower cost is the most valuable attribute of electrification, and again we find a greatly augmented capacity without the need of additional tracks.

It is not my purpose to make an exhaustive comparison of the relative advantages of steam and electric operation. That has been done often and well by others. What I have said about the expanding opportunities for electrified service is by way of illustration to emphasize my plea that the question should always be viewed in its broader aspect and not hampered and restricted within any narrower limitations than properly belong to it.

I am going to assume, then, the broadest possible treatment and to suppose that every electrification project is to have its pros and cons most fully examined. The real and vital question then is, "How far will this lead us?" "To what extent may we expect complete electrification of all our roads?" Parts of a number of them have already been equipped. Many of these are numbered among our prominent roads, successful corporations which have had the advice of the most highly skilled executives and engineers, and which are progressive. The service performed on the electrified sections comprises practically every kind of railroad transportation. The Bluefield division of the Norfolk & Western R. R. in West Virginia is an example of an important coal road operating through the mountains. The Chicago, Milwaukee & St. Paul 440-mile main line, through Idaho and Montana, demonstrates what can be done by a transcontinental carrier on a large scale with through traffic, both freight and passenger. The New York, New Haven & Hartford R. R. 73-mile stretch between New York and New Haven shows how through freight and a heavy passenger traffic can be taken care of on the most congested four track section of an important eastern carrier and what is possible for complicated freight yard operation, while the New York Central and the Pennsylvania out of New York City are splendid examples of our greatest modern passenger terminal electrifications. There are of course many other electrifications, but even if there were not, those named are of a character to command the respect and attention of the railroad world. Now, every one of these projects has been successful. Every one has justified itself. Nearly every one in its present scope represents an extension of the zone initially electrified, the most convincing evidence possible as to what views the operating companies hold regarding these several projects. Railroad officials are generally glad to give others the benefit of their experience so it is reasonably safe to say that operating statistics are available covering long enough periods so that the results to be expected from any proposed undertakings may be predicated on established facts and not upon theories. In the light of present day knowledge, therefore, what answer can we make to the question "Should all railroads be electrified?"

Taken together in 1910 there were in the United States 240,000 miles of railroad main line regardless of the number of tracks. Of this mileage approximately 1250 or one half of one per cent has been electrified or is today in process. The remaining 99½ per cent comprises of course, roads performing every variety of service. They range from the back country branch line built by some over enthusiastic promotor and now perhaps, operated as part of a large system, only because operation cannot be avoided and regularly contributing its annual deficit, up to the most important through arteries of travel upon which the commerce and industry of the nation depend. Every sort of community is served; every kind of railroading has its place in this vast aggregation of effort and the variables in the problem are so numerous and their nature often so profound as to well daunt the courage of one who seeks to formulate them for incorporation in a general statement. Fortunately or unfortunately, depending on the point of view, it has been my lot to have to deal with this electrification problem from both sides. At one period from the standpoint of an intimate affiliation with the development and manufacture of electrical apparatus and at another from that of one charged with official responsibility on the railroad's behalf. I am a thorough believer in the virtues of electrification and an enthusiast about the wonders which it can accomplish but I also have a keen appreciation of the almost infinite variations in the railroad problem and a very wholesome respect for the dollar. I do not believe that all railroads will ever be electrified. I am not sanguine even that all the tracks of any one really big system will be so equipped in our time. It is a question of economics, and the results will not justify the expenditures even when considered with such broad vision as that which guided the Pennsylvania in spending millions to put their passenger terminal in New York City without the prospect of a direct return. Electrification will increase the track capacity. But there are thousands of miles of railroad that have sufficient capacity now, frequently several times over, and where the wildest stretch of imagination fails to picture a future need of this kind. Electrification works wonders in suburban and interurban passenger service. I have ridden for hours across the western prairies without seeing a single town, much less a city where these advantages would count. Electrification effects marked economies in fuel, in maintenance, in labor and otherwise through a long list; but electrification calls for a heavy investment and unless these economies bulk large enough, the interest on such investment will wipe them out and turn the enterprise into a losing venture. I do not believe the cause of electrification is helped by undue optimism on the part of its advocates. Rather should there be an enlightened partisanship, enthusiastic where enthusiasm is justified but tinged with the sober conservatism of the man who has to put his own dollars to work.

There need be no discouragement to the electrical engineer in the views just given, nor to the railroad man who has looked toward the new motive power for salvation. There are so many cases where electricity should be used, where its advantages are clear and conclusive, that once the railroads escape from the financial slough of despond in which they are now wallowing and are again able to get capital for their needs, there will not be enough engineers, there will not be enough electric factories in the country to serve them. Every big system has need of electricity somewhere. For some small roads it may mean the difference between solvency and bankruptcy. I electrified a short derelict line for the New Haven Road between Meriden and Middletown, long before given over into the one-train-a-day-annual-deficit class, and turned it into a good earner.

There can be no rule established. Generalities are sure to be misleading but electrification is now firmly intrenched and successful. It is recognized by railroads generally as an effective agency with great possibilities and one which is particularly valuable for certain specific purposes. Time alone will tell how broad its application is to be but I am confident we can await developments with tranquility assured that the art is in a healthy condition and that progress will be along the right lines.

DISCUSSION ON "SOME POSSIBILITIES OF STEAM RAILROAD ELECTRIFICATION AS AFFECTING FUTURE POLICIES (TOWNLEY). BOSTON, MASS., MARCH 14, 1919.

F. H. Shepard: We all know that electrification is a large but not impossible problem. Every hardshell railroad man can tell you that electric service, wherever it is adopted, always gives that section of the railroad better service.

Now, the transportation problem in the United States is probably the greatest problem we have. The railroads are the greatest employers of labor. The expeditious and regular movement of traffic is a matter which concerns the individual activities of everyone of us. We all know how our industries were halted when our lines of transportation were severely affected, due to the embargoes imposed during the last two years, and especially how serious this was in the case of our war activities.

The development of the United States is dependent entirely upon the freedom of transportation. For the proper development of our industries and for the success of our undertakings it is necessary that the broadest vision be exercised in the expansion of our railroad facilities.

If railroads are to expand in the proper way, it is necessary that the compensation for their product be adequate. Unfortunately we have been carrying on for a dozen years in this country a policy of unwarranted domination and witless regulation which has resulted in the starvation of our railroads.

The major expansion of a railroad is determined by the conditions of the future. No terminal pays for itself the first year, but terminals are necessary to secure proper transportation facilities. Revisions of line call for large expenditures, but they are not always immediately remunerative. If the return to railroad corporations is sufficient to carry them along, and to insure a healthful condition of finance, the railroads are ordinarily keen enough to take account of the situation ahead of them and look into the future. I think this starvation, more than anything else, has been the cause for the halt in railroad electrification.

The question in its entirety is a vast one, and although we will never see all the railroads electrified, none of us can even now say when we will see extensive electrification. However looking at the proposition in general terms, I think it may be said that the cost involved is something of the order of that which would be required to duplicate the present way, or tracks which now exist. That involves an expenditure which, if applied to include all the railroads, would amount to something like $15,000,000,000. This expenditure of course, is unthinkable at the present time, but gives an idea of the size of the problem.

The traffic on the railroads of the country doubles about each twelve years. Electrification can, in any particular situation, double the capacity of a railroad. The cost to move traffic primarily consists of two items: First, the rent for the investment, comprising the returns to the stockholders and bondholders, and second, labor and material which is practically all the product of labor. Now, electricity is a tool, and should be so considered, when analyzing electrification. It is a relatively new tool, unlimited in extent as to the amount of power that can be applied to a train. Mr. Townley shows pictures of locomotives having 4000 continuous h. p., and 7000 h. p. during acceleration. I know of projects which contemplate the use of 8000 h. p. continuously for a single train, also of tests in which the input has reached 8000 h. p. for a single locomotive and 15,000 kw. for a single train. These are relatively large figures.

The use of electricity as power in industry is fundamentally a labor-saving device. The great expansion in England after the Napoleonic wars was due to the use of steam which supplanted labor. Electricity is more flexible and can secure in all industry and in railroad electrification as well, more effective use of labor.

All steam railroad operation has been built up around the steam locomotive. The steam locomotive requires attention at about 100-mile intervals, terminals have been built up, and the application of labor has been determined, by the limitations of steam locomotives, so that the present method of assigning labor has thus grown up and along with it a lot of arbitraries all of which prevent, in the substitution of electricity for steam power, the new tool from showing all the fundamental advantages it should produce.

It is quite conceivable, for instance, that a through train can take its electric locomotive and run 500 miles. The operation of an electric locomotive does not impose the burden upon the operator of that train that is imposed upon him in the case of the steam locomotive. The electric locomotive, properly operated, and adequately designed for the service, will run without frequent attention and at a very low minimum of expense for maintenance and upkeep as well. We all know that a motor in a factory, provided it is oiled and occasionally wiped off, will run almost forever. Your fan motors do the same thing. It is reasonable to picture an electric locomotive performing service in a somewhat comparable way. The electric locomotive does not possess all the variables which influence the proper performance of the steam locomotive—such as the kind of water, the grade of fuel, the condition of the flues, firebox, etc., the skill of the fireman or of the locomotive driver, all of which limit the ability of the steam locomotive to perform successfully, during shorter or longer periods. The steam locomotive is a 100 per cent engine when it comes out of the shop. Its capacity is never as good after it has seen service.

The economies of electrification which will be possible when this new tool is thoroughly developed and understood will be greater than we now imagine. The possibilities of railroad electrification I consider are scarcely scratched.

I have in mind a rather extensive project on which the cost of fuel was actually less than the contemplated cost for electric power, but the other advantages, by a rather more extensive revision of service than is ordinarily contemplated, show an overwhelming advantage for the electric proposition.

I think Mr. Townley has touched a very important point in saying that our vision has been rather too limited as to the possibilities of electric power in electrification, and that great developments will follow a better analysis, based upon the possibilities of revision of railroad service.

W. S. Murray: Epitomizing the paper itself, it seems that its fundamentals are: that we have been prone not to combine the railroad and the electrical man in the past. In the future we must have a composite man, with two eyes, one eye for the railroad conditions and the other eye for the electrical conditions. That is a very important part which has been omitted in the past.

Another point in Mr. Townley's paper is that we must not try to electrify all the railroads in the United States. That is certainly patent, for the various reasons presented by Mr. Townley. He has pointed out that in all forms of electrification, pertaining to terminals, freight, passenger and switching service, in fact in all services the electric locomotive has performed very successfully, and that is significant.

Now, the thing in electrification to which I would like to draw particular attention is this: If we selfishly think of the electrification of one road or two roads, it is very true that we may reap a great many economies from that electrification, but it seems to me that is not the right way to go after this matter. If we are going to make electrification a success in this country we must do district or regional electrification.

In 1914 I had the pleasure of discussing with one of our eminent Institute men a plan of electrification which I think will appeal to you. It comprehended a district of electrification. There are six roads on the west side of the Hudson River, just outside of New York City—the Lehigh Valley, the Central of New Jersey, Delaware, Lackawanna & Western, the Pennsylvania, the West Shore, and the Erie. It seemed to us at that time that there was opportunity offered to work out a Power and Equipment Company with the idea of applying standard equipment to these six railroads, for handling electric passenger, freight and switching service within division zones of the six roads. A power station of 200,000 kw., or whatever capacity might be necessary was contemplated for location central to the roads.

At that time the railroads were much embarrassed, as they are now, in the matter of finanace, but the thought occurred that the railroads could be relieved of the financial burden through the agency of equipment trust bonds to cover the cost of locomotives and multiple unit cars and the contracts for power would be a guarantee for the bonds necessary to build the power station thus making the only cost to the railroads the building of their over head contact systems.

It was contemplated that unit shopping facilities would be arranged to cover the equipments of all the roads and the building of these shops would be merged into the banking syndicate who would underwrite the concern to be known as say, the Power and Equipment Co.

I thought and pondered a great deal over that electrification problem and it rather automatically expanded itself from a district problem into a regional problem. I have given much thought to Secretary Lane's proposed super-power generation, transmission and distribution plan for application in the regional district between Boston and Washington. By such an arrangement is offered the opportunity of propelling all trains in this region by electricity, and at the same time supplying all industrial concerns with light and power. I am most thoroughly in accord with the Secretary's advances in regard to this matter. It contemplates a trunk line transmission system between Boston and Washington, the route of which will lie sufficiently removed from the coast, as a military precaution, and which will include the use of steel transmission towers supporting wires bearing 3-phase current at a potential, say, of 220,000 volts, these lines, probably three or four in number, lying parallel to each other and on private right of way.

The super-power transmission would tie together all of the stations, steam and hydroelectric, which latter might be built on the rivers flowing to the coast between Washington and Boston; and large power houses to be constructed at the mouths of mines. Into this super-transmission trunk line system, would be fed this highly economical power.

You might say, right away—what would be the advantage of such a line, and could we not, on account of the very large stations we have developed, produce power as economically, and why, therefore, go to this investment in the super-power transmission line?

Now, our operating investigations show that with units of 25,000 to 40,000 kw. we can safely depend on being able to produce a kw-hr. for something between 10 lb. and 15 lb. of steam. I venture to say that over that regional district the amount of steam required to produce a kw-hr. as applied to the railroads and to the factories will average 25 lb. to 30 lb. and consequently, there would be offered an enormous saving in the production cost of power. This is not, of course, to preempt the use of any large stations that are already in existence as they, if their efficiencies justified would contribute their quota of power. And even those plants of lesser efficiency than the super power plants to be built would serve two most important functions: *First*, contributing power during the peak hour of service and acting as voltage regulators thus equalizing the potential over the regional zone; and *second*, during the periods of lesser load offering their capacity as synchronous condensers so that the idle current would be absorbed by them, thus permitting the super-power plants to work their great units of forty to sixty thousand kilowatts up to their maximum and economic generating capacity.

A great number of bills, as you all know, have been held up due to the rather disorganized condition in which Congress found itself at the close of its last session, and this bill,together with the others, was sidetracked, but it is hoped that the new Congress will take the matter up just as promptly as possible and appropriate a sufficient amount of funds to make the investigation.

Now, I said a little while ago that I would like to speak of something other than electrification, or at least speak of it in connection with electrification. It is this—and it is the progenitor of electrification—the amount of coal that we are using unnecessarily in this country, and particularly in this so-called North East Atlantic Coast section, is criminal. I was talking with Professor Breckenridge of Yale University, who you know has the chair of Mechanical Engineering there, and he told me that we were mining from 75,000,000 to 100,000,000 tons of anthracite coal per annum, and that in seventy-five years our supply of coal would be gone. He told me, in nearly the same breath, also, that they were already seriously at work trying to develop bituminous coal furnaces for house heating, in order that this situation could be taken care of when the anthracite coal was gone. He followed these remarks by saying—"Murray, you may not know that over a very short period of years we have by leaps and bounds gone up from the mining of 100,000,000 tons of bituminous coal to over 500,000,000 tons."

Now, I ask you to visualize this matter. I know from our Government reports that today 33 per cent of our cargo space is taken up by coal shipments. I venture to say that the percentage in New England is 40 per cent.

Now, what is the reason? A very great deal of it can be put down to improper power application by virtue of burning two pounds of coal where one is only necessary, and probably three pounds. I think I can speak with some authority so far as the railroad end of this matter is concerned, as I have put in ten pretty intensive years in the matter of determining just what the coal consumption was as between steam and electric service, for freight, passenger and switching engines.

Now, what are the figures? With the units in the Cos Cob

Station of the New Haven Road averaging 4000 kw. it was shown that, beyond any peradventure of doubt, the saving in passenger service was 2 to 1, in freight service 2.5 to 1, and in switching service 3 to 1. So, you can see right away, from the railroad side, that the coal saving would be very prodigious, with the super-power transmission, as the energy received into the transmission lines would come, not from units of the size I have mentioned, but from units of ten times that size. In fact, I think I can visualize such a line, and I believe that we all in this room will see this thing some day,—such a super-power line picking out districts which will warrant substations, ranging from 100,000 to 150,000 kw., or maybe 200,000 kw., and distributing to them its highly economic power. I know that is a pretty large order and involves a large sum of money, but in the same breath I want to say we are on the verge of very large things. Do you realize that lack of power is now limiting our industrial expansion on the Atlantic seaboard? And when I say to you that industrialism, which so tremendously depends upon the railroads for shipment of raw commodities, has got much of its cargo space preempted for the carrying of coal, why is it that we cannot with reason view the construction of a common carrier system for power thus lifting the coal off the roads and automatically creating the cargo space for the transportation of raw commodities and finished products.

The world is full of big problems, and I believe there are just as many big men in the world to solve these problems. I want to offer you a slogan, which has come to me, and it is this—We spent billions of dollars for destruction, for preservation; now let us spend billions of dollars for construction, for conservation.

And there comes to me another slogan as I visualize this transmission line which we must back Secretary Lane in building, and that is: The primaries of today will be the secondaries of tomorrow.

We are developing today on the Housatonic River, with special reference to the storage of water, power stations that will contribute over 350,000,000 kw-hr. yearly and this is just a little section, in that tremendous industrial district of the Naugatuck Valley. What does that mean in national conservation? Why, it means an equivalent of 500,000 tons of coal a year. That will be one of Connecticut's contributions to conservation.

During the war I had the privilege and the opportunity of making a report not a great while ago to the Government in the matter of what should be done to take care of the shipping load in the Philadelphia district. The Hog Island situation was such that additional capacity was absolutely necessary in that district at peak load. There is a river up in Pike County, Pennsylvania, called the Wallenpaupack, which we found had a basin which would hold seven billion five hundred million cubic feet of water; the physical conditions offered a draw down of twenty-six feet. The head is 300 feet, and the penstock would have to be about 5500 feet in length. There was opportunity to develop anywhere from a twenty thousand to a one hundred thousand kw. plant. There was 80,000 to 90,000 kw. required at Philadelphia. The rainshed of that river is 240 square miles with something like 31 or 32 inches of rainfall per annum. The cost of that development and the 102-mile transmission line from Philadelphia was $12,000,000., and the cost of a corresponding steam plant of equal capacity at tidewater in Philadelphia, was $13,000,000, and the actual figures which the operating account showed, put to the credit of the hydroelectric station to supply that power in any form wanted, (kw. or kw-hr.,) $1,000,000 per annum. That meant the steam plant had to be amortized at $20,000,000, before it began to compete with the hydroelectric station.

There are quantities of resources in the form of water powers along the coast line between Washington and Boston, and the system proposed could be extended to Portland, Maine, in one direction, and down to Richmond, Virginia, in the other; but just as Mr. Townley has pointed out, the *sine qua non* as to whether or not electrification is applicable—I do not care what form of traffic it may take—whether it is used for transportation or for factories—traffic density is the thing that is going to settle the question. It is by the reliability and economy of the electric drive, over the steam drive, that the dollars are saved, and as has been pointed out by Mr. Shepard, you must consider the fixed charges that the investment must be burdened with, before you can give a concluding answer to the problem.

Now, I want to leave with you this thought. In discussing this matter of the super-power line with a number of engineers, it has been a significant fact that I have not found a single dissenting voice, and everyone has agreed that it is the proper thing to do. Somebody might say—Why, Secretary Lane's plan is all imaginary. Now, that is wrong, and yet I will confess in the same breath that there is imagination in it. I think a man who has not imagination needs a guardian.

No doubt there are in this room many holders of patents. You had to imagine the piece of machinery for which you secured the patent, before you could make it, and so in this case of the super-power line, it had to be imagined—the rest of it is entirely practicable. There is nothing in it that is impracticable. What I have said about the financial return can be proved in advance, before the scheme is put into operation, so, let us make the investigation and find out; do not let us get too far behind England—England is doing just the same thing. I admit England has not anywhere near the area the United Stsates has, but that is not the point. We have got as much area right in this district, this regional district I am discussing with you gentlemen, as England has, and we have the advantage that we can extend that district, and if we have to compete with England, maintain the American standard of wage and living. I am thinking that we must have some super-power scheme of generation, transmission and distribution of power to our factories, by which the necessary economies can be effected and thus put us in position to compete with the world's trade.

I would like to say that this nation owes a job to every returning soldier, be he returning from Europe or from his post in the United States. There are two million sons of America returning from over the seas. Can they not fairly ask—Having you been thinking of us over here while we have been fighting for you over there? Millions of dollars must be spent in factory and in field during this reconstruction period and by these factories, supplied by power from this super-power transmission and distribution line, it will be possible for us to create new national wealth and automatically devise ways and means for the employment of American labor, this will provide us also with a fair answer to the fair question of the returning soldier.

Before I sit down I would like to say just about two words about a paper which I read, written by Dr. Arthur D. Little. The paper is called "Developing the Estate." It is printed in the *Atlantic Monthly* of March. If I could be mean enough to take any more of your time, I would ask that I might read that article to you. What I have said is a patch. What he says covers the field, and I commend that paper to you gentlemen.

Carl Schwartz (read by J. B. Taylor): Ten years ago the impression prevailed quite generally that the highly successful operation of several systems of steam railroad electrification then completed was all that was necessary to open the door for the replacement of steam by electric motive power on a large scale. Few electrical engineers realized, in those days, that the cost of the necessary electrical equipment constituted only a small fraction of the total capital cost of steam railroad electrification. Many other essential items of expense and very large ones indeed, were either overlooked or not fully appreciated; for example, the rebuilding of tracks, the installation of new signal systems, the extensive additions to maintenance facilities, the changes in terminal and switching arrangements, elimination of telephone interferences, etc., etc.

Since that time the question has been in most cases, how can the necessary return be secured for the additional large investment required for electrification? Generally applied, and as matters stand today, electrification would result in an increase in the cost of transportation, because the additional burden of capital expense cannot be compensated for by a corresponding decrease in the cost of operation and maintenance.

It is immaterial to a shipper of freight whether steam or electric motive power is used, nor does a trunk line passenger care what type of locomotive propels his train. Trunk line electrification in general would largely benefit interests other than the railroads, the traveling public or the shipper. Only in the surburban zone of commuters' traffic, by affording increased speed and frequency of operation combined with greater convenience and comfort, does the value of electrification appear to the passenger.

Who, therefore, is to pay the bill?

Purely from a railroad standpoint, electricity has one important advantage over steam motive power, namely, it is a most effective means of increasing the capacity of the transportation machine, and where this applies to the extent of producing additional revenue so as to compensate for the additional investment, electric operation readily justifies itself.

One trouble has been, and it is today, that the balance sheets of the railroads have not shown a favorable relation between revenue and expenditures; otherwise, many roads would have been more favorably disposed towards electrification in spite of the heavy expense involved and notwithstanding animated discussions of the past as to which system was the best to adopt.

Having in mind that, in general, the electrification of steam railroads may be promoted by devising means for reducing capital cost requirements and by simplifying installation as well as operation and maintenance, the writer submits the following points for consideration:

1. The value of a clear line of distinction between:

(a) The generation, distribution and supply of power as a finished product to the transportation machinery.

(b) Railroad electrification proper. This is to cover only means for the operation of the transportation machinery.

Under this classification, item (a), namely, the supply of the necessary power, would be a matter for an industry already well established in the special business of commercial power production with all economic advantages accruing from a pooling of loads of a widely diversified character.

2. The value of standardization of all motive power means along the broadest lines. The writer believes that the time has arrived when this can be done, and that it does not necessarily mean the exclusion of special systems which have proven their merit.

John B. Taylor: Referring to the third paragraph of Mr. Schwartz's discussion, the sentence ending with the words "locomotive propels his train." I can hardly agree with that statement, because I do not think it is correct. I know that many parties, in taking a transcontinental trip, prefer to take the Chicago, Milwaukee & St. Paul, because for a number of miles it is electrified. They have the opportunity to see the mountain scenery free from smoke and nuisances, and that really is a large factor in their determination to take that line.

Mr. Schwartz says at the end of the same paragraph "Only in the surburban zone of travel."

Just why he concedes it in surburban travel and not in transcontinental travel, I do not know.

N. W. Storer: Mr. Townley is right in his statement that the electrical engineers are very largely responsible for the slow progress which has been made in electrification. We are responsible, as he says, in the first place, because we made promises, estimates, which were not fulfilled. It was impossible to fufill them. He is also right in saying that we did not promise enough. I do not believe there is a man in the room here now who can with any degree of accuracy or comprehensiveness say what must ultimately be the result of electrification in this country. I do not think there is anyone with broad enough vision to comprehend all of the advantages and the things which can be developed from a complete electrification of our big systems. These are simply possibilities of years of evolution. They have got to grow, and they will grow just as any other problem of electrification, of power-using apparatus, has grown.

One reason, as I see it, why the electrical engineer has fallen down, so to speak, in his work, is that he has not been close enough to the railway situation—the electrical engineer has seen just one part of the problem, and he has sought to solve that part. Possibly he thought he was solving the whole thing when he undertook to change the motive power. That is the thing that was uppermost in his mind, to get a locomotive or motor equipment which would handle the traffic which the steam locomotive has been doing. Of course, that, we now know, is a very small matter. We must thank our esteemed railway friends for holding back on the job of electrification. We have got to thank them for their conservatism, although it has gone pretty hard with us sometimes, to appreciate its value, but the steam railway man is one who is distinctly from Missouri. He must be shown exactly what he is going to get before he invests any large amount of money in it. There are occasionally men with very broad vision, who will go ahead and undertake things in a pioneer way, but, taken as a whole, the steam railway man must learn through his own experiences.

All who have had any experience in electrifying roads will understand that very thoroughly. We see the mistakes which they are making, we see them attempting to operate electric locomotives on exactly the same basis as steam locomotives have been operated, and we know they are going to have trouble. We tell them so, but it makes absolutely no difference, and they go ahead and operate them, until something breaks down. They have a lot of breakdowns. They have trouble, so much trouble that they begin to appreciate where they were wrong, and they go ahead and take steps to change the situation.

I believe that the whole method of operating the roads, which, as has been pointed out, has grown up around the steam locomotive, will have to be changed. I think that the changes will be so vast that we cannot really understand them at this time, because the power is so different and its application requires entirely different control.

One of the first things that the railway man must understand is the value of proper distribution of his power requirements throughout the day, twenty-four hours a day. With electric power we can on many roads supply power enough at the head of a train or at the head and rear of a train, to handle all of the traffic with one operation per day, perhaps one train would do the whole thing, but everybody knows that that would be a foolish thing to do. It would mean equipping the entire road for that particular maximum peak load, whereas, to get the most economical results, so far as the equipment and the distribution system and the substations, and locomotives, too, are concerned, you would want to distribute them over a very much larger period of time.

When it comes to finding just where the limit is I think we have got to strike a happy medium between the cost of labor, which increases with the number of trains, and the cost of the investment necessary to equip the road and handle the maximum number of trains. It costs money, for instance, a great deal of money, to equip the road to handle the fleets of trains which are sometimes sent out by a western road, maybe loaded with perishable freight, cattle or fruit of some kind. They want to send out a train every ten or fifteen minutes, or every half hour, for six or eight hours, and then the work is done, and they may not have any more freight for another week. The application of electrical operation to such a proposition is clearly out of the question. It would not pay at all to equip the road for such heavy service at such infrequent intervals. The load must be properly distributed. Lines like that, as I take it, are the ones where the conditions are arbitrary rather than real, as Mr. Townley has pointed out.

There is another thing—we are sometimes told that we must be able to empty a yard or clear a track of congestion, and perhaps send a train out every five minutes until the yards and tracks are all cleared. Consider what that means in cost of insurance—that is what it is—simply insurance against delays. Then maybe a delay of possibly four or five or six hours due to a wreck—the traffic piles up—and there are a half-dozen trains on the siding—but why would the avoidance of a delay of a half hour or an hour longer, in getting all of these trains out of the way, be worth double the cost of the substation and overhead equipment supplying that immediate section of line?

These are some of the points that occur to one in considering the electrification of our roads.

There is another point which has been touched upon in the discussion, and that is with reference to a general electrification of all of the roads of the country. I do not think we need to discuss particularly the possibilities of electrifying all of the roads of the country. We have troubles enough trying to electrify one-half of one per cent of the mileage. If we had ten per cent of the roads on our books at the present time, with contracts for electrifying them, we would be overwhelmed with business. It would multiply the necessary numbers of engineers by fifty, or possibly one hundred. All we need to consider at the present time is the most crying necessity in the railroad field—the tunnels, the heavy grades, and the big passenger terminals.

Speaking of some of the advantages in the big terminals, for instance, Mr. Townley mentioned the advantage that could be taken of the aerial rights of the railways, and that has always appealed to me as one of the big things, to enable a railway occupying very valuable ground in a large city to cover their tracks over completely with buildings.

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I have felt that it would be quite possible to build immense warehouses over these tracks in the congested parts of the city where the freight could be removed aerially. For instance, is there any reason why freight cars should not be run in on an elevator in one of these buildings right over the track, and shoved up to a higher floor, unloaded where it is not occupying the track at all—run off the track entirely and unloaded—and maybe loaded up with new freight in the same place and quickly sent out?

Our biggest trouble has been in the terminals situation—the terminal congestion around the big seaboards was simply monumental, and it was responsible for the unprecedented delays in the operation of our transportation system and the breakdowns which took place a year ago. Cars were held for weeks, in some cases, where a day should have been sufficient to unload them and send them back over the line to earn more money.

The vision which Mr. Murray has unfolded to our view here today is most inspiring. I wish I were as tall as he is—if I were I feel I might see over some of the intervening space and obstacles in the way and see the day, even clearer than he has pointed out, when this will be an accomplished fact. All of these things which he has pointed out to us are easily within the range of possibility, and what we must look after in the immediate future is the conservation of our resources. We owe it to the future generations to be more economical in the handling of the resources of the country, because in a few years many of these commodities, like coal and iron, will become much more expensive, and they will become so scarce that it will be absolutely imperative to develop other means of supplying the power requirements for the country. One of the greatest things we can do is to exercise all of our ingenuity in saving power. It is said that a public benefactor is one who can make two blades of grass grow where only one blade grew before, and along the same line, if Mr. Murray, or some of the other engineers in that line of work, can make one pound of coal do as much work as two or three pounds can do now, one can see what that means when it is multiplied by the hundreds of thousands of tons of coal which are used every day.

I have been immensely interested in the economies that have been introduced in many of our street railroads, and steam railroads as well, in the saving of power, saving of coal. In the case of the street railroads a saving of ten to twenty per cent in the saving of power consumption has been brought about by different means of operation, introduction of some of these coasting clocks or watt-hour meters or similar devices, something to put a check on the motorman, as by this means they can easily save ten per cent, and possibly in some cases fifteen per cent of the power consumption.

The use of a more efficient form of control, as by controlling the speed, partly by means of field variation, known as field control, will save ten per cent of the power that is ordinarily required to operate street cars and trains on elevated railways, and if it becomes possible, as I believe it will in the near future, to introduce regenerative control in the case of some of these frequent stopping trains, there will be a possibility for another ten, or fifteen, or twenty per cent saving. We have got to be open minded in regard to propositions like that, and must seize every opportunity to bring them about.

The cost of electrification is very large, but it does not look one-hundredth as big to me as it did four years ago. When we consider that Great Britain spent enough per day during the years of the war to have built the Panama Canal in possibly ten days, and our country was spending enough in four or five days to electrify all of the terminals in Chicago, what does the cost of electrification amount to? It is a drop in the bucket, and does not amount to anything.

Our people have become so used to giving, in the last few years especially, that I believe they are going to insist on these improvements, not simply because they are going to save money, but because they are going to require better service, and it is going to be necessary because of the saving in labor. It will increase the productiveness which we must have in order to avail ourselves of the luxuries that are possible in this age.

W. B. Potter (read by S. T. Dodd): Progress in any branch of scientific endeavor would be automatic, if it did not involve some change in well established practise, and usually the physical changes involve an expenditure of money. I believe this is especially true of railroad development. Mr. Townley's reference to these requisites is worthy of careful consideration.

Electrification necessitates no radical change in the handling of transportation but minor changes are usually essential in order to secure the full benefit made possible by the increased power, speed, and continuity of service in the individual unit as compared with steam. The objection is sometimes cited that electrification as compared with steam does not provide equal facilities for handling an emergency congestion of traffic. This is so affected by other conditions as not to be always true, but, granting this truth in some instances, it is an emergency only and should be regarded as such. This condition I believe is more than offset by features of electric railway operation which go far towards removing the possibilities of congestion. Consider one item alone: that the electric locomotive is at its best during coldest weather, under conditions in which the steam locomotive is most limited in its capacity. This feature was especially noticeable during the winter of 1914 and 1915 on the C. M. & St. P. electrified section. During this time there was only partial electrical operation and there were long periods in the Rocky Mountain division when the thermometer stood at 40 degrees below zero. Numerous instances occurred during this cold spell of steam locomotives being frozen up and with their heavy freight trains standing dead on the main line. Under these circumstances the electric locomotives had an opportunity to demonstrate their freedom from temperature limits by repeatedly going out and clearing the line of dead trains and engines.

The investment for equipment is undoubtedly the most influential factor affecting the advancement of electrification. Herein exists the duty and opportunity of the electrical and the mechanical engineer. With the engineer rests the responsibility for devising and utilizing such apparatus and equipment as will insure the best economic return in successful service.

A subject of present consequence and of vital importance in the future is the fact that electrification operated from hydraulic power offers the only known method of conserving our limited supply of coal and oil. Even where hydraulic power is not available a saving of over two-thirds of the fuel, and much of the fuel haulage, can be obtained by the erection of steam power plants suitably located and furnishing electric current from coal burned in modern power houses. An indication of the coal consumption and the traffic lost incident to burning fuel on a small scale in individual units is given by the fact that nearly one-fourth of the coal mined in the United

States is used on the tenders of our steam locomotives, that five per cent of the ton mileage moving over our railroads is occupied with hauling of this coal for railway purposes and that seven per cent of the ton mileage is occupied with hauling this coal again in tenders back of the steam locomotives.

To repeat the statements above in a little more concrete form I may say that the coal and equivalent oil used on steam locomotives in the United States for the year 1914 was equivalent to approximately 140,000,000 tons of coal. The ton miles moved during the same period, excluding the tonnage of locomotive tenders and excluding 75 per cent of the railway coal, amounted to 930,000,000,000 ton miles. If the same tonnage had been moved electrically, basing our figures on a power consumption of 40 watt hours per ton mile, the annual power consumption would have been 37,200,000,000 kw-hr. If this electric energy had been obtained from steam driven power houses with an efficiency of 2.2 pounds of coal per kw-hr., there would have been required 40,000,000 tons of coal instead of the 140,000,000 actually used. This shows a net saving of 100,000,000 tons of coal in one year. This is based on generating the current from steam driven power houses, whereas a still greater saving of coal would have been shown proportionately to the extent we assume hydraulic power to be used.

There are frequently misconceptions as to the amount of power involved in railway electrification. The figures given in the last paragraph form a basis for illustrating this rather emphatically. If the tonnage moved in the United States on steam railroads could have been moved by an expenditure of 37,200,000,000 kw-hr., this would have been equivalent to an average load of 4,250,000 kw. This is not an exorbitant amount as compared with the power stations already installed. The data for the year 1917 indicate that the power station capacity in the United States for central stations, electric railways, and isolated plants, is of the order of 20,000,000 kw. In other words, there is already installed electric power station capacity equivalent to four or five times the power which would be required for operating all the railroads in the United States if the same were electrically operated.

While we agree with Mr. Townley that it is very improbable that all the railroads in the United States will ever be electrified, it is at the same time interesting to note what the power demand of these roads would be and the saving in coal which would result from their electrification.

The further development of central distributing stations with either fuel or water as their source of power, will afford the most direct means of conserving coal. The existence of such large central stations located at strategic points throughout the country, even if primarily installed for railway use, would of itself further develop the use of electricity for various industrial purposes, and by so doing would affect the coal consumption more widely than in the item of railway coal alone.

George F. Sever: I want to bring to you a message from the Pacific Coast where I have been stationed for about seven months. Coal is almost an unknown quantity in that part of the world. One thinks only in terms of oil and water.

There oil is used up and down the Coast for electrical generation and industrial operations, and by the railroads. The railroads use somewhere between 30,000,000 and 50,000,000 barrels of oil per annum, while public utilities in California have used in 1918 about 10,000,000 barrels of oil. Our new commercial shipping fleet is asking for 34,000,000 barrels of oil for the year for carrying our commerce over the seas. Besides these amounts there are about 40,000,000 barrels of oil used for other industries up and down the Pacific Coast.

California itself produces about 110,000,000 barrels of oil per annum, while the whole country, at the present time produces about 350,000,000 barrels. This means in California where much of the oil is used for the operation of the railroads, that hydroelectric development must occur not only for the prospective operation of the railroads over the mountain passes, but also for the operation of the electric public utilities that are using approximately 10,000,000 barrels of oil per annum.

The Oil Administrator for the Pacific Coast, as well as the Federal Oil Administrator, feels that the oil production of the United States will not increase at a high rate. As a matter of fact, they contemplate that it will decrease. The price of oil in 1914 was between 70 and 85 cents a barrel, and at the present time in California, at tidewater, the price is approximately $1.60, and in other parts of the Pacific Coast it ranges from $1.60 to $1.85 per barrel. Every public utility that has available undeveloped hydroelectric power should substitute this for its oil-burning operations, down to a certain limit.

I made careful investigations of hydroelectric possibilities and I advocated developments which, had the war lasted two or three years longer, would have been made to meet the growth due to war needs and those of the different industrial communities. It was clearly shown that over $3,500,000 could readily be saved by the installation of hydroelectric developments in California,

There are three mountain passes of the railroads that can be readily electrified, the Siskiyou in the north, the Sierra Nevada in the east, and the Tehachapi in the south, where both the grades and the loads are heavy.

This country should take every step to conserve fuel oil and the railroads especially should be carefully studied in an endeavor to co-operate in this respect.

Last spring, I made an extensive investigation of the hydroelectric power possibilities in New England in connection with the War Industries Board. At that time there was set forth to the Board the necessity of delivering all the available hydroelectric power in New England into the present transmission trunk lines, increasing and delivering to the same lines from the south and east all the steam-generated kilowatt-hours from the present steam-stations.

The number of tons of coal used by public utilities and manufacturing establishments in New England is now so large, that if all the hydroelectric kilowatt hours available were developed, their substitution for steam generated kilowatt hours would afford slight opportunity for transmitting any hydroelectric kilowatt hours outside of the confines of New England itself.

Charles F. Scott: Mr. Townley has presented a large view of a very large subject. He points out that it is the removal of the limitations of the steam locomotive which is the great thing in railway electrification. This is also the great thing in electrical work of many other kinds; electricity not merely does better what was done before but its great value lies in the new methods possible, due to the removal of prior limitations.

The removal of the limitations in the mechanical transmission of power have made possible the generation of power in new ways, both in great steam power stations and in the large utilization of water power. In its applications to lighting, to power in industries, to commercial heating, it is the new possibility of the effective application by which limitations in other methods are overcome and results of a new kind are secured which bring out the higher values in substituting electricity for other agencies. We have been principally concerned in the past as to how well the electric locomotive could do what the steam locomotive did, usually under some particularly trying situation such as tunnel service, but now Mr. Townley invites attention to the changes in railway operating methods which may bring out the full advantages of electrification.

At the convention at Great Barrington seventeen years ago we were wondering how we could use a voltage above 500 or 600 volts in railway service. Soon the single-phase system appeared, presenting one method, then followed the direct-current and now both are in operation and we can hardly tell which is the direct-current and which is the alternating-current system by the character of the service which is performed.

Another large vision has been presented to us. There has been a change in the attitude of engineers. A number of years ago the American Society of Mechanical Engineers was received by the Mayor of the city where the convention was held and in a fine address he greeted them as a Society of Stationary Engineers, the men who run engines. We have been addressed by a Governor today and what does he say? He says that we are those who minister to the general welfare;

that we have a goal towards which we are working which is the realization of the aspirations of mankind; that we are men engaged in making this world a better place to live in. It is very heartening to be greeted in such terms and it puts upon the engineer a great responsibility.

I wish to propose, Mr. President, the following resolution, and will move its adoption:

RESOLVED, That this meeting heartily endorses the investigation of the power resources and requirements of the North Atlantic coast region which is advocated by Secretary Lane, and that the Board of Directors be requested to consider the passage of suitable resolutions, and the offering of the facilities of the Institute, in support of the project.

George F. Sever: I second the resolution.

(The resolution was put to vote and unanimously carried).

George Gibbs (by letter): I was especially glad to see Mr. Townley bring out a point which needs discussion, that is, the best manner of obtaining coordination between the electrical engineer and the railway operating man, so that a railway electrification may represent all the advantages to be derived from improved motive power and operating methods combined.

In the early days of electric traction (not so long ago, either) railroad men were principally concerned with the question whether electric apparatus was capable of performing heavy railroad service, and electrical engineers were busy weeding out defects and limitations in apparatus. It was during this period that much heated discussion arose among engineers as to the best electric system for general adoption. The settlement of this particular question at the time was, to say the least, premature, as has been demonstrated by the parallel development since of two or more systems having great flexibility. Even now, when it has been conclusively proven that electric traction by more than one system is technically feasible, evolution is still in process, and standardization of system and a discussion of standardization apparatus, except as regards certain general features should, I think, be postponed.

The question has really now changed from one of technical to one of financial feasibility; in other words, railway men wish now to know under what conditions heavy steam railway electrification is a good proposition as regards operating and financial results. There are few railroad companies who can, in these times, face the very heavy expenditures required for electrification except to secure immediate and large operating economies either directly or by an increase in the capacity of the road, or in the stimulation of new business, any one of which factors or all taken together producing added net revenue sufficient to at least pay fixed charges on the new investment. Of course there have been a few cases where absolute necessity determined electrification to make possible an important railway improvement, for instance the underground terminals and tunnel systems in New York City; but aside from such cases, the essential thing is a demonstration of the financial feasibility before general adoption of this modern improvement in operating means can be expected.

As the Institute members doubtless know, the attention of our Government was directed during the war toward electrification, as a possible way of increasing the capacity of the railways for emergency service; also for conservation of our fuel supply. It may be that some important electric projects would in this way have been launched, as war measures, had the war continued, especially as results and not costs were then all-important. But it was quite evident that the Railroad Administration was somewhat at a loss to determine just what it should do in the matter, in view of the rather conflicting views expressed in and out of railway circles by technical and non-technical advisors. This brings me back to the main point brought out by Mr. Townley, as of importance for us to now direct our attention to, viz: we must find some way of attacking the railway electrification problem with a combined technical and transportation knowledge.

I cannot too strongly emphasize the fact that any general electrification of the entire railway system of the country is an economic absurdity by any present electric system. But there are, on the other hand, a number of instances where electric traction is indicated as necessary or advantageous. In some cases the direct savings will be sufficiently attractive to warrant the expenditure, but more often the deciding factor should be the indirect savings produced. The determination of these indirect savings generally involves important alterations in operating methods, facilities, etc., and it is therefore essential that engineers who are called upon to report upon electrification projects should be familiar with transportation methods and possibilities as well as with the electro-technical side of the problem. I regard unfamiliarity with the actual workings of railways as the chief handicap of the purely electrical expert in giving advice to companies who are considering the adoption of this great modern improvement in haulage means.

John Murphy: In Canada, as many of you know, we had a very serious coal shortage extending over two years, and a power shortage in part of Ontario extending over one and one-half years. It was my lot to be fairly close to these two shortage situations, and the question of railway electrification consequently came somewhat forcibly to my attention.

My view of the urgency of railway electrification as a measure of conservation can probably best be summarized by the following statement:

Only three or four per cent of the mechanical energy (represented in the heat value of the coal) is available in the shape of power at the wheels of a steam locomotive. Is it not our duty to try to save a great deal of that 96 or 97 per cent which is now wasted? Railway electrification can save from 50 to 66 per cent of the railway coal when the electrical energy comes from central steam stations. Water power can save it all.

But for the generosity and the wisdom of the U. S. Fuel Administration there would have been much suffering in Canada and many might have died from the cold during the past two or three winters. Our ships, as well as our other transportation facilities could not bring us coal because they were carrying men and materials to Europe to fight the common enemy of humanity. The officials of the U. S. Fuel Administration acting upon the advice of Canada's Fuel Controller, Mr. C. A. Magrath, generously and, as I said, with wisdom, alleviated a great part of the burden that was upon us when they sent us coal. In return, power from our hydroelectric developments came to the U. S.

The railways use such a large percentage of the output of the mines, and they use it so wastefully, that it would seem imperative to compel them to show reasons why their busiest lines should not be electrified at early dates.

H. D. Jackson (by letter): I heartily concur with Mr. Murray in the possibilities of what he terms a super-transmission line between Boston and north of Boston and Washington and south of Washington. The economies to be effected by the installation of such a transmission line will, I believe, very quickly pay for the investment and result in extremely cheap power for railway and other large industrial propositions. This, however, will not solve one of the big problems that Mr. Murray made a point of: that is, the use of coal. One of our largest uses of coal is for heat, although of course its uses for power are very, very great. But in most cases where power is generated coal is used with fairly good economy. I can look out of my window and see in all directions smoking chimneys, and it is these chimneys where the greatest waste occurs. The super-transmission line will in no sense eliminate this waste. In fact, it is my belief that cheap power will result in most cases in an increase in the waste of coal, as heating plants are notoriously wasteful in this way. One of the real items which is before all of us engineers is an improvement in the burning of coal as well as an improvement in the use of power.

C. Townley: Mr. Shepard very happily epitomizes the situation by saying that all steam road operation has been built up around the steam locomotive, and it of course naturally follows—as I have tried to demonstrate—that the removal of many of the limitations of the steam locomotive completely transforms many fundamental questions in railroad operation. As Mr. Shepard has put it "Electricity is a tool" and it becomes at once the duty and privilege of the engineer to make the proper use of it.

Mr. Murray in making some very interesting comments on the possibilities of cooperative action by two or more railroads operating adjacent tracks by reason of which all rails in a given district would be electrified, emphasizes a fact which has now come to be very generally recognized, namely, that it is not economical for a railroad to generate its own electricity. Both the load factor and also the plant or output factor of the load of an electrified steam road are very poor. Therefore not only is the generation of current solely for that load likely to be relatively inefficient but the investment and consequently the fixed charges per unit of output will be extremely high. The cure for this condition is the combination of the railroad load with other loads so that advantage may be taken of the diversity factor thereby made available, an arrangement obviously to be brought about through a power company which supplies energy, not only to the railroad but also to industry.

Mr. Schwartz's communication contains the following statement: "Generally applied—as matters stand to-day—electrification would result in an increase in the cost of transportation." I am reluctant to believe that Mr. Schwartz really means quite that. Not even the most sapient can tell whether the electrification of a given project will result in an increase or in a decrease in the cost of transportation until after the most careful scrutiny of all existing conditions. If, however, Mr. Schwartz has drawn his general conclusions from the results obtained by the New York Central electrification his comment is not surprising. As is well known coincident with electrification the New York Central built their magnificent New York terminal costing many millions of dollars. They undertook extensive enlargement of their terminal trackage including the construction of two train levels and they made other improvements of various kinds, both to their roadway and to their equipment, all designed to take advantage of every opportunity for improvement, but many of the changes in no wise compelled by electrification. No doubt the construction accounts were kept in modern and approved fashion but I do not believe it to be humanly possible to correctly allocate every expenditure and to say that such an item was due entirely to electrification and such another not at all. Mr. Schwartz later says "trunk line electrification in general would largely benefit interests other than the railroads, the travelling public and the shipper". With this statement we can all agree but if as it seems to me probable Mr. Schwartz intended also the inference that trunk line electrification would benefit the traveling public and the shipper and would not benefit the railroad, I feel that the inference should not be passed over. Such a conclusion obviously could not be based on the New York Central operation because that railroad has no trunk line electrification. If it be not based on the New York Central experience, it would be interesting to know on what experience it is based because the two, perhaps most prominent examples of trunk line electrification, namely, that of the Chicago, Milwaukee & St. Paul and that of the Norfolk & Western Railroad, offer a contrary experience. It would be interesting if Mr. Schwartz could give us further information as to the basis of his sweeping generalization.

Mr. Storer's expression of appreciation to the railroads for deferring electrification is pertinent and timely. We have not always felt that way but now we know that too early an attempt to electrify before we really were ready would have brought forth such a multitude of troubles as would have—for a long time—gone far to discourage other roads from contemplated undertakings. In referring to the possibility of handling freight cars by lifting and cross transfer, Mr. Storer has touched upon a question of large possibilities. It is well known that of the time elapsing between the shipment and receipt of freight more is consumed in yards and terminals than in actual transportation. The possibilities of elevation of freight may some day be developed sufficiently to over-shadow in importance the value of electrified transportation itself.

Mr. Potter has given us some impressive figures as to coal consumption. These are always interesting as illustrations.

Major Sever's recital of the tremendous increase in both the use and price of oil once again emphasizes the virtue of economy. It is gratifying to reflect in this connection that the modern power house of today produces a unit of electric energy with about one-half the fuel required some 18 years ago and that the limit of improvement in this direction has not yet been reached nor is it in fact in sight.

Mr. Gibbs succinctly sums up the situation when he says that the question of electrification has now become one of financial rather than of technical feasibility. In other words no one longer doubts that electricity can perform all the transportation duties of steam, the questions for consideration being solely whether a given project will show a satisfactory return on the investment and can the necessary money be obtained. It is extremely gratifying to me to find no dissenting voice and to be able to close with an expression of appreciation for the many distinguished engineers who have been interested enough to discuss my paper.

Presented at the 348th Meeting of the American Institute of Electrical Engineers, Boston, Mass., March 14th, 1919.

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ELECTROMAGNETIC THEORY OF THE TELEPHONE RECEIVER

With Special Reference to Motional Impedance

BY A. E. KENNELLY AND H. NUKIYAMA

ABSTRACT OF PAPER

The theory of the telephone receiver here offered is based upon the motional impedance circle which has been published in various chapters during the last few years. The new theory, which is stated under definite limitations, is a further development; taking into account the m.m.f. produced by the vibration of the diaphragm in the permanent magnetic field. The motional power is thus shown to be derived partly from the testing alternating current and partly from changes in power expended in the magnetic circuit. The motional impedance circle may therefore also be regarded as a power circle, with components along three different axes of reference.

Object of the Paper. It is well known that every telephone receiver possesses a certain difference of vector impedance between the "damped" and "free" conditions, such that, when plotted in polar coordinates, over a suitable range of alternating-current frequency, gives rise to a "motional-impedance locus", which locus is approximately circular, and is known as its "motional-impedance circle".

The motional impedance circle of a telephone receiver affords valuable material for determining the principal mechanical and electrical constants of the instrument.*

In the theory of the motional-impedance circle, as hitherto published, certain phenomena have been left out of consideration for the sake of simplicity; and especially that component of alternating magnetic flux passing through the coils which is due to the displacement of the diaphragm from its zero position when freely vibrating. The result has been that the motional-impedance circle failed to reveal the power expended parasitically (by both hysteresis and eddy currents) in the magnetic circuit. This defect in the theory has already been pointed out by Eccles.†

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*Bibliography, No. 9.

†Bibliography, No. 24.

It is the object of this paper to supply the above-mentioned omission, and thus to offer a closer approximation between the theory and the observed behavior of the telephone receiver as an electromagnetic motor. It should be pointed out, however, that the theory here presented is still far from being complete, and that much more work, both theoretical and experimental remains to be executed, analysed and reported, before the theory can be regarded as satisfactorily established.

Limitations of the New Theory. The following limitations should be noted in regard to the stage of attainment aimed at in the theory here presented.

1. The alternating current supplied to the receiver is of small amplitude, and of single frequency; so that the magnetic flux passing through the coils and polar surfaces may be regarded as a normal permanent flux, plus a small sinusoidal variation. All the alternating mechanical, magnetic and electric quantities, are therefore assumed to be simple harmonic functions of the time.
2. Throughout the range of frequency considered, the flux in the magnetic circuit of the coils retains the same magnitude and phase with respect to the constant exciting alternating current. This means that the influences of magnetic skin effect and hysteresis in the magnetic circuit remain substantially constant.
3. The proportional effect of a small increment of m.m.f. in the magnetic circuit, due either to a small increase in exciting current, or to a small diminution in air gap at the poles, shall be the same, both in magnitude and in phase.
4. That over the range of frequencies considered, the "force-factor" A, the "equivalent mass" m of the diaphragm, the "mechanical resistance" r of the same, and the "stiffness coefficient" s, remain substantially constant.

The theory is presented with relation to a series of impedance measurements already published* on a particular bipolar Bell receiver "*C*"; so that these measurements offer, in a particular case, a check upon the application of the theory.

Magnetic Circuit and Vibrating System. In Fig. 1, it is assumed that the two poles N and S are permanently magnetized, as though under the action of a constant-current magnetomotive force $\mathfrak{F}_0$ gilberts. Under the influence of the permanent magnetic flux Φ_0 maxwells in the magnetic circuit, the diaphragm becomes flexed from its horizontal plane position and comes to rest at a certain zero position at the same distance from each pole. Since vibrational displacements occur on each side alternately of this reference position of rest, we may reckon a displacement therefrom of x cm. towards the poles as positive, and away from the poles as negative. The total number of turns on both coils together, is taken as N. The active surface of each pole, or of the diaphragm opposite thereto, is taken as S sq. cm., and since the polar gap is small by comparison with the linear dimensions of this surface, we may neglect the magnetic fringe at the polar edges.

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*Bibliography 17, pp. 422–428 and 449–450.

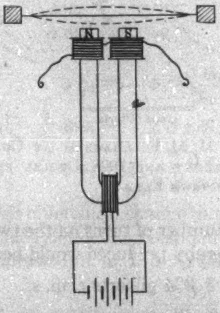

FIG. 1—SIMPLIFIED MAGNETIC CIRCUIT OF BIPOLAR RECEIVER

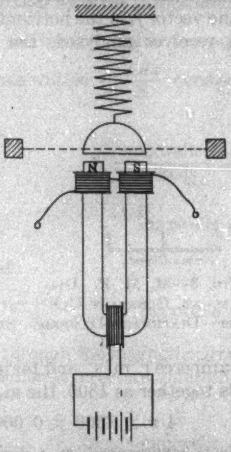

FIG. 2—SIMPLIFIED MECHANICAL SYSTEM OF BIPOLAR RECEIVER

In Fig. 2, the mechanical vibration system is indicated as simplified into an equivalent mass m grams, under the retractive influence, so far as concerns alternating stresses, of s dynes per cm. of displacement. The frictional resistance force of this system to motion is taken as simply proportional to the velocity of displacement; so that if $\dot{x}$ represents the velocity $\frac{dx}{dt}$ of the mass m, the retarding force is $-r\dot{x}$ dynes, where r is the mechanical resistance of the mass in dynes per unit velocity, or dynes per kine.

The simplification in the vibrational system represented in the transition from Fig. 1 to Fig. 2 seems justified from experimental results.

Magnetomotive Force Damped and Free. When the diaphragm is damped, or prevented from vibrating, a simple alternating current of I r. m. s. absamperes will develop a corresponding simple alternating m.m.f. of $\mathfrak{F}_1$ r. m. s. gilberts, in phase with I. This may be represented as in Fig. 3, by the plane vector f_1 in the horizontal or reference-phase position. In the receiver analysed, the testing current I was 0.000204 absamperes r. m. s., and taking the number of turns on the two coils together as 1300, the m.m.f. thereby produced would be

0
1
2
r.m.s.Gilberts

FIG. 3—M. M. F. DIAGRAM AT CONSTANT CURRENT—DIAPHRAGM DAMPED

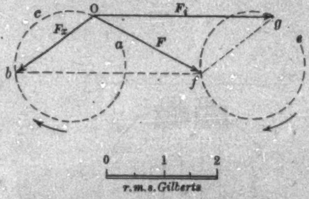

FIG. 4—M. M. F. DIAGRAM AT CONSTANT CURRENT AND 1028 CYCLES PER SEC. DIAPHRAGM FREE

$$4\pi \times 1300 \times 0.000204 = 3.334 \text{ gilberts r. m. s.}$$

This is represented, to a scale of 1.0 gilbert per linear cm. in Fig. 1, as a vector 3.33 cm. in length.

When the diaphragm is free, or allowed to vibrate, the same exciting current will produce the same m. m. f. as before; but a new periodic m. m. f. will be developed in the magnetic circuit, owing to the periodic change in diaphragm displacement, and length of airgap; so that the reluctance in the circuit of the permanent magnet will be periodically varied accordingly.

Let $\mathfrak{F}_0$ = the m. m. f. of the permanent magnet (gilberts)
$\mathfrak{R}_0$ = the reluctance in the permanent magnetic circuit (oersteds)
Φ_0 = the magnetic flux across the airgaps in the circuit (maxwells)

Then in the magnetic circuit of the permanent magnet, there will be a permanent magnetic flux which, neglecting magnetic leakage through the surrounding air, will be

$$\Phi_0 = \frac{\mathfrak{F}_0}{\mathcal{R}_0} \qquad \text{maxwells} \quad (\mathbf{1})$$

If the diaphragm is now caused to vibrate, by reason of independent variations of this magnetic flux due to alternating current in the coils, the reluctance in each airgap of area S sq. cm. will be varied by $-\dfrac{dx}{S}$ oersteds, if dx is the displacement of the diaphragm from its normal position, towards the pole; so that the total change in reluctance will be $-\dfrac{2\,dx}{S}$ oersteds.

But differentiating (**1**)

$$\frac{\partial \Phi_0}{\partial \mathcal{R}_0} = -\frac{\mathfrak{F}_0}{\mathcal{R}_0^2} = -\frac{\Phi_0}{\mathcal{R}_0} \qquad \frac{\text{maxwells}}{\text{oersteds}} \quad (\mathbf{2})$$

or

$$\partial \Phi_0 = -\Phi_0 \frac{\partial \mathcal{R}_0}{\mathcal{R}_0} \qquad \text{maxwells} \quad (\mathbf{3})$$

but $\partial \mathcal{R} = -2\,\partial x/S$; so that

$$\partial \Phi_0 = \Phi_0 \cdot \frac{2\,\partial x}{S\,\mathcal{R}_0} = \mathcal{B}_0 \cdot \frac{2\,\partial x}{\mathcal{R}_0} \qquad \text{maxwells} \quad (\mathbf{4})$$

This differential change in the flux through the magnetic circuit is such as would correspond to the action of a certain differential impressed m. m. f. $\partial \mathfrak{F}_0$, such that

$$\frac{\partial \mathfrak{F}_0}{\mathcal{R}_0} = \partial \Phi_0 \qquad \text{maxwells} \quad (\mathbf{5})$$

Consequently, substituting in (**4**)

$$\partial \mathfrak{F}_0 = \mathcal{B}_0 \cdot 2\,\partial x \qquad \text{gilberts} \quad (\mathbf{6})$$

If, then, we know $\mathcal{B}_0$, the normal mean flux density across the airgaps, an alternating displacement of ∂x cms. r. m. s. towards the poles at reference phase, will generate an equivalent vector m. m. f. according to (**6**) in phase with ∂x. This is shown in Fig. 4 by the plane vector $\mathfrak{F}_x$. The phase of $\mathfrak{F}_x$ with respect to $\mathfrak{F}_i$ cannot be assigned until we know the phase of the diaphragm displacement ∂x, but means for determining this phase difference will be indicated later on. The locus of $\mathfrak{F}_x$ is indicated in Fig. 4. It is a "displacement-admittance" locus.

At a very low frequency, $\mathfrak{F}_x$ would occupy the plane-vector position *O a*. At a very high frequency, it would disappear towards *O c*. It would, therefore describe the locus *a b c* in the direction of the arrow.

The resultant m. m. f. in the circuit is the plane vector sum of $\mathfrak{F}_i$ and $\mathfrak{F}_x$, and is indicated in Fig. 4 by $\mathfrak{F}$. At low frequencies, the size of $\mathfrak{F}$ exceeds the size of $\mathfrak{F}_i$; but towards higher frequencies, the size of $\mathfrak{F}$ falls below that of $\mathfrak{F}_i$. The effect, therefore, of the vibration of the diaphragm is to change the constant impressed a-c. m. m. f. $\mathfrak{F}_i$ to a correspondingly varied m. m. f. following the vector locus *e f g*, as the frequency is changed.

In Figs. 3 and 4, the vectors represent, to scale, the approximate values of $\mathfrak{F}_i$, $\mathfrak{F}_x$ and $\mathfrak{F}$, as derived from experimental tests of the selected instrument.

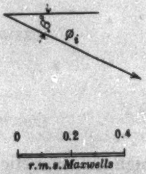

FIG. 5—FLUX DIAGRAM AT 1028 CYCLES PER SEC. — DIAPHRAGM DAMPED

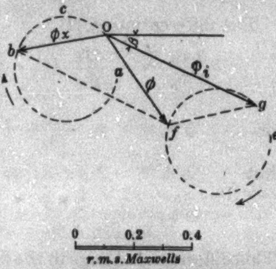

FIG. 6—FLUX DIAGRAM AT 1028 CYCLES PER SEC. — DIAPHRAGM FREE

Magnetic Flux, Damped and Free. In Figs. 5 and 6, the planevector alternating magnetic fluxes are represented with the diaphragm damped and free. When damped, as in Fig. 5, the alternating m. m. f. of $\mathfrak{F}_i$ r. m. s. gilberts at reference phase, produces an alternating magnetic flux of ϕ_i r. m. s. maxwells, lagging in phase $\beta°$ behind the m. m. f. The reason for the lag is partly on account of magnetic hysteresis, and partly on account of magnetic skin effect. This angle $\beta°$ (25.3 deg. in Fig. 5) can be determined from the observed motional-impedance diagram, on the assumption already made that β is half the depression angle of the motional-circle diameter. In certain cases, β is found to exceed 54 deg. although it is, more commonly in the neighborhood of 30 deg.

In Fig. 6, on the assumption already made that the lag β° is the same to both $\mathfrak{F}_i$ and $\mathfrak{F}_s$; *i. e.*, that ϕ_s lags β° behind $\mathfrak{F}_s$, we obtain the fluxes ϕ_i and ϕ_s with their resultant ϕ, as well as the approximate loci over which they travel under change of impressed frequency, β being taken as substantially constant over that range.

Actual observation shows that the angle of lag β_1 between $\mathfrak{F}_i$ and ϕ_i is not, in general, identical with the angle of lag β_2 between $\mathfrak{F}_s$ and ϕ_s, but their sum is 2 β, and the discrepancy is here neglected.*

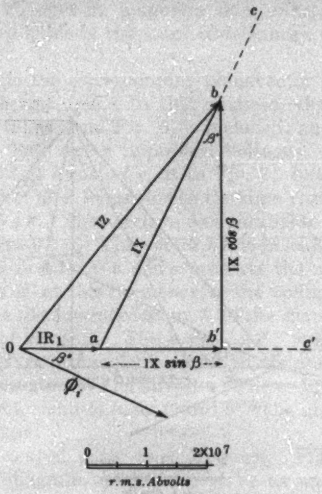

Fig. 7—E. M. F. Diagram at 1028 Cycles per Sec.—Diaphragm Damped

Electromotive Forces, Damped and Free. Fig. 7 shows the alternating e. m. f. planevector diagram when the diaphragm is damped. With constant exciting current strength I r. m. s. absamperes, and constant assumed resistance R_1 absohms in the winding, the vector $I\,R_1$ r. m. s. volts, expended in overcoming the winding resistance, will theoretically remain constant at all frequencies. The voltage expended in overcoming the reactance of the winding will be $I X$ r. m. s. volts, at a leading slope of $90 - \beta°$; because this voltage must be in leading quadrature to the flux ϕ_i. The resultant voltage $I Z$ is that which must be applied to the terminals of the receiver, in order to maintain in its winding the required constant current strength I. The locus of the impressed voltage $I Z$ on the preceding assumptions, will therefore be the straight line *a b c*.

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*Bibliography 18.

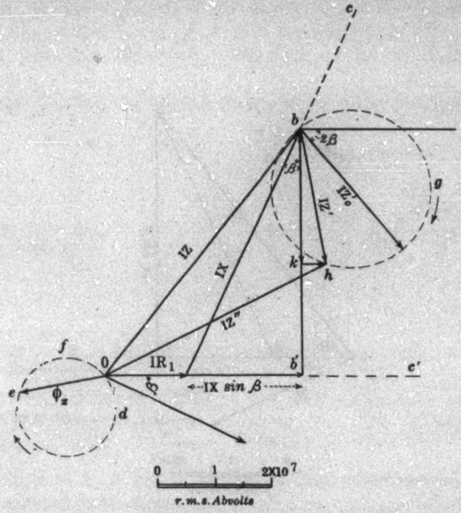

Fig. 8—E. M. F. Diagram at 1028 Cycles per Sec.—Diaphragm Free

The inductive voltage or voltage expended in overcoming the counter e. m. f. of induction is *a b* on the diagram and is expressed by $j\, I\, \mathcal{L}\, \omega$ abvolts ∠, where $\mathcal{L}$ is a vector inductance of slope $\nwarrow \beta°$ as defined by the relation

$$\mathcal{L} = 4\, \pi\, N^2\, \mathcal{P} \qquad \text{abhenries} \angle \quad \textbf{(6a)}$$

where $\mathcal{P}$ is a vector permeance expressed in maxwells per gilbert ∠, or 1/oersteds ∠. The corresponding apparent resistance voltage is $O\, b'$ traversing the locus $a\, b'\, c'$. The apparent reactance voltage is $b'b$. This apparent reactance voltage, as obtained from impedance measurements with damped diaphragm, is

$$I\,|X|\cos\beta = I\,|\mathfrak{L}|\,\omega\cos\beta \qquad \text{abvolts} \quad (7)$$

Here the symbol $|\mathfrak{L}|$ denotes the size of $\mathfrak{L}$ as a scalar.

The apparent inductance $\mathfrak{L}$ of the winding should also remain constant. Actually, observations show that the apparent inductance diminishes as the frequency is increased, owing to the influence of changes in magnetic skin effect, which are assumed to be negligible in the stage of telephone theory here aimed at.

Fig. 8 represents the corresponding planevector e. m. f. diagram, with diaphragm free. In this diagram, the additional periodic flux ϕ_x, taken from Fig. 6, is included, and runs over the locus $d\,e\,f$. The vector impressed voltage $I\,Z''$, travels over the straight line locus $a\,b\,c$, as in Fig. 7; but at its end, there is added an e. m. f. locus due to the time variation of ϕ_x, which is a circle $b\,g\,h$. This circle moves parallel to itself, along the lengthening line $a\,b\,c$. The vector $b\,h$ is in leading quadrature to ϕ_x, in the position $O\,e$, and represents the motional impedance voltage $I\,Z'$ at this frequency, or the voltage expended in overcoming the displacement e. m. f. of the diaphragm considered as a motor element. The e. m. f. $I\,Z''$ which must be impressed at receiver terminals, in order to maintain constant current I, is represented by $O\,h$ abvolts. The component $O\,h'$, is the cophase component of e. m. f. and $b'\,k$ the quadrature or reactive component.

Impedances, Damped, Free and Motional. Figure 7, the damped voltage diagram, might also serve as an impedance diagram, if its various vectors are divided by the constant current I, and if, therefore, these vectors are read off to an appropriate scale of absohms. In Fig. 9, the transition is made, however, to a convenient resistance scale. The apparent resistance is $O\,b = R_1 + |X|\sin\beta$ absohms, the apparent reactance $b'\,b = |X|\cos\beta$ absohms, and the impedance with damped diaphragm

$$Z = O\,b = R_1 + |X|\sin\beta + j\,|X|\cos\beta = R_1 + X \quad \text{absohms} \angle \qquad (8)$$

As the impressed frequency increases, the impedance travels over the locus $a\,b\,c$. In Fig. 1 of the paper referred to, (Bibliography 17), the vector damped impedance locus makes an angle of $\beta = 25.°3$, very nearly, with the reactance axis, between $f = 700 \sim$ and $f = 1500 \sim$.

Fig. 10 shows the corresponding impedance of the instrument with the diaphragm free. The vector Ob pursues the same path abc as in Fig. 9; but at its end, there is added a traveling circular locus ghk. The impedance of the instrument is Z'', and we have the vector relation.

$$Z' = Z'' - Z \qquad \text{absohms } \angle \quad (9)$$

The vector path of Z', plotted separately, is the stationary circle

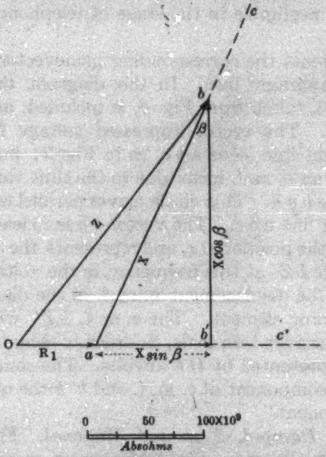

FIG. 9—IMPEDANCE DIAGRAM—DIAPHRAGM DAMPED

$b\,g\,h$, and this circle is obtained from experimental observations at constant current. It passes through the origin of coordinates like the circle of Fig. 27. It has a diametric depression angle, or lag angle, of $2\,\beta$, because the diameter being produced at resonant frequency, the displacement velocity $\dot{x}$ of the diaphragm is then in phase with the flux ϕ_i, which is $\beta°$ behind the current, and the displacement x is then in lagging quadrature with the velocity or $(90° + \beta)$ behind the current. But ϕ_x is $\beta°$ behind the displacement x (Fig. 6) and is therefore $90° + \beta°$ behind the velocity $\dot{x}$ or $90° + 2\,\beta°$ behind I. The e. m. f.

$I\,Z'$, Fig. 8, due to the variation of ϕ_x, being in leading quadrature therewith, is then 2 $\beta°$ behind I, and Z', at resonance, is thus 2 $\beta°$ behind the reference phase, or resistance axis.

Every telephone receiver has its own particular motional impedance circle, which is very sensitive to changes in temperature or in mechanical adjustment. Motional impedance circles differ:

1. In their size, or diametral length.
2. In their depression angle 2 β.
3. In the distribution of frequencies around the circle.

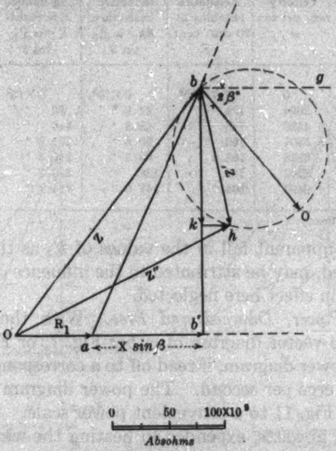

FIG. 10—IMPEDANCE DIAGRAM—DIAPHRAGM FREE

Sharply resonant instruments will pass from the lower quadrantal frequency f_1, to the upper f_2, over half the circle, with a change of only a very few cycles per second. On the other hand, bluntly resonant instruments may require a change in frequency of several hundred cycles per second, to pass over the same range of locus.

Having found by observation the depression angle 2 β of the motional impedance circle, the angle β is known, and the construction of Figs. 9 and 10 may be carried out, with the angle at $b\,b'$ made equal to β. Knowing the apparent resistance $O\,b'$, and the apparent reactance $b'\,b$, at various impressed frequencies, as well as $\beta°$, we can readily compute R_1 and $\mathfrak{L}$ from (7) and (8). R_1 should theoretically be substantially constant under the assumption that $\mathfrak{L}$ is constant. For the instrument here considered, the following Table gives the values of R_1 as computed from the observed resistances with damped diaphragm, and the angle β assumed constant at resonant value.

Frequency cycles per sec.	Angular Velocity rad. per sec. ω	Measured Resistance absohms at 20 deg. cent. R	Active component of inductive resistance $R_h = \omega \mathfrak{L}_0 \sin \beta$	Reactive component of inductive resist- $X = \omega \mathfrak{L}_0 \cos \beta$	$R_1 = R - R_h$ absohms
0	0	86.7×10^9	0×10^9	0×10^9	86.7×10^9
429	2694	124 "	41.9 "	89 "	82.1 "
702	4420	145 "	68.8 "	146 "	76.2 "
923.4	5801	162 "	90.5 "	191.5 "	71.5 "
*1015	6378	168.4 "	99.5 "	210.5 "	68.6 "
1306	8202	189. "	128 "	270.7 "	61. "
1507	9460	204 "	147.6 "	314.1 "	56.4 "

*Resonant frequency.

The steady apparent fall in the values of R_1 as the frequency was increased, may be attributed to the influence of changes in magnetic skin effect here neglected.

Electric Power, Damped and Free. With the diaphragm damped, the vector diagram of either Fig. 7, or Fig. 9, might serve as a power diagram, if read off to a corresponding scale of abwatts, or ergs per second. The power diagram is repeated, however, in Fig. 11 to a convenient power scale. $O\,a$ is active power, $I^2 R_1$ abwatts, expended in heating the winding. $I^2 X$ is the vector power $a\,b$, expended in the magnetic circuit. It follows the locus $a\,b\,c$, as the frequency is varied. The active component $a\,b' = I^2 \mid X \mid \sin \beta$, is dissipated thermally in the magnetic circuit, in hysteresis and eddy currents. The reactive component $b'\,b = I^2 \mid X \mid \cos \beta$ is utilized in alternately storing and releasing energy in the magnetic circuit.

When the diaphragm is freed, the power conditions are indicated vectorially in Fig. 12. $O\,a$, as before, is the thermal power $I^2 R_1$, dissipated in the winding. The power delivered to the magnetic circuit is $a\,h$, which is in leading quadrature to the resultant flux ϕ, see Fig. 6. The active component $a\,h'$ includes the power expended in hysteresis and eddy currents, and also the active mechanical power delivered from the magnetic circuit to the diaphragm. The reactive component $h'h$ includes power utilized in storing and releasing magnetic energy in the magnetic circuit, and also power utilized in storing and releasing reactive mechanical energy in the diaphragm.

Mechanical Velocity, Damped and Free. When the diaphragm is damped, the vibrational velocity of displacement must be zero. When, however, the diaphragm is free, the velocity of displacement $\dot{x}$ will have a definite magnitude and phase. If we assume the vibromotive force $f_i = Og$ r. m. s. dynes, Fig. 13, to remain constant at all frequencies, the locus of the vector velocity $\dot{x}$ will be the circle $O\,a\,b\,c$. This mechanical relation has its complete electric analogy in the simple alternating-current circuit of Fig. 14, where a simple harmonic e. m. f. E, of constant r. m. s. magnitude, and at standard phase, supplies the simple circuit $C\,R\,L$ at successively varied frequencies. All of the resistance of the circuit, assumed constant, is located in the resistor R, all of the inductance in the inductor L, and all the capacitance in the condenser C. At a very low frequency, the impedance of the circuit will

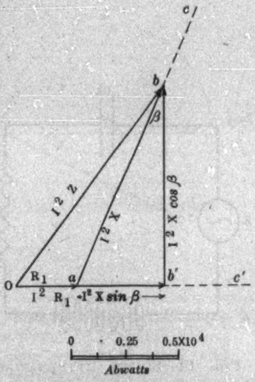

Fig. 11—Electric Power Diagram—Diaphragm Damped

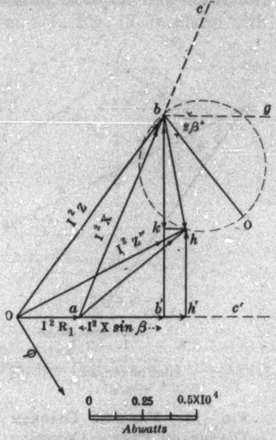

Fig. 12—Electric Power Diagram—Diaphragm Free

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be large and entirely due to the condenser C. The alternating current will be very feeble and will lead the impressed e. m. f. by 90°. As the frequency is raised, the impedance diminishes, both in size and in slope. The current strength will therefore increase in magnitude, and come more nearly into phase with E. As is shown in Fig. 15, when the resonant frequency of the circuit is attained, the current will be a maximum at I_0, and will be in phase with E. Finally, as the frequency becomes very high, the current will disappear, and lag 90° behind E. The locus of the vector current will be the circle $O\ a\ b\ c$, Fig.15.

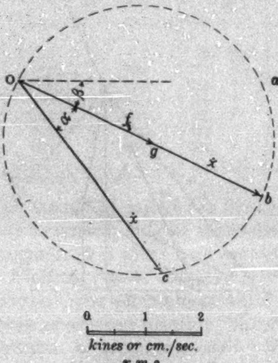

Fig. 13 — Velocity Diagram —Diaphragm Free — Constant V. M. F. and Varied Frequency

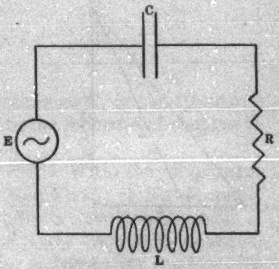

Fig. 14—Simple Alternating-Current Circuit Under Constant E. M. F. and Varied Frequency

If, in the mechanical system of the diaphragm, the equivalent mass replaces the inductance L, the stiffness s of the diaphragm, replaces $1/C$, and the mechanical resistance to motion r replaces R; also the vibromotive force f_i replaces E; then the vibrational velocity $\dot{x}$ will correspond both in magnitude and phase to the alternating-current strength I. The phase lag α of the velocity is then equal to the slope of the mechanical impedance

$$r + j\,(m\,\omega - s/\omega) \qquad \text{mechanical ohms} \angle$$

Mechanical Displacement. The mechanical displacement of the diaphragm at elongation corresponds, in the electrical system of Fig. 15, to the displacement of electricity at cyclic elongation. It is represented in Fig. 16 by the vector $O\,c$. This vector is in lagging quadrature to the vector $O\,c$ of Fig. 13, at one and the same impressed frequency. The locus of x is the displacement curve $a\,b\,c\,O$.

Force Diagram Damped and Free. If the diaphragm is damped, any single vibromotive force (v. m. f.) f_i, Fig. 17a, will develop a counter v. m. f. f_c, which will make the resultant force zero; so that no displacement results. If, however, the diaphragm is free, there will be four forces called into action, in similarity to those in the corresponding electrical case of Fig. 15; namely, the impressed single v. m. f. f_i, Fig. 17, in phase with the flux ϕ_i; the frictional resisting force $-r\,\dot{x}$ dynes, the inertia force $-j\,\omega\,m\,\dot{x}$ dynes, and the elastic force $j\,\dfrac{s\,\dot{x}}{\omega}$ dynes. These are represented in Fig. 17 by the vectors $O\,a$, $O\,b$, $O\,c$ and $O\,d$. Since $O\,c$ and $O\,d$ must be mutually opposite, their difference $O\,e$, combined with $O\,b$, must balance the impressed force $O\,a$. This relation determines the position and magnitude of the velocity $\dot{x}$. We have already seen that as the frequency is changed from a small to a large value, the locus of $\dot{x}$ is a circle about O. At mechanical resonance, $\dot{x}$ will have a maximum size and zero slope, or will coincide with f_i in direction. The

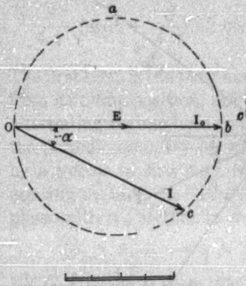

FIG. 15—CURRENT DIAGRAM IN SIMPLE A-C. CIRCUIT, UNDER CONSTANT E. M. F. AND VARIED FREQUENCY

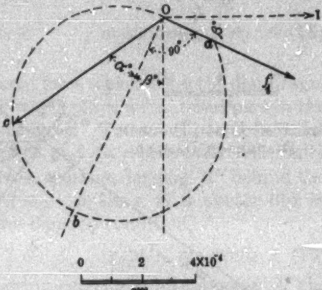

FIG. 16—DISPLACEMENT DIAGRAM—DIAPHRAGM FREE

displacement x, not indicated in Fig. 17, will always have the direction $o\,c$, 90° lagging behind $\dot{x}$.

The vector force equation is in the rotatable vector diagram of Fig. 17.

$$m\ddot{x} + r\dot{x} + s\,x = f_i \qquad \text{r. m. s. dynes} \angle \quad (10)$$

or since x is a simple harmonic displacement of the type $x_m\,\epsilon^{j\omega t}$,

$$j\,\omega\,m\,\dot{x} + r\,\dot{x} + \frac{s\,\dot{x}}{j\omega} - f_i = 0 \qquad \text{r. m. s. dynes} \angle \quad (11)$$

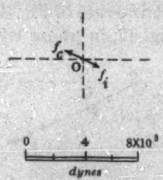

FIG. 17A—FORCE DIAGRAM —DIAPHRAGM DAMPED

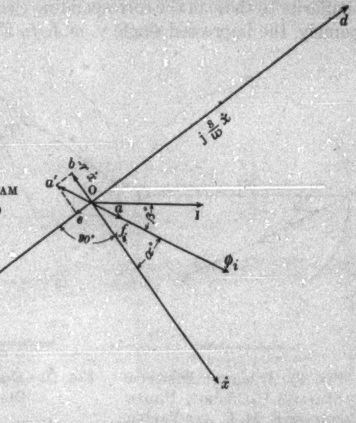

FIG. 17—FORCE DIAGRAM FOR THE IDEAL CASE OF A DIAPHRAGM FREE TO VIBRATE UNDER A SINGLE IMPRESSED V. M. F. f_i R. M. S. DYNES

or $-j\,\omega\,m\,\dot{x} - r\,\dot{x} + j\,\dot{x}\,s/\omega + f_i = 0 \qquad$ r. m. s. dynes ∠ (12)
whence

$$\dot{x} = \frac{f_i}{r + j\,(m\,\omega - s/\omega)} \qquad \text{r. m. s.} \ \frac{\text{cm.}}{\text{sec.}} \angle \quad (13)$$

which corresponds completely to Ohm's law in the vector simple alternating-current case.

The specific magnetic force acting at any time on the diaphragm, towards the magnetic poles, is $\frac{\mathfrak{B}^2}{8\,\pi}$ dynes per sq.

cm., where $\mathcal{B}$ is the total magnetic flux density. This flux density comprises the permanent magnetic flux density $\mathcal{B}_0$ gausses, and the alternating flux density $\mathcal{B}$ instantaneous gausses. The total magnetic force is thus

$$\frac{2\,S}{8\,\pi}\,(\mathcal{B}_0 + \mathcal{B})^2 = \frac{S}{4\,\pi}\,(\mathcal{B}_0 + \mathcal{B})^2$$

$$= \frac{S}{4\,\pi}\,(\mathcal{B}_0^2 + 2\,\mathcal{B}_0\,\mathcal{B} + \mathcal{B}^2) \qquad \text{instantaneous dynes} \quad \textbf{(14)}$$

Since $\mathcal{B}^2$ is very small by comparison with $\mathcal{B}_0^2$, it may be neglected in the summation, and the instantaneous force in the diaphragm is

$$\frac{S}{4\,\pi}\,(\mathcal{B}_0)^2 + \frac{S}{4\,\pi}\cdot 2\,\mathcal{B}_0\,\mathcal{B} \qquad \text{instantaneous dynes} \quad \textbf{(15)}$$

The first term is the permanent force which flexes the diaphragm from its plane position, and which plays no material part in the alternating regime here considered. The second term includes the periodic quantity $\mathcal{B}$, which may be represented vectorially by a vector of size $\mathcal{B}_i$ r. m. s. gausses, lagging $\beta°$ behind the exciting m. m. f. $\mathcal{F}_i$. But $S\,\mathcal{B}_i = \phi_i$ the r. m. s. vector flux in phase with $\mathcal{B}_i$, so that in the damped condition,

$$\phi_i = \mathcal{F}_i\,\mathcal{P} = 4\,\pi\,N\,I\,\mathcal{P} \qquad \text{r. m. s. maxwells} \angle \quad \textbf{(16)}$$

where I is the r. m. s. exciting current in absamperes, N the total number of turns including both coils, and $\mathcal{P}$ a vector permeance which possesses a lag angle $\beta°$. Consequently from **(15)**

$$f_i = 2\,\mathcal{B}_0\,N\,\mathcal{P}\,I = A\,I \qquad \text{r. m. s. dynes} \angle \quad \textbf{(17)}$$

or in a bipolar instrument $A = 2\,\mathcal{B}_0\,N\,\mathcal{P}$ r. m. s. dynes per absampere $\angle$ **(18)**

A is a vector force factor with a lag angle of $\beta°$.

We have thus far supposed that only one periodic force f_i acts on the diaphragm. There is, however, a second periodic force introduced by the flux ϕ_x, due to the displacement m. m. f., the magnitude of which is proportional, by **(6)**, to the maximum cyclic value of the displacement, and which has the same phase as that displacement.

We have already shown in **(6)** that the displacement m. m. f. is

$$\mathcal{F}_x = 2\,\mathcal{B}_0\,x \qquad \text{r. m. s. gilberts} \angle \quad \textbf{(19)}$$

This may be considered as similar to the m. m. f. of current

$$\mathcal{F}_i = 4\,\pi\,N\,I \qquad \text{r. m. s. gilberts } \angle \quad (\mathbf{20})$$

so that a certain number of turns N' multiplied by the r. m. s. vector displacement, would produce the same m. m. f. as the actual number of turns N multiplied by the corresponding r. m. s. vector current. Consequently, in the case of a bipolar receiver

$$N' = \frac{\mathcal{B}_0}{2\,\pi} \qquad \text{equivalent turns} \times \frac{\text{absampere}}{\text{cm.}} \quad (\mathbf{21})$$

If we consider the ratio

$$\frac{N'}{N} = p \qquad \frac{\text{absampere}}{\text{cm.}} \quad (\mathbf{22})$$

which, in the instrument considered, was 0.168,

$$\mathcal{F}_x = 4\,\pi\,N'\,x \qquad \text{r. m. s. gilberts } \angle \quad (\mathbf{23})$$

and from (**17**)

$$f_x = p\,A\,x = 2\,p\,\mathcal{B}_0\,N\,\mathcal{P}\,x \qquad \text{r. m. s. dynes } \angle \quad (\mathbf{24})$$

It may be noted, in passing, that just as in the case of a vector electromagnetic inductance

$$\mathcal{L} = A/p = 4\,\pi\,N^2\,\mathcal{P} \qquad \text{abhenries } \angle \quad (\mathbf{25})$$

$$\text{or ergs per absamp}^2 \angle$$

so
$$p\,A = 4\,\pi\,N'^2\,\mathcal{P} \qquad \frac{\text{r. m. s. dynes}}{\text{cm.}} \angle \quad (\mathbf{26})$$

and is a quantity bearing resemblance to an inductance. From (**25**) and (**26**)

$$A = 4\,\pi\,N\,N'\,\mathcal{P} \qquad \frac{\text{r. m. s. dynes}}{\text{absamp}} \angle \quad (\mathbf{27})$$

so that, in this sense, the force factor A bears resemblance to a mutual inductance.

The additional force f_x, due to ϕ_x, may be introduced into the problem either (**1**) by vector summation with f_i, to form a vector resultant force f, corresponding to ϕ in Fig. 6; or (**2**), by retaining f_i as the sole active force on the diaphragm, and introducing the action of f_x in the form of a mechanical impedance. For practical purposes, the latter method is to be preferred, just as in the simple alternating-current circuit, the effect of a counter e. m. f. of self induction is usually treated as an impedance, instead of being combined vectorially with the impressed e. m. f. in the numerator of Ohm's law.

The force f_x lags behind the displacement x by the angle $\beta°$, according to the relation

$$f_x = p\,A\,x = \frac{p\,A\,\dot{x}}{j\,\omega} = -j\,p\,A\,\frac{\dot{x}}{\omega} \quad \text{r. m. s. dynes} \angle \qquad (28)$$

or $90° + \beta°$ behind the phase of the velocity $\dot{x}$.

In Fig. 18, the force f_x is represented in a particular case with the diaphragm free. Here the m. m. f. $\mathfrak{F}_i$ is taken as of reference phase, with f_i lagging $\beta°$ behind it. This force f_i must now equilibrate four forces; namely, the elastic force $O\,d$, the inertia force $O\,c$, the frictional force $O\,b$, and the new magnetic force $O\,e$, whose components, along the $O\,b$ and $O\,c$ directions, are indicated as $O\,e'$ and $O\,e''$ respectively. With $\mathfrak{F}_i$ at reference phase, the locus of the new force $O\,e$ is a displacement-admittance locus like $a\,b\,c$ in Fig. 4; but considered solely with reference to the velocity $\dot{x}$, $O\,e$ always lags with respect thereto by $90° + \beta°$. The components $O\,e'$ and $O\,e''$ may be defined as $-\,r'\,\dot{x}$ and $\dfrac{-\,j\,s'\,\dot{x}}{\omega}$ respectively; where r' is a virtual frictional resistance, and s' a virtual stiffness coefficient.

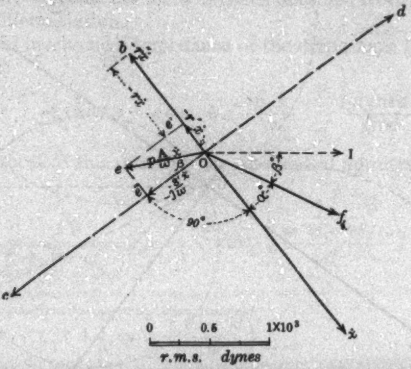

FIG. 18—FORCE DIAGRAM INCLUDING MAGNETIC FORCE DUE TO DISPLACEMENT—DIAPHRAGM FREE ENLARGED SCALE

The diaphragm therefore behaves as though its frictional resistance were increased by

$$r' = \frac{p\,|A|}{\omega}\sin\beta \qquad \frac{\text{dynes}}{\text{kine}} \qquad (29)$$

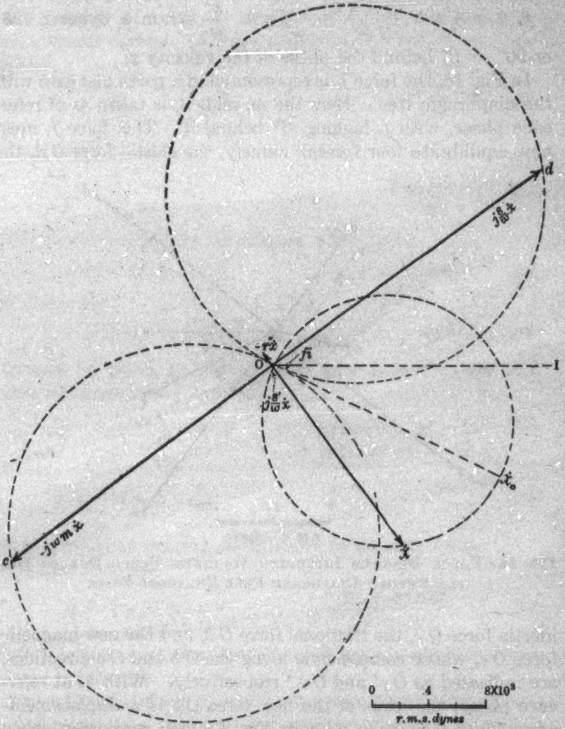
d
$j\frac{s}{\omega}\dot{x}$
$-r\dot{x}$
O
fi
I
$p\frac{s'}{\omega}\dot{x}$
$\dot{x}_0$
$\dot{x}$
$-j\omega m\dot{x}$
c
0
4
8X10³
r.m.s. dynes

FIG. 19—FORCE DIAGRAM—COMPLETE—INCLUDING MAGNETIC FORCE DUE TO DISPLACEMENT

while the stiffness of the diaphragm is apparently diminished by

$$s' = p\,|A|\cos\beta \qquad \frac{\text{dynes}}{\text{cm.}} \qquad (30)$$

The values of r' and s' can be assigned, when A, β, and p are known. In the instrument considered, at resonance, $r' = 68.8$ dynes per kine and $r = 199$ dynes per kine, r' diminishing, according to our theory, as ω is increased. Also s' is 9.3×10^5 dynes per cm. as against $s = 376 \times 10^5$. According to our theory, s' is thus only about 3 per cent of s, at all frequencies.

In Fig. 19, the loci of the inertia and stiffness forces, $O\,c$ and $O\,d$, are represented to scale, without including therein the components $O\,e'$ and $O\,e''$, Fig. 18, of the added magnetic force f_s. The directions of motion in the loci, with increase of frequency are all clockwise. The locus of $O\,d$ is a displacement-admittance locus, obtained from the circular locus of $\dot{x}$ by time integration; whereas the locus of $O\,c$ is obtained from that of $\dot{x}$ by time differentiation.

The total mechanical impedance of the diaphragm from (**13**) and (**28**) is

$$z' = r + j\,(m\,\omega - s/\omega) + j\,\frac{p\,A}{\omega} \qquad \frac{\text{dynes}}{\text{kine}}\ \angle \quad \textbf{(31)}$$

The reciprocal of this quantity, or mechanical admittance y' is

$$\begin{aligned} y' &= 1/z' \\ &= \frac{1}{\sqrt{r''^2 + (m\,\omega - s''/\omega)^2}} \angle \tan^{-1}\left(\frac{m\,\omega - s''/\omega}{r''}\right) \\ &= \frac{1}{\sqrt{(r + r')^2 + \left(m\,\omega - \dfrac{s - s'}{\omega}\right)^2}} \\ &\quad \angle \tan^{-1}\left(\frac{m\,\omega - \dfrac{s - s'}{\omega}}{r + r'}\right) \qquad \frac{\text{kines}}{\text{dyne}}\ \angle \quad \textbf{(32)} \end{aligned}$$

The values of r' and s' are to be taken from (**29**) and (**30**). This vector admittance y', multiplied by the constant impressed mechanical force f_i, due to the constant alternating current, gives the mechanical velocity $\dot{x}$, or

$$\dot{x} = f_i\,y' = A\,y'\,I \qquad \text{r. m. s. } \frac{\text{cm.}}{\text{sec.}}\ \angle \quad \textbf{(33)}$$

and the r. m. s. displacement x by

$$x = -j\,f_i\,y'/\omega = -j\,A\,y'\,I/\omega \qquad \text{r. m. s. cyclic cm. } \angle \quad \textbf{(34)}$$

The actual maximum cyclic displacement of the diaphragm is then

$$x_m = |x|\sqrt{2} \qquad \text{max. cyclic cm.} \quad (\mathbf{35})$$

Motional Electromotive Force Diagram. Referring to Fig. 16, the vector displacement Oc is determined by equation (**34**), and the vector velocity Oc in Fig. 13, by equation (**33**), in terms of the motional constants r, r', m, s and s', as well as A. The displacement flux ϕ_x, Fig. 8, is then obtained through (**23**) or

$$\phi_x = \frac{p\,A\,x}{N'} = 4\,\pi\,N'\,x\,\mathcal{P} \qquad \text{maxwells} \angle \quad (\mathbf{36})$$

The e. m. f. produced by ϕ_x in the coils is the motional e. m. f. and is

$$I\,Z' = j\,\omega\,\frac{p\,A\,x}{N'}\,N = j\,\omega\,A\,x$$

$$= A\,\dot{x} = A^2\,y'\,I \qquad \text{abvolts} \angle \quad (\mathbf{37})$$

It may be noted that since $\dot{x}$ is a velocity in kines, the vector dimensions of A are not only expressed by dynes per absampere, as in (**18**) or (**27**), but also by abvolts per kine, as in (**37**).

The vector motional e. m. f. $I\,Z'$, Fig. 8, is in leading quadrature to $O\,e$, the displacement flux. Since in (**37**), this vector is directly proportional to the velocity $\dot{x}$, which has been shown to travel over a circular locus, when, as in (**13**), a single periodic force is impressed upon the diaphragm, the motional e. m. f. $I\,Z'$ must also have a circular locus, as shown in Fig. 8, when the second force f_x is negligible; *i. e.*, when p in (**28**) may be taken as zero. When p is appreciable, the subsidiary constants r' and s' are introduced into the motional equations (**32**) and (**33**). It will be seen that s' merely reduces the stiffness coefficient s of the diaphragm by the same amount at all frequencies; but r' varies with the frequency inversely, as shown by (**29**). The theoretical effect of r' is therefore to distort the locus of the motional e. m. f. $I\,Z'$, from a circle, to an extent depending on the magnitude of $p\,A$, and also upon the range of frequency necessary for the execution of the circle; *i. e.*, upon the bluntness of mechanical resonance. Since in a bipolar instrument

$$p\,A = \frac{\mathcal{B}_0^2\,\mathcal{P}}{\pi} \qquad \frac{\text{dynes}}{\text{cm.}} \angle \quad (\mathbf{38})$$

an instrument which has a relatively powerful permanent magnetic flux density $\mathcal{B}_0$ in the airgaps, and also large magnetic-circuit permeance, is one in which the force f_s tends to be greatest, by (**28**). This, with a large value of β and a wide range of ω for executing the circle, would give rise to the largest distortion. In practise, the distortion is usually not sufficiently serious to be very noticeable, probably because the range in ω within which half of the circle is executed, is ordinarily only a few hundreds of radians per second. In the case considered, $\omega_2 - \omega_1 = 305$.

It is evident, moreover, from (**37**) that the lagging displacement of vector $I\,Z'$ with respect to I, is that of $A^2\,y'$, and since, at resonance, the slope of y' vanishes (**32**), it will then be the displacement of A^2, or $-\,2\ \beta°$, and $A^2 = Z_0'\,r''$.

At resonance, the size of $I\,Z'$, the motional e. m. f. will, by (**37**) be

$$I\,Z_0' = \frac{A^2\,I}{r''} \qquad \text{abvolts} \angle \quad (\mathbf{39})$$

Motional Impedance. Referring to Fig. 10, the motional impedance executes a circular locus similar to that in Fig. 8, neglecting the disturbing influence of r' at different frequencies. At resonance, its size is

$$Z'_0 = A^2/r'' \qquad \text{absohms} \angle \quad (\mathbf{40})$$

and its slope is that of A^2 or $-\,2\ \beta°$.

From an experimental standpoint, the motional-impedance circle is of primary importance, as, from it, the motional constants may be derived, if the maximum cyclic displacement at resonance

$$x_m = |\,x_0\,|\ \sqrt{2} \qquad \text{cm.} \quad (\mathbf{41})$$

can be measured.

Technique for Deriving the Constants A, m, r'', r', r, s'', s, p and β. (1) The motional-impedance circle is obtained from observations of the apparent resistance and reactance of the instrument, both free and damped, with constant testing current, under adjustably varied frequency. This gives the resonant maximum value Z'_0 and its slope $-\,2\ \beta°$ at ω_0 the angular velocity of apparent resonance; also* the quadrantal values ω_1 and ω_2, or their equivalents. We thus obtain the total damping constant

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*See technique in Bibliography 17, p. 469.

$$\Delta = \frac{\omega_2 - \omega_1}{2} = \frac{r''}{2\,m} \qquad \text{hyps per sec.} \quad (\mathbf{42})$$

and

$$\omega_0 = \sqrt{s''/m} \qquad \text{rad. per sec.} \quad (\mathbf{43})$$

(2) The amplitude of displacement x_m over the poles at resonance is measured for the measuring current I. Hence

$$A = \frac{I\,Z'_0}{j\,x_0\,\omega_0} = \frac{I_m\,Z'_0}{j\,x_m\,\omega_0} = \frac{I\,Z_0'}{x_0} \qquad \text{abvolts per kine} \angle \quad (\mathbf{44})$$

Then from (**40**)

$$r'' = A^2/Z_0' \qquad \text{dynes per kine} \quad (\mathbf{45})$$

Hence by (**42**)

$$m = \frac{r''}{2\,\Delta} \qquad \text{gm.} \quad (\mathbf{46})$$

Also by (**43**)

$$s'' = m\,\omega_0^2 \qquad \text{dynes per cm.} \quad (\mathbf{47})$$

Having ascertained the value of the vector damped inductance $\mathfrak{L}$, at resonant impressed frequency, during the measurements of damped impedance, we find by (**25**)

$$p = A/\mathfrak{L} \qquad \text{absamp per cm.} \quad (\mathbf{48})$$

We can then compute the value of the resistance component r' of f_x by (**29**) at resonance

$$r' = \frac{p\,|A|}{\omega_0} \cdot \sin\,\beta \qquad \text{dynes per kine} \quad (\mathbf{49})$$

and hence r, neglecting changes in r' with frequency,

$$r = r'' - r' \qquad \text{dynes per kine} \quad (\mathbf{50})$$

Next s' can be computed from (**30**) and s found by the relation

$$s = s'' + s' \qquad \frac{\text{dynes}}{\text{cm.}} \quad (\mathbf{51})$$

The total number of turns N is supposed to be known, from which $\mathfrak{B}_0$ is obtainable through the bipolar relation

$$\mathfrak{B}_0 = 2\,\pi\,p\,N \qquad \text{gausses} \quad (\mathbf{52})$$

The equivalent number of turns N' for computing the displacement m. m. f. is

$$N' = p\,N \qquad \frac{\text{absamperes}}{\text{cm.}} \qquad (53)$$

assuming that N is a pure numeric. The quantities $p\,A$, $\mathcal{F}_i$, ϕ_i and f_i are then easily found.

Motional Power. If a simple periodic m. m. f. of $\mathcal{F} = 4\,\pi\,N\,I$ r. m. s. gilberts ∠ at the reference phase of I r. m. s. absamperes, acts upon a leakanceless magnetic circuit of vector permeance $\mathcal{P}$ maxwells per gilbert ∠, the vector magnetic flux thereby produced, neglecting harmonics, which are powerless, is

$$\phi = \mathcal{F}\,\mathcal{P} \qquad \text{r. m. s. maxwells } \angle \qquad (54)$$

The phase of the flux will have the slope $-\ \beta°$ of $\mathcal{P}$, with respect to the phase of the exciting current and m. m. f. The e. m. f. which will be induced in the coil will be

$$E = -\,j\,\omega\,N\,\phi = -\,j\,\omega\,N\,\mathcal{F}\,\mathcal{P} \quad \text{r. m. s. abvolts } \angle \qquad (55)$$

The power of this e. m. f., expressed electrically, is

$$P = E\,I = -\,j\,\omega\,N\,\phi\,I \qquad \text{abwatts } \angle \qquad (56)$$

the slope of I being taken as zero.* The real component of P will be active power, expended thermally in the magnetic circuit, and the quadrature component will be reactive power expended in cyclically storing and releasing magnetic energy in the circuit. But (**56**) may be written**

$$P = -\,j\,\frac{\omega\,\mathcal{F}\,\phi}{4\,\pi} = -\ j\,(f/2)\,\mathcal{F}\,\phi \qquad \text{abwatts } \angle \qquad (57)$$

where $\mathcal{F}$ is taken as at standard phase, and ϕ has the lagging slope of $\beta°$ with respect thereto. The negative sign attached to the expressions in (**56**) and (**57**) indicates that the power is absorbed in the magnetic circuit.

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*Bibliography 7.

**The power exerted in a magnetic circuit by a simple alternating m. m. f. $\mathcal{F}$ r. m. s gilberts, at zero slope, on a simple alternating flux ϕ r. m. s. maxwells ∠, is, in the C. G. S. system,

$$P = \frac{\mathcal{F}}{4\,\pi}\cdot\frac{d\,\phi}{d\,t} \qquad \text{abwatts } \angle \qquad (\mathbf{57a})$$

where the real component is average active power, and the imaginary component max. cyclic reactive power. Here $\frac{d\,\phi}{d\,t}$ corresponds to electric current.

In Fig. 20, the vector m. m. f. $\mathfrak{F}_i$ is drawn to the same reference phase as I. The flux ϕ_i, thereby produced, with the diaphragm damped, is indicated as lagging $\beta°$ behind $\mathfrak{F}_i$. The induced e. m. f. E, lags $90° + \beta°$ behind $\mathfrak{F}_i$. The vector power $E\,I$, taken with I as of zero slope, is $O\,a$. Its real component, $O\,b$, is dissipated. The undissipated or reactive component is $b\,a$.

In Fig. 21, the r. m. s. m. m. f. and r. m. s. flux are represented without reference to e. m. fs. Their vector product $\mathfrak{F}_i\,\phi_i$, taking $\mathfrak{F}_i$ as of standard phase, would be along the dotted line $O\,c$. Taking $-f/2$ times this product, as called for in (**57**), we obtain, to a power scale, the vector $O\,d$, Fig. 22.

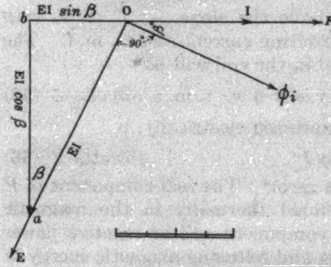

FIG. 20—DIAGRAM OF A R.M.S. M.M.F., THE R. M. S. FLUX THEREBY PRODUCED, THE CONSEQUENT E. M. F. AND THE VECTOR POWER

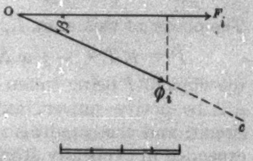

FIG. 21—FUNDAMENTAL RELATIONS OF POWER IN A MAGNETIC CIRCUIT CARRYING A R. M. S. FLUX $\phi\mathfrak{F}_i$, UNDER AN IMPRESSED R. M. S., M. M. F.

The cosine product of $f\,\mathfrak{F}_i/2$ and ϕ_i is reactive magnetic power, and the sine product, or vector product, active power liberated from the magnetic circuit. These conditions are opposite to those in the alternating electric circuit, where the cosine product is active and the sine product reactive electric power. In order to interpret this vector magnetic-circuit power correctly, we may rotate the axis $O\,X$ of reference, through $-90°$, into the position $O\,A$. The component $Oc = -(f/2)\,\mathfrak{F}_i\,|\phi_i|\sin\beta$ is then the average active power, absorbed by the magnetic circuit, and liberated from it, usually as heat; while the reactive component $O\,b = -(f/2)\,\mathfrak{F}_i\,|\phi_i|\cos\beta$ is power absorbed in storing and releasing magnetic energy.

If, however, we consider the vector power delivered by the electric circuit to the magnetic circuit, it will have the opposite sign to $O\,d$, or will be $O\,d$ reversed, and equal to the vector $O\,d'$, whose components are $O\,c' = + (f/2)\,\mathfrak{F}_i\,|\phi_i| \sin \beta$, liberated active power and $O\,b' = + (f/2)\,\mathfrak{F}_i\,|\phi_i| \cos \beta$ reactive power engaged in storage.

In Fig. 23, we have the vector magnetic power delivered from the electric circuit with the diaphragm damped. The real component is $O\,a'$ and the reactive component $a'\,a$.

When the diaphragm is free, we have the displacement m. m. f. $\mathfrak{F}_x$ and its flux ϕ_x introduced into the magnetic circuit, as shown in Fig. 24. The resultant of these two m. m. fs. is $\mathfrak{F}$, and that of the two corresponding fluxes is ϕ. This magnetic-circuit vector diagram bears a close resemblance to the electric-circuit vector diagram for the simple case of an alternator driving a synchronous motor through the impedance of a connecting circuit. The m. m. fs. of Fig. 24 then correspond to e. m. fs., flux-frequencies to currents, and their respective permeance vector ratios to admittance ratios. We have

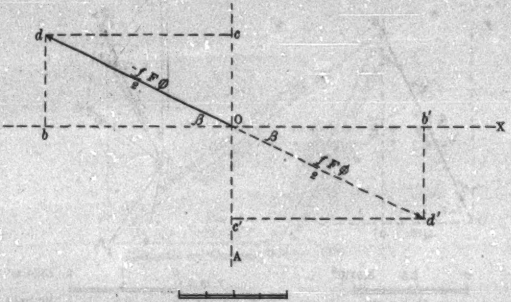

FIG. 22—POWER RECEIVED FROM AND DELIVERED TO A MAGNETIC CIRCUIT CARRYING A R.M.S. FLUX ϕ LAGGING $\beta°$ BEHIND AN IMPRESSED R. M. S. M. M. F. $\mathfrak{F}_i$ AT FREQUENCY f CYCLES PER SEC.

$$\mathfrak{F}_i + \mathfrak{F}_x = \mathfrak{F} \qquad \text{r. m. s. gilberts} \angle \quad \textbf{(58)}$$

If we multiply this equation by the vector resultant flux ϕ, considered for that purpose as of standard phase, we obtain

$$(f/2)\ \mathfrak{F}_i\,\phi + (f/2)\ \mathfrak{F}_x\,\phi = (f/2)\ \mathfrak{F}\,\phi \quad \text{abwatts} \angle \quad \textbf{(59)}$$

The first term is here the vector power input P_i'' delivered to the magnetic circuit by the exciting current. The second term, reversed in sign, is the vector power P_m liberated from the magnetic circuit into pure mechanical power of the diaphragm, both active and reactive, and corresponding to vector mechanical power given to a synchronous motor in an electric circuit. The term $(f/2)\mathfrak{F}\,\phi$ is the vector power P_r'' expended thermally within the magnetic circuit in overcoming its reluctance. The equation (**59**) may therefore be written

$$P_i'' - P_m = P_r'' \qquad \text{abwatts} \angle \quad (\mathbf{60})$$

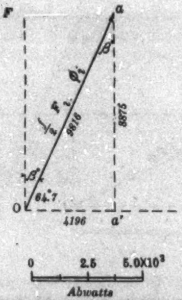

Fig. 23—Power Delivered to Magnetic Circuit at 1028 Cycles per Sec.—Diaphragm Damped

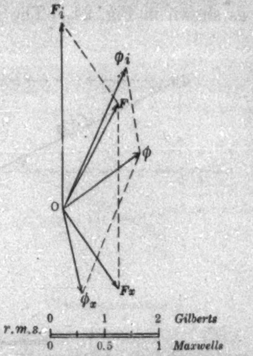

Fig. 24—Diagram of M. M. F' and Fluxes in Magnetic Circuit—Diaphragm Free

In the instrument referred to, these vector values at 1028 ∼ were as shown in Fig. 25, where $O\,b$ is the magnetic input of $P_i'' = 6438\ \underline{/36^\circ.8}$ abwatts, with an active component of 5156 abwatts, liberated both in heat and in mechanical power, and a reactive component of 3856 abwatts, engaged in the reactive storage of magnetic energy. P_m or the vector $O\,a$, is the power expended on the diaphragm mechanically, 3349 $\underline{/0.^\circ 7}$ abwatts, almost entirely in the active form, nearly 3349 abwatts being developed in vibratory motion $\dot{x}^2\, r$, against mechanical resistance r and 40 abwatts in reactive mechanical power; *i. e.*, power consumed in exchanging potential energy cyclically from the elastic to the inertia type. The vector $a\,b$ or P_r'' is the power expended in the magnetic conducting circuit by the total flux ϕ in overcoming the vector reluctance $\mathcal{R}$ of that circuit. At this frequency, it was 4199 $/\underline{64^\circ}.7$ abwatts, with a real component $a\,b'$ of 1795 abwatts, expended thermally in hysteresis and eddy currents. The reactive component $b'\,b$ of 3797 abwatts is expended in maintaining the activity of cyclically storing and releasing magnetic energy of the type $\frac{\mathcal{B}^2}{8\,\pi\,\mu}$ ergs per cu. cm. in the magnetic conducting circuit.

When the diaphragm is damped, the magnetic circuit conditions are shown in Figs. 21 and 23. Here the magnetic input at 1028 ∼ is 9816 $/\underline{64^\circ}.7$ abwatts, with no counter m. m. f. of motion. The active component $O\,a'$ is 4196 abwatts, expended entirely in heat leaving the circuit. The reactive component is 8875 abwatts, engaged in maintaining the cyclic storage and release of magnetic energy.

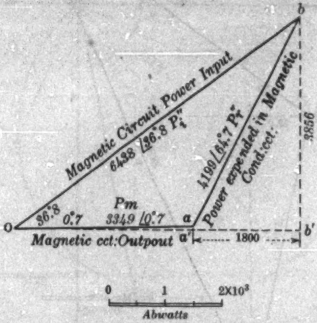

FIG. 25—MAGNETIC CIRCUIT POWER DIAGRAM—DIAPHRAGM FREE

The motional magnetic power diagram is obtained as in Fig. 26 by subtracting the magnetic power input damped P_i, from the magnetic power input free, P_i''. The difference, at the frequency of 1028 ∼, is 5108 $\overline{\diagdown}$ 79.°2 abwatts, with an active component of 957 abwatts, representing an increase of the power taken from the electric supply circuit and a negative reactive component of 5018 abwatts, representing a diminution in magnetic power of storage.

It should be observed that the active component $O\,c =$ 957 abwatts is much smaller than the power $P_m = 3349$ abwatts, (Fig. 25) put into the diaphragm. The difference of 2392 active abwatts is diverted from the power previously wasted in the magnetic circuit of 4196, Fig. 23, only 1800 abwatts (*a b′* Fig. 25), being wasted thermally when the diaphragm is freed. Consequently, the motional power diagram does not reveal on its real axis all the power delivered to the diaphragm; but only that part which is supplied from the electric circuit. The remainder has to be made up from power saved in hysteretic and eddy loss, due to the reduction in magnetic flux by the motional counter m. m. f., especially towards higher frequencies.

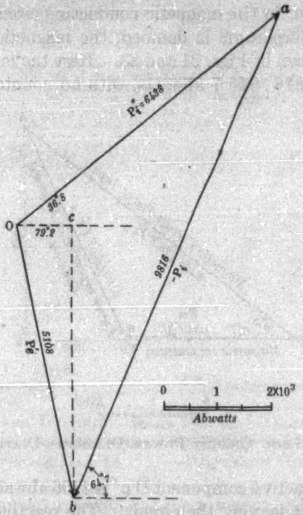

FIG. 26—DIAGRAM OF MOTIONAL POWER IN MAGNETIC CIRCUIT—DIAPHRAGM FREE

Turning to Fig. 27, the motional power diagram, the circular locus there represented may be considered as the motional impedance circle of Fig. 10, changed in scale, however, for convenience, in order to permit powers to be read off it. If, however, a fractional scale of power is accepted, the motional impedance circle may be interpreted directly as a motional power diagram. In Fig. 27, *O a* is the vector corresponding in phase to the motional impedance vector at the frequency considered, in this case 1028 ∼. There are three pairs of co-

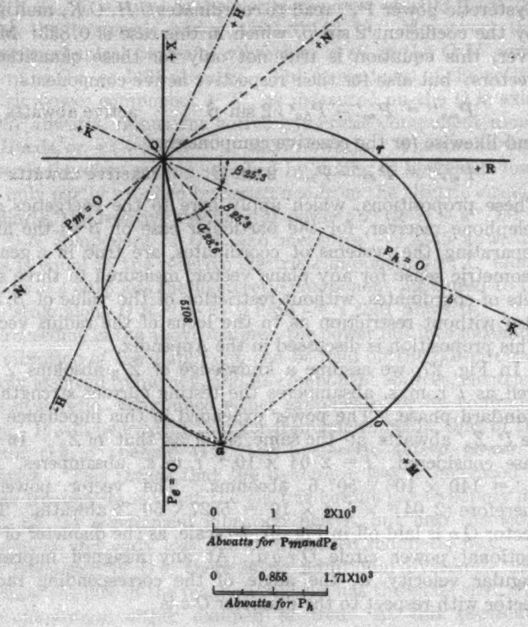
+H
+N
+X
+K
O
r
+R
β 25°.3
β 25°.3
α 28°.6
Pm = O
P_h = O
-N
5108
b
-K
-H
a
o
+M
P_e = O
0
1
2X10³
Abwatts for P_m and P_e
0
0.855
1.71X10³
Abwatts for P_h

Fig. 27—Motional Power Diagram

ordinate axes; namely (**1**) *O R*, at standard phase, for the electric circuit power, and *O X* its perpendicular, (**2**) *O H*, inclined at $-\beta°$ from *O X*, and its perpendicular *O K*, for components of hysteretic loss in the magnetic circuit, and (**3**) *O M*, at a slope of $-2\beta°$ with *O R*, and its perpendicular *O N*, for mechanical power of vibration in the diaphragm.

At any vector motional power $O\,a$, we have the following vector relation

$$P_m'' = P_e' - P_h' \cdot 2 \sin \beta \qquad \text{abwatts } \angle \quad (61)$$

Or, in language, the vector mechanical power P_m'', as read to coordinates $O\,M$, $O\,N$, is equal to the electrically delivered vector power P_e', read to coordinates $O\,R$, $O\,X$, less the hysteretic power P_h', read to coordinates $O\,H$, $O\,K$, multiplied by the coefficient $2 \sin \beta$, which in this case is 0.855: Moreover, this equation is true not only for these quantities as vectors; but also for their respective active components,

$$P_{ma}'' = P_{ea}' - P_{ha}' \cdot 2 \sin \beta \qquad \text{active abwatts} \quad (62)$$

and likewise for the reactive components

$$P_{mr}'' = P_{er}' - P_{hr}' \cdot 2 \sin \beta \qquad \text{reactive abwatts} \quad (63)$$

These propositions, which apply here to the energetics of a telephone receiver, for the particular case of β as the angle separating the systems of coordinates, are true in a general geometric sense for any plane vector, measured to three such sets of coordinates, without restriction of the value of β, and also without restriction as to the locus of the radius vector. This proposition is discussed in the Appendix.

In Fig. 27, we assume a knowledge of Z'_0 absohms $\angle$, as well as I r. m. s. absamperes the testing current strength, at standard phase. The power expended in this impedance will be $I^2\ Z_0'$ abwatts at the same slope as that of Z_0'. In the case considered, $I = 2.04 \times 10^{-4}$ r. m. s. absamperes, and $Z_0' = 140 \times 10^9 \measuredangle 50^\circ.6$ absohms. The vector power is therefore $(2.04)^2 \times 1.4 \times 10^3 = 5827 \measuredangle 50.^\circ 6$ abwatts. This vector $O\,o$ is laid off in Fig. 27, to scale, as the diameter of the motional power circle $O\,r\,o\,a$. At any assigned impressed angular velocity ω, the angle of the corresponding radius vector with respect to the diameter $O\,o$ is

$$\alpha = \tan^{-1}\left(\frac{m\,\omega - s''/\omega}{r''}\right) \qquad \text{degrees lag} \quad (64)$$

In the case considered, with $f = 1028\sim$, and $\omega = 6460$, $m\,\omega = 5827$, $s''/\omega = 5681$, $r'' = 268$; so that

$$\alpha = \tan^{-1} 0.545 = 28.^\circ 6.$$

The vector position $O\,a$, corresponding to $1028\sim$ can thus be laid down in the circle. Its length is 5108 abwatts. With respect to the $R\,X$ coordinates it is $5108 \angle 79.^{\circ}2$. It is $5108 \angle 28.^{\circ}6$ to $M\,N$, and $5108 \angle 143.^{\circ}9$ to $H\ K$. Here the electrical vector is $5108 \angle 79.^{\circ}2$ abwatts, the mechanical vector $5108 \angle 28.^{\circ}6$, and the hysteretic vector $P_h = P_h' \cdot 2 \sin \beta$

$$= 5108 \angle 143.^{\circ}9 \times 2 \sin 25.^{\circ}3 = 4369 \angle 143.^{\circ}9.$$

The proposition of (**61**) states the mechanical power of $5108 \angle 28.^{\circ}6$, is equal to the electrical power of $5108 \angle 79.^{\circ}2$, minus the hysteretic power $4369 \angle 143.^{\circ}9$. Moreover, the active mechanical power of vibration, $\dot{x}^2 r''$ is the $O\,M$ component of $O\,a$, or 4484 abwatts. This is made up of the active electric power component $I^2 R'$, as measured on the $O\,R$ axis, of 957 abwatts, minus the active hysteretic component along the H axis, or $-\ (-\ 3537) = +\ 3537$ abwatts. The mechanical power is therefore mainly supplied by saving in hysteresis loss, and only partly supplied by assistance from the electric circuit. Similarly, the reactive mechanical power $j\,\dot{x}^2\,(m\ \omega - s''/\omega)$ is the $O\,N$ component of $O\,a$, and measures $-\,j\,2445$ abwatts. This is equal to the reactive electric component $j\,I^2\,X'$ along $O\,X$ or $-\,j\,5018$ less the hysteretic component measured to its own scale along $O\,K = -\ (-\,j\,2591) = j\,2591$ abwatts.

Proceeding in the above manner, the power put into mechanical vibration of the diaphragm can be read off the power circle at any assigned frequency, and its component sources, electrical and hysteretic, determined either graphically or by computation. It will be evident that for frequencies below that of the vector $O\,b$, β° behind $O\,R$, the hysteretic active power is positive. This means that the hysteretic loss is increased at these frequencies, so that the electric circuit has to supply not only all the mechanical vibration power, but also the extra hysteresis and eddy loss due to increased magnetic flux. At the frequency of $O\,b$, there is no additional hysteretic loss, because the flux in the magnetic circuit has the same size, with the diaphragm either damped or free. At this particular frequency, all the active mechanical power is supplied directly from the electric circuit. At frequencies above that of $O\,b$, the active mechanical power is partly supplied by hysteresis saving. At the vector point c, where the circle cuts the reactance axis, there will be no extra power supplied by the circuit to maintain the mechanical output of the diaphragm; but all of this power will be paid for by saving in hysteresis loss, when the diaphragm is released.

The total active mechanical power developed in the diaphragm will be

$$P_m'' = \dot{x}^2 (r + r') = \dot{x}^2 r'' \qquad \text{abwatts} \quad (65)$$

and the active power put into vibration

$$P_m = \dot{x}^2 r \qquad \text{abwatts} \quad (66)$$

Similarly, the active power put into motional hysteresis loss,

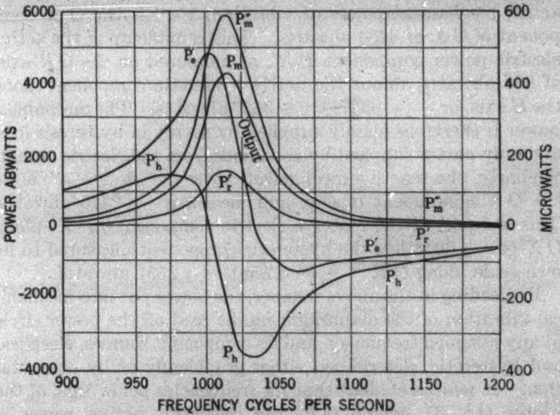

Fig. 28—Magnetic Circuit Motional Power Disbtribution at Different Frequencies. P_m'' Gross Mechanical Active Output P' Active Electric Power Increases P_h Change of Hysteretic Power. P_r' Active Mechanical Power Expended Against r' in Displacement. P_m Net Active Mechanical Output Free. $P_m'' = P_e' - P_h = P_r' + P_m$.

by reason of the vibration of the diaphragm in the magnetic field will be

$$P_r' = \dot{x}^2 r' \qquad \text{abwatts} \quad (67)$$

The mechanical efficiency of the diaphragm, as a motor, at resonance, is therefore

$$\eta_0 = \frac{r}{r''} = \frac{r}{r + r'} \qquad \text{numeric} \quad (68)$$

In the case considered, $\eta_0 = 0.74$.

Strictly speaking, according to our theory, r' is a function of ω, see (**29**), but unless the resonance is very blunt, the changes in r' and in η, with change in frequency, are not likely to be important.

Fig. 28 shows the distribution of the various active powers, in the instrument considered, over a range of frequency from 900 ∼ to 1200 ∼. Here $P_m = \dot{x}^2 r$ is the active mechanical power output in sound and in overcoming internal mechanical bending friction of the diaphragm. $P_r' = \dot{x}^2 r'$ is active mechanical power expended in the magnetic circuit in hysteresis and in eddies, by reason of the vibration of the diaphragm in the permanent magnetic field. $P_m'' = \dot{x}^2 r'' = \dot{x}^2 (r + r') = P_m + P_r'$ is the total active mechanical output of the diaphragm. P_e' is the extra active electric power taken from the electric circuit and put into mechanical vibration when the diaphragm is freed. The saving of active damped power put into the magnetic circuit when the diaphragm is freed is $- P_h$. Consequently

$$P_m'' = P_e' - P_h.$$

The following tabular statement gives the computed analysis of the constants of the particular instrument considered, partly taken from the Kennelly-Affel paper of 1915, already referred to, and partly as developed in this analysis:

TABLE OF NUMERICAL DATA CONCERNING ANALYZED RECEIVER

Mechanical	
Area of each pole, cm × cm.	1.14×0.199
Distance separating poles, cm.	0.686
External diam. of diaphragm, cm.	5.52
Diameter of clamping circle, cm.	5.0
Thickness of diaphragm over japan, cm.	0.031
Weight of diaphragm, gm.	4.397
Number of turns including both spools.	1300
Electrical	
D-c. res. at 20 deg. cent. absohms R_1	86.7×10^9
Inductance at resonant frequency $\mathfrak{L}_0$ abhenries.	3.65×10^7
Current through receiver I r. m. s. absamp.	2.04×10^{-4}
Resonant frequency apparent, f_0 cycles per sec.	1015
Resonant angular vel: ω_0 radians per sec.	6378
Motional impedance circle diameter Z_0' absohm.	140×10^9
Slope of permeance at resonant frequency β. degrees	$-25°.3$

Max. amplitude of vibration at resonance x_m cm.	10.35×10^{-4}
Effective cyclic amplitude vibration at resonance x_0 cm.	7.32×10^{-4}
Max. cyclic velocity at resonance x_0 r. m. s. cm. per sec or kines	4.666
Quadrantal angular velocities ω_1, ω_2 radians/sec.	6227 6532
Total decrement Δ hyps. per sec.	149
Force factor A dynes /absampere or abvolts/kine.	$6.12 \times 10^{6}\ \overline{\angle}\ 25.^\circ 3$
Total mechanical resistance r'' dynes/kine.	268
Equivalent mass in grams.	0.902
Total stiffness coefficient s'' dynes/cm.	36.7×10^{6}

Recomputed Electromagnetic Data

Ratio of numbers of turns p absampere/cm.	0.168
Mechanical resistance of magnetic reaction r' dynes/kine	68.8
Pure mechanical resistance r dynes/kine.	199
Stiffness coefficient of magnetic reaction s' dynes/cm.	0.928×10^{6}
Pure mechanical stiffness coefficient s dynes/cm.	37.63×10^{6}
Equivalent number of turns per cm. displacement N' gilberts/cm.	218
Normal mean polar flux density $\mathcal{B}_0$ gausses.	1369
Normal total polar flux Φ_0 maxwells/pole	311.0
Equivalent permanent magnetic reluctance $\mathcal{R}_0$ oersteds	$0.582 \angle 25.^\circ 3$
Equivalent permanent magnetic permeance $\mathcal{P}_0$ (oersteds)$^{-1}$	$1.718\ \overline{\angle}\ 25.^\circ 3$
Equivalent permanent m. m. f. gilberts.	181
M. M. F. of testing current (2.04×10^{-4}) r. m. s. gilberts $\mathfrak{F}_i$.	$3.334 \angle 0^\circ$
Flux in magnetic circuit due to current ϕ_i r. m. s. maxwells	$5.73 \angle 25.^\circ 3$
Mechanical force due to current I f_i, r. m. s. dynes, $v\,m\,f$	$1249\ \overline{\angle}\ 25.^\circ 3$
Magnetic power input at 1028 ~ with current I, P_i abwatts,	$9816 \angle 64.^\circ 7$ $= 4196 + j\,8875$
Total mechanical impedance at $f = 1028$, z' dynes/kine	$305.2 \angle 28.^\circ 6$
Mechanical velocity at $f = 1028$. $\dot{x}$. r.m.s. cm/sec.	$4.091 \angle -53.^\circ 9$
Cyclic mechanical displacement at $f = 1028$. x r. m. s. cm.	$6.332 \times 10^{-4}\ \overline{\angle}\ 143.^\circ 9$
M. M. F. due to displacement, at $f = 1028$ and for current I, $\mathfrak{F}_x$ r. m. s. gilberts.	$1.735\ \overline{\angle}\ 143.^\circ 9$
Resultant m. m. f., at $f = 1028$. $\mathfrak{F}$ r. m. s. gilberts	$2.174 \angle -28^\circ$
Flux due to displacement, at $f = 1028$. ϕ_x r.m.s. maxwells	$2.980\ \overline{\angle}\ 169.^\circ 2$

Resultant flux at $f = 1028$. ϕ r. m. s. maxwells. 3.757 ⦢ 53.°2
Magnetic input diaphragm free, at $f = 1028$.
P_i'' abwatts ....6438 ∠ 36.°8
=5156 + j 3856
Free magnetic power, or total power absorbed in
magnetic circuit, at $f = 1028$ ~, abwatts ∠ P_r'' =4199 ∠ 64.°7
= 1795 + j 3797
Mechanical output power of diaphragm, P_m at
1028 ~ abwatts ∠ .... = 3349 ∠ 0.°7
= 3349 + j 40
Total mechanical power of diaphragm at 1028 ~
P_m'' abwatts ∠ = 5108 ⦢ 28.°6 = 4484 − j 2445
Motional electric power of diaphragm at 1028 ~
P_e' abwatts ∠ = 5108 ⦢ 79.°2 = 957 − j 5018
Hysteretic power at 1028 ~ P_h abwatts ∠
= 5108 × 0.855 = 4369 ⦢ 143.°9 = − 3537 − j 2591
Mechanical efficiency of diaphragm at apparent
resonance, η_0 numeric = 0.74
Net efficiency of receiver, freed, and at apparent
resonance numeric, = 0.4
Maximum motional impedance per unit of continuous-current resistance Z_0'/R_1 numeric ∠ = 2.04 ⦢ 50.°6
Force factor per root of continuous-current res.

$$A/\sqrt{R} \quad \frac{\text{dynes} \sqrt{\text{absohms}}}{\text{abvolts}} \angle = 23.36 ⦢ 25.°3$$

Effect of a Changing the Number of Turns in Receiver of given Winding Space. If the windings of a given receiver are allowed to occupy a definite constant volume, and also if the ratio of bare to covered wire diameter is kept constant over the range of wire diameters selected for the winding, so that the same volume and weight of copper will exist no matter what the size of wire; then it is evident that if we reduce the wire diameter one-half, the number of turns N will be quadrupled and the d-c. resistance R_1, at constant temperature, increased sixteen fold. The effect of this will be to increase N, A, $\mathfrak{F}_i$, and ϕ_i, 4 times, leaving $\mathfrak{F}_0$, $\mathfrak{B}_0$, $\mathcal{P}$, and β, unchanged. The quantities $\mathfrak{L}$, Z, Z_0', P_e, and P_{m0}, will be all increased 16 times, (neglecting variations in r') or in direct proportion to the resistance. Consequently the ratio of active mechanical power output $\dot{x}^2 r/I^2 R_1$, where R_1 is the d-c. resistance of the instrument, will remain substantially unchanged for all values of R_1 within the range for which the copper in the winding space remains constant. On this theory, changing the winding of a receiver, other things remaining the same, increases the mechanical output at resonance in proportion to the resistance, for a fixed exciting current strength, but for a given amount of $I^2 R_1$ power loss in the winding, this mechanical power output remains substantially constant.

Since Z_0' the maximum or diametral value of the vector motional impedance increases, according to this theory, directly with the d-c. resistance of the winding, different receivers should have their diametral impedances Z_0' reduced to a common basis of comparison by taking the ratio Z_0'/R_1. The sensitiveness of a receiver is proportionate to this ratio. The ratio is independent of the strength I of the testing current. An instrument of large diametral impedance per unit of d-c. resistance is therefore an instrument of inherently large electromagnetic sensitivity, no matter what the resistance of the winding may be. This ratio calls for no other measurements than that of the motional impedance circle or the essential sector thereof, and the d-c. resistance at normal temperature.

Similarly, the force factor A should be divided by $\sqrt{R_1}$, in order to bring it to a common basis of comparison among various receivers.

It should be observed that the diametral or maximum size of the motional impedance Z_0' is not produced at the frequency of strict mechanical resonance of the diaphragm. True mechanical resonance is produced when the angular velocity is such that the mass reactance $m\ \omega$ is numerically equal to the elastic reactance s/ω. In the motional-impedance circle, however, the apparent resonance and diametral impedance are produced when $m\ \omega = \dfrac{s''}{\omega} = \dfrac{s - s'}{\omega}$. The fictitious component s', due to the displacement flux, thus slightly lowers the apparent resonance, or causes the maximum velocity, displacement and impedance to occur at a frequency a little lower than that of true mechanical resonance. In the caseconsidered, mechanical resonance would be produced at $\omega_0 = \sqrt{s/m} = \sqrt{37.63 \times 10^6/0.902} = 6459$ radians per sec.; corresponding to 1028 ∼; whereas the apparent resonance would be produced at $\omega''_0 = \sqrt{s''/m} = \sqrt{36.7 \times 10^6/0.902} = 6379$, corresponding to 1015 ∼. This displacement of resonance must occur in the motional impedance circle, whatever method of including the effects of the displacement flux φ_s is adopted. Apparent resonance occurs at the frequency of mechanical resonance as modified by magnetic reaction, which acts in such a manner as to reduce the stiffness coefficient of the diaphragm. The angle of displacement $h\,b\,o$ in the motional impedance circle, Fig. 10, between the vector of true mechanical resonance and apparent resonance, is equal to the angle α between ϕ and ϕ_i at the angular velocity ω_0' of true mechanical resonance; because at ω_0' the velocity $\dot{x}$ is in phase with the resultant flux ϕ.

A bibliography of the subject, without pretensions as to completeness, is appended, in so far as relates to the motional-impedance aspect of telephone-receiver theory.

It may be noted that the maximum mechanical output $\dot{x}_0^2\,r''$ at apparent resonance is entirely active power, and is numerically equal to $I^2\,|Z_0'|$ abwatts or the square of the testing current multiplied by the size of the diametral impedance. This constitutes a simple method of determining the maximum mechanical output of the diaphragm, including frictions and hysteretic loss due to vibration. The gross efficiency of the receiver at apparent resonance is therefore $\dfrac{I^2\,|Z_0'|}{I^2\,R_0''} = \dfrac{|Z_0'|}{R_0''}$.

In this case, this efficiency reaches $\dfrac{140}{257.3} = 0.544$. This must be multiplied by the $\eta_0 = 0.74$ to give the net efficiency of the receiver at apparent resonance of 0.40. This is the ratio of active mechanical output of the diaphragm, including internal frictions and acoustic delivery, to the active electric power consumed at terminals, with the diaphragm free. With the receiver pressed against the ear, the condition is intermediate between damped and free.

SUMMARY OF RESULTS

1. The theory presented in the earlier papers mentioned in the bibliography were lacking in not taking into account the m. m. f. and flux, of diaphragm vibratory displacement in the magnetic field.

2. The present theory purports to include the above mentioned elements, under the form of an apparent increment r' in mechanical resistance, and an apparent diminution s' in the stiffness coefficient, the latter being independent of the frequency, but the former varying inversely as the frequency.

3. The effect of r' is to introduce a slight distortion of the motional impedance and power diagrams from the strictly circular locus, assuming the force-factor A and other constants

to remain substantially the same at all frequencies; but the distortion is not serious from a practical standpoint.

4. The m. m. f. of vibratory displacement may be expressed in terms of a coefficient N', a fictitious number of turns. The ratio p of N'/N is a constant of the instrument.

5. The depression of the motional-impedance circle diameter is 2 $\beta°$ where $\beta°$ is the lag angle of the damped magnetic flux behind the exciting current.

6. The power analysis depends directly upon the m. m. f. and flux diagram, without any necessary reference to the e. m. fs. produced. The power in the magnetic circuit is then found to be similar to the power in a simple alternating-current circuit, containing an alternator, a synchronous motor, and a simple connecting impedance.

7. The observed motional impedance circle may be converted into a corresponding motional power circle, by a suitable alteration in scale of linear dimensions. By introducing three sets of coordinate axes, mutually displaced by $\beta°$, the power output P''_m, can be read off in terms of the electric power input P'_e, and the hysteretic power input P_h. The same relation holds for active and for reactive components.

8. At the slope $-\ \beta°$ in the power circle, the mechanical power is all electric. At the slope $-\ 90°$, it is all hysteretic.

9. The active power mechanically expended in the magnetic circuit by the diaphragm as a motor, against hysteresis and eddy currents $\dot{x}^2 r'$, can be separated from that expended against friction and air resistance $\dot{x}^2 r$. The mechanical efficiency is

$\frac{r}{r + r'}$, which varies somewhat with the frequency. It tends to increase slightly with frequency, according to this theory.

10. The behavior of a particular instrument analysed some years ago in a preceding paper is investigated in detail, at a particular frequency, in accordance with the theory.

11. The gross efficiency of a receiver at apparent resonance is $\frac{|Z'_0|}{R''_0}$ or the ratio of the motional impedance diameter to the apparent resistance of the instrument at this frequency with the diaphragm free.

Appendix

Proposition that the Plane Vector

$$|\rho_m| \overline{\backslash\alpha} = |\rho_e| \overline{\backslash 2\ \beta + \alpha} \qquad - 2 \sin\ \beta\, |\rho_m| \overline{\backslash 90^\circ + \alpha^\circ + \beta}$$

In Fig. 29, *O A* is a planevector, or geometrical complex quantity of any convenient size ρ, taken at standard phase, or zero slope. It may be denoted by ρ_A · *O B* is a similar plane vector of the same size, distinguished by the symbol ρ_B. Its slope is $- 2\ \beta^\circ$. *O c* is a vector denoted by ρ_c of size ρ and drawn perpendicular to *O d*, the bisector of the angle *A O B*.

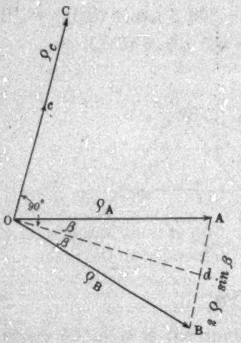

FIG. 29

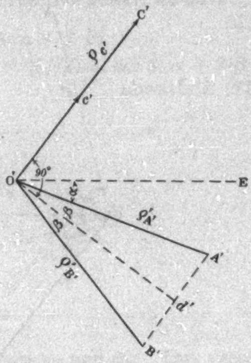

FIG. 30

Then, because *B A* expressed vectorially is $|\rho|\, 2 \sin\ \beta\ \underline{/\, 90^\circ - \ \beta}$ and is equal to the vector *O c*, we have the vector relation

$$\rho_B = \rho_A - 2 \sin\ \beta \cdot \rho_c \qquad \text{length units } \angle \quad (\mathbf{69})$$

If now, as in Fig. 30, we rotate the whole vector system in the plane of reference through the angle $- \alpha^\circ$ with respect to *O E*, a fixed line of reference, the effect will be the same as though the vector system remained fixed in the plane; but the line of reference, *O E* was rotated through an angle of $+ \alpha$. This rotation of a reference line cannot, however, affect the vector relation expressed in (**69**) when those vectors are taken to the new reference line.

But the three vectors of (**69**) may be regarded as one and the same vector ρ, Fig. 31, expressed each with respect to three initial lines $O'' m$, $O'' e$, and $O'' h$, and then denoted by ρ_m, ρ_e and ρ_h respectively. Thus in Fig. 32,

$$|\rho_m| \diagdown\overline{\alpha} = |\rho_e| \diagdown\overline{2\ \beta + \alpha} - |\rho_h|\, 2 \sin \beta \diagdown\overline{90^\circ + \alpha + \beta} \quad \text{length units } \angle \quad (\mathbf{70})$$

Since this relation holds for the vectors themselves, it must also hold for their real components, and for their imaginary components respectively.

The power circle of Fig. 27, corresponds to Fig. 31; while the interpretation of (**62**) and (**63**) are referable to the construction of Fig. 32.

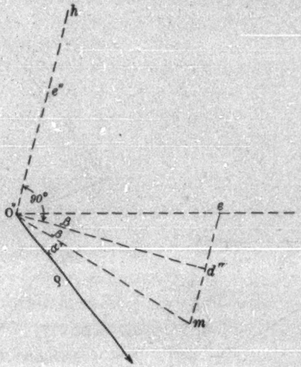

FIG. 31

Referring to Fig. 24, in the analogy of a simple a-c. circuit in which a generator of $\mathfrak{F}_i$ r. m. s. abvolts terminal e. m. f. supplies an external circuit of fixed admittance $\mathcal{P}$ mhos ∠, with a current of ϕ_i r. m. s. absamperes. To the condition of diaphragm damped, corresponds the analogous condition of zero c. e. m. f. in the synchronous motor. To the condition of diaphragm free, corresponds the analogous condition of e. m. f. $\mathfrak{F}_z$ r. m. s. abvolts in the synchronous motor armature, which would produce the current of ϕ_z r. m. s. absamperes if operating independently in the circuit, at assumed constant frequency.

The vector change in electric power P'_e given by the alternator to the circuit when the diaphragm is suddenly freed, will be the power exerted by the e. m. f. $\mathfrak{F}_i$ on the current ϕ_x. The vector mechanical power P_m'' given to the motor by the current ϕ_i will be the power of ϕ_i on $\mathfrak{F}_x$.

Converting these electric powers into their analogous magnetic powers, and remembering that in the magnetic circuit a vector a-c. m. m. f. $|\mathfrak{F}| \angle \gamma°$ r. m. s. gilberts, producing a vector alternating flux $|\phi| \angle 0°$ r. m. s. maxwells, develops a vector power in the magnetic circuit of $|(f/2)\ \mathfrak{F}\,\phi| \angle 90° - \gamma°$ abwatts, of which the real part is average active power, and the imaginary part is maximum cyclic reactive power, we have

$$P'_e = |(f/2)\ \mathfrak{F}_i\,\phi_x| \angle 90° - (90° + \alpha + 2\ \beta)$$
$$= |(f/2)\ \mathfrak{F}_i\,\phi_x| \overline{<} \alpha + 2\ \beta = |(f/2)\ \mathfrak{F}_x\,\phi_i| \overline{<} \alpha + 2\ \beta \quad \text{abwatts} \angle \quad (71)$$

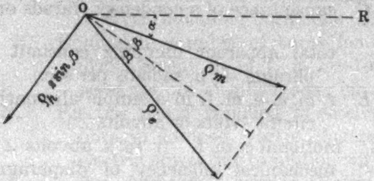

FIG. 32

where $90° + \alpha$ is the angle between ϕ_i and $\mathfrak{F}_x$. This vector power has a circular locus, because $|(f/2)\ \mathfrak{F}_x| \overline{<} \alpha$ has a circular locus.

Also

$$P_m'' = |(f/2)\mathfrak{F}_x\phi_i| \angle 90° - (90° + \alpha) = |(f/2)\mathfrak{F}_x\phi_i| \overline{<} \alpha \quad \text{abwatts} \angle \quad (72)$$

This vector mechanical power must also have a circular locus under varying frequency.

Consequently,

$$P_e' - P_m'' = |(f/2)\ \mathfrak{F}_x\,\phi_i| \overline{<} \alpha + 2\ \beta - |(f/2)\ \mathfrak{F}_2\,\phi_i| \overline{<} \alpha \quad \text{abwatts} \angle \quad (73)$$

and this by (70) is

$$P_h = |(f/2)\ \mathfrak{F}_x\,\phi_i| \cdot 2 \sin\ \beta \overline{<} 90° + \alpha + \beta \quad \text{abwatts} \angle \quad (74)$$

a vector hysteretic power having also a circular locus.

LIST OF SYMBOLS EMPLOYED

For simplicity of numerical treatment and universality of application, the units employed are those of the C. G. S. magnetic system.

A	force factor (dynes per absampere ∠* or abvolts per kine ∠)
α	angle of mechanical effective impedance; *i. e.*, mechanical impedance including magnetic displacement reaction (degrees)
$\mathfrak{B}_0$	magnetic flux density in the airgaps due to permanent magnet (gausses)
$\mathfrak{B}_i$	r. m. s. magnetic flux density due to current (r. m. s. gausses ∠)
$\mathfrak{B}$	instantaneous alternating flux (gausses)
β	mean angle of lag between magnetic flux and m. m. f. (degrees)
C	capacitance of a condenser (farads or abfarads)
$\Delta = \dfrac{r''}{2m}$	total apparent damping constant of a diaphragm (hyp. radians per sec.)
E	r. m. s. e. m. f. in a simple alternating-current circuit (volts or abvolts ∠)
$E = I Z'$	motional e. m. f. (r. m. s. abvolts ∠)
$\eta = r/r''$	mechanical efficiency of diaphragm, corresponding to armature efficiency in a synchronous motor (numeric)
η_0	mechanical efficiency of diaphragm, or friction and acoustic mechanical resistance to total mechanical resistance, at apparent resonance (numeric)
$\mathfrak{F}_i$	m. m. f. due to constant exciting alternating current (r. m. s. gilberts ∠)
$\mathfrak{F}_x$	m. m. f. due to vibratory displacement of diaphragm in permanent magnetic field (r. m. s. gilberts ∠)
$\mathfrak{F}$	resultant vector m. m. f. in magnetic circuit (r. m. s. gilberts ∠)
$\mathfrak{F}_0$	m. m. f. due to permanent magnet (gilberts)
f	impressed frequency (cycles per sec.)

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*The sign ∠ after a unit signifies that the quantity is a plane-vector or complex quantity as distinguished from a scalar or simple numerical quantity.

f_0,	impressed frequencies of apparent resonance (cycles per second)
f_c	reactive mechanical force of clamping (dynes ∠)
f_i	vibromotive force on diaphragm due to alternating current I alone (r. m. s. dynes ∠)
f_x	vibromotive force due to vibratory diaphragm displacement in permanent magnetic field (r. m. s. dynes ∠)
I	strength of constant small alternating testing current (r. m. s. absamperes)
$j = \sqrt{-1}$	
L	Self inductance of an electric circuit (abhenries or henries)
$\mathcal{L}$	Self inductance of a receiver winding with diaphragm damped (abhenries ∠)
m	Equivalent mass of a diaphragm (grams)
N	Number of turns in the winding of a bipolar receiver, including both coils (numeric)
N'	Equivalent number of turns for expressing a displacement m. m. f. in cm-turns (absamperes per cm. or gilberts per cm.).
P_e, P_e', P_e''	Electric power supplied to receiver with diaphragm damped, vector change in supplied electric power due to motion with diaphragm free, and total supplied electric power with diaphragm free. (abwatts or ergs per sec. ∠)
P_h	Increase in magnetic circuit power loss due to motion of diaphragm (abwatts ∠)
P_h'	Apparent increase in magnetic circuit power loss due to motion of diaphragm, as measured in the motional power circle diagram along the hysteretic axis before applying change in power scale abwatts ∠
P_i, P_i''	Damped magnetic power input (abwatts ∠), and input power from electric to magnetic circuit with diaphragm free (abwatts ∠)
P_m, P_m''	Mechanical output power of diaphragm against pure mechanical res. r (abwatts ∠) and total mechanical output power of diaphragm against total resistance of $r'' = r + r'$ (abwatts ∠)
P_{ma}, P_{ea}, P_{ha}	Active or dissipatory components of mechanical, electric, and hysteretic power (abwatts)

P_{mr}, P_{er}, P_{hr}	Reactive or non-dissipatory components of mechanical, electric and hysteretic power (j abwatts)
$P_r' = x^2 (r' + j s'/\omega)$	Motional power or power expended against reactive force of diaphragm displacement (abwatts $\angle$)
P_r''	Free magnetic power, or vector power of resultant m. m. f. $\mathcal{F}$ on resultant magnetic flux ϕ, or total absorbed in magnetic circuit (abwatts $\angle$)
$p = N'/N$	Ratio of equivalent displacement turns to exciting turns (absamperes per cm.)
$\mathcal{P}$	Permeance of magnetic circuit (oersteds^{-1} $\angle$ or maxwells per gilbert $\angle$)
ϕ_i	Magnetic flux due to constant exciting current (maxwells $\angle$)
ϕ_x	Magnetic flux due to displacement x in magnetic field (maxwells $\angle$)
$\phi = \phi_i + \phi_x$	Resultant magnetic flux (maxwells $\angle$)
R_1	Continuous-current resistance of receiver winding (absohms)
R	Apparent resistance of receiver to alternating current with diaphragm damped (absohms)
R_h	Increment of apparent resistance due to hysteresis and eddy currents, with diaphragm damped (absohms)
R'	Resistance component of motional impedance Z' (absohms)
R''	Total apparent resistance of receiver with diaphragm free (absohms)
R''_0	Total apparent resistance of receiver at apparent resonance (absohms)
r	Mechanical resistance of diaphragm including internal frictions and acoustic or air resistance (dynes per kine)
r'	Hysteretic resistance of magnetic circuit to motion of diaphragm, or apparent increase in friction of diaphragm due to displacement flux losses in magnetic circuit. (dynes per kine.)
$r'' = r + r'$	Total mechanical resistance to motion of diaphragm (dynes per kine)

R_0	Reluctance of magnetic circuit to permanent flux (oersteds)
S	Surface area of one pole neglecting magnetic fringe. (sq. cm.)
s	Effective stiffness coefficient of diaphragm to vibrational displacement (dynes per cm.)
s'	Apparent diminution in stiffness coefficient due to displacement in magnetic field. (dynes per cm.)
$s'' = s - s'$	Total effective stiffness of diaphragm including magnetic displacement (dynes per cm.)
t	Time elapsed from a certain epoch (seconds)
X	Reactance of a receiver winding with damped diaphragm (absohms ∠)
X'	Reactive component of motional impedance Z' (j absohms)
X''	Reactive component of receiver impedance Z'' with diaphragm free (j absohms)
x	Displacement of diaphragm towards poles from normal flexed position (r. m. s. cm. ∠)
x_0	Displacement of diaphragm at apparent resonance (r. m. s. cm. ∠)
$x_m = \|x_0\| \sqrt{2}$	Maximum cyclic displacement of diaphragm at apparent resonance (max. cm.)
$\dot{x}$	Velocity of diaphragm r. m. s. kines ∠ or r. m. s. cm. per sec. ∠
$\dot{x}_0$	Velocity of diaphragm at apparent resonance (r. m. s. kines ∠)
$\ddot{x}$	Acceleration of diaphragm (r. m. s. kines per sec. ∠)
$y = 1/z$	Mechanical admittance of a diaphragm when the magnetic reactive force due to displacement is neglected (kines per dyne ∠)
y'	Apparent mechanical admittance of diaphragm, including magnetic reactive force due to displacement (kines per dyne ∠)
Z	Impedance of a receiver winding with diaphragm damped (absohms ∠)
$Z' = Z'' - Z$	Motional impedance of winding (absohms ∠)
Z''	Impedance of winding with diaphragm free (absohms ∠)

Z_0'	Motional impedance at apparent resonance, and at diametral or maximum size. (absohms $\angle$)
z	Mechanical impedance of a diaphragm with reactive magnetic force neglected (dynes per kine $\angle$)
z'	Effective mechanical impedance of diaphragm including reactive magnetic force (dynes per kine $\angle$)
$\omega = 2\,\pi\,f$	Impressed angular velocity (radians per sec.)
ω_0'	Impressed angular velocity at true mechanical resonance (radians-sec.)
ω_0	Impressed angular velocity at apparent mechanical resonance (radians per sec.)
ω_1, ω_2.	Lesser and greater quadrantal angular velocities (radians per sec.)
r. m. s.	Contraction for root-mean-square.
$\lvert Z \rvert$	Size or mere numerical value of a vector quantity Z.

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THE ABSOLUTE MEASUREMENT OF THE INTENSITY OF SOUND

BY ARTHUR GORDON WEBSTER

Abstract of Paper

This paper includes a description of a series of acoustical researches extending over a period of twenty-eight years. The properties of vibrating bodies and the subject of elastic hysteresis are discussed. Two fundamentally important instruments for the absolute measurement of sound have been developed and the theory given. The first is the standard of sound, called the phone, which is capable of reproducing at any time a sound of the simplest character and which permits the output of sound to be measured in watts of energy. The second is an instrument called the phonometer for measuring a sound in absolute measure. This instrument is now practically as sensitive as the human ear. Two essential features are the small damping of the vibrating system which results in extreme sensitiveness, but at the same time in great selectiveness, and the capacity for being tuned. Each of these instruments is fitted with a variable volume resonator and tuning over a range of about two octaves is accomplished by varying the volume of the resonator and by changing the tension of the wires to which the piston is attached. The determination of the space distribution of sound and of the effect of disturbing bodies, and the measurement of the reflecting coefficient of surfaces have been accomplished. The phonotrope is a third instrument designed and used to find the direction of a source of sound, for example a fog signal.

THE profession of acoustical engineering is a new one that has arisen during the last few years. The late Professor Wallace C. Sabine, of Harvard University, attracted the attention of the public to the very great importance of physical measurement and thoroughly scientific design in the construction of auditoriums which until his time had remained an entire mystery to architects.

The writer has been interested for many years in the problem of the measurement of sound in absolute units, involving the design of an instrument which is capable of determining, at any point in space, the pressure in the air wave as a function of the time. In order to carry out this object three things are necessary.

First, the construction of a standard of sound which will enable a given sound of the simplest possible character, namely, that in which the pressure varies as a simple harmonic function of the time, to be reproduced at any time, and which permits an output of sound to be measured in watts of energy. This problem has been solved by a number of persons including Professor Ernst Mach and Dr. A. Zernov of Petrograd.

Second, the invention of an instrument capable of measuring a sound of the simple harmonic character described, also in absolute measure; that is to say, in such a way that the amplitude of pressure variation may be measured in absolute measure, as dynes per square centimeter, or in millionths of an atmosphere. This problem has also been successfully treated by a number of investigators, among whom may be mentioned Professor Max Wien, Lord Rayleigh and Lebedeff.

Third, a step in the process remains which is perhaps equally important as the first and second. Given the invention of the proper source of sound, which I have called a phone, because it is a sound and nothing else, and a proper measuring instrument, which I call a phonometer, there still remains the question of the propagation and the distribution of sound in space between the phone and the phonometer. Any measurements made in an enclosed space will be influenced by the reflection from the walls, and even if we had a room of perfectly simple geometrical form and were able to make the instruments of emission and reception work automatically without the disturbing presence of an observer, it would still be impossible to specify the reflecting power of the walls without a very great amount of experimentation and complicated theory. Nevertheless, this is exactly what Sabine did. He did it, however, by employing as a receiving instrument the human ear. Those who have made experiments upon the sensitiveness of the human ear for a standard sound will immediately doubt the possibility of making precise measurements by the same ear at different times, and particularly of comparing the measurements made by one ear with those made by another. Nevertheless, Sabine attained wonderful success and was able to impart his method to pupils and colleagues who carried on his methods successfully.

To proceed with the question of disturbing objects, one should take his phone and his phonometer to an infinite distance from all objects, which is manifestly impossible. The plan which I followed was to attempt to get rid of all objects except an infinite space covered with a surface the reflecting coefficient of which could be measured. I found the proper weather and the proper space on the golf links of a country club where conditions appeared to be favorable. Several days were spent in determining the reflection coefficient. This was done about ten years ago, whereas with the instrument available to-day it could be done in a very few minutes.

The history of my investigations is briefly as follows: In 1890 I proposed to use a diaphragm made of paper or some similar substance which should be placed, shielded on one side, at the spot where the sound was to be measured. Upon the center of the diaphragm was cemented a small plane mirror. In close juxtaposition and parallel with this was the plane side of a plano-convex lens which viewed in the light from a sodium flame was to give Newton's rings, or as we now call them, interference fringes. Of course, when the sound falls upon the diaphragm, the fringes vibrate rapidly and disappear from sight. I then determined to use a stroboscopic arrangement which would permit the viewing of the fringes as they slowly shifted their position. All sorts of difficulties occurred. The light from the sodium flame was not strong enough to permit the fringes to be seen through the stroboscopic device. It was not at that time easy or perhaps possible to make an electric motor that should drive a stroboscopic disk at a constant speed. One of the first difficulties was to devise a method of controlling a synchronous motor. This was accomplished by means of interrupting the current by a tuning fork.

Later I made the acquaintance of Professor A. A. Michelson and of his remarkable optical interferometer, and I immediately saw that one of the difficulties which I had met, namely, the difficulty of adjustment of the lens so that it should not strike the vibrating plate, could be overcome at once, as the two mirrors in the Michelson interferometer could be any distance apart. Also the trouble due to the faintness of the light disappeared, for in Michelson's instrument white light could be used, and it was possible to use gas, incandescent or arc light with good effect.

In 1896 I put the subject in the hands of a student who produced an instrument which was successful. The examination of the substance suitable for a membrane or a diaphragm consumed much time, and all organic substances such as parchment, paper or anything of that sort had to be discarded because it was impossible to keep the tension constant under the changes of moisture in the air. Metal was found to be quite out of the question on account of the difficulties due to thermal expansion. We next decided upon thin glass diaphragms which we afterward found were used by phonograph companies. A slight change of temperature, however, such as was made by breathing upon the diaphragm, would sometimes produce a change of pitch of several tones, so I next decided to clamp the glass diaphragm upon a ring sawed out of plate glass and cement it on with silicate of soda. I finally found that it was possible to get mica so flat that it was superior to the best possible glass, and this gave me the best results up to that time.

Let me now state briefly the qualities which the diaphragm must have in order to be successful. It is well known to physicists that a flat diaphragm can vibrate in an infinite number of modes which the physicist calls normal vibrations. In the simplest mode the diaphragm vibrates so that every point moves in the same direction. The natural period of this vibration is the lowest that the diaphragm can have. Next is a mode in which the central portions of the membrane move in one direction while all those outside of a certain circle, which we call a nodal line, move in the opposite direction. Other vibrations have radial, as well as circular, nodal lines on the opposite sides of which the diaphragm is moving in opposite directions. When a mass is screwed to any point of the diaphragm its motion is entirely changed. And any apparatus that may be attached to the diaphragm and required to do work, such as the carbon button of the telephone, or anything that interferes with the free response of the diaphragm to the variations of the air pressure will alter the action in such a way as to make it extremely difficult of interpretation. My idea, then, was to make a diaphragm which did not do any work at all, but merely carried the mirror which moved practically by itself.

Now, in spite of the great variety of forms and vibrations, it can easily be shown that under proper circumstances the diaphragm will move essentially as a whole, and may, therefore, be compared to a single body which has three characteristic constants: First, its mass; second, its stiffness, defined as the force required to produce a unit deflection of its central point; and third, and most difficult to control, the coefficient of damping, which is defined as the force of resistance that will be exerted from any cause when the velocity of the body or the central point of the diaphragm is unity. This damping must be due to a large number of causes. In the first place, the resistance of the air will cause damping. In the second place the energy of the sound waves that are radiated from the vibrating diaphragm will cause damping. In the third place and by far the most important, the bending of any body is resisted by elastic forces and we therefore have the phenomenon of elastic fatigue. In order to settle the question "what is the best material from which to make tuning forks", which has been variously answered as tool steel, bronze, bell metal, quartz, etc., I perceived that it was necessary to have an exact method of experimentation.

In striking a steel bar supported by a string at two points it will be observed that the high overtones emitted by the bar rapidly die away, and it occurred to me that if the bar were thrown into one normal vibration there would be a rate of damping for every normal vibration, and that by stating the rate of natural damping the elastic hysteresis could be studied. This was demonstrated experimentally by my assistant Mr. James L. Porter with very successful results as far as the experiment went. The mathematical theory has been lacking up to the present time. Two theories are possible, first the theory of solid viscosity, and second the theory of heredity, or elastic hysteresis. The attempts to compute the results of our experimental measurements are now approaching completion, and we soon shall be able to give a theory of elastic hysteresis. My conclusion is that fused quartz is the best substance from which to make tuning forks.

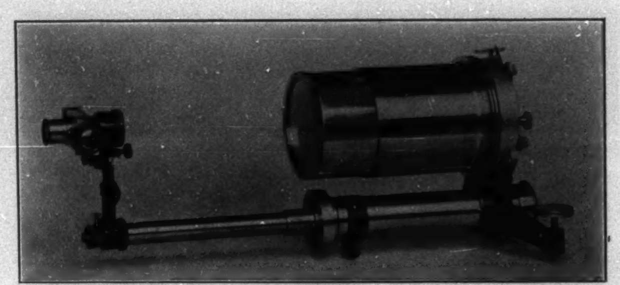
This digression from the investigation is made in order to show the very great difficulties that this subject presented.

The illustrations (shown in the] lecture, but‸[not] reproduced) show the apparatus as it was built several years ago. It was mounted upon a heavy stand made of bronze, covered in at the back by a heavy bronze cover, through which protruded through air-tight fittings the shafts turning the screws of the interferometer adjustments, three in number. Upon the front of the instrument was attached a resonator properly tuned and at the side was a small incandescent lamp with a straight filament arranged horizontally, an image of which was projected by a lens upon the first miror of the interferometer. A telescope was focused upon this image, giving an image in the reticule of the horizontal straight filament. This was crossed by the vertical interference fringes. The objective of the telescope was carried by one prong of a tuning fork which oscillated vertically and this, combined with the horizontal oscillation of the fringes due to the vibration of the diaphragm, resulted in a figure in the form of an ellipse. On tuning the two sounds nearly together this ellipse could be caused to go through its various phases as slowly as desired. At the moment when the ellipse degenerated into an inclined straight line the reticule, which was tilted by an observer, was brought into line with the interference band and the tangent of the angle was read off on a straight scale. The pressure in the sound wave was then proportional to the number read off on the tangent scale. The apparatus was used in this form for a good many years, a precursor of it being shown at the Congress of Physics at Paris in 1900.

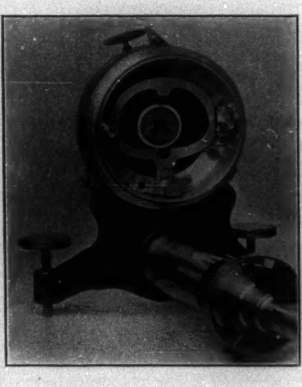
Professor D. C. Miller's phonodeik, which is a very beautiful instrument, was designed for quite other purposes than the phonometer of the speaker. It was intended to photograph sound curves of any sort and by means of it Professor Miller has obtained most beautiful reproductions of speech, and sounds of musical instruments. Unfortunately, this instrument does not adapt itself in the least to absolute measurements. One objection to Professor Miller's instrument is that it employs a spring made of rubber material. This spring has to be calibrated whenever measurements are taken, and it is well known that the properties of soft rubber are far from constant.

The speaker next undertook to devise a better mode of reading the amplitude of vibration of his diaphragm. In the first place, the mirror which was rotated by the diaphragm in Professor Miller's apparatus, instead of being carried on an axis in jewels was placed on a thin steel torsion strip which could be made at far less expense and could be rapidly adjusted. The straight filament lamp was now viewed through a telescope into which the mirror focused the filament in the reticule of the eyepiece. A magnification of about 1200 to 1500 is used. The instrument was also used photographically just as the old instrument had been used to photograph the motion of the interference fringes. Photographing moving interference fringes, however, was attended by many experimental difficulties.

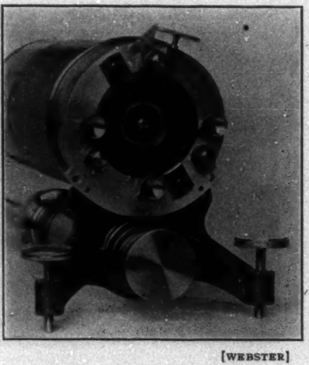
A new instrument in the form described was made in 1913, for



PHONOMETER



PHONOMETER—INTERIOR

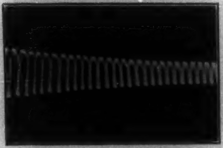


[WEBSTER]

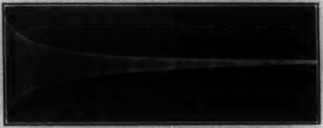
PHONOMETER—FRONT VIEW

PLATE XXXIV.
A. I. E. E.
VOL. XXXVIII, 1919

Quartz Tuning Fork



Damping of Quartz Tube



Damping of Phonometer

Phonotrope

[WEBSTER]

Phonotrope—Tuning Device

Professor Louis Vessot King,* of McGill University, who had a commission from the Canadian Government to make experiments upon fog signals and had secured permission to investigate the great siren at Father Point on the St. Lawrence. Professor King spent the whole summer at Father Point in making the first survey of an acoustical field ever made. The Canadian Government had furnished him a large steamer with which he went out every day, blew the "criard" as the natives call it, the very perfect siren which can be heard thirty miles away in good weather, and explored the gradient of sound under all meterological conditions, studying the wind, the temperature of the air and of the water, etc. This spurred the speaker on to undertaking the same thing in this country. It was impossible to get from the Lighthouse Board the loan of a steamer, but permission was given to go on any lighthouse steamer going along the coast of Maine. The first step was to tune the phonometer for the different pitches, and at that time the only method of tuning the diaphragm was to load upon it small pieces of wax. This, although it worked perfectly, seemed very clumsy, and not an engineering method. It was therefore decided to redesign the whole instrument so that it could be tuned continuously. It occurred to me that this could be accomplished by furnishing potential energy to the diaphragm by means of a string parallel to it, and bearing against it by means of a bridge, and tunable like a violin string. The very first attempt showed this to be entirely successful. Then the idea immediately presented itself that if a string could be tuned continuously why use the energy of bending at all and why not get entirely rid of all difficulties of elastic fatigue. Since that time the diaphragm has been entirely discarded and replaced by a stiff disk supported upon three strings in tension. The disk is made of aluminum or mica and is carried by a little spider made of aluminum containing three clamps to hold the wire. The wire is made of steel and is under tension about steel pegs, two of which are turned roughly by means of a screw driver, the other by means of a lever actuated by a micrometer screw. We have now a very perfect instrument at least ten times as good as the one Professor King had.

A few figures will show the advantages in the reduction of the damping. In a system of one degree of freedom the amplitude of the vibration for the most perfect tuning is inversely proportional to the quotient of the damping coefficient *k* divided by the mass *m*. When we began with the good glass diaphragm we had a value of *k* something like 150. In 1913 the mica diaphragm lent to King had reduced this to 30. With the phonometer as built now we have reduced this to 1.34. In all these cases the mass was about the same so that so far as the sensitiveness goes, the instrument is more than one hundred times as good as it was ten years ago. The instrument now competes in sensitiveness with the human ear and may be tuned to pitches varying by perhaps two octaves. If I strike a tuning fork and allow its amplitude to die away, a person looking into the eye-piece of the instrument cannot say with certainty whether he can see the sound or hear it the longest. Unfortunately, however, this very great sensitiveness obtained by a small damping is attended with a very great disadvantage. The instrument is extremely selective and if the tuning varies by a very small amount the amplitude falls off very greatly.

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*King, Phil. Trans. Ser. A. Vol. 210, pp. 211-293-1919.

Within the last three months I have devised a plan of doing away with this, and making an artifcial ear. For, as you know, the human ear has a wonderful sensitiveness, not for all sounds, but to sounds of a frequency of perhaps 30 per second up to perhaps 30,000 per second, attaining a maximum sensitiveness for frequencies of from 1000 to 1500. We know how this affects the telephone and how the damping of the telephone and microphone disks enables the response to be carried over a large range, so that they answer very well for the frequencies involved in speech. Nevertheless, the sounds of *s*, *f*, *t*, and certain others, on account of the very high harmonics involved, are most difficult to transmit telephonically.

Having described the construction of the diaphragm, the new piston with wires, and the method of reading the vibration by means of the inclined mirror which is also reduced to absolute measure by means of the interferometer temporarily attached, or more simply by displacing the diaphragm by a micrometer screw, I come to the theory of the instrument. The resonator into which the air enters, and the hole which is now constituted by an annular opening around the disk, furnishes through the movement of the air in and out an additional degree of freedom. We are then in the possession of a system with two degrees of freedom, statically coupled by means of the increased pressure in the resonator when the piston is forced in. The theory of such a system under the action of the periodic force is well known and has been treated by Professor Max Wien in great detail. If the frequency in 2 π seconds be n and if the mass be m, I shall call the stiffness minus the product of the mass and square of the frequency the uncompensated stiffness or mistuning. If this were equal to zero for a system of one degree of freedom we should have resonance. In the case of two degrees of freedom I plot the mistunings horizontally and vertically, and the amplitude of response as a third co-ordinate. I thus get a surface that may be described as two mountain peaks with a pass between them, the summit of the pass being when both mistunings are zero; but this is by no means the highest point of the region. To attain either summit we must mistune both systems by a certain amount which is porportional to their two dampings. The addition of the resonator to this instrument may produce an increase of sensitiveness of fifty times. You now see, I hope, how we have obtained an engineering instrument, everything in which is measurable in absolute measure. It is obvious that the piston may be made the diaphragm of the telephone, that the instrument may be used by the psychologist, the engineer, the physiologist, for instance in a stethoscope, and in many other applications.

I turn now to the source of the sound, the phone. With the advent of the tuneable diaphragm came the new phone. It is very light. The amplitude of the diaphragm is measured by a microscope. I use a hot wire vacuum tube as a source of alternating current, tuneable at will, and tune the phone to it. We thus have in small compass a very perfect set of acoustical instruments.

The third portion of my investigations involved a determination of the coefficient of reflection of the ground. In order to accomplish this the phone is set at a convenient height and the phonometer at a convenient distance. The latter is then moved back and forth at the same height, when it is immediately found that interference between the phone and its image in the ground sets in, producing a variation of the intensity of the sound. Different curves have been plotted for different coefficients of reflection. When the reflection is zero or the ground is acoustically perfectly black we have a rectangular hyperbola. The existence of the minimum is obvious to the most unskilled observer. We found the coefficient of reflection of grass, or gravel surface, to be about 95 per cent. I may say that the whole measurement of the instruments of the two ends and the transmission between checks up with an accuracy of probably better than two per cent. With this apparatus all sorts of experiments have been performed. By attaching to the phonometer a long glass tube or antenna it has been made possible to explore all sorts of places, such as the field within a horn or a tube lined with absorbent substance. The theory has been always completely verified. In order to examine the transmission of sound a piece of substance is clamped between two heavy cast iron rings cemented against a hole in a brick wall which will exclude all sound except that that comes through the fabric. The transmission through doors, windows, walls, and telephone booths may be carried out very quickly, and the coefficients of absorption and reflection determined.

Finally, there has resulted an instrument for determining the direction of a fog signal blowing in the fog which I call a phonotrope; since the heliotrope turns toward the sun, the radiant of heat, so the phonotrope turns toward the radiant of sound. This instrument consists of two equal horns bringing the sound to the opposite side of the same disk. It is arranged to be rapidly tuned to the whistle, and when the whistle blows the band of light spreads out; the whole instrument is then revolved until the band reduces its width to zero when the whistle is directly ahead. This instrument was taken to Pensacola to see whether it would determine the direction of an aeroplane in the night. It was found to be as sensitive as the ear, but owing to Doppler's principle, the continual coming and going of the aeroplane changed the pitch so as to put it out of tune. A new modification that I have devised will obviate this I hope.

I have now given you briefly and without any mathematics an account of the principles which I think must always be involved in any measurements of sound. I have always been very anxious to join forces with Professor Miller and to calibrate his instruments so as to render his wonderful results of serious interest to the physicist. I am also glad to co-operate with all my colleagues whether engineers, physicists, physiologists or physicians. It will be a great pleasure to me to know that this apparatus may be of use in solving any of the multitude of questions that confront us.

The plates annexed clearly show the three instruments and several damping curves.

The following list of papers *read* is annexed. Only those marked with an asterisk have been printed, except in abstract.

PAPERS ON SOUND

1. B. A. A. S. Toronto, Aug. 1897. "A New Instrument for Measuring the Intensity of Sound".

2. A. A. A. S. Boston, Aug. 1898. "A New Instrument for the Measurement of the Intensity of Sound".

3. A. A. A. S. Washington, Jan. 1903. "A Portable Apparatus for the Measurement of Sound".

4. American Phys. Soc., New York, April 1902. "An Apparatus for the Quantitive Study of Sound."

5. Am. Phys. Soc., New York, April 1902. "Vibrations of Rotating Wires, Spindles and Shafting."

6. Nat. Acad. Sciences, Boston, Nov. 1906. "Acoustic Measurements".

7. Nat. Acad. Sci., New York, Nov. 1907. "Rayleigh's Disk as an Absolute Measure of Sound".

8. Am. Acad. Arts & Sci., Boston, May 1908. "Absolute Measurements of Sound".

9. Am. Phys. Soc., New York, Oct. 1908. "Distribution of Sound from the Megaphone".

10. National Acad. Sci., Washington, April 1910. "On the Distribution of Sound from the Megaphone or Speaking Trumpet".

11. Nat. Acad. Sci., Washington, April 1910. "A New Method for the Study of Elastic Hysteresis."

12. B. A. A. S., Sheffield, Aug. 1910. "A Complete Apparatus for the Measurement of Sound".

13. Am. Phys. Soc., Washington, Dec. 1911. "Elastic Hysteresis in Metal Bars".

14. Am. Phys. Soc., Washington, Dec. 1911. "Another Instrument for Photographing Sound".

*15. Am. Acad. Arts & Sciences, Dec. 1910. "The Wave Potential of a Circular Line of Sources".

16. Am. Math. Soc., Poughkeepsie, Sept. 1911. "The Wave Potential of a Circular Line of Sources".

17. Am. Phys. Soc., Cleveland, Jan. 1913. "Some Points Concerning Absolute Measurements in Sound".

18. Am. Phys. Soc., New Haven, March 1913. "Forced Vibration of a Circular Plate".

19. Nat. Acad. Sci., Baltimore, Nov. 1913. "A New Portable Phonometer".

20. Nat. Acad. Aci., Baltimore, Nov. 1913. "The Transmission of Sound Through Porous Materials".

21. Am. Phil. Soc., Philadelphia, April 1913. "Some Observations of the Transmission of Sound Through Walls."

22. Am. Physical Soc., Washington, April 1914. "A New Phonometer".

23. Am. Phys. Soc., Philadelphia, Dec. 1914. "A New Standard Phone and Phonometer for any Pitch".

24. Nat. Acad. Sci., New York, Nov. 1915. "Experiments and Theory of Conical Horns".

25. Nat. Acad. Sci., New York, Nov. 1915. "Instruments for the Measurement of Sound".

26. Nat. Acad. Sci., New York, Nov. 1915. "An Instrument for Finding the Direction of a Fog-Signal".

*27. Am. Phys. Soc., Columbus, Dec. 1915. "Mechanical and Acoustical Impedance, and the Theory of the Phonograph".

28. Am. Phys. Soc., Columbus, 1915. "Impedance of Conical Horns".

29. Am. Phys. Soc., Columbus, Dec. 1915. "The Phonotrope, a new Instrument for Finding the Direction of an Acoustical Ray".

*30. Am. Math. Soc., Cambridge, Sept. 1916. "On Acoustical Impedance, and a Theory of Horns".

31. Am. Acad. of Arts and Sci., Boston, Nov. 1916. "Practical Tests of a New Phonotrope".

32. Nat. Acad. Sci., Baltimore, Nov. 1918. "An Automatically Played Brass Instrument".

33. Am. Phys. Soc., Baltimore, Dec. 1918. "A Mechanically Blown Wind Instrument".

Discussion on "Electromagnetic Theory of the Telephone Receiver" (Kennelly and Nukiyama) and "On the Absolute Measurement of the Intensity of Sound" (Webster). Boston, Mass., March 14, 1919.

John B. Taylor: Certainly the matter of tone that Prof. Webster spoke of, the acoustical analysis of musical instruments, is intensely interesting. We have been trying to discover why a bassoon, which is a very complicated instrument, is absolutely unwritten up in a technical way. It is very gratifying to hear Prof. Webster say that he can measure these sounds and noises in quantitative terms.

If Prof. Webster had been at the last meeting of the Institute in New York City, when there was a discussion on telephone interference, and this new interference standard, after all, is a matter of noise, going into the Institute Standardization Rules, he would have been able to contribute something instructive. A question was asked at the meeting about the quantitative measuring of noise in the telephone field, and I do not recall that the answer to that question was very illuminating.

T. E. Shea: According to Dr. Kennelly's theory, I believe, the action of a telephone receiver may be likened somewhat to that of a single-phase transformer, except that the secondary circuit generates oscillating mechanical power instead of oscillating electrical power. The authors of the paper have, in effect, combined the equivalent vector diagrams of the mechanical circuits with the vector diagrams of the electrical circuit; and the resulting diagrams they have obtained are similar to those ordinarily used in explaining the theory of the transformer.

But the wide range of frequencies considered has introduced several new elements, regarding which the authors have made assumptions. One of these assumptions is that the current passing through the receiver is constant. Another is that the angle of lag of the flux behind the impressed magnetomotive force, called by Dr. Kennelly the hysteretic angle β, is also constant.

I would like to ask Dr. Kennelly if, when the current through the receiver varies over a wide range, as it does in practise, the hysteretic angle β would not also vary greatly. I realize that Dr. Kennelly has not attempted to push his theory beyond the limits of constant current, but I think the impression has been given that the angle β is independent of the other variables of telephone receiver theory.

B. A. Behrend: I wish to call attention to the fact that the term "motional impedance" was first suggested by our honorary member, Mr. Oliver Heaviside, to whom we also owe the terms "inductance", "reactance", and "impedance".

R. L. Jones: Professor Webster has described and shown three exquisitely designed acoustical instruments which should be very serviceable in certain types of sound researches. Both the phone and the phonometer are characterized by very sharp tuning, however, and although tunable, an appreciable period of manipulation is required to adjust precisely to a new pitch. In much work, as for example in many studies of speech and music, we are interested in a whole range of frequencies or a flow of rapidly changing pitch. In such cases we need a sensitive phonometer of uniform frequency characteristics. Such apparatus is provided by a properly designed and calibrated combination of electrostatic telephone transmitter and vacuum tube amplifier, as has been described in recent volumes of the *Physical Review*.

The paper by Professor Kennelly and Mr. Nukiyama is primarily a pictorial treatment of the electric and magnetic relations of the telephone system by means of vector diagrams. I believe it will broaden the consideration if we discuss the matter algebraically for the case of a relatively simple receiver.

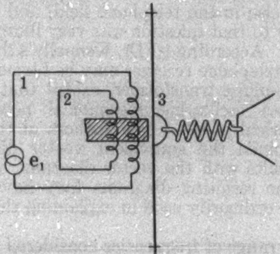

FIG. 1—SIMPLE RECEIVER WITH EDDY CURRENTS IN THE CORE

Let us assume that straight line laws hold and in general make the other usual simplifying assumptions just as the authors do. In addition assume that for the frequencies under consideration hysteresis is negligible compared to eddy current losses and that the eddy current losses in the diaphragm are negligible compared to those in the cores. From experience these assumptions appear reasonable as a first approximation.

Fig. 1 then shows the condition schematically where 1 is the driving circuit, 2 the schematic equivalent of eddy current circuits in the cores, and 3 the mechanical or diaphragm circuit. The following nomenclature will be used:

a = inductance or mass
b = resistance, electrical or frictional
c = stiffness
Z = impedance, electrical or mechanical
$\dot{x}$ = current or velocity

Subscripts refer to the networks of the coupled system, 1 the driving circuit, 2 the eddy current circuit, and 3 the diaphragm or mechanical circuit.

Then from the figure we may write down the well known force equations. There is an electromotive force, e_1, let us say, working in the driving circuit, none in the eddy current circuit, and no external forces working on the diaphragm.

$$\begin{aligned} e_1 &= Z_{11}\overset{\circ}{x}_1 + Z_{12}\overset{\circ}{x}_2 + Z_{13}\overset{\circ}{x}_3 \\ 0 &= Z_{21}\overset{\circ}{x}_1 + Z_{22}\overset{\circ}{x}_2 + Z_{23}\overset{\circ}{x}_3 \\ 0 &= Z_{31}\overset{\circ}{x}_1 + Z_{32}\overset{\circ}{x}_2 + Z_{33}\overset{\circ}{x}_3. \end{aligned} \tag{1}$$

The relations of the mutual impedances appearing in these equations are as follows:

Z_{12} and Z_{21} are the mutual impedances between the driving and eddy circuits

Z_{13} and Z_{31} are the force and reaction factors between the the driving circuit and the diaphragm

Z_{23} and Z_{32} are the force and reaction factors between the eddy circuit and the diaphragm

$$\begin{aligned} Z_{12} &= Z_{21} \\ Z_{13} &= -Z_{31} \\ Z_{23} &= -Z_{32}. \end{aligned} \tag{2}$$

The mutual impedances of a transformer are of the same sign since flux due to current in either winding is opposed by that due to induced current in the other winding. In considering the mutual relations between the receiver diaphragm and either the driving or eddy current circuits, however, it is seen that whereas flux due to diaphragm motion induces currents which set up an opposing flux, flux due to current in the driving winding produces a motion of the diaphragm which tends to increase the flux still further. Hence in the receiver the force and reaction factors are of opposite sign.

By means of (**1**) and (**2**)

$$Z_1 = \frac{e_1}{\overset{\circ}{x}_1} = Z_{11} - \frac{2\,Z_{12}\,Z_{13}\,Z_{23} + Z_{12}^2\,Z_{33} - Z_{13}^2\,Z_{22}}{Z_{22}\,Z_{33} + Z_{23}^2} \tag{3}$$

If the diaphragm is damped so that $\overset{\circ}{x}_3 = 0$, then from (**1**) and (**2**)

$$Z_{1d} = Z_{11} - \frac{Z_{12}^2}{Z_{22}}. \tag{4}$$

By subtraction the motional impedance of the receiver,

$$Z_{1m} = \frac{\left(Z_{13} - Z_{23}\dfrac{Z_{12}}{Z_{22}}\right)^2}{Z_{33} + Z_{23}\dfrac{Z_{23}}{Z_{22}}}. \tag{5}$$

If there were no eddy currents this would reduce to

$$Z_{1m}' = \frac{Z_{13}^2}{Z_{33}} \,. \tag{6}$$

the form familiar to us as the square of the force factor divided by the mechanical impedance of the diaphragm. Comparing (**5**) we see that the force factor with eddy currents has the value

$$B_{13} = Z_{13} - Z_{23}\,\frac{Z_{12}}{Z_{22}} = b_{13} - b_{23}\,\frac{j\,\omega\,a_{12}}{b_2 + j\,\omega\,a_2} \tag{7}$$

$$= \left(b_{13} - b_{23}\cdot\frac{\omega^2\,a_{12}\,a_2}{b_2^2 + \omega^2\,a_2^2}\right) + j\,b_{23}\cdot\frac{\omega\,a_{12}\,b_2}{b_2^2 + \omega^2\,a_2^2} \tag{8}$$

The effect of eddy currents is evidently to decrease the force factor with increasing frequency, and to cause a lag.

Substituting B_{13} in (**5**),

$$Z_{1m} = \frac{B_{13}^2}{Z_{33} + Z_{23}\,\dfrac{Z_{23}}{Z_{22}}} \,. \tag{9}$$

Substituting primary constants for Z's it may be seen that Z_{1m}

$$= \frac{B_{13}^2}{\left[b_3 + \dfrac{b_2\,b_{23}^2}{b_2^2 + \omega^2\,a_2^2}\right] + j\left[\omega\left(a_3 - \dfrac{a_2\,b_{23}^2}{b_2^2 + \omega^2\,a_2^2}\right) - \dfrac{c_3}{\omega}\right]} \tag{10}$$

Centering attention on the denominator it may be seen that the eddy current may be considered as increasing the mechanical resistance by an amount equal to

$$b_2 \cdot \frac{b_{23}^2}{|Z_2^2|} \,.$$

Similarly it may be considered as decreasing the effective mass by an amount

$$a_2 \cdot \frac{b_{23}^2}{|Z_2^2|} \,.$$

or as increasing the elasticity by a corresponding factor. Physically this second factor merely changes the apparent mechanical reactance.

Now let us turn to a consideration of efficiency relations in the simple receiver with eddy currents in the core. Writing down the power output of the diaphragm divided by the power input to the driving winding we have

$$\mu_{13} = \frac{|\dot{x}_3^2| \,.\, b_3}{|\dot{x}_1^2| \,.\, B_1} \qquad \textbf{(11)}$$

where B_1 is the apparent or free resistance of the driving circuit.

Solving for $\dfrac{\dot{x}_3}{\dot{x}_1}$ from (**1**) and substituting we have

$$\mu_{13} = \frac{\left| \left(Z_{13} - Z_{23} \dfrac{Z_{12}}{Z_{22}} \right)^2 \right| .\, b_3}{\left| \left(Z_{33} + Z_{23} \dfrac{Z_{23}}{Z_{22}} \right)^2 \right| .\, B_1} \qquad \textbf{(12)}$$

By further substitution from (**5**) and (**7**)

$$\mu_{13} = \frac{| Z_{1m}^2 |}{\dfrac{| B_{13}^2 |}{b_3} \,.\, B_1} \qquad \textbf{(13)}$$

This expression is the true overall efficiency of the receiver at any frequency in terms of the motional impedance, the free resistance, the force factor and the mechanical resistance. If we multiply this last expression by the ratio of the apparent mechanical resistance (**10**) to its true value, we get

$$\mu_{13}' = \frac{| Z_{1m}^2 |}{\dfrac{| B_{13}^2 |}{b_3 + b_2 \dfrac{b_{23}^2}{| Z_2^2 |}} \,.\, B_1} = \frac{| Z_{1m}^2 |}{| Z_{1m/0} | \,.\, B_1} \qquad \textbf{(14)}$$

Since the first factor of the denominator, the ratio of the square of the force factor to the apparent mechanical resistance, is the motional impedance at resonance, at resonance this reduces to

$$\mu_{13/0}' = \frac{| Z_{1m/0} |}{B_1} \qquad \textbf{(15)}$$

This is what Dr. Kennelly calls the "gross efficiency at apparent resonance." It is very readily determined from the motional impedance diagram, and is susceptible of fairly precise determination. On the other hand, it applies simply to a single frequency, and is a value that, for the comparison of different receivers, is generally applicable only in case the receivers are quite similar in their natural frequencies and ranges of response. In the case of telephones for wireless telegraphy, one is interested in response at substantially a single frequency and for such instruments this apparent efficiency taken in conjunction with the natural frequency gives a very good measure of the relative performance of different types. It may be of interest to note that for nearly a year past the U. S. Signal Corps specification for wireless telephones has had a requirement stated in terms of these quantities. In the telephone problem, however, we are interested in the whole range of speech frequencies running from 200 or 300 cycles up to 3000 or above. In receivers for reproducing speech, therefore, whether transmitted by wire or by radio, we cannot depend upon the relative measures given by expression (**15**), but must apply (**13**) to the whole range of frequencies, or get some other equivalent measure.

To retransform μ_{13}' to μ_{13}, we have to multiply by the ratio of the true mechanical resistance to the apparent value. The latter appears in the denominator of (**10**). This ratio,

$$\eta = \frac{b_3}{b_3 + b_2 \frac{b_{23}^2}{|Z_2^2|}} \qquad \textbf{(16)}$$

as pointed out by Dr. Kennelly, measures the vibratory efficiency of the diaphragm, showing the fractional part of the total vibratory power that is of a mechanical nature.

These results are a small part of the consideration given to this subject in the Western Electric Research Laboratories at New York. It may be of some interest to cite one or two experimental results for the same case as that assumed.

An experimental receiver of the ordinary type was constructed in which the eddy current effects were reduced to a minimum. The cores were soft iron in a form which is the equivalent of very fine laminations and for the diaphragm silicon steel was chosen on account of its high specific resistance. The case was made of Portland cement, this being chosen because we wished to avoid eddy currents in the case and to use a material with about the same coefficient of expansion as iron so as to keep the airgap constant with temperature.

In addition to the driving winding a 100-turn coil was put on close to the pole tip which could be short-circuited at will to represent an eddy current circuit. Measurements on this circuit enabled us to predict its effect on the impedance of the driving circuit. In accordance with the foregoing formulas the predictions could be checked by measurement. To cite simply one piece of data, with the 100-turn coil open, 2 β was about 4 deg., with it closed, 25 deg. The corresponding computed value was 23.5 deg. A second model had the artificial eddy circuit wound as a twin winding, in parallel with the driving circuit. With this arrangement for eddy currents the lag angle 2 β went from a very small angle to 130 deg., and the motional impedance was diminished from 31 ohms to 4 ohms.

A. G. Webster: This is what I call practical engineering. I should like to point out, as a matter of history, that the first problem of this sort solved was solved by James Clark Maxwell in his famous paper in 1865. Of course there are here three lines of equations instead of two. You know how Maxwell understood the impedance, as we call it now, because of the invention of the word of Oliver Heaviside—that the presence of the secondary increases the resistance of the primary and decreases the inductance. We all know that now, but we did not know it then. Of course, then came Lord Rayleigh, who described the reactions of any mechanical system.

Harold S. Osborne: Engineers who are dealing with power transmission rather than with telephone work may feel more at home with the motional impedance diagram of the telephone receiver if they can consider that it is not an entire stranger but the first cousin of an old friend. I refer to the circle diagram of the induction motor in which, as in the telephone receiver, the secondary moves. In the circle diagram of the induction motor, as indicated in Fig. 2, the secondary current corresponding to a given load is represented by a chord of the circle *O I*. The secondary current is a measure of the reaction of the moving secondary on the magnetic flux of the primary winding. Similarly, in the motional impedance diagram of the telephone receiver, the reaction of the moving diaphragm on the fixed winding is represented by a chord of the circle *O V*, (Fig. 3).

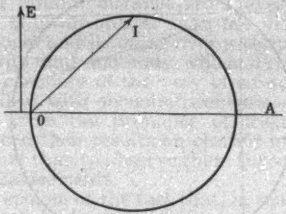

Fig. 2.

There is a further close relationship between these two diagrams. The variable element seems to be different in the two cases because in the case of the induction motor we are considering variations of load, and in the case of the telephone receiver variations of frequency. However, in the induction motor variations in load mean variations in frequency of the secondary current, and one starts with zero frequency at point *O*, and goes half way around the circle to infinite frequency at point *A*. The other half circle corresponds to negative frequencies. In the case of the telephone receiver the law of variation in frequency in passing around the circle is, of course, different. In this case, starting with zero frequency at point *O*, one passes around the circle and at infinite frequency completes the circuit, arriving again at point *O*.

The difference in the use of the circle diagram for the induction motor and for the telephone receiver illustrates one difference between the problems of the power engineer and of the telephone engineer to which I should like to call attention. In the case of a power system, one is interested in studying the system under conditions of constant impressed e. m. f. and varying loads of a given frequency. In the case of the telephone system the load is, in general, constant but the frequency is varying.

The importance of a very wide range of frequencies in telephone work is, I think, an old story even to engineers who are not engaged in telephone work. Over thirty years ago Oliver Heaviside pointed out the effect of variation in efficiency of the telephone receiver with frequency, and the effect of that on the telephone quality. In those days people were still marveling over the fact that this wonderful little instrument could reproduce speech, and that they were astonished at what seemed to them, perfection of articulation, and by the fact that one could even recognize the voice of different persons. However, Mr. Heaviside stated that to the critical observer the telephone quality was very far from perfect. As he expressed it, "the disk does not do it, for want of deadbeatness." The fact that telephony is practicable, he claimed, is not so much a triumph of the electrical art, as it is a triumph of the human brain, which has been trained for years, and, in fact, for centuries, in interpreting the very much distorted sounds that come to it.

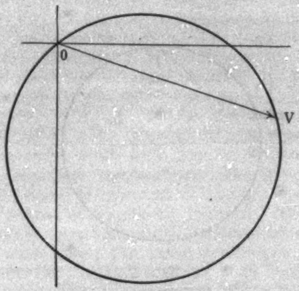

FIG. 3.

I do not believe that there is a general appreciation of the very large amount of distortion of speech which takes place in an ordinary room. Prof. Webster has told us that the reflection from the earth is 95 per cent. From a wooden floor or walls of a building it must be very great. The reflected waves cause tremendously large variations in the magnitude of the different components of the speech sounds, and large differences in different parts of the room. In addition, there is the effect of room noises. Beyond that, there are personal factors which perhaps cannot be called merely physical matters, such as slovenliness of speech on the part of the talker, and defective and wandering attention on the part of the listener.

Now, although large advances have been made since Heaviside's day, the telephone receiver still is very far from a perfect reproducer of the energy supplied to it. However, when one considers the distortions to which ordinary speech is subject, and the fact that our ears through training are capable of understanding this distorted speech very well, it is not so surprising that we should be able to detect the distorted speech of the telephone circuit. It is probable that the difference in quality of conversation between listening in a room like this, and listening an infinite distance from everything, as Prof. Webster puts it, is much greater than the difference between listening in the room and listening over a telephone circuit.

The instruments and methods which Prof. Webster has shown to us are certainly of the very greatest interest to any one who has to deal with acoustic problems. However, electrical engineers, from the prejudice of their training, very much prefer to read their results on electric instruments when it is possible, rather than to observe them through a microscope as mechanical displacements.

Mr. Jones has spoken of the fact that, in many cases, we are very much interested in measuring sound with an instrument which is not highly selective, but which, on the contrary, responds so as to give as nearly as possible a faithful reproduction of a complicated wave shape, that is, an equal response to the different harmonic components of the wave. A good solution of this problem has been discussed in two papers which were printed in the *Physical Review* in July, 1917, which I would like to call to your attention. These are the "Thermophone as a Precision Source of Sound," by H. D. Arnold and I. B. Crandall, and "A Condenser Transmitter as a Uniformly Sensitive Instrument for the Absolute Measurement of Sound Intensity," by E. C. Wente. By means of the instruments described in these papers it is possible to set up a system for transforming acoustic into electric waves and vice versa with an efficiency of conversion which can be made sensibly uniform throughout the range which is important in telephonic work. Of course, these instruments give very small responses, but in these days of vacuum tube amplifiers that is not a difficulty.

Mr. Taylor has spoken of the possibility of using Prof. Webster's instruments to study the characteristics of interfering currents in telephone circuits. However, for such complicated wave shapes as those or for the study of the characteristics of telephone speech, instruments giving a uniform response, such as are described in the papers just mentioned are very much more valuable than instruments which are sharply tuned to particular frequencies.

A. G. Webster: I was very much interested in these papers when I heard them read. I went to New York with some qualms as to whether I had been anticipated. When I saw the amplifying tube I said, "That let's me out." Who knows whether the amplification of that tube is the same for all these frequencies. I said to Major Whitehead last year in his laboratory—"I have seen this wonderful description of the amplifier. Does it do it?" I will not tell you what he said, but those of you who have heard of experiments of that sort, will, I am sure, say that the electrical engineer is mistaken. We know that this is an exact thing, no hysteresis or eddy current. This is a perfection of simplicity, and therefore I recommend it. As to its being selective, it is selective. You can tune it to any one of these sounds and make a photograph and analyze it.

I believe I can give more satisfactory answers to all of these telephone engineer's queries than can be got by the instruments he gets up himself. They are handy, no doubt, and all that. I remember Lord Kelvin seeing one of my instruments several years ago, and he said "It was important that sound could be measured by electrical reading apparatus." I do not do it that way.

A. Press (communicated after adjournment): "May I draw the attention of the authors to a work of mine which antedates that of M. Marius Latour on "Hysteresis and Eddy Current Losses in Iron Plate." This paper appeared in *The Electrician* of London for July 23, 1915. It not only gave a solution of the plate problem but it also indicated a general method for solving problems in which what amounts to a difference of phase between H and B occurs. Since then a simpler method has been developed for dealing with all the harmonics set up by iron in any electrical circuit, the manuscript of which is now in press. Distortions and resonances due to hysteretic harmonic components can be very effectively handled by the methods indicated.

A. E. Kennelly: In reference to the set of equations which Dr. Jones has so clearly explained, it is interesting to observe how his algebraical and our graphical presentations of the behavior of the telephone receiver converge towards similar conclusions.

In response to Mr. Shea's question, the angle β is the angle between the vector damped impedance locus and the reactance axis of coordinates. Strictly speaking, this angle β is a function of the impressed frequency, and increases with increasing frequency. Nevertheless, it usually varies but little within the range of frequency included between the quadrantal points of the motional impedance circle. Such variations as occur outside of those limits have relatively small disturbing influence.

In Fig. 1 of the Kennelly-Affel paper* (Bibliography 17) appears the locus of the damped impedance of the particular telephone receiver which is analysed in this paper presented today. In its Fig. 4 is the motional impedance circle. The depression angle 2β of the circle's diameter (see also Figs. 10 and 27 of todays paper) is approximately 50°.6. In Fig. 1, the angle β which the locus *ABD*, between *B* and *D*, makes with the reactance axis is very nearly 25°.3. The fact that this angle β as measured on the diagram of the original paper is very nearly half the angle of diametric depression of the motional impedance circle was only discovered after the paper here presented was written, and serves as a check upon the provisional theory. A complete theory, however, would call for a recognition in the change of β with impressed frequency.

We are all indebted to Dr. Webster for his exhibition here of this ingenious acoustic apparatus. The fact that he has been able to reduce the damping coefficient to so low a value as has been mentioned is a very valuable achievement in itself.

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PRESENT STATUS OF INDUSTRIAL LIGHTING CODES

BY G. H. STICKNEY

ABSTRACT OF PAPER

In order to protect workers from accident and eye-strain, industrial lighting codes have been adopted in four states and in Federal establishments. Similar action is under consideration in several other states and there is prospect of extension throughout the country.

Investigation and experience indicate the need of government regulation of factory lighting. When adopted by industrial commissions under authority granted by Legislatures, the codes become in effect state law. Variations in the codes as adopted are less than might appear, some features being experimental. The existing codes correspond in essentials to the Illuminating Engineering Society code, on which they are based.

Mandatory regulations are necessarily limited in function to the assuring of safety. Higher standards are essential for efficient production. Popular education in which electrical and illuminating engineers can cooperate, is an important feature of future development.

INTRODUCTION

THE industrial lighting codes are expressions of the "Safety First" movement in terms of legal regulations directed to the lighting of factories. The prime function of these codes is the safeguarding of life, limb and vision of industrial workers. No argument should be necessary to enlist the support of all good citizens to such a humanitarian effort. In addition, the codes seem likely to teach practise which will enhance the earning power, not only of the workers themselves but also of the industrial plants. This incidental result seems likely to be of great economic importance in the coming period of international competition. Sufficient progress has been made in the application of these codes to indicate that such regulations will ultimately be in force throughout the country. Through their relation to lighting practise, these codes seem destined to have a broad influence on electrical engineering, and it is the purpose of this paper to inform electrical engineers regarding the scope and tendencies of these codes, as interpreted from a close association with the work and with those responsible for it.

Industrial lighting codes, based on the Illuminating Engineering Society's code, are now in force in the states of Pennsylvania, New Jersey, New York and Wisconsin. In several other states similar codes have been drawn up and are under consideration for adoption. In others, bills looking to the adoption of the code have been introduced before the legislatures. As a war measure, the Advisory Commission of the Council of National Defense, through its Committee on Labor, appointed a sub-committee known as the Divisional Lighting Committee, to encourage the movement. The sub-committee, working through state representatives, has approached the authorities in each of the states to encourage the adoption of such codes. As the result of this activity, progress has been made toward the introduction of an industrial lighting code or a safety code, by the Departments of Labor, Industrial Accident Commissions, or other agencies charged with the enactment and enforcement or regulations in regard to factory lighting.

Through cooperation of the Bureau of Standards and the Federal Inspectors of Safety, a lighting safety code was adopted during the war, in all Federal establishments, including arsenals, munition plants and navy yards.

NECESSITY

The necessity for such regulation is well explained in the introduction to the new edition of the Wisconsin code, which reads as follows:

> Insufficient and improperly applied illumination is a prolific cause of industrial accidents. In the past few years numerous investigators, studying the cause of accidents, have found that the accident rate in plants with poor lighting is higher than similar plants which are well illuminated. Factories which have installed approved lighting have experienced reductions in their accidents which are very gratifying.
>
> Of even greater importance, poor lighting impairs vision. Because diminution of eyesight from this cause is gradual, it may take the individual years to become aware of it. This makes it all the more important to guard against the insidious effects of dim illumination, of glaring light sources shining in the eyes, of flickering light, of sharp shadows, of glare reflected from polished parts of the work. To conserve the eyesight of the working class is a distinct economic gain to the state, but regardless of that, humanitarian considerations demand it.
>
> Finally, inadequate illumination decreases the production of the industries of the state, and to that extent, the wealth of its people. Factory

managers, who have installed improved illumination, are unanimous in the conviction that better lighting increases production and decreases spoilage.

Illuminating engineers have recognized for a long time that inadequate lighting was responsible, in a greater or less degree, for industrial accidents, and that good lighting was a potent means of accident prevention. Extensive data, accumulated by John Calder[1], R. E. Simpson[2], and others provide convincing proof of this condition.

The loss of visual power, through improper lighting, cannot be so readily recorded, and hence is not so directly substantiated by actual data. It is, however, recognized as a serious menace by oculists[3] and others. Because it incapacitates the skilled worker just when he should be most valuable, and further, because the danger is not readily recognized by those immediately responsible for industrial lighting, the necessity for governmental supervision is even greater for this purpose than for the prevention of mechanical accidents.

In spite of the recent general improvement in industrial lighting practise, the increasing need of ample assurance in this matter is evident from the present tendencies of manufacturing, among which may be mentioned the following: the grouping of larger number of employees; the extension of all night processes; increasing use of dangerous machinery; intricacy of processes and specialization of workers requiring sustained visual application.

HISTORICAL

Practically all the codes in force or under preparation are based on the "Code of Lighting Factories, Mills and other Work Places" of the Illuminating Engineering Society[4], and conform closely to it in all essentials.

This code was prepared by the cooperative action of the Committee on Lighting Legislation, under the chairmanship of Mr. L. B. Marks, and the former Committee on Factory Lighting, under the chairmanship of Professor C. E. Clewell.

The Illuminating Engineering Society was called upon in 1913 to assist in formulating the lighting section of the labor law of New York State. It was found impracticable then to incorporate definite lighting specifications in the law, and so provision was made in the law for the later adoption of such regulations, by the Labor Commission. In preparation for the Commission's requirement, work of drafting a code was undertaken. The problem proved a very difficult one and many modifications were found necessary, even after its first publication. Various societies, many engineers and state commissioners pointed out desirable changes, which were incorporated from time to time. Even at the present time, although the code is a very valuable working instrument, it is considered, by those who have had most to do with it, to be under development and liable to slight changes and amplifications, with the development of the art and with increased experience in application of the various provisions.

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[1] For references, see Bibliography.

In 1916 at a meeting in Philadelphia[6], Dr. J. P. Jackson and Colonel L. T. Bryant, industrial commissioners of Pennsylvania and New Jersey, respectively, became interested, and after introducing some practical improvements, their respective commissions took action which resulted in the adoption of the code in the summer of 1916. The New York Commission adopted the code with some modifications and put it in force July 1st, 1918, some of the features being tentative for a year.

The Wisconsin Commission began issuing lighting orders in 1913, but these orders did not contain any specification as to intensity of illumination on the work. After thorough investigation the orders were superseded by the present code, which was issued in May 1918.

The author has just received word, as this paper goes to press, that the legislature of the State of Oregon has passed a bill establishing an industrial lighting code.

SCOPE

In the states which have adopted codes, the action has been by industrial or labor commissions, under the authority granted them by legislature, to promulgate rules in the interest of safety of industrial workers. The codes are, therefore, backed by the state police power.

Since the function of the Commissions is limited to insuring safety, the codes require only such lighting as is necessary for that end. Under these circumstances the codes cannot demand the higher standards of illumination desirable for economical production[6]. That the intensities specified are minimum limits, consistent with safety rather than good practise, is recognized, and in connection with all the codes, higher values representing more desirable practise are recommended.

It has seemed to be the experience of labor commissions that at least 90 per cent of the manufacturers cordially desire to comply with their regulations. The principal difficulty so far encountered in applying the codes, has been to make clear to the layman just what is required and how it can be secured. It has, therefore, been found expedient to publish educational sections supplementary to the codes proper. Such sections explain the principles involved and make suggestions for securing and maintaining good installations. The recent revision of the Wisconsin code treats rather comprehensively of the selection and location of lighting equipment, illustrated by charts.

By joint action of the Pennsylvania and New Jersey Commissions, a course of lectures[7] on the interpretation of the codes was given to their inspectors at the University of Pennsylvania in the spring of 1918. A similar lecture[8] was given to factory inspectors of the Board of Labor & Industries of Massachusetts in October 1918. Doubtless this will be repeated and the method applied elsewhere. The New Jersey Commission has held meetings of contractors and manufacturers, to assist in the educational work.

In all the codes, the employment of expert engineering or architectural advice is recommended.

SPECIFICATIONS

The principal lighting specifications common to all states which have adopted the code, are intensity, glare limits, and distribution. It is also usual to require provision of emergency lighting, watchman's circuits and certain switching provisions. In some cases these features are omitted from the codes, being provided for in the law or elsewhere.

A comparison of the codes adopted or under preparation shows a remarkable uniformity in the specification of these factors. There is some variation, especially as regards experimental features, but this is of a nature likely to help rather than retard the development of codes toward excellence and uniformity.

In drawing the specifications, it has been constantly recognized that they must be kept simple and practicable so that conformance could be determined without recourse to lighting experts or bulky and expensive instruments. On the other hand, it is desirable to make the specification as definite as possible, minimizing the part left to individual judgment and thus avoiding controversy and prejudice.

DAYLIGHT

While it has been considered desirable to require that new buildings and extensions be so constructed as to provide adequate daylight, it has been found very difficult, if not impracticable, to lay down a definite rule, and in most of the codes it has been assumed that the purpose was accomplished if suitable artificial light were required where and when daylight is unavailable.

Daylight is subject to a wide variation throughout the hours of the day, seasons of the year, with changes in weather and for different directions of window exposure, etc. In any building the natural lighting may be affected by surrounding construction on property not under the control of the owner. A workroom having ample daylight when erected may be darkened by later construction. In a dense manufacturing center, *e. g.*, downtown Manhattan, it would be impracticable to realize so good a condition, especially on lower floors, as might be reasonably required of a suburban factory. Some reasonable provision can undoubtedly be reached, which will at least require the minimization of glare and provide for wide distribution of light, by the use of suitable awnings, shades, refractive windows, etc.

The Wisconsin Commission has adopted the following rule in this connection:

> Order 2111. Natural Light. Windows, skylights, saw-tooth or other roof lighting construction of buildings shall be arranged with reasonably uniform bays and the glass area so apportioned that at the darkest part of any working space, when normal exterior daylight conditions obtain, (sky brightness of 1.5 candle-power per sq. inch) there will be available a minimum intensity equal to twice that of Order 2112 for artificial light.
>
> Awnings, window shades, diffusive or refractive window glass shall be used for the purpose of improving daylight conditions or for the avoidance of eye strain wherever the location of the work is such that the worker must face large window areas, through which excessively bright light may at times enter the building.
>
> Note. The intensity requirements for adequate daylighting are much higher than those for adequate night lighting, because in general, under daylight conditions, the light coming to the eye from all the surroundings in the field of vision is much brighter than at night, and hence a correspondingly more intense light must fall on the object viewed.

It is the almost universal experience of illuminating engineers and others, that it is possible to operate satisfactorily with a lower intensity of artificial than natural illumination.

The reason for this has not been satisfactorily explained; but with daylight the relatively high illumination of walls and other surrounding objects, compared to the illumination on the work, is undoubtedly a contributory cause.

In the earliest draft of the Illuminating Engineering Society's code, a ratio of 3 to 1 between natural and artificial lighting requirements was specified. Later this was changed to 2 to 1, and still later, in the absence of definite data in confirmation of the necessity of higher daylight intensities, artificial lighting was required when daylight intensities fell below the values specified for artificial lighting. The Wisconsin code and one or more of those under preparation, have adhered to the ratio of 2 to 1, which figure has also been used in the Illuminating Engineering Society's School Lighting Code. Further experience is apparently necessary before any final relation can be determined.

It is probable that in different conditions of daylight and of artificial light, the relative value varies. It is not likely, at least in the near future, that actual foot candle measurements will become the criterion for turning on the lighting, so that an accurate determination of this ratio does not seem to be of immediate importance.

The Illuminating Engineering Society, Pennsylvania, New Jersey and New York codes are essentially artificial lighting codes and do not attempt to specify daylighting. On the other hand, serious conditions of glare are not infrequent in the natural lighting of workrooms, and the extension of the code to mitigate such conditions seems to have considerable merit.

From a hygienic standpoint, sunlight has desirable features distinct from its relation to vision, so that it is desirable to have daylight available wherever possible.

There are some processes which inherently require the exclusion of daylight. Moreover, in the large cities workrooms, such as engine rooms, pipe shops, etc. have been established in basements, apparently without serious harm to the workers. On a larger scale, workrooms have been established above ground, which require artificial lighting throughout the day.

Unquestionably, all visual requirements can be provided with proper artificial lighting. It is, therefore, doubtful if a mandatory requirement of daylight for all places is justifiable. One of the codes under preparation, but not yet adopted, meets these conditions by simply specifying the lighting features necessary without distinction as to whether the light is natural or artificial.

INTENSITY SPECIFICATION

In all the codes which have come to the author's attention, intensity is specified in foot-candles. While at first some difficulty was anticipated in explaining these values and providing for their measurement, the development of an inexpensive and simple "foot-candle meter" has facilitated the application of these specifications.

For yards, passageways, aisles and stairways, the intensity on a horizontal plane, at or near the floor level, is assumed for measurement.

For various operations, the intensity is required at the work and would naturally be measured in the plane of any surface requiring vision of the character indicated. In all cases the limits are given as minima, not averages.

It has been generally recognized in selecting the intensity levels that there are no definite critical points, where a variation of a small percentage, one way or another, would cross a definite line between safety and danger. In determining the various intensity levels, lighting experts have drawn on their experience, supplemented by special investigation. It inspires confidence to note that the American values correspond rather closely to those determined independently in England after a very exhaustive investigation under governmental auspices[9].

The intensity specifications naturally fall into two groups: those required over general spaces and those required for work operations. The former are primarily for the prevention of accident, while the latter have the added element of conservation of vision, especially in the higher intensity classes.

A lower standard is allowed out-of-doors than indoors, on account of the difference in the usual character of surroundings and the nature of travel. The large out-of-door areas must, for economic reasons, have a low intensity requirement, and it would not be reasonable to demand much more than ordinary street lighting or moonlight intensities. The value of 0.02 foot-candle, therefore, appears in all the codes. For interior lighting a minimum of 0.25 foot-candle has been required for all traversed spaces.

New York and Wisconsin have included a classification to cover elevators, washrooms, dressing rooms, etc. at 0.5 foot-candle. This has not appeared in the earlier codes, but seems to be a useful addition.

The lowest intensity for manufacturing under the Illuminating Engineering Society, Pennsylvania, New Jersey and Wisconsin codes is 1.25 foot-candles. New York has adopted 1 foot-candle for the corresponding class, *i. e.*, rough manufacturing. While this value is, in the opinion of some engineers, lower than desirable, there is doubt if there is evidence to warrant the standardization of the fractional value.

The New York Committee has also adopted a new level of 0.5 foot-candle for the handling of materials and other very rough operations, and this step is being included in some of the codes in preparation in other states.

The higher steps are the same in all codes being located at 2.3 and 5 foot-candles, according to the fineness of vision required by the operations.

While it is true that there are operations requiring more than 5 foot-candles, they are exceptional and usually performed by highly skilled specialists. As such individuals are likely to be able to protect themselves, it has not seemed necessary for governmental authorities to take action.

Exceptions arising from the nature of a few processes are necessary to the illumination intensity requirements, as they appear in most of the codes. The omission has been due to the belief on the part of certain commissions that the exceptions were so infrequent and obvious that they can be taken care of readily and thus avoid misunderstanding on the part of others not entitled to exception.

One of the codes under preparation includes a class of zero intensity, while another lists the exceptions in a note following the intensity rule. This reads:

> "Note: Some exceptions to the Intensity Rule:
>
> a. There are some operations that are performed in comparative darkness, as for example, photographic processes in the dark room.
>
> b. There are some operations that are best observed by their own light as in the parts of the process of working glass.
>
> c. Some operations are best observed by the "silhouette" method of lighting in which the work is seen against a lighted background in a comparatively dark room, as in some processes of working with dark threads and lamp filaments. In all such cases in which work is of necessity carried on in comparative darkness, special precautions should be taken to properly safeguard the workmen.

A limiting feature of the intensity specification is the rather indefinite descriptive classification of the operations assigned to the various levels. The need of more definite descriptions is well known to all who have undertaken the work of drawing up specifications. It is sometimes important that an inspector or a manufacturer be able to tell without question within what class any particular operation falls. This, however, is not so serious as a superficial consideration might indicate, being somewhat ameliorated by the fact that manufacturing economy should dictate a much higher intensity than required by the codes.

Many efforts have been made to render the specifications more definite, but the problem is much more difficult than appears at first. There are many degrees of fineness in the same type of process as carried on in different shops or even in the same shop. Lathe work in machine shops, for example, varies in fineness all the way from very rough work down to watchmakers' fineness. An article is made by one manufacturer, say to 0.01 inch of accuracy, while another making nominally the same article works to 0.001 of an inch or finer.

Several proposals have been made, among which may be mentioned:

a. To make the intensity table simply an index of intensity levels, each indicated by an arbitrary designation (a, b, c, etc.) without any description of the process covered. Supplement this with a complete list of all manufacturing processes, using the symbol to designate the particular class to which each process is assigned.

b. To supplement the present classification with a detailed list of standardized processes.

The latter scheme (b) seems to have some advantages provided it is not extended too fast so as to lead to confusion through repeated changes. The New York Commission has adopted tentatively for a year, a list of this sort, but has found it necessary so far to use in many cases the qualifying terms "rough work", "medium work" "fine work", which make the specification in such cases but little more definite than the general classification.

The first plan (a) has some ardent advocates, but has been objected to on the ground that such a list implies an accuracy of description that does not really exist. Another objection is that the space occupied by such a list seemed likely to overemphasize the intensity specification as compared with other code specifications. It seems quite probable that this method will be tried out in one state, in which case its relative merit will be better determined by experience.

In general, it seems desirable for the ultimate good of all that most states adhere to the broad descriptive specification, while a few experiment with the extensions and determine their practicability. They will then be more readily able to adopt any method which shall have been found especially meritorious.

GLARE LIMIT SPECIFICATION

The establishment of proper limits for glare is probably more important than even the matter of intensity. While the provision of sufficient light is fundamental, it is also better understood and has a more obvious relation to production. It seems to be generally agreed that more employes suffer from glare than from insufficient light. Certain lighting conditions are readily recognized as glaring, while others are equally recognized as free from objectionable glare. But when it comes to drawing a definite limit between danger and safety, our present inability to measure or accurately define objectionable glare makes it necessary to utilize a qualitative rather than a quantitative specification.

The I. E. S. Pennsylvania and New Jersey codes simply require that lamps be "suitably shaded to minimize glare," with an explanatory note that glare from lamps or unduly bright surfaces produces eyestrain and increases accident hazard. This is supplemented by the requirement (under "Distribution of Light on Work") that "sharp contrasts of intensity" on the work be avoided.

The New York code requires that "exposed bare lamps, located less than 20 ft. above the floor, shall be provided with shades, reflectors, diffusing glassware or other accessories, to eliminate or minimize glare." Sharp contrasts of intensity on the work must also be avoided.

The Wisconsin Code goes a little farther and says,

> Lamps suspended at elevations above eye level less than one-quarter their distance from any position at which work is performed, must be shaded in such a manner that the intensity of the brightest square inch of visible light source shall not exceed seventy-five candle power.
>
> Exception: Lamps suspended at greater elevations than twenty feet above the floor, are not subject to this requirement.
>
> Note: (a) Glare from lamps or unduly bright surfaces produces eyestrain and increases the accident hazard.
>
> The brightness limit specified in this order is an absolute maximum. Very much lower brightness limits are necessary in many interiors illuminated by overhead lamps, if the illumination is to be satisfactory.

In some cases the maximum brightness should not exceed that of the sky (1.5 to 3.0 candlepower per square inch).

Note: (b) Where the principal work is done on polished surfaces, such as polished metal, celluloid, varnished wood, etc., it is desirable (but not mandatory at present) to limit the brightness of the lamps in all downward directions to the amount specified in this order.

For local lighting the Wisconsin code establishes the limit of 3 candle power per square inch, whenever visible from any working position. This lower limit is selected "because the eyes are more sensitive to strong light received from below."

There is a difference of opinion among illuminating engineers as to the proper values of candle power per square inch to be applied to this specification, while some think a better principle of specification can be determined. It has been suggested that glare be specified in terms of contrast of brightness, perhaps expressed in ratios. Some experimental work has been done along this line and at least two experimenters have made up experimental instruments for measuring ratios of brightness. It is recognized that there is a considerable range in the permissible contrast for different processes for different directions, especially of elevation, and for different intensities.[10]

It therefore seems likely that sometime in the future the codes will have some sort of glare classification corresponding to the intensity classification. But advance in such a direction cannot be made until further investigation and development has provided a reliable basis.

Considerable space has been devoted to the engineering limitations of the present glare specifications, with the view of interesting experimenters and engineers in the solution of the problem. On the other hand, the present specifications must not be regarded in any way as a failure. They are practical, working specifications which will do much to improve lighting conditions. They provide the means of eliminating practically all conditions which constitute a serious menace, and while the lack of a defined limit may in some cases be taken advantage of by either the manufacturers or the inspectors, such instances are likely to be quite exceptional. In reality the present specification is more definite than many existing legal requirements in force regarding matters other than engineering.

DISTRIBUTION OF LIGHT

The New Jersey code reads: "Lamps shall be installed in regard to height, spacing, reflectors or other accessories, so as to secure a good distribution of light on the work, avoiding objectionable shadows and sharp contrasts of intensity."

The distribution of light is partly taken care of in the intensity and glare limit specifications. That is, the requirement that intensity shall not fall below a certain minimum, coupled with economic considerations, tends to avoid a wide variation in intensity, while the latter also requires the elimination of dense shadows on the work. As worded in most codes these rules do not call for the elimination of bright striations of light, or, in brightly lighted rooms, of shadows which may interfere with vision but still have an intensity above the minimum limit.

The distribution of light rule covers these omissions, by calling attention to the relation of lamp equipment and location to light distribution. The lighting effects referred to, if objectionable, are so obvious that they are readily recognized when attention is called to them. Therefore, the rule seems likely to accomplish its purpose without much difficulty.

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In the Wisconsin code a further requirement is made that the local lighting shall be supplemented by a moderate intensity of overhead lighting, except when reflection from light colored surfaces produces a general illumination. This renders more definite a feature than is implied by the general requirement of the rule.

EMERGENCY LIGHTING

Panics, which have occurred in crowded rooms when the lighting has failed at the time of a fire, explosion or other emergency, have emphasized the necessity of avoiding any probability of darkness at such a time. There is, however, a wide variation in the actual needs of different workrooms. For example, where a large number of women workers occupy an upper floor surrounded by dangerous conditions, inflammable materials, etc. continuity of lighting is exceedingly important. But where a small number of employes are scattered over a large ground floor area, with convenient exits and without dangerous conditions, safety can be assured without an elaborate provision for continuity of lighting. Moreover, such provision as would be necessary in the first instance, might be prohibitively expensive for the second.

It has, therefore, seemed necessary to make only a general provision in the code, with the expectation that rulings would be made to cover specific cases, or that the detailed requirements would be supplied later by those having suitable experience. It is more a problem of safety engineering than of illuminating engineering.

The general requirement of the rule, common to most of the codes is: "Emergency lamps shall be provided in all work space aisles, stairways, passageways and exits. Such lamps shall be so arranged as to insure their reliable operation when through accident or other cause the regular lighting is extinguished." In some states the rule has been expanded to include instructions for insuring reliable operation—separate mains or sources of energy being required.

For example, the New Jersey code requires—

> Emergency lighting systems, including all supply and branch lines, shall be entirely independent of the regular lighting system and shall be lighted concurrently with the regular lighting system and remain lighted throughout the period of the day during which artificial light is required or used.
>
> Emergency lighting systems shall be supplied from a source independent of the regular lighting system wherever possible. This source of supply and controlling equipment shall be such as to insure the reliable operation of the emergency lighting system when through accident or other cause the regular lighting system is extinguished. Where a separate source of supply cannot be obtained for the emergency lighting the feed for emergency lights must be taken from a point on the street side of all service equipment. Where source of supply for the regular lighting system is an isolated plant within the premises an auxiliary service of sufficient capacity to supply all emergency lights must be installed from some outside source, or suitable storage battery; or separate generating unit may be considered the equivalent of such service.

It is apparent that the question of insuring the source of supply is one which is open to considerable discussion, involving, as it does, the reliability of the generating station and various classes of supply lines. Not only do the characteristics of service received from electric central stations and isolated plants have to be considered, but in some cases gas service and other illuminants.

It is evident that conditions of reliability vary considerably in different instances and that leeway must be allowed according to the merits of the particular cases. Where central station service is employed, it has usually been assumed that independent wiring to the street main provides a sufficient safeguard.

There is a tendency in some quarters to require the emergency lighting to be in operation whenever the regular lighting is in use. This comes from the apprehension of deterioration of unused equipment as well as the anticipation of failure to put the emergency lighting into operation instantaneously. In many cases simultaneous operation may involve an excessive expense. The provision of an automatic system, with reasonably frequent inspection and tests, should provide ample protection.

Where the system is not electric the automatic control may present more of a problem.

Most of the codes do not specify what intensity of lighting shall be provided. In New York State the emergency lighting is provided for in the law, while the code specifies one-quarter foot-candle as the intensity to be provided. This value, which might be implied from the intensity table, is being considered for inclusion in other state codes. It seems to be the reasonable value to require for exits, stairs, hallways and passageways, and also for the main aisles of large workrooms. It might, however, involve an unnecessarily high investment in lines and equipment, to provide such an intensity in all work space aisles.

SWITCHING AND CONTROLLING APPARATUS

This provides for the location of controlling switches so that at least pilot or night lights may be turned on at the main point of entrance.

Some commissions have considered it important that watchmen and others should be able to go about safely without lanterns, while others believe that the carrying of a lantern is sufficient. The former certainly has advantages, but may in some instances be unnecessarily expensive.

It would appear that the rule in this form is satisfactory on the assumption that an industrial commission would make exceptions where unnecessary hardship is shown.

ENFORCEMENT

Attention has been called to the readiness and willingness of the industries to comply with commission regulations, if they are understood.

Other limitations of the code have already been discussed, but none of these has presented more of an obstacle than the fact that many of those by whom the regulations must be applied are not versed in the principles of light and illumination, and have but little idea of the qualities and quantities necessary for the definition of lighting conditions.

Every effort has been made to simplify the rules and make them understandable to others than engineers. Some of the commissions have included simple definitions of terms used. Some have attached explanatory treatises and articles on methods of designing lighting installations. But there is still a need for more popular education. The possibilities of rendering the codes more definite and accurate in the future depend in a considerable degree on such education.

Undoubtedly the commissions can take care of their inspectors but the small manufacturers, foremen and others responsible for the construction and operation of industrial lighting installations need assistance.

The codes and their appendixes will undoubtedly provide effective mediums, but the help of those professionally related to the lighting practise is needed. It is earnestly hoped that the individual members of the American Institute of Electrical Engineers and the Illuminating Engineering Society, will inform themselves regarding the principles of the code specification and not only offer constructive criticism but also help in the educational effort of applying the regulation.

CONCLUSION

The author is inclined to look with apprehension upon laws or regulations emanating from the professions whose business they affect most directly. There has sometimes seemed to be a tendency in such legislation to favor the profession. Such tendencies are presumably more due to the professional viewpoint or prejudice, than to any intent to be unfair.

Having been connected from the first with the committees responsible for the illuminating engineering features, it is only fair to say that the danger of prejudice has been anticipated and carefully guarded against.

Especial care was exercised in the make-up of the committees and the solicitation of criticism, not only to represent the viewpoint of every phase of illuminating and other engineering, but also the light user and light purchaser. The engineer, the scientist and the practical constructor and operator were all included.

On the other hand, it is even more dangerous to legislate regarding such a technical subject without the guidance of the profession. In lighting we have numerous examples of such legislation as, for example, in the headlight laws for railways and automobiles. Many of these laws are not effective or reasonably enforcible, their meaning is not clear, while they not only impose unnecessary hardships, but have in some cases prescribed a dangerous rather than a safe condition.

The industrial lighting codes, besides avoiding both of these dangers, have been so formulated as to encourage uniform action throughout the various states of the Union. The continuation of the code development along the present lines is therefore of importance not only to those directly interested in the electrical and illuminating phases, but to the country as a whole.

In discussing the codes an attempt has been made to emphasize the principles involved rather than the details. Changes in details may be expected, but any considerable change in principles seems unlikely. No one is more conscious of the limitations of the code in its present form than the illuminating engineers who have been active in its development, but all who have had anything to do with the work regard it as a valuable working instrument. The author feels safe in saying that they consider it highly important to any state that it be adopted and actively applied as soon as possible. The greatest possibility for future development is through the experience and popular education following its enforcement.

The author has tried to avoid the expression of personal opinion as far as practicable. He has sometimes found it desirable to express his understanding of the views held by committees and their members, but this has been informal and unauthorized and should be so understood.

He herewith acknowledges with thanks the helpful assistance in the way of comment and criticism of Mr. L. B. Marks and Professor C. E. Clewell, to whom perhaps more than to any other individuals we are indebted for the initiation of these constructive regulations.

The author also acknowledges the assistance of Mr. J. A. Hoeveler and others.

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DISCUSSION ON "PRESENT STATUS OF INDUSTRIAL LIGHTING CODES" (STICKNEY), NEW YORK, N. Y., APRIL 11, 1919.

C. A. Adams: While the subject of "Safety Codes" is perhaps not as broadly interesting to the rank and file of our membership as some suppose, it is one of those things which are being forced upon our attention more and more as time goes on and shows the necessity for taking into account in a serious way the comforts and safety of the user of the things which we produce, or at least have some part in producing and installing.

The Safety Code movement, while in some branches it has been in existence quite a number of years—twenty years or thereabouts—is really only in its infancy. At first we did things as a matter of course because we were interested in the job, and finally we have been forced to take into account the interests of the users, but in so doing we meet with certain other serious difficulties, difficulties in establishing the codes and difficulties in enforcing them, I think it is here that not only electrical engineers and illuminating engineers, but engineers in general, may take considerable pride unto themselves in that they secured the provision that men trained, in some degree, in engineering, must be employed in these matters.

The great difficulty in the establishment of codes is to not only get them right, but to get the agreement and consent of all those that are involved in them, so that in a matter of this sort it is not only things which have to be considered, but it is the public and users, it is labor unions and labor, broadly speaking. So that I think that those who have to do with this work, those who have given so much time and thought to it, may feel to a considerable degree comforted, if they do not get much money out of it, in that they are doing a real and vital public service.

I am sure that we all appreciate the amount of disinterested and very serious effort that has been put on a paper of this sort, and back of that the work which has been done on the codes themselves. There are one or two things that Mr. Stickney has said that drew my attention. For instance, in the early part of his paper he said the manufacturers or employers were glad to adopt these codes, because as a rule they meant economies. It would seem natural that a manufacturer would be glad to adopt such codes in the interests of the workers, even though they did not mean economies. I recall a serious blow which I received one day in discussing certain safety devices with a large manufacturer when he objected to the introduction of safety devices on a very great number of machines he had, explaining that it was cheaper to allow the accidents to occur and pay the damages. Fortunately, that is not the attitude of many of our manufacturers at the present time, but there are still a few who take that point of view.

C. E. Clewell: The importance of legislation along the lines on which Mr. Stickney has so ably presented the subject, seems to me to be evidenced by the interest which has been taken in the several states where these codes have recently been enacted into law. When the subject is viewed from the purely legislative side it would seem that the responsibility for drafting these regulations in such a way that they will be understood both by the manufacturers and the factory inspectors, who are intimately associated with the enforcement of the codes, must rest almost entirely with the state departments concerned.

Mr. Stickney has pointed out the very interesting fact that the various labor departments where these codes have recently been put into effect have depended very largely upon the Illuminating Engineering Society for guidance in drafting these regulations in such a way that they will be suitable to the understanding of those concerned.

I have met and talked intimately with quite a number of factory inspectors, both in Pennsylvania and New Jersey, and have received the general impression that these codes, as obvious as they may appear at first glance, contain matters which, to some extent, are new and the situations created are different from the kinds of things which these inspectors have been accustomed to handling in the past, with other regulations governing the factory equipment. On this account it seems to me that just as soon as there is placed in the hands of an inspector a set of regulations which he is called upon to enforce, and at the same time in the enforcement of which he must depend upon his own judgment, that the thing tends to fail of its purpose in so far as the inspector is not posted on the fundamental principles which are back of these regulations.

The demand which has been placed on the Committee on Lighting Legislation of the Illuminating Engineering Society by these various safety departments, in their requests for something definite, has resulted in a method of procedure which, as far as I can see, is the only method which could have been followed by these committees in the development of the codes as Mr. Stickney has outlined.

Three things stand out in looking over Mr. Stickney's paper: First, that the enforcement of these codes and their ready acceptance on the part of the different labor departments and manufacturers, calls for a certain amount of education; second, that the inspectors who are really the go-betweens, as far as the labor department and the manufacturer is concerned, should have in their hands suitable devices by which they can check up their own judgment when they come to these specifications as to quantity of light and glare which Mr. Stickney has emphasized, and third, that it is almost essential, at least from the standpoint of those who are most intimately concerned with these regulations, to reduce as far as possible the element of judgment on the part of the inspectors, and one of the suggestions which has been made has been the compilation of more elaborate lists of various kinds of work and of the intensities corresponding to the same, so that an inspector may know readily how to place in its proper group any given class of work with which he may be confronted in his rounds of inspection.

The first of these points, the matter of the presentation of the principles contained in the codes, has been handled by a supplementary statement in addition to the regulations proper, which contains much information on lighting and the fundamental principles which govern these regulations, and another item, and what seems to me to be a vital point, is the matter of gathering these inspectors together—because the inspectors as they run are not a fixed group, due to changes which are going on all the time— and a recurrence of these gatherings could be planned, at which the inspectors would receive instruction along the lines of these fundamental principles at first hand. The physical measurement has been cared for, to some extent, by the foot-candle meter. As to the matter of covering all the requirements that come up, we must remember that with long lists of working operations and accompanying tables of intensities, the attention of the inspectors and the manufacturers is likely often to be drawn primarily to the matter of quantity, when there is quite a good deal of feeling on the part of those observing the conditions in the industries, that the quantity of light is not necessarily the most important item, and that the elimination of glare is by and of itself often a more important thing to attack than the matter of quantity.

Another item which I notice Mr. Stickney has dwelt upon, and which is somewhat different from these other items involved in the application of the code is the movement on the part of the New Jersey authorities toward the establishment of a Museum of Safety, so that the manufacturers throughout the state can see assembled in one structure, typical types and examples of lighting which will give them some idea of the kind of things they should watch for in systems of this kind.

The Lighting and Illumination Committee of the Institute, under whose auspices this meeting has been held, is sensible of the opportunity which such a general session has offered for discussion of this topic, and I think we all realize, after hearing Mr. Stickney's paper, that he has not only emphasized the scope and necessity of these codes, but has brought out in an interesting way the relations which they bear both to the manufacturers and to the engineering profession generally.

E. B. Rosa: I wish to emphasize particularly the educational value of this class of work and of the codes after they are issued. It seems as though the educational value would be much greater than the legal value—the benefit to those who are glad to avail themselves of these codes is greater, probably, than the benefits to those who have recourse to that law. I do not mean by that, that they should not be adopted by legal authority, but I mean to emphasize the very great educational value, first to those who take part in the preparation of the code—it stimulates the study of and education in the subject, and it is of educative value to those who participate in the preparation of the codes; in the second place it is of very great value to engineers to know about these things; and in the third place it is of great educational value to employers, managers and superintendents, those who are responsible for the lighting and the care of manufacturing plants and other places where lights are needed; and finally, of educational value to the employes, to those who are working in these factories.

While we speak of the compilation as a code, and think of the legal force of it, and the necessity of getting it into form suitable for adoption by legal authority, I think, after all, the educational value of the code is properly more important than anything else.

It is well not to put such codes into force too soon. We believe, in the Bureau of Standards, that these codes—lighting codes or whatever they are—shall be put out and given a trial, and those who are concerned with them given considerable opportunity to become familiar with them, so that the various features of their application may be developed before it is attempted to enforce them strictly. In other words, leave the matter of enforcement to those who are responsible for their enforcement—until they have full opportunity to become familiar with the codes and adapt themselves to the conditions.

C. B. Auel: The lighting codes, as is stated in the paper, are an expression of the "Safety First" movement in terms of legal regulations, and with their object everyone without exception will be in full accord. In the main they are excellent; but, regardless of their excellence they, unfortunately, represent individual states only and we shall accordingly probably have inflicted on us eventually as many different lighting codes as there are states in the Union, thus swelling unnecessarily the ever increasing burden of manufacturers and others, especially those doing an inter-state business, in trying to keep abreast of them. Another thing that should be borne in mind is that safety codes can be turned out, comparatively speaking, over night and are also capable of amendment in the same manner, and as they always involve expense while sometimes missing the object sought, they should be most carefully considered from every viewpoint, before being promulgated. Still another feature that should be mentioned is that all codes are practically a series of instructions to the employers and largely ignore the fact that there are frequently equally important instructions which should be issued to the employes, if the codes are to be successful. Mr. Stickney remarks that the experience of State commissions shows at least 90 per cent of manufacturers cordially desire to comply with their regulations. This is, of course, as it should be, but what manufacturers want to see is about 90 per cent of such commissions agree among themselves and if this can be accomplished, more nearly 100 per cent of manufacturers instead of only 90 per cent will agree with them.

Surely the lumens are no different out in Wisconsin from what they are in New York, or even in Massachusetts, nor should the requirements for good lighting be any different, and it ought accordingly to be quite possible to get the American Engineering Societies' Standards Committee, in conjunction with the Bureau of Standards at Washington, to formulate a single "League of Nations" code for general use throughout the country, and this without any state losing its sovereign rights.

It is certainly pitiful to see a great big nation like this which prides itself on its breadth of view, its vision, and on its being in the front rank of every forward movement, wasting such vast amounts, both in time and money, in having its work done piecemeal, the same problems solved, the same duties duplicated in various parts of its domain, as is evidenced in so many ways, for example, by our State laws including the safety codes. Through the efforts of the American Society of Mechanical Engineers, we have secured a Boiler Code which bids fair to become universal, so why can we not corral these lighting codes, which are commencing to be turned loose and turn them into one code for the community at large?

Referring to the Pennsylvania code, as published, and with which the speaker is best acquainted, the preamble starts off with paragraph entitled "Petitions" showing how to proceed in case one desired to appeal from any of its provisions. The one appeal the speaker would make, as already pointed out, and in so doing he believes he pleads for many others, is "O Lord, deliver us from all State codes." He speaks more feelingly in this case because immediately under the paragraph on "Petitions," is one entitled "Penalties," with fines and imprisonment staring you in the face. Having been duly warned as to the consequences impending for violation of the code or failure to carry it out, there follows a paragraph entitled "Application," which, for literary fog, can hardly be surpassed. It reads, "All Safety Standards of the Industrial Board in force at the adoption of this Standard, or adopted after this Standard, applicable in any way to the Lighting Industry (What is the Lighting Industry by the way?) or the operation of the Lighting Industry, or to the apparatus used in or around (how large a radius is included in 'around,') or in connection with the Lighting Industry, shall be considered as applying to the

Lighting Industry, and to such operation and apparatus." It is quite clear from this (and it is the only clear thing about it), and taken in connection with the penalties, where most of those having anything to do with industrial lighting in Pennsylvania, are going to land.

The code itself is short as far as its technical side is concerned and the appendix contains some very good matter.

It is apparent from the wording that effort has been made to allow as much latitude as possible in the matter of intensities, elimination of glare, and spacing of lamps. As pointed out by Mr. Stickney, this is due to the fact that suitable values have not been determined which will accomplish the desired result and without working unnecessary hardships on those chiefly concerned.

Under the paragraph on "Intensities Required" is given a table of the minimum as well as the desirable intensities required for different situations and classes of work. Mr. Stickney states that "every effort has been made to simplify the rules and make them understandable to other than engineers;" but, it has been the speaker's observation that illuminating engineers have apparently been recommending values for factory lighting altogether too low, based largely no doubt on a lack of knowledge of actual working conditions; for example, it takes as good light to read a micrometer, a finely divided rule, or a blue print, in the shop, as it does to read a slide rule or a tracing in the office. Therefore, even if the work be classed as rough, but involves the doing of such things as have been enumerated, then the lighting should be arranged on a higher base than the work itself would seem to require. Again, the loss of light from dirty and old lamps and reflectors is glossed over too briefly, and the novice usually has to learn by more or less costly experience that in order to maintain the proper degree of illumination he must allow a wide margin over and above the values set down in tables.

The paragraph dealing with "Shading of Lamps" gives no idea of what may be considered as "Glare" or the value of a "minimum of glare". It is a matter of common knowledge that a light source of high intrinsic brilliancy within the range of vision not only prevents comfortable and accurate vision but may cause permanent injury to the eye if allowed to continue any length of time. The eye, due to the position of the sun at its maximum intensity, has become accustomed to bright light from overhead, yet the retinal fatigue resulting from snow underfoot on a bright winter day is sufficient evidence of its weakness in this respect. When lamps are mounted sufficiently high, the shading of the lights is not important except to distribute it properly, but when the mounting height is lowered this feature is one of very great importance, and the paragraph in question should accordingly be given more consideration than some of the other sections which have possibly been over-emphasized.

Still, again, the paragraph entitled "Distribution of Light on Work" reads "Lamps shall be so installed as regards to height, spacing, reflectors or other accessories, as to secure a good distribution of light on the work, avoiding objectionable shadows and sharp contrasts of intensity." This taken in conjunction with the intensities given in the tables, would seem to compel the use of overhead lighting exclusively and thus to eliminate localized lighting, whereas, much shop work is of such nature that anything short of individual lamps will prove quite unsatisfactory.

If the paragraph relating to emergency lighting, not only in the Pennsylvania but in the New Jersey code, really means what it says, namely, that every establishment where workers are employed, regardless of what it is, where or how built, or for what purpose it is used, must be equipped with two entirely independent systems of lighting, then we may as well expect to have incorporated in due course in this, that, or the other code, a provision that all machine tools shall have an extra base, that all tables and seats shall have extra sets of legs, that all buildings shall have an extra foundation, except of course in Wisconsin, where no foundation at all will be permitted, since the lighting code in that remarkable state distinctly says every room must have daylight.

Again, a literal interpretation of the Pennsylvania Code would imply that an emergency system should provide an intensity equal to that of the regular system with an equal number of lamps used. The other extreme would be to furnish an emergency lamp merely as a marker in each working space, aisle, or exit, according to the custom in theatres and public buildings—to serve as a guide rather than to provide illumination. While the latter would prove somewhat inadequate, a duplicate system would be quite unnecessary. It would seem that a rather low intensity such as that specified for yards and roadways, should satisfy every requirement for emergency lighting.

The code further requires that emergency lights shall operate concurrently with the regular system, meaning presumably that the emergency lights must be automatically turned on whenever the regular system is in general use. This would be unreasonably burdensome in a great many cases where plants are already wired and equipped for adequate ordinary lighting, without provision for concurrent and independent emergency lighting, and would involve the use of considerable power unnecessarily. Such plants would be compelled not only to rewire their buildings with separate emergency circuits, but to increase their lighting load and equipment beyond those limits which have already been found to give satisfactory illumination. In some classes of service where emergency lights are really necessary, it has been customary to control such circuits by relays actuated by the regular circuit;

the relay operating and causing the emergency circuit to be lighted when, for any reason, the operation of the regular circuit is interrupted. This feature avoids the need of using power simultaneously and unnecessarily in the emergency circuit. The reliability of the emergency circuit may be insured by periodical tests just as fire alarm boxes and equipment are inspected and tested.

Such a radical requirement as "Emergency Lighting" should, it seems to the speaker, certainly be made advisory rather than mandatory, at least until the various states shall have agreed upon a plan which has proven itself by actual experience to be feasible.

M. G. Lloyd: The author of the paper made a reference to the work of the Federal Safety Engineers, and perhaps a word in explanation of that would be of interest. The country had not been long in the war before the Government officials recognized the necessity of conserving human resources. In the summer of 1917, through the initiative of the U. S. Employes' Compensation Commission, there was started a survey of the conditions with respect to safety in the Government war plants, that is to say, the arsenals and the navy yards. As a result of the conditions disclosed in that survey it was decided to appoint safety engineers at each one of these plants, and that was done. The safety engineers, soon after their appointment, arranged a series of meetings at which they got together and drafted some rules to be enforced in their respective plants, with the idea of preventing accidents. Before they got very far along in this work they called upon the Bureau of Standards to assist them in drawing up these rules and codifying them, and in that way the Bureau took a hand in the work and in conjunction with them drafted a series of Federal safety standards which were utilized at these plants.

Had the war continued longer than it did, it is probable that an attempt would have been made to have these safety standards applied to a larger field, that is to say, in munitions plants operated by private companies. The obvious method of securing such enforcement in plants not directly controlled by the Government would have been through the incorporation in Government contracts of proper clauses requiring conformity with the safety rules. With the war ending as it did, the matter, of course, was not pressed further.

The requirements for lighting which were incorporated in these safety standards were included in a set of Standards for Building Construction. These Federal Safety Standards have never been printed as yet. They were simply gotten out in mimeograph form for distribution among the Federal Safety Engineers for their own use and for the use of contractors supplying equipment for the use of the Government, who had to conform with certain of the requirements of these rules. In these Federal Lighting Standards the code of the Illuminating Engineering Society was closely followed, but there were one or two items that were somewhat different. The author has referred to the introduction of the requirement for a half-foot-candle in the codes of New York and Wisconsin for certain places, such as toilets and wash rooms, and that provision was put in these Federal standards over a year ago, and that intensity was also applied to elevators and elevator landings, an item which has not been included in the Wisconsin code.

There was also a requirement covering daylight illumination as well as the artificial lighting, and there is a paragraph I want to call attention to, on account of Mr. Auel's remarks with regard to the effect of dirt, which rapidly decreases the illumination intensities, if it is not taken care of. That clause I might read, as follows: "Artificial lighting equipment and windows shall be periodically cleaned and kept in order so that the illumination intensities do not fall below the minimum value specified: Artificial lighting equipment shall be so installed as to be readily accessible for cleaning." It is recognized that that is a very important item in obtaining suitable values of illumination.

After these Federal Standards were completed we found that there was considerable demand from the outside for copies of them, showing that there are many persons who want some guide to follow for safe practises in private plants. As has already been mentioned this evening, there are many employers who have to be forced to provide suitable conditions, but there are many others who are always seeking information as to how they can better the conditions in their plants.

As a result of this demand it was concluded at the Bureau of Standards that it would be desirable for us to proceed to elaborate these Federal Standards so as to make them more complete and a more suitable series of codes for general use. We have started that work, which has been referred to by our presiding officer, and have four national industrial safety codes now under way.

In considering the requirements in the different industries from the safety standpoint, it may be desirable to know what suitable illumination intensities would apply in the different industries, and I am consequently very much interested in the attempts that have been made in New York State and in Ohio to determine the suitable values in separate individual industries. I hope to see considerably more work done in that direction.

Another point where lighting standards touch the work of the Bureau of Standards is in the National Electrical Safety Code, which has been worked upon at the Bureau for a number of years, and of which a tentative edition for trial application has been in print for over two years. We are now engaged in revising that code and attempting to make it more complete and improving the provisions wherever possible. The present edition of the National Electrical Safety Code has some rather general requirements for illumination. For instance, with reference to generating stations and substations, there is a requirement for an emergency source of illumination as it is one of the places where it is considered necessary that complete darkness should not result from any disturbance with the usual lighting system. An exception is made to that requirement in the case of automatic substations, which are, of course, in condition to operate without an attendant.

There are certain limitations to illumination in storage-battery rooms. For instance, there is a requirement that sockets in that location shall be of the keyless variety, with the idea of preventing any spark which might ignite hydrogen gas and cause an explosion, since such gas is liable to be present in a room of that description. The requirements, however, for intensities of illumination are not very definite; they are covered by such terms as "good" or "sufficient" or "adequate." In the next edition of the Code we hope to get information which will guide those responsible for providing suitable illumination in that class of rooms. We called upon the Illuminating Engineering Society for help in this connection, and I want to express our appreciation of the co-operation which we have received. A committee of the Society carried out an investigation and has made a report giving some values of illumination which they consider suitable requirements to insure safety in such situations. They have recommended, as usual, a minimum requirement and given also a value representing what is considered good modern practise.

ILLUMINATION VALUES FOR POWER PLANTS.

Approved by the Council of the Illuminating Engineering Society, November 4, 1918.

	Minimum Foot-candles	Modern practise Foot-candles
Switchboard instruments, gauges, switches, etc.	1	2 to 4
Switchboards with no exposed live parts.	½	1 to 2
Storage battery room	½	1 to 2
Generating room, boiler room, pump room (at machinery or exposed live parts)	1	2 to 4
Stairways and passageways (measurements made at floor level)	½	1 to 2
Stairways and passageways (measurements made at floor level) where there is moving machinery, exposed live parts, hot pipes, etc.	1	2 to 4

The illumination values to be those for the working surfaces. (Measurements to be made on the vertical, horizontal or intermediate plane, as the case may be.)

Any traversed space to be illuminated to not less than one-quarter of a foot-candle minimum (measured at floor level).

I might call attention to the fact also, in view of Mr. Auel's comment, that in the Electrical Safety Code there is provided a set of operating rules which the employee is required to follow in his work in order to do his share in avoiding possible accidents.

Before closing, I want to mention the Oregon law, to which the author referred, and a copy of which I have here in hand. This law, as he mentioned, does not include any definite quantitative values for illumination, but confers upon the Commissioner of Labor the duty of specifying them. It makes general requirements that places shall be adequately and properly lighted; it covers requirements to minimize glare; it provides for suitable distribution of the light, and has a requirement for emergency lamps. There is also the usual requirement for the control of the lighting system. There is a rather unusual provision, however, in empowering the Commissioner of Labor to specify the minimum intensity. This section of the law states that the Commissioner is authorized to establish minimum values. It then goes on to say that he shall be guided in so doing by the recommendations of the Illuminating Engineering Society, but before the specified values can become effective, he is required on his own initiative to appoint a commission of three persons, one to represent manufacturing interests, one to represent electrical workers, and one to be an electrical engineer. This Commission is to hold hearings and it is expressly stated that this Commission shall have power, independently of the Commissioner of Labor, to establish, rearrange or readjust the schedule of limiting values and rules as above set forth, that is, as designated by the Commissioner himself; in other words, there is a court of review, we might say, provided to approve or amend the schedule provided by the Commissioner of Labor, supposedly with the recommendation of the Illuminating Engineering Society.

There is one other point about the Oregon law that seems to me rather important. Most of the State commissions which have acted in regard to the lighting code have general powers for making requirements along the line of safety for factory workers. In the case of Oregon, however, this is a particular separate law, conferring this power on the Commissioner of Labor, to make these requirements. It is of interest to observe that in this law there is no reference to safety, and that apparently the Commissioner is given powers to determine minimum values upon any basis that he may consider fit, not necessarily upon the safety basis. That would seem to open the way for him to make requirements which would be higher than would be ordinarily deemed sufficient merely to provide safety.

Ward Harrison: I do not believe any of us fail to sympathize with Mr. Auel's view that he would like to see a National

Code covering the whole subject. At the same time, I think he will acknowledge that the situation as far as the Lighting Code is concerned, is far better than in the case of the tax laws, or almost any other kind of State legislation with which he may be familiar. Then too, as Mr. Stickney has said there would be an emphatic dissent at the present time from any intimation that we knew all about lighting, and just how universal and permanent legislation on the subject should be framed. There are a great many things that we do not know. The various states in the supplements to their respective codes—most of the differences in the codes are in the supplemental matter—can try out different plans and thus find which one works most equitably and effectively. We shall make more progress in this way than if all were exactly uniform from the beginning.

When the subject of lighting codes was first proposed in some states the attitude of a number of large employers was, that of opposition. I know of one case where they said, "We have competent engineers to take care of our lighting; let the other fellow do likewise. This Code will be nothing but one more law added on to all the rest of the industrial regulations. Why should we further it?" At this point one of his associates from another plant replied, "We must not forget that the interests of the large and small manufacturer, so far as labor is concerned, are identical. It is possible that your plant is so well lighted throughout that the eye-sight of your workers will never be impaired and that no accidents will happen attributable to this cause. Yet, you must not forget that the entire community is your labor supply. If the small plants are poorly lighted and if as a result of that there are maimed workers or workers with impaired vision, these men, when in the course of time they go to work in the larger plant, will certainly reduce your average efficiency. We are in favor of having all plants in this state properly lighted."

On the general question of lighting intensities it might be said that, up to this time we have been for the most part in the same condition in which people found themselves a century or more ago in regard to heating. Then they were dependent upon open fires, and temperature-measuring instruments were a rarity. If you stood close to that fire it almost burned you up and yet other portions of the room were often below 40 deg. fahr. Very nearly the same effect is had in lighting by the use of bare drop lamps with no general illumination.

At the time thermometers came into general use people not only began to appreciate what the temperature variations were, but to demand temperatures all over the house which were comfortable; the foot-candle meter and similar measuring instruments should likewise lead to the same conception of lighting intensities. Unfortunately, we have no satisfactory instrument for measuring glare, as yet; in fact, as the author has stated, we do not know just what we want to measure. However, decided progress is being made along these lines and in two or three years we may well expect some form of glare meter which will measure glare almost as readily as the footcandle meter measures illumination. When this is accomplished we shall be more nearly ready for a national code.

John Vogt: In New York State we have a code which we think is about as good as any, and this evening we have not received as many knocks as some of the other states. I believe if these other states had conferred upon the different Departments of Labor, law or rule making power there would be more uniform codes promulgated.

Back in 1897 the labor law of New York State contained provisions for proper lighting of factories and the proper lighting of aisles, stairways and stair halls. In 1903 provisions were added to provide for adequate light in basements and mercantile establishments where women and children were employed. In 1912 and 1913 forty-six sections were added to the labor law as a result of an investigation by the State Factory Commission and gave authority to the Industrial Board of the Labor Department to make suitable regulations as to standards for temperature, ventilation, lighting, etc. About two years ago the Industrial Committee on Lighting was formed and we got up this code to regulate the lighting of factories. Many sections of the labor law relate to the proper sufficient and adequate lighting of stairs, stair halls, stairways, workrooms, storerooms, elevator entrances, elevator cars, washrooms, toilet rooms, etc. These words "sufficient, proper and adequate" meant nothing to the ordinary inspector who is called upon to enforce the provisions of the law. In those intensities which we have worked out we thought it best to follow the Illuminating Engineering Society's Code, and in addition thereto to make a large number of photometric tests ourselves. We have a corps of engineers in the department which is capable of doing this kind of work, and more than ten thousand lighting tests were made for the Code Committee. Since then, there have been more than five thousand tests made, so that these tests play a very important part on the amount of light which is specified in our Code.

One thing that interested me in this paper was relative to the education of inspectors and the suggestion that the inspectors might not be able to carry out the work. Our inspectors are regularly drilled, regularly given lectures. Every two weeks inspectors of our department are called together, and have been for the last three months in the Metropolitan Hall, and given lectures in connection with the American Museum of Safety in matters of this kind. We have particularly given four or five lectures to the factory inspectors from the practical standpoint of our law, not alone on illumination, but sanitation and safety, the removal of dust, fumes and gas. We realize that the Code as it stands to-day is embryonic. It is partly mandatory, but not mandatory insofar as other hearings to be held in the various states, relative to the amounts of light to be provided at processes of manufacture are concerned.

To give you some idea of what has been done in the past year as to orders issued and compliances secured in the regular work of the Department, I have gone over the statistics and find that during the month of July, 1918, when the Code went into effect, relative to that part which is mandatory, that is, the intensities required in washrooms, toilet rooms, aisles, passageways, storerooms, etc., leaving out the part which will be mandatory after July 1, 1919, after the hearings are held at which manufacturers are invited and others express their opinion at these hearings, which are to be held in the cities of Buffalo, Rochester, Utica, New York and some other points, as to whether we are right or not, or whether the Code is proper, the inspectors issued during July 105 orders relating to lights and secured three compliances; in August of 1918, 135 orders issued and 67 compliances secured; September, 174 orders issued and 278 compliances secured; October, 64 orders issued and 88 compliances secured; November, 546 orders issued and 156 compliances secured; December, 446 orders issued and 205 compliances secured; January, 1919, 608 orders issued and 517 compliances secured; February, 765 orders issued and 50 compliances secured, a total of 3,290 light orders issued and 1,569 compliances secured, and only 92 waivers made. These statistics include the work only up to March 1st. Our statistics are not ready for the later months. These orders were issued out of a total of 56,236 inspections of the regular type and 4,791 special inspections. This gives you some idea as to the character of work and the quantity of work that the Department of Labor, now the Industrial Commission, has done.

Total orders issued up to March 1st, 3,290 and compliances secured, 1569, that is, about twice the number of orders issued as compliances secured, giving about 1500 or 1600 compliances still to be secured. I have not had time to segregate these orders to determine whether they were for glare or for improper distribution, etc. From what I can remember, most of them were for the elimination of glare or minimizing of glare.

Many other orders have since been secured, no doubt, but I have only given statistics up to the 1st of March. These orders go out, and the parties upon whom they are served are given a definite time in which to comply with the law. Manufacturers may ask for an extension of time. In that case the order still is pending. In the case of all orders that are pending, compliance will be insisted upon, and if the ordinary method does not secure it the Court will be resorted to. We have a capable legal bureau and these things are threshed out in Court if necessary.

W. P. Hurley: It seems to me that if the codes in themselves are in advance of the popular education, it is the work of the promotion of these codes that needs to be taken in hand, in order to make them of the greatest efficiency. We may get ourselves into the position of the states of Wisconsin and Massachusetts, with their insurance laws, where they passed useful and effective laws, providing for state insurance, but they fell flat because the popular education was not sufficiently advanced on the subject for the people to make use of them.

One of the most interesting points, that has not been touched on here to-night, is how to bring these codes before the public, and it seems to me we must first appeal to the self-interest of the user, that is, the manufacturer, and next the self-interest of those who can with profit to themselves bring the codes to the attention of those manufacturers, and keep up this education, otherwise the burden must be on the Society as a scientific body, and on the Labor Bureaus of the different states. Even if the different states handle it, it means the popular education of the users, and this work will have to be undertaken by them. It would mean that the inspectors, even, must be educated to tell when the law is violated, and they would have to organize to the extent of telling the men how to design installations, which is usually considered beyond their province.

The present time is particularly ripe for the education of all the interests concerned in the lighting industry—by that I mean the manufacturers of the incandescent lamps and the fixtures and the wiring contractors, as well as the plant departments themselves of these manufacturers. It is particularly suitable at this time because we have the gas filled lamp practically in its infancy, as far as application is concerned. The educational work should be with the contractors and dealers. This would be found to react at once in favor of the Code, because it would give them a legal standing and something to work by that could be published and made use of.

W. T. Blackwell: This subject has been considered from practically every phase but there is one point which I would like to bring up and that is the instrumentality through which we are to enforce the code. We have at the present time codes in four or five states, therefore, we must take into consideration how the lighting measures in the codes are to be enforced. The Department of Labor inspectors, upon whom it is encumbent to enforce the requirements of the codes, are as a rule appointed through Civil Service examinations. We as a Society have fathered these codes—therefore, we should take the initiative in taking measures toward the end that the inspectors may receive proper information which will enable them to do their work properly. Last year a series of lectures were arranged at the University of Pennsylvania, under Professor Clewell, for the training of the inspectors of the States of Pennsylvania and New Jersey. Owing to the short period covered by these lectures only a meagre idea of the requirements of good lighting could be touched upon.

Many of the codes have a monetary penalty for non-compliance with requirements and in consequence an inspector who is not thoroughly familiar with lighting is therefore not in a position to determine whether the prevailing conditions in a plant come within the minimum requirements of the code and therefore hesitates about placing a violation on the installation. It seems to me that having evolved the code we should bend our efforts toward the education of the inspectors as has been suggested by the previous speakers. This perhaps can best be done by placing in the hands of the inspectors some form of a manual which would be comparable to the National Electrical Code or the Fire Prevention Manual adopted by the National Board of Fire Underwriters, and also the National Fire Prevention Bureau.

The foot-candle meter provides a ready means of checking up the intensities of illumination, provided the inspections are made at night, but if the inspector is to use this instrument for measuring daylight intensities it will then be necessary to have the instrument equipped with a daylight screen. This, of course, does not offer an insurmountable obstacle but it occurs to me that this instrument offers the best solution to settling differences of opinion which may arise between the inspector and the superintendent of a plant. It would therefore seem desirable that the use of footcandle meters properly fitted to the requirements of the situation should be encouraged in those states where codes are in force.

In order to thoroughly understand the situation and the conditions under which such inspectors work, we must place ourselves in their position or otherwise I fear that the lighting legislation now incorporated in the Labor Codes of several states will become inoperative through lack of enforcement. I, therefore, hope that the Society will take some measures toward formulating an inspection manual for the use of the inspectors in those states having codes.

E. G. Perrott: A point in connection with the Code which I think should be amplified, is the question whether the minimum of illumination should be stated in the Code, or whether it should be left to the discretion of the Labor Department, or the Commissioner who enforces the code.

I wish to mention something which might be considered as paralleling in its operation the lighting codes, that is, the building codes of the various cities. We have in Philadelphia a Building Code under which we are suffering, due to the fact it was adopted in 1854, while it was suitable for that period, from the standpoint of the knowledge which existed at that time, whereas the knowledge of building construction has advanced to such an extent that it no longer applies. The law is a State law and cannot be changed by local authority, cannot be changed by anybody that has anything to do with building construction, and as it requires State action, it is a very difficult matter to get the State Legislature to repeal old acts and reframe them to suit new conditions.

It appears to me that in the state of the art as it is to-day we are in a much better position if we omit from the Code itself the actual illumination we desire for the various parts of the building and leave that to the commissions or to the labor departments to establish as has been done, I believe, in the state of Oregon.

T. F. Foltz (communicated after adjournment): Although there has been a lighting code in force in the State of Pennsylvania for nearly three years, it is only during the past year that definite arrangements have been made by the Department of Labor and Industry to apply its requirements in more than an advisory way. This was due largely to war conditions which affected to a large extent the industries in this State and made difficult the detailing and training of inspectors for this work.

With the revision of the code about a year ago and the course of excellent lectures given to the inspectors last year at the University of Pennsylvania through the courtesy of Professor Clewell, the real work of applying the provisions of the code commenced. These lectures were supplemented by talks at inspectors' meetings of the different districts into which the state force of factory inspectors is divided. It soon became apparent, however, that it was more practicable to detail for this work, men who by training and aptitude, could be drilled into the work of judging and advising on good lighting arrangements for industrial plants. These men are being given special instructions on the subject of factory lighting and on the use of the recently developed compact and convenient foot-candle meter with which each man is equipped. Although the training of these men is not yet completed, results would indicate that we are on the right track.

Thus far it has seemed inadvisable to use the police powers to secure compliance with the provisions of the lighting code; the method used, therefore, is one rather of education and co-operation. It has been found that the average employer, especially the one who does not have a business big enough to have experts on his staff, knows little or nothing about good and bad lighting; although experience shows that he is, on the whole, anxious to correct conditions when he is shown how to do so. He is conversant with requirements for machinery guarding, dust and fume removal, etc., and these requirements can be laid down in concrete form as to dimensions, materials, etc. To him, a requirement for a 5-foot guard is clear, but he has yet to learn the meaning of an illumination intensity of 3 foot-candles. As a means of educating the employer, there was issued with the last edition of our lighting code, an appendix containing much of the information in the Illuminating Engineering Society "Code of Lighting Factories, Mills and Other Work Places."

Some employers can be won over best by the money argument. When they are shown the ridiculously low cost of a good lighting system, as compared with the saving they can make in better and greater output, less spoilage, etc., they regard it as a shrewd business proposition to comply. The means of gaining compliance must be determined after a careful analysis of local conditions as the inspector will find them; and our experience would indicate that compliance with our requirements secured through co-operation and education produces more lasting results than that obtained by the exercising of mandatory powers.

A strict interpretation of the legislation which gives this department its authority would seem to limit its requirements to those just sufficient to prevent accidents to the workers. Illumination adequate for this purpose, obviously, is not enough for satisfactory and efficient work. However, those powers might be interpreted to include the authority to call for conditions that would reduce eye strain, industrial fatigue, etc. Such a broad interpretation, then, would permit requirements for lighting which would be sufficient, not only to prevent accidents, but also to result in truly adequate illumination for all purposes.

Mr. Stickney's paper is a most comprehensive analysis of this subject. He not only shows that considerable progress has been made on the subject of lighting legislation, but he also frankly admits that certain phases are still susceptible to much improvement. For instance, the subject of glare, an element most difficult of treatment in a code, is still in need of further study. To our mind the treatment of glare is as important, if not more so, than that of illumination intensities. It is futile to provide a sufficient quantity of illumination unless the lights are so disposed and equipped as to minimize glare. The requirement that lamps under a height of 20 feet be specially equipped, does not begin to cover the case as there is nothing said about intrinsic brilliancy, size of unit, contrast of brightness, or whether or not the light source is within the field of vision. Limiting the candlepower per square inch of light source does not cover the case as a certain light source in a comparatively dark field may be objectionable, whereas, the same light source in a light field may not be annoying. Thus far, the California requirement takes into account the greatest number of factors affecting glare, particularly that referring to the light source being within field of vision. To cover the case adequately, a rule or formula would have to take into account the intrinsic brilliancy of the light source, contrast of brightness, distance and angle at which the light source is viewed, and the nature of the work done. It is obvious that the complexity of such a rule or formula would militate against its adoption. It is indeed unfortunate that some mechanical means to evaluate glare has not yet been developed to the extent that has the meter for measuring foot-candle intensities.

No attempt is made in our code to give detailed requirements as to the extent and means of securing emergency lighting. Local condition will have to determine whether the emergency lighting will be from an independent lighting plant, separate transformers, separate fuses, from a separate power circuit, or from gas or even oil lamps. The manufacturing processes and the nature and occupancy of the building are also determining factors. The inspector is compelled to exercise fine judgment after making a careful survey of local conditions.

Much credit is due the Illuminating Engineering Society and its individual members for the rapid progress recently made on the subject of lighting legislation, and it is noted with considerable satisfaction that they intend to make further progress along this line. The Pennsylvania Department of Labor and Industry will welcome developments that will increase the practicability of its lighting code.

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James R. Cravath (communicated after adjournment): Like other legislation, industrial code lighting legislation is largely a problem of education. First, the experts formulating such codes must thoroughly educate themselves on the conditions to be met. Next, they must pass this information along in usable form to the non-technical legislator and user. Before such codes become thoroughly effective, those responsible for the enforcement of the law must be educated as well as those upon whom the law must be enforced. Such education means that the factory inspectors and superintendents, operating under the law, must understand something of the illumination unit with which they are dealing. Fortunately simplification of instruments for measuring illumination has recently placed this matter on a much more satisfactory basis than heretofore, so that education in what a foot-candle of illumination really means can be carried on a sufficiently large scale to accomplish the desired results within a reasonable time.

Specifications with regard to glare, as the author indicates are rather difficult to formulate at this time, when one considers the prime reason for such legislation is safety. As far as safety goes, lighting conditions are tolerable which would by no means come up to reasonable hygienic requirements. For example, a glaring light source may be far enough above the line of vision not to interfere with clearness of vision and so it would be exempt under any "safety first" regulations. However, it might be a very bad source of eye fatigue or strain. As this would pertain to the health of the workers, it would also be a justifiable subject of legislation even though not necessary for safety from accidents. The Wisconsin code, which as the author says, is the only one giving definite glare limit specifications, is nevertheless very lenient. Seventy-five candlepower per square inch is over 3600 millilamberts or considerably brighter than an acetylene flame. The difficulty of formulating glare regulations is of course great. Our knowledge of what glare limitations are desirable is probably greater than our knowledge of how to lay down such limitation in a code which must take in such an infinite variety of conditions, as an industrial lighting code.

G. H. Stickney: The extensive discussion which this paper has evoked is a gratifying indication of the wide interest in the subject. The general unanimity of opinion expressed seems to indicate a remarkable agreement among engineers with the line of procedure. There is considerable opportunity for debate as to how far certain specifications should go. I am convinced that more experience in applying the codes is the only reliable basis for determining these questions very much further.

We can all agree with President Adams that manufacturers should welcome safety provisions for humanitarian reasons, independent of ordinary factors of economy. While the Commissions have reported a broad attitude on the part of manufacturers, it is undoubtedly true that the enthusiastic acceptance of the codes by manufacturers will be encouraged by the realization that the codes make for economy rather than expense. Thus, the educational work in which Prof. Clewell and Dr. Rosa have expressed so much interest, will be even more important than might otherwise appear.

The whole trend of the discussion is to emphasize the need of wider popular education in regard to the factors of illumination. It would seem as if there were a need of going even further than anything which has been projected. We may well urge that the values and characteristics of light be taken up more thoroughly in the teaching of physics, not only in colleges, but also in high schools, and that the units of light might even be treated in connection with the teaching of measurements in arithmetic. We can hardly expect that the relations of light quantities will be as broadly studied as the common measures of length, volume, weight, etc., but they might very reasonably be included in the training of all technical students.

I note Mr. Auel is opposed to States' rights. This principle certainly has made the code problem a much more difficult one in this country than it was in England, where there was but one government to deal with. The difficulty, however, as regards lighting codes is a small matter compared with the corresponding confusion in other classes of state law, as has been pointed out by Mr. Harrison.

The question of Federal *versus* State enactment is certainly too big a problem for the Committee on Lighting Legislation to undertake to secure modification. The committee has wisely accepted the existing methods of law making and has done the best it could under these conditions. That there should be a general uniformity of action in all states is of course important, and in this connection it is very fortunate that the Committee has been able to establish a leadership which keeps it in touch with the activities in the different states and enables it to exert a strong standardizing influence.

As pointed out in the paper, the variations in the code so far adopted are not fundamental and are largely confined to undetermined factors, where the experimentation, if properly followed up, should lead to progress followed by uniformity of enactment. All the commissions encountered have seemed to be composed of remarkably sincere, able men, seeking the best interests of the people.

It is true that in one or two instances there seemed to be a tendency on the part of state committees to introduce variations as evidences of individual accomplishment. It does not seem likely, however, that such variations will seriously compromise the ultimate status of the codes. In some ways the variation may be of benefit if the leadership can be maintained. At any rate it cannot be entirely avoided.

No report of hardship has come to me in regard to the enforcement of the Pennsylvania Code and I believe that a fair and common sense interpretation of the rules is likely to be made by the Commission.

In regard to the question of cleaning lighting equipment, it is my recollection that a foot note appeared in at least one of the early drafts of the Illuminating Engineering Society's code, pointing out that the values of required intensity were minima; that lighting equipment should be cleaned and that in the design of lighting installations, at least 25 per cent allowance should be made for depreciation. I believe this note was transferred to the Advisory Section at the suggestion of a State Commissioner. At that time there was a tendency for explanatory notes to accumulate in the code proper and it seemed advisable to eliminate them as far as possible, confining the rules to the conditions to be maintained and treating the methods of securing them in the Advisory Section. The low intensity values have been explained in the paper. These values have been criticised by some as being too high; by others as being too low. They seem to be about right as safety provisions, in view of the present state of the art.

The practicability of the intensity specification was greatly enhanced by the foot-candle meter, which has been mentioned by several in the discussion. The instrument devised by

Dr. Sharp provides a simple inexpensive means by which reasonably accurate measurements can be made by relatively inexperienced observers. Such observers are liable to make greater errors in reading the more complex, though more accurate higher priced instruments.

The present foot-candle meter is being distributed at cost by lamp manufacturers in the belief that a better understanding of intensity will be favorable to the development of the art. It is understood that other illuminometers are under development by instrument makers which, in addition to the present type, should result in a wide distribution of suitable instruments.

Referring to Mr. Perrott's comment regarding the flexibility of intensity values, it should be noted that in every instance so, far these values have been adopted by the Commissions and not by act of Legislature. The Commissions have the authority to make changes from time to time. Oregon is the only state in which the Legislature has written any of the definite rules.

I think we are all agreed that the codes should be so authorized that the Commissions can make changes in the rules as necessitated by the advance in the art and without recourse to legislation.

The question raised by Mr. Cowles and others, in regard to the emergency circuits, is an important one. If the specification as written in some of the codes is too literally interpreted, there seems to be considerable possibility of inflicting unnecessary and burdensome expense, especially upon small industries. It is, of course, impracticable to write a specification for each individual set of conditions. Commissioners with whom I have discussed the question seem to think it preferable to specify for the dangerous conditions and allow exceptions where the duplication of service is shown to be unnecessary. Personally I think it would be desirable to recognize such exception in the rule and make provision so that those entitled to exceptions will understand that they can secure them without difficulty. Consideration should be given in each case to the necessity of the emergency lighting, the liability of failure of lighting service, its extent, its relation to other emergencies and the practicable provisions which can be made to avoid undue hazard. It seems reasonable that wherever emergency lighting is necessary it should be separated from the power circuits in such a way that an overload on the power circuit cannot cause entire failure of the lighting.

The need of more definite glare specifications has been emphasized by Mr. Foltz and others. The Committee has been keenly aware of this need. It has been suggested that rules be made for each type of lighting equipment. On account of the variety of reflecting equipment used with the incandescent lamps, this would be quite an undertaking;

moreover, it is almost impossible to draft rules of this type so that they will be fair to all illuminants. There would, therefore, be considerable possibility of stirring up competitive recrimination which might seriously interfere with the general acceptance of the rule.

A definite measurement of the glare effect, independent of the particular equipment, seems to offer a more helpful solution. What is now needed is a simple, inexpensive and reasonably accurate instrument for measuring glare. With such a device available, the illuminating engineering features of the codes could readily be carried to definite conclusion.

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UTILIZING THE TIME CHARACTERISTICS OF ALTERNATING CURRENT

BY HENRY E. WARREN

Abstract of Paper

By the invention of a very small, simple and reliable self-starting synchronous motor, in conjunction with convenient means for regulating the average frequency of an alternating circuit, there has been developed a new field of usefulness for electric power. It is now feasible to drive all kinds of timing devices such as clocks, graphic instrument movements, time recorders, etc., directly from the lighting circuits. Remarkable accuracy is obtained, and the amount of care is minimized by the elimination of winding and regulating.

MORE than a decade ago the possibility was recognized of increasing the accuracy of the alternations of current distributed for light and power to a degree where it might serve through synchronous motors to drive ordinary clocks. The system proposed, which was said to have been actually tried in Germany, was inadequate in using motors which required to be manually started and in the mechanism for enabling control at the central station.

Within a few years the author, impressed by the great opportunity to extend the use of alternating current into the field of clocks and various timing devices, recognized that the first requirement was a very small reliable self-starting synchronous motor. After many experiments the motor shown in phantom view in Fig. 1 was developed. This has a small bipolar field magnet, fitted with shading coils on its poles, and is excited by a winding *A* taking current from a single-phase circuit. Located in the field with a relatively large air gap is a disk rotor *B* with projecting pins *C*, the rotor and pins being made of hardened magnet steel. The rotor is mounted upon the shaft *D*, which is vertical to reduce wear of the bearings; and the relation between the rotor and the field is such that the weight of the former, which is only 1/20 of an ounce, is magnetically supported, so that when running it floats without appreciable end thrust. Motion from the rotor shaft is transmitted through the worm thread *E* to the gear *F* and thence by another similar reduction to the terminal shaft *G*. This gearing is enclosed in a nearly air-tight case *H* which contains an ample amount of lubricating oil. The worm thread on the rotor shaft forces this oil through the upper bearing and distributes it to all rubbing surfaces. The space around the rotor is guarded from entrance of dust and loss of oil by a tight cap of non-magnetic metal which is sealed into the case.

The rotor of this motor starts from rest and runs at synchronous speed because of the magnetic remanence in the hard steel. The initial starting torque is due to the fact that the poles induced in the disk by the rotating magnetic lines of the field persist locally during part of an alternation after the inducing magnetism has advanced, and by ordinary magnetic attraction tend to follow the field. The following reverse magnetic alternation overpowers these local poles in the rotor disk and sets up new ones, but the disk has moved meanwhile. At first there is a progressive shifting of the poles through the mass of the rotor. As the speed increases the driving torque continues strong until actual synchronism is reached, when the local poles in the rotor remain in step with the rotating field and no longer shift progressively through the mass of the rotor. In order for this action to take place, it is necessary to proportion and design the field, airgap, and rotor so as to maintain certain very definite relations among them, especially with regard to the distribution and intensity of the magnetic flux. Otherwise the magnetic poles of the rotor will shift at synchronous speed and the rotor will slip.

The rotor pins *C* considerably increase the synchronous torque of the rotor without interfering with its starting torque, but they must be properly proportioned and arranged to accomplish this result. Material such as is used for permanent magnets serves better than ordinary carbon steel for pins and rotors. Apparently the higher the remanence the greater the output.

The general appearance of the motor is shown in Fig. 2. The overall dimensions are 2 3/16 inches high, 2 11/16 inches wide and 2 5/8 inches deep. The weight is 15 oz. One of these motors consuming about two watts will deliver at its terminal shaft, making one rev. per min., slightly more than one-millionth of a horse power. Its efficiency is therefore one twenty-fifth of one per cent. Nevertheless, this minute amount of power is more than ample to drive clocks and various forms of timing devices. In fact it represents several times the output of a strong spring clock movement. These synchronous motors will start and run indefinitely at absolute synchronism when connected in circuit of the approximate frequency for which they are designed. Their synchronous operation is not affected by very considerable changes in voltage. For example, they will operate correctly when the voltage is 50 per cent above or below its normal value, but a motor designed for 60 cycles requires certain changes in its proportions in order to operate properly at 25 cycles. There is, however, no difficulty in supplying motors to operate at any standard frequency.

The starting torque of these motors is more than 50 per cent of their running torque. If overloaded beyond capacity they will generally come to rest and will not continue to run below synchronous speed. The period during which a motor will run without attention has not yet been determined, but some of them have been operating continuously 24 hours per day for more than 2½ years. It is evident that the lubricating oil which they contain should be renewed every year or two and if this is done, there seems to be no reason why the motors, when driving light loads such as clocks, should not run for many years. The oil can be easily renewed but the motor has its parts sealed together when it is built. The only place where the oil can escape is through the clearance space around the terminal shaft, but this is above the oil level when the motor is in operation. Owing however to variations in air pressure and temperature changes, a very slight amount of oil is occasionally forced out at this point.

The next requirement in order to improve existing distributing systems so that they might be used for the operation of clocks and other timing devices was an instrument which could be used in place of the present frequency meters to standardize the frequency at the generating station. The result achieved is shown in Fig. 3, which illustrates the type of master clock that has been installed in many of the principal power stations in the eastern part of the United States, and which has been adopted for controlling the frequency.

This clock contains two complete movements, one regulated by a pendulum and the other driven by one of the self-starting synchronous motors which have been described. On the large dial of the clock there are two hands, one of them black and the other gold. The black one represents the time as determined by the pendulum. This pendulum has a rod of invar and its rate is adjusted by two methods. One used in astronomical timepieces consists in adding or subtracting weights on a small platform carried by the pendulum rod. These weights are numbered to indicate the seconds per day which they will alter the rate. The other method is electromagnetic and consists in passing a very minute measured current through flat coils located below the pendulum bob. Within the pendulum bob there is a permanent magnet, the field of which cuts across the conductors of the coils. This combination of parts permits an exceedingly accurate adjustment of the rate of the clock. It is easy by this means to make the clock run a fraction of a second per day faster or slower.

The other hand which is mounted upon the large dial and is gold in color, is driven directly by gearing from one of the self-starting synchronous motors. The gear ratio between the hand and the motor is such that at normal frequency the gold hand will revolve at exactly the same rate as the black hand, and if they are started together they will remain so. These hands make a complete revolution in five minutes.

The operators at the switchboard are instructed to look at the clock occasionally and, if they find that the gold hand is gaining over the black hand, to lower very slightly the speed of the turbines. If the gold hand is running behind the black hand, the turbine speed is to be slightly increased. Small momentary variations in the frequency do not perceptibly affect the relative position of the gold and black hands, and since in general these fluctuations are equally likely to be positive and negative, there is shown only the error which is cumulative. Any real tendency of the speed to run either higher or lower than its normal value will gradually bring about a change in the relation of the two hands. This is really the only error in the frequency which should be corrected by the operator, because the governors of the turbines will automatically take care of the momentary fluctuations. The master clock may be considered as a device to maintain a true base line for the frequency.

The physical effort required to maintain this correct base line is very much less than has formerly been expended in some large systems in trying to keep the frequency correct by following the indications of an ordinary frequency meter. The reason for the difference is that the frequency meter shows only the instantaneous value and is consequently fluctuating above and below the average value. These fluctuations are only momentary as has been pointed out before, but there is no way by which the operator can tell whether the indication of the meter when he looks at it represents a high point in the frequency curve or a low point. Consequently, he is obliged to assume that it represents the average frequency, which it rarely does. Therefore when he adjusts the speed according to a reading of the frequency meter, he may make the average value worse than it was before. The next time he looks at the meter, the needle may happen to be on the same side of the average, in which case the operator assumes that he did the right thing previously. Or it may happen to be on the opposite side, when he concludes that it needs new adjustment in the reverse direction. The more frequently these adjustments are made, the more necessary they apparently become. The difference between determining frequency by the frequency meter method and that of the master clock may be made clearer by the following somewhat analogous illustration:

Suppose that a lake, the surface of which is covered with waves, is to have its depth measured. One method is to go out in a small boat and drop a sounding line overboard. The observer will be able to tell the depth of the water within a foot or two, but the readings which he makes will be constantly varying on account of the motion of the waves. This corresponds to the frequency meter method. A much more accurate means of determining the depth of water in the lake would be to enclose a considerable area of its surface in a structure such that the waves could not enter and the surface of the water would be smooth and level inside. Then a reading by a sounding line would instantly give the true depth. This corresponds to the master clock method.

With the master clock method, the only error that appears is a real one and is due to the tendency of the average frequency to run either high or low. This tendency in a large station may not become manifest for a considerable time interval, sometimes as long as 30 minutes or more. Therefore it is not necessary for the operator to make adjustments nearly as often as he did formerly where he depended upon the frequency meter. Perhaps it would be fair to estimate that one-half or one-third as many adjustments are needed where the master clock is used.

The only care which must be given the master clock is to compare it every few days with accurate time signals sent out from Washington or some astronomical laboratory. When it has been carefully regulated it should run within a very few seconds a week. If conditions are favorable one of these clocks will keep within one or two seconds per week. The winding of the movement which drives the pendulum is done automatically by the same motor which is geared to the gold hand on the large dial. As a safe-guard a second motor which drives a clock hand moving over a small dial is installed in the same case. In an emergency this hand can be used to regulate frequency by comparison with the black hand on the large dial. There should also be installed on the wall near the master clock a secondary clock with a second hand driven by one of the small synchronous motors so as to check at all times the frequency on the system. The master clock itself should stand upon a reasonably firm foundation so that it will not be subject to excessive vibration. Its motors, which require only four watts, may be connected to any nearby instrument transformer which can be depended upon to remain in circuit continuously. The effect of controlling the frequency by one of these master clocks may or may not be apparent on the frequency meters which have been in use. This will of course depend upon the error of these instruments. If however the average frequency is measured by more sensitive means, for example, by counting the total number of alternations during the day and determining the average mathematically, the effect of the master clock will at once be apparent. Any ordinary frequency meter can hardly be expected to keep the error in the average frequency below one or two per cent, while the master clock will easily, over a 24-hour period, keep the error in the average frequency below 1/100th of 1 per cent.

Ordinary frequency meters are subject to errors principally of the following kinds:

1. Calibration.
2. The effect of use and time.
3. Variations due to temperature.

Of the foregoing errors those due to calibration and use may be overcome by frequent recalibration, provided an accurate standard is available, which is not usually the case; but the errors due to temperature are very difficult indeed to eliminate

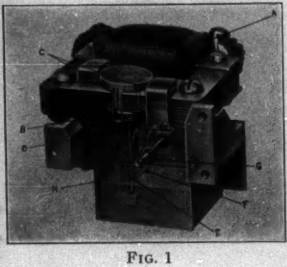
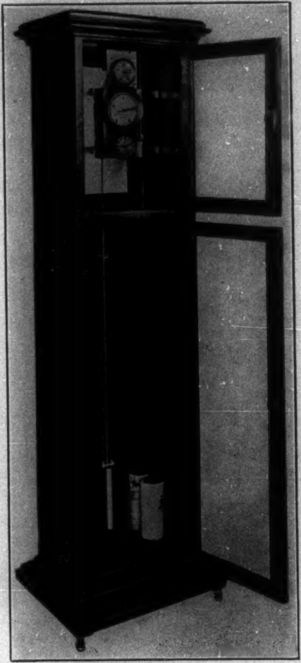


Fig. 1



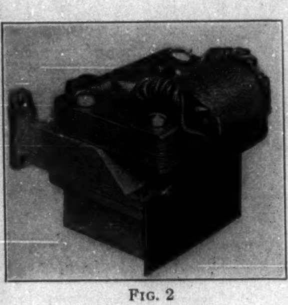


Fig. 2

Fig. 3

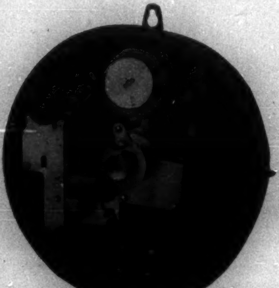


Fig. 5

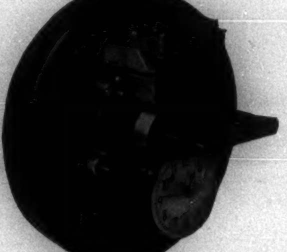


Fig. 6

[WARREN]

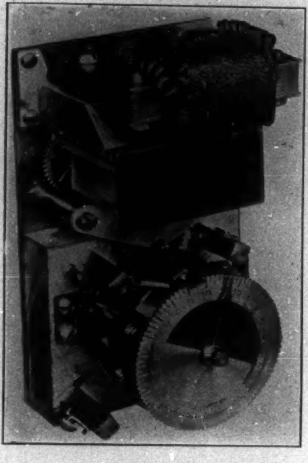


FIG. 7

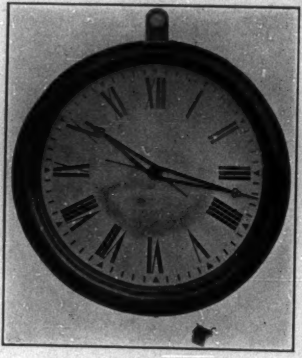


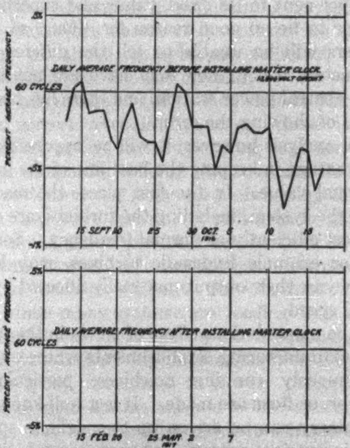
FIG. 8

FIG. 10

FIG. 9 [WARREN]

and these errors alone will ordinarily justify the master clock method. There is no reason why the frequency of a great system should be dependent upon the temperature at the switchboard, which has generally been true.

In Fig. 4 is shown an interesting comparison between the frequency obtained on a very large system by means of first class frequency meters which were often checked and conscientiously watched, and the frequency on the same system at a later period when the regulating was done by means of a master clock.



BEFORE AND AFTER

The effect of the installation of a master clock is here strikingly illustrated. Note the marked irregularities with the best regulation possible under the old system.

FIG. 4

The errors in the former case were principally due to variations in temperature, and these during the whole year covered a considerably wider range than shown in the curve, which covers only one month. These two curves do not represent all the variations in frequency during each day, but only the average value of the frequency as determined by dividing the total number of alternations during the day by the time in seconds. They may be said to form the base lines of the frequency. In one case the base line is very irregular and differs materially from its normal value, and in the other case it is a horizontal line having true normal value with an error too small to show on the plot. Experience has shown that where master clocks have been installed on various systems the average frequency before the master clock was started has usually been as much as one or two per cent above or below normal, although of course it was impossible to determine this until the clock began to operate.

The question will immediately arise whether there is any advantage in trying to hold the average frequency nearer than one or two per cent to its exact value, and superficially there might appear to be no good reason for doing so. Certainly the customers will be unable to tell the difference, because frequency meters which they may have are not better than those at the central power station, and there has been no other easy method of showing the error.

On careful analysis however, it will be apparent that there is a real advantage in keeping the base line of the frequency at its true normal value. In the first place, the machines connected with the system, including the turbines, are so designed as to run most efficiently at normal frequency. Some of these machines, for example hydraulic turbines, may have either their efficiency or their output materially affected by deviation from normal speed.

Perhaps the most obvious effect of errors in the frequency is upon those manufacturing establishments whose output comes from continuously running machines, particularly where textiles, paper, or flour are made. It is a well-known fact that textile machinery can be driven up to a definite speed with a constantly increasing output, but not beyond this without injury to the product. Now whether induction or synchronous motors are driving such machinery, the speed will depend directly upon the frequency. Consequently, if the average frequency is even one per cent lower than the most efficient speed for the machines, the output of the mill per day will be diminished by just that same percentage, and if the frequency is higher the product will suffer injury. The fact is of course recognized that it is necessary to allow a certain margin for safety between the average operating speed and the maximum speed to take care of temporary variations caused by load fluctuations or otherwise, but this same margin must exist in any case. It is clear that the output of a cotton mill connected to the system where the curve in Fig. 4 was made would be greater during the summer season when the frequency was nearly normal than later in the season when the frequency dropped, because of the temperature coefficient of the frequency meter. In a number of textile mills connected to a single system, the value of the lost output in dollars and cents due to this cause would soon exceed the trifling cost of the master clock equipment necessary to standardize the frequency. In paper mills the evenness and quality of the paper depend upon the accuracy and uniformity of the speed, so that there is a very obvious financial advantage in maintaining correct frequency. Doubtless many other classes of manufacturing are similarly affected, but the above examples are sufficient to show the financial desirability of the new method.

Beside the advantage to the user of electric power in maintaining the correct average frequency, there is a definite psychological advantage to the operating engineers and managers of power companies in being able to assure themselves and all of their customers that they are maintaining the frequency upon which they have agreed. A still greater gain from the standpoint of the power producer results from the ease with which systems having such standardized frequency may be connected in parallel without the slightest disturbance. Generally, where many systems are feeding into a large network, the individual variations in the frequency meters are such that it is necessary to make adjustments by raising or lowering the speed, sometimes to a very perceptible extent, before an individual station can come into synchronism with the network; but where master clocks are installed it is only necessary to wait until the machines have the right phase relation, because there will be no measurable difference in the average frequency of the independent station and the network when they are running independently. As at the present time the tendency is very clearly to increase the interconnection of big power producers towards the ultimate goal of a complete union extending all over the country, it is obvious that preliminary standardization of the frequency is a step in that direction.

Passing now from the advantages of master clock control which affect existing service, one comes to gains which open an entirely new field of service. Just as soon as the frequency of a system has been controlled by means of a master clock, it becomes feasible to connect anywhere in the line small self-starting synchronous motors like Fig. 2 and use them to measure time accurately. The most obvious use of such motors is to drive clocks, and this is being extensively done. Such clocks may ultimately supersede a large proportion of the spring- or weight-driven clocks that are in use today. Obviously the new clocks never need to be wound nor regulated and they will keep time in a manner which is most astonishing to one familiar with the poor time-keeping qualities of an ordinary clock. On a well regulated system these motor-driven clocks can be depended upon to remain correct within a few seconds, month after month. This requires that provision be made against interruptions of the service, which will be mentioned later.

To the electric light companies the great immediate advantage of the new kind of service is the ability to substitute these self-starting synchronous motors for the troublesome and unreliable clock movements that are used at present to drive graphic meters, maximum demand recorders, and similar devices. It is well known that these various kinds of meters require a large amount of attention to keep them wound, regulated and in repair. This is particularly true of the maximum demand recorders, which are distributed widely over large systems and thus require a great deal of traveling to keep them in operation. Substituting motors does not of course eliminate all attention, because it is still necessary to change the record sheets occasionally; but in many cases the recording time can be extended either by using larger paper rolls or a slower feed, and in all cases the attention for winding and frequent regulating is eliminated. It might be expected that these meters would, when motor-driven, give much more accurate records, and this has been found true. In the first place, the motors are far more powerful than the clocks and they are not affected by the varying friction of the paper feed. Therefore it has been found feasible to make better records by using more tension on the paper. Frequently such recording devices are used to analyze the cause of disturbances, and in such cases it is of great importance that the records should be synchronous. This is impossible where ordinary clock movements are used. Where there is a difference between two records, there is always more or less doubt whether this is due to an error of the clocks or not; but where synchronous motors are driving the recorders there need be no uncertainty in this respect. On account of the very small size of the motors shown in Fig. 2 it has been found easy to substitute them in all kinds of recorders where spring-driven clocks have been used. Fig. 5 shows a complete movement driven by one of these motors, suitable for a maximum demand recorder. There are certain instruments in which periodic contact making is required, and in these, this motor is equally well adapted to replace the clocks. Fig. 6 shows a printometer equipment whereby a circuit is closed for a fraction of a second every fifteen minutes. This interval is as exact as the regulation maintained by the master clock at the power station.

There is another class of instruments which have not come into as general use as they deserve, very largely because of the objectionable features of spring-wound clocks. These are time switches used to control sign lighting, street lighting, and for other purposes. Several forms of time-switch operated by motor have been designed which are in many ways more satisfactory than the ordinary types of time switches. Fig. 7 shows one of these switches with a capacity of 15 amperes designed for controlling street lights. The outside dimensions of this switch are as follows: height 4¾ inches width 2 7/8 inches depth 5 5/8 inches. It will be recognized that this is very much smaller than any kind of clock-driven time switch; it is in fact so compact that when encased it may be placed inside the iron pole of a street light. The advantages of such a switch are obviously the elimination of the winding and regulating, which have always involved a great deal of attention.

In the field of pure timing devices, the most important use of the motors is probably the driving of ordinary clocks. In some respects this is the easiest adaptation of the motor, because the power required is exceedingly slight and the necessary mechanism comparatively simple. As an illustration of the small power output needed, Fig. 8 shows two clocks, the one above with a 2½-in dial and the one below with a 36-in. dial, the latter a comparatively large clock for indoor use. Nevertheless the tiny motor which is also shown in the photograph is equally adapted for driving either of these clocks. Indeed, this small motor can drive still larger clocks. In order to indicate the almost inconceivably small power required to drive the hands of a clock through well-made gearing the following measurements were made:

Diameter clock dial	Hands	Horse power required with ample margin for safety
4 inches	unbalanced	0.0000000017
8 "	unbalanced	0.0000000022
12 "	unbalanced	0.0000000027
14 "	balanced	0.0000000014
Safe output of motor at synchronous speed		0.000001

These tests were made by measuring that torque which was more than ample to drive the train of gears and the hands in their most strongly resisting position. From these figures it is clear that any moderate size clock requires an amount of power which is negligible as compared with the output of one of these small motors.

In the driving of clocks it is necessary to make provision against interruptions in the supply of current, which would of course show as lost time on the dials, owing to the temporary stoppage of the motors. In a large modern power system, especially in the neighborhood of big cities, the interruptions are infrequent and generally of short duration, but they must be reckoned with. Two different devices are provided to safeguard these motor driven clocks against interruptions in the current. The simplest of these devices consists of an indicator which can have two positions. So long as the current flows continuously the indicator is not apparent on the face of the clock, but after an interruption lasting more than a few seconds this indicator appears on the dial and remains there even after the current is restored, thus showing and stating that the clock should be reset. For a brief interruption of the current the indicator, after appearing momentarily, returns to its hidden position. The device is magnetic and exceedingly simple and reliable.

The other means for guarding against interruptions consists in providing an auxiliary spring driven movement connected by differential gearing to the hands of the clock. This movement is at rest so long as the current flows through the motor, because its pendulum of magnetic material is prevented from vibrating by the field magnet of the motor. When an interruption occurs however, the pendulum is released so that the spring driven movement can drive the hands for the time being. Just as soon as the current is restored the pendulum is again stopped and the hands are driven through the motor. The spring movement is capable of driving the hands of the clock for 24 hours. It has not been deemed necessary to provide a very accurate auxiliary movement in these clocks, because it is not expected that the current will be interrupted for long intervals, and the error during a short interruption is very slight even if the auxiliary movement does not keep particularly accurate time. As shown by the illustration of the works for the auxiliary type in Fig. 9, this movement occupies very little room, and can be enclosed in any ordinary case. It is expected that as their use becomes widespread, these clocks will be available in all kinds of cases and at prices which compare favorably, considering their advantages, with common spring-driven clocks. After trying them people quickly realize their great advantage as compared with all other kinds. These advantages may be summed up briefly: Extreme simplicity; elimination of winding and of regulating; far greater precision in time-keeping; the ability to get time service from any lamp socket; low cost of maintenance; the avoidance of expensive and troublesome local master clocks and local independent wiring systems; the elimination of batteries, charging sets, relays, etc; and the avoidance of contact mechanism in the clocks themselves.

The service which these clocks give is so valuable in comparison with the cost of the current which they consume that their use can be extended far beyond the field that can be economically covered by existing systems for distributing time. Barring interruptions, the accuracy of time-keeping of the clocks is measured by the care given to the master clock at the power station. On the Boston Edison system where these clocks have been in use for about 2½ years, the service is so good that the clocks seldom vary more than 5 or 6 seconds from the true time. This is a degree of precision which is closer than can be read unless they are provided with second hands. Moreover, this remarkable quality of service really requires, as has been stated before, less attention from the station attendants than had previously been given where frequency meters were used for regulating purposes. The clocks are not absolutely noiseless but they are quieter than any ordinary clock which ticks. It is hoped that ultimately the very slight humming sound which they emit can be entirely suppressed, but this is to most people practically unobjectionable.

The clocks are not affected by temperature changes unless subjected to cold so intense as to thicken the lubricating oil. Different kinds of oil are used for different kinds of service. In some cases ordinary kerosene oil has been employed. This will enable the motor to run at very low temperature. The lubricating qualities of the oil do not seem to be of importance, because the friction of the moving parts is so exceedingly slight. Variations in voltage do not affect the operation of the motors in driving the clocks. A drop in potential of 50 per cent will not trouble the time-keeping qualities of the motor; but with auxiliary movements some difficulty has been experienced when the potential falls to a point where the pendulum of the auxiliary movement is released or permitted to vibrate while the motor also continues to run. In such a case the clock tends for the time being to gain. This could rarely happen in a first class distributing system. The objection does not apply to the indicating type of clock.

The field magnet winding for the motor is so well insulated layer by layer and from the ground that it will stand severe surges of potential due to lightning discharges. The windings are usually designed for 110 volts but potential above 2000 volts has been applied briefly in tests without causing damage. The fire risk of the clocks is evidently negligible and they are approved by the Underwriters' Bureau.

An important use for these clock motors appears to be the driving of time recorders, cost keeping devices, time stamps etc., for offices and factories. There are thousands of such appliances in manufacturing establishments which at the present time are operated by spring driven clocks or in some cases electrically by impulses from local master clocks. Spring driven clocks are objectionable because they require winding and regulating, and where a number are used in the same factory they cannot be depended upon to remain in synchronism. The electrically driven recorders installed hitherto are objectionable on account of the great expense of installation which requires wholly independent wiring and the trouble and cost of maintenance of batteries, charging devices, etc. Because of the ability to connect with the ordinary lighting wires of a factory, these time recording devices may, if driven by the new motors, be located almost anywhere without extra cost. The entire burden of winding and regulating and a large part of the cost of maintenance of the system are eliminated. It is in many cases feasible to attach the new motors to the old movements by a simple device such as a closely wound spiral spring serving as a coupling as shown in Fig. 10.

The indications are strong that as the district supplied by alternating current with accurately controlled frequency extends and this new service becomes ultimately available everywhere, there will develop many new and unforeseen uses for these small self-starting synchronous motors. Combining the functions of a wonderfully accurate clock with those of an exceptionally simple and reliable electric motor, a new instrumentality is available which can be adapted to cooperate with many kinds of existing devices and lead to the development of others for entirely new purposes.

Presented at the 35th Annual Convention of
The American Institute of Electrical Engineers,
Lake Placid Club, N. Y., June 24, 1919.

COOPERATION

PRESIDENT'S ADDRESS

BY C. A. ADAMS

INTRODUCTION

SINCE the last Annual Convention of this Institute the greatest war in the world's history has come to an end. Millions of lives have been sacrificed. Millions of men have been crippled in greater or less degree. Wealth to the extent of many billions of dollars has been destroyed. Most of the nations involved have been seriously impoverished. The whole social, economic and industrial order has received a severe shaking up and has in some countries been completely overturned. Widespread dissatisfaction with the old order prevails in nearly all of the countries involved in the war. Unrest is generally prevalent, and, however unreasonable the demands of the restless may seem, there probably *is* a reason. In any case "it is a condition not a theory that confronts us."

The old order has failed and can never wholly return. What will be the new order? What are we going to do about it? Are we as engineers, educated men, who pride ourselves on our breadth of vision, who complain because we are not properly recognized in large questions affecting the public welfare, going to crawl back into our little cabins and leave the steering of the ship to those we consider incompetent? Are we going to use our influence to keep the ship on the old course, in spite of the fact that it led to the rocks? Or are we going to ask the same question we ask when our engineering plans result in failure, namely, why? Are we going to scrutinize the ships' log to see why she ran on the rocks and then revise the sailing orders accordingly?

These are some of the questions, the consideration of which has urged me to deviate somewhat from the custom established by my predecessors, of discussing either Institute activities and policies or general engineering questions, in order to present a much broader question, of interest to us not only as engineers but also as citizens, a question which has repeatedly arisen in connection with my work for the Institute during the past two years.

Moreover this year the important question of Institute activities and policies is receiving a thorough discussion through our Development Committee before which we will all have an opportunity to express our views at this Convention.

The subject of *cooperation* to which I wish to call your attention has a decided bearing on several important Institute functions, and although it will be discussed largely in connection with these functions, it is the principle involved and its broad bearings on other questions as well, that I wish to emphasize.

My reason for selecting this subject is partly because of my recent experience in applying the principle, but more because the broad application of the principle seems to offer at least a partial answer to the questions propounded above, and also because this application is of peculiar interest to the engineer.

I am not proposing to cast aside the traditions of our fathers, nor the experience of the past, but rather to make use of that experience.

I do not propose any revolutionary program, but merely the hastening of the natural evolutionary development by the application of a little perspective and common sense. This does not involve altruism except insofar as a vision reaching somewhat beyond the end of ones nose may be considered altruistic. It is merely a step forward, a step of progress, which may prevent our being forced into some revolutionary stride which is doomed to failure and can mean only destruction and disaster as long as it endures.

COOPERATION IN GENERAL

The history of human society is the history of cooperation, at first between the individuals of a small group such as a family, then the cooperation of the small groups with each other to form a larger group such as a tribe, then of these larger groups to form a nation, then of several small nations into one larger nation, and now we are seriously considering a League of Nations.

The reason for these various increases in the size of the unit, is not an altruistic ideal, but the knowledge that each party to the cooperation will gain thereby, not only in strength, but also industrially and economically through a more extensive division of labor and an increase in the average productivity per unit of labor.

Every commercial transaction of the greatest or of the least magnitude, between individuals or groups of whatever size, is made on the assumption that it is profitable to both parties concerned, otherwise it would not ordinarily be made. Every such transaction involves a certain degree of cooperation, a certain degree of faith or confidence in each by the other. The more the mutual confidence and the cooperative spirit, the freer and the more efficient will be the interchange and the greater the mutual profit. This is merely the simplest possible illustration of cooperation to show its fundamental nature. Industrial cooperation from which my illustrations are drawn and with which engineers are mostly concerned, usually involves more than this simple and somewhat restricted cooperation between buyer and seller, it involves cooperation all along the line, internal and external, between producers, between consumers, and between producers *and* consumers.

The obstacles to cooperation, particularly between the larger units, are of two varieties, material, and human or intellectual. The chief material obstacles are space coupled with imperfect means of communication and transportation, differences in language, differences in coinage, weights and measures, and duties or tariffs. Some of these bear obviously only on international cooperation. The chief intellectual obstacles are tradition, custom, prejudice, suspicion, distrust, jealousy, narrow minded and short sighted selfishness or greed, or in general ignorance and lack of understanding one of the other. Most of this latter group bear upon minor or internal as well as upon international cooperation.

Most of the obstacles of both types may be looked upon as barriers which keep individuals or groups apart and thus prevent that mutual understanding which makes cooperation possible. It is my desire not only to point out the importance and value of cooperation, but also the part that we as engineers can play in helping to break down these barriers. My meaning can most easily be made clear by illustrations.

Standardization. Consider first the work of engineering and industrial standardization. The work of the Standard Committee of this Institute is second to none in its own particular field or in any other, either in this country or in any other country, in spite of the fact that the ground covered is a peculiarly difficult one. Why this success? Simply because of the *cooperative* spirit which has prevailed. Conflicting interests come together, pool their experience, share the work, place their cards on the table face up, and finally agree to abide by the resulting rules.

Does any one of these interests regret the results? Does any one feel that it has been a waste of time and money, that it has been chasing an illusory ideal, that it was visionary or impractical, that it was not a financially profitable investment in the long run?

But it is not only profitable to the producer, it is even a greater gain to the consumer by a reduction of cost, the elimination of at least some of the confusing and conflicting claims of competing salesmen, by rendering more definite the purchasers knowledge of the capacity of a machine of given rating and thus making possible a more suitable and accurate selection of a machine for a particular purpose. Thus it is an asset of no mean value to the industry and to the nation as well.

But this cooperative spirit does not always prevail, in fact rarely in the degree found in our Standards Committee. Until very recently the French National Committee of the International Electrotechnical Commission contained none of the designing or manufacturing engineers, it being assumed that the latter would be biased and partisan in their attitude. As a result the manufacturers formed a syndicate with a committee and standardization rules of its own, which were and even now are to some extent the rules generally employed in France although much inferior to our own. In some cases the American rules are specified. Even now the French rules originate in the Manufacturers Syndicate and are passed on to the French National Committee for their approval, with a consequent loss of time and efficiency.

As the work of our own Standards Committee expanded we found it overlapping the work of other Societies in adjacent fields, and cooperative relations were established. This failed from the lack of any suitable and generally acceptable cooperative machinery. A movement was then started by the Institute more than two years ago to organize a central body (the American Engineering Standards Committee) to coordinate all the work of engineering standardization being done by the various engineering organizations. The objects of this organization are to avoid overlapping, duplication of effort and the confusion of conflicting standards; to ensure the employment of an effective and sound procedure in the development of standards, to the end that a standard when promulgated may be not only workable but also acceptable to all concerned, and the only accepted standard of its kind; to stimulate desirable standardization and at the same time to shut off premature or ill-advised attempts at standardization.

The consummation of these objects will mean: an enormous saving of effort in the production of standards, a tremendous increase in the value and usefulness of standards, and, when the work is brought up to the needs of the time, an incalculable saving in the cost of production and the productivity of labor. International standardization will be a tremendous stimulus to foreign commerce; but such international standardization is in most cases exceedingly difficult without a central authoritative body with which similar foreign bodies may cooperate. Several instances of this difficulty which have actually arisen during the past two years could be cited, if time permitted. These have resulted in hopeless confusion, misunderstandings, and waste of time and money.

Great Britain has its Engineering Standards Association which, during its eighteen years of life, has demonstrated its reason for existence and its value to industry and to the Government. Its work in air craft standards alone, which it is doing on behalf of the Government, involves some fifty odd subcommittees.

France now has its Permanent Standards Commission a little more than a year old, and just getting under way.

Holland also has its Normalization Bureau for the same purpose, and Switzerland is just starting a movement in this direction.

With all these National Bodies our American Standards Committee has already established cordial cooperative relations.

The American Engineering Standards Committee, although organized last October is now in process of reorganization. But the success of any such movement is in direct proportion to the degree in which the cooperative spirit prevails, and the two years consumed before the first organization, and the present delay in the reorganization is due largely to some of the obstacles to cooperation mentioned in the second group above.

The conflict is chiefly and quite naturally between the desire to accomplish a highly desirable result and the desire on the part of each cooperating element to avoid paying the price, which may consist in a very slight loss in independence or prestige, although in the present case this will be negligible. It is the fear rather than the fact that causes the trouble. But even if the sacrifice were much more than it is, the price would be small in proportion to the result.

It is the generous cooperative spirit for which I am pleading, rather than the timid fearful spirit of him whose vision is of such short range that he dare not take a man's size step for fear of stubbing his toe, who holds the little present so close to his eye that it shuts out the whole landscape of the future, whose immediate self interest or small group interest is so dominant as to hide anything beyond.

Look around at the strikingly successful men of this country; rarely will you find one of the timid fearful type. He has vision even if sometimes of a very material nature; he sees a worthwhile goal some way ahead and starts for it; if he makes mistakes he tries again with added experience. The man who never makes a mistake never gets far. I am not advocating slap-dash methods, but rather more of the positive and less of the negative attitude, more constructive and less destructive criticism, more of the forward looking progressive spirit, and less of the visionless, timid, reactionary spirit.

To return for a moment to the American Engineering Standards Committee and its reorganization, may we not look to every member of this Institute to take part in a campaign of education and in the development of that wholehearted spirit of cooperation which will mean the support rather than the hindrance of many well meaning but timid objectors to this movement, which seems to promise so much in the development of our industries, in the increase in the productivity of labor, and in the breaking down of at least one of the barriers of international misunderstanding.

Research. Another fertile but as yet little cultivated field for cooperation is that of *industrial research.* Here we have a field where the obstacles of the second group are specially dominant, particularly short sighted self-interest and corporate interest, and the timid desire for secrecy.

That many of the most important and revolutionary developments of modern industry are dependent upon research needs no proof. It is not so obvious to many however that cooperation in industrial research is to any large extent feasible at the present time.

First let me point out that the line often drawn between scientific and industrial research is a very hazy one, namely between research the purpose of which is merely to extend the bounds of human knowledge and that intended primarily to meet an industrial need or to solve an industrial problem. In other words the difference is one of purpose rather than of method. But I think that, particularly since the war, most of our so called pure scientists have become greatly interested in the industrial value of their scientific work. Therefore when I speak of industrial research I include a very large part of the research work now being conducted in this country.

Consider for a moment the advantage of cooperative research Few individual corporations except the very large ones can afford either a suitable research laboratory or competent men to conduct the work. The result is that most of the research work is superficial and much of it misleading. The total cost is many times greater than of a comprehensive cooperative research led by the foremost experts in the particular field. Moreover some of the most vital and fundamental researches are either beyond the capacity of even the largest corporation laboratories or have to be postponed for those offering more chance of immediate returns.

I am fully aware of the difficulties arising from patentable developments, but these can readily be handled by suitable agreements between the parties to the cooperation.

After some experience in this field I am convinced that thorough going cooperation in industrial research would mean not only a reduction of the cost of research to a small fraction of its present figure, a much more rapid industrial development of the country as a whole and a material increase in the productivity of labor, but also an increase in the profits of every intelligent party to the cooperation.

Again I wish to point out that it is not so much a question of altruism as of a far sighted self-interest and patriotism.

Many other illustrations of cooperation in the engineering field could be cited, but those already briefly discussed will be sufficient for the present purpose.

Reverting now to the opening paragraphs of this address, let us consider some of the broader aspects of cooperation, particularly as they bear upon the great world problems which are insistent in their demand for solution.

Serious cooperation of any kind between widely separated groups is practically impossible without the mechanism of transportation and communication provided by the work of the engineer. Also with this mechanism developed even to its present degree of perfection, cooperation becomes not only possible but also desirable and necessary, even between comparatively widely separated groups. Moreover the problems which confront the world at this time are largely a direct result of this mechanism. Are we as engineers going to leave their solution to others? Are we going to be satisfied to play alone in our own little backyard and never look around us even after most of the fences have been removed by our own work. Are we going to fail to take an interest in our neighbor who has much that we need, from whom we have much to learn, and by cooperation with whom we can profit broadly? If we do offer cooperation is it to be with the provision that we gain all and sacrifice nothing? Are we going to be satisfied to sit back and criticize destructively the details of a cooperative agreement, priding ourselves on our critical faculty, however valuable that may be at times, when the real basis of the criticism is either intellectual pride or short sighted fear that we, as individuals or as a group, may lose a little of our independence or prestige; when the issue at stake is in some cases of such paramount importance (had we only the vision to see it) as to overshadow our very existence.

Let us rather make use of the logical habits of thought and freedom from bias which *should* at least be the result of our training, and take our part in the solution of the problems we have helped to create. Let us see the goal ahead and start for it without thought of fear or favor, as surefootedly as possible, but with that generous cooperative spirit which will surmount many an apparently impassible barrier, critical of details if so minded, but generously and constructively so.

Now just a word as to a specific and very serious problem, namely, that arising from the world-wide unrest in the labor group. That this condition exists must be admitted by the most optimistic. To ignore it is suicidal. It is difficult to characterise this condition in a few words as there are so many degrees of radicalism and of revolutionary spirit represented; but it may be safely stated then on the average, labor is unreasonable in its demands and seems to fail to realize that it cannot receive more wealth than it creates, or at best it has an exhorbitant idea as to the value of what it creates. For example in England labor is not only demanding higher wages but according to all reports is reducing the hourly output as well. There seems to be a prevailing notion in labor circles that the excess profits of the capitalist constitute a limitless supply which only needs the tapping; or to put it in another way, the labor group is trying to use its organized power as a monopoly, and to insist on "all the traffic will bear". The revolutionary element goes even farther.

It is very easy to criticise and condemn this attitude because of its unreasonableness, looking down from our superior height of intelligence. It is very easy to assume that it is merely a matter of showing the laborer the error of his ways and of forcing him into line. But would it not be wiser at this time to examine our own attitude with equal care, to make sure that we are setting an example which if followed by labor would solve the problem? Is demanding all the traffic will bear under the pressure of organized labor any worse than the same policy carried out by the capitalist whenever he has the monopoly grip. Unfortunately the two cases are not treated the same by our laws; labor monopoly is not subject to the same limitations and responsibilities as is capital monopoly; that disparity should be removed. But there are two forms of monopoly other than of labor which are not only not limited by but are actually fostered and protected by our laws, and in which the only limitation ever dreamed of by the owner is, "all the traffic will bear"—These are patents and titles to land.

Our patent system has a worthy motive, but who will claim that the commercial value of a patent bears any relation to the brains, industry and expenditure that entered into its development, or that in general the patent system works for justice to all those who share in the development of a patent. It is often a matter of pure accident as to which of two men who have conceived the same idea at about the same time reaps the reward, and it often goes to the less worthy. Often the *originator* of *an idea* makes a much less important contribution than the man who makes it a commercial product. But to point out the defects and the injustices of our patent system would take all day. This is another realm in which broadgage cooperation would eliminate much of the unfair and gambling aspects of the patent system, and distribute the rewards in a much more equitable fashion.

Of all sources of monopoly our present land system is the most unfair, the most dangerous, the most far reaching in its influence. Under it a land owner is paid a premium for holding valuable land idle, for forcing a community to spend its money to build around his land and to take its time to travel around or past his land. The sight (or unimproved) value of land in this country constitutes about one-half of our national wealth, yet the landowning class as a group taken from the beginning of the settlement of the country, have been made a free gift of that wealth, which is the product of the industry of society as a whole. This is a statement which cannot be controverted.

Private ownership of land seems to me desirable, it has a wholesome stabilizing influence, and the injustice of our present system can be largely removed without sacrificing any of the real advantages of private ownership, by shifting taxation from individually created wealth to community created wealth which is the sight value of land *i. e.* by taking for the community what the community creates and leaving to the individual what he creates, by shifting taxation from industry to privilege. But this is a long story by itself and I touch upon it at this time only as one of a group of suggestions as to possible ways in which we can assist in progressive rational *evolution* of society rather than encouraging and inducing a *revolution* by a reactionary policy. I assure you that none of these suggestions can be called other than conservative at this time.

Are we sincere in our desire to be more recognized in our broader capacity as citizens and to be consulted in matters of broad public interest; if we are not sufficiently interested in those matters to make an intelligent study of them; if we are satisfied to accept without analysis the prejudices and propaganda of a biased group, however intellectually superior that group may consider itself; if we cease to utilize our logical habits of thought, our habit of analysis, of demanding the reason why, as soon as we step outside of the realm of those physical laws which force us to think straight? Are there no laws in this other realm of human relations which are just as inexorable as the physical laws with which we are so familiar? Is there no law of compensation which is the counterpart of our law of conservation of energy? Are we going to be allowed to go unpunished for crooked or careless thinking in this other realm any more than in the purely physical realm?

What can we expect of labor in the way of intelligent fairmindedness if we, with our superior advantages encourage the perpetuation of institutions which work for serious injustice to the many for the benefit of the few, or even if we allow them to persist because of our lack of intelligent fairmindedness.

My opinion as to the seriousness of the present world situation, as to the importance of the problem before us, is not mine alone but that of many thinking men in this country and in others. I have talked with many influential men in France and England and have found a striking unanimity on this point.

May we not as engineers utilize our training and experience, our method of thought of which we are so proud, to encourage clean, straight, unbiased and fairminded thinking, and to encourage that broad generous cooperative spirit which will mean so much in breaking down the barriers which separate classes within a nation, as well as the nations themselves, and which is essential to the solution of the world problem which is knocking so hard at our door?

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TRANSMISSION LINE RELAY PROTECTION

BY H. R. WOODROW, D. W. ROPER, O. C. TRAVER AND P. MACGAHAN

FOREWORD

As a result of the large development which has been made in the relay practises in the last few years, the Protective Devices Committee felt it was desirable to make an analysis of the transmission line protective relay situation throughout the country and prepare a summary to bring before the profession, experiences of the operating companies with this protective apparatus. With this in view questionnaires were sent out to sixty-one operating companies asking for their experience and present practise in regard to relay protection.

Replies were received from 32 of the operating companies and from these the following general analysis was made of the practise of relay protection for transmission lines.

The nomenclature used by the different operating and manufacturing companies for the various classes of relays is not in harmony and the Protective Devices Committee undertook the work of standardizing the nomenclature which it is hoped will be adopted by all parties so that any particular class of relays will be given a definite name. The following nomenclature has therefore been proposed by the Protective Devices Committee and this nomenclature is used throughout this paper.

PROPOSED RELAY NOMENCLATURE

Electric Protective Relay. An intermediate device by means of which one circuit is indirectly controlled by a change in conditions in the same or other circuits. The relay is ordinarily equipped with contacts to open or close an auxiliary circuit.

Directional Relay. Any relay which functions in conformance with direction of power or voltage or current or phase rotation, etc.

Power-Directional Relay. Any relay which functions in conformance with direction of power.

NOTE: This includes both uni-directional relays with single-throw contacts and duo-directional relays with double-throw contacts. The reason this name is preferred to "reverse power" is that the device is frequently used to function under normal direction of power. Furthermore, in some cases the normal condition of the system may permit power to flow in either direction. Relays for use in either alternating or direct-current circuits are to be classed as power-directional relays.

Polarity-Directional Relay. Any relay which functions by reason of a reversal of the normal direction of polarity.

Phase-Rotation Relay. Any relay which functions by reason of a reversal of the normal direction of phase rotation.

Current Relay. Any relay which functions at a predetermined value of the current. These may be either over-current relays or under-current relays.

Voltage Relay. Any relay which functions at a predetermined value of the voltage. These may be either over-voltage relays or under-voltage relays.

Watt Relay. Any relay which functions at a predetermined value of the watts. These may be either over-watt relays or under-watt relays.

Frequency Relay. Any relay which functions at a predetermined value of the frequency. These may be either over-frequency relays or under-frequency relays.

Temperature Relay. Any relay which functions at a predetermined temperature in the apparatus protected.

Open-Phase Relay. Any relay which functions by reason of the opening of one phase of a polyphase circuit.

Differential Relay. Any relay which functions by reason of the difference between two quantities such as current or voltage, etc.

NOTE: This term includes relays heretofore known as "ratio balance relays", "biased" and "percentage differential relays".

Locking Relay. Any relay which renders some other relay or other device inoperative under predetermined values of current or voltage, etc.

Trip-Free Relay. Any relay which prevents holding in an electrically operated device such as a circuit breaker while an abnormal condition exists on the circuit.

Auxiliary Relay. Any relay which assists another relay in the performance of its function and which operates in response to the opening or closing of its operating circuit.

Signal Relay. An auxiliary relay which operates an audible or a visible signal.

QUALIFYING TERMS AS APPLIED TO RELAYS

Notching. A qualifying term applied to any relay indicating that a number of separate impulses are required to complete operation.

Inverse Time. A qualifying term applied to any relay indicating that there is purposely introduced a delayed action, which delay decreases as the operating force increases.

Definite Time. A qualifying term applied to any relay indicating that there is purposely introduced a delayed action, which delay remains substantially constant regardless of the magnitude of the operating force. (For forces slightly above the minimum operating value the delay may be inverse).

Instantaneous. A qualifying term applied to any relay indicating that no delayed action is purposely introduced.

Where relays operate in response to changes in more than one condition, all functions should be mentioned.

Present Day Practise of Transmission and Tie Line Relay Protection

THERE was a time when the protection of an electrical transmission system with the single generating station and its radial feeders was not a matter of very grave importance. That time, however, has passed and the multiplicity of generating stations with tie lines and parallel feeders have greatly complicated the once simple problem. Furthermore the practise of inter-connecting large transmission systems has brought about conditions which require very serious consideration when laying out a protective relay scheme. An effective sectionalizing relay scheme is necessary in order to secure both continuity of service and maximum efficiency of transmission, and the most complicated system can be effectively relayed if all the conditions are thoroughly analyzed and proper relays, carefully set for both time and current values, are applied.

Most of the large systems of the present day have grown out of a comparatively small beginning, stations and lines being added as required, without taking into consideration the relay problem. As a consequence this problem has, in a great many cases, become rather difficult and it has become evident that the application of protective relays for sectionalizing trouble is one of the principal features to be considered in determining the design of a transmission system. This point should be borne in mind whether additions are being made to an existing system or whether an entirely new system is being designed. Numerous instances could be given, if space permitted, whereby a slightly different design in extending lines and adding stations would have given a much more simple and effective protective scheme.

The same argument applies to the design of an entirely new system, particularly if it is likely to become very complex, and in so far as practical, possible extension should be considered in the original layout so that various methods of protection can be studied and their advantages weighed against possible disadvantages of a design to which they can be applied. It will be found in practically every case, that one of the many sectionalizing schemes now in use can be combined with a proper layout of stations, transmission and tie lines to give proper selective action. Thus the many difficulties encountered when attempting to apply relays to a system laid out without regard for its protection may be avoided. It is therefore of paramount importance that the selective features of various types of relays be thoroughly analyzed before adapting them to any particular system.

The most perfect relay system cannot be expected to give the best results unless the man in charge of its installation and operation be thoroughly competent and familiar with the various precautions to be taken. It is absolutely essential that he know the characteristics and peculiarities of the relays as well as the scheme he is using. Results of experience have shown that an experienced man will secure more satisfactory results from an inferior type of relay than a less capable man can secure from the best equipment obtainable.

In the early days of the development of electrical transmission systems it was the rule to operate all lines radially so that a much simpler and less accurate relay was sufficient for the service required. But when an attempt was made to operate these lines in parallel it became apparent that a more reliable and accurate relay should be used. It was then that the phenomenal relay development which has taken place in the last few years was begun in order to meet the requirements of the operating companies with their various designs of systems. In order to secure the isolation of the proper feeder in case of trouble on these different systems, it has been necessary to bring out designs of several relays.

In the early days it was the practise in applying protective relays to set them in such a way as to prevent overloading the feeders. However, experience has shown that it is not practical to set relays in this way and expect them to give proper selectivity under heavy short-circuit conditions. For this reason the tendency now is to set the relays for short-circuit conditions to disconnect a defective feeder from the system, and depend on the operator to prevent overloading of feeders. Experience has shown that the overloading of a cable for a reasonable length of time is not as serious a matter as it was once considered to be, and it is sometimes desirable to load a cable beyond its rated capacity in order to prevent interruption of service to important customers.

One of the principal factors controlling the settings of relays is the amount of short-circuit current available under various conditions as outlined later in this paper.

The proper type of relays to be selected for protecting transmission lines is dependent upon whether the system is operated radially, in parallel by groups, or in loop, and the time allowable for holding a short circuit on the system. In general, however, the most simple form of relay protection where several parallel feeders are involved is the straight inverse-time current relays which are set to give times of operation which are progressively longer as the source of power is approached. In some cases, however, where an extended system is considered, the time requirements for the relays approaching the generating station are so long that it is considered dangerous or inadvisable to hold short-circuit currents on for this period. We believe a delay of two seconds to be the extreme time a dead short circuit should be allowed to remain on a system, except under very unusual conditions.

The introduction of directional relays at points where power is feeding into a station will materially reduce this maximum time setting as these relays can be given a short time setting as outlined later under the heading "Typical Applications of Current and Directional Relays."

Another method of shortening this time is by the use of the balanced system in which case current can only flow through the relay at time of short circuit as outlined later under the heading "Balanced Protection Schemes."

Another method has been developed for shortening this time by the combination of an under-voltage relay with the inverse-time-current relay as outlined later under "Combination of Under-voltage and Over-current Relays for Protecting Transmission Lines."

For relay protection on tie lines, between generating stations in general, the conditions are somewhat similar to the straight transmission line relay protection if the additional precautions are taken to prevent operation on heavy momentary surges of power that are likely to exist between two or more generating stations at time of synchronizing or momentary disturbances on the system. The relay desired here is dependent to a large extent upon the method of system operation, as in some cases an operating company will want the tie lines to open, in case of trouble on a portion of the system, whereas other companies want these tie lines held rigidly in service in all cases excepting in case of trouble on tie lines themselves. In the first case shorter time settings are given to the relays and the application of straight inverse time current relays with the possible application of directional relays, is all that is required. In the second case, however, application of some form of a balanced relay, as described later may become necessary. If the tie lines are not to stay in indefinitely, inverse-time current and directional relays with long time settings would give a very suitable operation and allow the tie lines to open in case of more serious trouble on either side of the system.

SAVINGS EFFECTED BY A PARALLEL OPERATION OF FEEDERS

The savings which can be effected by a parallel operation of feeders depends in a large degree upon the design of the transmission system and the ratio of the capacity of the line to the capacity of the individual synchronous converters or transformers, which are supplied by this feeder. In one system supplying converters varying from 500 kw. to 4000 kw. in size together with step-down transformers in substations of 1500 and 3000 kw. capacity and also industrial substations on the premises of customers ranging from about 500 kw. to 4000 kw. in capacity, it was estimated that if the feeders could be operated in parallel then a saving of 20 per cent could be made in the amount of investment. As the installation in question had a book value of about $5,000,000, there was a possible saving estimated at $1,000,000. This company has been operating feeders in multiple for about two years. During this period they have realized nearly 40 per cent of the possible saving in the investment in feeders, and this has been secured by an actual reduction in the number of feeders, notwithstanding a considerable increase in the maximum load.

The same company also reports in the four years preceding the installation of the relays permitting parallel operation of feeders that they averaged 27 burn-outs per year which could not be definitely ascribed to external causes, while in the two years since the feeders have been operated in parallel, the corresponding figure was 16 burn-outs. This would indicate a reduction of 40 per cent in the burn-outs of cables due to operation of the feeders in multiple, although the cables were actually more heavily loaded. While the time is rather too short to accept these figures as final and conclusive, it apparently indicates that the operation of feeders in parallel reduces the number of troubles due to internal causes.

TESTING AND SETTING RELAYS

In order to secure the best results from the relays, it is desirable to make a complete check on the current and time settings of each relay on the first installation and at least once every six months thereafter. Although the time for making these tests is determined to a large extent by the operating conditions, we feel that the best results will be obtained if a general plan is worked out for performing these tests at regular intervals.

If selective operation is to be obtained from the relays the greatest care and precautions must be taken in making these tests. The tests should be performed under conditions as nearly equal to the operating conditions as possible which makes it desirable to include the wiring and complete relay equipment in this test.

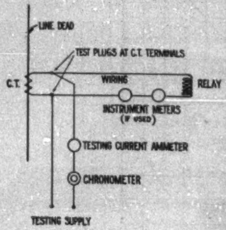


FIG. 1

In this test, wherever practicable, the test current should be applied to the primary of the current transformer. Where it is impracticable to do this the testing current may be supplied to the secondary terminals of the current transformer with the current transformer left in circuit as a shunt as shown in Fig. 1. The results obtained under these conditions very closely approximate those under short circuit and check the relay wiring and current transformer, as well as the relay. This will detect any short circuit in the current transformer or wiring and any open circuit in the wiring but open circuits in the transformer coils should be determined by a separate continuity test.

The degree of accuracy obtained when the relay is tested alone is dependent upon the type of transformer used and the secondary load. The error in this method is due to the high magnetizing current of transformers (especially with the low-ratio single-turn type on heavy secondary loading) on the heavy short-circuit conditions. As an example, a 40-volt ampere secondary load on a current transformer at full load imposes a 16-kv-a. load under short-circuit conditions of twenty-times full load. This, in many cases, is above the saturating point of the transformer and the ratio breaks down badly. The accompanying curves in Fig. 2 illustrate these conditions.

The time-current curve of a relay is affected by the nature of the secondary load or by the current transformer characteristics. It is therefore desirable to test each individual relay with current such as would be occasioned by short circuit. At least three points along the curve should be taken, one of which is approximately 150 per cent of the minimum value, one at as near maximum short-circuit current as the testing equipment will permit, and one intermediate point. The time should be taken by a chronometer or cycle counter which will eliminate the personal equation of the operator. The method of timing by a stop watch is entirely inadequate for close relay settings.

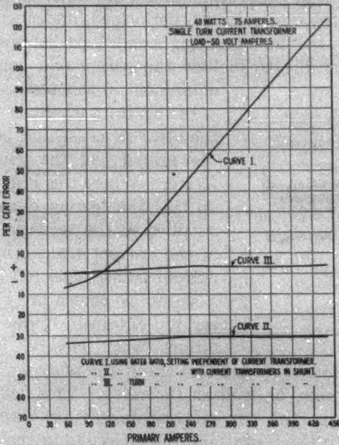


FIG. 2

An additional inspection should be made about every month for continuity of trip circuits (where this is not indicated by lamps) and for dirt or mechanical defects, and where conditions will permit, the relay should be operated to trip the circuit breaker. In case of bellows relays the bellows should be oiled with neatsfoot oil to prevent hardening.

DETERMINATION OF RELAY SETTINGS

For selecting the settings on a relay, it is necessary to analyze carefully the fault currents which will be obtained on the circuits in question under various operating conditions. The limitations on these conditions are usually determined by the number of circuits in operation and the maximum and minimum number of generators connected to the system.

It is desirable to have the relays so set that selective action will be obtained under all probable conditions. It is therefore necessary to check the relay settings against the minimum conditions to be sure that they will operate under minimum connected generating capacity and to also check, the curve for maximum short-circuit conditions and allow a sufficient time between the operations of the relays, to permit the opening of the oil circuit breaker and to cover inaccuracies in the relays themselves.

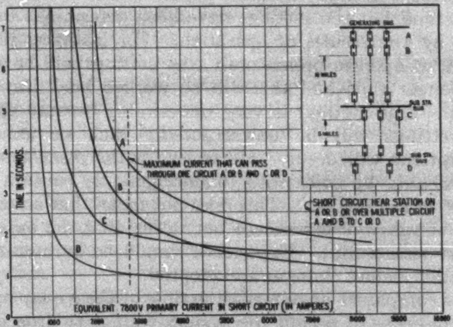


Fig. 3—Time-Current Curves of Inverse-Time-Current Relays

Where a large number of circuit breakers are in series and selectivity is desired between the several breakers, it is possible to apply, at the various points, relays having different characteristics in order to reduce the time that a short circuit is held on the system near the generating station. The time characteristic curves in Fig. 3 will illustrate a specific case where this method was adopted.

You will note from the curves that the maximum short-circuit current that can pass through relays *A*, *B*, *C* and *D*, is 2800 amperes and that the relays at this point are set selective with approximately ¾ of a second between each one successively. Greater currents may pass through circuits *A* and *B* with the short circuit on these feeders near the generating station, and the time characteristic curve being much steeper at this point will allow the relays to open the circuit very quickly. Although the currents through relays *C* and *D* may be much higher than 2800 amperes, these conditions represent times when more than one circuit (*A* and *B*) are operated in parallel and a proper selective action is therefore given with ¾ of a second between consecutive relays under any condition of operation.

The actual determination of short-circuit conditions is a rather difficult problem where a network is considered. Various formulas and calculating tables have been developed for simplifying this work which will give a sufficient accuracy for practical purposes.*

The relays in many systems have not been set according to this method but the settings have been determined to a large extent by the results of operation. For instance, if a relay appears to operate prematurely, the settings will be adjusted for a longer time and this will be worked back through the system. This method is very undesirable because either the final settings will be unwarrantedly high or unnecessary interruptions will occur. Moreover, in the meantime results will probably be sufficiently discouraging to cause a lack of interest in the matter. This does not mean that advantage is not to be taken of experience but rather that the basis should be more substantial. Experience should be harmonized with calculation before changes are adopted.

Some companies have made the time of their relay operation longer in order to permit the system voltage to drop in value, thereby reducing the rupturing strains on the oil switches. We point out, however, that considerable precaution should be taken when this is done as the prolonged current may do more damage than the greater rupturing arc in the switch.

METHOD OF TRIPPING

The use of circuit-opening relays for a-c. tripping has not given very satisfactory results, particularly where severe short circuits are possible. It is very desirable to have a definite uninterrupted supply of power for tripping. This has brought into use in many stations the storage battery for this purpose, which has proved to be reliable even in very severe system disturbances.

G. E. Review, October 1916 and February 1919; *Elec. Journal*, July 1916.

Some companies have connected their control and tripping circuits to the excitation bus thereby obtaining the advantages of direct current, but sometimes this has led to considerable trouble due to the unstable conditions of this source, especially under a-c. short circuits. It has resulted occasionally in a complete disabling of the control system on account of abnormally high induced voltages on the excitation bus with single-phase short circuits on the a-c. generators. Great precaution should be taken against these conditions and for best results we feel that an auxiliary supply should be used if possible for obtaining at all times a constant power for the tripping circuits.

TYPICAL APPLICATION OF CURRENT AND DIRECTIONAL RELAYS

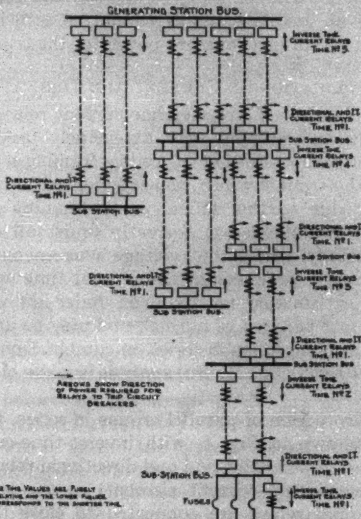
The combination of inverse-time-current relays with directional relays is becoming very extensively used in cases of parallel operation of lines or of feeders forming a ring system.

Some of the trouble which was experienced in the early days was mainly due to the difficulty in obtaining a directional relay which would operate correctly under all conditions of faults, especially where low voltage was encountered. The directional relays produced at the present time are apparently giving very satisfactory results, even under low voltage conditions down to 1 per cent of normal, and may be applied to loop or parallel group systems in series, in combination with inverse-time-current relays, if a careful analysis is made of the proposed installation.

For the protection of parallel groups in series, the relays are applied as shown in Fig. 4, with inverse-time-current relays placed at the transmitting end and directional relays (combined with an inverse-time-current element) at the receiving end. The directional relays are connected to close their contacts for power flowing from the bus to the line, and these contacts are placed in series with those of the inverse-time-current relays of the same circuit so that the circuit breaker is tripped only in case of power flowing in the direction away from the bus and of a magnitude exceeding a certain given amount for a certain time. Any time delay required is provided by the adjustments of the inverse-time-current relay element.

The settings should be made to suit each individual application and sufficient selective time leeway should be allowed to cover the time of operation of the oil circuit breaker, as well as the maximum probable error in the relay.

For protecting the loop system both the inverse-time-current and directional relays (with inverse-time-current element) are used as shown in Fig. 5. Each line, at each station, is provided with inverse-time-current relays. Where these inverse-time-current relays are given the lower time settings, directional relays are also used with the tripping contacts connected in series, so that both relays must operate before the circuit breaker is tripped.

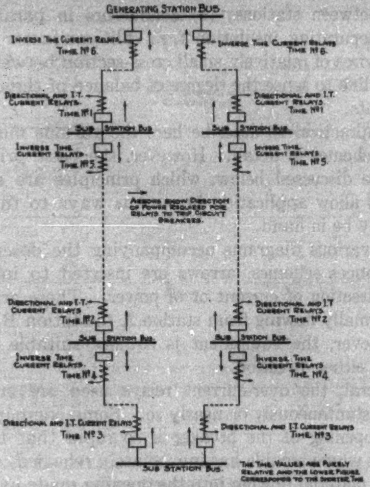

GENERATING STATION BUS.
INVERSE TIME CURRENT RELAYS TIME Nº 5.
DIRECTIONAL AND I.T. CURRENT RELAYS TIME Nº 1.
SUB STATION BUS.
INVERSE TIME CURRENT RELAYS TIME Nº 4.
DIRECTIONAL AND I.T. CURRENT RELAYS TIME Nº 1.
SUB STATION BUS.
DIRECTIONAL AND I.T. CURRENT RELAYS TIME Nº 1.
SUB STATION BUS
INVERSE TIME CURRENT RELAYS TIME Nº 3
DIRECTIONAL AND I.T. CURRENT RELAYS TIME Nº 1.
SUB STATION BUS.
DIRECTIONAL AND I.T. CURRENT RELAYS TIME Nº 1.
SUB STATION BUS
ARROWS SHOW DIRECTION OF POWER REQUIRED FOR RELAYS TO TRIP CIRCUIT BREAKERS.
INVERSE TIME CURRENT RELAYS TIME Nº 2
DIRECTIONAL AND I.T. CURRENT RELAYS TIME Nº 1.
SUB-STATION BUS.
THE TIME VALUES ARE PURELY RELATIVE AND THE LOWER FIGURES CORRESPOND TO THE SHORTER TIME.
FUSES
INVERSE TIME CURRENT RELAYS TIME Nº 1.

INVERSE TIME CURRENT AND DIRECTIONAL RELAYS AS APPLIED TO PARALLEL FEEDERS.

FIG. 4

Reference to the time settings given in the diagram (these are purely relative) will show that they are graded on the same basis as a single series of tandem connected substations, assuming that one end of the loop is open at the generator bus, and neglecting those breakers which would be incoming at the various substations. It should be noted that when the other end of the loop is considered open, the substation breakers which in the first case were incoming will now be outgoing and vice versa. The directional relays are also installed if the time of an incoming line is lower than an outgoing line further from the source. This must be checked separately for each end, in turn, considered opened at the generator station.

In order to secure the operation of the directional relays under bad power factor conditions, the connections are made so that the maximum torque is obtained with lagging power factor. This may be accomplished by several schemes of connections of the potential and current elements, which schemes may not be interchangeable with different types of directional relays. Some operators have found it advantageous when using directional relays with step-down transformers (particularly if not paralleled on the primary side) or reactors, to connect the voltage elements of the relays to the bus side of this apparatus in order to give a higher voltage in case of


Generating Station Bus.
Inverse Time Current Relays. Time No 6.
Inverse Time Current Relays Time No 6.
Directional and I.T. Current Relays. Time No 1
Directional and I.T. Current Relays. Time No 1
Sub Station Bus
Sub Station Bus.
Inverse Time Current Relays. Time No 5.
Inverse Time Current Relays Time No 5.
Arrows show Direction of Power Required for Relays to Trip Circuit Breakers.
Directional and I.T. Current Relays Time No 2
Directional and I.T Current Relays. Time No 2
Sub Station Bus.
Sub Station Bus.
Inverse Time Current Relays. Time No 4
Inverse Time Current Relays Time No 4
Directional and I.T. Current Relays Time No 3.
Directional and I.T. Current Relays Time No 3.
Sub Station Bus.
The Time Values are Purely Relative and the Lower Figure Corresponds to the Shorter Time

INVERSE TIME CURRENT AND DIRECTIONAL RELAYS AS APPLIED TO A RING SYSTEM.

Fig. 5

short circuit, thereby obtaining better operating conditions for the directional relays.

BALANCED PROTECTION SCHEMES FOR TRANSMISSION LINES

A great many operating companies have installed on a portion of their lines, protective equipment which is designed to be inoperative so long as the currents in different circuits or at different points in the same circuit are substantially equal. To this end, they have made use of (a) fully insulated lines in parallel between stations; (b) conductors in parallel having reduced separating insulation *i. e.*, split-conductor cable and (c) pilot wires of relatively small cross-section between the two ends of a line to sense the degree of balance of the currents at these points.

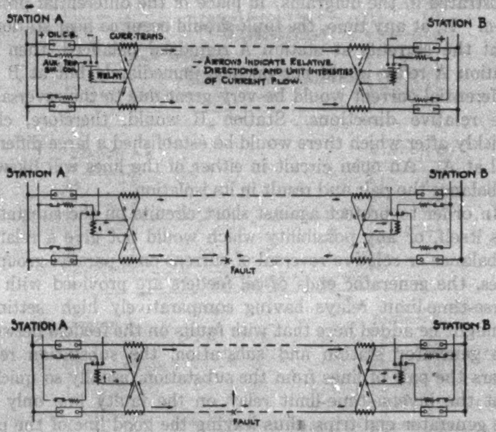
It is impractical to describe here the various modifications of these schemes in detail. However, the basic principles involved are discussed below, which principles are sufficiently flexible to allow application in various ways to the problem which may be in hand.

In the various diagrams accompanying the descriptions of these balances schemes, arrows are inserted to indicate the relative direction of current or of power. These arrows show power normally flowing from station A to station B. In each case, however, the equipment is equally suitable for power flowing in either direction.

In general, the over-current relays used are adjusted to operate instantaneously or nearly so. Some companies report that the opening of the breaker is so rapid that there have been cases where a cable has immediately rehealed, due to hot insulating material flowing into the opening made by the fault. Usually the point of breakdown has been actually located by subsequent inspection.

Balanced Protection of Two Parallel Lines (Not discriminating). An exceedingly simple scheme of balanced protection is illustrated in Fig. 6. This has given excellent results in a system which is so arranged that the lack of discrimination on the part of the relays between the sound and the injured line of a pair is not very vital. On the system referred to, each substation having such relay scheme, is provided with power over at least two different groups of lines. The complete, though temporary, interruption of one of these sources does not therefore kill the bus in question, and no hardship is imposed greater than the inconvenience of determining the good line preparatory to replacing it in service without its mate. This inconvenience is to a considerable extent compensated for by the freedom of the equipment from all alternating-current potential connections. While these potential connections, in present day relays, do not occasion the misgivings they were responsible for in older types, their elimination is a point not to be overlooked.

The balanced equipment as shown in Fig. 6 consists of "cross-connected" current transformers in the similar phases of the two lines, to the equi-potential points of the secondaries of which an over-current relay is connected. Reference to the arrows will show an assumed normal current flow, so long as the currents are equal and in the same direction in the two lines, there will be no current to flow through the relay coil, as demonstrated by the arrows on the secondary circuits. A through short circuit will not, therefore, improperly open these circuit breakers providing, of course, the line characteristics are such that a suitable balance is maintained.


STATION A
STATION B
+ OIL C.B. –
CURR-TRANS.
AUX. TRIP. SH. COIL
RELAY
→ ARROWS INDICATE RELATIVE DIRECTIONS AND UNIT INTENSITIES OF CURRENT FLOW.
– +
STATION A
STATION B
FAULT
STATION A
STATION B
FAULT

FIGS. 6, 7 AND 8—BALANCED PROTECTION FOR TWO PARALLEL LINES (NOT DISCRIMINATING)

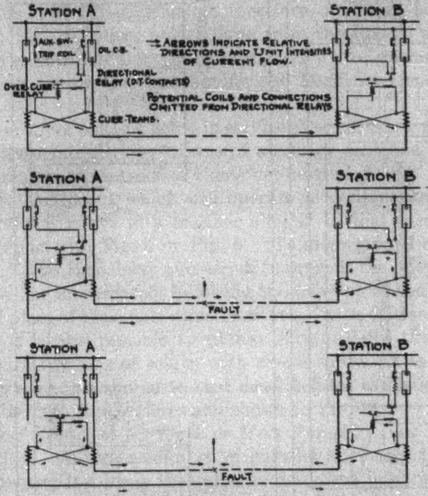
Assume a fault as shown in Fig. 7. In this case there is an unbalance of currents at substation A and a relative reversal of current at substation B. The differential of the secondary currents pass through the over-current relays, being an arithmetical difference at substation A and an arithmetical sum at substation B. Due to the differential current in the relay coils, the latter cause contacts to close, which trip both circuit breakers at each end of the two lines. Following such operation it is reported to be the practise to find the healthy line and connect it in service with time over-current protection (not illustrated in the diagrams) in place of the differential protection. If, at any time, the fault should occur so near station B that the currents at station A remained balanced, then the station A relay would not operate immediately but at B the differential current would be very great due to the reversal in the relative directions. Station B would, therefore, clear quickly after which there would be established a large differential at A. An open circuit in either of the lines will likewise unbalance the pair and result in its isolation.

In order to protect against short circuits on the substation bus itself, or any possibility which would not give a relative unbalance or relative reversal of current on a parallel group of lines, the generator ends of all feeders are provided with inverse-time-limit relays having comparatively high settings. It might be added here that with faults on the feeders between the generator station and substation, the substation relay clears the pair of lines from the substation, usually so quickly that the inverse-time-limit relay on the faulty line only at the generator end trips, thus leaving the good line of the pair connected at the generator substation.

Assume now a restricted fault in one of these lines, as in Fig. 8, such that the power direction remains throughout as formerly although the intensities have changed. It will be noted that the resulting difference in the currents in the two lines is reflected in the secondaries where it appears as a current through the relay coils and, causing the contacts to close, trips both circuit breakers at each end of the two sides.

Balanced Protection for Two Parallel Lines. (Discriminating). In order to obtain discriminating action in the relay equipment for short circuits in either of two parallel lines, direction relays are included in the scheme previously shown ln Fig. 6. With this modification the arrangement is as illustrated in Fig. 9. It should be noted that in case of an open circuit in one of these, selective isolation of the broken line is not obtained. The resulting unbalance would cause the opening of the sound feeder at one end and of the broken circuit at the opposite end, in case of sufficient current flow, as determined by setting of the time-current relay.

As in the previous case, so long as a balanced condition is maintained between the corresponding phases in the two lines, there will be no current in the differential or relay circuit. When a fault occurs, however, the vectorial unbalancing in the main circuits will be indicated in the relay coils as illustrated in Figs. 10 and 11. In these figures the arrows may also be considered as indicating direction of power flow. The moving contact of the directional relay will travel in the direction of the arrow.


STATION A
STATION B
AUX. SW.
TRIP COIL
OIL C.B.
ARROWS INDICATE RELATIVE DIRECTIONS AND UNIT INTENSITIES OF CURRENT FLOW.
DIRECTIONAL RELAY (D.T. CONTACTS)
OVER-CURR. RELAY
POTENTIAL COILS AND CONNECTIONS OMITTED FROM DIRECTIONAL RELAYS
CURR. TRANS.
STATION A
STATION B
FAULT
STATION A
STATION B
FAULT

FIGS. 9, 10 AND 11—BALANCED PROTECTION FOR TWO PARALLEL LINES (DISCRIMINATING)

Looking at the matter from another view point if we assume for the moment that only one line is in service, then the current

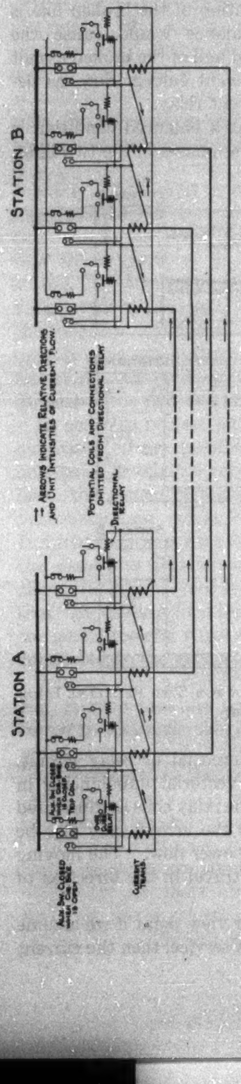

STATION A
STATION B
ARROWS INDICATE RELATIVE DIRECTIONS AND UNIT INTENSITIES OF CURRENT FLOW.
POTENTIAL COILS AND CONNECTIONS OMITTED FROM DIRECTIONAL RELAY
DIRECTIONAL RELAY
AUX. SW. CLOSED WHEN CKT. BKR. IS CLOSED.
TRIP COIL
OVER LOAD RELAY
AUX. SW. CLOSED WHEN CKT. BKR. IS OPEN
CURRENT TRANS.

FIG. 12

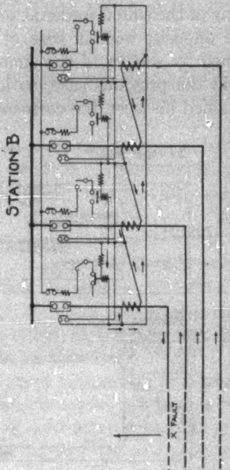

STATION A
STATION B
X FAULT

FIG. 13—BALANCED PROTECTION FOR THREE OR MORE PARALLEL LINES (DISCRIMINATING)

and potential connections to the directional relay should be so made that for power flowing from the bus to the line the contacts would close on the side to trip the breaker of the line in service. When both lines are working, the circuit having the greater flow of power from the bus to the line will control the operation of the directional relay and, therefore, trip the breaker of the line in trouble which always carries the greater power from the bus to the line.

It should be borne in mind that power flowing from the line to the bus is of negative value, therefore, for cases such as shown in Fig. 10, where power flows from the line to station B bus over both circuits, the circuit having the greater flow from bus to line is the one having the least flow from line to bus.

In the event of one line being in service alone, it is the general practise automatically, by means of auxiliary switches on the circuit breakers, to introduce time-over-current relays in the protective equipment which will operate in conjunction with the directional relays in a manner similar to that described in connection with Fig. 4 or Fig. 5 (Tandem groups or loop circuits). This time delay also serves to prevent the opening of the sound line immediately following the tripping of the faulty one. In place of the single directional relay with double throw contact, it is also possible to obtain substantially the same results with two sets of relays with single throw contacts arranged in a manner similar to that described for the protection of three or more parallel lines and shown in Fig. 12.

Balanced Protection for Three or More Parallel Lines. (Discriminating). Where a number of parallel lines are involved the underlying feature is the same as described in connection with Fig. 9, although the treatment of the matter is necessarily different. Here, as shown in Fig. 12, the current transformer secondaries are connected all in series in a loop circuit so that, when the primary currents are all equal the secondary currents will also be equal and will circulate through the loop as one current of the magnitude of each, and practically none will pass through the coils of the over-current and directional relays which are connected across each current transformer secondary.

Fig. 13 illustrates a fault in one line such that power is reversed at station B in this line from the direction taken by the remaining sound lines. It will be observed that only in the case of the injured line is the power direction through the directional relays (as indicated by the arrows) such that the contacts will close. At station A this is because there is a preponderance of positive power flowing from the bus into this particular line. At station B the injured line is the only one having positive outward flowing power.

Inasmuch as the success of the scheme depends upon the relatively low impedance of the loop circuit as compared to the impedance of the coils, it will be apparent that some method should be provided to eliminate the useless impedance injected in the loop circuit by the relays of a dead feeder. This is usually done automatically by short-circuiting, by means of auxiliary switches on the circuit breakers or by auxiliary relays controlled by such auxiliary switches, the relay equipment of the line whose circuit breaker is open.

It will be noted that when all but one line is out of service, the relay equipment of this last line will be short-circuited by the auxiliary switches or relays referred to in the preceding paragraph and accordingly the last line will be non-automatic unless some means is provided to open the loop circuit. This opening of the loop may be accomplished manually or it may be done automatically with considerable complication of auxiliary switches, etc., which complication is usually considered inadvisable. When the loop has been opened, each feeder will be left with over-current and directional protection at values determined by the settings of the over-current relays. Various other, also somewhat complicated, means may be used for inserting additional relays for the protection of this last line which on account of their variety will not be described here.

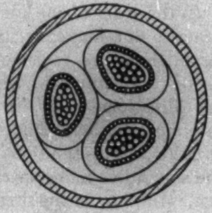
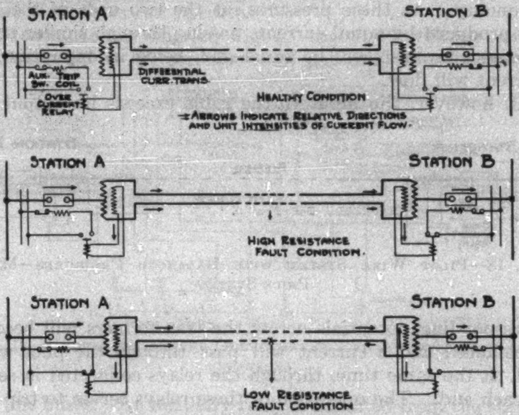


FIG. 14

If an open circuit should occur in one of the conductors or if, when putting another line into service only the breaker at one end is closed, an unbalanced condition will result which may tend to open the good lines in use if the current flowing at the time, is sufficiently great. This danger becomes relatively smaller as the number of lines involved is increased because the secondary unbalancing will be inversely proportional to the number of lines in service. For instance, if the over-current relays are set to operate at the normal load of each feeder and if four lines are in, (3 continuous and one open at one point) an overload of three times normal on each feeder would be required before any trouble could be encountered.

One installation of this scheme is described in a paper by Mr. Philip Torchio, "Relays for High-Tension Lines," TRANS. A. I. E. E., Vol. XXXVI, page 361, March 1917.

Split-Conductor Protection. Split-conductor cables are being tried out in this country by two operating companies who report good results to date. The cable is comprised of six conductors, each of the three phases being "split" into two halves. A section of the cable used is shown in Fig. 14.


STATION A
STATION B
AUX. SW.
TRIP COIL
OVER CURRENT RELAY
DIFFERENTIAL CURR. TRANS.
HEALTHY CONDITION
⇉ ARROWS INDICATE RELATIVE DIRECTIONS AND UNIT INTENSITIES OF CURRENT FLOW.
STATION A
STATION B
HIGH RESISTANCE FAULT CONDITION.
STATION A
STATION B
LOW RESISTANCE FAULT CONDITION

FIGS. 15, 16 AND 17—SPLIT-CONDUCTOR SYSTEM PROTECTION

The healthy and faulty conditions are illustrated in Figs. 15, 16 and 17. The action of the relays is similar to that described in connection with the two-parallel-line equipment shown in Figs. 7 and 8, excepting that under end fault conditions, additional devices (such as are described in the paper referred to in the next paragraph) are required to provide the necessary unbalancing. Repetition here will therefore be unnecessary.

The above greatly curtailed description is intended to cover in a simple way the underlying principle of split-conductor protection. For more specific information the reader is referred to the paper by Mr. W. H. Cole, "Split-Conductor

Cable—Balanced Protection," TRANS. A. I. E. E., Vol. I. 1918, page 757.

Pilot Wire System with Balanced Pressures. Adaptations of the Merz-Price protective system have been used by two operating companies. One of these reports very satisfactory results, the other is not so favorable due to an inherent difficulty in obtaining the proper degree of balance.

Fig. 18 illustrates the scheme of connections. The current transformers in each end of the circuit have their secondaries connected in opposition so that when current flows through the feeder, a potential is produced across each current transformer secondary. As these pressures, at the two ends of the line, are produced by equal currents passing through similar transformers they also will be equal and, being in opposition, no current will flow.

If, however, the input to the cable exceeds its output the corresponding potentials across the transformers will become unbalanced and a current will pass through the pilot wires and, at the same time, through the relays connected in series at each end. The operation of these relays serves to trip the circuit breakers.

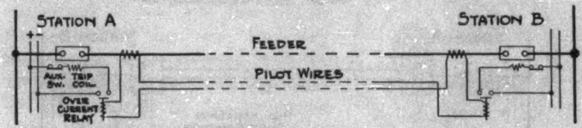


FIG. 18—PILOT WIRE SYSTEM WITH BALANCED PRESSURES—MERZ-PRICE SYSTEM

A modification of this method of protection is illustrated in the paper by R. F. Schuchardt: "Protective Relays" TRANS. A. I. E. E., Vol. XXXVI, page 383, 1917.

COMBINATION OF UNDER-VOLTAGE AND OVER-CURRENT RELAYS FOR PROTECTING TRANSMISSION LINES

One of the companies reported an application of a combination of under-voltage and over-current relays for their system which has given them very satisfactory results.

In this scheme all the circuit breakers in any section are mechanically locked in the closed position by means of a magnetically operated lock. It is controlled by a voltage relay which releases the lock when the voltage falls below a predetermined value (approximately 70 per cent). Upon the occurrence of a short circuit the voltage drops and when it has fallen below the predetermined point the voltage relays act to unlock the circuit breakers which are then free to be opened by the over-current relay or other over-current device. Thus, in most cases, only the breakers in the affected section can open on short circuit. It sometimes happens, however, that when two stations are close together, breakers in an unaffected section may be unlocked but usually the over-current device will act to open the proper ones only and clear the trouble before the others can operate.

Referring to the diagram Fig. 19 the under-voltage relays are connected to the secondaries of potential transformers on the line buses. An auxiliary relay is actuated from the station service transformer and is controlled by the under-voltage relays. In its turn this auxiliary relay controls the locks on the circuit breakers. One set of under-voltage relays, with the auxiliary relay, may control a number of locks on a number of circuit breakers. The operation is as follows:

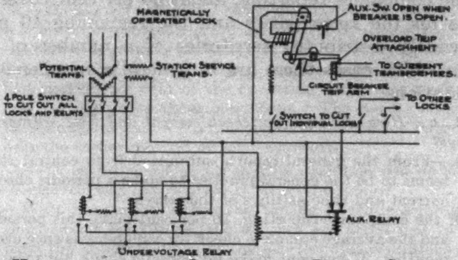

Magnetically Operated Lock
Aux. Sw. Open when Breaker is Open.
Overload Trip Attachment
Potential Trans.
Station Service Trans.
To Current Transformers.
Circuit Breaker Trip Arm
4 Pole Switch to Cut Out All Locks and Relays
To Other Locks
Switch to Cut out Individual Locks
Aux. Relay
Undervoltage Relay

Fig. 19—Undervoltage and Overcurrent Relay—Scheme of Connections

When the line potential drops below a predetermined value, one or more of the under-voltage relays drop and short-circuit the auxiliary relay through a resistance which causes this auxiliary relay to open. This, in turn, releases the latch and leaves all the circuit breakers under its control free to operate in case of over-current. Thus, so long as the potential remains above the predetermined value, no breaker can be opened by its automatic over-current feature.

This scheme, illustrated in Fig. 19, is reported as giving successful results on an over-head system covering 4000 square miles where there is a large number of stations located on an average of approximately 10 miles apart. As the lowest voltage occurs nearest the short circuit, it is probable that only the circuit breakers in the immediate vicinity will be unlocked and allow the circuit breakers to be automatically opened if the current is of sufficient magnitude.

APPENDIX

In reply to a general questionnaire which was sent out to sixty-one operating companies, information was received from thirty-two giving analysis of their relay protective schemes, from which the following summary was made.

Of the companies reporting, about 60 per cent cover supply systems for the larger cities and the remaining 40 per cent cover widely distributed territories of a number of smaller cities in which cases there are in general a number of water power stations.

Ques. 1. *a.—How often do you check current and time settings on your relays?*

Ans.—From the general reports submitted from central stations it seems to be the general practise to make a periodic check on the current and time settings of the relays.

Of the companies reporting, 70 per cent make this periodic check and the average complete check for each relay is once every three months; 27 per cent make no periodic check but most of them report making checks after cases of trouble and at intervals depending on operating conditions; 3 per cent report that they make no check whatever on the relay system.

Ques. 1 *b.—Have you a regular routine for making tests and inspection; What is it?*

Ans.—Approximately 70 per cent of the operating companies reporting have a regular routine of making tests.

Although the reports are not very definite, some of the larger companies have found it desirable to disconnect the circuit from service and test the relays by applying the testing circuit to the current transformer secondary terminals with the complete relay circuit left intact so that current transformer is left in shunt with the relay circuit under test. This method is found to give more accurate test results under varying conditions of various loads, relay circuits and different types of current transformers.

A few of the companies make their tests on the relays with the feeder alive which necessitates disconnecting the trip circuits and for the period of test the feeder circuit breakers are left non-automatic. This test may be made at relay terminals or at current transformer terminals in which case a check is made of the wiring.

Ques. 1. *c.—Do you calibrate with a stop watch, or do you use some other instrument? What is it?*

Ans.—For good relay calibration it is apparently becoming the practise to use some form of an automatic chronometer or cycle-counter in which the personal equation of the tester is eliminated. Of the companies reporting, 63 per cent have adopted this method of calibration where 37 per cent still hold to the old form of the stop watch.

Ques. 1. *d.—Do you make a more frequent inspection for dust and dirt and mechanical defects? At what intervals?*

Ans.—Of the reporting companies 65 per cent make a more frequent inspection of the relays for dust, dirt and mechanical defects and these additional inspections vary from periods of one week to one month.

Ques. 2. *a.—Do you test and adjust relays with the current according to the setting or to the current that will probably flow at the time of short circuit?*

Ans.—There is a growing tendency to test the relays with currents approximating the short-circuit conditions as borne out by the fact that 50 per cent of the operating companies reporting are now testing under these conditions. The results obtained by these companies show that this method is a marked improvement over the earlier method of small currents.

Ques. 2. *b.—In adjusting relays for selective operation how do you determine the current and time settings of the several relays?*

Ans.—With companies, where the testing is made approximating short-circuit conditions, the settings are arrived at in general from the determinations of the short-circuit conditions and the relays on which selective operation is desired are set to give selectivity under these conditions with allowances made in time settings for the actual operation of circuit breakers, the minimum time being determined by the estimated requirements for preventing operation on system load and surges.

In some companies the settings are determined by a "trial process" which is the result of the actual operations obtained under trouble conditions. Others use either the calculating table (See *G E Review*, October 1916), or Mathematical Calculations—(See *G E Review*, June 1916 and *Electrical Journal*, July 1916) for determining these conditions while others report the advisability of installing recording instruments for determining these conditions in a purely practical manner.

Ques. 2. *c.—In determining the current settings of your relays do you consider the probable short-circuit current with all generators in operation or with reduced generating capacity such as corresponds to the average or minimum load?*

Ans.—Of the operating companies reporting their settings determined by short-circuit conditions, 62 per cent determine their settings with the minimum generating capacity and in addition some of these consider also the maximum generating conditions, and 25 per cent consider only the maximum generating conditions, 13 per cent consider the average generating conditions.

Ques. 3. *a & b.—Do you use a-c. or d-c. for tripping oil circuit breakers on transmission lines? Is it satisfactory?*

Ans.—Of the replying companies 80 per cent use d-c. tripping current and most of them find it satisfactory and preferable and of the 20 per cent that use a-c. current only in one case did they state that they believed the a-c. to be preferable.

Storage batteries, and in some cases primary batteries, have been used for giving direct current for tripping.

Some of the central stations have had trouble with their circuit-breaker control circuits when connected to the d-c. excitation bus, due to the surges produced on the excitation system in cases of heavy short circuits on the high-tension system.

Ques. 4—*Do you connect watthour meters or indicating wattmeters to the same current transformers as your relays?*

Ans.—In 30 per cent of the cases neither watthour nor indicating wattmeters are connected to the same current transformers as the relays.

In 35 per cent of the systems the relays are connected to the meters in cases where a high degree of meter accuracy is not required.

In 35 per cent of the systems, the relays are connected with the meters with no restrictions mentioned.

Ques. 5—*Do you attempt to operate underground or overhead transmission lines with a ground on one phase, and if so, for how long before disconnecting?*

Ans.—Of the systems answering the questionnaire 67 per cent are grounded and the ground line is automatically disconnected.

Of the remaining systems, which are ungrounded, 25 per cent operate for short intervals (no longer than necessary and in no case longer than 30 minutes) with a ground on one line and 75 per cent disconnect the grounded line immediately.

Ques. 6.—*What is the longest time (in seconds) as determined by your relay settings and switch constants, that a short circuit is permitted to exist on your transmission lines before the switch is opened automatically?*

Ans.—The average maximum time as reported from the operating companies on which a short circuit is kept on the system is $3^1/_3$ seconds. This time varies with different companies from ¼ of a second to as high as 15 seconds.

We wish to point out that the above is not the time on which the heaviest short circuit is held on the system which cases are undoubtedly much shorter than given above. In some cases where the longer time is permitted, there has been trouble experienced due to synchronous apparatus falling out of step. Two seconds is apparently a good limit for heavy short circuits.

Ques. 7.—*About what percentage of your transmission line troubles are ground and what percentage are short circuits between phases?*

Ans.—Of the systems reporting, an average of 70 per cent of the troubles are started from ground. There is considerable variation in the reports; for example, some give as high as 90 per cent to ground while others give as low as 25 per cent to ground. We feel this discrepancy is probably due to the fact that some of the operating companies have no records to show what portion of the trouble did actually start from the ground.

Ques. 8.—*Do you ordinarily keep all transmission lines in service or do you keep some lines dead and in reserve?*

Ans.—Of the operating companies reporting 93 per cent keep all of their good transmission lines in service without any held in reserve with the exception of two cases where with overhead high-voltage transmission lines one of two parallel lines is taken out of service for reserve during lightning disturbances and in one of these cases the line is grounded at this time.

Ques. 9.—*Where you have several schemes of relay protection that accomplish practically the same purpose, will you kindly state your preference and your reasons?*

Ans.—Of the companies reporting 13 per cent prefer the combination of time-current and directional relay schemes, 13 per cent the balanced pair, 6 per cent split conductor, 6 per cent straight inverse-time-current protection, and 3 per cent balanced relay with pilot wires.

And of the remaining systems there was only one relay scheme in operation from which no comparison could be drawn.

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Ques. 10.—*Have you any scheme of relay protection which in your opinion is particularly effective for your conditions? If so will you kindly state in some detail the desirable features of this system?*

Ans.—Of the companies reporting, 32 per cent feel that some form of a balanced relay protection is well adapted for their conditions. Of this 32 per cent, 6½ per cent prefer the split-conductor scheme, 6½ per cent, two lines operated as a unit with balanced overload protection: 16 per cent, two lines in multiple with the balanced directional and current relays for disconnecting a single defective line only; and 3 per cent use pilot wires.

22 per cent find that the current-directional relay combination gives particularly good results for their conditions and some of these find that the connection of the potential circuits of the directional relay to the substation bus side of transformers, where not in parallel on high-tension side, give preferable operating conditions, as a higher voltage is maintained under short-circuit conditions.

3 per cent find special ground relays desirable (Refer to A. I. E. E. TRANSACTIONS p. 361 vol. 26).

6 per cent prefer definite time-limit relays as they have given very satisfactory service.

3 per cent prefer no automatic protection on very high-tension lines.

3 per cent find that a combination current and under-voltage relay system has solved, for their conditions, difficulties that they have not been able to overcome with any other scheme.

Balance report no special conditions.

Ques. 11—*Have you some installations or schemes of operation for which you find difficulty in securing a satisfactory scheme of relay protection? If so, please describe?*

Ans.—Several companies reported that they have had difficulty in finding a proper relay protection in cases where they have taps on the tie lines between stations. This is particularly true where the protection of these lines would have been of the balanced-relay type were it not for the taps.

Other companies at present using the balanced-current relay protection, which automatically disconnects both feeders in case of trouble on either, on parallel lines would like to find a simple relay scheme which would disconnect only the damaged line of the pair in case of trouble on one of the lines.

Some difficulty has also been experienced in finding the proper relay protection for tie lines between stations where very heavy momentary surges may exist with the flow of power in either direction without any actual trouble on the tie lines themselves.

Large variations in generating equipment, in operation with water power systems, caused the operating companies difficulty in finding a relay arrangement which would provide protection under all these different conditions.

Ques. 12.—*Do you attempt to operate all of your transmission lines in parallel by groups? Or in parallel?*

Ans.—Of the companies reporting 60 per cent operate with the major part of their lines in parallel and 40 per cent operate with some of their lines in parallel by groups and in some other cases the lines are strictly radial.

Ques. 13—*Have you in operation a balanced-relay system with pilot wires?*

94 per cent of the companies reporting use no balanced-relay system with pilot wires. In one instance it was reported that pilot wire scheme had been tried out but its use discontinued.

Ques. 14 *a. b. & c.*—*Do you use a balanced-relay scheme consisting of split-conductor cable or lines in multiple?*

When trouble occurs on one line of two in multiple is the good line again placed in service while repairing the other line?

What protective scheme do you have on the single line during the repairs on the line in trouble?

Ans.—72 per cent of the reports indicate that no balanced scheme of any kind is used.

6 per cent use split-conductor protection of the six-conductor cable type.

3 per cent use a similar scheme excepting that the six conductors comprise two separate three-phase circuits. For this scheme it was reported that after the two lines had been disconnected, due to a fault in one, the good line was found by test and put back in service as quickly as possible with inverse-time-limit protection at each end.

19 per cent report other schemes of balanced protection with directional relays in which selective isolation of the injured line is attempted.

Ques. 15 *a. b. & c.—What are the principal difficulties that you have experienced with relay protective system?*

Have you experienced any false operations or failures of relays to operate?

Have you been able to attribute such difficulties to defects in the relay, the design of the relay protective system, the difficulties in making proper connections, or to the irregular operation of oil circuit breakers? Please answer in some detail?

Ans.—Practically all the companies report trouble with the operation of their protective relay system. Of these troubles there are 46 per cent reported due to the difficulties in the relay, 20 per cent to improper application of the relays, 15 per cent to improper settings of the relays, 9 per cent to defective wiring, and 10 per cent to defective operation of the oil circuit breakers.

Several of the companies reporting trouble with the relays find that they were due to the chattering of the bellows type of relay in which cases some of the trouble had been eliminated by replacing this relay with other time-current types and in some cases it was eliminated by the installation of a saturating reactor in shunt with the relays. The shunt reactor eliminated the trouble and gave a desirable characteristic time curve at a considerably lower cost than required to replace the relays.

Some defective operations of directional relays are reported, in most cases of which the relays were of the old type.

Some companies report a change in calibration with time and although these cases are not very explicit, the trouble could probably be avoided by a more frequent check and inspection of the relay.

Of the troubles due to improper wiring, some of these are attributed to a wrong connection of the potential element of directional relays and some to a wrong connection of the balanced or differentially connected relays and the remainder is due to poor workmanship.

Ques. 16—*What is the maximum current that can flow into a short circuit on a transmission line just outside the generating station?*

a. The maximum instantaneous current?

b. The maximum sustained current?

Ans.—Report covers the experience from thirty-one operating companies with systems varying from 25,000 to 455,000 kv-a. whose short-circuit currents, at any point, vary in value from 100,000 to 1,650,000 kv-a. and sustained short-circuit kv-a. values vary from 20,000 to 340,000 kv-a.

GENERATING STATION DATA

Of the systems reporting, 67 per cent operate with the grounded neutral. Of these 40 per cent are grounded without resistance, 23 per cent grounded through low resistance and 4 per cent are grounded through a high resistance where special ground relays are required for selective operation on grounds.

Tie Line Relay Protection

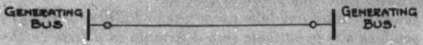
Generating Bus
Generating Bus.

Diagram A

About one-third of the companies reporting had tie lines that could be classified under this connection. Of these—

55 per cent of the lines were protected by means of inverse-time-current relays only.

5 per cent use balanced protection with pilot wires.

5 per cent use split-conductor installations.

35 per cent were provided with directional and current relay protection.

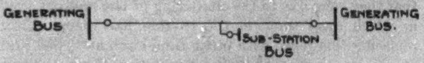
Generating Bus
Sub-Station Bus
Generating Bus.

Diagram B

About one-fifth of the companies reporting had tie lines that could be classified under this connection. Of these—

85 per centof the lines reported were provided with inverse-time-current relays only.

15 per cent were provided with current and directional protection.

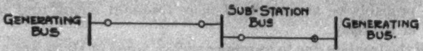
Generating Bus
Sub-Station Bus
Generating Bus.

Diagram C

About one-fourth of the companies reporting had tie lines that were classified under this connection. Of these—

50 per cent of the lines reported were protected by inverse-time-current relays only.

25 per cent by current and directional relays.

10 per cent were provided with balanced protection using pilot wires.

15 per cent were provided with balanced pair protection.

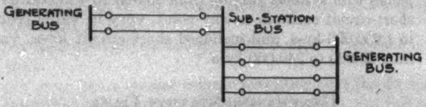
Generating Bus
Sub-Station Bus
Generating Bus.

Diagram D

About one-tenth of the companies reporting had tie lines that were classified under this connection. Of these—

65 per cent of the lines reported were protected by means of inverse-time-current relays.

35 per cent were equipped with current and directional relays.

TRANSMISSION LINE RELAY PROTECTION

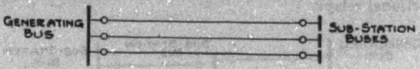
GENERATING BUS
SUB-STATION BUSES

DIAGRAM E

About one-eighth of the reporting companies have lines that were classified under this type. Of these—

75 per cent of the lines were protected by inverse-time-current relays.

25 per cent of the lines were protected by directional relays and current relays.

(Even on a radial system the secondary connections of the step-down transformers may be in parallel and cause a feed back in case of trouble which makes a directional relay desirable.)

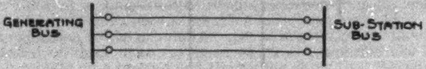
GENERATING BUS
SUB-STATION BUS

DIAGRAM F

About one-half of the companies have lines in this classification. Of these—

40 per cent of the lines were reported to have current and directional protection.

55 per cent were equipped with inverse-time-current relays only.

5 per cent were protected by balanced relays of the interconnected directional type.

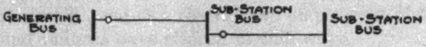
GENERATING BUS
SUB-STATION BUS
SUB-STATION BUS

DIAGRAM G

About one-tenth of the companies have lines under this classification of which one-half are protected by inverse-time-current relays and one-half by balanced connections.

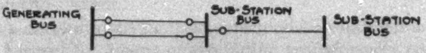
GENERATING BUS
SUB-STATION BUS
SUB-STATION BUS

DIAGRAM H

About two-fifths of the companies have lines under this classification. Of these—

55 per cent were reported to have inverse-time-current relay protection.

30 per cent were equipped with directional and current protection.

7½ per cent were equipped with balanced relay protection with pilot wires.

7½ per cent were equipped with balanced relay protection scheme using directional relays.

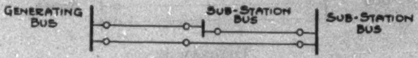
GENERATING BUS
SUB-STATION BUS
SUB-STATION BUS

DIAGRAM I

About one-eighth of the companies have line classifications in this list. Of these—

75 per cent were reported to be protected by means of current relays and directional relays.

25 per cent by means of inverse-time-current relays only.

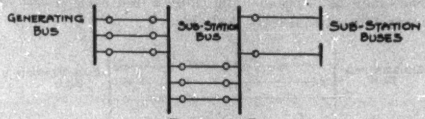
GENERATING BUS
SUB-STATION BUS
SUB-STATION BUSES

DIAGRAM J

Two companies reported lines in this classification and in both cases straight inverse-time-current relay protection was used.

Presented at the 35th Annual Convention of
The American Institute of Electrical Engineers,
Lake Placid Club, N. Y., June 24, 1919.

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GROUNDING THE NEUTRAL OF GENERATING AND TRANSMISSION SYSTEMS

BY H. R. WOODROW

ABSTRACT OF PAPER

Four different high-tension transmission systems in the New York district have been grounded at different times during the past four years, all of which are operating with the grounded neutral at the present time. These systems were all grounded in a different manner and the results obtained from the effect of grounding showed that the troubles have been reduced.

In addition to the grounding, special ground relays have been installed on three of the systems which disconnected the feeder very quickly on a ground and the results show that the addition of this quick acting relay further reduces the stresses on a system.

6600-VOLT 25-CYCLE SYSTEM

IT had been the policy of The New York Edison Company to operate its high-tension feeders radially with the system neutral ungrounded, but with selective ground indicating relays on all feeders to locate a ground on any feeder. This method of operation gave the operators time to disconnect defective grounded feeders from the system before developing into short circuits, and as a result 70 to 80 per cent of the system ground failures were cleared in this way.

During the later years, however, it became desirable to study means of paralleling feeders at the substations and of reducing the time limit for disconnecting the defective feeder to avoid short circuits following a ground on account of the increased capacity current of the greatly larger mileage of cables on the system.

After a careful analysis it was decided to establish a definite ground current by means of grounding transformers connected to the system bus and connect the selective ground relays to trip the oil circuit breakers automatically.

The method of grounding through zig zag-connected transformers was adopted as the latest generators installed are delta-connected, from which a neutral cannot be brought out. The limiting resistances were connected in the primary side of the grounding transformer and of a value to limit the total ground current with both generating stations operating in parallel to 400 amperes. The selective ground relays, which are operated from sheath type current transformers, are adjusted for operation on 140 amperes minimum and on 300 amperes in one second.

The above described system was placed in operation in the early part of 1918 and the accompanying Fig. 1 will show the results which have been obtained. Although the time that this system has been grounded is too limited to draw any definite conclusion, the total system troubles have slightly decreased and of these total troubles the number which have developed into short circuits is reduced to 60 per cent of those before grounding.

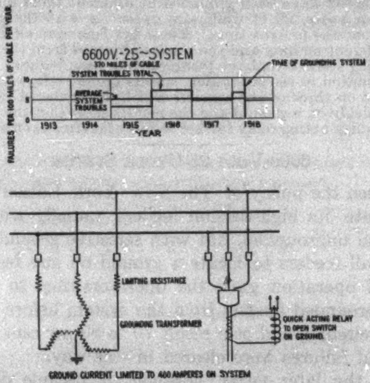
6600V.-25~SYSTEM
370 MILES OF CABLE
SYSTEM TROUBLES TOTAL
TIME OF GROUNDING SYSTEM
AVERAGE SYSTEM TROUBLES
FAILURES PER 100 MILES OF CABLE PER YEAR
10
5
0
1913
1914
1915
1916
1917
1918
YEAR
LIMITING RESISTANCE
GROUNDING TRANSFORMER
QUICK ACTING RELAY TO OPEN SWITCH ON GROUND
GROUND CURRENT LIMITED TO 400 AMPERES ON SYSTEM

Fig. 1

7800-Volt 60-Cycle System

The new 201st Street station of the United Electric Light & Power Company was placed in service with the same general scheme of operating with an ungrounded neutral with selective ground indicating relays. In the first few years of operation, however, several generator failures were experienced and at the recommendation of the manufacturer the system was grounded by means of the generator neutral through a two-ohm resistance, which would allow a ground current of approximately 2000 amperes to flow.

The results of the system troubles before and after grounding are shown in Fig. 2, and although no general system disturbances were experienced before the grounding, there has been an apparent reduction in the number of system troubles since grounding.

16,000-Volt System

When the 16,000-volt system was adopted for supplying the northern territory of the 25- and 60-cycle systems, step-up and step-down transformers were provided on each feeder, thereby isolating the high-tension side of each feeder. During the first two years of operation the number of cable failures was very large, most of which occurred in the cable splices.

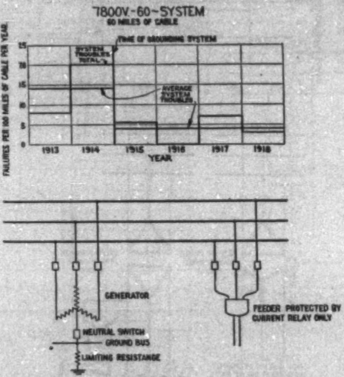

Fig. 2

To remedy this trouble a number of splices on the 60-cycle system were changed and the transformer connections were changed on both the 25-cycle and 60-cycle systems to make a solid ground on the high-tension side. The systems were operated in this manner for a period of approximately two years and no material change in the number of failures occurred. During this time, every ground on the system was a short circuit due to the solidly connected neutral, thereby materially increasing the stresses on the transformers and cables. It was decided to change the method of grounding and provide quick-acting relays to disconnect a feeder practically instantaneously on a ground.

The method adopted for grounding these feeders as shown in Fig. 3 followed the suggestion of Dr. Creighton of using potential transformers with the neutral grounded and the secondaries connected in delta with a quick-acting relay in the loop.

In the year and a half that this system has been in operation, the results have been encouraging although the number of system troubles, as shown in Fig. 3, was only reduced by approximately 28 per cent; the number of short circuits have been reduced by 82 per cent. In 73 per cent of the cases in the present method of operation the failures are disconnected from the system before short circuit occurs.

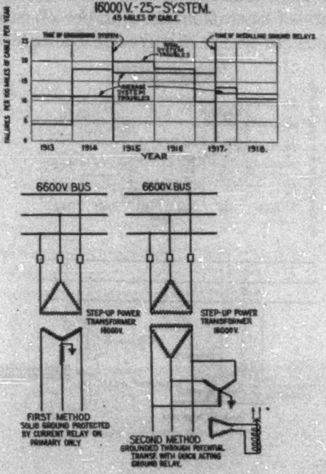
16000 V.-25~SYSTEM.
45 MILES OF CABLE.
FAILURES PER 100 MILES OF CABLE PER YEAR
TIME OF GROUNDING SYSTEM
TIME OF INSTALLING GROUND RELAYS
TOTAL SYSTEM TROUBLES
AVERAGE SYSTEM TROUBLES
25
20
15
10
5
0
1913
1914
1915
1916
1917.
1918.
YEAR
6600V. BUS
6600V. BUS
STEP-UP POWER TRANSFORMER 16000V.
STEP-UP POWER TRANSFORMER 16000V.
FIRST METHOD
SOLID GROUND PROTECTED BY CURRENT RELAY ON PRIMARY ONLY
SECOND METHOD
GROUNDED THROUGH POTENTIAL TRANSF. WITH QUICK ACTING GROUND RELAY.

Fig. 3

These figures are the results on the 25-cycle system only, as a number of changes in the cable joints have been made on the 60-cycle cables and the results would be confused as it would be impossible to determine what portion was due to betterment of insulation and that due to the method of grounding.

25,000-Volt 25-Cycle System

In the installation of 25,000-volt 25-cycle cables, the neutral of the transformers at the generator end was solidly grounded.

The type of joint used in this cable was of the oil-filled type as described by Mr. Torchio before the Institute, and practically no failures have resulted in the splices. In the first fifteen months of operation, however, a number of cable failures occurred, some of which could be traced to defective manufacture of cable. To relieve the stresses on these cables it was decided to install grounding relays connected to a sheath transformer for disconnecting the cable very quickly on a ground, as the method of operation of these feeders necessitated a long time setting on the normal current relays.

These cables have been in operation with this method of protection for three years and, as shown in Fig. 4, the number of failures has decreased until no failures have occurred during the past year.

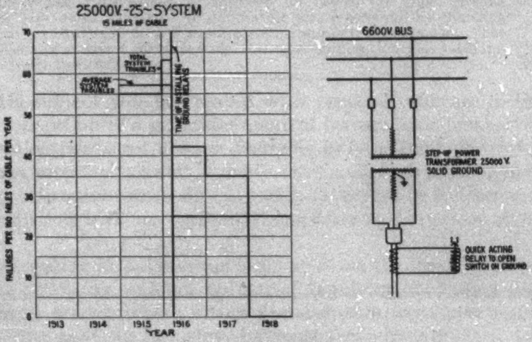

Fig. 4

Conclusion

Our experience has shown unquestionably that the stresses on the cables are relieved materially by grounding, but we further believe that it is desirable to limit this grounding current as much as possible, thereby reducing the stresses on the system and particularly the burning of lead sheath and grounding connections. Also the installation of grounding relays, which can be set to disconnect a feeder more quickly than the straight normal current relays, will further greatly relieve the potential stresses due to a prolonged grounding current which occurs even with a grounded system.

Presented at the Thirty-fifth Annual Convention of the American Institute of Electrical Engineers, Lake Placid Club, N. Y., June 24, 1919.

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GROUNDED NEUTRAL TRANSMISSION LINE

BY W. E. RICHARDS

Abstract of Paper

The paper describes the conditions on the system in Toledo which was originally delta-connected. Serious trouble was experienced when a short circuit occurred especially with synchronous apparatus connected to the line which invariably failed due to the voltage drop occasioned by the severe short circuits on the delta system. This trouble was overcome by changing the transmission to a Y system with the neutral grounded. The effect of a short circuit with the new connection has been decidedly minimized.

THE subject will be treated with especial reference to the operation of a grounded-neutral transmission line having several elements not always desirable in handling high-tension voltage within city limits, namely, extremely high and long-span construction connected directly with an extensive underground transmission with its various connections to conversion apparatus.

The system in Toledo was built more on the line of developments following existing industrial requirements than a prearranged system where future requirements were anticipated.

In this case, an industry, by reason of circumstances, was located some four miles from the generating station. Conditions were such that it was not desirable for the customer to generate his own supply. His requirements started with a demand of from two to four thousand kilowatts, with the ultimate expected demand of eight or ten thousand. As the industry has exceeded their expected increase, the demand on this system will shortly run to sixteen thousand kilowatts, and possibly greater.

Geographical conditions and city ordinances determined that this transmission should be in part purely underground. There was fortunately located on the route selected, several substations which were badly in need of additional energy and such other developments that the company had in mind, that it was decided to install a 23,000-volt 25-cycle system, it being felt that this was the limit of successful high-voltage underground operation. Certain features presented themselves at this time which made it desirable to have the transformers delta connected. This feature was thoroughly discussed and at that time no serious objections presented themselves and it was put into operation the early part of the year of 1915.

Some trouble developed due to weak spots in cable and joints but these faults were surprisingly low, considering the quantity of cables that were put into service at this time.

The system rapidly settled down to a successful continued operation and no faults developed that did not have a reasonable explanation, such as fault in material and joints, and joints not properly made.

Since correcting this condition the system has operated successfully with reference to the general plan, but a serious condition arose when a short circuit occurred, particularly disastrous to synchronous apparatus connected with the system, which would invariably fail by reason of the voltage drop occasioned by the severity of a short circuit on the delta system. The cables suffered severely, and in fact, in some cases the covers of the manholes blew off, taking on more the nature of an explosion if the short circuit occurred in the manhole.

It was then decided to change this transmission to a Y system with the grounded neutral. The transformers being constructed for delta operation, it was necessary to take the core out of the case, cut the coils apart and parallel them for 23,000 volts Y. A five ohm-resistance was inserted, which afterwards was removed, thereby operating the neutral directly to ground. The results of a cable failure now has a decidedly different effect.

In some cases no superficial evidence is present at the point of failure, and when such evidence is present, it is not of a severe nature to destroy adjacent cables and in no way presents the disastrous appearance as when trouble would occur with a system operating delta.

It is true that by experience a better understanding of protective apparatus resulted in changes to such apparatus which were beneficial, but it was felt that the successful operation of this system was largely due to the change from delta to Y grounded.

Referring to the Figs. 1, 2 and 3, the system is graphically shown, and presents some unique features as before mentioned.

Fig. 1 is the general scheme of the transmission, carrying

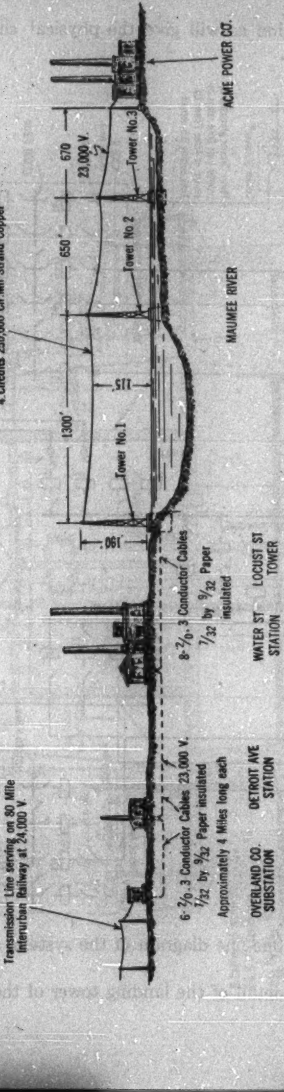

Transmission Line serving on 80 Mile Interurban Railway at 24,000 V.
6-2/0. 3 Conductor Cables 23,000 V. 7/32 by 9/32 Paper insulated Approximately 4 Miles long each
OVERLAND CO. SUBSTATION
DETROIT AVE STATION
WATER ST STATION
LOCUST ST TOWER
8-2/0. 3 Conductor Cables 7/32 by 9/32 Paper insulated
190'
Tower No.1
1300'
115'
3 Ground Wires 0.5in. Strand Steel
4 Circuits 250,000 Cir.Mil Strand Copper
MAUMEE RIVER
Tower No 2
650'
Tower No.3
670
23,000 V.
ACME POWER CO.

FIG. 1—GENERAL SCHEME OF TRANSMISSION

such information as will give the physical characteristics of the system.

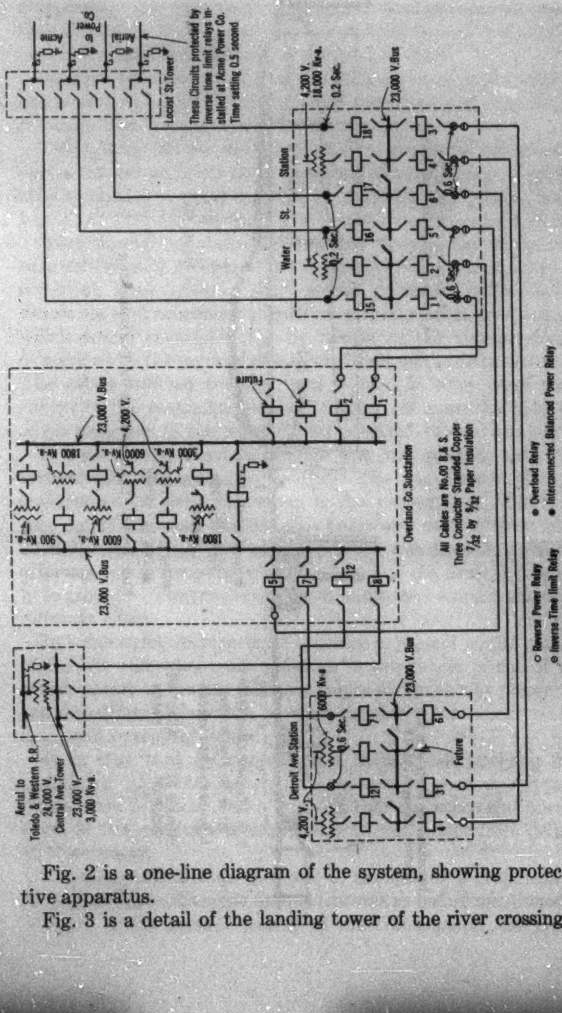

Acme
to Power Co.
Aerial
Locust St. Tower
These Circuits protected by inverse time limit relays installed at Acme Power Co. Time setting 0.5 second
Water St. Station
4,200 V. 18,000 Kv-a.
0.2 Sec.
23,000 V. Bus
0.6 Sec.
Future
23,000 V. Bus
4,200 V.
1800 Kv-a.
6000 Kv-a.
3000 Kv-a.
900 Kv-a.
Overland Co. Substation
All Cables are No. 00 B.& S. Three Conductor Stranded Copper 7/32 by 9/32 Paper Insulation
○ Reverse Power Relay
⊖ Inverse Time limit Relay
● Overload Relay
● Interconnected Balanced Power Relay
Aerial to Toledo & Western R.R. 24,000 V.
Central Ave. Tower
23,000 V. 3,000 Kv-a.
Detroit Ave. Station
4,200 V.
6000 Kv-a.
0.6 Sec.
23,000 V. Bus
Future

FIG 2—HIGH-TENSION SYSTEM OF THE TOLEDO RAILWAYS & LIGHT CO.

Fig. 2 is a one-line diagram of the system, showing protective apparatus.

Fig. 3 is a detail of the landing tower of the river crossing.

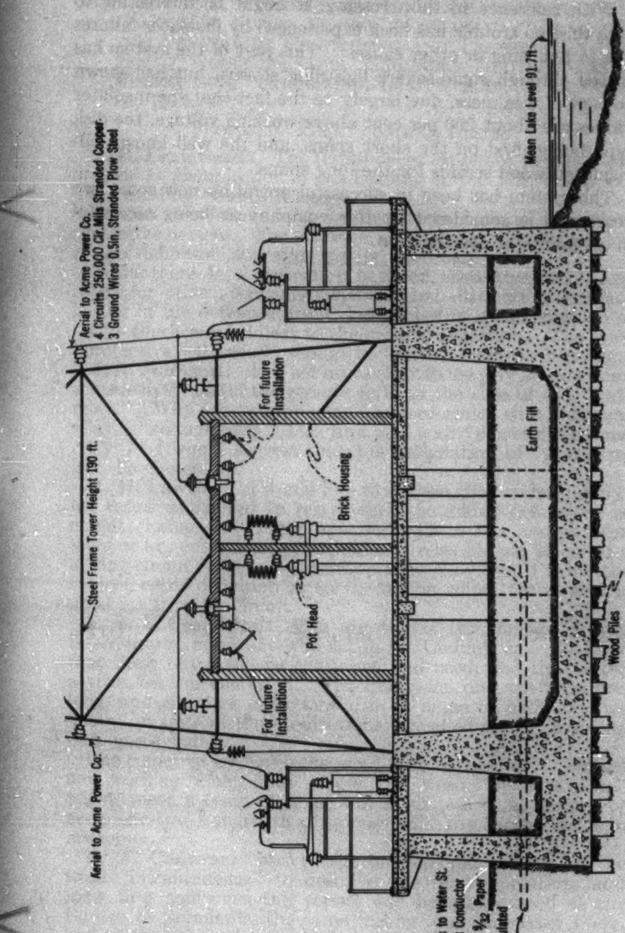

Aerial to Acme Power Co.
4 Circuits 250,000 Cir.Mils Stranded Copper.
3 Ground Wires 0.5in. Stranded Plow Steel
Steel Frame Tower Height 190 ft.
Aerial to Acme Power Co.
For future Installation
For future Installation
Brick Housing
Pot Head
Earth Fill
Wood Piles
Mean Lake Level 91.7ft
8-Cables to Water St. Sta. 2/0-3 Conductor 1/32 by 9/32 Paper Insulated

FIG. 3—SECTION AT BASE OF TOWER NO. 1

With reference to this crossing, it might be interesting to note that no trouble has been experienced by insulator failures due to lightning or other causes. This part of the system has passed through some severe lightning storms, but has shown no sign of weakness, due largely to the fact that the insulator values are about 200 per cent above working voltage, the disk type being used on the short spans, and the well known oil-insulated wood strains for the long spans.

The system has been in successful operation now some two years and is considered by this company as being successful in all points of construction.

DISCUSSION ON "TRANSMISSION LINE RELAY PROTECTION" (WOODROW, ROPER, TRAVER, MACGAHAN), "GROUNDING THE NEUTRAL OF GENERATING AND TRANSMISSION SYSTEMS" (WOODROW), AND "GROUNDED NEUTRAL TRANSMISSION LINE" (RICHARDS), LAKE PLACID, N. Y., JUNE 24, 1919.

PROPOSED RELAY NOMENCLATURE

J. R. Craighead: The terminology expressed in this paper is open to an objection in the heading of the first definition. The heading reads, "Electric Protective Relay," and the definition that follows is so worded as to include other than protective devices. For instance, a telegraph relay is included in the definition according to its wording.

While there is a distinction between protective relays and relays for other operating purposes, that distinction does not belong in the definition itself, but rather in a classification of relays which is subsidiary to the general definition. The word "protective," therefore, should be omitted from the heading.

J. A. Johnson: It is not quite clear to me why an exception is made in the use of the generic term in the case of the "watt relay." Why not "power relay" in conformity with "current relay," "voltage relay," etc., and also with "power directional relay"? I would suggest that the designation of "power relay" be chosen.

A. H. Lawton: I would like to suggest that under qualifying terms as applied to relays there be added definitions of circuit closing and circuit opening. There has always been more or less confusion as to whether a relay should be called a closing circuit relay or a circuit opening relay, and I think the subject needs definition as to the action which the relay has upon its tripping circuit.

E. E. F. Creighton: The question of terminology is very interesting to every one who sat in the Committee. We hope that these terms will be continued and used, and thus avoid a great deal of confusion. The discussion on each term was long, and although objections can be made to each one, we finally, in spite of those objections, accepted the term as the best one possible.

One comes to me now—the word "instantaneous." We are now working along lines where the word "instantaneous" might mean a tenth of a second, a sixtieth of a second, or even one-two hundredth of a second. Instantaneous is a misnomer.

H. T. Conover: Mr. Creighton deplored the use of the word "instantaneous" to describe the relay operation, and some time ago when this report was being discussed at the Bureau of Standards, the same feeling was expressed there, and at that time the word "immediate" was suggested as a substitute for the word "instantaneous." I would like to offer that to the Committee as a suggestion. While the word "immediate" leaves something to be desired, it certainly has not the rigor and the absolute characteristics that the word "instantaneous" has and would be more liberal and allow for an operation that is rapid but at the same time does not take place in zero time.

O. C. Traver: The term "Electric Protective Relay" was proposed in an effort to restrict the field to be covered by the other terms, which followed it. Possibly, some other preferable means of accomplishing this purpose will be substituted at a later time.

There seems to be but slight preference between the generic terms "Watt Relay" and Power Relay." Even if "Power" should be the better when used alone, the combined term "Over Power" is considered very objectionable on account of previous usage.

Circuit closing and circuit opening contacts might well be defined.

Although objections may be raised to the term "Instantaneous," it must be remembered that in the physical world there is no such thing as instantaneous action in the absolute sense. Practically, universal usage sanctions "Instantaneous" as applied to relays, and as no confusion results there seems to be no reason for changing to some other term, possibly no more accurate.

PRESENT DAY PRACTISE OF TRANSMISSION AND TIE LINE RELAY PROTECTION

J. R. Craighead: This paper emphasizes the fact that we cannot specialize too narrowly on one branch of engineering without inviting trouble in other allied branches. The transmission line engineers have for a long time chiefly given their attention to the questions of efficient transmission, safety and protection in other senses than relay protection, and have consequently allowed relay protection to be developed as a separate art which must be adapted to transmission lines constructed without consideration of relay protection.

J. A. Johnson: As networks become larger by the interconnection of systems, the necessity for better means of isolating faults with a minimum of disturbance becomes greater. The delay of seconds after the occurrence of a fault and the use of reactance are, after all, but make-shifts adopted on account of the limitations of the circuit breaker and ought not to be, and, in my opinion, will not be permanently tolerated. Some other means must be found to accomplish the purpose, and I believe that we must look for it in improvements in the circuit breaker itself. It may well be that some entirely new and revolutionary principle of operation will be the means of accomplishing the result, but in any event I am convinced that a big advance in the effective isolation of faults by more efective circuit breaking devices is on the books for the near future.

I would invite the attention of inventors to this problem. We have accomplished much, but we must guard against the self-satisfied attitude, keep in mind the ideal to be attained, and strive forward to the day when the size limitations of transmission networks will disappear and the isolation of faults take place instantaneously and entirely without disturbance to operation of unfaulted circuits.

I believe that the future system should be the reverse of the present practise in that the time of operation of relays should become shorter as the generating station is approached, as the disturbance to the system is greater with short circuits near the generating station, and consequently such trouble should be cleared from the system in the shortest possible time.

P. Torchio: Referring to Mr. Johnson's remarks, I fully appreciate, and I think most of us appreciate, the work of development of the Protective Devices Committee. We have with us at present only two solutions to cope with the requirements of handling large powers, and the two are either the balanced system, including split conductor or pilot wire, or the other system developed by American engineers in the last few years, depending on selectivity by timing relays.

Mr. Johnson gave a knock-out to the development of this system of protection because necessarily if a system of that kind depends on having the longest time of selectivity on the relay near the station, that must be the last to go. Mr. Johnson is probably right in aiming at an ideal goal, that ideal goal being the development of a circuit breaker that will open any kind of short circuit, and any amount of power. He does hope that such development will be made, but to my knowledge there is not now on the market a circuit breaker that will cope with the stresses imposed by opening a circuit as met in a central station of large size.

Now, with the failure of the large circuit breaker, we have followed another line, to reduce the amount of power that will go into a short circuit. If we reduce the amount of power that will go into a short circuit, and that is the only practical way that we now have available, then we can use the selective relay that these engineers are developing.

So I do not want to discourage this American development because I believe that at present it is the right line to follow. We have no other alternative from an economical standpoint, and I believe that from an engineering standpoint it can be made successful.

J. A. Johnson: I had no intention whatever of casting aspersions on the present relays; on the other hand, I have every admiration for and desire to express appreciation of the work that has been done. I believe that under present conditions, and with the apparatus that is at present developed, the lines that we are following are right. They are the best that we can do.

The ideal purpose will perhaps be accomplished in cable systems by means of the differential principle, which in a great many cases, enables faults to be disconnected before they develop into short circuits but on the other hand in high-tension overhead lines, I firmly believe that some new development of circuit breakers, carrying with it a new development of relays, has got to be made and I simply want to encourage progress in that direction, not at all to disparage the progress that has been made up to the present time.

D. W. Roper: I would like to add a word about the maximum delay of relay operation and its effect on the synchronous apparatus.

Some years ago, in the course of switch experiments by the Commonwealth Edison Company, some large generators were made with the reactance coils in series. Experiments were made to determine the effects on synchronous apparatus in case of short circuits at the bus and at some distance electrically from the bus. The experiments showed, and subsequent experiments have verified, that a short circuit on the bus will disconnect the synchronous apparatus, regardless of how quickly you open that short circuit.

The types of breakers that we have available apparently would not open the short circuit quick enough to prevent disconnecting the synchronous apparatus in the substation. If, however, the short circuit, instead of being on the bus is a comparatively short distance away—a hundred or two feet on a cable—so that you get a perceptible amount of resistance in series in the arc, then the synchronous apparatus will not be disconnected if the breaker on the transmission line operates properly and in the correct time.

There are, however, other effects that have to be safeguarded and one of them is the effect of the burnout on other lines in adjacent conduits. Our experience indicated that in the days when we had the instantaneous relays, the ordinary burnout resulted in a small hole in the lead sheath of the cable, perhaps half an inch or an inch or two in diameter, depending upon the location with respect to the station.

As we introduced a time element into the operation of the breakers, we got larger burns on our cable so that with these delayed actions of two seconds or thereabouts, instead of having a small hole in the cable we now very generally involve all three failures in the cable burnout before the breaker opens, and sometimes there is perhaps two or three feet missing. That is one of the many important things that we now have to guard against in these burnouts.

If you have an eight-second limit, as one man has suggested, I venture to state that on the system of the Commonwealth

Edison Company, we would in nearly every case involve other cables in the conduit by burns through the conduit system into adjacent ducts. Our experience has indicated that to be the case with very much less than eight seconds, where the relay not operating properly had delayed the action to perhaps three seconds.

P. H. Adams: We are using a maximum setting of two seconds on the lines of the Public Service Electric Company of New Jersey, under short-circuit conditions. These relays, on overloads as high as 100 per cent, operate at from six to ten seconds. I believe that the Committee intends that the two seconds value should apply to short-circuit conditions and not to overload conditions.

R. N. Conwell (by letter): Many operating companies are undoubtedly making trial installations of newly developed relays or relay schemes to fulfill certain conditions in their systems. I would suggest that complete reports to the Protective Devices Committee on the operation of these trial installations would prove of great value if compiled and published annually, for in this way, wasteful duplication would be eliminated and real progress obtained. The Protective Devices Committee should also maintain a list of trial installations so as to afford opportunity for simultaneous tests of several different types of relays or relay schemes for the same conditions.

The co-ordination of transmission, station and substation design, and the application of protective relays is necessary in all systems, but as systems increase in size, the matter increases in importance. A new line loses much of its commercial value if it is so connected into the system that trouble cannot be properly sectionalized. Tie bus or section reactances do not completely fulfill their functions if the transmission system is so interconnected that these reactances are shunted by lines, thus increasing the requirements for discrimination on the part of the relays, in addition to subjecting much of the transmission system to needless strains under short-circuit conditions. Careful grouping of lines from the standpoint of relay protection as well as operation will undoubtedly result in the minimum cases of trouble, minimum short-circuit current, minimum discrimination required of the relays, and minimum interruptions to service.

Careful analysis of many systems will reveal the fact that more effective protection may be obtained by changing operating schedules to conform with relays already installed. The possible slight reduction in efficiency in transmission will be more than compensated for by the reduction in the number of interruptions.

The effectiveness of any application of relay protection is dependent upon complete cooperation between those having charge of operation and those having charge of the protection of the system. The most carefully planned application of relays for transmission line protection, and the most carefully calculated settings will fall far short of the requirements if the system is not normally operated in such a way that the relays may function as designed and set.

A. W. Copley (by letter): In this paper the authors have laid emphasis upon the desirability and importance of taking into account the relay system to be employed when a transmission system is being planned. At times, however, departure from standard relay practise may be warranted, because of certain advantages in a type of system for which relay protection has not been entirely developed. The recurrence of similar cases finally brings the new development into the standard class.

C. P. Osborne (by letter): The average operating engineer is prone to let well enough alone, especially during such times as we have been passing through in the last few years when it was difficult to secure financial assistance for betterments.

The company with which the writer is connected cannot boast of its relay protection. However, in a country where there is a small amount of lightning and where weather conditions are more or less uniform, I feel that protective devices are not as important as in a more severe climate. My experience is that it has been only in recent years that manufacturers have put out protective relays which are a real protection to a transmission system. The Bellows type relay has proven one of the most satisfactory in the writer's experience, but it must be properly taken care of and tested periodically and even then its protection is not of the best. The experience of a great many companies certainly shows that the a-c. relay does not give the protection to transmission systems or tie lines that the d-c. controlled relay does. One of our greatest troubles with high-tension oil switches, where a-c. trips are used in series with the line for protection, is the inaccuracy of the relay. The writer has found it necessary in some cases to change the mechanism to get a relay to properly trip, and our opinion is that the manufacturers have not as yet gotten an a-c. relay for high-tension transmission lines which is accurate enough to give the operating engineer assurance that his lines are protected and his circuit breakers will open when line trouble is experienced. Where there are four or five high-tension lines generating from stations feeding into a network at the main distributing point where all tie lines are in parallel, it is too often the case that trouble on one high-tension line will cause disturbances which will affect the whole system. Especially is this true if one line should have a swinging ground and we have frequently found this to be the case where insulators are breaking down. I am a firm believer in running grounded neutrals on high-tension lines where it is possible.

The authors of the paper bring out a point in the first paragraph of the fourth page of the paper which seems to the writer is putting the reliability of the relays up to the operator instead of making them more or less fool proof and automatic. I do not believe the operator should in any way be responsible for the action of the relays, except that the relays must be kept in good working condition.

It is difficult for operating companies to keep up to date on relay installations as any installation which is two or more years old is more or less out of date. The advances in recent years make so many different changes and different kinds of relays for protective apparatus that the operating companies are often at sea to know just what to order for giving the best protection to their lines.

P. M. Lincoln: I think the work that this Committee is doing is one of the most important that we have before our Institute. The thing which our transmission systems and supply systems must have above everything else is reliability. Service must be reliable and as systems increase in size we have two elements that are working against that quality. In the first place, the very fact that the systems increase in size and extent of itself makes them more vulnerable because of the greater amount of apparatus and equipment and lines which are exposed to the possibility of trouble spreading. In the second place, the need of greater reliability becomes greater as the system increases in size. If the larger systems cannot give service without the possibility of continual interruptions, the service loses its value. I certainly hope that the Committee will continue its work and give us in the future, as they have in this report, resumés of the advances that are made from time to time in protection of systems.

C. P. Steinmetz: Broadly and bluntly speaking, the problem of relay protection essentially consists in producing a simple, reliable and automatic device to disconnect an abnormal circuit without interfering with the operation of any normal line. *This problem has not yet been solved.* The formidable difficulty in the way of solution of the problem is that the abnormal currents or voltages exist not only in the abnormal circuit element but also in many or most of the normal circuit elements. And therefore, any device depending on the existence of abnormality of current and voltage would not inherently discriminate between the normal and abnormal circuit element, and cannot solve the problem.

The attempt of solution thus has been made discriminating by the degree of abnormality, which means time element. This is the present solution which naturally is limited, as you all know. Therefore, the problem on the whole is to discriminate between normal and abnormal circuit elements, and it still stands unsolved as the most formidable and most important problem before you.

E. T. Moore: I presume the discussion of this paper from an industrial man's standpoint is rather different from that of a distributing company. However, the functioning of a relay is common to both viewpoints and I therefore venture a question regarding the protection of a feeder line carrying energy to an electric furnace. Are the present types of relays capable of giving necessary protection? Certainly no type except the induction type, and even these will not always function in the same time and in the same way in each instance of overload.

N. L. Pollard: I would like to emphasize the necessity for each system to have some one man who is thoroughly familiar with the relay situation to have charge of the relays. It is certainly a waste of money to install expensive relays and then neglect to properly maintain them. The man in charge of the relays should be properly equipped with suitable timing and calibrating instruments.

R. F. Schuchardt: It seems very strange how long it took for us to appreciate the marked effect of these relays on the investment. We were all apparently too willing to wait for the coming of perfection instead of using the best that was available, and having the development take place by experience rather than expecting too much from the manufacturing engineer.

Perhaps five or six years ago, we in Chicago came to the conclusion that we must realize some of that saving in investment which is possible by fairly complete interconnection of the transmission system. We tried what was best at that time. We built an artificial line by using some of the existing lines in part and adding artificial sections in a substation and tried out what was then available. As a result, certain improvements were made and we put the improved devices on our system.

What is now found on the Chicago system is the result of that experiment and because of the extent of that system with its 800 odd miles of underground transmission lines, the experience may be of interest since sooner or later other companies will grow and will have to meet the same problems that we have had to solve.

During the war we were forced to look for more opportunities to save on our investments, so we put into effect more interconnections requiring relays, even though we knew they were not perfect. A description of the system used in Chicago was contained in a paper before the Institute some time ago. Some of you may be familiar with it, and the main elements are contained in this report. The system is a combination of time-element selective relays, unidirectional relays and balanced relays.

It is a rare case now when the relay system fails. We do have times, occasionally, when more than the faulty line is disconnected but those are not by any means a very large proportion of the total failures.

Now, I want to emphasize particularly one statement made in the report with reference to the ability of the men employed to maintain the relays because it is only by having the proper talent applied at that end and learning from their experience that the manufacturers will be able to know wherein to improve.

I would like to see the Protective Devices Committee serve throughout the year as a clearing house. Such a scheme was recently put into effect by the Electrical Apparatus Committee of the N.E.L.A. in which the member companies send to that Committee, descriptions of all cases of apparatus operation that they think might help to indicate in which direction the development should be.

J. C. Parker: I would like to emphasize the remarks of Mr. Schuchardt. The matter of relay protection is one, of course, of distinct importance in the engineering of our power stations, primarily of importance only because it involves insurance of continuity of service, not because of its protection of the apparatus itself, protection of the apparatus is of importance only insofar as it involves the continuity of service.

If we keep that thoroughly in mind, our question as to the relative speed of operation of relays close to a fault or close to the generating apparatus will automatically solve itself.

Mr. Schuchardt called attention to the fact that we are a little tardy in availing ourselves of the best that has been developed to date, possibly through a spirit of false economy or possibly through the desire to wait for the ultimate ideal of perfection in our apparatus. I rather wonder whether that is not due to the fact that as engineers we are too little inclined to recognize the ultimate object in all of our large power stations which, as in all engineering enterprises, is an economic one. There is no factor that is more significant in the capacity of a central station organization to hold and retain business at a satisfactory earning rate than continuity of service.

I have in mind the instance of one corporation which a few years ago was forced to accept a reduction in its receipts for power service from two large customers which, on a five-year contract, aggregated $50,000, a condition which will doubtless continue through the rest of the history of that concern. The reduction in earnings was due absolutely to lack of continuity of service. Obviously, such a concern might have spent easily a half million dollars, if necessary, in securing continuity of operation.

I know at the present time of another very large and very important concern which faces the loss of several important consumers because of its failure to isolate faults through adequate relay protection. If we keep in mind the commercial significance of continuity of service and keep in mind the actual dollars involved in that sort of thing, I believe we will not hesitate to spend fairly large sums of money in using the at present confessedly imperfect protection devices rather than waiting for absolute protection.

H. C. Don Carlos: There is one point brought out in the paper which I wish to question and that is the limit of two seconds which was set as a maximum time for relays. In the Ontario Power Company's plant there has been maintained a time setting of eight seconds on the generators, and on the cables supplying the hydro system from the Ontario Power Company we have maintained a time setting of not less than four seconds. We have never yet had a case of cable failure on account of this long time setting, so our experience would not agree with the recommendation in the paper for short time setting. As a matter of fact on an extensive transmission system it is impracticable, I believe, to maintain such a short time setting of relays, as you will readily see that we could not graduate the relays on long feeders and get selective action with the difference in time that we would have to give to the different relays at the different stations. It has been our experience that we could not maintain a setting of less than three-quarters of a second difference in time between relays and get any results. I do not believe that with present type of relays and circuit breakers they can be depended upon to duplicate their action. The relays are not accurate enough in their time relay and the circuit breakers will take longer to operate one time than another. I would say that this difference in time between the different successive relay settings would be necessary.

D. A. McKenzie: I would like to know how the value of two seconds was arrived at as an ultimate limit. The synchronous apparatus would be off the system before that time had elapsed and the extension of the time limit to allow a relay further out on the system to operate would not increase the damage due to heating very much.

F. L. Hunt: Regarding the matter of maximum time setting on relays, I feel quite sure that circumstances under which we have to operate will control that. We use three and four second settings right along.

F. C. Hanker: The paper prepared by the Protective Devices Committee will go a long way toward establishing uniform practise in relay protection and brings out very clearly and definitely the importance of giving full consideration to this phase of the problem when laying out transmission systems.

The one big advance that is evident is the development of a relay with a dependable time element that maintains its accuracy. As late as 1907 the very lack of such a dependable time element forced the adoption of a control system on the New Haven electrification, involving the installation of control wires over the electrified section. The additional cost was probably on the order of $40,000 to $50,000 and could have been avoided if we had had the modern types available.

The importance of the information that has been submitted is recognized by everybody and it is suggested that the Committee for the ensuing year continue the work and include other applications such as balanced protection where currents are out of phase, as in two-phase three-phase transformer banks. The usual schemes for these involve more or less complicated wiring and special arrangements should be investigated. Similarly, cases where parallel transmission lines are involved, having tap connections at sub-stations, should be carefully analyzed.

O. C. Traver: Regarding Mr. Johnson's comments, it would be very nice to have a system which would reduce the time of operation as you approach the generators, but at the present time in the art there are no relays which will perform this function excepting so far as inverse time delay will accomplish the result. We agree with him entirely on the development of breakers for giving us quicker time. It certainly is something that needs much attention.

While it is true, as Mr. Johnson suggests, that selective relaying must sometimes be handicapped on account of circuit breakers of insufficient interrupting capacity, still the question of cost will always be a deciding factor, the loss in revenue due to poor service and of equipment due to delayed action, being balanced against the increased capital outlay. The evidence submitted by Prof. Parker, though relating primarily to relay equipment, indicates how one can misinterpret the effect of these details on dividends.

On the other hand, the use of current limiting reactors, we consider in general, an aid to good relaying in that they tend to insure ample potential for directional relays and provide a more nearly definite amount of fault current for all conditions of operation.

Dr. Steinmetz stated that the relay problem still stands unsolved. That is true much the same as if the same remark were made concerning any other matter of engineering, relay perfection does not exist. This is also the case with the greater portion of power station equipment in about the same degree. Relays of a reasonable accuracy are available. The question has resolved itself into a matter of application more than of manufacture.

A number have spoken of the maximum interval of two seconds. There has been discussion on both sides, to increase it and to decrease it. The maximum interval is largely a function of the system and the conditions which have to be met.

A two-second delay maximum is considered advisable; first, on account of possible damage to equipment, particularly in the larger metropolitan systems, and second; on account of the greater liability of synchronous apparatus falling out of step.

The damage to equipment may involve overhead lines as well as cables. Arc over of an insulator, if cleared promptly, frequently leaves that insulator uninjured, whereas an additional delay might cause the complete loss of the line at an embarrassing time. The effect of sustained shorts circuits on a cable was brought out in the discussion by Mr. Roper.

SAVINGS EFFECTED BY PARALLEL OPERATION OF FEEDERS

H. C. Albrecht: I do not understand what relation reduction of 40 per cent of the burnout of cables on one particular system has to parallel operation. To me this seems to be but a coincidence. "After parallel operation was adopted, the number of burnouts was reduced by 40 per cent."

E. A. Hester: Regarding Mr. Albrecht's statement with reference to the case cited in which a saving had apparently been effected by operating feeders in parallel it may seem coincident since only one incident was given. However, it has been called to our attention by at least two other companies that as soon as they began to operate their transmission lines in parallel they noticed a very decided reduction of transmission line trouble and consequently a reduction in maintenance cost. The committee made inquiries of several operating companies regarding their experience along this line but most of them replied that they had never made the change over from radial to parallel operation with relays, one method or the other having been used from the beginning. That the same should be experienced by three companies who had made such a change and at a time when their transmission systems were rapidly growing and becoming more complex would indicate that a reduction in maintenance cost at the same time that parallel operation was adopted is more than a coincidence.

TESTING AND SETTING RELAYS

H. C. Albrecht: I question whether or not it is practicable to test on the primary of the current transformer. It is expensive and takes considerable time. I would like to have more information as to where this is being done and in what manner. I am somewhat astounded at the curves in Fig. 2 showing the error in current transformer testing under various conditions. I notice that this curve refers to a single-turn current transformer. It was my impression that one of the standard lines of these transformers could be loaded up to 400 per cent load with only 2 per cent error, while this particular curve shows 70 per cent error for four-times load. I also question whether this transformer was not a bus type transformer, and whether we should not differentiate between single-turn and bus-type transformers, and it seems to me that this is a very poor record for even the bus-type transformer.

PLATE XXXVII
A. I. E. E.
VOL. XXXVIII. 1919

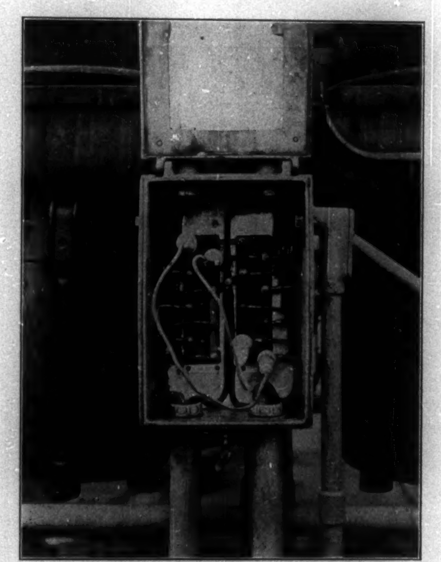
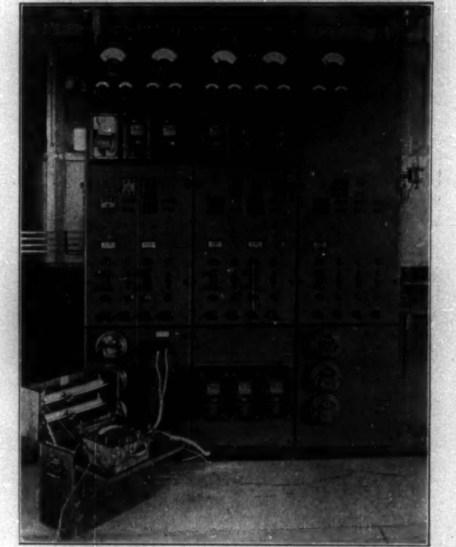


Fig. 1—Pull Box on Oil Switch Showing Test Leads Connected in Shunt to Relay Leads at Current Transformer Terminals



[CONWELL]

Fig. 2—Front of Line Panel Showing Relay Testing Set Connected to Test Leads

PLATE XXXVIII.
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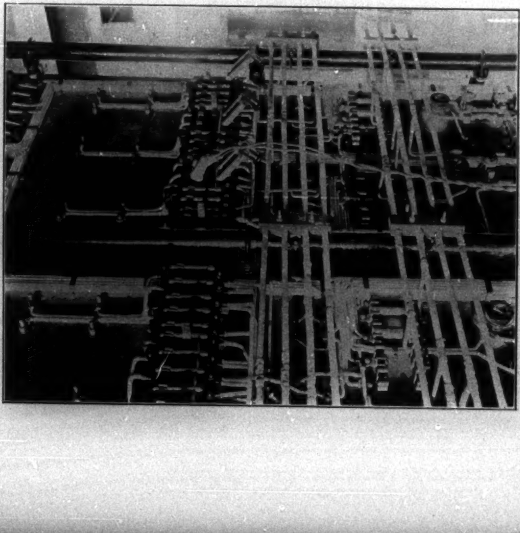
FIG. 3—BACK OF LINE PANEL, SHOWING LEADS TO CYCLE COUNTER CONNECTED TO TRIP CIRCUIT OF RELAY—SWITCH IN CURRENT LEADS CLOSED

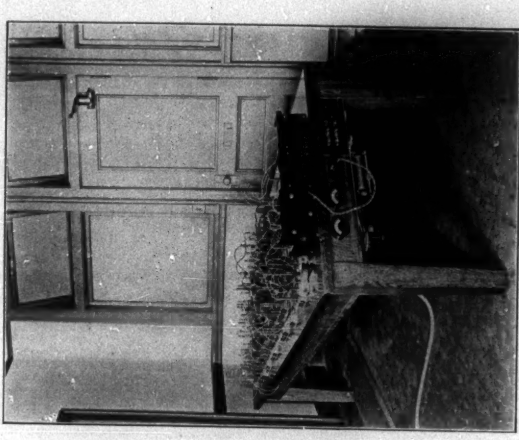
[CONWELL]

FIG. 4—CALCULATING TABLE USED FOR THE DETERMINATION OF SHORT-CIRCUIT CURRENT VALUES

E. G. Merrick: It is not necessary to reduce the time settings of the relays near the source of power below two seconds for protection to the generators and transformer equipment, as a modern generator and transformer will withstand short-circuit current at the terminals for several seconds without difficulty.

The improved relay situation of the Montana Power Company, as mentioned by Mr. Lincoln, is the result of a recommendation of a Committee as taken from test table data and the present operation shows 90 per cent perfect operation. The present main difficulty is due to high resistance grounds, which do not give sufficient current to trip out the relays, and this feature will probably have to receive further consideration in the future.

R. N. Conwell (by letter): The test schedule which should be adopted for relays will probably be best determined by testing the relays very frequently at the start and keeping a record of all tests. An inspection of the test record will clearly indicate the time interval which should be allowed between tests, the interval varying with types of relays, location and duty.

The test method described on pages 801 and 802 has been used on the Public Service Electric Company's system with considerable success on relays connected to high-turn ratio bushing-type current transformers, used in conjunction with power-directional and current relays on 26,400-volt transmission lines. Fig. 1 shows one of the pull boxes mounted on the side of each outdoor oil circuit breaker, the terminals of the bushing-type current transformers being brought to a terminal board to which the relay leads are connected and a pair of test leads running from binding posts on the switchboard, through each pull box is connected to two studs on the same terminal board in each pull box. The relay may then be tested in shunt with the current transformer by connecting the testing supply with necessary control apparatus and meters to the binding posts at the switchboard See (Figs. 2 and 3) and at the oil switch, connecting with jumpers, the test leads to the studs, to which the current transformer leads and relay leads are connected.

Frequently, relays are installed and cut into service without any final check, other than inspection, being made of the wiring. Experience has shown that even when a standard color scheme is used for the secondary leads, in fully fifty per cent of the new installations, the relays will be found to be cross-phased, a ground connection improperly located, or some other mistake made which will result in faulty operation. With the complicated wiring necessary for the more recent applications of balanced protection, the problem of phasing out becomes difficult.

A single-phase, four-quadrant power factor meter, reading from zero to unity, lag and lead, has been used for work of this character, the results obtained being such as to warrant the general adoption of this test method. If currents are to be phased, the potential coil is energized by connection to any convenient 110-volt a-c. source of the required frequency. The current coil is then connected successively in each circuit to be phased, direct and reversed readings being taken from which the relationship of the currents may be determined. Currents in phase will give the same deflections on the power factor meter. If check readings are not obtained the vector diagrams may be drawn from the readings and the error in wiring determined. Although the meter used has the usual cosine scale, a scale marked in degrees would eliminate the necessity of converting cosines to the equivalent angles.

For the phasing of potentials, the current coil of the meter is energized with an artificial load, test current being obtained as above, the potential coil being connected to the circuits to be phased.

Two conditions must be fulfilled to obtain satisfactory results: (1) the excitation of the meter coil used for reference must remain constant in direction and value throughout any series of tests and (2) sufficient torque obtained to give correct indications on the meter. Generally, half load on the meter with full potential is sufficient.

C. P. Osborne (by letter): There is not enough attention paid to checking and time-setting of relays by most operating companies and unless this is followed up by one certain engineer they are very apt to be neglected. The regular routine testing is, in my opinion, the only way to keep relays in a workable condition. The inspection of relays should be made by every operator on shift every day. This is just as important as it is for the operator coming on shift to look over the minutes and discuss with the operator going off shift any difficulties which he may have had or which may be on at the time of the change in shift. Relays should be tested under operating conditions wherever possible, thereby not only checking the apparatus under load conditions, but also giving the operator a chance to get familiar with the workings of the apparatus. The writer does not believe that watt-hour meters should be connected in series with relays where it can be avoided.

N. A. Carle: I think the method outlined will be of assistance in eliminating a few more interruptions due to inaccuracies in current transformers.

D. A. McKenzie: One gentleman has questioned the accuracy of the curve in Fig. 2 with reference to the operation of current transformers. I do not get the distinction between the single-turn current transformer and the bus-bar type but would think that both of these transformers would have similar characteristics. In my opinion the 75-ampere current transformer of a single-turn type would not give satisfactory results and should not be used in any relay protective scheme regardless of voltage rating or secondary load, unless possibly all the current transformers in the series were of the same type and carried the same secondary load.

I would like to ask if any company has made an investigation of the standard instrument-type current transformer under short-circuit conditions to determine the comparative shapes of the wave form on the secondary current as compared to the primary and also to determine the phase relation between these two. With a current transformer carrying a secondary load of 40 volt-amperes at five amperes secondary current and operating at twenty times full load and attempting to deliver 16 kv-a. the iron in the transformer would have been saturated long before this limit was reached and the wave form considerably distorted. This would have some effect on the use of directional relays containing potential elements. It would appear to me that wave distortion would affect their operation.

I do not believe it is practicable to set relays from the primary side of the current transformer and it should not be necessary to subject all your equipment to short-circuit strains for periodic testing.

I think that the current transformer may be tested at relatively low values of current and ammeters placed in series with the secondary winding will bring an indication of the normal operation of the current transformer and continuity of secondary wiring. It might be desirable to test the current transformer each time after testing the relay, and I believe that a relay should be tested immediately after each operation regardless of the time of the routine test.

The maximum indicating ammeter as mentioned by Mr. Craighead is a desirable piece of apparatus for obtaining operating conditions under short circuit.

H. R. Woodrow: Some of our most severe system disturbances have been due to failures in current transformers and we have therefore adopted the single-turn type of transformer for our generating stations where very heavy short-circuit currents are to be expected.

The record shown in Fig. 2 of the paper is for a standard 75-ampere single-turn current transformer without any special precautions being taken for accuracy and although this is an unusually low current for a single-turn transformer it is a type which is to be found on the system. For standard 400-ampere single-turn type of transformer the error would be in the order of four to five per cent. The accuracy is largely dependent upon the amount of secondary loading and, as a matter of fact, the length of the secondary wiring changes the characteristics of the relay settings even where identical relays are used. We have even found conditions, where the secondary loading is so high on a low-rated single-turn current transformer, that the relay was practically inoperative as insufficient current could be obtained due to the saturation of the current transformer.

As pointed out by Mr. McKenzie, a series ammeter in the secondary circuit of the current transformer will indicate continuity of service but it will not show the change in the numerous contacts in the circuit which may come loose and thereby require abnormal secondary loading of the current transformer which will entirely change the characteristics of the relay setting.

We have made investigations determining the wave shape of the secondary current of a current transformer up to 200 amperes (secondary current) and found that the wave became very peaked but with the installation of directional relays, if proper potential connections are made, this wave distortion will not change selectivity of direction, which is all that is required of a directional relay

The single-turn and bus-type transformers are in the same class giving considerably greater mechanical strength but a tendency to poorer accuracy especially in the lower current types.

I agree with Mr. Merrick that most generators and transformers are capable of withstanding short-circuit currents for a period of two seconds as regards heating but many are not capable of withstanding the mechanical force produced under short-circuit currents.

As pointed out by Mr. Conwell considerable care should be taken in the connections of directional and balanced relays as an improper connection will invariably result in failure of proper selectivity.

O. C. Traver: I might say concerning the bushing transformers that the single turn is considered just the same whether it is directly around the core or whether it goes around by a circuit of several miles; the difference is so slight that it is scarcely noticeable.

I would like to make a plea in favor of using a higher turn ratio on bushing transformers. From the manufacturers' standpoint, time after time we have to ask the operating company if we cannot use a higher tripping point for the relays. He may think that we are trying to put something over on him, but almost invariably it is for this same reason which has been brought out several times this morning—namely, that the accuracy of a bushing transformer with about 70 amperes turns (that is, 70 amperes with a single primary turn) is exceedingly poor, especially when used with a time relay. If instantaneous action is involved, it does not make much difference. So, in general, if you can start at 150 amperes for your minimum operating point when using your relays with a bushing transformer, you will begin to get reasonably good results.

If you can, as Mr. Woodrow has just mentioned, use 300 amperes instead, you will begin to get accuracy equal to the best accuracy you can get in an instrument transformer, and sometimes much better on account of the much greater amount of iron that is used in the single-turn transformer.

DETERMINATION OF RELAY SETTINGS

H. B. Vincent: The paper speaks of the successful relay settings being three-quarters of a second. I would like to ask if this is sufficient in all cases. Is it understood that the Consolidated Gas, Electric Light & Power Company of Baltimore, and others use a shorter value very successfully, and I also raise the question—"Does this entail anything special in the relay or in the testing of the relay and breaker?"

J. R. Craighead: The improvement that has been made within the last few years in the accuracy and speed of operation of oil circuit breakers has rendered the shorter times spoken of by Mr. Vincent practicable in a number of cases, and in order to obtain the benefit of these shorter times a corresponding improvement in relays is required. This improvement has already made progress but there is still room for further advance along that line in order to keep the oil circuit breaker, and the relay in such relation that we will not waste time in either. I believe it is quite practicable to cut the differential of time somewhat below the three-quarters of a second mentioned with satisfactory results.

A thorough understanding of conditions is the most necessary thing in the application of relays, and the difficulty of getting that understanding has been mentioned in the paper. The best way of getting the data is the practical method of obtaining actual measurements when possible. Until recently, this has been impracticable under operating conditions. By the development of a new instrument called an "Impulse Ammeter," records are now made of the approximate highest effective value reached during a short circuit on an operating line. This device is an indicating ammeter with a current scale high enough to cover the short-circuit current value. Under the pointer is a record paper which under short circuit, is punctured by a spark discharge from the pointer, thus recording the highest reading of the instrument during the short circuit.

H. C. Albrecht: The Philadelphia Electric Company adopted a short time ago the use of the cycle counter in setting relays, and since that time on our 60-cycle relays we have been using a time interval of twenty-five cycles or five-twelfths of a second. That is based on allowing fifteen cycles or a quarter of a second to the oil circuit breaker operation, and ten cycles in addition for inaccuracies in relays, etc.

I would like to question whether too much attention is not paid to reduction in bus voltage on short circuits, as our experience has shown that the sustained short-circuit value is not very much reduced from the instantaneous value.

Regarding the question of whether to set relays for short-circuit conditions, I question whether it is necessary to test at currents approximating maximum short circuits, as most relay curves flatten very much before this point, and it is the practise of our company, for instance, to set the relays at the point where the curve starts to flatten.

N. L. Pollard: Mr. Don Carlos questioned the minimum setting of relays on lines in tandem. The Public Service Electric Company maintain a difference of four-tenths of a second between the settings of successive relays and get very satisfactory results. It would seem to me that one-half of a second would be sufficient for almost any type of relay that we have on our system.

Will Mr. Don Carlos tell us what type of relay he referred to? The relays that we are using are of the induction type, and we have no difficulty at all in operating with a difference in the time setting of four-tenths of a second.

R. N. Conwell: The Public Service Electric Company is using a calculating table (See Fig. 4) similar to that referred to on page 804. Such a table is necessary if successful relay operation is to be obtained on large interconnected networks. In many of these systems the mathematical calculation of the short-circuit value of current is practically impossible, or at least, very tedious. The table quickly supplies information relative to the effects of changes in operation or design. The work of making a complete analysis of a system is not much greater than that necessary for relay settings so that the rupturing duty required of circuit breakers and other valuable data may be obtained with the same set up and at the same time.

R. F. Schuchardt: In connection with the time-element relays used in Chicago, our cable breakdowns now are more severe than they were in earlier days before we had so much power or so much ability to get energy into a fault. That may be due to the increase in power or to the increase in time setting or perhaps both.

D. A. McKenzie: Referring to Fig. 3 of the paper, it is interesting to note that selectivity between breakers *B* and *C* changes at about 4200 amperes and I would like to inquire if the breaker capacity was the determining feature in this. Should it not be necessary to have those curves so adjusted that they do not cross.

I was interested to see that three-quarters of a second was considered a permissible interval between relays. That appears rather short on account of the different types of circuit breakers which would be used, some of which may operate faster at one time than another, depending upon adjustment since last operated and to mechanical condition.

I believe that the relay problem for protection of electric steel furnaces is an important one and can be solved if the wave form of a primary current and the distortion in current transformers under short-circuit conditions are considered.

H. R. Woodrow: Several have questioned the advisability of allowing three-quarters of a second time interval between consecutive relays for selective operation as shown in curves of Fig. 3. There are a number of conditions which effect the allowable time interval. First, the type of relay, its characteristics and reliability; second, time of operation of the oil switch (time from closing of relay contacts until switch ruptures the arc); and third, the care with which the relays are set and kept in operating condition.

As pointed out, several companies have found a shorter time allowance desirable but I believe the conditions require a very careful analysis, especially where a time allowance below three-quarters of a second is considered.

It is interesting to note from Mr. Craighead's discussion that a maximum indicating ammeter will soon be placed upon the market which will give the operating companies a more accurate indication of the currents experienced under short-circuit conditions.

I would consider that if the relays are calibrated out to the flat point of the curve, as mentioned by Mr. Albrecht, it would give conditions approximating those under short circuit. We do not feel however that proper selectivity can be obtained unless the calibration point is carried out to this value.

Referring to Mr. McKenzie's questions regarding the changes in characteristics of the different circuits, shown on Fig. 3, would say that these characteristic curves were made in this manner to reduce the time which a short circuit would be held on the system near the generating station and still give selectivity with the lower values of current when a short circuit occurs at the end of the line. The dotted line in curve 3 shows the current and selectivity when the short circuit occurs on the end feeder.

METHOD OF TRIPPING

A. H. Lawton: In connection with the statement that the use of circuit-opening relays for alternating-current tripping has not given very satisfactory results, I beg to inquire what has been the experience with automobile storage batteries for tripping purposes? Has any contact difficulty been experienced due to low voltage, or has the system been entirely satisfactory?

N. A. Carle: The previous speaker asked about the use of the automobile storage battery. We have been using automobile storage batteries in a 1000-kv-a. substation for about two years and have had very good satisfaction.

H. C. Don Carlos: In regard to the use of automobile storage batteries for a source of tripping current, we have used these rather extensively for the last two years and we have had very good results with them and we have recently used the ordinary dry cell for tripping current very successfully and the indications are that the maintenance cost on the dry cell will be very much lower than with the storage battery and will give equally as good results.

I might say that we have used both the lead cell and the Edison battery for tripping current and I don't know that we are in a position to say just at the present time whether one would give better results than the other or how the maintenance and operation charges would compare with the two.

D. A. McKenzie: I believe that the lead cell is going to be more economical in maintenance than the Edison cell, but as far as my experience goes the dry cell will be quite satisfactory for the purpose of tripping switches in isolated stations. I believe that there are some companies that are using dry cells for tripping even switches with 125-volt direct-current control with, as far as I know, perfectly satisfactory results.

H. B. Vincent: No mention is made in the paper of using circuit-closing relays with alternating current by introducing an auxiliary relay.

E. A. Hester: Those who have expressed themselves on the subject are apparently of the same opinion as regards the use of the automobile storage battery as a source of tripping current. And in so far as we have been able to determine from cases which have come under our observation the service obtained has been very satisfactory.

Referring to Mr. Lawton's question as to whether difficulty is experienced with contacts at such low voltage, we have known of but little trouble from this cause and when such trouble has appeared it has been overcome by means of a small auxiliary switch so constructed that a very positive and high contact pressure was obtained. This solved the difficulty and yet allowed the use of sensitive relays for selective action.

As regards the maintenance and operation of lead cells and Edison batteries we do not have comparative costs, though as stated by Mr. McKenzie, it is very likely the lead cell will be more economical. It is also very interesting to note the statements made by Mr. Don Carlos and Mr. McKenzie that dry cells have actually been used for tripping purposes and have given satisfaction even on 125-volt control circuits.

It is stated by Mr. Vincent that no mention was made in the paper of using circuit-closing relays with alternating current by introducing an auxiliary relay. From this we understand that he refers to tripping directly from current transformers on the power circuit. The chief reason for not including this was that a complete description of the relays used and of direct-trip relay schemes would have been necessary in order to make such a discussion of any value. Therefore since the Protective Devices Committee hopes to conduct some investigations on special problems, among which is that of direct tripping, it was decided not to try to incorporate this in the paper. However we may say that while several satisfactory methods and schemes have been worked out, direct tripping is not yet on the same footing with the use of a separate source. In our opinion it is possible to devise some means of utilizing the energy from current transformers which will be satisfactory for tripping. The circuit-opening scheme has never proven very satisfactory, more because of contact trouble than any thing else, so there is evidently only the alternative of using a circuit closing relay with auxiliary device.

One scheme which has been used with a certain amount of success is that of taking the tripping current from a voltage transformer connected to the station bus. The disadvantage is that when trouble occurs the voltage is likely to fall so low that sufficient energy can not be supplied by the transformer to trip the circuit breaker. It would therefore appear that since such a scheme is likely to fail on the worst cases of trouble, just when it is most needed, its use would never be justifiable.

TYPICAL APPLICATION OF CURRENT DIRECTIONAL RELAYS

D. A. KcKenzie: The study of this paper indicates that there is apparently no field for the definite-time relay, even for protection of parallel groups of lines in tandem or for the ring system, which cases would appear to me to be the most logical places to use this type of relay where progressive time settings are required. I am of the opinion that the definite-time relay should not be used in transmission line protection but it is a question in my mind whether the definite-time relays should not be used for generator protection or for protection against bus bar troubles. The current setting would be very large under these circumstances and would be determined by the sustained short circuit obtainable from the generator.

E. A. Hester: Mr. McKenzie states that in his opinion definite-time relays should not be used for transmission line protection. We understand this statement to refer to relays having a strictly definite-time element and agree with Mr. McKenzie from this point of view. However it must be remembered that practically all relays now manufactured are made with a minimum definite-time element and in most cases this is adjustable. This feature has proven itself to be very valuable and while not a definite-time relay in a stricter sense of the word it is often applied as one, current setting being made at such values that operation is on the definite part of the curve. The progressive time settings are obtained by adjusting the definite-time element.

As regards the use of definite-time relays for protection against generator or bus bar troubles we believe it best to use some form of a definite-time relay though the time interval allowed should not be very long. As stated by Mr. McKenzie the current setting will necessarily be very high and for this reason an adjustable minimum definite inverse-time-limit relay can be used to an advantage as the operation will come on the definite part of the curve.

BALANCED PROTECTION SCHEME FOR TRANSMISSION LINES

H. C. Albrecht: On balanced protection in order to obtain continuity of service for two parallel lines, without the discriminating method as shown in Figs. 6, 7 and 8 of the paper, it would be necessary to use four lines between stations, which I do not believe is feasible in many cases.

The discriminating scheme shown in Figs. 9, 10 and 11 for the operation of two or more parallel lines, seems to be full of merit, particularly where the feeders are used between generating stations.

I would like to hear something in the discussion regarding experience with split-conductor cables (which seem to be rather expensive) being used by only two systems. I would appreciate having first-hand information as to this experience up to date.

D. A. McKenzie: The mechanically opposed type of differential relay has not been mentioned and I know of one instance where this type of protection is operating very satisfactorily. This type of relay is not complicated with any potential elements and therefore should have a considerable field of application.

The balanced type of relay used with the split-conductor cable system seems to give quite satisfactory results, although it is handicapped by considerable increase in the cost of cable as the company using it is extending this system considerably.

N. L. Pollard: Someone raised the question as to what results had been obtained with split-conductor cables. We have ten split-conductor cables in service at the present time, and the relay operation to date has been perfect. Since these cables were put in service with proper relays, several insulation failures have occurred and the operation of the relays was perfect in each case. These failures occurred not only between the outer conductor and sheath but also between the inner and outer conductor of a phase.

R. N. Conwell (by letter): It is to be noted that the trend of relay development is toward the adoption of the balanced principle as affording the best method of sectionalizing trouble and cutting the faulty feeder out in minimum time. At the present time, however, most of these schemes are open to one or more serious objections and the limitations of each scheme should be carefully considered before application is made. Some of these objectionable or limiting features are as follows:

1. There seems to be a sharp line of demarkation between protective schemes applicable to paired lines and those applicable to three or more lines, requiring an entirely new installa-

tion of relay protection for three lines when the number of lines in parallel is increased from two to three.

2. The balanced principle applied to parallel feeders will not, as a rule, clear bus or other faults originating within stations or substations, since the fault current under these conditions generally divides equally between the lines, thus maintaining a balance. Additional over-current protection must be provided if faults of this character are to be cleared automatically.

3. When applied to paired radial feeders, some balanced schemes fail with the occurrence of a fault at the substation end of a feeder, due to equal division of fault current between feeders.

4. When practical application is made of seemingly simple balanced relay schemes, the resulting secondary wiring frequently becomes complicated, introducing other weak links in the chain of protective apparatus. Certainly the tendency should be towards simplicity in apparatus and wiring, if ruggedness and uniform performance is to be obtained.

We have under construction or in operation on the Public Service Electric Company of New Jersey system four complete installations of relays, with several modifications in two cases, which may be considered as on trial. It is hoped that sufficient data will be available after a year's operation to make an accurate comparison of the relative merits of the installations. Such comparison is not possible at the present time, owing to the difficulty in obtaining correct settings in one installation, and lack of operation in two others, the fourth being under construction.

A. W. Copley (by letter): A type of transmission system not covered by the authors, but which is frequently met with, has involved the development of a protective layout possessing certain features which can be advantageously applied to the more usual type of system involving parallel feeders.

On certain transmission lines, particuarly those for the transmission of power to railway substations, located rather close together, it is desirable to operate the substation bus on taps from the transmission lines, rather than to sectionalize the lines. This introduces a problem, which is not taken care of by the schemes mentioned in the paper. It is necessary in this case to disconnect one transmission line, in case of trouble due either to ground or short circuit, from the bus at every substation, as well as at the ends of the lines. Power directional relays used in connection with breakers between the lines and bus will adequately care for short-circuit conditions, as a short circuit on one line will draw power from the other line through the substation bus, and power flow through the breaker between the affected line and bus will be in the reversed direction from normal.

For the solution of the ground protection problem, balanced protection between the two lines is unreliable on account of the possibility of the normal currents in the two lines being unbalanced. The desired protection may be obtained by the use of power directional relays, connected differentially between the conductors from bus to line and with potential coils supplied by the voltage between an artificial neutral and ground.

With this connection, true directional action takes place on the occurrence of a ground, as the direction of the potential applied to the relay is always dependent upon the particular conductor which is grounded. For instance, on a single-phase system, if a ground occurs on one conductor current in the differentially connected relay will be in one direction, and if the ground occurs on the other wire it will flow in the opposite direction. The potential coil, when connected from neutral to ground will have its potential reversed in a corresponding manner under such conditions. The arrangement for a single-phase circuit is shown in Fig. 5. Three-phase protection will be similar to it. The scheme has been used very successfully on the electrifications of the P. R. R. between Philadelphia and Paoli, and between Philadelphia and Chestnut Hill.

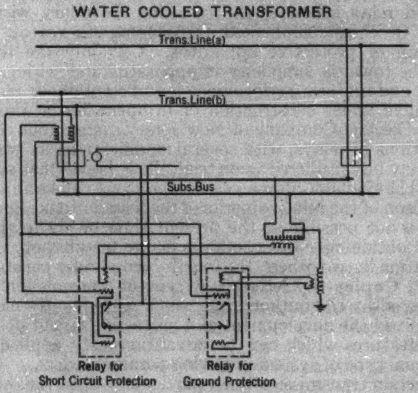

Fig. 5

In order to apply the scheme it is only necessary to establish an artificial neutral at the substation by the use of potential transformers or resistances and to connect a potential transformer from this point to ground, making the connections as indicated. It will be noted particularly that there is neither current nor potential applied to the relay when the lines are clear of ground, but when a ground occurs both current and potential are simultaneously applied in the proper direction to cause selective action.

The scheme is also applicable to the protection of ordinary parallel transmission lines when the neutral of the system is grounded through high resistance, and consequently the current flow incident to the occurrence of a ground may be so low that it is of the order of the unbalanced current which might be normally expected in the parallel lines. In such a case the protection, as shown by the authors in Fig. 9, 10 and 11, will not be satisfactory, but the scheme which has been described will give full protection.

O. C. Traver: With reference to Mr. Albrecht's remarks concerning the balanced scheme shown in Fig. 6 it might be mentioned that this arrangement is applicable where the four lines are installed for other reasons. In the instance cited in the paper the substations were in loop formation and each bus was connected by a pair of lines to each of two different stations from which power was normally available. All the lines were primarily needed for service.

Mr. Conwell states that for balanced schemes an entirely new installation of relays is required when increasing the number of lines from two to three. This is true of some schemes but not of all. Space did not permit of a description of the various possibilities in detail in the paper although in some cases references were made to results which would be obtained with varying numbers of lines in service. For example the scheme illustrated in Figs. 12 and 13.

Regarding Mr. Conwell's objection to complication in balanced schemes, it should be borne in mind that when transmission systems become more complicated, successful protective equipment must also involve more complication. This may be in the primary connections, the secondary connection or in the contact and trip coil connections. The degree of this complication and the least objectionable variety must be decided in each case on its own merits.

The scheme described by Mr. Copley was not reported in the replies to the questionaire. As the intention of the paper was to catalogue actual experience any scheme which did not have some operating company to stand sponsor was not included. We are greatly indebted for this addition which otherwise could not have appeared in the record. The mechanically balanced relays referred to by Mr. McKenzie were not mentioned for the same reason.

COMBINATION OF UNDER-VOLTAGE AND OVER-CURRENT RELAYS FOR PROTECTING TRANSMISSION LINES

J. A. Johnson: In connection with the combination of undervoltage and over-current relays, I would like to suggest a modification which appears to me to be simpler and less expensive in application. Instead of having the under-voltage relay operate locks on each individual circuit breaker, let it simply control the supply of current to the tripping bus. The circuit breakers are then left free to be operated by the control switch, or by hand, but cannot trip automatically unless the voltage relay has operated to connect the tripping bus to the source of tripping current.

This plan, in its essential features, is in use by one of the power companies at Niagara Falls in connection with its ground relays. The system is a radial one, each feeder having a relay connected in the neutral of its star-connected current transformers. This relay is selective on grounds, the generator neutral being grounded through resistance but the relay may also operate in case of single-phase switching between two feeders at the receiving end.

To obviate this, a relay was connected in the generator neutral, arranged to control the supply of current to the tripping bus. No feeder switch can be tripped by its ground relay unless current is actually flowing in the generator neutral.

It seems to me that this system of combining the effects of under-voltage and over-current for relay operation has great possibilities, and is worthy of more general application than it has yet had.

D. A. McKenzie: As far as I know the application of a master relay for similar purpose at the Ontario Power Company's plant at Niagara Falls has been very satisfactory.

E. A. Hester: We are much interested in Mr. Johnson's proposed modification of this under-voltage and over-current scheme and can see no reason why it will not give the same results as the one described in the paper. If anything the modification is more desirable since it leaves the circuit breaker free for manual operation which is certainly one thing in its favor.

It is interesting to note that this scheme of controlling the trip circuit by means of a relay has actually been successfully used on another application. We are familiar with the ground relay scheme although we have never heard of any trouble due to single-phase switching. However it is easy to see that the installation of a relay in the generator neutral to close the control circuit would prevent tripping from this cause.

We agree with Mr. Johnston in his opinion that the possibilities of combination under-voltage and over-current protective relays are great and believe the next few years will bring forth developments which will overcome its present difficulty and make its application more general.

GROUNDING THE NEUTRAL

P. M. Lincoln: I note in this report that apparently the efficacy of the grounded neutral has been fairly well demonstrated. There are two cases which have been cited in this report in which testimony is given that grounding of the neutrals has given better protection and better operation. The two cases cited today are those of the New York Edison Company and the company out in Toledo.

This has been a thing which I have studied for many years and have advocated, that is, the grounding of the neutral. It is my opinion that reliability of service can be very considerably increased if the neutrals of the systems be grounded and the larger the system the more desirable such a move becomes.

I happened some years ago to personally come into contact to considerable extent with the system of the Montana Power Company. That system has been growing rapidly. It is the system, as you all know, which supplies power to the Chicago, Milwaukee & St. Paul for their electrification across western Montana. Years ago they operated delta-connected transformers and ungrounded. At first they had very little trouble, when the system was small, but as the system increased in size and capacity they began to have considerable difficulty. A ground or a short circuit at one point would apparently cause a blow-up of apparatus maybe hundreds of miles away and after studying the matter, the engineers of that company decided to adopt the grounded neutral for their system. They did this about 18 months or two years ago and the testimony from the engineers operating that system has been very definitely to the effect that the troubles which they have had have been very materially decreased on account of the presence of that grounded neutral.

F. D. Newbury: I wish to say a word in favor of the use of the grounded neutral from the standpoint of generator protection. In advocating the grounded neutral in order to protect apparatus, we have often encountered the feeling on the part of some operators that this increased protection was only secured at the expense of more frequent interruptions to service. That was an argument that a generator designer could not very well combat, so I am particularly pleased to see in both of these papers evidence that grounding neutrals is an advantage, even from the standpoint of service. With that argument removed, I hope that there will be a continued growth in the practise of grounding neutrals.

N. A. Carle: In regard to grounding the neutral, I would like to ask if anyone has had experience on moderate sized systems entirely overhead, operating at 13,000 volts with the grounded neutral, and if such systems have been using the ground wire on their lines or not.

R. F. Schuchardt: This paper by Mr. Woodrow reminds me very much of the vote by states on an amendment to the Federal Constitution. The system in Chicago was, I think, the first state to ratify the grounding of the neutral. Back in 1901 and 1902 certain investigations made by George Eastman, at that time head of our testing department, brought out very conclusively to our minds that grounding the neutrals of the system was the only proper way to operate. One by one the other states have ratified and even New York is now in the fold.

It might be worth while to state that the system in Chicago is sectionalized, that is, we limit the total capacity which we connect to a single section, separating the sections through reactors. That limit at the beginning was 50,000 kw. and now is at least doubled and may grow as the system grows. On each section we ground only one of the units in operation. We connect each neutral to a neutral bus which is connected to a ground through approximately two and one-half ohms resistance.

Originally we grounded solid but our experience soon indicated that with the increased capacity, the destruction and general disturbance resulting from a fault became very undesirable and we have found from perhaps fifteen years' experience that that amount of resistance in the neutral, combined with the relay system which we now use, is quite satisfactory.

E. E. F. Creighton: In regard to the grounded neutral, going back to history, the following may be of some interest to those who advocated a non-grounded neutral under certain reservations. The arcing ground suppressor which was available a good many years ago made it possible to operate a non-grounded neutral system with greater continuity of service because of the fact that you could operate the system for any length of time if that neutral was solidly grounded by a switch. In that development, which was aided by the Public Service Electric Company of New Jersey more than any other company, the radial system was in use and extremely good results were reported from the use of the suppressor.

For reasons of economy, as have been pointed out in the papers today and as all the operating engineers know, lines are being connected together at both ends and in loops, and no one has yet devised a way of localizing where the arcing ground is taking place, unless a radial system is used.

As Dr. Steinmetz pointed out, there is nothing in the current anywhere that would indicate which one of the many feeders was grounded. Therefore, the method of protecting by the use of the arcing ground suppressor fails because it is impossible to open the circuit that is at fault. Since all systems are going more and more to closed loops in one form and another, the arcing ground suppressor has not been found a permanent solution.

Naturally, with a grounding of the neutral the practise has gone to the suppression of the arc by the use of a fuse in series, following the same principle and practise as was first outlined in the arcing ground suppressor. In that development, however, it has been found impossible to use a fuse, since the fuse takes a time to operate depending upon the amount of power or the amount of current sent through it. When the system is heavily loaded and all the power is available, the fuse will blow in a very short time. That is the time when the fuse should hold the longest in order to extinguish the greatest volume of arc. On the other hand, at some odd hour of the night, or of the week, the power is at a minimum, the same fuse will hold for an abnormally long time; in fact, long enough to trip the circuit breaker so it has been found that the fuse is undesirable.

F. L. Hunt: About a year ago we grounded the neutrals on our system and we feel quite certain that the operation of the relays is much more definite and positive than it was before.

N. L. Pollard: It might be interesting to some of you to know that we are going to ground the neutral of the Public Service Electric Company's system at Newark, N. J. We came to a decision about three months ago that by grounding the neutral, we would eliminate many of our troubles.

About seven or eight years ago we seriously considered grounding the neutral, but since our system was operating differently at that time, and since we would have been obliged to establish several artificial neutrals. We finally decided against grounding. At this time, we installed an arcing ground suppressor, which apparently eliminated some of our troubles for the time being, at least. As our system grew larger, our troubles naturally increased until we decided recently that grounding the neutral was absolutely necessary.

We have two hundred and fifty miles of cable and four hundred miles of overhead lines in the Northern Division, which is the part of the 60-cycle system which we anticipate grounding.

A. E. Silver: I want to add a word of testimony to the benefits of the grounded neutral system. In my work, I have been associated with, at least three systems that were initially constructed and operated isolated from the ground which have been changed over to grounded neutral. In one case, with a Y connected system it could be done very easily; in the other two cases it was necessary to put on artificial neutrals. However, in each case the results have been very good indeed. We have gotten much better service, fewer punctures of insulation, fewer breakdowns of bushings and so on, so that we feel very thoroughly convinced from our own practise that the grounded neutral is practically the only system for high-voltage transmission. I might say that these systems that I have mentioned were each of 60,000 volts or higher.

F. C. Hanker: The paper analyzing the operation of the New York systems and giving the actual results for periods prior to and immediately following the installation of neutral grounding resistances is of particular value as confirming the contention of engineers relative to the advantages of neutral-grounding. The relay committee has presented a paper outlining the more general relay problems and it is suggested that they extend their activities to include a survey of the different systems that are operating with the grounded neutral.

There has always been a hesitancy among operating engineers to grounding the neutral due to the feeling that continuity of service would be seriously affected. That is one feature that is easily understood, whereas the contention that the operation of an ungrounded system with a partial ground was dangerous and likely to have more serious results than the interruption of service on individual feeders was not so self evident.

The information given in the paper prepared by the Committee on "Relays and Protective Devices" would indicate that the operators are recognizing the condition that exists. We believe that a review of the experiences of these various companies as suggested, will be of advantage in convincing operators on systems that are ungrounded of the advantages of this method of operation.

Four different methods of grounding the neutral are described. The first scheme using zigzag transformer and limiting resistances operates on relatively small currents and is applicable to systems where sheath-type transformers can be used. A modification of this arrangement using three current transformers connected in star and arranged with a low-current relay in the neutral connection will give similar operating results.

The second method outlined by Mr. Woodrow is probably one that is more generally used, the resistance varying from zero, corresponding to a solid ground, to six to eight ohms.

Probably the reason this method is the one most generally used is that it can be applied to existing systems with minimum change. It involves, of course, the use of current transformers in all three phases of each feeder in order to insure protection of all conductors.

Where the feeders are of relatively small capacity the adoption of this scheme does not require excessively heavy ground currents and avoids unnecessary damage to apparatus and cables in cases where grounds occur. A combination using this method on the small current feeders and adopting the relay connected in the neutral of the current transformers will probably meet the needs of the majority of installations and still avoid radical changes in the lay-out.

H. R. Woodrow: Mr. Hanker states that the three current transformers connected in star, with a low-current relay in the neutral, will give results equal to the special grounding transformers and I would like to point out that extra precautions are necessary as the third harmonics, produced in the three current transformers star-connected will cause considerable current to flow in the neutral, and although this arrangement is practical, special precautions in regard to this must be taken.

On a high-resistance ground where they have had trouble with the standard relay protection, I feel that a special ground relay should be given very serious consideration. This is very simple to do and we have adopted it throughout our system, with very good results. The grounded feeders are disconnected from the system before short circuit develops in 85 per cent of the cases.

The method of grounding is dependent to a large extent upon the system. We have shown four different methods in which we have grounded various parts of our system, and in each case the method of grounding was adopted because of the special condition under which that system must operate. It might not be clear that in some of these cases the method will operate only on a radial system.

In Fig. 3 the grounding there by means of potential transformers can only be accomplished where the feeder is isolated at both ends by power transformers. The method as shown in Fig. 1 necessitates the use of a ground relay in the substation, where everything is in parallel operation.

ANNUAL REPORTS OF TECHNICAL COMMITTEES

ANNUAL REPORT OF THE POWER STATIONS COMMITTEE

To the Board of Directors,

IN answer to the request of the Board's Committee on Technical Activities that suggestions be offered for improving the technical work of the Institute, we incorporate our suggestions in this Committee's report for the year 1918–1919:

(A) AS TO PLAN OF WORKING OF TECHNICAL COMMITTEES

Can improvements be made in the co-ordination of the work of the authors of Institute papers and that of the Technical Committees?

In the past the great majority of papers in our PROCEEDINGS were secured either from direct offers of authors or through the solicitation of Committees.

Lately it has been the custom to refer every paper, before acceptance, to the Meetings and Papers Committee, which practically consists of a Chairman and a Secretary, with the Chairmen of the Technical Committees as members. Unsolicited papers are referred by the Meetings and Papers Committee to one of its members for endorsement or criticism, the member to report being the Chairman of the Technical Committee most interested in the subject of the paper in question. Papers solicited by a Technical Committee are accepted upon the recommendation of its Chairman, without previous submission to the members of his Committee, except by title.

In practically all cases, therefore, papers are not submitted and discussed by any Committee in advance of presentation before the Institute.

Hence, in the past, with few exceptions, the main work of the Technical Committees has been confined to secure especially qualified authors to write papers on subjects in which the Committee was interested. While the papers are nominally submitted under the auspices of this or that Committee, in fact the Committee itself never has an opportunity of acquainting itself with them. It follows that the discussion in the open meetings is greatly made up by comments and criticisms of

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NOTE:—No reports made by Traction and Transportation Committee and Electrical Machinery Committee.

members of the same Committee under whose auspices the paper is presented.

The suggestion arises: *Could not material advantages be secured by the Committee discussing the paper before it is presented at the general meeting of the Institute?*

In fact, should not the Committee share in the responsibility with the author or write, as an introduction, a brief statement of the opinions and conclusions of the Committee, either endorsing the author's paper in full or qualifying any point of divergence, giving its reasons and evidence to the contrary views?

If this course were followed, the papers and the discussions would improve, because the author would gladly revise, condense or modify his draft, if that would make his paper more clear and complete, eliminating small inaccuracies, superflous matter or statements susceptible to misconstruction, and adding information of value to corroborate the conclusions.

Also when views fundamentally and radically at variance are prevailing on a given subject, the Committee should secure different papers, carefully discussed and prepared to convey comprehensive, clear and substantiated statements of the facts from the different angles.

The suggestion is not to curtail the freedom of the writers, but only to assist them with whatever constructive criticism the Committee can give.

The result would be that the Institute PROCEEDINGS would become more condensed, valuable and readable than under the present method, and the importance of the Committee's work immeasurably enhanced.

The suggestion is in the direction of creating an organic administration of the technical work of the Institute, which will automatically expand to reach out for, and divulge in crystallized form, the information and experience of more and more members. This work would grow and widen from year to year along lines of greater benefit to, and closer co-operation with, each member of the Institute.

In the interest of effectiveness, this suggestion possibly would displace the newly organized Committee on Technical Activities. The respective Chairmen of Technical Committees as members of the Meetings and Papers Committee should, through its Chairman or individually, report to the Board of Directors in matters pertaining to papers, reports, co-operation with sister societies or institutions, etc.

(B) AS TO SUBSTANCE OF STUDIES OF COMMITTEES WITH SPECIAL REFERENCE TO THE POWER STATIONS COMMITTEE

There exists considerable indefiniteness as to what field a Committee, like the Power Stations, should cover. A power station presents a great complexity of features, like those of design, mechanical and electrical equipment, operation, efficiency of generation, production costs, etc. Many of these subjects, for instance, are strictly mechanical, though perhaps of preeminent importance only in electrical power stations.

Take, as an illustration, the steam turbine. Its design and construction are carried out by mechanical engineers and frequently discussed by the Society of Mechanical Engineers. Similarly, the features of operation, efficiency of generation, and production costs are mainly objects of study by committees of associations of operating companies most vitally interested in the subject. Features of design of electrical apparatus would, under the present organization of committees, come under the jurisdiction of specific committees, like Electrical Machinery, Measuring Instruments, Protective Devices.

Without extending further this analysis, one readily sees that, if we are bound too closely by these limits of existing specialization, we lose the opportunity of doing constructive work in a broad sense. We must, therefore, overlap in other fields under the limitation of not conflicting with other activities in so far as our work would be mere duplication of efforts already satisfactorily covered by others.

To illustrate this principle, let us outline one of the studies which this Committee has had under consideration for the last two years and which will shortly be presented to the Institute. The subject is "Limits of speed and power of single-shaft steam turbo generator units." Three papers are being prepared, two covering the steam end and one the electrical end. It is hoped that these papers will give an idea of limits of power and efficiencies of machines of existing types, *i.e.* a single-shaft steam turbine or a single-shaft generator for steam turbines. Since the study was first considered, its realization having been retarded by the war, there occurred three or four serious mechanical failures of steam turbine wheels which have created considerable concern to the users and aroused a widespread interest in one phase of this theoretical subject. The details of such occurrences can, for obvious reasons, only be discussed by the manufacturers and the users. This has been done but, to a large extent, the findings are properly considered confidential matters, often having a legal aspect, which cannot be made a subject of investigation and report by this Committee. One the other hand, the users would greatly benefit by any work of this Committee that, regardless of specific experiences, would outline the possibilities and limitations of power capacity of single-shaft turbo generators for different speeds permissible by the given frequency. The erroneous remark is often heard that it is unsafe to go beyond a certain limit in the capacity of units for central stations. In reality, units of twice or three times the power of the largest existing units can be manufactured to have the same or even greater factor of safety. Possibly such units may be more costly, or less efficient, or too heavy, or of too large dimensions, or be undesirable in situations where they would concentrate all the power of a station in two or three units instead of five or six or more, but we should not dismiss the study of the possibility of such machines simply from the wrong assumption that they cannot be made as safe as smaller capacity units, because in some situations the larger units may yet prove to be of advantage.

In any event, if they are undesirable, we should know why they are so. The papers under preparation will discuss these features and the Committee believes that such study will be of benefit to manufacturers and users if the study is intelligently carried out.

As we stated before, this departure of invading some of the fields of other technical societies or associations must be done with the understanding that the Committee will not attempt to duplicate the work of others, like, for instance, the Prime Movers Committee of the N. E. L. A. The latter, as a committee of an industry, can obviously fully investigate every operating feature or defect of apparatus and secure information to which this Committee cannot properly have access. On the other hand, features of engineering and design of machines can only constructively be discussed with the co-operation of users and the principal assistance of engineers thoroughly familiar with calculation and design of machinery.

It would appear to us, therefore, that there is a clear line of demarcation for the field of activities of this Committee, which, if directed along the plan first suggested, offers the promise of giving material aid to all the interests concerned.

From this illustration we submit that, as to substance, the work of the Committees must in certain cases be broadened even to the extent, sometimes, of invading the field of other technical societies or committees of the Institute where this is necessary to make a comprehensive study of the subject, under the condition of not duplicating work already satisfactorily and more properly done by others.

(C) PROGRAM OF FUTURE ACTIVITIES

Other proposed work of this Committee for the ensuing year consists of:

A paper outlining the statistical items which are essential in reporting efficiencies of power plants—this to the end of standardizing and making possible comparisons between different plants, which are not possible at present on the basis of the information given in all the published reports of United States Bureaus, State Commissions or other Governmental bodies.

A comprehensive review of the relative merits of shunt and compound-wound exciters, steam and motor-driven exciters, etc. has been considered and is recommended for the incoming Committee.

While the war continued it was found impracticable to make progress along the study of a national policy for the generation and utilization of power, as referred to in last year's report, but some of the members have further studied and investigated the broad subject and will submit suggestions and recommendations to the Committee or the Board as soon as practicable.

PHILIP TORCHIO, *Chairman*

ANNUAL REPORT OF THE TRANSMISSION AND DISTRIBUTION COMMITTEE

To the Board of Directors,

THE Committee on Transmission and Distribution submits the following report for the year 1918–1919:

The need for further reducing the membership of the Committee, as pointed out by the preceding chairman was again brought out very forcibly during the past year.

The large and scattered membership made it impossible to call meetings and it was found necessary to conduct the work entirely by correspondence. This method is generally unsatisfactory, but was particularly so in this case because many of those appointed seemed to consider their connection with the Committee as purely honorary, involving no obligation on their part to enter actively into the work. Several of the members made no reply to any of the communications sent out during the year.

The number of papers presented during 1918–1919 was somewhat smaller than usual because activity during the war period was directed toward intensified production and after the armistice to general readjustment.

However, the papers which were presented dealt with subjects of timely interest and constituted a valuable addition to the literature on overhead and underground transmission.

At the November meeting, held in Toronto, Mr. S. Svenningson presented a paper dealing with the construction of a 110,000-volt transmission line crossing the St. Lawrence River. The preliminary investigation, leading to the adoption of a single 4800-ft. span with 350-ft. towers, is outlined, together with a general description of the design and construction of the towers, insulators and cables. The provisions for protection from ice and method of calculating sag is also included.

At this Annual Convention a session is devoted to the presentation of papers covering both overhead and underground transmission problems.

Messrs. W. S. Clark and G. B. Shanklin, in a paper on high-tension single-conductor cables for polyphase systems, set 25,000 or 30,000 volts as the limit for three-conductor cables and point out the adaptability of single-conductor cables for voltages up to 55,000. The paper compares the dielectric, inductive and general line characteristics of single-conductor and three-conductor cables, discusses the advantages and disadvantages of each type and states that the induced sheath voltage or current is the only factor in single-conductor operation which requires serious consideration.

Mr. R. W. Atkinson presents a paper on the dielectric field in an electric power cable. The data given cover, particularly, the field of three-conductor, three-phase cables, and is primarily the solution by physical measurements, of some of the geometric problems of this type of cable. Methods are outlined by which it is possible to determine from electrical measurements certain specific quantities such as the permittivity and resistivity of the dielectric of three-conductor cables.

The potential stress distribution in three-conductor cables is given, most of the data being based on measurements with electrodes in an electrically conducting liquid, thus approximating closely a homogeneous dielectric. Some additional potential measurements were made with exploring electrodes built into a three-conductor cable having commercial insulating material.

Mr. A. E. Silver in a paper on the problems of 220-kv. power transmission suggests 220-kv. as the logical voltage for the high-capacity, long-distance transmission of energy from distant coal fields and water power plants. The author discusses very fully the important problems introduced by large concentration of energy at high voltage and covers in detail the economic and technical considerations underlying the design of a 220-kv. system. The paper indicates that while further investigation and research are necessary for a complete solution, established principles of design and present types of equipment can, with minor changes, be adopted for 220-kv. transmission.

The points brought out in the paper will aid in working out this advance in the field of high-voltage transmission by promoting constructive discussion and directing research along the lines indicated.

Mr. F. W. Peek, Jr. has given us the results of his investigation of the operating and protective characteristics of various types of spark gaps. The paper shows that there is a marked difference in the relative lightning spark-over voltages of different gaps, as well as a great difference in the settings imposed by operating conditions. Both of these factors must be considered in comparing relative protective values.

It is pointed out that the combination sphere-horn having electrodes of points, horns and spheres gives very good protection over the whole range of frequency or wave front. The spheres discharge the very steep waves and the horns the moderate waves, the points taking care of continuous high-frequency, static, and waves of very slanting front.

The comparative protective values of various types of gap are shown graphically and some fundamental points in the design of bushings and insulators are pointed out.

RECENT DEVELOPMENTS

High-Tension Insulators. There have been no radical changes in the design of high-tension insulators during the past year, and efforts along this line have been toward perfection of details and improvements in reliability.

A great deal has been written on the subjects of insulator troubles, but the difficulties experienced have been no greater than might have been expected in high-voltage work. It is believed that a proper appreciation of the factor of depreciation of insulators and the inspection and replacement of faulty material will go far toward improving operating results.

Practise has clearly demonstrated that a line which is liberally designed as regards insulators will operate satisfactorily in spite of a considerable amount of depreciation. As a result there is little doubt of the tendency to go to considerably higher voltages, wherever there is sufficient load to warrant a conductor large enough in diameter to keep down the corona losses.

The past year has seen the installation across the Tennessee River of the longest span in the world, the very heavy stress occasioned by this span being carried on a single insulator of comparatively small size.

Underground Cables. The material progress which has been made in the last few years in the field of underground cables may be traced to the growth in power requirements incident to the war, as well as to the more exacting demands of cable users.

The recent practical developments are a natural outcome of many investigations dating back to the papers of Gorman in 1901, Raynor in 1912 and Klein in 1913, all of whom clearly demonstrated the start of corona in paper insulated cables at stresses of between 25,000 and 30,000 volts. Developments in the United States have been closely paralleled in foreign countries.

Great general interest is shown regarding the subject of dielectric losses. This matter has become of great commercial importance in the last few years due to the increased loading of the conduit systems. A great variety of opinions exists regarding the actual importance of losses and particularly as to the temperatures at which these are most important. Standardization of ideas on these questions is desirable.

Several methods, with numerous variations, are being used for making dielectric loss measurements, the results checking very closely. However, on account of the magnitude and nature of the quantities sought, all of the methods require more care and skill than do the more usual measurements on electrical apparatus.

Many investigators have studied the problems connected with the stresses in dielectrics. Since there are inherent limitations to the development of the usual three-conductor cable, proposals have been made to avoid these. One plan involves the use of single-conductor cables and another the use of the electrical equivalent of three single-conductor cables but under one sheath.

One direction of present day development is toward the standardization by operating companies of larger sizes of transmission cables. One company is planning to install a large amount of 11,000-volt cable with conductors of 600,000 cir. mils, and 22,000-volt with 350,000 cir. mils. While this is not larger cable than has been made in individual instances, it is larger than has been standard practise for any company in the past. Another company is making a practise of operating two cables as large as 4/0, 26-kv. as a single unit.

There has been an increase in the use of split-conductor cable for transmission line protection. Two large operating companies seem to have partly standardized on this type of cable for future installations.

Cable users are now fairly well informed as regards the operating temperatures of their cables. By comparatively simple means it is possible to obtain an accurate knowledge of conditions of an existing system. A large amount of data is in the possession of operating companies as regards temperature rises of conduit systems under different conditions of installation, and it would be of extreme value to the industry as a whole for such data to be made generally available. Existing systems which have poor cooling characteristics might be improved in some cases and the design of new systems would be greatly facilitated.

Operating companies understand much better than formerly the properties of cables which have to do with successful operation and there is an increasing tendency toward the elimination of non-essential features of specifications and for co-operation with manufacturers so that the latter will not be hampered in securing to the highest degree the essential properties some of which cannot yet be covered by specifications.

SUGGESTIONS FOR THE FUTURE

There has been considerable discussion during the past two years on the subject of dielectric losses in underground cables. Very little real progress appears to have been made, although several large cable users are planning to incorporate in their cable specifications a clause relating to dielectric losses.

Your Committee has been advised that cable manufacturers have been working on this problem with a view to standardizing equipment and methods of measurements, and it is recommended that succeeding committees give this matter their attention.

Allied very closely with the subject of dielectric losses, is the question of safe operating temperature of cables. Mr. R. C. Powell presented a paper which appeared in the PROCEEDINGS for August 1916, but very little additional information has been made available.

There is a growing tendency on the part of those operating high-tension transmission cables to base their current ratings on the heat dissipating characteristics of the individual duct line and there is no doubt that some progress has been made along these lines. It is very desirable that the Committee secure one or more papers covering this phase of underground transmission.

As regards insulators, it is very desirable that the results of tests conducted by Professor Ryan on the aging of insulators be made available, as this question is of vital interest in the operation and maintenance of high-voltage lines.

Referring again to the reduction in the membership of the Committee, your Chairman feels that the work would be greatly facilitated and the results of much greater value if the personnel were limited to not more than ten members.

E. B. MEYER, *Chairman.*

June 2, 1919.

ANNUAL REPORT OF THE INDUSTRIAL AND DOMESTIC POWER COMMITTEE

To the Board of Directors,

FOR two terms back, the Industrial and Domestic Power Committee has studied the general idea of investigating the application of electrical power to industry. During this term, the idea has been reduced to a definite plan, and the plan put definitely in motion.

It is impossible in this report to more than indicate the general features of the plan, its organization and personnel. Fourteen men, *i. e.*, the committee as appointed, started work in September, 1918. May 1919 finds the committee increased to 43, not including men who are helping in the work without committee recognition.

It was decided early in the term to actively prosecute the study of application of power in industries by the formation of sub-committees. Each sub-committee was to concentrate on its branch of the work, subdivide its activities as seemed wise and go right ahead. As it was questionable as to the correct final procedure, it was decided that the investigations would be based upon.

a. The application of electric power to specific processes,

b. The application of electric power to specific electrically driven machinery,

and that part of the sub-committees would work to one thought; the remainder to the other.

Nine sub-committees were formed, and have been under operation this term as follows:

1. Sub-committee on motors with particular reference to speed torque characteristics.

A. M. Dudley, Chairman.

R. H. Tillman	A. C. Lanier
Wilfred Sykes	John C. Parker

2. Sub-committee on Domestic Power Applications.

H. Weichsel, Chairman

James Dixon	C. L. Kennedy
A. F. Welch	A. M. Harrelson

Bernard Lester

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3. Sub-committee on Applications in Printing Industry.

W. C. Kalb, Chairman

W. E. Date	John D. Nies
J. C. Lincoln	Carl F. Scott

4. Sub-committee on Applications to Cranes and Hoists.

W. L. Merrill, Chairman

H. W. Eastwood	E. Friedlaender
R. H. McLain	

5. Sub-committee on Application to Machine Tools.

H. D. James, Chairman

T. E. Barnum	W. T. Snyder
R. H. Goodwillie	W. C. Yates

6. Sub-committee on Application to Passenger and Freight Elevators.

R. H. Goodwillie, Chairman

H. D. James	H. P. Reed
David L. Lindquist	Charles H. Roth

7. Sub-committee on Application in Textile Industry.

H. W. Cope, Chairman.

E. T. Foote	S. B. Paine
C. T. Guilford	D. H. Sadler

8. Sub-committee on Application in Cement Industry.

R. B. Williamson, Chairman

A. M. Dudley	C. A. Kelsey
H. Weichsel	Arthur Simon

9. Sub-committee on Application in Woodworking Industry.

L. E. Underwood, Chairman

Truman Hibbard	S. A. Staege

Under the plan, other committees will be formed and put under way as the work develops. Plans are also under way in the Office of the Secretary of the Institute to care for records from the sub-committee activities as they are turned in. The activities of the sub-committees are further interchanged through the Chairman of the Industrial and Domestic Power Committee, acting as an advisory personal clearinghouse.

The actual records of sub-committee work for the term are not numerous. Much time has been spent in planning the sub-committee work, determining its personnel and securing appointment thereof. Certain of the sub-committees have not advanced beyond this stage. All have submitted reports of activities for this term indicating full co-operative support from their associates and definite sustained interest in the work. In order to bring out more clearly the work in hand, excerpts from several of the reports follow.

From the sub-committee on the printing industry, Chairman Kalb reports the following divisions:

1. Press operation on raised surface or typographical printing, this to cover motor and control application for typical job presses, magazine and newspaper presses.

2. Contributory device operation relating to raised surface printing. This includes linotype, monotype, stereotype, electrotyping, conveyors or carriers, hoists, folders, paper cutters, tension devices and electric heating as applied to metal melting pots and bookbinding tools.

3. Types of presses used in surface and sub-surface printing, this to cover lithographic, planographic, rotogravure and embossing. This division will also include power application on bronzing machines and wood printing.

4. Contributory devices used in connection with surface and sub-surface printing and embossing.

Chairman Goodwillie indicates the work of the sub-committe on the elevator industry as follows:

Scope—The committee has undertaken the study of the electric elevator from the standpoint of its use, *i. e.*, fulfilling its purpose as a mechanism for raising and lowering vertically, passengers and freight in a hatchway.

The subject is being treated by considering comprehensively the requirements of elevator service and leading to the fulfillment of these requirements by the application of electric power to the motor and controlling devices.

The sub-committee on machine tool application has given much thought to its subject. Chairman James in part indicates his committee's plan as follows:

1. Control Equipment: This term will include the complete control apparatus, including disconnecting means and overload protection which is used for the control of the machine tool motor. The complete equipment may or may not form a single unit. Note: It has been common to speak of the machine tool controller as that part of the control equipment used for starting, stopping, reversing, or regulating the speed of the motor. In many cases, this controller did not include the disconnecting means and overload protection. It was, therefore, thought desirable to select a term which would include all of these items.

2. Application Requirements: It was agreed to divide the control equipment into two classes in reference to the features required for particular applications. These divisions are as follows:

   1. General Application Requirements: These requirements are common to all machine tool applications and it is recommended that they be incorporated in all machine tool control equipments. They are as follows:
      1. 1 Overload Protection.
      1. 2 Low-Voltage Protection.
      1. 3 Enclosure or Equivalent Protection.
      1. 4 Disconnecting means.
   2. Additional Application Requirements.
      2. 1 Armature Speed Control.
      2. 2 Field adjustment for Speed Control.

2. 3 Reversing or Non-reversing.
2. 4 Manual or Automatic.
2. 5 Dynamic Brake.
2. 6 Drift.

The next item on the program will be to compile a list of machine tool applications which comprise only the machines more commonly used. This list will then be analyzed and the additional requirements needed for each application specified.

The sub-committee on the cement industry through its Chairman, Mr. Williamson presented the results of earlier activities in an admirable paper read to the Institute at its regular meeting November 8th, 1918. This meeting was held under the auspices of the Industrial and Domestic Power Committee. Chairman Williamson reports that his committee is collecting such additional data as will supplement this paper and in course of time it may be advisable to publish either a revision of it, or an appendix. As an example, we hope before long to secure some information regarding the application of synchronous motors in this industry. In course of time this additional information should add considerably to the value of the paper.

As a committee we feel that our work, and its plan is not only possible but practicable but that two perhaps three years are necessary to determine this. We feel that this plan should be continued another year and recommend this. We also call attention that it takes much time to get together a working personnel and so far as possible, and where not inconsistent with other plans, it is recommended that this existing personnel be kept intact, the recommendation extending to the sub-committees' makeup.

We must call attention to our belief that a certain amount of definite interest in the Institute is being generated by this plan of extending work among a large group. We believe Institute men are anxious to contribute service if a method be indicated. I believe that our sub-committee chairmen are helping materially in this, and their activities have resulted already in bringing several desirable electrical engineers on to our roster.

The committee particularly appreciates the prompt help and attentions which our executive offices in New York through the instrumentality of Mr. F. L. Hutchinson, have constantly rendered.

H. G. PIERCE, *Chairman*

ANNUAL REPORT OF THE LIGHTING AND ILLUMINATION COMMITTEE

To the Board of Directors,

I beg to submit on behalf of the Lighting and Illumination Committee the following report for the year 1918–19.

The activities of the Committee for the current year were composed largely of the plans for a session of the Institute for a discussion of Industrial Lighting Codes, this subject being one of those suggested by last year's Committee, but which, due to the abnormal conditions which prevailed at that time, could not be arranged. As the outcome of these plans a paper was presented by G. H. Stickney at a joint meeting of the Institute and of the Illuminating Engineering Society on April 11, 1919 in New York under the auspices of this Committee of the Institute. At this meeting Mr. Stickney presented an outline of the general scope and reasons for these codes. The session was highly successful and promoted valuable discussion on the part of representatives of the lamp companies, the state labor departments and manufacturers in general.

Another feature of the Committee's work during the present year has been the suggestion which the Chairman was authorized to make to all the Institute sections throughout the country calling attention to the advantage to be gained by devoting one of the Section meetings at each center during the current year to some aspect of Illumination, preferably as a joint session with the local branch of the Illuminating Engineernig Society where such a local branch existed. This suggestion met with a widespread response and a large number of the Institute sections devoted one meeting of the year to some particular division of the lighting and illumination fields and a number of valuable papers and discussions on this topic were presented before the local sections. In some cases the Committee cooperated with the local sections by suggestions as to possible speakers for illumination subjects.

At one of its later meetings, the Committee, following the suggestion of the Board's Committee on Technical Activities, decided upon steps to be taken in plans for presenting one comprehensive paper before the Institute during the coming year on "An Analysis of Daylight Saving," and a symposium on "Systems of Electrical Distribution for Street Lighting" at a later date.

According to the custom of the Committee during the past few years, and also acting upon the suggestion of the Committee on Technical Activities, the following brief summary of progress in electric illumination during the past year, is herewith presented.

PROGRESS IN ELECTRIC ILLUMINATION

Street Lighting. One of the effects of the war was to curtail street lighting installations throughout the country, existing systems being extended only when absolutely necessary, and work on new installations being practically at a standstill during the war. With the signing of the armistice, renewed activity has become evident in this field, and present prospects for extensive developments appear bright.

To illustrate the effect of the war on curtailment of installation work, the construction work on extensive street lighting in Los Angeles and San Francisco were suspended during the war, but the work on these new systems was resumed almost immediately after the signing of the armistice. It may be added that the two installations here mentioned constitute, in themselves, notable achievements in the street lighting field. Other recent developments in street lighting include, among other cities, portions of Chicago, Denver and Dallas.

In the fixtures to be used with incandescent lamps in street lighting, there is an increased tendency to apply some form of prismatic refractor. New lanterns have been developed of both upright and ornamental bracket types, using dome shaped prismatic refractors with an outer globe, which has the effect of breaking up the concentrated rays without loss of the redirective value of the refractor. These units exhibit a bright, sparkling appearance which is considered desirable in street lighting if undue glare is avoided. They direct an unusually high proportion of the light to the street surface.

In recently designed street lighting systems there is an increased tendency to take advantage of the wide range in the incandescent lamps available for operation on circuits of the same kind, thus making it possible to select the lamp sizes according to the strictly engineering requirements of each given class of street and thus affording a far more effective illumination result for a given outlay than is possible with a single fixture and one given size of unit of either large or small size as the case may be.

Studies tending toward still greater flexibility in installation and extension of street lighting systems, and toward a reduction in the disproportionate investment charges which are necessary in the common form of series street lighting system, have led to the development of new plans and devices for current supply. A new automatic movable-coil constant-current regulator, in sizes up to ten kilowatts for pole mounting, and operation from a time clock or remote-control relay switch, is especially useful for the extension of street lighting at distances from substations.

Another promising development of street lighting is the operation of the street lamps from existing secondary multiple or house lighting circuits rather than from series circuits. This form of installation avoids the separate high-tension circuits, special transforming and regulating equipment and the cost of the high-voltage insulation for series systems. Small synchronous motor clock switches on the lighting circuits are available for controlling the lamps. Another proposed method is the use of simple remote control switches operated by a so-called pilot wire.

A great deal of interest has also been displayed in highway lighting, and there is a possibility that some of the states in placing contracts for road work will include the erection of suitable poles for the lighting of the state highways, on a larger scale than has been considered practical in the past.

The tendency for sharing the expense of ornamental street lighting between the city and property owners and merchants is illustrated by the action of the property owners and merchants to pay one-half of the total annual cost for the ornamental street lighting in the Triangle District of San Francisco. A defense of proportionate sharing of this expense on a 50 per cent basis for the city and 50 per cent basis for the merchants and property owners, based on Syracuse, N. Y. conditions, has recently been discussed in one of the leading engineering weeklies.

Under war time street lighting economies, one interesting statement has been advanced that the average streets in this country require an increase of illumination of 70 per cent, but that a considerable reduction of white-way lighting could be afforded under extreme conditions like those during the war. The small percentage which the fuel used for street lighting bears to the total fuel used in this country has also been a matter of discussion, brought on by war conditions.[1]

Street lighting curtailment under war conditions, proved unpopular in a number of localities and cases have been cited where the popular demand for regular lighting conditions in the streets of a city or town was so great during the period of curtailment, that the regular lighting conditions were restored under pressure of public sentiment. In certain cases where curtailment of street lighting was carried out, interesting problems of rebates during the curtailment period were created.

Saving coal by efficient street lighting was an alternative to curtailment and one case which has been reported in a part of Chicago's street lighting system indicates an annual saving of about 3000 tons of coal by the substitution of type C Mazda lamps for arc lamps.[2] The magnitude of the street lighting in the larger cities may be realized from the published figures in New York City, where there are 2800 miles of lighted streets at an annual cost of about $700,000. to the city.[3]

To the foregoing, there might be added a list of the new street lighting units and standards which have been developing recently and which have made available a great variety of effective equipment for use in this field.

Protective Lighting. While the ideas of flood-lighting the fronts of buildings and yard surfaces with light from projectors, is not new, its use during the war has received increased attention as a vital measure of plant protection. This has resulted in an unusual appreciation of light as a factor of usefulness for industrial plants and undoubtedly has had some effect in convincing manufacturers of the necessity of higher intensities of interior illumination.

Factory Lighting. One of the most notable influences of the war upon the lighting field has been the widespread acceptance of the fundamental value of good lighting to factory production. The pioneer tests of Mr. Wm. A. Durgin, of the Commonwealth Edison Company in Chicago, on the relations of higher illumination intensities to factory production rates, has brought out forcibly the interesting fact that illumination is a factor of

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1. J. R. Cravath in *American City*, October 1918, p 303.
2. G. T. Dunklin in *American City*, June 1918, p 517.
3. D. F. Atkins in *Modern City*, February 1918, p 13.

primary importance up to an intensity which makes vision possible, but even beyond this, an increase in the intensities to much higher values than those required for vision itself, bring about rather surprising results from what seems to be a stimulating effect of highly cheerful surroundings upon the employees, thus increasing production by percentages which are very large in comparison with the costs for the increased illumination.

The importance of factory lighting in war work was also emphasized by the appointment of a National Committee on Lighting to act as a sub-committee of the Advisory Commission, Council for National Defense, referred to by last year's Committee in its report. The publication of a Code of Lighting by this Committee on Labor, with a suggestion that the Code be put in effect in every state in the country, has resulted in bringing these regulations before the various state authorities from a new standpoint. The paper by Mr. Stickney, found in another issue of the Institute PROCEEDINGS, contains an excellent summary of the developments in these codes.

Motion Picture Lamps. Considerable development work has been directed to the production of a satisfactory motion picture lamp. The use of the incandescent lamp for motion picture work will undoubtedly become quite general.

Searchlights for War Purposes. The searchlight has been of the utmost importance in the experiences of the recent war. The 60-inch searchlights first used by the Allies together with the automobile power plant and the truck, brought up the weight of each to eleven tons. Specifications which were prepared by American engineers during the progress of the war were directed to a reduction of weight to make the equipment more practical for army work, to the use of a 60-inch metal mirror, and to the greatest possible simplicity.

The so-called Lynn searchlight is now said to be the most powerful searchlight in the world, giving a beam of an intensity of over three times that of the most powerful searchlights in use at the beginning of the war. Moreover, the spectrum of the light from these units approaches a daylight character so nearly as to make them very useful in revealing aeroplanes at great distances above the earth's surface.

The importance of the searchlight is indicated by the general conclusion of army engineers that the only satisfactory method of meeting an aeroplane bombing operation is by means of searchlight defense. In fact it has been stated that the city of Bruges was never bombarded at night by allied planes due to the protection afforded by seventeen enormously powerful searchlights.

Automobile Headlighting. The two 1918 reports of the Committee on Automobile Headlighting Specifications of the Illuminating Engineering Society and the corresponding discussion as published in the TRANSACTIONS of that Society, March 20, 1919, constitute a valuable fund of information on this subject. The Committee on Lighting Legislation of the Illuminating Engineering Society has recently formulated proposed regulations for incorporation in a model law in regard to headlamps on motor vehicles. The Specifications for Headlight Tests of New York State are also of interest in this connection.

C. E. CLEWELL, *Chairman*

ANNUAL REPORT OF COMMITTEE ON ECONOMICS OF ELECTRIC SERVICE

To the Board of Directors,

DURING the first half of this Institute year your chairman and three other members of the Committee were in military service. It will be readily appreciated by anyone who was in active service in engineering and construction lines that there was consequently no opportunity to carry forward the work of the Committee. This should not be considered as an index of the importance of a Committee on Economics of Electric Service to the Institute. During the past year the normal conditions in connection with electric service were so disarranged that there was little such a Committee might have done except record the effects of war conditions on the service, which might have become of historic value but which could not have had any considerable immediate usefulness. Last year the Committee took charge of a New York meeting of the Institute, presenting a paper on The Effects of War Conditions on the Cost and Quality of Electric Service which served its end in bringing people to a lively appreciation of the almost incredible effects the war conditions had on electric service and the great difficulties the managers of electric properties were forced to face in continuing effective service at reasonable prices during the war. The preceding year the Committee took charge of a meeting considering, in five papers, the matter of valuations of properties, and the production of effective inventories both for use in valuations in the regular operations of the properties.

Now that the war is over electric service will gradually become normal, both in the matters of cost and character, but it is not to be expected that conditions will return to those of pre-war times. It is therefore important that the Committee on Economics of Electric Service should keep careful account of the matters under its jurisdiction and by timely reports and papers enable the Institute to assist in bringing out from the rather chaotic war conditions sound and reasonable bases for the establishment of electric service under the new conditions. It is important also that this Committee of the Institute should work in cooperation with committees of like scope of other organizations, including those of a more or less commercial character, so that these economic questions both as to cost of service, quality of service and value of properties may be solved for the future from as broad a point of view as possible. The Institute's place in this work should be an important one as the recognized leader amongst purely technical societies having particular regard for electrical matters.

WM. B. JACKSON, *Chairman.*

ANNUAL REPORT OF THE PROTECTIVE DEVICES COMMITTEE

To the Board of Directors.

SHORTLY after the signing of the armistice last November, there were indications that the members were rapidly discontinuing their various war activities, and upon making an attempt to resume the work of the committee, which had been practically dormant during the war, it was found that the members of the committee and the members of the Institute with whom they were in correspondence, were in general, ready, willing and anxious to co-operate in the work of the committee. Work on the Relay Questionnaire was, therefore, pushed rather aggressively and was discussed in several meetings of a subcommittee and later in meetings of the Protective Devices Committee. The result of this questionnaire and of the committee work appears in the form of papers which are to occupy one of the technical sessions of the Annual Convention.

The future activities of the committee were discussed at some length at one of its meetings. The questionnaire had, in the opinion of the members of the committee, brought forth information so interesting and valuable that it was their opinion that similar investigations should be undertaken by the committee and that for this purpose the number of members on the committee should be increased so as to permit the investigation of more than one subject at a time. The subjects for investigation recommended by the committee, in the order of preference, are as follows:

1. Schemes of Relay Protection.
2. Oil Circuit Breakers.
3. Grounding of the Neutral.
4. Lightning Arresters.
5. Current Transformers.
6. Potential Transformers.

In connection with the work on the Relay Questionnaire it was brought out by several members of the committee and by several of the members of the Institute with whom we corresponded during the work of the questionnaire, that in a number of cases the operating companies were experimenting with various types of relays and schemes of relay protection, and that there were no facilities for co-operation or for comparing experiences so that each of the parties interested in such matters could get the benefit of the experience of all. In some cases it was found out that one company was at some expense investigating a particular type of relay for a particular purpose, and that another company a year or two earlier had made the same experiment under practically identical conditions and had found that the relay was not adapted for the purpose. It is, therefore, recommended that the Protective Devices Committee undertake to devise some scheme of facilitating co-operative research on subjects within the scope of the committee and that the scheme include some method of communicating, at regular intervals, to the engineers who are co-operating in this manner, such information as has been developed by all of the contributing members.

D. W. Roper, *Chairman.*

ANNUAL REPORT OF ELECTROCHEMISTRY AND ELECTROMETALLURGY COMMITTEE

To the Board of Directors,

TECHNICAL papers on electrochemistry and electrometallurgy, involving chiefly chemistry, physical-chemistry and metallurgy, are naturally discussed more intelligently, more critically and more effectively, in our sister society, The American Electrochemical Society, which includes in its membership most of the chemists, physical-chemists and metallurgists who are interested in such subjects. Hence papers on these subjects naturally drift to and belong to that society.

In connection with many electrochemical and electrometallurgical plants there are however features of a purely electrical character which chemists, physical-chemists and metallurgists are not able to discuss critically and intelligently, such as regulation, power factor, load factor, balancing of loads, different kinds of currents, long-distance transmission, relative advantages and costs of water power *vs.* steam power, etc. Papers on the latter subjects ought to go to the A. I. E. E., or perhaps better yet, be presented at joint meetings of these two societies. Although the Chairman of the present committee has endeavored to procure papers of this kind, he has not yet been successful.

The Chairman recommends that the above plan be adhered to, and that there should be no rivalry but rather a mutually beneficial cooperation between these two national societies. As joint meetings are difficult to arrange, it might be advisable that in case of a particularly good paper on an overlapping subject, arrangements be made that such a paper might become the joint property of both societies, be read and discussed at the meetings of both societies, and be published in both TRANSACTIONS.

CARL HERING, *Chairman.*

ANNUAL REPORT OF THE ELECTROPHYSICS COMMITTEE

To the Board of Directors,

IT has been the endeavor of this committee for the last few years:

1. To encourage original papers of high technical standard, marking advances in electrophysics.
2. To have each year a broad, interesting, general lecture, free from mathematics, dealing with modern physics.
3. To promote a more complete cooperation and mutual understanding between the engineer and the physicist.

This general policy has been endorsed by the present committee.

The following are the activities of the committee for the past year:

LECTURES

"Lectures on Research". Drs. Whitney and Millikan. October 1918. Philadelphia. Arranged by the Committee of 1917–1918.

"An Evening with the New Astronomy". Dr. John A. Brashear. Mid-winter Convention, 1919. New York. Arranged by Meetings and Papers Committee.

TECHNICAL PAPERS

A technical session devoted to electrophysics was held at the Mid-winter Convention. The following papers were read:

"Ionization of Occluded Gases in High-Tension Insulation", G. B. Shanklin and J. J. Matson.

"The Dielectric Strength of Air Films Extrapped in Solid Insulations". F. Dubsky.

"Abnormal Voltages in Transformers". L. F. Blume and A. Boyajian.

It is difficult for the engineer to keep up with the rapid evolution and progress in modern physics, as most of the work is clothed in mathematics and appears in philosophical magazines never read by engineers. The general lectures were arranged to overcome this, and have proved very successful and *popular*.

It is hoped that arrangements for a joint meeting with the American Physical Society can be made soon.

F. W. PEEK, JR., *Chairman*

ANNUAL REPORT OF THE TELEGRAPHY AND TELEPHONY COMMITTEE

To the Board of Directors,

A NOTEWORTHY development during the past year has been the drawing together of the arts of telegraphy, telephony and radio signaling into one subject of Electric Communication. War conditions have very materially aided this tendency. The enormous increase in traffic, due to the war, made it imperative that each method of rapid, long distance communication be developed to its utmost capacity.

With the entry of our country into the war, the requirements of the military and naval authorities imposed a certain amount of concentration in the direction of communication facilities, and, when on July 31, the Postmaster General was empowered to take control of all telegraph and telephone lines in this country, unity of direction was further advanced.

On December 1, the facilities of the two large commercial telegraph companies were consolidated, and on December 4, the marine cable systems of the various cable companies were taken over by the government.

From an engineering standpoint these developments had a retarding effect upon the plans of the private companies for apparatus and construction improvements; but several Boards appointed by the government were given opportunity to experiment with suggested technical improvements, and to distribute traffic loads where they might be moved in the most expeditious manner possible over the lines of the combined systems.

STANDARDIZATION

During the year a considerable amount of contributory work has been done along the line of standardizing telephone, telegraph and radio terms, and symbols.

RAILROAD COMMUNICATION

The Association of Railway Telegraph Superintendents was, during the year, absorbed by the American Railroad Association; but the engineering work carried on by the Association has not been neglected by the railroad telegraph and telephone officials. Indeed, the manager of the Telegraph Section of the Railroad Administration has been enabled to coordinate technical effort to the end that the railroads will undoubtedly benefit from the work done by technical committees.

LIGHTNING PROTECTION

During the year considerable improvement has been made in the design of lightning protective apparatus used to safeguard telegraph and telephone lines from lightning and accidental contact with foreign wires carrying higher potentials than the signaling circuits. The vacuum type of arrester has been improved in effectiveness and reliability.

PRINTING TELEGRAPHS

On the lines of the Western Union Telegraph Company the use of the multiplex printer system has been considerably extended. About 80 per cent of this company's wire traffic is now handled by means of the Multiplex, and the Morkrum printer duplex systems. The Postal Telegraph-Cable Company, on the other hand, which has experimented during the past twelve years with three different printing telegraph systems, on April 1, 1919 discontinued the use of printing telegraphs on all of its lines, reverting exclusively to the Morse manual system.

SUBMARINE CABLE SIGNALING

During the past year considerable improvement has been made in the traffic carrying capacity of submarine telegraph cables. The application of the cable relays and amplifiers invented by Heurtley, Dixon, Brown, Gulstad, and Bruce, to cable operation, has made longer direct-working possible, and has resulted in increased speeds in words per cable in unit time. On the cables operated by the Western Union Telegraph Company between Hearts Content, Newfoundland, and Valentia, Ireland, the printing telegraph system has been applied with satisfactory results.

[The following contributions to this report are submitted by the members of the Committee whose names appear under the various subheads.]

TELEGRAPH LINE CONSTRUCTION

R. E. CHETWOOD

No radical changes in the design or type of telegraph construction have been made during the past year. There has, however, been a decided tendency towards stronger construction, both for the pole line and for the wires. Stronger pole line construction has been obtained by using more poles per mile of line, by using stronger poles and by the use of as short poles as right-of-way conditions will permit.

Developments in the telegraph art which lessen the harmful effect on telegraph transmission of the high electrostatic capacity of conductors in cables have permitted the use of aerial and underground cables to a somewhat larger extent than formerly. Stronger and more reliable construction has thus been obtained in and near cities where, on account of right-of-way conditions, it is often necessary to carry the wires of two or more pole lines on one line.

As iron wire is a satisfactory conductor for many classes of telegraph circuits, both with manual or with automatic operation, a considerable increased strength in line wires has been obtained by using iron wires of large size, such as No. 4 and No. 6 B. w. g.

INTERFERENCE TO COMMUNICATION CIRCUITS FROM NEIGHBORING POWER LINES

A. W. COPLEY

A mutual interchange of information and co-operative work between power companies and communication companies toward the solution of inductive interference problems in particular cases, has resulted in improved conditions, and at the same time, caused the maintenance of friendly relations between the parties.

Conditions obtaining on the power circuits during regular operation are given the greatest attention working out the solution, as serious interference under such conditions is intolerable. Abnormal transient conditions, such as short circuits on the power lines, are of comparatively infrequent occurrence and short duration, and if protection of the communication circuits against interference at such times were to be generally attempted, the cost and inconvenience of the arrangements would be out of all proportion to the benefits gained. It is, therefore, considered reasonable to protect only against physical damage to the property or personnel of the communication company under such abnormal conditions.

For the prevention of interference in telephone circuits under operating conditions, co-ordinated transpositions of the power and telephone circuits give a solution in most cases. Wave-form correction in the power circuits by resonant shunts or other means, may sometime be resorted to, especially when transpositions are impracticable. There has been no standard in the past by which the tendency of a machine to cause telephone interference could be measured. The new telephone interference factor now defined in the Standardization Rules provides a measure of this tendency and its use will assist in the adjustment of interference difficulties.

Telegraph interference from transposed power circuits is so small as to be negligible, except during abnormal conditions. In cases of interference from power lines which cannot be transposed, such as trolley circuits, the use of resonant shunts in the telegraph circuits has given very good corrective results. Much has been accomplished in the prevention of interference from such circuits by a careful layout of the power system, particular regard to substation locations being given, so as to minimize the inductive interference in the communication lines.

LONG DISTANCE TELEPHONE TRANSMISSION

O. B. BLACKWELL

The features of perhaps greatest general interest at this time in long distance telephone transmission are telephone repeaters, loading, and multiplex-telephony.

The vacuum tube amplifier has been highly developed for use in telephone repeaters and has become the preferred form of amplifier for such use. The transmission gains which may be obtained with such an amplifier in two-way operation depend in each case on the electrical conditions of the line in which used. This has had a large effect on cable and line construction.

In the loading art it has required the development of coils having especially great magnetic stability and uniformity. Such electrical stability is also particularly important on long circuits which are composited for Morse operation to prevent the telegraph interfering with the telephone transmission.

The repeater and loading developments have considerably extended the use of telephone cables for long distance transmission. They have, however, made it imperative to keep open wire circuits as free as possible from cable in order to prevent such electrical irregularity from reacting on repeater operation.

A multiplex telephone circuit has been put into commercial use between Baltimore and Pittsburgh, giving four circuits over a single pair of wires in addition to the ordinary telephone circuit over the same pair. This multiplexing depends on the use of "carrier currents." In order, however, to be effectively transmitted over telephone circuits, these carrier currents are of comparatively low frequency and the ratio between them and the telephone frequencies being transmitted is, therefore, low. This brings in peculiar problems in the handling and separating of these different frequencies on the one pair of wires.

The vacuum tube plays an important part in this system, not only as an amplifier but also as a modulator and demodulator. The transmission of these frequencies over telephone circuits makes more severe requirements on the circuits than do ordinary telephone frequencies in view of their larger attenuations, the greater effect on them of lengths of cable and their increased tendency to crosstalk. The multiplex system promises to have a considerable field in comparatively long distance telephone service.

TELEPHONE OUTSIDE PLANT PRACTISES

W. R. McGOVERN

At the present time the best practise in outside plant construction is represented by an all cable and twisted pair distribution in the medium and large exchange districts. In the small exchanges where estimates of future growth do not show the need along any given route for more than from 10 to 15 circuits, it is more economical to use crossarm construction with galvanized wire. Even in these exchanges it is desirable to place a small amount of cable in the immediate vicinity of the exchanges.

In those exchanges where cable and twisted pair wire are used, the cable portion of the plant should be so designed, and supplied with distributing terminals, that the amount of twisted pair wire is held at a minimum. In thickly populated areas the twisted pair wire should appear only as the connecting link between the pole route and the subscribers premises.

The dividing line between the underground and the aerial portion of the cable plant is a variable determined by local conditions, but in general the circuits should be placed underground, if estimates of growth for several years in the future show that they will exceed a few hundred in number.

Among the tendencies of telephone construction is the more extended use of fine-wire cable, it being theoretically practicable to use 24-gage, or smaller circuits in the zones near the exchanges. The use of small-gage wire in the aerial cable plant will result in economy; and a more extended use of fine-wire cable in the underground plant will conserve conduit space as well.

Another tendency is towards the use of cable terminals of such a type that potheads are unnecessary. The so-called sealing chamber type of terminal allows paper cable to be taken directly into the terminal, and its use will become more extended in the future.

STATUS OF AUTOMATIC TELEPHONY

ARTHUR BESSEY SMITH

There is at the present time a general recognition that automatic telephony has arrived. It has been established that automatic switching is very flexible, offering a number of solutions for every traffic problem.

The switching apparatus has become fairly well standardized as to mechanism, electrical circuits, processes of manufacture and maintenance. The recent trend of development has been along the line of increasing the factor of safety, making more definite the standards of adjustment, and increasing the life of parts subject to wear.

Automatic switching is a recognized efficiency device for many classes of service. The private automatic exchange is widely used in commercial institutions, either isolated or with trunks to the public system. It is maintained by ordinary electrical employees.

The rural automatic telephone has become firmly established. It is served by party lines from a city central office or from a small switchboard at the center of the rural area. Common battery is successfully used for voice transmission as well as for dialling.

Single office and multi-office exchanges up to 70,000 lines are in use and have proved their adequacy. The application of automatic switching to metropolitan areas is a recognized possibility.

The automatic switching of toll lines is established. It is possible to dial directly from the originating toll board to the called party's telephone. This has increased the time-efficiency of toll lines.

ARMY SIGNALING

E. B. CRAFT

The telephone has become an essential part of the modern war mechanism. Because of the vast territory and enormous masses of men involved in military operations of the past four years, the military telegraph has assumed an entirely new character.

We now find networks of wires and cables reaching from all points of the fighting zone back to bases of supply often hundreds of miles away. These networks make use of the most modern developments of the communication art, such as high speed printing telegraphs and telephone repeaters for long distance circuits.

While necessarily of a temporary type of construction, these wires systems with their exchange centers, compare in magnitude and complexity with those of our large industrial centers.

Probably the most important development and one which will have the most far reaching effect upon the art in the future, relates to the development and application of thermo-ionic devices. The design and manufacture of these devices commonly known as vacuum tubes, has been placed upon a basis which probably could not have been reached within a decade under normal peace time conditions.

Some of the more important applications comprise the following:

Radio telephone for communication to and between airplanes while in flight.

Determination of direction by radio compass.

Signaling through the ground by means of leakage currents.

Overhearing of enemy operations through the use of energy amplifying devices.

As a result of the intensive development of the past two years there has been made available to the engineer, a device which will in the future undoubtedly play an important role not only in the communication but in other branches of the engineering world.

RADIO SIGNALING

JOHN STONE STONE

Radio engineering has continued to feel the powerful stimulus of the World War.

Secrecy has necessarily veiled a considerable amount of the original discoveries and inventions made in this field as well as the complete statistics which would disclose the enormous expansion of the manufacturing industry in this line during the past year, because all the development and manufacture were necessarily conducted for the military and naval services and under strict governmental observation and regulation.

The general tendency has been towards a more systematic and sound engineering development of radio communications. Sustained-wave-train radio telegraphy has, to a great extent, replaced the older and less satisfactory damped-wave-train, or spark system, while radio telephony has been developed into a most practical form for air plane service as well as for land and ship stations.

These advances have largely been the fruit of careful quantitative studies of the audion, resulting in the possibility of accurately predetermining its design when it is to be used either as an oscillating generator, detector, amplifier or modulator.

Advances have been made in minimizing "interference" and the amount of radiated power which can be effectively "modulated" by the voice has been increased with a considerable resulting increase in the range of radio telephony.

DONALD MCNICOL, *Chairman*

ANNUAL REPORT OF THE MARINE COMMITTEE

To the Board of Directors,

THE status of the application of electricity to merchant vessels was such that at the beginning of this season's Institute work it was deemed necessary to greatly enlarge the membership of this Committee. In previous years the shipbuilding industry comprised about five or six fairly large size steel shipbuilding plants. The representatives of these companies as well as the representatives of the few manufacturing concerns which supplied the shipyards comprised the Marine Committee. These gentlemen were fully informed as regards the methods of installation and the various requirements of the classification societies, ship owners, naval architects, etc., so that standard requirements in the nature of governing rules were hardly necessary. The war brought about a great expansion in the shipbuilding industry with the outcome that a great many men were brought into the newly organized plants to perform the work of electrical applications. It seemed fitting to bring these new men into contact with the more experienced members of the Institute and to prepare for all concerned some guiding regulations to make for the proper development of electrical installations on board merchant vessels.

Seven meetings of the committee were held. The first few meetings were used for the purpose of organizing both the committee itself and the work to be done. To this end sub-committees on the two main questions pertaining to the work were appointed. These main committees were called the Ship Installation Committee and the Electric Ship Propulsion Committee. The Ship Installation Committee was further subdivided into a Power Apparatus Committee, a Lighting Committee, and a Distribution Committee.

The sub-committees of the Ship Installation Committee prepared the necessary details pertaining to the recommendations and reported in conference to the Ship Installation Committee. In turn the Ship Installation Committee laid the results of its work before the Marine Committee proper. I, therefore, have the pleasure of submitting for the consideration of the Board of Directors the report of the Merchant Ship Installation Committee which establishes the Rules and Requirements as approved by the Marine Committee.

This report speaks for itself as regards the activity of the Ship Installation Committee. It may be stated that the work of the Ship Propulsion Committee has shown a similar amount of work but the nature of its investigations preclude the incorporation of the data that have been made, as the status of this application does not admit of the definite formulation of Rules and Requirements. I am attaching, however, as an addenda to this report a considerable amount of exceedingly interesting and important material which cannot at this time be put into print but will remain in the files of the Institute for the work of the Committee next year.

The undertaking of this work as is always to be expected grew with the investigations made by the sub-committees so that the work of the Ship Installation Committee is by no means complete. There are five items which though acted upon by the Power Apparatus Committee and the Lighting Committee were not considered by the main committee. There are seven items still to be completely considered by the sub-committees. In general, these subjects include such questions as the following:

(completed by sub-committee)

Control equipments.
Alternating-current apparatus control.
Radio installations.
Running lights, etc.
Fire-alarm systems.

(not completed by sub-committee)

Tabulation of generating sets.
Storage battery installation.
Gyro-compass installation.
Gyro-stabilizing installation.
Tabulation of wires and conductors.
Appliances in vicinity of compass.
Inspection report to Classification Societies.

The data pertaining to these questions are not to be printed but are attached as an addendum to this report for the benefit of the committee next year.

The attendance at the meetings of this committee have all been large and representative. The committee membership as a whole is to be congratulated upon the work accomplished. The chairmen of the main committees as well as the sub-committees have worked earnestly and faithfully. It is to be expected that next year, by following the work here begun, that the Institute will be enabled to lay down the first requirements for marine installations that have ever been made. It is suggested that next year the concluding work of the committee should be done in conference with the Classification Societies and the Bureau of Standards.

H. A. HORNOR, *Chairman*

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ANNUAL REPORT OF COMMITTEE ON USE OF ELECTRICITY IN MINES

To the Board of Directors,

I BEG to submit the following report regarding the activities for the past year of the Committee on the Use of Electricity in Mines.

The Committee provided papers for one session of the Philadelphia Meeting at which time a paper on "The Use of Electric Power in the Mining of Anthracite Coal", by J. B. Crane, Vice President, Lehigh Navigation Company, and a paper Drum Shapes as Affecting the Mine Hoist Duty Cycle and Motor Rating," by F. L. Stone were presented.

Having been Chairman of the Schenectady Section during the past year has prevented me from devoting as much time to the Committee work as I would liked to have given to it. However, I have given considerable thought to the question of making the activities of this Committee felt among mining men, and while I have no definite plans to recommend for doing so, I do believe that closer cooperation between the various technical societies, both national and local branches, will prove very beneficial. As the Committee on Development has this whole matter in hand at the present time I have taken no action with reference to it other than a talk with our local representative on this Committee concerning the question.

K. A. PAULY, *Chairman,*

ANNUAL REPORT OF THE IRON AND STEEL INDUSTRY COMMITTEE

To the Board of Directors,

THE increase in the use of electrical energy in the Iron and Steel Industry in the last few years has been remarkable and partly responsible for the prominent position the United States has taken in furnishing steel products to the entire world.

Over 500 motor equipments are in use in this country, driving main rolls with a maximum horse power of over 850,000, while the total horse power of auxiliary motors exceeds 1,350,000, making a total of 2,200,000 h. p., which requires about 650,000 kw. in generator capacity. In addition, electric furnaces require approximately 250,000 kw. in generator capacity, or a grand total of 900,000 kw. generator capacity in the iron and steel industry.

This is by far the largest amount of electric energy required in any one industry, and from all indications will continue to increase considerably in the future.

The activity of the Iron and Steel Industry Committee has mainly been confined to arranging joint meetings with the Association of Iron and Steel Electrical Engineers for the purpose of presenting and discussing technical papers of interest to members of both societies.

Practically all members of the A. I. E. E. especially interested in the steel industry are members of the A. I. & S. E. E., and attend the latter's technical monthly meetings and yearly conventions.

While members of the A. I. E. E. are primarily interested in the design and development of electrical apparatus, the membership of the A. I. & S. E. E. consists mainly of men having responsible charge of electrical machinery in the steel industry and is primarily interested in the installation and operation of electrical apparatus.

It is suggested that both societies closely co-operate, keeping in touch with the latest and best practise and progress in the art, and avoid duplication of effort as much as possible.

One of the main subjects to be taken up jointly by the two

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societies is "Standardization." In general, the Standardization Rules of the A. I. E. E. have been adopted by the A. I. & S.E.E., however they cover primarily standardization of terms and conditions which characterize the rating and behavior of electrical apparatus without going into details of construction.

The Standardization Committee of the A. I. & S. E. E. feels that for its work, standardization rules should also refer to such details and dimensions as will reduce cost and complexity and not retard progress in design and development. In its *Proceedings* of 1917, Mr. H. D. James, then Chairman of "Co-operative Standardization," expresses himself very ably in regard to advantages of Standardization as follows:

> Those interested in the application and manufacture of electrical apparatus realize the many advantages in standardizing equipment. Not all, however, stop to analyze these advantages, and in order to bring these to your attention, the Standards Committee wish to present the following advantages of standardization:
>
> 1. The best average equipment is obtained when a considerable number of qualified persons is willing to agree upon a particular arrangement. Such an agreement is based upon the available facts and when the personal bias of individuals is eliminated.
> 2. The adoption of a standard by an association tends to establish engineering and commercial practise in that particular field. The first approximation towards standards crystallizes criticism and suggestion, so that the tentative standard may be improved.
> 3. A standard equipment enables operating men to interchange parts of an equipment between different machines and reduces the number of spare parts required.
> 4. A standard equipment enables the manufacturer to carry this equipment in stock, so that prompt delivery can be made. It also reduces the misunderstandings which sometimes arise regarding the details of such an equipment.
> 5. A standard equipment permits the manufacturer to organize his work so that a better product can be obtained at a reduced cost.
> 6. A standard equipment saves the time of an executive. It enables him to delegate his authority to a subordinate where the choice of the equipment is limited to recognized standards.
> 7. Standard equipment simplifies the work, of the purchasing departments. Where standards have been agreed upon, price forms can be issued and a great deal of delay incidental to the purchase of this apparatus will be avoided.
> 8. In making plans for the application of electrical equipment to the various types of machinery required, the draftsmen will have available standard outline drawings, together with weights and dimensions, so that he may proceed expeditiously with his work.
> 9. Where equipment is agreed upon, the question of the mechanical interchangeability of parts can be more readily considered; for instance, the mechanical dimensions of a grid resistor frame.

10. Standard equipment will concentrate the attention, and increase the number of research investigations to improve this standard equipment.

11. The adoption of standards indicates the stability of any art, showing that it has passed through its initial stages and has become a recognized factor in the industry. This results in placing engineering upon a higher plane and commands the respect for those engaged in this particular art.

12. It is important that the electrical engineers of the Iron and Steel Industry take an active part in national standardization, so that the requirements of their industry may receive proper consideration in parallel with the standardization work of other organizations.

Standardization forms the basis of a great deal of our work. We are accustomed to take for granted many of our standards, such as weights and measures, screw-threads, pipe fittings, standard time, standard railroad gage, etc. In every direction we see evidences of standardization. If it were not for these every-day standards, it would be very difficult for us to do business or to design apparatus. Standardization of motors, controllers, cranes, etc., is the carrying out of this general idea, which has existed for generations among civilized people.

Manufacturers of electrical apparatus should get together for real co-operation, to a much greater extent than they have done in the past, and not stop in the standardization of characteristics, voltages, frequencies, ratings, torque, etc., but begin to standardize mechanical dimensions, shafts, gears, pulleys, etc. They are generally in sympathy with the subject of standardization, but think it is the duty of the user of electrical apparatus to first decide on the proper standards and then co-operate with them, so as not to interfere with the progress of the art.

The problem then before us is to find the proper standards; the first step towards solving this problem should be to make general requirements for the installation and operation of electrical apparatus in the steel industry as uniform as possible.

It is suggested that the Iron and Steel Industry Committee take up this work jointly with the A. I. & S. E.E., and formulate a plan by which sufficient practical and theoretical data can be collected through the members of both societies, and finally draft standard rules for the installation and operation of electrical apparatus in the iron and steel industry. A great amount of detail work and time will be required to carry out this plan, and the closest co-operation of all members of both societies interested in this subject will be required.

It is recommended that the "Board of Directors" of the A. I. E. E. confer with the Executive Committee of the A. I. & S. E. E. in regard to this subject.

E. FRIEDLAENDER, *Chairman.*

ANNUAL REPORT OF THE INSTRUMENTS AND MEASUREMENTS COMMITTEE

To the Board of Directors,

THE Committee during the past year has had the difficulties and handicaps incidental to the war period to contend with, but these were borne by all similar committees and associations so that nothing is gained by amplifying them in this report.

Several meetings were held and in addition an effort was made to transact as large a volume of the business of the Committee as possible by correspondence. The Committee has been responsible for the preparation of several papers on subjects relating to the functions of the Committee and while some of these papers have been completed, others are still in preparation. It has not been possible, however, to present any of these papers due to the crowded calendar of meetings. It is strongly recommended, however, that a session be set aside, possibly at the next Mid-winter Convention, for a group of papers on the subjects within the scope of this Committee. Justification for such a session is based on the success of the similar session held during the initial year of this Committee's activity.

The Committee through its membership was active with the Bureau of Standards working with the War Department to standardize purchase specifications for electrical instruments and accessories for overseas service, and joined in other governmental activities within the functions of the Committee.

The demands of the Government for standard instruments, for special designs of the standard types and for instruments to meet requirements new to the art, required a development of far reaching magnitude. The Institute was largely represented in the personnel engaged in this work and early in 1918 the output met the needs of the Government in all its special fields and met as well the unusual demands of the country.

The appeal by the Government to manufacturers heretofore specializing on a particular line of instrument or accessory to undertake the manufacture of instruments or accessories foreign to their regular production, resulted not only in increased production but the enlisting of resources and individuals new to a given problem. Not only did this produce the increased quantity desired by the Government but also resulted in a greatly improved product from a technical standpoint.

Space does not permit other than a passing reference to these developments, some of which are briefly summarized herewith:

HIGH SENSITIVITY MEASURING DEVICES

Prior to the war thermoelectric couples had been employed for measuring low values of alternating current. These couples were almost exclusively of German manufacture and not extremely accurate nor sensitive. There have since been developed and put on the market, thermoelectric couples of American manufacture which are superior in every way and are now available for the accurate measurement of alternating currents of any frequency. Their sensitivity is such that a current of one milliampere will produce a deflection of 3 millivolts in a direct-current voltmeter.

The recent perfection of the thermo-ionic tube has put into the hands of the physicist and engineer, a new device for use in measurements of precision. By means of thermoelectric amplifiers, very feeble currents may be greatly amplified and their properties studied.

A peak voltmeter has been developed which makes use of the unilateral properties of the thermo-ionic tube. With this device it is possible to measure the instantaneous potential between any two points in an electrical circuit, without conuming any part of the power in the circuit and without disturbing its electrical constants.

Vacuum gages of high precision have been developed, making use of these tubes. Pressures from 0.001 millimeter down can be directly measured.

Quantity production of hot-wire ammeters.

Bureau of standards Type D decremeters.

Wavemeters.

Consideration of the foregoing and the uses to which they were put in Government service, will make it apparent that some of the very significant developments, while valuable material for the Institute, could not properly during the major portion of the year, be made the subject of papers or reports.

The widespread use of maximum demand apparatus requiring accurate timing and synchronous operation renders very interesting the development of the Warren timing element and system.

The Warren self-starting synchronous motor constitutes a very accurate time-keeping element in the space of 2⅛ by 2⅝ by $2\frac{9}{16}$ inches capable of delivering more power than many of the present spring driven clocks, and this without the usual weekly or monthly winding. It is, therefore, particularly adapted for use in graphic and demand meters, time switches, etc. where the advantages of a time-keeping element with such characteristics are obvious. An even more important asset, for certain applications, than any of those mentioned, is that it is possible to synchronize the records of graphic or demand meters—a feature which heretofore has been very difficult to accomplish.

The small timing element can be installed in any device connected to the alternating-current mains of a company where constant speed and synchronous operation are desired. The element itself, when combined with an accurate clock, is used as a master clock at the central station both for providing uniform time throughout the system and for regulating frequency.

S. G. Rhodes, *Chairman*

ANNUAL REPORT OF EDUCATIONAL COMMITTEE

To the Board of Directors,

FROM September 1918 to January 1919, all the principal colleges in the country were under a military rule in connection with the Student's Army Training Corps. During that time the activities of the institution of learning and their policies were governed entirely by the War Department and its Educational Committee. Before the armistice it looked as if such an arrangement would continue throughout the whole academic year, and for this reason, your committee did not make any effort to plan its work, merely keeping itself in readiness for an emergency.

After the disbandment of the Student's Army Training Corps the colleges found themselves facing many difficult problems on account of extremely confused scholastic records of many students, mixed schedules, lack of instructors, and the necessity of finishing the academic year in such a way that a large number of students, including those discharged from the regular army, could start the next academic year with a clean slate. The committee did not feel, therefore, that the time was opportune for discussing any general educational policies, or sending out questionnaires, until conditions became more normal.

At one time during the war there was considerable talk about radical changes in technical education, as part of the general plan of reconstruction after the war. It seems, however, that the colleges of engineering are essentially going back to the pre-war standards, methods, and curricula. An investigation recently made by one of the leading technical journals, among the principal schools of engineering in the country, has shown an overwhelming majority of the deans and professors to be in favor of no radical changes from the ante-bellum methods.

Educational reconstruction, if any, will have to follow and not precede the general economic and industrial reconstruction. The country as a whole first has to decide what is technically and economically right, what is feasible to accomplish and what kind of men and women will be needed as leaders under the new regime. The first steps of this reconstruction will have to be carried on by the present college graduates, and the coming generation will have to be educated in accordance with the newly laid principles of justice and brotherhood, and with sound principles of scientific engineering.

One important direct contribution of the war to the problem of education has been the introduction of elaborate psychological tests for analyzing a person's intelligence, fitness and caliber for a particular kind of work. Some leading universities have either introduced this system to test their students, or are planning to use it in addition to regular examinations. This is only a beginning towards a great new principle, namely that of fitting education to the individual, instead of trying to fit every individual to the same stereotyped courses of instruction.

If there is a recommendation that the retiring committee cares to make to its successors, it is that the new committee should add its influence to the general acceptance of the sound principle that the principal purposes of education are self-realization, inspiration for co-operative service, and the development of intelligence and moral sense, rather than an indiscriminate cramming with undigestable information. Psychological tests made before, during, and after the college course are among the first steps in this direction, and the new committee might well devote its effort to an adaptation of such tests for the needs of electrical and mechanical engineering.

V. KARAPETOFF, *Chairman.*

Presented at the 35th Annual Convention of the American Institute of Electrical Engineers, Lake Placid Club, N. Y., June 24, 1919.

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HIGH-TENSION SINGLE-CONDUCTOR CABLE FOR POLYPHASE SYSTEMS

BY W. S. CLARK AND G. B. SHANKLIN

ABSTRACT OF PAPER

Three-conductor underground cable is limited to a line potential of about 25,000 or 30,000 volts. The transmission and distribution of large amounts of power at this voltage involves heavy investment in copper.

The only factor that limits the voltage rating of single-conductor sheathed cable is over-all diameter. Line voltages of 44,000 and 55,000 volts are perfectly practical and mean a saving in copper over three-conductor cable of as much as 70 per cent. Induced sheath voltage or current, depending upon whether sheath is open- or short-circuited, is the only factor in the operation of single-conductor cable that needs serious consideration.

In this paper the dielectric, inductive and general line characteristics of three-conductor and single-conductor cable are compared. The advantages and disadvantages of each type of cable are brought out in a way that will aid in deciding the merits of individual problems.

INTRODUCTION

SINGLE-CONDUCTOR sheathed cable has been used only to a limited extent for conducting polyphase energy. This has been but a natural result of conditions. Multi-conductor cable has generally met underground service conditions satisfactorily and there has been a sufficient overlap with overhead high-tension service to make one of the two meet all but a very few exceptional cases.

Single-conductor cable is more economical than multi-conductor only when there are large quantities of high-tension power to be handled. It is only within recent years that steadily increasing units of power have reached figures that make the substitution of single-conductor cable for multi-conductor an attractive proposition.

Another feature favorable (although not welcome from an economical standpoint) to a more extended use of high-tension single-conductor cable is the tendency towards abolishment of overhead lines in densely populated districts.

COMPARISON OF MULTI-CONDUCTOR AND SINGLE-CONDUCTOR CABLE

The use of multi-conductor cable for transmitting and distributing large amounts of power economically is limited in two ways; by the maximum allowable voltage, and conductor cross section, or current carrying capacity.

The highest operating voltage in ducts is about 30 kv. This voltage will probably never be safely exceeded to much of an extent in ordinary three-conductor cable, because of the tangential stresses first pointed out by Höchstadter.[1]

Conductor cross section is limited to about 250,000 or 300,000 circular mils. Larger conductors than this would make the cable unwieldy, difficult to manufacture, transport and install. The radius to which it could be safely bent would not be small enough for installation purposes. For a rise of 30 deg. cent. a 300,000-cir. mil, 30-kv., three-conductor cable will carry approximately 230 amperes per conductor under favorable conditions. This represents 12,000 kv-a. per cable. The copper loss will be 6.54 watts per duct-ft. per cable at 55 deg. cent. With 12/32-in. by 6/32-in. thickness of insulation and 9/64-in. lead sheath the outer diameter is 3.63 in. Since this is the highest voltage and largest cross section that it is practical to use we may regard this design as having the least amount of copper and lowest I^2R losses for the given amount of power to be transmitted.

Now, let us compare this limiting size of three-conductor cable with different designs of single-conductor cable. Comparison is best made by reference to the design data in Table I. Cable (a) is the three-conductor cable discussed above. Cable (b) is single-conductor cable, has the same voltage to neutral as (a) and the same total copper cross section, also the same current density. The kv-a. capacity and copper loss per duct-ft. are therefore the same. The two must be distinguished between however, as (a) is a separate three-phase circuit, while (b) is only one leg of a three-phase circuit. It is assumed that the total amount of power to be transmitted will require three cables or some multiple of three, of either type and the cables will each be in a separate duct. It is then easy to see how closely the conditions coincide.

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[1] "Three-Core Cables" E. T. Z. Heft. 47, 1915. Abstract in *London Electrician*, 1916, p. 209.

TABLE I

COMPARISON OF THREE-CONDUCTOR AND SINGLE-CONDUCTOR CABLE THREE-PHASE Y CONNECTION

Cable	Total copper cross section cm.	Core diam. in.	Cond. diam. in.	Thick. insul. in.	Lead sheath in	Overall diam. in.	3 ϕ volt to neut. Kv.	Current per cable	Kv-a. capacity per cable	$I^2 R$ loss per ft. per cable watts	Ratio a/b	Kv./cm. G_{max}
					Three-conductor cable.							
(a)	3 x 300,000	0	0.631	12/32x6/32	9/64	3.63	$30/\sqrt{3}$	3 x 231	12,000	3 x 2.18	......	......
					Singl-eeonductor cable.							
(b)	900,000	0	1.09	17/32	1/8	2.40	$30/\sqrt{3}$	693	12,000	6.54	0.505	18.3
(c)	275,000	1.16	1.30	1 3/32	9/64	3.76	$55/\sqrt{3}$	378	12,000	6.32	0.370	19.5
(d)	550,000	1.00	1.30	1 3/32	9/64	3.76	$55/\sqrt{3}$	378	12,000	3.00	0.370	19.5

An advantage of (b) over (a) is in its smaller over-all diameter. The cost of installation would be less, especially as regards simplicity of joints, and the cable itself would be cheaper because of less insulation and lead. Another advantage is that (b) can be more safely overloaded with less danger of deterioration developing, as there are no tangential stresses.

The disadvantages of single-conductor cable depend upon individual cases and involve rather complicated considerations such as: transposition at manholes, necessity of breaking the continuity of lead sheath or accepting the additional loss introduced by short-circuited sheath. This will be dealt with later in a way that will aid in deciding the merits of each individual case.

We would not especially recommend the substitution of cable (b) for (a) although there probably are cases, such as short tie-ins, where it would be advantageous. The real advantage of single-conductor cable lies in the opportunity for much higher operating voltage with consequent reduction in copper cross section and increase in line efficiency.

Cable (c) is designed to transmit at $55/\sqrt{3}$ kv. the same amount of power as (a) and (b) with the same loss per duct ft. There is, therefore, no gain in line efficiency, but the amount of copper has been reduced to less than one-third of that needed in (b).

If the limiting feature is line loss rather than investment in copper the copper cross section can be increased until the proper blaance is reached. In cable (d) the copper cross section is doubled and the I^2R loss thereby even more than halved because of the lower operating temperature. Cable (c) is designed to operate at 55 deg. cent. and (d) at about 37 deg. cent., 12,000 kv-a.

Liberal copper cross section is a good investment in any cable and particularly so in this type, for the additional cost involved is solely that of the bulk copper, which can be realized upon when the cable is scrapped. Reference to the design data of (c) and (d) will show what is meant by this. They both have exactly the same conductor diameter, which is fixed by the voltage rating and not by the amount of copper. The conductor is filled out to this diameter by wrapping the strands over a core, usually made of hemp rope. The only dimension that is different in (c) and (d) is the diameter of this core. Obviously, (d) is good for 41 per cent overload in an emergency.

Cable (d) with only 550,000 cir. mils of copper can transmit 41 per cent more load than (b), which has 900,000 cir. mils copper, for the same temperature rise.

DESIGN OF SINGLE-CONDUCTOR CABLE

The most important point to be considered in designing a high-tension cable is the maximum voltage stress. In a recent paper[2] a method of determining the maximum allowable stress is worked out in detail. For paper cable impregnated with mineral hydro-carbon base compound the maximum working stress is placed at between 19 and 20 kv-eff. per cm. (48.3 and 50.8 volts per mil.) The formula for calculating the maximum stress in single-conductor cable is

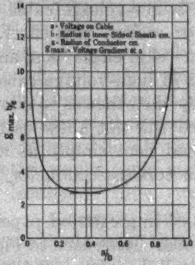

FIG. 1—SHOWING THE MOST EFFICIENT CROSS SECTION OF SINGLE-CONDUCTOR CABLE

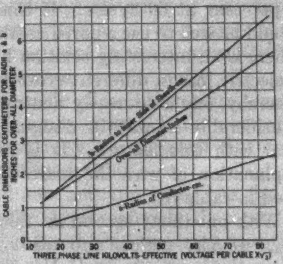

FIG. 2—SINGLE-CONDUCTOR CABLE—SMALLEST OVER-ALL DIAMETER FOR GIVEN VALUE OF G_{max}

G_{max} . = 19.5 kv./cm.

$b/a = \epsilon = 2.718$

$$g_{max} = \frac{e}{a \log_e b/a}$$

where

e = impressed voltage.

a = radius of conductor in cm.

b = radius to inner side of sheath in cm.

A curve is given in Fig. 1, calculated with the above formula and assuming different values of the ratio a/b. The most efficient cross section of cable, that is, the smallest cross section for any given conditions, is determined from the minimum ordinate of this curve. Ratio a/b found in this way has a value of 0.370, or b/a is exactly equal to the naperian logarithm base, $\epsilon = 2.718$.

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[2] "Ionization of Occluded Gases in High-Tension Insulation," by G. B. Shanklin and J. J. Matson. TRANS. A. I. E. E., 1919, p. 489.

A safe working stress is selected at 19.5 kv. per cm. (49.5 volts/mil.) With $a/b = 0.370$ the most efficient dimensions of cable are calculated at various voltage ratings and are given as curves in Fig. 2.

Cable (b) in Table I, has a conductor diameter that cannot be reduced to the most efficient value because of the large cross section of copper involved. The "skin effect" is rather high,—7 per cent at 60 cycles. If this were a low-voltage cable it would be well to use a cored conductor to reduce "skin effect." There is an interesting relation between "skin effect" loss and dielectric loss in different cross sections of high-voltage cable, whereby, one is so nearly neutralized by the other as to allow considerable lee-way in designs. Space will not allow us to deal with this feature. It is mentioned here simply as an explanation of why the more desirable coreless conductor is used in cable (b).

With one thirty-second in. thickness of impregnated paper, $g_{max} = 18.3$ kv. per cm. and ratio $a/b = 0.505$. Reference to Fig. 1 shows this ratio to be high. The overall diameter is 2.4 in., whereas the minimum diameter for the same voltage rating in Fig. 2 is 2.15 in. However, the over-all diameter of 2.4 in. is sufficiently small to be easily handled and any further reduction would not be of great importance. When feasible, it is better not to stress a cable up to the maximum limit of 19.5 kv. per cm. That is why cable (b) is stressed to only 18.3 kv. per cm. The factor of safety is not only greater but due to lower dielectric loss, the cable can safely operate at a higher temperature.

With higher-voltage cable, over-all diameter is of greater importance, in fact, it is the only thing that limits the voltage rating. The practical limit of over-all diameter is about 3.9 in. (9.9 cm.) at the very most. From Fig. 2 the corresponding voltage rating is $57/\sqrt{3}$ kv. Cables (c) and (d) in Table I are rated at $55/\sqrt{3}$ kv., the nearest standard voltage to this. It would not be practical to use cable rated at, say, $66/\sqrt{3}$ kv. The over-all diameter would then be 4.46 in. (11.33 cm.) With special machinery for manufacture and special means of transportation and installation it would be possible to use such cable, but the extra expense involved would hardly warrant the benefit derived.

GRADING OF SINGLE-CONDUCTOR CABLE

It, no doubt, has occurred to the reader that since the over-all diameter is limited, further gain in voltage rating could be accomplished by grading the insulation. Osborne[3] and Beaver[4] demonstrated the advantages, from a theoretical standpoint, of graded cables. Actually, however, from a practical standpoint grading has few possibilities.

If all other factors are ignored grading is of benefit only if:

(a) there are high grade insulating materials of reasonable cost to select from, whose permittivities cover a fairly wide range, and individually have consistent values of permittivity that do not vary in different batches, or with ageing, etc.

(b) these materials have the same allowable working stress, regardless of their individual permittivities. If the allowable stress varied directly with permittivity it is apparent that even better results could be obtained by grading.

Neither of these two conditions actually hold. As far as (a) is concerned there are only three materials that can be considered, and of these three, one, rubber, introduces doubtful features. Aside from its high cost there are questions of its stability and length of life when used in composition with other materials, especially in such extremely high-voltage cable. The truth is, rubber has never been given a thorough try out in high-voltage cable because of its prohibitive cost.

The three materials mentioned and the range of their 60-cycle, relative permittivities at 55 deg. cent. are listed below:

Rubber, 4.6 to 6.0
Varnished cambric, 3.8 to 5.2
Impregnated paper, 2.8 to 4.0

A better representation of permittivity characteristics can be obtained from the dielectric loss data which will be given in this paper and those in two papers read before the Institute in 1917[5] and 1919.[2]

It is apparent from the above values that rough grading could be accomplished by using these three materials, but, condition (b) upsets this combination. In the paper, "Ionization of Occluded Gases, etc.,"[2] the safe maximum working stress for impregnated paper is placed at 19.5 kv. per cm. (49.5 volts per mil) while that for varnished cambric is only 16.0 kv. per cm. (40.6 volts per mil) because of its higher permittivity. Rubber insulation is not dealt with in that paper. We have made tests on rubber insulated cable, however, and found that its corresponding stress is a little less than that for varnished cambric, 15 kv. per cm. (38.1 volts per mil).

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[3] "Potential Stresses in Dielectrics" by H. S. Osborne, TRANS. A. I. E. E., Vol. XXIX, p. 1553.

[4] "Cables" by C. J. Beaver, *Journ. I. E. E.*, Vol. 53, 1915.

[5] "Insulation Characteristics of High-Voltage Cables" by Clark and Shanklin, TRANS. A. I. E. E., 1917, p. 447.

It follows from all this, that practically no advantage is gained by an attempt to grade with these materials, because their maximum allowable working stress is inversely proportional to their relative permittivities. It is much better to have the insulation a homogeneous, compact mass of impregnated paper and aim towards a uniformly high grade product with low temperature coefficient and dielectric losses.

Beaver, in his paper on "Cables,"[4] advocates the metallic inter-sheath method of grading as furnishing means of overcoming the almost universal objections to the variable permittivity method. He proposes connecting these inter-sheaths to transformer taps and holding them at some definite, predetermined potential. As far as we know, this has never been tried out in practise, and we doubt if any operating company would be willing to assume the risk involved. The chief objections to this method are:

a. The inter-sheath must necessarily be of flimsy construction and there is danger of damaging it or even breaking its continuity during transportation and installation. High local potentials would concentrate at this point, resulting in failure.

b. The length of line would be very limited, as the inter-sheath could not carry the heavy charging current in long lines.

c. This method is based on the supposition that it is desirable to equally stress the whole insulation cross section. Actually, it is not desirable, because of the high dielectric loss involved.

d. Weak spots in the different layers would be lined up over the whole length. Failure of one layer would throw full potential on the others, also the short-circuit current might fuse the inter-sheath at some point far distant from the original point of failure.

The only other way of increasing voltage rating without prohibitive increase in over-all diameter is by stressing the cable above the safe limit we have assumed of 19.5 kv. per cm. This value is just about a maximum of what popular experience has shown to be conservative practise. It is more than a coincidence that recent laboratory investigations[2] check it very closely. The only radical departure from this practise we know of has been tried in Germany. In several instances underground cables have been worked at stresses much higher than 19.5 kv. per cm.[6] Just how successfully these experiments worked out over a reasonable period of time we do not know.

The single-conductor cable referred to in Litchtenstein's paper operated at a stress of 42 kv. per cm. At the time his paper was published it had operated intermittently for twenty months, however, the frequency was only 16 $^2/_3$ cycles and the current only 2.4 per cent of full-load current.

DIELECTRIC LOSSES

When estimating the watts loss in a conduit line the dielectric loss is often ignored. This is a great mistake. Even in the highest grade cable this loss is an appreciable part of the total, although, as a rule, a small part. The higher the voltage rating and operating temperature the more nearly it approaches the copper loss. In poor grade cable this loss may be even more than the copper loss. Consequently, the very highest grade of impregnated paper is absolutely necessary for high-tension cable.

The amount of space that can be given to dielectric energy loss characteristics is necessarily limited in this paper for the inductive characteristics of single-conductor sheathed cable and the general line characteristics must also be dealt with. We will limit the data presented to three designs of single-conductor cable and then show an empirical method of calculating the dielectric characteristics for any cross section of cable either single- or three-conductor.

The loss measurements were taken by the "Compensated Dynamometer Wattmeter Method," which is described in detail in an article published in 1916.[7] The energy loss data given

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[6] "Underground High-Tension (60-kv.) Cables" by Leon Litchtenstein E. T. Z. Jan. 1913, Abstract in *Elec. Rev.* March 7, 1913.

"30,000 Volt, Three-Phase Cable System, Berlin" by W. Pfaunkuch, E. T. Z., April, 1916.

[7] "Compensated Dynamometer Wattmeter Method of Measuring Dielectric Energy Loss", by G. B. Shanklin, *General Electric Review*, Oct. 1916.

TABLE II.
CABLE SAMPLE NO. I.

Dia. over bare copper strands 1.186 in. (3.012 cm.).

Insulation—26/32 in. (2.064 cm.) thickness paper filled with mineral hydro-carbon base compound.

Temp.	Freq.	e kv.	i_d $\times 10^{-3}$	w_d	ρ_{av} $\times 10^{10}$	K_{av}	C $\times 10^{-10}$	$\cos \theta_d$ per cent
23 deg. cent	60	12.01	0.276	0.0219	148.3	3.11	0.610	0.660
		18.01	0.411	0.0497	147.2	3.09	0.607	0.673
		21.00	0.489	0.0671	148.0	3.14	0.616	0.654
		23.99	0.559	0.0878	147.2	3.14	0.616	0.657
		26.98	0.630	0.1118	146.8	3.15	0.619	0.657
		29.97	0.700	0.1372	147.2	3.16	0.621	0.655
		32.96	0.772	0.1959	125.0	3.17	0.623	0.758
		35.95	0.840	0.2960	98.3	3.16	0.621	0.980
		41.93	1.018	0.6790	58.3	3.28	0.645	1.592
		47.90	1.200	1.3320	38.7	3.39	0.666	2.320
61.5 deg. cent	60	12.01	0.292	0.404	8.05	3.26	0.640	11.50
		18.01	0.440	0.920	7.95	3.28	0.643	11.61
		21.00	0.513	1.258	7.90	3.28	0.643	11.70
		23.99	0.589	1.637	7.90	3.28	0.645	11.61
		26.98	0.660	2.090	7.82	3.28	0.644	11.72
		29.97	0.732	2.560	7.89	3.28	0.644	11.68
		32.96	0.807	3.100	7.82	3.28	0.645	12.01
		35.95	0.888	3.890	7.49	3.31	0.650	12.20
		41.93	1.053	5.680	6.96	3.38	0.663	12.88
		47.90	1.246	7.750	6.64	3.49	0.686	13.01
60 deg. cent	25	11.99	0.136	0.304	10.60	3.60	0.707	18.68
		17.98	0.206	0.689	10.55	3.64	0.717	18.62
		20.97	0.240	0.940	10.52	3.64	0.715	18.68
		23.97	0.273	1.211	10.64	3.64	0.714	18.50
		26.96	0.310	1.538	10.68	3.66	0.720	18.40
		29.95	0.343	1.901	10.60	3.64	0.715	18.50
		32.94	0.380	2.300	10.58	3.67	0.722	18.40
		35.92	0.414	2.790	10.42	3.66	0.720	18.80
		41.91	0.494	3.980	9.92	3.76	0.738	19.28
		47.90	0.591	5.640	9.14	3.93	0.772	19.90
100 deg. cent	60	12.01	0.333	1.300	2.50	3.56	0.700	32.45
		18.01	0.501	2.730	2.65	3.58	0.703	30.32
		21.00	0.588	3.780	2.60	3.60	0.707	30.60
		23.99	0.670	4.970	2.60	3.60	0.706	31.00
		26.98	0.769	6.12	2.65	3.67	0.721	29.55
		29.97	0.849	7.750	2.60	3.64	0.716	30.50
		32.96	0.940	9.440	2.56	3.66	0.719	30.50
		35.95	1.037	11.890	2.45	3.70	0.728	31.90
		41.93	1.250	17.650	2.24	3.80	0.747	33.80
		47.90	1.482	23.500	2.19	3.94	0.775	33.10

TABLE II. *(continued)*

Temp.	Freq.	e kv.	i_d $\times 10^{-3}$	w_d	ρ_{av} $\times 10^{10}$	K_{av}	C $\times 10^{-10}$	$\cos\theta_d$ per cent
100 deg. cent	25	11.99	0.169	0.827	3.91	4.16	0.816	40.90
		17.98	0.254	1.880	3.89	4.17	0.820	41.30
		20.97	0.296	2.550	3.88	4.17	0.820	41.10
		23.97	0.339	3.330	3.88	4.17	0.820	41.00
		26.96	0.380	4.280	3.82	4.16	0.818	41.90
		29.95	0.422	5.190	3.88	4.17	0.820	41.00
		32.94	0.469	6.290	3.88	4.21	0.827	40.70
		35.92	0.519	7.680	3.78	4.25	0.836	41.20
		41.91	0.630	10.970	3.60	4.43	0.870	41.60
		47.90	0.753	14.980	3.44	4.65	0.911	41.50
60 deg. cent	60	12.01	0.283	0.288	11.25	3.18	0.623	8.45
		18.01	0.431	0.644	11.35	3.22	0.634	8.29
		21.00	0.509	0.875	11.35	3.27	0.642	8.20
		23.99	0.579	1.134	11.40	3.25	0.639	8.15
		26.98	0.651	1.339	11.37	3.25	0.639	8.20
		29.97	0.722	1.780	11.35	3.25	0.639	8.24
		32.96	0.801	2.245	10.89	3.28	0.645	8.52
		35.95	0.877	2.840	10.25	3.30	0.649	9.01
		41.93	1.036	4.270	9.25	3.36	0.660	9.86
		47.90	1.232	6.070	8.48	3.45	0.677	10.30
60 deg. cent	25	11.99	0.130	0.253	12.82	3.47	0.683	16.22
		17.98	0.195	0.573	12.70	3.46	0.680	16.39
		20.97	0.230	0.775	12.76	3.52	0.691	16.10
		23.97	0.263	1.011	12.76	3.51	0.691	16.05
		26.96	0.298	1.262	12.80	3.52	0.691	15.78
		29.95	0.331	1.580	12.76	3.53	0.694	15.92
		32.94	0.366	1.931	12.65	3.55	0.697	16.02
		35.92	0.402	2.330	12.45	3.58	0.704	16.12
		41.91	0.486	3.400	11.61	3.71	0.730	16.69
		47.90	0.574	4.960	10.38	3.82	0.750	18.10
23 deg. cent	60	12.01	0.281	0.0262	124.0	3.16	0.621	0.777
		18.01	0.424	0.0595	123.0	3.17	0.623	0.782
		21.00	0.495	0.0796	124.5	3.18	0.626	0.767
		23.99	0.565	0.1038	124.5	3.18	0.626	0.767
		26.98	0.640	0.1318	124.0	3.20	0.629	0.762
		29.97	0.710	0.1684	120.0	3.20	0.629	0.794
		32.96	0.780	0.2760	88.6	3.20	0.629	1.072
		35.95	0.854	0.4130	70.5	3.20	0.629	1.347
		41.93	0.995	0.8290	47.7	3.20	0.629	1.990
		47.90	1.179	1.4590	35.3	3.32	0.653	2.580

in this paper can be considered as supplemental to those in the two papers previously referred to [2] and [5] and are dealt with in the same way.

In Table II, 25- and 60-cycle data are given for a high-tension single-conductor cable (No. 1) having the following specifications.

Conductor. 58 strands, each 0.104 in. diameter in two layers, on a core 0.770 in. (1.956 cm.) diameter. Total conductor diameter 1.186 in. (3.012 cm.) Cross section 625,000 cir. mils

Insulation. 26/32 in. (2.064 cm.) thickness paper filled with mineral hydro-carbon base compound.

Sheath. 0.101 in. (.257 cm.) thickness of lead.

Over-all Diam. 3.02 in. (7.67 cm.)

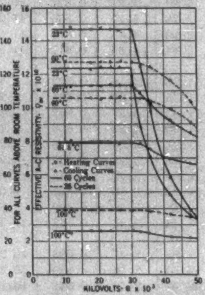


FIG. 3—CABLE NO. 1—SINGLE-CONDUCTOR (SEE TABLE II)

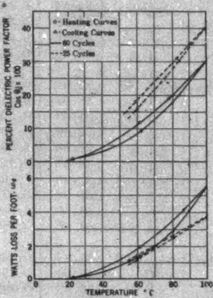


FIG. 4—CABLE NO. 1—SINGLE-CONDUCTOR (SEE TABLE II) CURVES PLOTTED AT 25.4 kv.

The measurements in Table II are given consecutively in the same order in which they were taken in the laboratory. The nomenclature and the method of calculating the different values can be found in Appendix A. In Figs. 3 and 4 curves are given of effective a-c. resistivity vs. impressed voltage, watts loss per ft. vs. temperature, and dielectric power factor vs. temperature. These curves show the more important characteristics. The other characteristics may be obtained from Table II. Space does not allow the presentation of them all in the form of curves.

Referring to Fig. 3 the resistivity (ρ_{av}) is constant up to the impressed voltage at which internal ionization starts, in other words, the dielectric loss varies as the square of the voltage

TABLE III
CABLE SAMPLE NO. 2

Dia. over smooth conductor surface 1.311 in. (3.33 cm.)

Insulation—24/32 in. (1.905 cm.) thickness paper filled with mineral hydro-carbon base compound.

Temp.	Freq.	e Kv.	i_d $\times 10^{-3}$	w_d	ρ_{av} $\times 10^{10}$	K_{av}	C $\times 10^{-10}$	$\cos\theta_d$ per cent
24 deg. cent.	60	12.01	0.309	0.0196	185.2	3.06	0.679	0.527
		18.01	0.468	0.0442	184.6	3.10	0.689	0.525
		21.00	0.549	0.0603	184.0	3.11	0.692	0.524
		23.99	0.628	0.0789	184.0	3.13	0.695	0.524
		26.98	0.707	0.0996	184.0	3.14	0.697	0.523
		29.97	0.785	0.1420	159.0	3.13	0.695	0.604
		32.96	0.864	0.2218	123.2	3.13	0.695	0.779
		35.95	0.940	0.3550	91.6	3.11	0.692	1.050
		41.93	1.125	0.8900	49.7	3.21	0.714	1.890
		47.90	1.327	1.8500	31.2	3.19	0.735	2.920
60 deg. cent.	60	12.01	0.328	0.298	12.15	3.26	0.724	7.55
		18.01	0.492	0.674	12.08	3.26	0.724	7.60
		21.00	0.580	0.925	12.00	3.29	0.730	7.60
		23.99	0.660	1.211	11.95	2.28	0.730	7.64
		26.98	0.744	1.529	12.00	3.28	0.730	7.63
		29.97	0.829	1.911	12.00	3.29	0.732	7.71
		32.96	0.910	2.410	11.90	3.28	0.730	8.03
		35.95	0.990	2.940	11.05	3.27	0.726	8.26
		41.93	1.190	4.650	9.51	3.38	0.750	9.34
		47.90	1.400	6.910	8.34	3.47	0.772	10.32
60 deg. cent.	25	11.99	0.150	0.251	14.40	3.56	0.790	13.95
		17.98	0.227	0.559	14.51	3.59	0.796	13.72
		20.97	0.269	0.768	14.40	3.65	0.810	13.62
		23.97	0.304	1.006	14.35	3.60	0.798	13.96
		26.96	0.345	1.260	14.40	3.63	0.805	13.68
		29.95	0.387	1.565	14.40	3.69	0.814	13.50
		32.94	0.423	1.906	14.35	3.65	0.809	13.68
		35.92	0.464	2.304	14.09	3.66	0.814	13.90
		41.91	0.551	3.480	12.69	3.75	0.831	15.05
		47.90	0.655	5.350	10.77	3.87	0.858	17.09
100 deg. cent.	60	12.01	0.372	1.16	3.13	3.57	0.793	25.90
		18.01	0.561	2.56	3.20	3.60	0.799	25.25
		21.00	0.656	3.64	3.06	3.61	0.803	26.30
		23.99	0.746	4.63	3.13	3.60	0.799	25.80
		26.98	0.850	5.85	3.13	3.64	0.804	25.60
		29.97	0.950	7.10	3.18	3.66	0.813	25.00
		32.96	1.032	9.55	3.08	3.68	0.816	26.95
		35.95	1.173	11.79	2.81	3.74	0.831	27.90
		41.93	1.410	17.70	2.50	3.83	0.851	30.00
		47.90	1.665	25.50	2.26	3.94	0.875	32.00
100 deg. cent.	25	11.99	0.188	0.87	4.20	4.17	0.926	38.20
		17.98	0.280	1.92	4.24	4.14	0.917	38.20
		20.97	0.330	2.57	4.32	4.20	0.931	37.00
		23.97	0.379	3.37	4.28	4.21	0.935	37.20
		26.96	0.426	4.26	4.28	4.21	0.935	37.20
		29.95	0.472	5.26	4.28	4.20	0.931	37.30
		32.94	0.529	6.61	4.32	4.26	0.947	37.90
		35.92	0.579	8.09	4.01	4.25	0.944	38.90
		41.91	0.699	11.98	3.68	4.37	0.971	40.90
		47.90	0.836	16.72	3.45	4.57	1.013	41.70

TABLE III (*continued*)

Temp.	Freq.	e Kv.	i_d $\times 10^{-3}$	w_d	ρ_{av} $\times 10^{16}$	K_{av}	C $\times 10^{-10}$	$\cos \theta_d$ per cent
60 deg. cent.	60	12.01	0.318	0.219	16.60	3.15	0.700	5.72
		18.01	0.478	0.490	16.64	3.16	0.700	5.69
		21.00	0.555	0.665	16.70	3.15	0.700	5.71
		23.99	0.631	0.870	16.64	3.15	0.700	5.75
		26.98	0.719	1.101	16.64	3.17	0.704	5.70
		29.97	0.800	1.381	16.64	3.18	0.706	5.76
		32.96	0.882	1.759	16.37	3.19	0.708	6.05
		35.95	0.964	2.222	14.65	3.20	0.711	6.43
		41.93	1.156	3.690	12.00	3.29	0.729	7.64
		47.90	1.370	6.040	9.56	3.40	0.755	9.22
60 deg. cent.	25	11.99	0.147	0.190	19.00	3.49	0.774	10.81
		17.98	0.220	0.430	18.85	3.49	0.774	10.89
		20.97	0.253	0.590	18.75	3.45	0.765	11.11
		23.97	0.290	0.769	18.80	3.45	0.765	11.08
		26.96	0.330	0.973	18.80	3.50	0.775	10.93
		29.95	0.363	1.199	18.80	3.45	0.765	11.02
		32.94	0.402	1.452	18.70	3.49	0.774	10.97
		35.92	0.446	1.811	17.90	3.55	0.789	11.30
		41.91	0.532	2.879	15.36	3.63	0.804	12.90
		47.90	0.634	4.810	12.00	3.75	0.833	15.90
29 deg. cent.	60	12.01	0.301	0.0241	150.0	2.99	0.665	0.670
		18.01	0.460	0.0538	151.0	3.05	0.676	0.649
		21.00	0.539	0.0731	151.5	3.06	0.679	0.648
		23.99	0.612	0.0959	151.0	3.05	0.676	0.652
		26.98	0.701	0.1220	151.0	3.00	0.666	0.645
		29.97	0.774	0.1548	146.0	3.08	0.684	0.668
		32.96	0.859	0.2190	124.8	3.12	0.691	0.775
		35.95	0.935	0.3310	98.2	3.11	0.689	0.985
		41.93	1.118	0.7360	60.0	3.18	0.707	1.579
		47.90	1.323	1.7750	32.5	3.31	0.734	2.800
29 deg. cent.	25	11.99	0.133	0.0213	170.0	3.35	0.744	1.340
		17.98	0.209	0.0476	170.5	3.34	0.741	1.269
		20.97	0.245	0.0654	169.0	3.35	0.744	1.278
		23.97	0.274	0.0846	170.5	3.28	0.728	1.290
		26.96	0.311	0.1074	170.0	3.32	0.736	1.284
		29.95	0.350	0.1348	167.2	3.36	0.744	1.285
		32.94	0.386	0.1782	153.1	3.36	0.744	1.405
		35.92	0.416	0.2425	134.0	3.33	0.739	1.620
		41.91	0.496	0.5740	77.0	3.41	0.756	2.760
		47.90	0.590	1.2440	45.4	3.54	0.786	4.410

up to this point. From here on ionization introduces additional loss and ρ_{av} drops rapidly. The minimum voltage at which ionization starts is 30 kv. and the voltage stress at the conductor is

$$g_{max} = \frac{30}{a \log_e b/a} = 23.1 \text{ kv. per cm.}$$

Since this cable is designed to operate at $44/\sqrt{3} = 25.4$ kv., or at a voltage stress, $g_{max} = 19.5$ kv. per cm., it is apparent that there is no danger of internal ionization during operation. The divergence between the 25- and 60-cycle watts loss increases with temperature (Fig. 4). The power factor, of course, is higher at 25 cycles. No 25-cycle measurements at room temperature are given but will be for the other two cables.

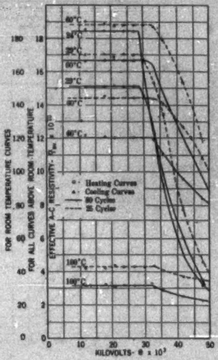


FIG. 5—CABLE NO. 2—SINGLE-CONDUCTOR (SEE TABLE III)

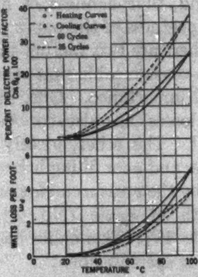


FIG. 6—CABLE NO. 2—SINGLE-CONDUCTOR (SEE TABLE III) CURVES PLOTTED AT 25.4 KV.

In Table III measurements are given for cable No. 2, having the following specifications:

Conductor. Same as that for cable No. 1 except that a 1/16 in. (0.1578 cm.) lead jacket was placed over the bare strands to give a smooth surface. Conductor diameter 1.186 + 0.125 = 1.311 in. (3.33 cm.)

Insulation. 24/32 in. (1.905 cm.) thickness paper filled with mineral hydro-carbon base compound.

Sheath. Same as cable No. 1.

Over-all Diameter. Same as cable No. 1.

The test on this cable was made to determine by comparison with No. 1, the benefit derived from a smooth surfaced conductor. The ρ_{av} vs. E curves in Fig. 5 give a minimum ionization voltage of 28.5 kv., which corresponds to a voltage stress at the conductor, g_{max} = 22.5 kv. per cm. This is even less than the voltage stress in cable No. 1 and would indicate that there is practically no difference between stranded and smooth surfaced conductors. The watts and power factor curves in Fig. 6 show No. 2 to have slightly less loss than No. 1. This is only a variation that often occurs in different batches and has nothing to do with the type of conductor. It is due to slight differences of the compound, thoroughness of filling, and compactness of the paper wrappings.

There are two factors involved in high-tension cable that are of equally vital importance. One, high dielectric losses, has already been emphasized. The other, loosely wrapped paper and poor filling is best brought out by the measurements on a special length of cable (No. 3) given in Table IV. This cable had the following specifications:

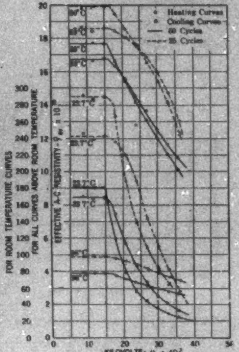


Fig. 8—Loose Wrappings (See Table IV)

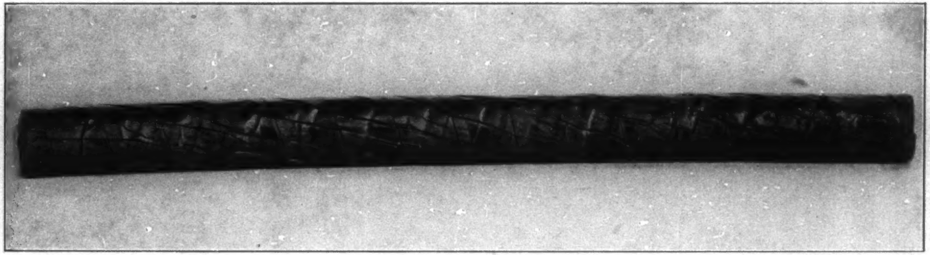
Conductor. Same as cable No. 1.

Insulation. 22/32 in. (1.746 cm.) wrapped on loosely and poorly filled with mineral hydrocarbon base compound.

Sheath. 9/64 in. (0.357 cm.) thickness of lead.

The first few wrappings of paper were loosely applied with the remaining wrappings applied in the usual way. The length was then vacuum treated and filled with compound, but not left in the compounding tank for the usual period of time. Consequently, the spaces between the loose wrinkled wrappings were not well filled. These wrinkles are plainly shown in Fig. 7, the photograph being taken after all but the first two paper wrappings were removed.

The ρ_{av} vs. E curves in Fig. 8 show decidedly more ionization than occurred in Cables Nos. 1 and 2. Ionization starts at



[CLARK AND SHANKLIN]

FIG. 7—CABLE NO. 3—WRINKLING OF IMPREGNATED PAPER WRAPPING NEAR CONDUCTOR
(TWO WRAPPINGS ONLY REMAINING)

TABLE IV.

CABLE SAMPLE NO. 3

Dia. over stranded conductor surface same as Cable No. 1.

Insulation—22/32 in. (1.746 cm.) thickness paper loosely wrapped and partly filled with mineral hydro-carbon base compound.

Temp.	Freq.	e kv.	i_d $\times 10^{-3}$	w_d	ρ_{av} $\times 10^{10}$	K_{av}	C $\times 10^{-19}$	$\cos\theta_d$ per cent
23.7 deg. cent	60	9.00	0.221	0.0118	170.0	2.96	0.651	0.595
		11.99	0.296	0.0214	168.0	2.97	0.653	0.601
		14.99	0.374	0.0331	169.0	3.01	0.660	0.590
		17.98	0.449	0.0540	148.9	3.02	0.663	0.669
		20.97	0.525	0.0976	112.0	3.02	0.663	0.885
		23.95	0.599	0.1651	86.2	3.02	0.663	1.153
		26.94	0.679	0.2730	66.2	3.04	0.668	1.495
		29.93	0.760	0.4440	50.2	3.07	0.675	1.952
		32.91	0.844	0.6510	41.3	3.10	0.681	2.345
		35.90	0.925	0.9710	33.0	3.12	0.685	2.920
23.7 deg. cent	25	8.98	0.094	0.0082	246.0	3.12	0.664	0.971
		11.97	0.126	0.0148	240.0	3.16	0.671	0.981
		14.97	0.158	0.0230	242.0	3.16	0.671	0.976
		17.98	0.191	0.0342	235.0	3.18	0.675	0.998
		20.99	0.222	0.0499	220.0	3.16	0.671	1.074
		24.01	0.257	0.0761	188.2	3.21	0.682	1.233
		27.05	0.296	0.1270	143.2	3.28	0.697	1.585
		30.11	0.342	0.2030	111.0	3.40	0.723	1.978
		33.21	0.401	0.3340	82.0	3.62	0.770	2.505
		36.34	0.498	0.5870	56.0	4.11	0.874	3.240
59 deg. cent	60	9.00	0.228	0.120	16.72	3.05	0.670	5.86
		11.99	0.306	0.216	16.60	3.08	0.676	5.86
		14.99	0.385	0.334	16.72	3.09	0.679	5.80
		17.98	0.461	0.492	16.32	3.09	0.679	5.94
		20.97	0.540	0.705	15.51	3.10	0.681	6.22
		23.95	0.620	0.979	14.56	3.12	0.685	6.58
		26.94	0.702	1.328	13.62	3.14	0.690	7.01
		29.93	0.785	1.770	12.56	3.16	0.695	7.54
		32.91	0.871	2.307	11.64	3.18	0.700	8.03
		35.90	0.960	2.958	10.83	3.22	0.709	8.57
59 deg. cent	25	8.98	0.098	0.107	18.65	3.26	0.693	12.20
		11.97	0.132	0.193	18.45	3.29	0.700	12.10
		14.97	0.165	0.301	18.56	3.28	0.696	12.20
		17.98	0.199	0.436	18.42	3.29	0.700	12.21
		20.99	0.234	0.609	18.05	3.30	0.704	12.42
		24.01	0.271	0.815	17.54	3.36	0.714	12.50
		27.05	0.313	1.078	16.88	3.43	0.730	12.72
		30.11	0.363	1.416	15.91	3.58	0.763	12.95
		33.21	0.421	1.859	14.74	3.76	0.799	13.28
		36.34	0.529	2.518	13.05	4.20	0.894	13.10
98 deg. cent	60	9.00	0.240	0.514	3.92	3.12	0.686	23.80
		11.99	0.321	0.924	3.87	3.13	0.688	24.00
		14.99	0.405	1.441	3.87	3.16	0.695	23.60
		17.98	0.489	2.150	3.74	3.19	0.700	24.45
		20.97	0.575	3.110	3.52	3.20	0.704	25.60
		23.95	0.662	4.260	3.34	3.22	0.707	26.90
		26.94	0.754	5.710	3.17	3.24	0.711	28.16
		29.93	0.852	7.530	2.96	3.29	0.724	29.50
		32.91	0.952	9.710	2.77	3.32	0.729	31.00
		35.90	1.047	12.340	2.62	3.32	0.729	32.80

TABLE IV. *(continued)*

Temp.	Freq.	e kv.	i_d $\times 10^{-3}$	w_d	ρ_{av} $\times 10^{10}$	K_{av}	C $\times 10^{-10}$	$\cos\theta_d$ per cent
98 deg. cent	25	8.98	0.118	0.415	4.84	3.62	0.770	39.20
		11.97	0.160	0.722	4.93	3.70	0.786	37.60
		14.97	0.199	1.149	4.84	3.66	0.780	38.50
		17.98	0.242	1.684	4.77	3.73	0.794	38.70
		20.99	0.289	2.410	4.56	3.79	0.805	39.80
		24.01	0.334	3.270	4.37	3.81	0.809	40.75
		27.05	0.386	4.440	4.10	3.87	0.823	42.50
		30.11	0.445	5.770	3.90	3.99	0.849	43.10
		33.21	0.514	7.440	3.68	4.17	0.887	43.60
		36.34	0.623	9.650	3.41	4.65	0.987	42.60
59 deg. cent	60	9.00	0.226	0.115	17.60	3.02	0.664	5.64
		11.99	0.299	0.202	17.76	3.01	0.661	5.63
		14.99	0.376	0.317	17.67	3.02	0.664	5.64
		17.98	0.454	0.484	16.61	3.04	0.668	5.94
		20.97	0.529	0.711	15.40	3.04	0.668	6.44
		23.95	0.609	1.008	14.14	3.06	0.674	6.91
		26.94	0.690	1.380	13.10	3.09	0.679	7.43
		29.93	0.773	1.851	12.03	3.11	0.684	8.00
		32.91	0.859	2.430	11.05	3.15	0.692	8.60
		35.90	0.951	3.180	10.10	3.19	0.701	9.31
59.5 deg. cent	25	8.98	0.096	0.102	18.85	3.18	0.676	11.78
		11.97	0.130	0.178	20.00	3.23	0.686	11.40
		14.97	0.163	0.280	19.95	3.24	0.688	11.48
		17.98	0.195	0.413	19.47	3.24	0.688	11.78
		20.99	0.229	0.591	18.52	3.24	0.688	12.88
		24.01	0.265	0.814	17.61	3.27	0.695	12.82
		27.05	0.308	1.100	16.54	3.38	0.718	13.21
		30.11	0.355	1.461	15.40	3.50	0.744	13.68
		33.21	0.420	1.931	14.20	3.75	0.797	13.84
		36.34	0.521	2.680	12.27	4.26	0.905	14.14
23.7 deg. cent	60	9.00	0.215	0.111	181.0	2.87	0.631	0.574
		11.99	0.288	0.198	180.2	2.91	0.639	0.574
		14.99	0.360	0.310	180.2	2.90	0.637	0.574
		17.98	0.435	0.674	119.4	2.92	0.641	0.862
		20.97	0.509	1.389	78.8	2.93	0.643	1.304
		23.95	0.586	2.542	56.0	2.96	0.651	1.811
		26.94	0.669	4.110	44.0	2.99	0.658	2.280
		29.93	0.750	6.250	35.6	3.03	0.666	2.780
		32.91	0.830	9.190	29.2	3.05	0.670	3.360
		35.90	0.921	12.900	24.8	3.10	0.682	3.900
23.7 deg. cent	25	8.98	0.092	0.0070	291.0	3.06	0.649	0.840
		11.97	0.124	0.0124	288.0	3.10	0.659	0.835
		14.97	0.155	0.0193	289.5	3.10	0.659	0.830
		17.98	0.187	0.0302	266.0	3.12	0.663	0.900
		20.99	0.219	0.0581	189.0	3.12	0.663	1.268
		24.01	0.252	0.1024	139.8	3.15	0.668	1.690
		27.05	0.292	0.1661	109.5	3.24	0.688	2.100
		30.11	0.336	0.2718	83.0	3.34	0.710	2.685
		33.21	0.400	0.4230	64.8	3.61	0.767	3.190

a minimum voltage of 15 kv., which corresponds to a stress at the conductor, $g_{max} =$ 12.95 kv. per cm. Since the cable is supposed to operate at 19.5 kv. per cm. it is evident that it would soon give trouble. The thickness of gas space estimated for this sample in the paper on internal ionization previously referred to[2] was found to be seven times that in average good cable. It is interesting to note that at voltages below ionization the watts loss per cm.[3] and the power factors are lower than those in the better cables (Nos. 1 and 2). The permittivity is also lower. This is caused by poor filling and gas spaces, since no loss is experienced in the entrapped gases. A cable made up of unfilled paper would have even less loss if well dried and provided the voltage was below ionization. This characteristic is often deceptive. The watts loss, alone, does not indicate the grade of material, unless there is reasonable assurance that it is compact and well filled.

Predetermination of Dielectric Loss

Dielectric loss characteristics (up to the voltage at which ionization starts) can be predetermined for any cross section of cable with a fair degree of accuracy, provided sufficient measurements are available on cables made of the same materials and by the same process to obtain a good average of the dielectric power factor and permittivity. The loss variation in different batches of well made cable is not over 25 per cent which allows a difference of 12.5 per cent one way or the other from the estimated average loss.

Sixty- and 25-cycle curves of power factor and permittivity vs. temperature, for both single- and three-conductor paper cable filled with mineral hydro-carbon base compound, are given in Figs. 9 and 10. To be on the safe side, the power factor curves given are actually 10 per cent higher than the averages of a large number of tests. They are what might be termed "high average" curves. It will be noticed that the power factor is about 10 per cent higher for three-conductor cable than for single-conductor. We believe that this difference is due to the jute filler rather than three-phase stress, for we have been unable to detect any difference between the three-phase loss calculated from single-phase measurements on three-conductor cable, as described in a previous paper[5] and that actually measured with three-phase voltage.

The permittivity of three-conductor cable is also higher.

The jute filler doubtless has something to do with this. The greater part of the difference, however, we believe to be due to error in the approximate formulas (6 and 7 in Appendix A) used in calculating K_{av} from the measured values of c_1 and c_4 This error cancels itself out when K_{av} is converted back again to c_1 and c_4, as it must be in preliminary calculations, and is therefore of no importance. Knowing the power factor and permittivity, the loss and all other dielectric characteristics in any cross section of cable can be estimated from the formulas in Appendix A.

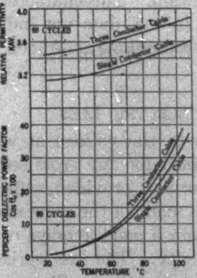


FIG. 9—PAPER CABLE FILLED WITH MINERAL HYDRO-CARBON BASE COMPOUND

These curves are 10 per cent higher than average found from large numbers of tests on different batches—they apply at all voltages below internal ionization

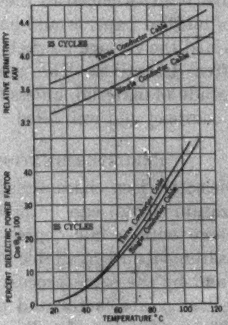


FIG. 10—PAPER CABLE FILLED WITH MINERAL HYDRO-CARBON BASE COMPOUND

These curves correspond to the 60-cycle curves in Fig. 9

INDUCTIVE CHARACTERISTICS OF SINGLE-CONDUCTOR CABLE

Dielectric characteristics are essentially functions of line voltage, and inductive characteristics of line current. The induced sheath current when sheath is short-circuited, or induced voltage when open-circuited, brings in a complication in dealing with which we had considerable trouble during the initial stages of the work with single-conductor cable.

The formulas for calculating the inductive characteristics developed in Appendix B give very accurate results. In fact, the accuracy reached is greater than can be obtained by actual measurements, even when these are taken with extreme care.

The formulas are completely developed theoretically without approximations. Berg,[8] in dealing with the same subject gave short approximate formulas, but they apply only within certain limits not met with by the cable cross sections and spacings considered in this paper.

The theoretical work in Appendix B was checked in the laboratory. A number of tests were made with which we can deal only briefly. Fig. 11 gives a diagram of the test circuit. The two parallel lengths of cable were each 50 ft. in length, laid out straight on wooden horses 3½ ft. high. The horses were made without iron nails and there was no iron within their height of the cables. A uniform magnetic field resulted. A flexible loop of cable connected the far ends of the conductors and the ends of the sheaths were bonded by a straight conductor of low resistance perpendicular to the cable lengths (a switch was inserted in the sheath bond so that external resistance could be added to the sheath circuit when desired). By extending the flexible loop out far enough its inductive effect on the sheath bond was eliminated. The drop leads were connected as shown, with corresponding pairs twisted together and arranged to reduce inductive effect to a minimum.

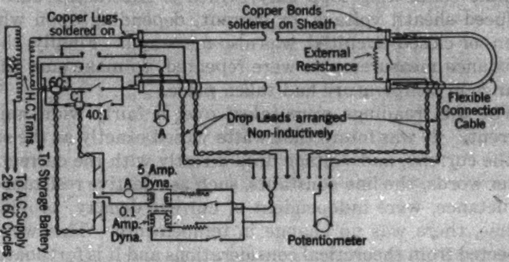

Copper Lugs soldered on
Copper Bonds soldered on Sheath
External Resistance
Flexible Connection Cable
22:1
160:1
CT
40:1
H.C.Trans.
A
Drop Leads arranged Non-inductively
To Storage Battery
To A.C.Supply 25 & 60 Cycles
5 Amp. Dyna.
A
0.1 Amp. Dyna.
Potentiometer

FIG. 11—CONNECTION DIAGRAM OF HIGH-CURRENT SINGLE-CONDUCTOR CABLE TEST

Current transformers stepped the current down for ammeter and wattmeter readings. Watts were read with a dynamometer wattmeter having a constant of 0.27×10^{-6}. Another very sensitive dynamometer (constant 0.013×10^{-6}) was used as a voltmeter. Induced sheath current and wattmeter current were read with carefully calibrated portable ammeters. By a double-throw switch arrangement either a-c. or d-c. readings could be taken, the d-c. voltage drop being measured with a potentiometer. The accuracy of d-c. resistance readings was greatly facilitated by the opportunity to check readings of the total circuit against the sum of the different sections.

---

[8] "Constants of Cables and Magnetic Conductors" by E. J. Berg, TRANS. A. I. E. E., Vol. 26, p. 555.

The general procedure was first to measure the d-c. resistance for determination of copper and sheath temperatures. Then a-c. readings of current, watts and voltage drop were taken over a range of current that would give seven-point curves. The conductor readings were taken on the total circuit (two cable lengths plus connecting loop at far end) and then on the flexible connecting loop only, which is subtracted out. The induced sheath voltage, or current, depending upon whether open- or closed-circuited, was also measured. Finally, the d-c. resistance measurements were repeated, to make sure that no change in temperature had taken place.

The a-c. readings were taken over a fairly wide range of current. It was found that watts varied exactly as the square of the current, and voltage drop directly with the current. In other words, the line constants, such as effective resistance and inductance, were independent of current density, provided of course, there was no change in temperature. This would be expected from theoretical considerations and it is fortunate that it is so, for it allows accurate and complete calculations of line characteristics as will be shown later.

There are two sources of error in this method of test;

a. phase-angle errors, due to inductance in the current transformer and wattmeter circuit.

b. distorted wave shape.

The power factors at which the measurements were taken were fairly high. In the worst case the phase-angle error introduced was only about 4 per cent. By measurement of the current transformer phase angle and inductance of wattmeter circuit it could be largely corrected for. This source of error can be considered negligible when proper care is taken. Errors from distorted wave shape are much more difficult to overcome. The only way they can be corrected for is by using a perfect sine wave of current, which is difficult of attainment with such high values of current. Our wave shape was of the ordinary commercial type. Errors in the measurements given in the following tables were due to variances from a sine wave.

TABLE VI.—CABLE NO. 2 (See Table III.)
ALL VALUES PER 1000 FT. OF ONE CONDUCTOR TO NEUTRAL.

SHEATHS CLOSED-CIRCUITED (GROUNDED IN TWO PLACES.)

Temp deg. cent.		Inches spacing D	Freq.	Measured d-c. resistance			r_c	r_{eff}		$L_{eff} \times 10^{-3}$		Z_{eff}		Ratio I_s/I_c	
Cond.	Sheath			Sheath	r_s	Cond.		Meas.	Calc.	Meas.	Calc.	Meas.	Calc.	Meas.	Calc.
26.0	26.0	24.0	60	0.1161	0.1835	0.017295	0.01736	0.03950	0.03736	0.2140	0.2155	0.0899	0.0895	0.330	0.3305
25.0	25.0	24.0	60	0.1160	1.5990	0.017210	0.01731	0.02123	0.01987	0.2383	0.2340	0.0925	0.0904	0.0415	0.0401
27.0	27.0	24.0	25	0.1146	0.1869	0.01738	0.01738	0.02180	0.02102	0.2242	0.2290	0.0411	0.0405	0.1455	0.1415
25.0	25.0	24.0	25	0.1153	1.9560	0.01727	0.01727	0.01759	0.01736	0.2378	0.2341	0.0412	0.0408	0.01395	0.01364
26.0	26.0	6.15	60	0.1150	0.1951	0.01728	0.01738	0.02390	0.02278	0.1480	0.1490	0.0606	0.0606	0.1710	0.1669
25.0	25.0	6.15	60	0.1140	1.981	0.01724	0.01734	0.01881	0.01789	0.1531	0.1508	0.0608	0.0596	0.01725	0.01665
22.0	22.0	6.15	25	0.1150	0.5890	0.01699	0.01699	0.01807	0.01731	0.1512	0.1509	0.0289	0.0293	0.02255	0.0232
23.0	23.0	6.15	25	0.1146	1.937	0.01705	0.01706	0.01751	0.01716	0.1510	0.1509	0.0295	0.0292	0.00735	0.00706

SHEATHS OPEN-CIRCUITED (GROUNDED IN ONE PLACE ONLY.)

Temp. deg. cent.		Inches spacing D	Freq.	r_{d-c}	r_c		$L_{eff} \times 10^{-3}$		$M \times 10^{-3}$		Z_{eff}		Ratio e_s/E	
Cond	Sheath				Meas.	Calc.	Meas.	Calc.	Meas.	Calc.	Meas.	Calc.	Meas.	Calc.
27.4	26.7	24.00	60	0.01738	0.01790	0.01746	0.2398	0.2341	0.1759	0.1701	0.0921	0.08995	0.720	0.714
24.6	24.7	24.00	25	0.01718	0.01741	0.01718	0.2382	0.2341	0.1737	0.1701	0.04115	0.04055	0.658	0.657
24.0	24.0	6.15	60	0.01714	0.01784	0.01722	0.1539	0.1509	0.0899	0.0869	0.0607	0.05950	0.560	0.552
23.3	23.7	6.15	25	0.01710	0.01730	0.01710	0.1518	0.1509	0.0895	0.0869	0.02947	0.02920	0.476	0.467

A comparison between measured and calculated values of resistance and inductance for two different cross sections of cables No. 2 and 4 are given in Tables V and VI. The method of calculation and nomenclature are in Appendix B. Before the various characteristics can be calculated the resistances of the sheath and conductor must be known for any assumed temperature. These are given in Fig. 12 for the cable considered in Table V. A 12-ft. length of the cable was placed in an electric oven and d-c. measurements taken of conductor and sheath resistance over the temperature range indicated. Ignoring the spirality effect of the strands the resistance of the conductor is closely represented by copper of 98 per cent conductivity. This value of conductivity is used in Formula 4, Appendix B, for calculating the 25- and 60-cycle effective resistances r_c as given in Fig. 12.

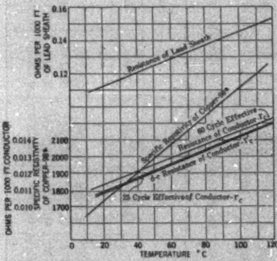


FIG. 12—D-C. AND A-C. RESISTANCE OF CONDUCTOR AND SHEATH AT VARIOUS TEMPERATURES FOR CABLE NO. 4 (TABLE V)

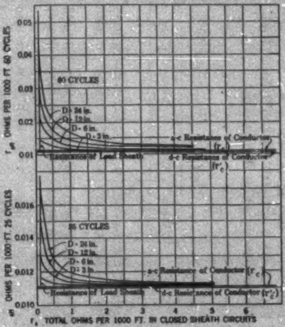


FIG. 13—RELATION BETWEEN EFFECTIVE RESISTANCE AND TOTAL RESISTANCE OF CLOSED SHEATH CIRCUIT—ONE CABLE TO GROUNDED NEUTRAL (TABLE V) 25 DEG. CENT.

The calculated values of effective resistance r_{eff} and effective inductance L_{eff} in Table V for cable No. 4 are given in the form of curves in Figs. 13, 14 and 15. Before plotting the curves the temperature was corrected to 25 deg. cent. from Fig. 12 and by Formulas 2 and 3, Appendix B.

The curves of r_{eff} vs. r_s in Fig. 13 are very interesting. They

Copper area 1,007,000 cm.
Dia. of hemp core........ 0.594 in. (1.51 cm.)
Dia. of copper............ 1.346 in. (3.42 cm.)

Thick. of insulation........
Thick. of sheath..........
Over-all dia..............

Temp. deg. cent.		Spacing inches		Measured d-c. resistance			
Cond.	Sheath	*D*	Freq.	Sheath	r_s	Cond.	Mea
26	26	24	60	0.1130	0.1455	0.01103	0.011
25	25	24	60	0.1145	0.7575	0.01100	......
25	25	24	60	0.1145	1.3400	0.01100	......
23	26	24	60	0.1150	1.9050	0.01091	0.011
25	25	24	60	0.1145	3.6550	0.01100	0.011
28	28	24	25	0.1158	0.1465	0.01112	0.011
25	25	24	25	0.1145	0.7575	0.01100	......
25	25	24	25	0.1145	1.3400	0.01100	......
25	25	24	25	0.1145	1.9050	0.01100	......
25	25	24	25	0.1145	5.0000	0.01100	0.011
25	27	12	60	0.1153	0.1455	0.01100	0.011
25	25	12	60	0.1145	0.7575	0.01100	......
25	25	12	60	0.1145	1.3400	0.01100	......
23	25.5	12	60	0.1148	1.9050	0.01091	0.011
25	25	12	60	0.1145	3.6550	0.01100	......
27	27	12	25	0.1153	0.1520	0.01109	0.011
25	25	12	25	0.1145	0.7575	0.01100	......
25	25	12	25	0.1145	1.3400	0.01100	......
25	25	12	25	0.1145	1.9050	0.01100	......
25	25	12	25	0.1145	5.0000	0.01100	......
23	26	6	60	0.1150	0.1455	0.01091	0.0111
25	28	6	60	0.1158	0.7575	0.01100	0.0112
25	27	6	60	0.1153	1.3400	0.01100	0.0112
25	28	6	60	0.1158	2.2500	0.01100	0.0112
25	25	6	60	0.1145	5.0500	0.01100	0.0112
24	27	6	25	0.1154	0.1625	0.01095	0.0110
25	25	6	25	0.1145	0.7575	0.01100	......
25	25	6	25	0.1145	1.3400	0.01100	......
25	25	6	25	0.1145	1.9050	0.01100	......
25	25	6	25	0.1145	12.9750	0.01100	0.0110
23	24	3.024	60	0.1140	0.1530	0.01091	0.0112
25	25	3.024	60	0.1145	0.7575	0.01100	......
25	25	3.024	60	0.1145	1.3400	0.01100	......
24	26.5	3.024	60	0.1152	2.2500	0.01095	0.0112
25	25	3.024	60	0.1145	3.6550	0.01100	......
23	23	3.024	25	0.1138	0.1610	0.01091	0.01095
25	25	3.024	25	0.1145	0.7575	0.01100	......
25	25	3.024	25	0.1145	1.3400	0.01100	......
25	25	3.024	25	0.1145	1.9050	0.01100	......
25	25	3.024	25	0.1145	5.0000	0.01100	......

SHEATHS OPEN-CIRC

Deg. cent. temp.	Inches spacing *D*	Freq.	$r_{d\text{-}c}$ meas	r_c	
				Meas.	Calc.
25	24	60	0.0110	0.01125	0.01150
25	24	25	0.0110	0.01105	0.01110
25	12	60	0.0110	0.01115	0.01150
25	12	25	0.0110	0.01105	0.01110
25	6	60	0.0110	0.01120	0.01150
25	6	25	0.0110	0.01100	0.01110
25	3.024	60	0.0110	0.01120	0.01150
25	3.024	25	0.0110	0.01105	0.01110

(1.11 cm.)
(0.3175 cm.)
(6.375 cm.)

TABLE V. (CABLE NO. 4)

VALUES GIVEN PER 1,000 FT. OF ONE CONDUCTOR TO NEUTRAL

Sheaths Closed-Circuited (Grounded in Two Places.)

	r_{eff}		$L_{eff} \times 10^{-3}$		Z_{eff}		Ratio I_s/I_c	
Calc.	Meas.	Calc.	Meas.	Calc.	Meas.	Calc.	Meas.	Calc.
01153	0.03930	0.03857	0.1858	0.1960	0.07955	0.08310	0.4290	0.4310
01150	......	0.01780	......	0.2285	......	0.08800	......	0.0912
01150	......	0.01508	......	0.2295	......	0.08800	......	0.0518
01135	0.01225	0.01393	0.2340	0.2300	0.08810	0.08800	0.03840	0.0370
01150	0.01245	0.01282	0.2280	0.2300	0.08470	0.08800	0.01860	0.0190
01121	0.01738	0.01667	0.2190	0.2225	0.03900	0.03880	0.2060	0.1935
01110	......	0.01210	......	0.2300	......	0.03815	......	0.0381
01110	......	0.01173	......	0.2300	......	0.03800	......	0.0216
01110	......	0.01154	......	0.2300	......	0.03795	......	0.01512
01110	0.01129	0.01127	0.2215	0.2300	0.03690	0.03780	0.00532	0.00579
01150	0.02865	0.02885	0.1690	0.1710	0.06895	0.07050	0.3460	0.3480
01150	......	0.01526	......	0.1876	......	0.07205	......	0.0665
01150	......	0.01364	......	0.1877	......	0.07205	......	0.0381
01141	0.01255	0.01291	0.1935	0.1880	0.07195	0.07205	0.02780	0.0281
01150	......	0.01228	......	0.1879	......	0.07205	......	0.01461
01119	0.01440	0.01437	0.1835	0.1840	0.03250	0.03175	0.1470	0.1450
01110	......	0.01175	......	0.1879	......	0.03176	......	0.0290
01110	......	0.01147	......	0.1879	......	0.03165	......	0.01662
01110	......	0.01136	......	0.1879	......	0.03155	......	0.01170
01110	......	0.01120	......	0.1879	......	0.03150	......	0.00446
01141	0.02115	0.02080	0.1360	0.1393	0.05660	0.05625	0.2630	0.2500
01150	0.01295	0.01337	0.1470	0.1455	0.05625	0.05635	0.0483	0.0496
01150	0.01190	0.01186	0.1430	0.1455	0.05510	0.05610	0.0224	0.0228
01150	0.01175	0.01172	0.1460	0.1455	0.05623	0.05600	0.0157	0.0167
01150	0.01164	0.01163	0.1455	0.1455	0.05600	0.05600	0.00715	0.00744
01105	0.01250	0.01255	0.1455	0.1450	0.02575	0.02605	0.0960	0.0951
01160	......	0.01143	......	0.1455	......	0.02605	......	0.0207
01100	......	0.01129	......	0.1455	......	0.02600	......	0.0117
01100	......	0.01123	......	0.1455	......	0.02600	......	0.00823
01100	0.01110	0.01112	0.1440	0.1455	0.02520	0.02595	0.00122	0.00121
01141	0.01465	0.01446	0.1013	0.1028	0.04045	0.04115	0.1390	0.1412
01150	......	0.01213	......	0.1040	......	0.04095	......	0.0288
01150	......	0.01161	......	0.1040	......	0.04090	......	0.0163
01145	0.01155	0.01160	0.1025	0.1040	0.04025	0.04085	0.00980	0.00969
01150	......	0.01159	......	0.1040	......	0.04085	......	0.00590
01101	0.01153	0.01154	0.1045	0.1040	0.01960	0.01985	0.0570	0.0564
01110	......	0.01121	......	0.1040	......	0.01980	......	0.0120
01110	......	0.01116	......	0.1040	......	0.01975	......	0.00678
01110	......	0.01115	......	0.1040	......	0.01975	......	0.00476
01110	......	0.01113	......	0.1040	......	0.01975	......	0.00182

(Grounded in One Place Only.)

ductance $f \times 10^{-3}$	Inductance $M \times 10^{-3}$		Impedance Z_{eff}		Ratio e_s/E	
Calc.	Meas.	Calc.	Meas.	Calc.	Meas.	Calc.
0.2295	0.1825	0.1840	0.0840	0.0872	0.798	0.800
0.2295	0.1813	0.1840	0.0330	0.0378	0.750	0.765
0.1880	0.1395	0.1418	0.0715	0.0717	0.729	0.745
0.1880	0.1450	0.1418	0.0320	0.0316	0.712	0.705
0.1455	0.1010	0.0999	0.0565	0.0560	0.683	0.672
0.1455	0.1015	0.0999	0.0254	0.0254	0.629	0.617
0.1040	0.0563	0.0577	0.0406	0.0408	0.524	0.533
0.1040	0.0565	0.0577	0.0194	0.0198	0.457	0.466

show that at 60 cycles and a spacing between cable centers of 6 in. (15.2 cm.), which is close to the usual spacing in ducts, a resistance equal approximately to ten times that of the lead sheath must be inserted in the ground bonds to reduce the effective resistance with closed sheath circuit to within 10 per cent of that with open sheath circuit. At 25 cycles the same percentage is almost reached without any resistance at all in the sheath bonds, that is, with the sheath directly grounded. Two interesting conclusions are drawn from this:

a. In a 60-cycle line of the above cable it is better to break the continuity of the sheath into relatively short lengths (at every man hole or every other man hole) and directly ground each sheath length only once because of the high loss with short-circuited sheath. This can be conveniently done by having a common neutral wire running the length of the conduit line. The continuity of the sheaths can be broken by a well designed, moisture-proof sleeve unit. Such a unit could easily be turned out in quantity production and would take the place of the ordinary jointing sleeve. This method would be better than using a continuous sheath and grounding it in two or more places through a bond resistance. Nothing is gained and a good deal is lost by grounding through bond resistance. The resistance would absorb over 90 per cent of the induced sheath voltage and the sheath would remain at almost the same potential above ground as it does open circuited. Also, such a high resistance ground is not reliable and does not afford sufficient protection.

b. 25-cycle operation is more favorable to single-conductor cable than 60-cycle. In a 25-cycle line there is the choice of either grounding the sheaths direct, which amounts to the same thing as short-circuiting them, or breaking their continuity and grounding each length only once as described above. With short-circuited sheaths an additional loss of 14 per cent is introduced in cable No. 4. There will be no potential between sheaths under this condition, as the sheaths, themselves, absorb the induced voltage and remain at ground potential over their whole length. If this additional loss is undesirable the continuity can be broken and the induced sheath voltage, being proportional to the frequency, will be only 42 per cent of that at 60 cycles.

The idea, of course, in breaking the continuity of the sheaths is to keep the maximum sheath voltage within reasonable limits, so that leakage, especially in wet or flooded systems, will not give trouble. Just what would constitute a reasonable limit of sheath voltage depends upon the leakage resistance between sheaths and this depends upon the particular conditions and type of conduit and manhole construction used.

As each cable is enclosed in a separate duct the leakage between sheaths over the conduit length is negligible compared with the leakage at manholes, when well constructed tile or fibre conduit is used. It is at the manholes, then, that the greatest care must be taken to reduce leakage. The ducts should spread as they project into the manhole and the sheaths well protected. The usual methods of protection used to reduce fire risk at manholes as described by Carlton[9] should prove sufficient with a little additional care, such as applying a coating of waterproof paint to the bare sheaths before the protective covering is applied. Methods of reducing sheath voltage and leakage will be dealt with further under another heading.

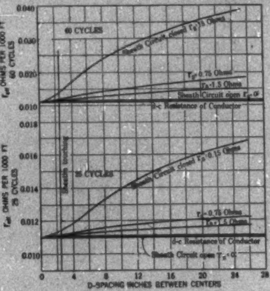


FIG. 14—25 AND 60-CYCLE EFFECTIVE RESISTANCE—ONE CABLE TO NEUTRAL (TABLE V) 25 DEG. CENT.

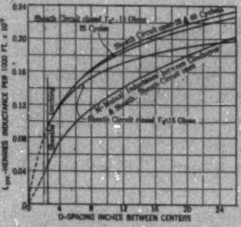


FIG. 15—EFFECTIVE INDUCTANCE—ONE CABLE TO NEUTRAL —(TABLE V) 25 DEG. CENT.

The effective resistance and effective inductance curves of cable No. 4 in Figs. 14 and 15 indicate clearly the importance of keeping the spacing between cables down to a minimum as far as the induced voltage or current in the sheath is concerned.

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[9] "Protection of Cables from Arcs due to the Failure of Adjacent Cables" by W. G. Carlton, A. I. E. E., 1904, p. 471.

It is surprising, though, to find that the self-inductance between conductors, and therefore the line regulation, compares very favorably with that of ordinary three-conductor cable For instance, cable (c) in Table I has an inductance per conductor to neutral of 0.145×10^{-3} henrys per 1000 ft. with a spacing of 6.74 in., while the equivalent three-conductor cable (a) in Table I, has a corresponding inductance of 0.113×10^{-3} henrys with only 1.38 in. spacing. Formula 8 in Appendix B was used in calculating these values. The difference in conductor cross sectional dimensions accounts for this unexpectedly close comparison.

It must not be assumed from the foregoing discussion that all single-conductor cables have as severe sheath inductive effects as Cable No. 4. This cable represents an extreme case. The sheath inductive effect for any given spacing depends upon the conductor cross section and the distance between sheath and conductor, that is, thickness of insulation. The percentage increase in effective resistance when sheath is short-circuited is tabulated below for three sizes of cable at a spacing of 6 inches.

Frequency	Cable No. 4	Cable No. 2	Cable (c)
25 cycles	14.4 per cent	8.8 per cent	4.5 per cent
60 cycles	96.0 per cent	48.0 per cent	22.0 per cent

These values were calculated by the method described in Appendix B and show clearly that it is perfectly feasible to operate the higher voltage cables at 25 cycles with sheaths short-circuited. Cable (c) might even have its sheath short-circuited at 60 cycles. This question will be taken up later under another heading.

SUMMARY OF SPECIAL INDUCTIVE TESTS

Several special tests other than those dealt with above were made to determine the effects of eddy currents in the open-circuited sheath and conductor, advantage of enameling the individual conductor strands and skin effect in the conductor. The results are outlined briefly:

a. By testing lengths of lead-sheathed cable and then repeating the measurements after the sheath was removed it was clearly proved that the additional loss due to eddy currents in the open-circuited sheath was too small to be detected. Cable No. 4 was the limiting design tested in this way, that is, it had the largest conductor cross section and least thickness of insulation.

b. Lengths of cable having an ordinary conductor of bare copper strands were compared with lengths made up in identically the same way, excepting that the individual copper strands were enameled. Not the slightest difference could be detected, even at the closest possible spacing, and with conductors up to 1,000,000-cir. mil cross section. This shows that eddy currents are not appreciable in ordinary bare stranded conductors. Evidently the thin oxide film that forms naturally on the surface of the strands furnished sufficient insulation to prevent cross currents.

c. The accuracy of our test methods was not great enough to allow a close check of the skin-effect formula (4) in Appendix B. As far as we are able to determine this formula holds nicely. It also checks very closely over the range needed in this work with the results obtained by Kennelly and Affel.[10]

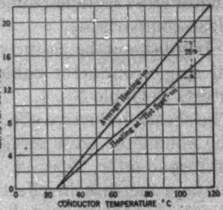

Fig. 16 — Representative Heating Characteristics in Conduit Line—25 Deg. Cent. Surrounding Soil Temperature

Note: The line calculations are based on these heating conditions

Line Characteristics of Underground Cables

Heating in Underground Cable. In determining the characteristics of underground cable lines there is only one factor, the heat dissipating ability of the conduit system, that introduces serious complications. Many papers dealing with the thermal conductivity of soils and the general theory of heating in underground cables have been published, both here and abroad. They have thrown a great deal of light on the subject but it can hardly be said that any method has yet been evolved with which individual systems can be dealt with theoretically.

In calculating line efficiency, regulation, etc. the *average* heat dissipating ability of the whole length of line must be known. In determining the maximum safe load the average is of no value, the *maximum* heating at the "hot spot" section of the line must be considered instead. Fig. 16 shows the average and

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[10] "Skin Effect Resistance Measurements of Conductors" by A. E. Kennelly and H. A. Affel, *Proc. Inst. of Radio Eng.*, Dec. 1916.

"hot spot" heating characteristics for a representative conduit line, based on the assumption of 25 deg. cent. surrounding soil temperature. The curves represent a very fair average of the conditions usually met with in underground conduit. They are useful in comparing different designs of cable for line efficiency, maximum safe rating, etc.

We would recommend very strongly that some such conditions of heating be adopted as a standard. Cables could then be rated and designs compared on the basis of this standard. Without some such basis to work on it is a very uncertain matter to modify cable ratings to suit individual conduit lines, or even to suit different seasons of the year in any given line. By keeping careful temperature records under actual operating conditions and comparing them with this standard we believe that operating companies could accumulate a valuable mass of empirical data that would clear up many of the uncertainties existing to-day.

Curves similar to those in Fig. 16 can be determined for any given conduit line by a heat run, just as made on any electrical unit such as a transformer or generator. The average temperature is obtained from resistance measurements of the conductor. The maximum temperature at "hot spots" must be found by exploring temperature indicators. Since these curves are practically straight lines only one or two load readings are necessary.

We do not believe that laboratory investigations of the thermal conductivity of soils, duct walls, insulation, etc. and a mathematical study of the heat paths will ever be of practical application. This abstract method of attack has proved and will continue to be a valuable and instructive adjunct to the more direct method outlined above. We are at present conducting a laboratory investigation along these lines and hope to describe the results in a future paper.

CALCULATION OF LINE CHARACTERISTICS

The general method of calculating cable line characteristics is explained in Appendix C. It applies to relatively long lines where the effects of charging current, line regulation, etc., are appreciable. Such elaborate and complete calculations are hardly necessary in dealing with short lines, where the usual approximate methods serve the purpose.

COMPARISON OF SINGLE- AND THREE-CONDUCTOR CABLE LINES

The ultimate purpose of the cable work so far dealt with in this paper has aimed towards placing the reader in a better position to compare the relative merits of single- and three-conductor cable lines.

As a final comparison, two exactly similar four-duct conduit lines will be assumed, placed side by side and each having heating characteristics corresponding to those given in Fig. 16. Three cables of the limiting design of 30-kv. three-conductor cable, given in Table I cable (a) will be placed in one of these conduit lines. The other line will have three 55-kv. single-conductor cables cable (c) designed for approximately the same loss per duct-ft. with open-circuited sheath. The general line characteristics of these two limiting types of cable will be calculated and compared at 60 cycles, following the method in Appendix C. Other details will also be discussed and compared.

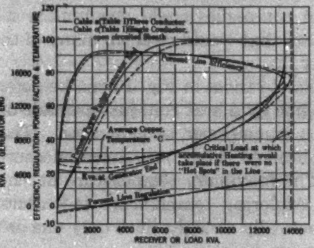

FIG. 17—COMPARISON OF LINE CHARACTERISTICS OF SINGLE- AND THREE-CONDUCTOR CABLES (SEE TABLE VII)—60-CYCLES—ALL VALUES PER CABLE—LENGTH OF LINE 50 MILES

The calculations for a fifty-mile line are tabulated in Table VII. They are based on the assumption of constant voltage at load or receiver end and a load power factor of 85 per cent. The more important characteristics are given in curve form in Fig. 17. There is little to choose between the two cables as far as general line characteristics are concerned. They have practically equal efficiencies, regulation, etc.

A cable line fifty-miles in length is exceptional. This length was chosen partly because it gives a severe comparison between the two cables and partly to show that there is nothing objectionable in such long lines other than the large charging capacity at no load. It amounts to approximately 5000 kv-a. or 100 kv-a. per mile of cable. High-potential testing would require at least 10,000 kv-a. unless made at a lower frequency. Few operating companies would have, or would care to invest, in the necessary a-c. equipment for this testing. A high-potential direct-current testing set would meet the conditions better than anything else.

In long cable lines such as considered the current and voltage, and hence the copper and dielectric losses, vary appreciably over the length of line. Fortunately at heavy loads where heating is a limiting factor the copper and dielectric losses vary in opposite directions, the increase in one heing more or less compensated for by decrease in the other. Consequently, if there were no inequalities in the heat dissipating ability of the duct lines the heating would be uniformly distributed and the copper temperature rise would be practically the same over the whole length of line.

The only inequalities we are concerned with are "hot spots," where maximum heating and temperature rise occur. These hot spots may be anywhere in the length of line and due to the complicated relationship between copper and dielectric losses there is always doubt as to just what position of hot spot would cause the severest heating.

In Fig. 18 we have accordingly assumed hot spots for the extreme conditions, that is, at the two ends of the line. The curves in this figure were calculated only for the single-conductor cable (c) by the method explained in Appendix C and the data in Table VII. The empirical formula for the dielectric conductance of cable (c) (Formula 22, Appendix C) found by the $\Sigma \triangle$ method is

$$g_d = 0.079 \times 10^{-9} + 0.306 \times 10^{-13}(T')^{2.65}$$

This formula holds only up to 90 deg. cent. but Fig. 18 shows that accumulative heating is reached at a temperature below this. Fig. 18 also shows that the worst position of hot spot is at receiver or load end. The critical load at which accumulative heating starts here is 11,200 kv-a. If the hot spot is at generator end the critical load is 11,800 kv-a. This difference is due to a greater increase in current, and hence copper loss, at load end over the correspondingly higher voltage and dielectric loss at generator end. A study of Table VII and Figs. 17 and 18 brings out many other interesting features which we will not deal with. It can readily be seen that the worst position of hot spot is not always at receiver end but depends upon the length of line, load power factor and relative dielectric and copper losses.

Since in this particular case the limiting position is at receiver end, the hot spot characteristics of cables (*a*) and (*c*) are compared at this point in Fig. 19. The calculation of these curves is very simple and was first suggested in the paper by Bang and Louis.[11] It furnishes a splendid means of comparing the merits of different cables and would be especially valuable if a standard duct of known heat dissipating ability was adopted, as previously recommended. Fig. 19 does not indicate any appreciable difference between cables (a) and (c). Cable (c) is slightly favored since it has a critical load of 11,200 kv-a., while that of cable (a) is 11,050 kv-a. The introduction of this hot spot at the receiver end, based on the assumption of 25 per cent less heat dissipating ability than the average, has lowered the critical load capacity of cable (a) 18.5 per cent and of cable (c) 19.5 per cent. The safe load ratings are therefore, lowered about the same amount.

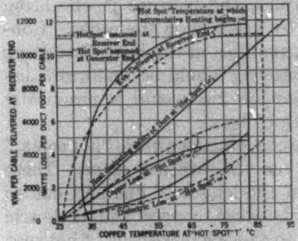

FIG. 18—"HOT SPOT" CHARACTERISTICS OF CABLE C (SEE TABLE VII AND FIG. 17)—60 CYCLES — SURROUNDING SOIL TEMPARATURE 25 DEG. CENT. —OPEN-CIRCUITED SHEATH

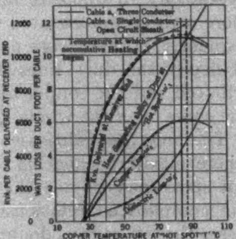

FIG. 19 — COMPARISON OF "HOT SPOT" CHARACTERISTICS OF SINGLE- AND THREE-CONDUCTOR CABLES (SEE TABLE VII) "HOT SPOT" ASSUMED AT RECEIVER END

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[11] "The Influence of Dielectric Losses on the Rating of High-Tension Cables," by Bang and Louis, A. I. E. E., Feb. 1917.

ADVANTAGES AND DISADVANTAGES OF SINGLE-CONDUCTOR CABLE

Both the line and hot spot characteristics of cables (a) and (c) are so closely similar that they can be neglected in a general comparison of the two types of cables. The comparison is thereby greatly simplified and allows a study of other important factors that might otherwise be over-shadowed.

In changing from (a) to (c) with open-circuited sheath there is a saving in copper of 70 per cent, a tremendous advantage. This is reduced to 66 per cent when the necessary neutral ground wire is added and a line of three cables only is considered. The saving will approach 70 per cent according to the number of cables in the line, as a single neutral wire can be made to serve any number of cables. Other advantages such as greater simplicity and reliability of joints and pot heads, and lesser possibility of insulation deterioration because of uniform radial stressing have already been dealt with.

There are two other advantages we would like to point out before taking up the disadvantages. One of these, and a very important one in large systems of almost unlimited short-circuit power is, that all short circuits from cable failures will be single phase. The chance of short circuits between phases is very remote. The majority of three-conductor cable failures ultimately form between phases and as every one knows they are a much more severe trial on the system.

The other advantage is in the possibility of substituting aluminum conductors. An equivalent aluminum conductor has a cross section ratio to copper of about 1.63 to 1. It would not be feasible to substitute aluminum in the three-conductor cable (a) because the conductor diameter is already the largest practical size. In cable (c) the substitution of aluminum merely means a reduction in the conductor core without otherwise affecting the dimensions. Whether this substitution would be desirable or not would depend upon the existing market price of aluminum and copper, although the difficulty of splicing aluminum conductors would counterbalance to some extent any advantage in cost.

The disadvantages of single-conductor cable that need be seriously considered are: unbalancing of phase inductance and induced sheath voltage or current, according to whether the sheath is open- or short-circuited.

The standard duct used today has a rectangular cross section.

This means that three single-conductor cables installed to transmit three-phase power must be spaced at the corners of a right angle triangle with consequent unbalancing of the phase inductance. In short lines this unbalancing would be of little consequence but in long lines transposition would have to be resorted to, the problem being handled in exactly the same way as in over head transmission.

By using special ducts of round cross section the cables could be spaced equilaterally. If the conduit line consisted of only one three-cable circuit there would be no need of transposition. If, however, more than three cables, six for instance, were closely spaced in one conduit mutual inductance between circuits would cause unbalancing to about the same degree as that caused by unsymmetrical spacing of three cables only.

It is not within the scope of this paper to deal with phase unbalancing or advise when transposition is necessary or when it is not necessary. A theoretical consideration of the subject would involve complicated and tedious mathematics and would be of little practical use. It is a subject that had best be left to the practical experience and judgment of operating engineers. Transposition can be made at manholes without much extra cost and to be on the safe side perhaps it should generally be used, especially so when parallel operation of two or more circuits is attempted.

Under a previous heading it has been shown that single-conductor cable can be operated at 25 cycles with sheaths bonded together and grounded in two or more phases. The short-circuited sheath current introduces an additional loss but this is a small item compared with the great saving in copper accomplished in changing over from three-conductor cable. This form of connection is shown diagrammatically in Fig. 20. (Connection No. 1.) The sheaths remain at ground potential over their whole length with an absence of bothersome leakage between sheaths and the special precautions needed to overcome it. Fig. 20 shows the sheaths bonded together and grounded at the ends in the same way three-conductor cable sheaths are, the only difference being the intermediate bonds, necessary to relieve the sheaths of the charging current that accumulates along their length. If the intermediate bonds are spaced every mile or two the loss due to charging current flowing along sheaths will be negligible. Greater care must be taken to assure reliable and permanent bonding and grounding than with three-conductor cable. If one of the sheaths was accidently freed of its bond connections electromagnetic and electrostatic induction would tend to raise it to a high potential. There would not only be danger of destroying the sheath or causing injury to workmen but there would also be danger of exploding accumulated gases by electrostatic sparking.

The 25-cycle effective resistance of cable (c) with short-circuited sheath and 6.74-in. spacing was calculated (see Formula 2, Appendix B) and found to be 6.3 per cent higher than effective resistance with open-circuited sheath at 25 deg. cent. and 5.3 per cent higher at 50 deg. cent. The decrease at higher temperature is due to increase in sheath resistance. There can be no question concerning the desirability of short-circuiting the sheath of cable (c) for 25-cycle operation. Also, the superiority in every way over cable (a) is very apparent.

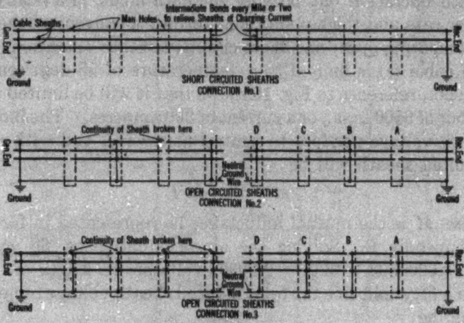
Intermediate Bonds every Mile or Two to relieve Sheaths of Charging Current
Cable Sheaths
Man Holes
Gen. End
Rec. End
Ground
Ground
SHORT CIRCUITED SHEATHS CONNECTION No.1
Continuity of Sheath broken here
D
C
B
A
Gen. End
Rec. End
Neutral Ground Wire
Ground
Ground
OPEN CIRCUITED SHEATHS CONNECTION No.2
Continuity of Sheath broken here
D
C
B
A
Gen. End
Rec. End
Neutral Ground Wire
Ground
Ground
OPEN CIRCUITED SHEATHS CONNECTION No.3

FIG. 20—SHOWING DIFFERENT METHODS OF BONDING AND GROUNDING SINGLE-CONDUCTOR CABLE SHEATHS

At 60 cycles, 25 deg. cent., the effective resistance is 30 per cent higher with short-circuited sheath and 26 per cent higher at 50 deg. cent. This means that approximately 28 per cent more copper in cable (c) would give practically the same line loss, efficiency, regulation, etc., with short-circuited sheath as that given in Table VII and Figs. 17, 18 and 19 for open-circuited sheath. Since the neutral ground wire would be eliminated it actually means only 15 per cent more copper for a three cable line. *There would still be a saving in copper* *of* 61 *per cent in changing over from cable* (*a*) *to cable* (*c*) *compared with* 66 *per cent saving when* (*c*) *is operated with open-circuited sheath.* This slight difference in copper saving is a significant and important feature and indicates very convincingly that, in this particular case, short-circuited sheath operation is much more favored even at 60 cycles.

A 28 per cent increase in copper changes the conductor cross section in cable (c) from 275,000 to 350,000 cir. mils. The data in Table VII and Figs. 17, 18 and 19 were calculated for a conductor of 275,000 cir. mils and open-circuited sheath rather than for a conductor of 350,000 cir. mils and short-circuited sheath, not with any intention of emphasizing open-circuited sheath operation but because the data holds, practically, for either case and the assumptions furnish a convenient method of comparing open- and short-circuited sheath operation.

If cable (c) is limited to a temperature of 55 deg. cent. at hot spot, reference to Fig. 19 shows that it will be limited to an output of 9500 kv-a., or a current of 300 amperes. The induced sheath voltage per foot to neutral at receiver end with open-circuited sheath will be

$$e_s = 2\pi f M I_r$$

where M is the mutual inductance as represented in formula 6, Appendix B. Solving the above equation at 60 cycles; $e_s = 0.0000302 \times 300. = 0.00906$ volts per ft. length of cable.

In a fifty-mile line the total induced sheath voltage to neutral would be

$$0.00906 \times 264{,}000 = 2390 \text{ volts.}$$

The problem remains to break the sheath up into the shortest practical sections and arrange the ground connections in such a way that maximum sheath voltage will be reduced to a minimum. Connections Nos. 2 and 3 in Fig. 20 show the two best methods of doing this. In No. 2 the sheath is broken at every other manhole with ground bonds at intermediate manholes. The sheaths are broken and bonded at every manhole in Connection No. 3. Assuming the manholes 500 ft. apart, a maximum distance for this size cable, the sheath voltage along the length of line is given in Fig. 21 for the two different connections.

For connection No. 2 the maximum voltage from sheath to ground is 4.53 volts, therefore the voltage between sheaths of any two cables is $\sqrt{3} \times 4.53 = 7.85$ volts, and the voltage

38
9

	Cable a	Cable c
Number of cables per line	3	3
Spacing between conductor centers (inches)	1.38	$\sqrt[3]{6\times 6\times 8.5} = 6.74$
Length of line (miles)	50	50
Weight of conductor copper per line (pounds)	2,202,000	672,000
Weight of copper in No. O, 8 and 8 neutral ground wire (pounds)	0	84,000
Total weight of copper per line (pounds)	2,202,000	756,000
Total weight of lead per line (pounds)	6,021,000	6,240,000

COM

60 CYCLES—ALL VALUES PER CABLE PER DUCT TO NEUTRAL—SURROUNDING SOIL 25

	Line constants				Volts to neutral		Current	
T deg. cent	r_c 10^{-4}	L 10^{-6}	g_d 10^{-9}	C 10^{-9}	e_r 10^3	e_g 10^3	I_r	I_g
						3 x 300,000 CM. THREE CONDU		
27.44	0.1235	0.03765	0.880	0.1596	17.32	16.85	0	273.2
27.44	0.1235		0.880	0.1596		17.40	122.4	237.0
27.5	0.1235		0.880	0.1596		17.47	130.0	236.8
28.0	0.1239		0.910	0.1597		17.71	188.7	241.5
35.0	0.1268		1.515	0.1607		18.91	434.1	371.8
45.0	0.1315		2.60	0.1625		19.71	589.1	505.2
55.0	0.1360		4.30	0.1646		20.25	681.6	694.0
75.0	0.1454		10.07	0.1695		20.95	769.7	701.0
90.0	0.1521		17.80	0.1739		21.10	780.6	746.0
100.0	0.1570		24.96	0.1769		21.25	777.5	770.0
					275,000 CM. SINGLE CONDUCTOR CABLE (C			
28.03	0.4016	0.145	0.320	0.05373	31.78	30.60	0	167.9
28.0	0.4016		0.320	0.05373		30.64	3.65	166.2
28.0	0.4016		0.320	0.05373		31.90	78.6	146.2
28.03	0.4016		0.320	0.05373		31.99	82.5	145.9
30.0	0.4043		0.380	0.05382		33.12	149.4	159.0
35.0	0.4125		0.470	0.05410		34.59	235.2	205.9
45.0	0.4272		0.830	0.05472		36.21	323.1	274.5
55.0	0.4425		1.350	0.05550		37.27	372.9	321.0
75.0	0.4730		3.040	0.05750		38.52	434.7	388.6
90.0	0.4960		5.300	0.05940		38.84	413.8	407.5

*NOTE: Generator power factor lagging.

TABLE VII

ON OF LINE CHARACTERISTICS—OF SINGLE AND THREE CONDUCTOR CABLES.

THE CABLES COMPARED ARE CABLES (A) AND (C) IN TABLE I.

nt. —SEE APPENDIX C FOR NOMENCLATURE.

Kv-a.		Power factor		Power		Line Efficiency per cent	Line Regulation per cent
	Gen.	cos θ_r lagging	cos θ_g leading	W_r 10^3	W_g 10^3		
BLE (CABLE A, TABLE I.)							
	4,570	0.850	0.0305	0	150.5	0	− 2.79
20	4,120		0.4741	1,802	1,952	92.4	0.46
50	4,140		0.5013	1,912	2,066	92.6	0.86
60	4,260		0.6960	2,778	2,962	93.6	2.20
10	7,020		0.9984	6,390	7,005	91.1	8.40
50	9,970		*0.9960	8,710	9,945	87.6	12.11
90	12,009		*0.9881	10,010	11,858	84.5	14.45
10	14,710		*0.9810	11,310	14,380	78.8	17.32
20	15,750		*0.9844	11,480	15,478	74.3	17.93
60	16,310		*0.9856	11,430	16,043	71.3	18.48
, TABLE I) OPEN CIRCUITED SHEATH.							
0	5,140	0.850	0.0365	0	187.8	0	− 3.86
15.9	5,100		0.0555	98.5	283.0	34.8	− 3.72
00	4,660		0.5060	2,126	2,311	92.0	0.376
20	4,660		0.5160	2,225	2,413	92.4	0.655
50	5,260		0.8240	4,038	4,346	92.9	4.05
70	7,120		0.9800	6,351	6,968	91.2	8.13
50	9,950		*0.9997	8,710	9,940	87.7	12.23
50	12,090		*0.9890	10,080	11,824	85.0	14.62
00	14,980		*0.9880	11,730	14,829	79.0	17.50
00	15,830		*0.9880	11,650	15,652	74.4	18.18

[CLARK AND SHANKLIN]

across the break in continuity of any one sheath is 2 × 4.53 = 9.06 volts. The only advantage of connection No. 3 is that the voltage across break in sheath is reduced by half. If unsymmetrical spacing of cables is used, or there are two or more circuits closely spaced in one conduit, unbalancing of inductance will increase the maximum sheath voltage. The increase will not be more than about 20 per cent.

These potentials are so low that it would not be a difficult matter to insulate the sheaths and reduce leakage to a negligible quantity even in wet or flooded conduit. However, when the 15 per cent increase in copper for short-circuited sheath operation of cable (c) is compared with the complications and expense of open-circuited sheath, such as, addition of neutral ground wire, the many breaks in continuity of sheath and the many bond connections that must be made absolutely secure, the balance in favor of short-circuited sheath stands out clearly.

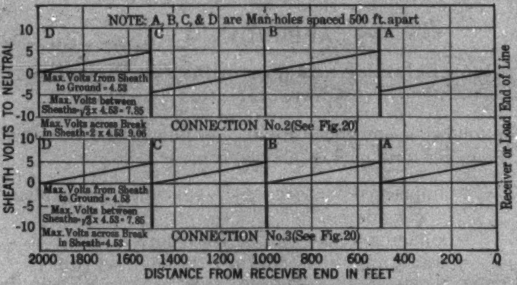

Fig. 21—Induced Sheath Voltage along Length of Single-Conductor Cable—Cable (c)—60 Cycles—300 amperes—Spacing 6.74 in.

With lower voltage, heavier current cable, cables No. 2 and 4 for instance, short-circuited sheath would mean an increase in copper of from 50 to 100 per cent, or more, for a 60-cycle line. The balance then turns in favor of open-circuited sheath, although the sheath potential is, unfortunately, somewhat higher than that given for cable (c) in Fig. 21. Potentials as high as 30 volts between sheaths might then have to be considered. The problem of insulating the sheaths would be increasingly difficult, but in some cases it would be the only alternative.

CONCLUSION.

The main features of single-conductor cable operation, as we see them, have now been presented. Some of the work is new, although the greater part is merely a compilation and elaboration of previous work by many different engineers. Where possible credit has been given. It is obviously impossible to give full credit in every particular.

The valuable advice and assistance of the following engineers directly associated with us in this work is deeply appreciated; Messrs. J. J. Matson, D. A. Ballard, W. C. Hayman, J. L. R. Hayden and H. H. Dewey.

APPENDIX A

Nomenclature and Formulas for Dielectric Characteristics

The following nomenclature and formulas are used throughout the paper for single-conductor cable.[12]

e = impressed voltage = $E/\sqrt{3}$ when three phase is considered.
i_d = amperes (dielectric) per ft. length of cable.
w_d = watts loss (dielectric) per ft. length of cable.
$\cos \theta_d$ = dielectric power factor.
c = capacity in farads per ft. length.
K_{av} = relative permittivity (average through thickness.)
K_a = capacity per cm.^{3} of air = 0.08842×10^{-12} farads.
r_d = effective a-c. insulation resistance per ft. length.
ρ_{av} = effective a-c. insulation resistance per cm.^{3}

$g = \frac{d\,e}{d\,x}$ = voltage gradient.

g_{max} = voltage gradient at conductor surface.
l = length in cm. (1 ft. = 30.48 cm.)
b = radius in cm. to inner side of sheath.
a = radius in cm. of conductor.
f = frequency.

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12. For derivation of these formulas see, "Insulation Characteristics of High-Voltage Cables," TRANS. A. I. E. E., June, 1917.

The dielectric circuit is considered as a parallel circuit, that is, the resistance and capacity are considered as being in parallel Then:

$$\cos \theta_d = \frac{w_d}{e\, i_d} \qquad (1)$$

$$c = \frac{i_d \sin \theta_d}{2\, \pi f e} \qquad (2)$$

For a single-conductor cable in which the thickness of insulation consists of one kind only:

$$K_{av} = \frac{i_d \sin \theta_d \,.\, \log_\epsilon b/a}{(2\, \pi)^2\, l f e\, K_o} \qquad (3)$$

$$\rho_{av} = \frac{2\pi\, l\, e^2}{w_d \log_\epsilon b/a} \qquad (4)$$

$$g = \frac{e}{x \log_\epsilon b/a} \quad \text{or} \quad g_{max} = \frac{e}{a \log_\epsilon b/a} \qquad (5)$$

THREE-CONDUCTOR CABLE

The calculations of dielectric characteristics for a three-conductor cable are greatly simplified if worked to neutral instead of between conductors.

e = impressed voltage to neutral = $E/\sqrt{3}$

i_d = amperes (dielectric) per con. per ft. length.

w_d = three-phase watts loss (dielectric) per cond. per ft. length.

$3\, w_d$ = total three-phase loss (dielectric) per ft. length of cable.

$\cos \theta_d$ = dielectric power factor (average for cable cross section.)

c_1 = capacity in farads per ft. between one cond. and sheath.

c_4 = capacity in farads per ft. between any two conds.

c_n = capacity in farads per ft. of one cond. to neutral.

K_{av} = relative permittivity (average through cross section.)

l = length in cm. (1 ft. = 30.48 cm.)

a = radius in cm. of conductor.

b = radius in cm. to inner side of sheath.
d = distance in cm. between axis of a and b.

Approximate formulas (**6**) and (**7**) for calculating the capacity of three-conductor cable, as derived by Alex. Russell, are given below:

$$c_1 = K_{av} \left(\frac{0.1693 \times 10^{-10}}{\log_\epsilon \dfrac{b^6 - d^6}{3\,(b^3\,d^2\,a)}} \right) \qquad (\mathbf{6})$$

$$c_4 = K_{av} \left[\frac{0.3384 \times 10^{-10}}{6 \log_\epsilon \left\{ \dfrac{d\sqrt{3}}{a} \cdot \dfrac{b^2 - d^2}{b^4 + b^2\,d^2 + d^4)^{1/2}} \right\}} - \frac{0.3384 \times 10^{-10}}{6 \log_\epsilon \dfrac{b^6 - d^6}{3\,(b^3\,d^2\,a)}} \right] \qquad (\mathbf{7})$$

It is apparent that the capacity between conductor and sheath c_1 is to ground or neutral. The capacity between conductors c_4 is converted to neutral by multiplying by three.

$$\therefore c_n = c_1 + 3\,c_4 \qquad (\mathbf{8})$$

$$\text{also, } c_n = \frac{i_d \sin\theta_d}{2\,\pi\,f\,e} \qquad (\mathbf{9})$$

$$3\,w_d = 3\,(e\,i_d \cos\theta_d) \qquad (\mathbf{10})$$

APPENDIX B

Theoretical Solution of Parallel Single-Conductor Cables with Non-Magnetic Sheaths

Since cable conductors are stranded the effect of this stranding on the calculated inductance although fairly small, must be corrected for. Referring to Fig. 22 an accurate way of doing this is to assume a solid conductor of the same cross section as the stranded and having an average radius equal to that of the stranded.

Let a_0 and b_0 be the actual inner and outer radii of the stranded conductor, a_1 and b_1 the radii of the equivalent solid conductor and A the cross section.

$$A = \pi\,(b_1^2 - a_1^2) \quad \text{and } b_0 + a_0 = b_1 + a_1$$

$$\left.\begin{aligned} \text{Therefore: } b_1 &= \frac{b_0 + a_0}{2} + \frac{A}{2\,\pi\,(b_0 + a_0)} \\ a_1 &= \frac{b_0 + a_0}{2} - \frac{A}{2\,\pi\,(b_0 + a_0)} \end{aligned}\right\} \qquad (1)$$

In Fig. 22 let us assume that the two sheaths are short-circuited together and the distance between the two end bonds is length l. (No current flows across the intermediate bonds if the resistance of the end bonds is negligible. If resistance is inserted in end bonds current will flow across intermediate bonds.)

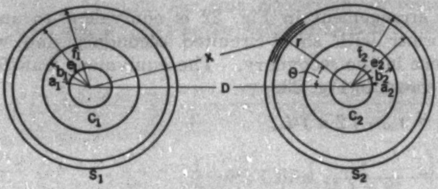

Fig. 22

It can be seen that flux set up by current in the sheath interlinks all of the current in the concentric conductor. Hence the sheath has no self-inductance. The bonds, however, have self-inductance but since they are short compared with length l the inductance is very small in comparison with the sheath resistance between bonds and can be neglected.

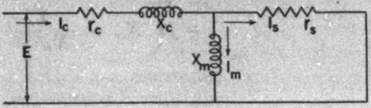

Fig. 23

In deriving the following formulas one cable only will be considered and all values worked to neutral. That is, only one-half of the circuit in Fig. 22 will be considered. The results can then be applied to either single-phase circuits or polyphase circuits having symmetrical arrangement of cables. A unit length of 1000 ft. of cable will also be considered.

Let:

r_c = ohms effective a-c. resistance of copper.
L_c = henries self inductance of conductor to neutral.
M = henries mutual inductance of conductor and sheath to neutral.
r_s = effective a-c. resistance of sheath and bonds to neutral.
i. e. r_s = total resistance of closed-circuit path to neutral.

The arrangement in Fig. 22 is equivalent to an ordinary transformer with short-circuited secondary and with zero reactance in the secondary. The equivalent transformer circuit is shown in Fig. 23.

$$Z_c = r_c - j\,2\,\pi\,f\,L_c$$

$$Y_m = \frac{j}{2\,\pi\,f\,M} \text{ and } Y_s = \frac{1}{r_s}$$

$$Y' = Y_m + Y_s = \frac{X_m + j\,r_s}{X_m\,r_s}$$

$$Z' = \frac{X_m^2\,r_s}{X_m^2 + r_s^2} - j\,\frac{X_m\,r_s^2}{X_m^2 + r_s^2}$$

$$Z_{eff} = Z_c + Z' = \left(r_c + \frac{X_m^2\,r_s}{X_m^2 + r_s^2}\right) - j\left(X_c + \frac{X_m\,r_s^2}{X_m^2 + r_s^2}\right) \qquad (2)$$

Where

$$r_{eff} = r_c + \frac{X_m^2\,r_s}{X_m^2 + r_s^2} \text{ and}$$

$$X_{eff} = 2\,\pi\,f\,L_{eff} = X_c + \frac{X_m\,r_s^2}{X_m^2 + r_s^2}$$

for open-circuited sheath $r_s = \infty$
therefore $Z_{eff} = r_c - j\,(X_c + X_m)$ (3)

The vector diagram of the short-circuit currents and voltages is given in Fig. 24.

If the constants r_c, r_s, M and L_c are known all of the characteristics can be easily calculated.

Effective resistance r_c is the a-c. value that would be measured with open-circuited sheath. It can in most cases be assumed as equal to the measured d-c. value. Eddy currents in stranded cable conductors have been shown both theoretically and by actual measurement to be negligible. In some cases, especially when coreless conductors are used, skin effect must be corrected for. Heaviside's formula for this correction as revised by Russell is given below (see *Philosophical Magazine* April, 1909, p. 545).

$$\frac{r_c}{r_c'} = 1 + \left(\frac{4\pi\mu\omega}{\rho}\right)^2 \left\{ \frac{(7a^2 - b^2)(a^2 - b^2)}{192} + \frac{b^2 a^4}{8(b^2 - a^2)} \log_\epsilon \frac{b}{a} - \frac{b^2 a^6}{4(b^2 - a^2)^2} \left(\log_\epsilon \frac{b}{a}\right)^2 \right\} \quad (4)$$

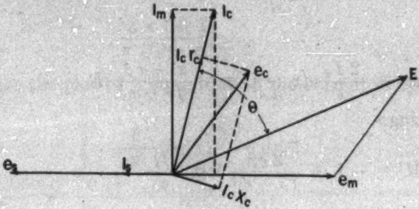

FIG. 24

where:

r_c = effective a-c. resistance.
r_c' = measured d-c. resistance
a = inside radius in cm. (for coreless cond. $a = 0$)
b = outside radius
μ = permeability = 1
ρ = specific resistance in C. G. S. units
$\omega = 2\pi f$

With open-circuit sheath there is a small circulating current in the sheath due to unsymmetrical flux distribution. This also adds to r_c but Berg has shown theoretically (see Berg's *Electric Energy*, p. 57) and actual measurements prove that it is inappreciable. We can, then, ignore the sheath entirely when open-circuited.

Effective resistance r_s is equal to the measured d-c. value. The high resistivity of sheath metal and the fact that the sheath is a comparatively thin shell eliminates skin effect.

Mutual Inductance M can be defined as the sum of the total and partial interlinkages of the flux from the conductor current (unit current) with the filaments of unit sheath current.

The formulas for total and partial interlinkages as used in developing M and L_c were derived by Alex Russell (see Russell's "*Alternating Currents*" Vol. I pp. 55 to 59.

Referring to Fig. 22:

$$-M_0 = M^{I} + M^{II} + (M^{III} + M_{s_2}) \qquad (5)$$

where M_0 is in C. G. S. units, so that henries per 1000 ft.,

$$M = 0.305 \times 10^{-4} M_0$$

$$M^{I} = -\frac{1}{I_c I_s} \int_{e_1}^{f_1} \phi c_1 \, i_{s_1} \text{ partial interlinkages.}$$

and:

$$\phi c_1 = \frac{2 I_c}{x} \, dx, \; - i_{s_1} = -\frac{x^2 - e_1^2}{f_1^2 - e_1^2} \, I_s$$

integrating:

$$M^{I} = -1 + \frac{2 e_1^2}{f_1^2 - e_1^2} \log_e \frac{f_1}{e_1}$$

$$M^{II} = -\frac{1}{I_c \, I_s} \int_{D \; f_1}^{D - f_2} \phi_{c_1} \, i_{s_1} \text{ total interlinkages}$$

where:

$$\phi_{c_1} = \frac{2 I_c}{x} \, dx \text{ and } - i_{s_1} = - I_s$$

integrating:

$$M^{II} = -2 \log_e \frac{D - f_2}{f_1}$$

$$M^{III} = -\frac{1}{I_c I_s} \int_{D - f_2}^{D + f_2} \phi_{c_1} i_{s_1} \text{ total interlinkages}$$

where;

$$\phi_{c_1} = \frac{2\,I_c}{x}\,dx \text{ and } -i_{s_1} = -I_s$$

integrating:

$$M''' = -2\log_e \frac{D+f_2}{D-f_2}$$

$$M_{s_2} = \frac{1}{I_c I_s}\int_{D-f_2}^{D+f_2} \phi_{c_1}\, i_{s_2} \text{ partial interlinkages}$$

In the sheath cross section S_2 consider a concentric ring of radius r and thickness dr. Then

$$x^2 = r^2 + D^2 - 2\,D\,r\cos\theta, \text{ keeping } r \text{ constant}$$

$$x\,dx = D\,r\sin\theta\,.\,d\,\theta$$

The current i_{s_2}' in this cylinder is equal to

$$\frac{2\,\pi\,r\,dr\,I_s}{\pi\,(f_2^2 - e_2^2)}\,.$$

Hence the part of $\Sigma\,\phi_{c_1}\, i_{s_2}$ due to the current in cylinder is

$$\int_{D-r}^{D+r} \frac{\theta}{\pi}\, i_{s_2}'\,.\,\frac{2\,I_c}{x}\,dx$$

for $\dfrac{\theta}{\pi}\, i_{s_2}'$ is the number of tubes of current in this elementary cylinder intercepted by a circle of radius x and

$$\frac{2\,I_c}{x}\,dx = \phi_{c_1}$$

Since $x\,dx = r\,D\sin\theta\,.\,d\,\theta$
this may be written

$$\frac{I_c i_{s_2}'}{\pi}\int_0^{\pi} \frac{2\,r\,D\sin\theta\,.\,d\,\theta}{r^2 + D^2 - 2\,r\,D\cos\theta}$$

$$= 2\,I_c i_{s_2}' \log\,(r + D) - 2\,I_c i_{s_2}' \log D, \text{ for the}$$

$$\text{part} - \frac{I_c i_{s_2}'}{\pi}\int_0^{\pi} \log\left(1 - \frac{2\,r}{D}\,\cos\theta + \frac{r^2}{D^2}\right) d\,\theta = 0$$

Therefore up to $D + r$ this elementary cylinder contributes;

$$2\, I_c\, is_2' \left[\log(r + D) - \log D\right] \qquad \textbf{(a)}$$

to $\Sigma\, \phi_{c1}\, is_2$. It also contributes from $D + r$ to $D + f_2$

$$\int_{D+r}^{D+f_2} is_2' \, \frac{2\, I_c}{z}\, dz = 2\, I_c\, is_2' \left[\log(D + f_2) - \log(D + r)\right] \qquad \textbf{(b)}$$

The total contribution by this elementary cylinder is $(a) + (b)$

$$= 2\, I_c\, is_2' \log(D + f_2) - 2\, I_c\, is_2' \log D$$

Substituting $\dfrac{2\, r\, dr\, I_s}{f_2^2 - e_2^2}$ for is_2' and integrating for $r = e_2$ to $r = f_2$ we get

$$M_{s_2} = 2 \log \frac{D + f_2}{D}$$

Adding all the components in (**5**) and simplifying, we get:

$$M_0 = \left(2 \log_\epsilon \frac{D}{f_1}\right) - \left(\frac{2\, e_1^2}{f_1^2 - e_1^2} \,.\, \log_\epsilon \frac{f_1}{e_1}\right) + 1 \text{ or,}$$

$$M = \left[\left(2 \log_\epsilon \frac{D}{f_1}\right) - \left(\frac{2\, e_1^2}{f_1^2 - e_1^2} \,.\, \log_\epsilon \frac{f_1}{e_1}\right) + 1\right]$$

0.305×10^{-4} henries per 1000 ft. of conductor (to neutral) (**6**)

Self Inductance L_c. The most convenient way of obtaining the short-circuit self inductance L_c of the conductors is to subtract M from the open circuit effective inductance L. When the sheath is open circuited it has no effect of any kind as already described. L is then actually the open circuit self inductance of the conductors and reference to Figs. 22 and 23 show $L = L_c + M$ where L is in henries per 1000 ft. and $= 0.305 \times 10^{-4}\, L_0$

$$L_0 = L_{c_1}{}^{\text{I}} + L_{c_1}{}^{\text{II}} + (L_{c_1}{}^{\text{III}} + M_{c_2}) \qquad \textbf{(7)}$$

The integral equations for the above components of L_0 work out in exactly the same way as those for the corresponding components of M_0, always giving the interlinkages $\phi\, i$ their proper sign. Thus:

$$L_{c_1}{}^{\mathrm{I}} = \frac{1}{I_c^2}\int_{a_1}^{b_1} \phi_{c_1}\, i_{c_1} \text{ partial interlinkages, where:}$$

$$\phi_{c_1} = \frac{2\, I_c}{b_1^2 - a_1^2}\left(x - \frac{a_1^2}{x}\right) dx \text{ and } i_{c_1} = \frac{x^2 - a_1^2}{b_1^2 - a_1^2}\, I_c$$

hence:

$$L_{c_1}{}^{\mathrm{I}} = \frac{2\, a_1^4}{(b_1^2 - a_1^2)^2}\log_e \frac{b_1}{a_1} + \frac{1}{2}\,\frac{b_1^2 - 3\, a_1^2}{b_1^2 - a_1^2}$$

$$L_{c_1}{}^{\mathrm{II}} = \frac{1}{I_c^2}\int_{b_1}^{D - b_2} \phi_{c_1}\, i_{c_1} = 2\,\log_e \frac{D - b_2}{b_1}$$

$$L_{c_1}{}^{\mathrm{III}} = \frac{1}{I_c^2}\int_{D - b_2}^{D + b_2} \phi_{c_1}\, i_{c_1} = 2\log_e \frac{D + b_2}{D - b_2}$$

$$M_{c_2} = -\frac{1}{I_c^2}\int_{D - b_2}^{D + b_2} \phi_{c_1}\, i_{c_2} \text{ partial interlinkages where the}$$

method of integrating is the same as that for M_{s_2}.
Therefore:

$$M_{c_2} = -\,2\log_e \frac{D + b_2}{D}$$

Adding all components in (7) and simplifying we get:

$$L_0 = \left(2\log_e \frac{D}{b_1}\right) + \left(\frac{2\, a_1^4}{(b_1^2 - a_1^2)^2}\,.\,\log_e \frac{b_1}{a_1}\right) + \left(\frac{1}{2}\,\frac{b_1^2 - 3\, a_1^2}{b_1^2 - a_1^2}\right), \text{ or}$$

$$L = \left[\left(2\log_e \frac{D}{b_1}\right) + \left(\frac{2\, a_1^4}{(b_1^2 - a_1^2)^2}\,.\,\log_e \frac{b_1}{a_1}\right) + \left(\frac{1}{2}\,\frac{b_1^2 - 3\, a_1^2}{b_1^2 - a_1^2}\right)\right] 0.305 \times 10^{-4}$$

henries per 1000 ft. of conductor (To neutral) (8)

Since there is no self inductance of the conductor beyond the sheath the value L_e can be independently obtained.

$L_{c0} = Lc_1^{\mathrm{I}} + Lc_1^{\mathrm{IV}} + (Lc_1^{\mathrm{V}} + M^{\mathrm{I}})$,

where Lc_1^{I} and M^{I} are the same components as defined above.

$$L^{\mathrm{IV}}_{c1} = \frac{1}{I_c^2} \int_{b_1}^{e_1} \phi c_1 \, ic_1 = 2 \log_e \frac{e_1}{b_1}$$

$$L^{\mathrm{V}}_{c1} = \frac{1}{I_c^2} \int_{e_1}^{f_1} \phi c_1 \, ic_1 = 2 \log_e \frac{f_1}{e_1}$$

The values of L_e worked out by these two different methods check exactly.

APPENDIX C

Line Characteristics of Underground Cables

The work is greatly simplified if only one cable is considered in the calculations of line characteristics, whether single- or three-conductor cables are used in the conduit line. Values can then be worked out per cable per duct. Further simplicity is obtained by considering all values such as voltage, inductance, capacity, etc. to neutral instead of between lines.

Single-Conductor Cable. When single-conductor cable is used the inductance depends upon the spacing between cables. Usually the duct arrangement requires that they be spaced at the corners of a right angle triangle. The phase inductance is then unbalanced. When this unbalancing is appreciable it can be conveniently compensated for by transposition at man holes. The equivalent spacing to be used in calculating inductance is;

$$D = \sqrt[3]{a\,b\,c} \qquad (1)$$

where a, b and c are the respective sides of the triangle. With proper transposition the proximity of other single-conductor cable lines can be ignored.

Three-Conductor Cable. The spacing between conductors is fixed and the conductors are symmetrically arranged, D is therefore the actual distance between conductor centers. Since all values are to be calculated per cable the assumed conductor cross section, line current, dielectric loss and capacity to neutral will be three times the actual values per conductor, and the assumed resistance and inductance to neutral will be one-third the actual values per conductor.

Formulas (2) and (3) below were developed by Dr. Steinmetz* for a line having distribution impedance and admittance.

$$\underset{\cdot}{E}_1 = \underset{\cdot}{E}_2\left(1 + \frac{ZY}{2}\right) \pm Z\,\underset{\cdot}{I}_2\left(1 + \frac{ZY}{6}\right) \tag{2}$$

$$\underset{\cdot}{I}_1 = \underset{\cdot}{I}_2\left(1 + \frac{ZY}{2}\right) \pm Y\,\underset{\cdot}{E}_2\left(1 + \frac{ZY}{6}\right) \tag{3}$$

The plus sign supplies when E_1 and I_1 are at or towards generator end and E_2, I_2 at or towards receiver load end. The negative sign applies when the directions are reversed.

In practically all cases the voltage is held constant at receiver end and value of load current and power factor assumed, letting the generator voltage, current and power factor come what they will. The above formulas then take the form:

$$\underset{\cdot}{e}_g = \underset{\cdot}{e}_r\left(1 + \frac{ZY}{2}\right) + Z\underset{\cdot}{I}_r\left(1 + \frac{ZY}{6}\right) \tag{4}$$

$$\underset{\cdot}{I}_g = \underset{\cdot}{I}_r\left(1 + \frac{ZY}{2}\right) + Y\,\underset{\cdot}{e}_r\left(1 + \frac{ZY}{6}\right) \tag{5}$$

Where:

$\underset{\cdot}{e}_r = e_r' + o\,j =$ receiver end voltage to neutral (constant).

$\underset{\cdot}{I}_r = i_r' + i_r''\,j =$ assumed receiver line current (lagging p. f).

$\underset{\cdot}{e}_g = e_g' \pm e_g''\,j =$ generator end voltage to neutral.

$\underset{\cdot}{I}_g = i_g' \pm i_g''\,j =$ line current at generator end.

$Z = l\,(r_c - j\,x)$ and $x = 2\,\pi\,f\,L$

$Y = l\,(g_d - j\,b_d)$ and $b_d = 2\,\pi\,f\,c.$

where;

l = length of line considered in ft.

r_c = resistance per ft. of conductor in ohms.

L = inductance to neutral per ft. of cond. in henries.

g_d = conductance to neutral of insulation per ft. in mhos.

c = capacity to neutral per ft. in farads.

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*"Engineering Mathematics" p. 204, McGraw-Hill Co.

Inductance L is the only line constant that is independent of duct temperature. The formulas for L (or L_{eff} when single-conductor cable with short-circuited sheath is used) are given in Appendix B, Formulas 8 and 2.

Resistance r_e varies with duct temperature. It is the effective a-c. resistance discussed in Appendix B, and therefore includes skin effect when this is appreciable. To be on the safe side the copper conductivity should be taken as 98 per cent in calculating skin effect by formula (**4**), Appendix B. Actually, the copper conductivity is usually between 99 and 100 per cent. In the majority of cases skin effect is negligible and the d-c. resistance sufficiently representative (with single-conductor cable and short-circuited sheath the resistance is represented by r_{eff}.) The standard formula for d-c. copper resistance is:

$$r = r_{25}\,[\,1 + 0.00378\,(T - 25)\,] \qquad \textbf{(6)}$$

where;

r_{25} = resistance at 25 deg. cent. (98 per cent copper).
T = copper temperature in deg. cent.

Conductance g_d of the cable insulation is an important line constant because of its rapid increase with temperature.

$$g_d = 1/r_d = w_d/e^2 \quad \text{(see Appendix A).}$$

It is much better to measure g_d for the particular cable used. However, if sufficient data are available from other cables of the same type g_d can be approximated, as described in the body of the paper and Appendix A. A small error is brought in by assuming the temperature, and consequently g_d, uniform through cable cross section. Usually, in duct operation, the sheath temperature is about 10 per cent lower than the conductor temperature. If necessary this error can be corrected for. In the majority of cases it can be ignored.

Capacity c should also be measured when possible, although it can be estimated closely by the method referred to above in connection with g_d. The variation of c with temperature is small in good grade impregnated paper, about 15 per cent from 25 deg. cent to 100 deg. cent. It is large enough to be considered if accurate results are desired.

Heating Characteristics of Duct System

Referring to Fig. 17, let;

w_d = *average* dielectric loss per duct. ft.
w_c = *average* copper loss per duct ft.
$w_t = w_d + w_c$ = av. total loss per duct ft.
T_1 = temperature of surrounding soil
T = *average* copper temperature as measured by by resistance drop.

Since w_t is a straight line function of T the formula is:

$$l\, w_t = A\, l\, (T - T_1) \tag{7}$$

where A is a constant and l is length of line in feet.

Relation Between Load and Average Temperature

For any assumed value of temperature T there is a certain value of continuous load that will satisfy the conditions. The problem remains to determine this load.

Impedance Z and admittance Y are calculated at the chosen temperature T and inserted in Formulas **4** and **5**. The receiver voltage e_r and power factor $\cos \theta_r$ are known and assumed to be constant at all loads. Then, in Formulas **4** and **5** the only unknown values are $\underset{\cdot}{e}_g$, $\underset{\cdot}{I}_g$ and the numerical value I_r, since;

$$\underset{\cdot}{I}_r = I_r \cos \theta_r + j\, I_r \sin \theta_r \tag{8}$$

We can, therefore, solve for $\underset{\cdot}{e}_g$ and $\underset{\cdot}{I}_g$ in terms of I_r.

Simplifying (**4**) and (**5**) they take the form:

$$\underset{\cdot}{e}_g = (a_1 + a_2\, I_r) + j\, (b_1 + b_2\, I_r) \tag{9}$$

$$\underset{\cdot}{I}_g = (a_3 + a_4\, I_r) + j\, (b_3 + b_4\, I_r) \tag{10}$$

where;

$$e_g' = a_1 + a_2\, I_r, \text{ and } e_g'' = b_1 + b_2\, I_r$$

$$i_g' = a_3 + a_4\, I_r, \text{ and } i_g'' = b_3 + b_4\, I_r$$

The power at generator end is:

$$W_g = e_g'\, i_g' + e_g''\, i_g'' \tag{11}$$

or, taking care to give the numerical values of the constants a_1, b_1, etc. their correct sign;

$$W_g = (a_1 + a_2\, I_r)\,(a_3 + a_4\, I_r) + (b_1 + b_2\, I_r)\,(b_3 + b_4\, I_r) \tag{12}$$

The power at receiver end is:

$$W_r = e_r\, i_r'$$

or

$$W_r = e_r\, I_r \cos \theta_r \tag{13}$$

The total line loss is:

$$l\, w_t = W_g - W_r \tag{14}$$

or, combining (7), (12) and (13) we have:

$$A\, l\, (T - T_1) = (a_1 + a_2 I_r)\,(a_3 + a_4 I_r) + (b_1 + b_2 I_r)\,(b_3 + b_4 I_r) - e_r\, I_r \cos \theta_r \tag{15}$$

in which I_r, being the only unknown can be solved for.

$$\frac{W_r}{W_g} \times 100 = \text{per cent line efficiency} \tag{16}$$

$$\frac{e_g - e_r}{e_g} \times 100 = \text{per cent line regulation} \tag{17}$$

RELATION BETWEEN LOAD AND HOT SPOT TEMPERATURE

Referring again to Fig. 17 let;

w_d' = dielectric loss per duct ft. at hot spot.
w_c' = copper loss per duct ft. at hot spot.
$w_d' + w_c' = w_t'$ = total loss per duct ft. at hot spot.
T_1 = temp. of surrounding soil
T' = copper temp. at hot spot.

Since w_t' is a straight line function of T' the formula is;

$$w_t' = A'\,(T' - T_1) \tag{18}$$

It now remains to represent w_d' and w_c' in terms of temperature, voltage and line current so that, for assumed values of any two of these the other can be solved for.

The current and voltage vary over the length of line, therefore, the hot spot temperature will depend upon its position in the line. If it is at receiver end where the voltage and power factor are constant and independent of current the solution is simple, as load current only must be solved for. Thus, for assumed values of T';

$$w_d' = e_r^2\, g_d \tag{19}$$

$$w_c' = I_r^2\, r_c \tag{20}$$

combining (18), (19) and (20):

$$A'\,(T' - T_1) = e_r^2\, g_d + I_r^2\, r_c \tag{21}$$

in which I_r, being the only unknown, can be solved for.

If the hot spot is at distance l from receiver end the solution is more complicated, for line voltage, power factor and current all depend upon the general line characteristics, as shown, for example, in Table VII. Assuming that e_g and I_g are known, temperature T' can be solved for as follows:

The empirical formula for g_d, found by the $\Sigma\triangle$ method, takes the form;

$$g_d = B + D\,(T')^n \tag{22}$$

where B, D and n are constants;

$$\therefore w_d' = e_g^2\,[B + D\,(T')^n] \tag{23}$$

From formulas (**6**)[14] and (**20**);

$$w_c' = I_g^2\,\{r_{25}\,[1 + .00378\,(T' - 25)\,]\,\} \tag{24}$$

combining (**18**), (**23**) and (**24**)

$$A'\,(T' - T_1) = e_g^2\,[\,B + D\,(T')^n\,] + I_g^2\,\{r_{25}\,[1 + 0.00378\,(T' - 25)\,]\} \tag{25}$$

in which T' being the only unknown can be solved for.

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[14] Formula (6) does not apply when single-conductor cable with short-circuited sheath is used. In that case a modification of Formula (**2**) in Appendix B, for r_{eff} must be used.

For discussion of this paper, see page 1017 of Volume II.

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THE DIELECTRIC FIELD IN AN ELECTRIC POWER CABLE

BY R. W. ATKINSON

ABSTRACT OF PAPER

The data given pertain particularly to the field of three-conductor three-phase cables when supplied with three-phase voltage, and are primarily the solution by physical measurements of some of the geometric problems of the three-conductor three-phase cable.

Data are given so that it is possible, from electrical measurements on three-conductor cable, to determine certain specific quantities as permittivity, resistivity, etc. of the dielectric of three-conductor cables in the same way as can readily be done for single-conductor cables, from geometric considerations.

Also, there is shown the potential and stress distribution in a three-conductor cable. The most extensive data are based on measurements made with electrodes in an electrically conducting liquid, thus simulating a homogeneous dielectric. Also exploring electrodes were built into some actual three-conductor cable, and measurements were made when three-phase voltage was supplied to the conductors.

IN the case of single-conductor cable, by means of simple mathematical calculation it is readily possible to calculate from the fundamental data, the unit quantities of the dielectric, such as permittivity, thermal or electrical resistivity, the corresponding properties of the cable itself. Also the voltage gradient at the conductor surface or elsewhere in the insulation may easily be determined in terms of the applied voltage and the cable dimensions, or the unit quantities may be determined from measurements upon cables.

Very few corresponding data for three-conductor cable are available. The reason for this is the difficulty of the mathematical work required for this solution and also the complicated form of those results which have been obtained. By means of electrical measurements in a tank of conducting liquid containing electrodes representing the conductors and sheath of a cable a solution is obtained of many of the problems of the three-conductor cable. Also there are reported the results of a few measurements of voltage gradients and of potentials in an actual three-conductor cable.

A great many data are available regarding the effect of stranding of single-conductor cable upon the gradient and the dielectric strength. We have added somewhat to the data now available on voltage gradient. For various reasons, the practical effect on dielectric strength of the increase of gradient due to stranding is not in proportion to that increase. Two reasons probably predominate: the change in gradient affects only a very small thickness of dielectric; it probably usually occurs that a better mechanical construction results when paper wrappings are applied to a stranded than to a solid conductor. Further data regarding this matter are of very considerable importance.

It is intended to leave most of the application of these data for presentation at a later time, though some of the directions in which it is useful will be pointed out briefly. It is important to distinguish from the beginning between two distinct uses for the data given. First, there is the correlation of the capacitance of the cable with the permittivity of the dielectric, the resistance of the cable with the resistivity of the dielectric, etc. Second, there is the determination of voltage gradient. Concerning the first there is a very definite and immediate application. The field of application of the second is much less definite though actually far more extended. The distribution of the voltage gradient in various parts of a cable is one of the several fundamental considerations governing the voltage rating of the cable.

Measurements of stresses and capacities, both three-phase and single-phase, were begun in the laboratory of the Pittsburgh factory of the company with which the writer is associated, in 1913, under the direction of C. W. Davis. At this time were begun the experiments with a tank of liquid, or solid electrolyte similar to that hereinafter described, and the results obtained were substantially in accordance with the results herein published. Later experiments were made at Perth Amboy in 1917 and again in the spring of 1919 by the writer. In the successive series of tests some of the earlier measurements were repeated and the field of study was extended, and apparatus was devised which would permit of a still more accurate measurement of the stresses than those obtained in 1913. This paper deals largely with the actual measurements made in the last two series of tests, though a brief summary of part of the earliest series is added.

DETERMINATION OF CAPACITY RELATIONS

The determination of capacity relations will first be described. The experimental work consisted in the measurement of resistance between electrodes placed in a tank of electrolyte with the electrodes so arranged that they occupied the same relative position to each other as do the separate metallic parts of a three-conductor cable.

The terms conductors and sheath will be applied to the electrodes representing them, and the terms conductor and belt insulation to that part of the electrolyte occupying the position that the conductor and belt insulation would occupy in a cable. Also the terms "filler" or "filler space" will be applied to the part of the section corresponding to that part of the cable section usually occupied by jute or paper "filler." The term cable will be applied to the complete set of electrodes and electrolyte in any one arrangement.

The capacity relations were determined in the following manner. The conductors and sheath were arranged in their relative positions in a wooden tub containing fairly high resistance water. Measurements were made of the resistance between electrodes when they were connected in various ways. The following were the connections used.

1. One conductor *vs.* another conductor. (Sheath and third conductor free).
2. One conductor *vs.* two conductors. (Sheath free).
3. One conductor *vs.* sheath. (Other conductors free).
4. One conductor *vs.* one conductor and sheath. (Third conductor free).
5. One conductor *vs.* the other two conductors and sheath.
6. Two conductors *vs.* sheath. (Third conductor free).
7. Two conductors *vs.* sheath and third conductor.
8. All conductors *vs.* sheath.

One of the same conductors was then placed in the center of the same sheath, thus forming the equivalent of a single-conductor cable, and a reading taken of resistance between conductor and sheath. The ratio of the resistances for the three-conductor connections to the resistance of the single-conductor to sheath gave the ratio of capacities for the three-conductor connections as compared to the capacity of a single-conductor cable having the same size conductor and sheath. The single-conductor capacity can easily be calculated. The matter may be explained in another manner. The measurement with concentric tubes furnishes data for ready calculation of the resistivity of the electrolyte. The three-conductor measurements thus are made with "dielectric" of known resistivity and conductors of known dimensions.

The above measurements were made with various proportions of belt to conductor insulation and with various ratios of conductor to sheath diameter.

The resistance measurements were made using single-phase 60-cycle current, by measuring the applied voltage and the energy consumed in the resistance of the electrolyte. For the latter measurement there was used the wattmeter connection of a Rowland dynamometer, the connection being such that the resistance of the fixed coils was included in the measurements. Approximately two volts from an auto-transformer was applied to the tubes, and watt readings were taken for the various connections, from which the effective resistance between the electrodes could be calculated. The dynamometer was calibrated by substituting a variable known resistance for the electrolyte.

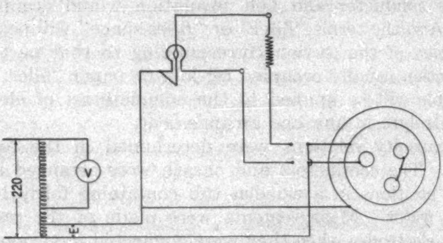

FIG. 1—WIRING DIAGRAM FOR DETERMINING CAPACITY RELATIONS

The bottom of the tub was covered with a layer of paraffine which prevented end effect due to possible conduction in the water-soaked wooden bottom, and also served as a flat level bottom on which to rest the tubes.

To check the general accuracy of the method and the applicability of the results, a large number of measurements were made with concentric tubes. Resistance measurements were made between many combinations of pairs of different sizes. Also after a measurement was made between such a pair of rings, a third one was placed between the two and concentric with them and measurements repeated. By this means it was shown that the resistances obeyed the theoretical law.

TABLE I.

Sheath Diam.	Cond. Diam.	Ratio thickness belt insulation to thickness conductor insulation
15.4 in.	3.62 in.	0.
"	"	0.25
"	"	1.0
15.4	2.50	0.
"	"	0.25
"	"	1.0
15.4	1.90	0.
"	"	0.25
"	"	1.0
11.8	2.5	0.
"	"	0.25
"	"	1.0
11.8	1.9	0.
"	"	0.25
"	"	1.0
9.9	2.5	0.
"	"	0.25
"	"	1.0
9.9	1.9	0.
"	"	0.25
"	"	1.0

These measurements showed that end effect actually had been eliminated by the initial arrangement.

Another disturbing factor was detected and eliminated through the agency of these tests. The tubes or rings seemed to have a high surface resistance possibly due to polarization.

Water of fairly low resistivity was being used (a very weak saline solution). The resistance was increased largely by the use of distilled water mixed with a very small amount of tap water. This however only partly removed the difficulty, as the surface resistance, though relatively smaller, was larger absolutely, and large enough to cause the results to be slightly in error. It was found that if the tubes were thoroughly polished with fine sandpaper or emery cloth at sufficiently short intervals this resistance could be entirely eliminated. In measurements of stress, described later, even a much lower surface resistance would have been of importance, but careful tests showed it was possible to entirely free the results from this type of error. Attempts were made to use some rings made of galvanized iron, but this material did not give satisfactory results and was abandoned.

Measurements were made with the sizes and arrangements of electrodes shown in Table I.

All tubes were of copper and 8½ in. high. The readings for different connections for any one cable checked the relations given by Russell within 2 per cent.

These relations are as follows;

TABLE II.

(1) Capacity between 1 and 2 $= 1/2\,(a - b)$
(2) " " 1 and 2, 3 $= 2/3\,(a - b)$
(3) " " 1 and S (2 and 3 insulated) $= \dfrac{(a - b)\,(a + 2b)}{a + b}$
(4) " " 1 and S, 2 (S insulated) $= \dfrac{(a - b)\,(a + b)}{a}$
(5) " " 1 and S, 2, 3 $= a$
(6) " " S and 1, 2 (3 insulated) $= \dfrac{2\,(a - b)\,(a + 2b)}{a}$
(7) " " 1, S, and 2, 3 $= 2\,(a + b)$
(8) " " 1, 2, 3, and S $= 3\,(a + 2b)$

With each cable, actual measurements were made of all the 8 combinations, though the tests on No. 1 and No. 5 were made with the greatest care. From these, the values of *a* and *b* were calculated, and following this, the values of the other six combinations. Check with measurements was made as indicated above.

APPLICATION OF DATA OF CAPACITANCE AND CHARGING CURRENT

The data of these measurements are shown only in the final form, Fig. 2. Specifically, Fig. 2 gives the charging current of different sizes of cable within the range of present practise. The values are based on a frequency of 60 cycles and a specific

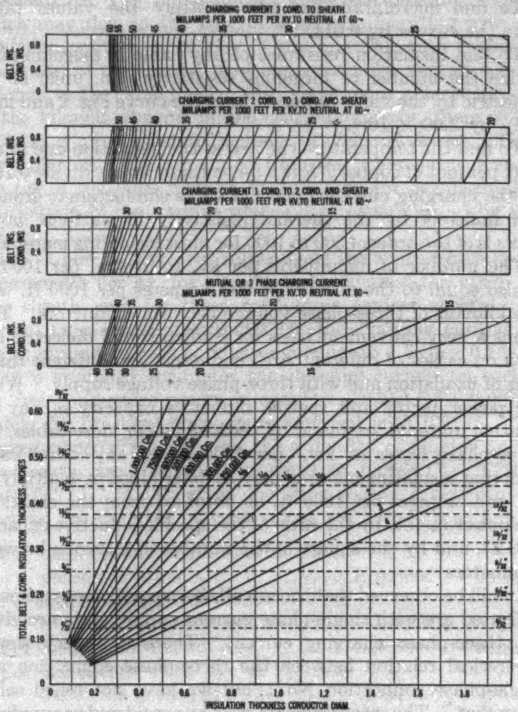

FIG. 2—CHARGING CURRENTS FOR ROUND THREE-CONDUCTOR CABLES BASED ON SPECIFIC INDUCTIVE CAPACITY OF 3.27 AND A FREQUENCY OF 60 CYCLES

Note:—The abscissa shown applies to all 5 sets of curves.

inductive capacity of 3.27, the latter being a somewhat arbitrary figure but actually about the normal value for impregnated paper.* By the use of constants given below, the chart may be used for the determination from unit quantities of the various other properties of a three-conductor cable.

To find microfarads per mile multiply the values taken from the curves by 0.014.

To find insulation resistance of any cable in megohm-miles divide the number of megohms resistivity (cm. unit) of the dielectric by the value obtained from the curve Fig. 2 and multiply by 2.06×10^{-5}.

To find the temperature rise across the insulation due to one watt per foot of copper I^2R in a 3-conductor cable, divide 0.1097 by the charging current given for three conductors to ground, and multiply by the thermal resistivity in watt-cm. units. (This is of the order of 500 to 1000 for most insulating materials.)

The number of milamperes per kv. to neutral per 1000 ft. is also equal to the number of volt amperes per 1000 ft. with one kilovolt of three-phase voltage between conductors. This forms a very convenient basis for comparison of dielectric loss data on cables of different size conductors and different thickness of insulation and with three-phase voltage supply. When the power factor and specific inductive capacity of any insulating material are found by tests on one or more cables, the three-phase losses in kw. per 1000 ft. on any other cable with that insulating material may be determined by multiplying the volt-amperes per 1000 ft. as obtained from the chart, by the power factor, by the ratio of the specific inductive capacity to 3.27, and by the square of the number of kilovolts between conductors.

It will be observed that though all the capacity measurements indicated are with single-phase current supply, data are given for three-phase charging current. There is a very simple theoretical relation between the three-phase connection and single-phase connection No. 1, but we have not relied solely upon that. The measurements may be considered as supplying information as to charging current of the single-phase connections only. By measurements on many cables it is known that the charging current per conductor with three-phase voltage is the same as that for single-phase voltage, connection No. 1, when the voltage from conductor to sheath is the same in each case. (In case of connection No. 1 the third conductor is at sheath potential, whether actually connected to sheath, or free). This is the theoretical relation said above to exist.

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*See Appendix A for instructions as to how to use Fig. 2.

Sometimes it is important to know the charging current which may flow in the case of a system operating with ungrounded neutral, when one of the lines becomes grounded. The value of the current is of importance for various reasons external to the cable itself. Also, on account of large increase in the kilovolt-amperes of the charging current, there will be at least a proportionate increase of dielectric losses. These particular data are calculated from the other data given by the use of the same method Russell gives for calculating the relationship of the various single-phase capacities. The data follow:

Let C_5 and C_8 represent respectively the single-phase charging currents per kv. for one conductor against the other two and sheath, and for three conductors against sheath. For normal three-phase operation then, the charging current per kv. to ground on each conductor is equal to $1.5\, C_5 - 0.167\, C_8$. With a grounded conductor the charging current on each of the free conductors per kv. is equal to $\sqrt{0.752\, C_5^2 + 0.027 C_8^2}$, which, for practical purposes is identical with $0.91\, C_5$. The charging currents per kv. on the grounded conductor and on the sheath, are, respectively, equal to $0.0866\, C_5 - 0.288\, C_8$ and $0.577\, C_8$. With a given operating voltage E between conductors, the charging current values are therefore as follows:

For each conductor on normal three-phase operation	$E(0.866 C_5 - 0.096 C_8)$
For each free conductor when one of the conductors is grounded	$0.91 E C_5$
For the grounded conductor	$E(0.866 C_5 - 0.288 C_8)$
For the sheath when one of the conductors is grounded	$0.577 E C_8$

The total volt-amperes due to the charging current will be, on normal operation $\sqrt{3}\, EI$, which is equal to

$$E^2\,(1.5 C_5 - 0.167 C_8)$$

and when one conductor is grounded

$$E^2 (1.5 C_5 + 0.167 C_8)$$

Let us take as an example a three-conductor cable with 4/0 conductors, having 6/32 in. insulation on each conductor and 6/32 in. on the belt, and at, say, an operating voltage of 11 kv.

C_5 = 0.0202 amperes per kv. per 1000 ft.
C_8 = 0.0365 amperes per kv. per 1000 ft.

The charging current with ordinary operation is, therefore, 0.154 amperes.

With one of the phases grounded, the charging current for each free conductor is 0.203 amperes, for the grounded conductor 0.077 amperes, and for the sheath 0.231 amperes.

The total volt-amperes for the case where the neutral is grounded will be 2930, and for the case where one of the conductors is grounded 4400. On the basis of the same power factor, the dielectric loss, as well as the volt-amperes of the charging current, in the second case will be 50 per cent higher than in the first.

EXPLANATION OF NATURE OF FIELD WITH THREE-PHASE VOLTAGE SUPPLY

Fig. 4 is presented largely to illustrate the general nature of the field in a three-conductor cable upon which three-phase voltage is impressed. There are two sets of curves shown—one showing lines of equal phase, and the other showing lines of equal maximum potential. Each set, to be complete, should comprise the entire 360 deg. of section, but, because of symmetry, the 120 deg. included by each set gives information which is exactly as complete. The method used for the determination of these data is not dissimilar to the one previously described in connection with the measurement of single-phase capacity relations.

The isophase line shows the phase relations of the potentials existing in the cable section. Taking the phase of the potential of the left hand conductor as the basis of reference, and calling it 0 deg., the phase of the potential of the right hand conductor will be 120 electrical degrees away. The phases of the potentials at all points included by the 120 geometric degrees between the radii through the centers of these two conductors are as indicated by the isophase lines. Another way of considering this set of curves is as follows: At some instant the 0 deg. line will be at zero potential. After the lapse of an interval of time represented by one electrical degree, the 1-deg. line will be at zero potential; after the lapse of another, and exactly equal interval, the 2-degree line will be at zero potential; until, finally, after a total interval of time, represented by 120 electrical degrees, has elapsed, the 120-degree line will be at zero potential. It will be noted that the change of phase is much more rapid (that is, a given distance corresponds to much more phase change) as the radius bisecting the center line of conductors (the 60-degree line) is approached. This difference is very pronounced at the sheath, less so at the conductors, and, as the center of the cable is approached, becomes nearly unnoticeable. At the very center the variation does not exist at all,—in other words, the electrical degrees coincide exactly with the geometric degrees.

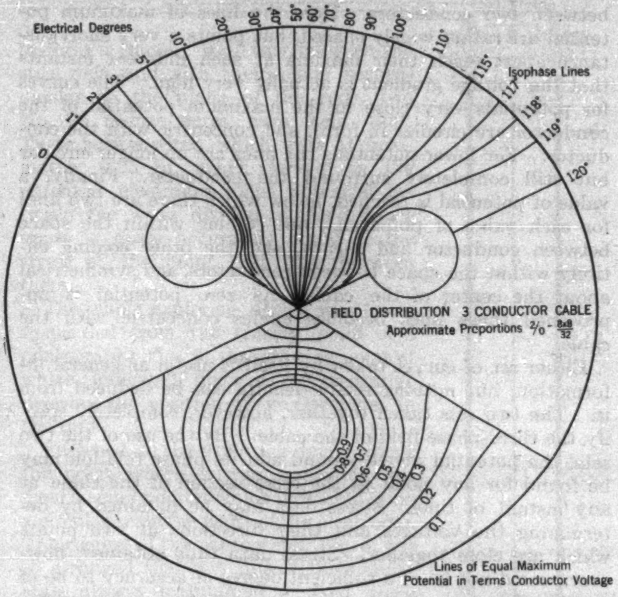
Electrical Degrees
0
1
2
3
5
10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
110°
115°
117°
118°
119°
120°
Isophase Lines
FIELD DISTRIBUTION 3 CONDUCTOR CABLE
Approximate Proportions 2/0 - 8x8/32
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Lines of Equal Maximum Potential in Terms Conductor Voltage

Fig. 4—Lines of Isophase and Lines of Equal Maximum Potential

The lines of equal maximum potential are the loci of points having the same maximum potential above ground at some time during the cycle. These curves should not be confused with equipotential lines, which represent instantaneous values of potential, and would be, for a three-phase field, different at each instant of time. Wide spacing between the lines of maximum potential does not indicate low stress. Particularly is this true near the position midway between two conductors. Here the lines of maximum potential are rather widely spaced, but points a very short distance apart reach their maxima at such different instants that the voltage gradient is actually very high. The curves for potentials very close to the maximum potential of the conductor are circular in form, and concentric with the conductor. For lower potentials the lines are no longer circular but still completely surround the conductor. Finally, a value of potential is reached, below which there are two lines for each value of potential,—one coming within the space between conductor and sheath, and the other coming entirely within the space between conductors, and symmetrical about the center of the cable. As zero potential is approached these lines becomes circles concentric with the cable.

Either set of curves taken by itself is useful as general information, but nothing really definite can be deduced from it. The two sets taken together, however, completely specify the three-phase field of the cable. By the use of the two sets, the potential above ground and its phase relation may be found for any point in the cross section of the cable at any instant of time. Stress data may be obtained by determining the voltages and their directions at two points which are close together. Stress data thus obtained, however, would not be of a sufficient degree of accuracy to be of great value.

For accurate stress determination it is necessary to measure the actual difference of potential between two points, a known small distance apart, rather than to correlate the voltages between the two given points and a given basis of reference. The method used for making these measurements is described in the next section. The data given in Fig. 4 might readily be calculated from the stress data recorded later, which were gotten by this method.

STRESSES AS MEASURED WITH ELECTROLYTE TANK

The stresses in the three-phase field were measured in the following manner: An exploring device was made up as shown in Fig. 5. Two steel rods about 1/16 in. in diameter and 8½ in. long were fitted into small hard rubber blocks so as to form a frame. Half way between the steel rods, and with their plane perpendicular to the plane of the rods, two No. 26 bronze wires were held taut. These bronze wires, which were approximately 0.25 in. apart, served as the potential electrodes. The potential between the wires divided by the distance apart is quite accurately equal to the stress at the point half way between them.

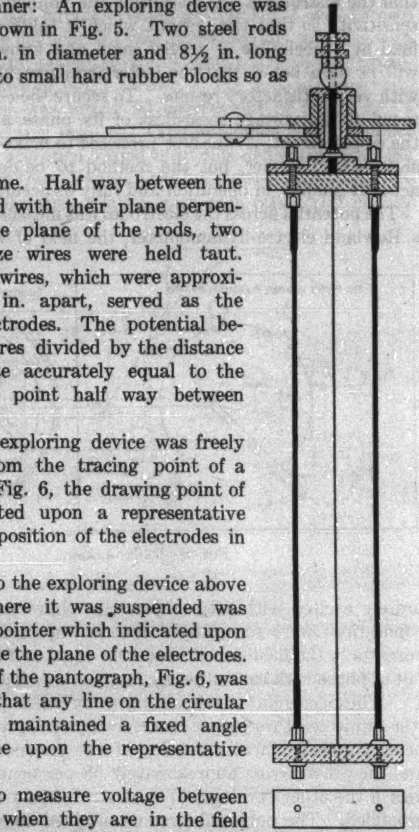

FIG. 5—EXPLORING DEVICE

The whole exploring device was freely suspended from the tracing point of a pantograph, Fig. 6, the drawing point of which indicated upon a representative drawing the position of the electrodes in the tank.

Fastened to the exploring device above the point where it was suspended was fixed a small pointer which indicated upon a circular scale the plane of the electrodes. The rigging of the pantograph, Fig. 6, was so arranged that any line on the circular scale always maintained a fixed angle with any line upon the representative drawing.

In order to measure voltage between these points when they are in the field between the three conductors, upon which is impressed three-phase voltage, it is necessary to measure the magnitude and phase displacement with respect to a fixed voltage, and to do so without distortion of the electrical field by the measuring current. The second requirement demands that a null method be used, or that the instrument for measuring the voltage be of sufficient sensitivity so that the current which it takes will distort the field by a negligible or a very small and known amount. It will be seen below that the latter arrangement has been used with very satisfactory results. To secure the correct measurement of the voltage regardless of its phase and to measure the displacement, it was first proposed to make use of a phase-shifting transformer, but the method to be described proved actually simpler in operation and fully as accurate in results.

The potential across the electrodes was measured by means of a Rowland electro-dynamometer, the field of which was separately excited with fifty amperes, as shown in Fig. 7. The connections were so arranged that it was possible to obtain currents in the field circuit that were equal and ninety degrees out of phase with each other by merely throwing a switch (No. 1). These currents were obtained by taking taps from a star-connected three-phase transformer, one side of the switch being connected across one leg of the transformer winding, and the other across approximately 58 per cent of the windings of the other two legs. The neutral of the transformer was grounded. The potential circuit contained 40,000 ohms non-inductive series resistance.

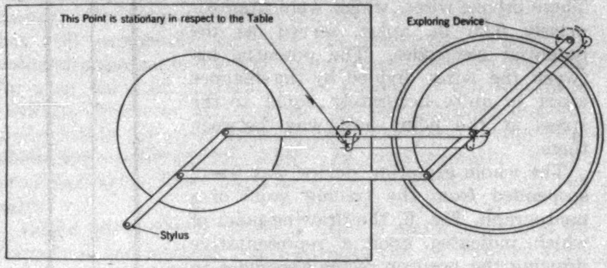

FIG. 6—PANTOGRAPH

The reactances of the fixed coil and of the auto-transformer were compensated by a capacity of 16 microfarads shunted across a variable resistance. The compensation was accomplished in the following manner: A voltage in phase with the voltage across one side of the main field switch was impressed on the moving coil circuit. When switch No. 1 was thrown on that side, the field current was approximately in phase with the current in the potential circuit, and a large deflection resulted. With switch No. 1 thrown in the opposite direction, the field current was nearly 90 degrees out of phase, and a very small deflection resulted. The resistance which shunted the condensers was then varied until this deflection was zero.

To insure that the voltages on the two sides of the switch were at the proper 90-degree relation with each other, the circuits were then interchanged, so that voltage from the other side of No. 1 switch was required to produce zero deflection. It was found that the phase relation was correct within ½ per cent.

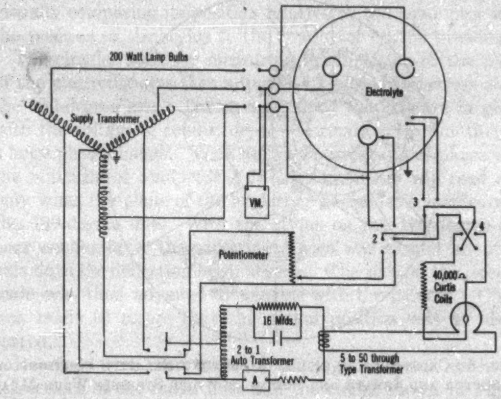
200 Watt Lamp Bulbs
Supply Transformer
Electrolyte
VM.
3
2
4
Potentiometer
40,000 Ω Curtis Coils
16 Mfds.
2 to 1 Auto Transformer
A
5 to 50 through Type Transformer

FIG. 7—WIRING DIAGRAM OF CONNECTIONS FOR STRESS MEASUREMENTS

The procedure of the stress measurements was then as follows:

An accurate template was made of thin Bakelite fibre, so that the conductors and sheath always maintained the same relative position.

A large circular piece of plate glass was placed in the bottom of the electrolyte tub and leveled by means of a spiri level. The electrodes were then placed in the tub and held in their positions by the template. The tub was then filled with water to within a half inch of the top of the electrodes.

Three-phase voltage of approximately 50 volts to neutral was then applied to the three conductors and the exploring device suspended in the water. The measurements were made in the field surrounding conductor *A*, whose voltage was in phase with the field current when the field switch (No. 1) was closed to the left.

A full size drawing was then made which exactly corresponded to the template, and various circles and lines were drawn upon it as shown on Fig. 8, the intersections of this line indicating the points at which it was desired to measure the stress.

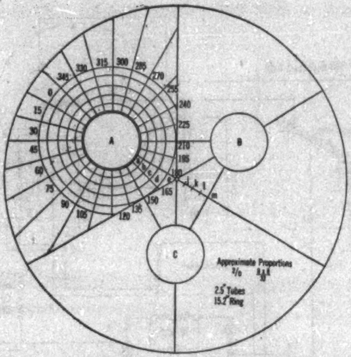

FIG. 8—CROSS-SECTION OF CABLE SHOWING RELATIVE POSITIONS OF CONDUCTOR AND SHEATH AND POINTS AT WHICH STRESSES WERE MEASURED

The drawing was then lined up as follows:

The exploring device was hung somewhere near the center of the cable. The two electrode-wires were short-circuited and the voltage between these and the sheath was measured by means of the dynamometer. At exactly the center of the cable the voltage to sheath is zero. The electrodes were moved about until the dynamometer indicated zero with the field switch on either side. The center of the representative drawing was then placed exactly under the drawing stylus and fixed to the board with a thumb tack.

The exploring device was then placed between the two conductors *B* and *C*. At a point exactly half way between these conductors the potential to sheath is in phase with the voltage of conductor *A* to ground. The switch was then thrown on the side that is out of phase, and the exploring device moved until the deflection was zero. The drawing was then swung about with its center as an axis until the line half way between conductors *B* and *C* was under the stylus. The drawing should then be exactly lined up with the tank. Checks were made by placing the exploring device near one of the conductors and visually comparing its position relative to the conductor with the position of the stylus to the conductor on the drawing.

The circular scale for measuring the direction of the plane of the electrodes was then adjusted by measuring stress along the 180-degree line. The stresses along this line are in phase with the voltage of conductor *A*. Normal to this line there is a large phase change. With the field current out of phase with the potential of conductor *A* the dynamometer will read zero only when the plane of the exploring electrodes coincides with the 180-degree line. With the stylus on the 180-degree line near conductor *A* the exploring device was turned about its axis until the deflection became zero. The zero on the circular scale was then adjusted to coincide with the pointer. Check was made to insure that the actual position was as above stated.

Readings were then taken by placing the stylus at the intersections of the radial lines and the circles shown in Fig. 8. Readings were also taken at the other points indicated. All readings were taken with the plane of the exploring electrodes radial to the conductor, except along the 180-degree line, where the stresses were also measured normal to the line.

Stresses close to the conductor were found by measuring between the conductor and a No. 40 wire fastened by hard rubber cleats 0.125 in. from the conductor. This conductor could be revolved in the template and the stresses on all sides of the conductor could be accurately compared.

To take a reading, the stylus was placed on a point on the drawing at which it was desired to measure the stress. The exploring device was rotated until its electrodes were radial to the conductor. Readings were taken of voltage and dynamometer deflection with the main field switch first on one side and then on the other. This gives both components of the stress as compared to the voltage to neutral of conductor *A*. From these readings the stress and phase angle could easily be calculated.

Voltage was read across the points indicated on Fig. 7. This voltage was frequently compared with the voltage directly on the conductors. The three-phase field would have been slightly distorted had it been attempted to keep the voltmeter connected to the conductor while making stress measurements, due to the slight additional drop across the series lamp bulbs.

Calibration of the instrument was made by means of a potentiometer connected as shown in Fig. 7. A deflection of the dynamometer of 8.08 divisions per volt was obtained with 40,000 ohms series resistance and 50 amperes field current. All results were placed on a common basis of field current and conductor voltage.

The deflections along the 180-degree line were plotted, and the average of this curve between the conductor and center of cable was found. The average voltage between the conductor and center of cable was the voltage on the conductor divided by the distance. The average deflection divided by the calibration constant and by the distance between the exploring electrodes must exactly check the average voltage. From this relation it is possible to find the exact distance between electrodes.

We have mentioned the possibility of field distortion due to the current taken by the apparatus used for measuring the potential difference of exploring electrodes. The effect of this upon the measurements described above was investigated in the following manner. With the exploring electrodes maintained in a given electric field in the tank, the resistance in the metering circuit was varied through a wide range. The reciprocals of the deflections were plotted as ordinates against the external series resistance as abscissas. The resultant "curve" was a straight line cutting the axis at a negative value. This value is equal to the effective resistance between the electrodes If this resistance be added to the external resistance in determining the calibration constant, we fully correct for whatever small distortion there may occur, for it follows from the above that the same voltage is obtained regardless of the series resistance, and therefore the value is the same as for infinite resistance, or zero current and distortion.

The calibration of the electrode fastened to the tube was found by measuring the stress in a single-conductor concentric field. The stress can here be quite accurately calculated and the distance between the No. 40 wire and the tube was found from the relation that the stress at a point half way between the wire and the conductor must equal the deflection divided by the calibration constant of the instrument divided by the distance between the wire and conductor. The internal resistance between this electrode and the conductor was found in the same way as for the other. The calibration constant to be used for any given condition is that given above, which applies to 40,000 ohms total resistance, divided by the ratio of the actual total resistance of the potential circuit (including internal resistance between electrodes) to 40,000. This ratio does not exceed 1.02 or 1.03.

All deflections were put on a common basis by dividing by the average deflection between the conductor and center of cable along the 180-degree line, thus giving the stresses in terms of the average along the 180-degree line from conductor to center of cable.

The two components of the stresses were added at right angles, and the angles by which the stresses lead the voltage of conductor *A* to ground were calculated.

The values were plotted in Figs. 9 to 18 inclusive, which will be described in detail below.

The field between the conductor and the sheath was next investigated. The electrodes were placed on the shortest line between conductor and sheath, and the deflection read. The other two tubes were then removed without disturbing the position of the remaining electrode or its voltage to ground. The deflection was not changed by the removal of the other conductors. Other spacings and sizes of tubes were used, but never did the removal of the two tubes change the amount of the deflection. From this, we can draw the conclusion that the field on the shortest line between the conductor and sheath is a single-phase field and is independent of the presence of the other two conductors.

Determination was then made of the effect of the amount of belt upon the stresses on the inside of the cable. Readings were taken with the tubes so arranged as to simulate a cable with equal belt and conductor insulation. Then without changing the position of the conductors, the sheath was replaced by a smaller one. This new ring was even smaller than one that would correspond to the sheath of an unbelted cable. Most of the readings were then repeated. The change in sheaths produced practically no change in stress within the portion of the field bounded by the three lines joining centers of conductors.

DESCRIPTION OF FIGS. 9 TO 16

Fig. 9 shows the stresses along the line joining the center of conductor *A* and the center of cable in terms of the average stress between the conductor and the center. This average represents the total voltage of conductor *A* divided by the distance between the conductor and center of cable.

In all the following curves, as well as most of the subsequent discussion, stresses are given in terms of this same unit. For a round conductor, this distance between conductor and center is equal to 1.154 times the conductor insulation (half the distance between surfaces of conductors) plus 0.077 times the conductor diameter.

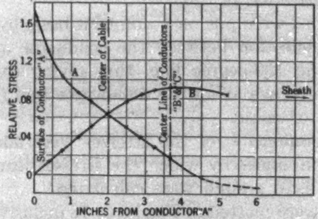

FIG. 9—STRESSES ALONG 180-DEGREE LINE OF FIG. 8 IN TERMS OF AVERAGE BETWEEN CONDUCTOR AND CENTER OF CABLE (SMALL TUBES)

Curve A—Stresses in direction of 180-degree line
Curve B—Stresses normal to 180-degree line

Curve *B* shows the stress normal to the line of centers of conductor *A* and cable. Here the maximum stress occurs near the line joining the centers of conductors *B* and *C*.

Fig. 10 shows the stresses around circle *a*, which is 1/16 in. away from the conductor. All stresses were measured radial to the conductor. It will be noticed that the maximum stress occurs on the line which passes through the center of the cable, though there is very little difference along the arc inclosed by the center lines of conductors.

Another maximum occurs at the outside of the conductor on the shortest line between the conductor and sheath. This maximum is dependent upon the total thickness of insulation, including conductor and belt.

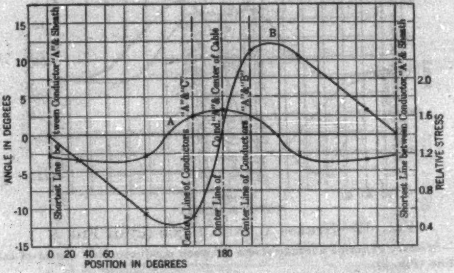

FIG. 10—STRESSES ON CIRCLE (A) OF FIG. 8

Curve A—Stresses in terms average stress between conductor and center of cable along 180-degree line (Fig. 8)

Curve B—Angle by which stress leads voltage of conductor A to neutral

Figs. 11, 12, 13 and 14 show the stresses on the various other circles shown. It will be noted that as the distance between the conductor and circle increases that the point of maximum stress moves toward the line joining centers of conductors. Also the point of maximum phase difference moves farther past the line joining centers of conductors.

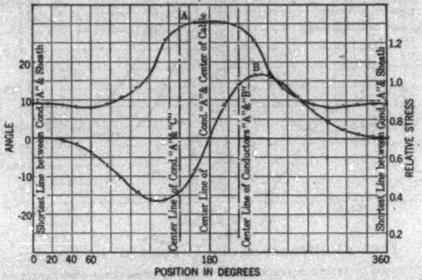

FIG. 11—STRESSES ON CIRCLE (B) OF FIG. 8

Curve A—Stresses in terms average stress between conductor and center of cable along 180-degree line (Fig. 8)

Curve B—Angle by which stress leads voltage of conductor A to neutral

Fig. 15 shows the stresses on the outside of the conductor along the shortest line from the conductor to sheath. This is a single-phase field and the maximum stress can be calculated by a formula given by Russell, as shown below. The value obtained from the curve checks within 2 per cent the calculated value.

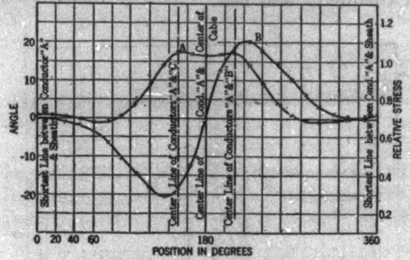

Fig. 12—Stress along Circle (c) of Fig. 8

Curve A—Stresses in terms average stress between conductor and center of cable along 180-degree line (Fig. 8)

Curve B—Angle by which stress leads voltage of conductor A to neutral

Fig. 16 is given in this paper because of its value in supplying data outside the range of the experimental values. The lowest curve gives the maximum stress toward the sheath of a three-conductor cable in terms of the average stress between conductor and sheath. The middle curve give

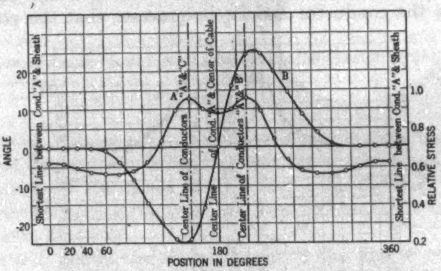

Fig. 13—Stress along Circle (d) of Fig. 8

Curve A—Stresses in terms average stress between conductor and center of cable along 180-degree line (Fig. 8)

Curve B—Angle by which stress leads voltage of conductor A to neutral for a single-conductor cable the maximum stress in terms of the average. The upper curve simply gives the ratio between the maximum stress of a single-conductor cable and that towards the sheath of a three-conductor cable having the same size conductor and the same distance between conductor and sheath. The ordinates for this last curve are obtained by dividing the ordinates of the middle curve by those of the lower curve. For the solution of all problems to which Fig. 16 is applicable, any two of the three curves shown give the complete data.

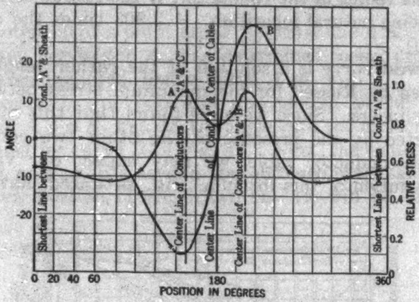

Fig. 14—Stress along Circle (e) of Fig. 8

Curve A—Stresses in terms average stress between conductor and center of cable along 180-degree line (Fig. 8)

Curve B—Angle by which stress leads voltage of conductor A to neutral

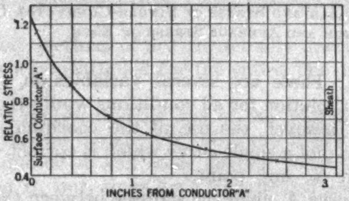

Fig. 15—Stresses along 0 Degree (Fig. 8) in Terms Average between Conductor and Center of Cable along 180-Degree Line (small tube)

The middle curve is a graphical solution of the common formula showing the relation between maximum and average stress of a single conductor cable. This is

$$\frac{\text{maximum stress}}{\text{average stress}} = \frac{R - r}{r = \log R/r}$$

r being the radius of the conductor, R the radius of the cable (= log) being natural logarithm, = 2.303 times the common logarithm).

The lowest curve is a solution of the rather lengthy formula given by Russell for the maximum stress between eccentric cylinders. Since we have found that this maximum is unaffected by the presence of the other two conductors of a three-conductor cable, we may expect our values to check a solution of this formula, and, as stated, have found they do so. There are several variables in this formula, and an exact solution for all conditions would require a series of curves. However, the use of the coordinates taken has enabled us to plot a single curve which is correct within about one per cent for any point.

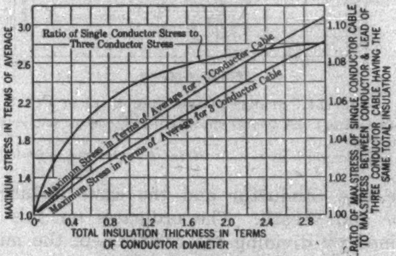

Fig. 16—Curves Showing Relation between Stress of Three-Conductor Cable and Stress of Single-Conductor Cable and between Maximum and Average Stress of Each

Fig. 17, Curve A, gives the stresses between the conductors on the line joining their centers. Curve B gives the angle by which the stress leads the voltage of conductor A to neutral.

The vector diagram, Fig. 18, was drawn primarily as a check on the method used for obtaining stresses. Incidentally, from it may be determined voltages between a conductor or the center of the cable, and any point on the line joining the centers of the two conductors.

The horizontal line *A X* is drawn to represent the direction of the voltage of conductor *A* to neutral. The arc *A Z B B* was obtained by the addition of potential differences obtained by the integration of stresses (shown in Fig. 17) along the line joining the centers of the conductors *A* and *B*.

The potential at a point along the center line *A B*, a short distance from conductor *A*, was determined, being proportional to the area included below curve *A* of Fig. 17. The vector representing this potential was plotted as the first step of the arc *A Z B B*, Fig. 18, the angle with the horizontal being determined by curve *B* of Fig. 17. Next, the voltage between this first point and a second was obtained in the same manner and a new portion of arc drawn in. (It will be noted that the individual sections of arc are actually drawn as chords). This process was continued for the entire line between conductors *A* and *B*.

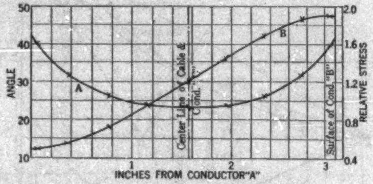

FIG. 17—STRESSES ALONG LINE JOINING CENTERS OF CONDUCTORS IN TERMS OF AVERAGE BETWEEN CONDUCTOR AND CENTER OF CABLE ALONG 180-DEGREE LINE (15.2 IN. RING—2.5 IN. TUBE)

Curve A—Stresses
Curve B—Angle by which stress leads the voltage of conductor A to neutral

The vector represented by any of the lines drawn from *A* and terminating on the arc *A Z B B* is the difference of voltage between conductor *A* and some definite point on the center line of *A* and *B*, but, since both the abscissa and ordinate of the point on the arc are used to represent potential, it is not possible to show on that curve alone the location of the point on the center line of *A* and *B*. Therefore, the line *A B* is drawn so that if a vertical line is drawn from any point on the arc to the line *A B*, the intercept between *A X* and *A B* represents the distance from the conductor *A* to the point the potential of which is represented by the chord drawn from *A* to the said point on the arc. Thus, the distance *B X* represents the distance of the conductor *B* from the conductor *A*, the potential of *B* being represented by *A B B*.

Also the line *W Y* is equal to half the line *B X*, and thus the chord *A Z* represents the voltage from conductor *A* to a point on the center line of *A* and *B*, and halfway between them.

The line *A O* represents the voltage between conductor *A* and center of cable, or ground. Also, the voltage to ground of any other point is represented by a line to the point *O*, thus *B B O* represents the voltage to ground of the conductor *B*, as determined by the measurements of stress given in Fig. 17.

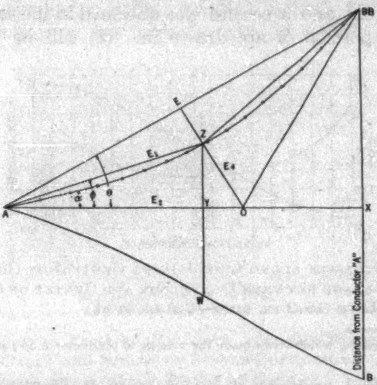

Fig. 18—Vector Diagram Obtained by Summation of Stresses along Line Joining Centers of Conductors

The value obtained from the mathematical integration of the stress along the line joining the centers of conductors *A* and *B* was found to be 1.72 times the voltage of conductor *A* to ground and at an angle of almost exactly 30 degrees from this voltage, a very satisfactory agreement with the actual values of 1.73 and exactly 30 degrees.

The voltage *O Z* is found either from this diagram or by the integration of the stresses along the line joining the center of the cable and the mid-point of the line joining the centers of conductors *B* and *C*. The values obtained by these two methods are 0.325 and 0.355, respectively, in terms of voltage of conductor *A* to neutral, and at an angle of 60 degrees from that voltage.

The voltage from conductor *A* to the point midway between conductors *A* and *B* on the line joining their centers is the chord *A Z*, and is equal to 0.87 times the voltage of conductor *A* to neutral, and is at an angle of 18.6 degrees from this voltage.

Data given thus far have applied to measurements on a single "cable." Similar measurements were taken of stresses with a different size of conductor. A sheath 15.2 in. in diameter was used and 5.82 inch conductors were so placed that the conductor and belt insulation were equal.

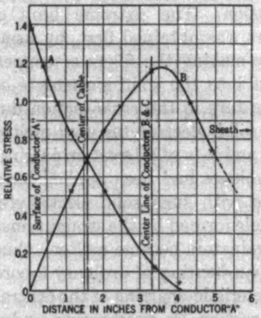

Fig. 19—Stresses along 180-Degree Line in Terms Average between Conductor and Center of Cable (large tubes)

Curve A—Stress along 180-degree line
Curve B—Stress normal to 180-degree line

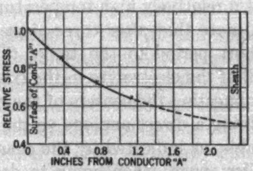

Fig. 20—Stresses along 0-Degree Line in Terms Average between Conductor and Center of Cable along 180-Degree Line for 15.2 in Ring and 3.82 in. Tubes

Not as many readings were taken for this "cable," Enough were taken however to show that the maximum stress occurred at the conductor on the line towards the center of cable and to determine stresses at the points of maximum importance. Stresses were taken all along the lines joining the center of conductor *A* and center of cable.

In Fig. 19, curves *A* and *B*, respectively, are shown the stresses along (parallel to), and normal to, the line joining center of conductor and center of cable.

Fig. 20 shows the stresses on the shortest line between the conductor and sheath. The maximum of this curve checks within 2.5 per cent the value calculated from Russell's formula as shown in Fig. 16.

USE OF SOLID ELECTROLYTE

In the Pittsburgh experiments of 1913 and 1914 use was made of solid as well as liquid electrolytes. Cable cross sections were prepared of solid electrolytes with cast-in exploring electrodes in which the specific resistivity of the electrolyte occupying the filler space differed from that of the electrolyte representing the conductor and belt insulation. The extreme limit of range of resistivity for the fillers was obtained first, by using solid insulators, and second, by making the fillers of copper.

Also, thermometers were cast into the solid electrolyte so as to note the change in temperature in the cross-section due to the energy loss. One of the solid electrolytes used melted at a relatively high temperature and it is of interest, in passing, to note that after three-phase voltage had been applied for a short period to the tank, filled with such solid electrolyte, all of the electrolyte in the triangle formed by the straight lines connecting the centers of the conductors became molten, thus giving a direct analogy to the over-heating of ordinary 3-core cable, as indicated by the characteristic charring of the central filler space, when going to destruction by overheating under three-phase voltages. This phenomenon has been noted by Mr. C. W. Davis as early as 1911.

The characteristic charring of three-core cable was observed by Höchstadter on what we understand was mineral base compound about the same time, although not reported to us until about 1913. It has also been noted on vegetable compound cable by Clark and Shanklin and by Bang and Louis, and reported by them in the A. I. E. E. TRANS. 1917, p. 431 and 447 and for some time engineers of operating and manufacturing companies have generally recognized this condition.

The electrolyte experiments of 1913-1914 were part of an investigation into the causes of this type of failure of three-core cable. Supplementing those experiments, covering the same period approximately, a comprehensive set of dielectric loss measurements were made at high and low voltage, both single-phase and three-phase, on mineral compound and vegetable compound three-core cable. As a result of these experiments, the opinion was reached that the arrangement of conductors and insulation in any ordinary three-core cable is such as to make it inherently weak, the tendency of the center to overheat being increasingly difficult to overcome the higher the operating voltage.

From the user's standpoint especially, however, it is very desirable to retain, insofar as possible, the advantages and economy resulting from having the three conductors under one lead sheathing rather than the more costly alternative method of using three single-conductor cables, each separately lead covered, with attendant disadvantages of extra cost due to the increased amount of lead sheathing, and the larger ducts or greater number required.

M. Höchstadter (*London Electrician*, May 19, 1916) has described a way to avoid the disadvantages of the ordinary three-conductor type of cable by converting the cable into what is substantially three single-conductor cables without the attendant disadvantages of disproportionate increase in cost. The conductors may be made sector shaped so that all the space within the single lead sheathing is economically used, thus securing a still further gain in economy, since it is obvious that three single-conductor cables separately sheathed could not conveniently be made in sector form and installed in such a way as would permit of their fitting together in order to occupy minimum duct space. The thin metal foil or tape suggested by M. Höchstadter is preferably made of copper, thus securing the most effective dissipation of heat.

As part of the characteristic charring of the central portion of a three-conductor cable of normal type is due to internal corona, it is obviously desirable to prevent this as far as possible, and what appears to be one of the most effective methods of preventing the occurrence of minute air or gas spaces in the highly stressed portions of cable dielectric is the suggestion of H. W. Fisher in 1911 to apply the previously dried and impregnated tape to the conductor while the conductor itself is submerged under oil. The adoption of this method should make internal corona, or ionization, of relatively little importance even at the highest voltages likely to be reached for many years to come.

Numerous tests were also made with the electrodes representing the conductors, of different shapes. Certain advantages were found as regards distribution of stress and dielectric loss with a conductor of approximately semi-circular form with slightly rounded corners, and with the approximately flat side toward the sheath. It is conceivable that such a special form would be available in special cases where large conductors are involved, but on account of the constructional difficulties and larger diameter of this form, a modified sector conductor is the most suitable practical alternative for regular use.

Referring again to the use of solid electrolytes, it may be of interest to report the use of such electrolyte in the form of pieces simulating simple forms of solid insulators, such as those made of porcelain, in use in certain types of cable terminals. By immersing such molded solid electrolyte in liquid electrolyte of higher resistivity there was obtained a direct representation of a porcelain insulator surrounded by air, and we were thus enabled to plot the equi-potential lines from the point where they leave the surface of the solid electrolyte. This method may be of interest to the designers of porcelain insulators, although complicated forms may prove themselves difficult to cast and make self supporting without undue deformation.

Using the same electrolyte method, and following the analogy of heat flow, isothermal lines for various shaped conductors in three-conductor cables were plotted in the Pittsburgh experiments of 1913-1914. These show that under the I^2R loss alone the center of a three-conductor cable will be lower in temperature than the temperature of the conductors by a small relative amount. In a cable using three sector shaped conductors, less relative difference was found between the temperature of the center of the cable and the temperature of the conductors than where round conductors were used.

Thornton has explored the field of a three-conductor cable by means of the Hele-Shaw method. He suggested the existence of a triple-frequency field at the center of the cable. In the same series of Pittsburgh experiments data were also secured regarding this point, and also later in measurements with actual cable, at Perth Amboy. All of the measurements made failed to show any such effect as described by Thornton. It is to be expected, however, that measurements carried on with cable at voltages above the ionization point would show pronounced harmonics in the potential wave of stresses at different points.

MEASUREMENT OF STRESSES IN ACTUAL CABLE

For this test a specially constructed cable was used. This cable was in nearly every way like a commercial three-conductor cable with round conductors. Its main differences were that at definite points very thin wires (No. 40 A. W. G.) were laid parallel to the conductor, and that the cable was assembled without "lay".

The cable consisted of 4/0 stranded conductors with 9/32 in. of paper insulation on each conductor and 5/32 in. on the belt. The separate conductors were made up as follows: Insulation was first applied to a thickness of 1/32 in. Eight No. 40 wires were then placed at equal intervals around the conductor, and parallel to its axis. The same was done after a total insulation thickness of 3/32 in., 6/32 in., and 9/32 in, had been applied. This made a total of 32 wires per conductor, or 96 for the cable. Immediately over each set of eight wires two layers of paper were applied by hand. The rest of the insulation was applied in the ordinary manner by machinery.

A three-conductor sector cable with the same size conductors and the same thickness of insulation as the other was also constructed in exactly the same manner. The number of No. 40 wires per layer, however, instead of being eight were only six,—one at each corner of the sector and one half-way between each pair of corners. The total number of wires for each conductor were then 24, and for the cable 72.

The small wires were placed where they were so as to furnish a means of measuring the potentials at these points. Hereafter, then, they will be termed "potential wires." The points at which the potential wires were placed will be termed "potential points." The three conductors of the cable will be designated as I, II, and III. The four layers of potential wires of each conductor, starting at the layer nearest the conductor, will be called, respectively, *A*, *B*, *C*, and *D*.

Potential data were obtained, in both the circular and the sector cable, on all four rows. Measurements of voltages on *A* and *B* were taken between each potential wire and its conductor, while values on *C* and *D* were taken between potential wire and sheath. Chiefly because data on *B* and *C* would give relatively little additional information, only results gotten on *A* and *D* are given in this paper.

The problem of measurement of the voltage of the potential wires in the three-conductor cable with three-phase voltage applied is quite similar to that encountered in the measurement of potentials and stresses in the electrolyte tank. As before, there is the problem of measuring voltages of various unknown phase relations. This was accomplished in a manner somewhat similar to the one previously described. The current which could be taken without field distortion was much smaller, so small that only a null method seemed available. Also, it was desirable to keep the detecting apparatus at ground potential.

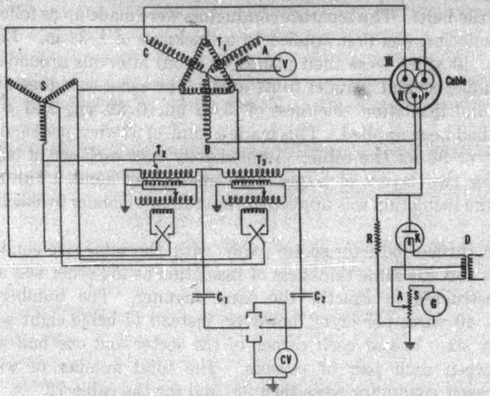

Fig. 21—Diagram of Connections for Determination of Potential Inside of Cable

S—three-phase supply
T_1—three-phase transformer supplying cable
T_2 and T_3—single-phase balancing transformers
V—Voltmeter for T_1
C V—crest voltmeter for T_2 and T_3
C_1 and C_2—condensers used with C V
D—detecting device
K—rectifier (Kenotron)
R—20 megohm resistance shunting K
A S—Ayrton shunt for G
G—d-c. galvanometer
P—potential wire

This explanation will make clearer the reasons for the methods adopted.

Fig. 21 shows the diagram of the connections used. The current supply was taken from a 40 kv-a. three-phase transformer, the primary being delta connected. The secondary is star connected and gives a rated voltage of 95. There are taps on each leg, of 21 per cent and 42 per cent of the full voltage. The delta primary of the three-phase high-tension transformer,

T_1, was connected to the 42 per cent taps, and as the normal rating of this transformer per phase is 440 to 25,000 volts, this gave a voltage per phase of approximately 2300, or a voltage between conductors of about 4000. This voltage was impressed on the cable.

The balancing voltages were obtained from two single-phase transformers, T_2 and T_3, each equipped with a regulating transformer, thus allowing a selectivity of voltage. The primary of T_2 was connected to the 21 per cent taps of the two phases of the supply which supplied phase A of transformer T_1, thus putting T_2 in phase with phase A. The primary of T_3 was connected between the 42 per cent tap of the third supply phase and neutral, thus making the voltage of T_3 90 degrees away from that of T_2. The normal rating of T_2 is 440 to 62,500 volts, with a capacity of 300 kv-a. There are 480 steps in the regulator, so that, with a primary voltage of about 20 volts there are six volts per step. The normal rating of T_3 is 440 to 50,000 volts, with a capacity of 200 kv-a. There are 400 steps in the regulator, so that, with a primary voltage of about 23 there are about seven volts per step. The secondaries of T_2 and T_3 were connected in series, the free end of T_3 being connected to one of the phases or to the neutral of T_1, and the free end of T_2 grounded. T_1 is not grounded at any point, so that the potential above ground of the phase of T_1, or of the neutral, to which the large transformers T_2 and T_3 are connected, is governed entirely by the voltage of these. This arrangement enables the detecting device to be always near ground potential.

To determine the potential at any point P_2, proceed as follows: Connect the potential wire at P to the detecting device. Connect the conductors of the cable to T_1 and the balancing transformers to either the conductor of P or to the sheath. Raise the voltage of T_2 and T_3 in the proper direction until the detecting device shows minimum deflection, which indicates that P is at ground potential. The voltage produced by T_2 and T_3 will then be accurately the difference of potential, in direction as well as in magnitude, between P and its conductor or sheath. This being a null method, the stress distribution, when balance is obtained, is in no wise altered by the measuring apparatus.

The detecting device consisted of a rectifier for which a 100,000-volt, 0.1-ampere kenotron was used in connection with a galvanometer equipped with an Ayrton shunt. The rectifier was shunted by a water resistance of the order of twenty megohms. The rectified current is necessarily unidirectional, and so the proper balance was considered obtained when an increase or decrease in voltage on either T_2 or T_3 increased the deflection. The minimum reading of the galvanometer was not zero, due doubtless mainly to the thermoelectric (Seebeck) effect between the plate and filament of the kenotron, but this did not alter the point of balance.

Voltages were measured as follows: The voltage of T_1 was measured by a voltmeter connected to a tertiary coil of the transformer. The voltages of T_2, and of the combination of T_2 and T_3, were measured by means of a crest voltmeter. By separate measurement the voltage per step of the regulator of T_3 was found to be quite uniform, and its value determined. The voltage of T_3 was therefore calculated from the number of steps on its regulator. Thus, the actual potential difference, as well as its two components, was accurately measured.

ACCURACY CHECKS ON MEASUREMENTS ON ACTUAL CABLE

To determine what error, if any, was due to end effect, the connector of the rectifier was brought within a quarter of an inch from the end of the cable and the full voltage applied to the cable by means of T_1, T_2 and T_3 being at their zero position. The galvanometer showed a deflection which would be c used by less than one step on either controller, indicating that the maximum error is less than 2 per cent in the measurement of the smallest voltage read.

The sensitivity of the detecting device was such that after balance was obtained, an unbalance of one step (6 or 7 volts) on either transformer would definitely increase the galvanometer deflection.

The voltages on T_2 and T_3 should be at right angles to each other. To check this point, both transformers were raised to the same potential and readings taken of the separate voltages and of the hypothenuse voltage. This check was made at several different values with the reversing switches of both transformers in the up position, with both in the down position, and with one up and the other down.

The measured and calculated values checked exactly in some cases, and in others varied by as much as 3 per cent. This variation probably accounts for some small phase angle errors in the results.

A single-conductor cable was prepared with potential wires in the same manner as the three-conductor cables, and was supplied with voltage from phase *A* of T_1, T_2 and T_3 were used as balancing transformers, and the potentials determined at various distances from the conductor. In the ideal case T_2 should have been exactly in phase with phase *A*, and so the balance point should have been with zero voltage on T_3. However, T_3 was required to the extent of changing the phase of T_2 about 1½ degrees on all the measurements. Correction was made for this error in plotting the curves.

The balancing transformers were connected successively with the three phases of T_1, the balancing transformers being grounded directly and T_1 being grounded through a condenser and the detecting circuit. The voltages were then balanced and readings taken on *V* and *C V*. These readings checked each other as closely as the instruments could be read.

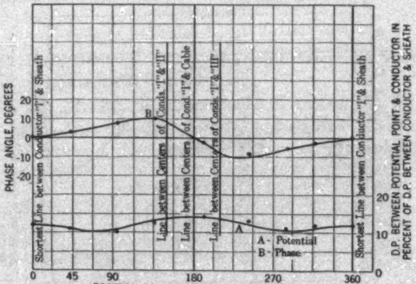

FIG. 22—PHASE AND POTENTIAL AT LAYER A, CONDUCTOR I, CIRCULAR CONDUCTOR CABLE

After the three-phase tests were performed, the cable with circular conductors was dissected, and one of the insulated conductors wrapped with foil, and measurements were made with single-phase on layer *A* of potential wires. Calculation from values thus gotten indicated these potential wires to be 0.031 in. away from the conductor. Micrometer measurements indicated this distance to be 0.032 in. The dissection also showed that the potential wires were quite uniformly spaced around the conductor, but that they were not absolutely parallel with its axis, the rotation in the entire length being not more than half the distance between wires.

Dissection of the sector cable showed layer *D* of potential wires to be placed quite closely to their proper position, but that layer *A* was altogether misplaced. The actual positions were therefore accurately measured and used in Fig. 25. Micrometer measurements indicated the insulation thickness between the conductor and the first layer of potential wires to be 0.325 in. at the small diameter and 0.335 in. on the long diameter.

The curves, Figs. 22 to 25 inclusive, are, as far as possible, drawn from values obtained on one conductor only. The circles indicating the points for this conductor have been filled in solid. On all curves except Fig. 22 points also have been shown which were gotten on the other conductors.

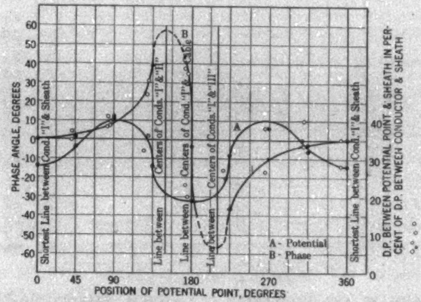

FIG. 23—PHASE AND POTENTIAL AT LAYER D, CONDUCTOR I, CIRCULAR CONDUCTOR CABLE

In the case of the circular conductor cable, the relative positions of the potential points were found by dissection, but the exact location of the group with reference to a fixed line of the cable, such as the line between conductor center and cable center, was found from preliminary plotting of the phase measurements. It was known that the zero of the phase curve must occur at the point nearest the center of the cable and again at the point nearest the lead. For the final curve the reference point of scale of abscissas was so taken that the phase curve would pass through zero on the said line.

In the case of the sector cable, the relative positions of the potential points were, as in the other case, found by dissection.

However, since the shape of the cable furnishes a reference point, there was no difficulty in determining the reference point of the scale of abscissas from the measurements of the dissected cable. The zero of the phase curve is here seen to pass zero on the line of centers of conductor and cable, as was assumed in the case of the round conductor.

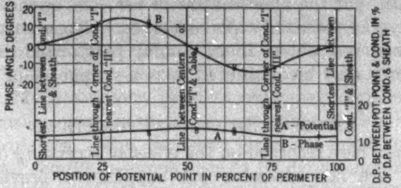

Fig. 24—Phase and Potential at Layer A, Conductor I, Sector Cable

Curves were drawn of phase and of potential on layers *A* and *D* of both the circular conductor (Figs. 22 and 23) and sector (Figs. 24 and 25) cables. The ordinates of the phase curves are in electrical degrees, positive and negative; positive representing out of phase in one direction and negative in the other, as in similar curves from the tank experiments. The ordinates of the potential curves are in per cent of the voltage between conductor and sheath. The abscissas of the curves on

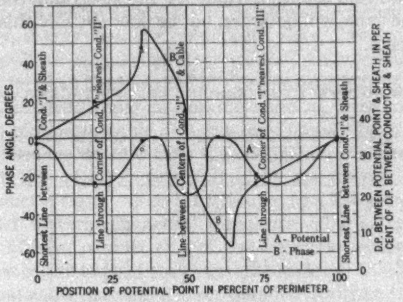

Fig. 25—Phase and Potential at Layer D, Conductor I, Sector Cable

the circular-conductor cable are in circular degrees, starting with 0 degrees at a point nearest the sheath, thus making the point nearest the center of the cable 180 degrees. In the case of the sector cable the zero is also taken at the point nearest the sheath, but the abscissas are in per cent of the perimeter at the layer of potential wires concerned. These curves should be symmetrical about their middle point, and any slight discrepancy may be attributed to non-symmetry of the three phases.

EFFECT OF STRANDING UPON STRESS.

The data here recorded on the effect of stranding on voltage gradient are not extensive but supplement previous data. Tests were made on a seven-strand and a nineteen-strand conductor, that is one containing six wires and one containing twelve wires in the outer row. The ratio of the stress in the case of the stranded conductor to that with a round conductor of the same area of section and with the diameter of the sheath the same in each case, is given to compare with the value determined mathematically by Jona. The value 1.23 found for seven-strand conductor checks exactly with the value given by Jona. We find the same value for the 19-strand, though Jona states the value will be larger, but he does not solve his equation for this case.

TABLE III

SHOWING EFFECT OF STRANDING UPON VOLTAGE GRADIENT

No. of Strands of conductor	$\frac{\text{Insulation thickness}}{\text{conductor diameter}}$	Ratio of stress with stranded conductor to that with round conductor	
		A	B
7	1.47	1.23	1.26
7	0.50	1.23	1.18
19	1.07	1.21	1.22
19	0.71	1.22	1.20

A. Same outside diameter over insulation, area round solid equal to total area of strands.
B. Same outside diameter over insulation, same diameter of solid and stranded conductor.

The most generally useful basis of comparison is the ratio of the stress of the stranded conductor to that of a round conductor of the same outside diameter and with the same insulation thickness. These values are given in the last column of the table. Little difference between the two styles of a stranding is apparent, though the data are not on exactly the same basis. On ordinary thicknesses of insulation the increase of stress due to stranding is roughly 20 per cent. For very thick insulation relative to conductor diameter, the increase is greater, and for thin insulation the increase is less.

Whitehead has shown that the effect of twisting the strands, as is actually done in cable practise, is to decrease the effect of stranding upon the gradient. Though his tests were with insulation thicknesses very great as compared to cable insulations, doubtless there is an important effect of this kind for thinner insulation. The data here given are upon untwisted strands, and should be supplemented by determination of the effect of twisting. Data here given are for single-conductor cables in a single-phase field.

PRACTICAL APPLICATION OF DATA

A brief outline of the manner in which the accompanying data may be applied to important problems is given.

The application of the resistance in electrolyte tank measurements has been indicated in the data given in connection with Fig. 2. This curve has been in use for some time in our laboratory and serves admirably to link together measurements of capacity, energy loss, etc. on different sizes, three-conductor cable as well as single-conductor cable. An interesting and important fact has been learned by the comparison thus allowed between the permittivity of the dielectric of three-conductor and single-conductor cable. It has been found that there is a definite difference, greater than the possible error of the curves, in the direction of lower permittivity for the dielectric of the three-conductor form. This is attributed to a low permittivity of the filler material in actual three-core cable as compared with the uniform permittivity of the dielectric occupying the filler spaces in the tank of electrolyte, as we know that the filler of an actual cable may have lower permittivity than the paper on the conductor, under certain conditions. A more comprehensive investigation of this has been begun. The bearing of this upon the stresses in the cable is apparent and will be discussed further.

The application of these data to temperature rise measurements of three-conductor cable is similar and of corresponding convenience and value.

A table is given comparing the stresses of a three-conductor cable under three-phase voltage with the stresses of single-conductor cables of certain specified insulation thicknesses. Tests are reported on two sizes, one with relatively thin and one with relatively thick insulation. It will be observed that the maximum stress next the conductor, toward the cable center, is so close to that in a single-conductor cable having the same insulation thickness as the distance between conductor and center of cable, that quite accurate calculation of the maximum stress of any three-conductor cable may be made readily from these data.

TABLE IV.—STRESSES AT IMPORTANT POINTS IN SECTION

	r	*A*	*B*	*C*
Max. stress at conductor surface on line toward center of cable..............	0.625 0.288	1.71 1.43	1.01 1.04	2.65 1.88
Stress at center of cable..............	0.625 0.288	0.65 0.685	0.385 0.496	1.01 0.902
Stress at mid point between conductors on center line of conductors in direction of center line..........................	0.625 0.288	0.92 1.18	0.545 0.855	1.42 1.55
Stress at edge of conductor insulation on center line of conductor and center of cable............................	0.625 0.288	0.77 0.83	0.455 0.602	1.19 1.09
Ditto but normal to the center line....	0.625 0.288	0.51 0.52	0.302 0.277	0.790 0.685

A. Stresses in terms of average stress between conductor and center of cable along line joining these points.

B. Stresses in terms of maximum stress of single-conductor cable having the same insulation as the distance between the conductor and center of three-conductor cable. This distance is equal to 1.154 times the conductor insulation plus 0.077 times the conductor diameter.

C. Stress in terms of minimum stress of single conductor having same insulation as *B.*

r. Ratio of conductor insulation to conductor diameter.

Data have been given showing the agreement between values determined from Russell and those measured, for maximum stress between conductor and sheath. A curve (Fig. 16) constituting a solution of Russell's formula is given.

Stresses at and near the center of the cable and at various points on the surface of the conductor insulation are seen to bear a fairly definite relation to the stress at the sheath of the same single-conductor cable. At the center, the stress is quite closely the same as that at the sheath of the single-conductor cable, though the stress at the other points is considerably greater.

When the stresses of the three-conductor cable are compared with those of a single-conductor cable, it should be remembered that in many parts of the three-conductor cable there is not a simple alternating field, but it may be considered to consist of two alternating fields superimposed at right angles and producing a rotating electric field somewhat analogous to the rotating magnetic field of an induction motor.*

We have also summarized in Table V the data on the actual three-conductor cable so that they may be compared with those given in Table IV, as the result of the measurements with the conducting liquid.

TABLE V

MEASURED STRESSES AT IMPORTANT POINTS IN ACTUAL CABLE

The data here given are for the circular conductor cable for which curves have been shown. For this cable $r = 0.532$.

		AA	*A*	*B*
Stress at conductor on line joining center of conductor and center of cable.	From data	.87	1.70	1.05
Stress at conductor on shortest line between conductor and sheath.....	From Data	.75	1.46	.90
	From Russell's Formula	.70	....	.84

A, *B* and *r*—same as table above.

AA. Stress in terms of maximum stress of single-conductor cable with insulation thickness equal to conductor insulation of the 3-conductor cable.

NOTE. Data shown in column *AA* were obtained by comparing the three-phase stresses with stresses obtained by actual single-phase measurements on one of the conductors of the three-phase cable after it had been dismantled. Data shown in columns *A* and *B* were derived from the values in column *AA*.

It will be seen that the stresses next the conductor are somewhat greater than those determined by the previous method. We are not able to state with certainty the reason for the difference, though we believe it too great to be accounted for by inaccuracy of measurements. If the permittivity of the filler material were higher rather than lower than that of the main insulation, it would result in giving such a result. As stated previously, it would be expected that this effect would be in the opposite direction, since for the low voltage at which measurements were made the filler was that thought to have a low permittivity. It is possible that the presence of the potential wires, due to the fact that they were not exactly parallel to the axis of the cable, distorted the field sufficiently to cause the increase of stress above the value found by the other method.

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*For instructions as to the use of Table IV in connection with Fig. 16 for obtaining stresses at different points in the section of a 3-conductor cable, see Appendix B.

It is important to consider the data on stresses in connection with the paper given before this Institute in February, 1919, by Shanklin and Matson. They conclude that the maximum allowable operating pressure for a given cable occurs at a certain gradient that is very definite for a given insulating material. If we accept this conclusion, we have at once from the data presented herewith a basis for the relative rating of cables with different sizes of conductor and different thicknesses of insulation. However, the application of the data by one of the same authors indicates that important modification of the said conclusion must be made. In the 1917 paper, a cable, No. 1/0 with 9/32 in. insulation on the conductors and 7/32-in. insulation on the belt, is mentioned as having operated successfully at 24 kv. for some years. The maximum voltage gradient in this cable is more than 27 kv. per cm.—much in excess of 22 kv., which they conclude in the 1919 paper to be the limit for that class of material, as based on experiments on cable presumably of much more recent manufacture, and therefore not of poorer construction. Of course, also, there are many other cables operating successfully at gradients much in excess of 22 kv. per cm.

In bringing out this point in connection wth the paper of Shanklin and Matson, I do not want to be misunderstood, as I consider their paper an important contribution to the problem of cable insulation; and though I feel that the theory offered must be subjected to material modification, yet it may be part of the foundation upon which may grow a material evolution in the cable industry.

While considering this paper by Messrs. Shanklin and Matson, it may be well to call attention to the possibility that, in some cases, the ionization which they found, occurred actually in the filler spaces rather than next to the conductors. It is not possible to give specific figures, as we do not now have sufficient data of stresses in the filler space with single-phase voltage, which is the condition under which their measurements were made.

SECTOR CABLE

We are not attempting to give as specific data on sector cable as have been given for round. Time and space make a barrier, and furthermore, specific data will have less general application, for the terms "sector" and "modified sector" as applied to cable conductors are given to shapes differing quite widely in form and thus in characteristics, and presentation of complete data on even one specific form of conductor would require even more space than for round conductors. Some approximate general statements will however be of value.

To determine capacitance of cables with conductors of sector shape, take the data for a cable with the same insulation thickness and with round conductors of about 50 per cent (two sizes A. W. G.) greater area than the sector conductor. Actually, for different insulation thicknesses relative to conductor sizes, and for capacities of the different combinations listed, there is much range in the ratio of areas of round conductors and sector conductors having the same capacitance, the range of this ratio being from about 1.2 to 2.0, hence the rule just given is not more than a first approximation though it will seldom give capacities greatly in error.

The measurements made on the sample of actual sector cable indicate that the maximum stress is somewhat lower than that for the corresponding round conductor cable, but tests in the electrolyte tank indicate that relatively slight change of shape can influence the value of maximum stress sufficiently to vary it from lower to higher than for round conductor. The stress on the outer surface next the sheath was in all cases much less than for the round conductor.

Attention has been called above to the fact that these data throw much light on the inherent weakness of the ordinary form of three-conductor cable. The stresses in the filler spaces and tangential to the layer insulation are somewhat less than the maximum, which occurs at the conductor surface. However, because of the weakness of the filler insulation and the weakness of the conductor insulation for tangential stress, these stresses become of great importance. Other stresses in the filler space are very large as compared with the average stress throughout the insulation, and large even as compared with the maximum values at the conductor surface. Improvement in this portion of the dielectric has not in the past kept pace with improvement in the insulation near the conductors and probably can still not do so in the future. Doubtless, therefore, stresses in this portion of the cross section will become more and more the limiting condition in cables of the usual three-conductor form. Furthermore, it must be borne in mind that the stresses given are for the condition of dielectric of uniform permittivity. The stresses in the filler space will exceed materially the values here found, where the permittivity of the filler material is lower than that of the remainder of the insulation.

These experiments were carried on in the factory of the Company with which the author is associated, and the writer wishes to express appreciation for the facilities extended by his Company, and for the cooperation of various individuals. Thanks are due Mr. C. W. Davis and Mr. H. W. Fisher for many valuable suggestions and comments, to Mr. Donald Simons in connection with the early work done in the Pittsburgh Laboratory, and to Mr. A. M. Hagen and Mr. L. Meyerhoff for their faithful and valuable assistance in the performance of experimental work and in the preparation of this paper.

APPENDIX A.

To obtain the charging current for any desired connection of a three-conductor cable from Fig. 2, proceed as follows:

On the lowest set of curves follow the horizontal line representing the value of total (belt plus conductor) insulation, until the curve for the given size conductor is intersected. This will give the ratio of total insulation thickness to conductor diameter. Now calculate the ratio of belt to conductor insulation for the given cable. From the point of intersection gotten above go vertically upwards until the horizontal line representing the calculated ratio of belt to conductor insulation, on the set of curves for the particular type of charging current desired, is reached. This point of intersection gives the value of charging current sought in terms of milliamperes per 1000 ft. at 60 cycles and a permittivity of 3.27.

For example, let it be required to find the charging current 3 conductors to sheath on a 3-conductor cable, having 4/0 conductors with 10/32 in. insulation on each conductor and 4/32 in. on the belt. The total insulation here is 14/32 in. and following the 14/32 in. line on the lowest set of curves we find that it intersects the 4/0 curve at a ratio of total insulation thickness to conductor diameter of 0.83. The ratio of belt to conductor insulation is 0.4. On the 0.4 line for 3 conductors to sheath we find that an abscissa of 0.83 corresponds to a charging current of 34.3 miliamperes per 1000 ft., which is the value sought.

APPENDIX B

The values given in Table IV puts us in position to determine stresses at various points in the cross-section of a 3-conductor cable when operating 3-phase. We will here outline the method for obtaining the stress at one particular point by the use of Table IV in connection with Fig. 16, but practically the same method can be used for obtaining stresses at other points in the cable section.

The particular point chosen is that point on the conductor surface which is nearest to the cable center. As stated in the paper, this stress is approximately equivalent to the stress of a single-conductor cable having the same conductor diameter and an insulation thickness equal to the distance from the conductor surface to the center of the cable. After finding this distance, the maximum stress at the surface of the conductor for this equivalent single-conductor cable may be found by the usual formula, or more conveniently by the middle curve of Fig. 16. As shown, however, in Column B of Table IV, this value is not absolutely correct, and a slight correction must be made depending upon the ratio of conductor insulation to conductor diameter, as shown in the above table. The correction for values of the ratio between those given in Table IV may be found by interpolation. Great care however must be used in exterpolating from these values, but, as very few commercial cables have ratios except between these limits, the question will ordinarily not arise.

To illustrate the above by example, let us consider a 3-conductor cable, No. 1 A.W-G. strand, 9/32 in. × 7/32 in. Putting sizes into decimals, we have a 0.328 in. diameter conductor, with insulation 0.281 in. plus 0.219 in. Obtain the equivalent single-phase insulation 1.155×0.281 plus $0.078 \times 0.328 = 0.350$. The ratio of equivalent insulation thickness to conductor diameter is $\frac{0.350}{0.328} = 1.07$, and from the middle curve of Fig. 16, for 1.07 we find the ratio of maximum to average stress is 1.88. The average stress with say 14 kv. between conductors is $\frac{14}{\sqrt{3} \times 2.54 \times 3.50} = 9.08$ kv. per cm. The maximum stress is therefore 1.88 × 9.08 = 17.01 kv. per cm. To make the final corrections referred to above, divide the actual conductor insulation by the conductor diameter, or $\frac{0.281}{0.328} = 0.856$. This is a higher value than either of the values given in the top of column B, Table IV. The correction is less than 1 per cent therefore and can be neglected.

To find the maximum stress in the direction of the sheath, for a three-conductor cable, either under three-phase voltage, or as is done in cable testing, under single-phase voltage, the method is very similar. Find the average voltage stress per cm. of the insulation between the conductor and the sheath. Find the ratio of the total insulation thickness (*i.e.* conductor insulation plus belt insulation) to conductor diameter and find the value corresponding to this ratio in the *bottom* curve of Fig. 16. Multiply the average stress by this figure and maximum stress at the surface of the conductor is obtained.

As illustration, let us take the same cable. The total insulation is 0.281 in. plus 0.219 in. = 0.500. The average gradient with 14,000 volts three-phase between conductors is

$$\frac{14}{0.500 \times 2.54 \times \sqrt{3}} = 6.35$$

$\frac{0.500}{0.328} = 1.52$, and the ordinate corresponding to this in Fig. 16, bottom curve is 2.02. 2.02 × 6.35 = 12.7 kv. per cm., which is the maximum stress at the surface of the conductor toward the sheath.

DISCUSSION ON "HIGH-TENSION SINGLE-CONDUCTOR CABLE FOR POLYPHASE SYSTEMS" (CLARK AND SHANKLIN) AND "THE DIELECTRIC FIELD IN AN ELECTRIC POWER CABLE", (ATKINSON), LAKE PLACID, N. Y., JUNE 27, 1919.

H. W. Fisher: The paper of Messrs. Clark and Shanklin takes me back to the year 1907 or 1908 when I had been making some experiments to determine the change of impedance on single-conductor lead-covered cables with and without lead cover short-circuited. I found in some cases the impedance was greater with the lead cover short-circuited; in other cases it was less. I discussed this matter with some of our very prominent engineers at the time and they said my experiments must be wrong as they would expect the impedance to decrease as it does in a transformer with the secondary short-circuited.

About that time, I spent a considerable time in trying to find a competent young man to take charge of the Experimental Dept. of my Company and I selected Mr. R. W. Atkinson whom I asked to give a solution of the above mentioned problem. In a paper which I read at Frontenac, in the year 1909, I presented Mr. Atkinson's graphical and mathematical solution of the problem with which the solution, in the paper we are discussing, is in accord.

Just about that time we were asked to quote on a 60-mile transmission in South America where it was very advisable to install cables instead of aerial lines on account of tropical conditions, and as we had previously been making experiments on dielectric loss, commencing about the year 1907, we were prepared at that time to discuss the problem fully.

The problem was as follows:—Frequency, 25 cycles; proposed transmission voltage, 42,000; power factor of load, 80 per cent; system operated with grounded neutral; number of circuits, 2 at least, and line loss to be 5 per cent.

Our calculations very soon showed we could not use three-conductor cables for the service and therefore we had to figure on single-conductor cable. We considered 60,000-volt and 42,000 volt transmission, and owing to the very much higher cost of 60,000-volt cables, we found that 42,000 volt-transmission was the more economical. It was impossible to operate at 5 per cent loss and the figured loss was between 12 and 15 per cent for the different kinds of cable.

The recommendation that stresses in the insulation of commercial high-voltage paper insulated cables made under present day conditions be limited to so low a value as 19.5 kv. per cm. is not in accordance with our experience. I know of a number of cases in operation where the stress at the surface of the conductor is considerably more than that, and I do not think that there should be a limit in the stress specified, because this would tend to curtail development in the art by manufacturers who might be able to improve their products so as to furnish cable which could be operated at a considerably higher stress at the surface of the conductor.

C. W. Davis (read by H. W. Fisher): Höchstadter in his paper of 1910, published in the *E. T. Z.*, discusses at length the merits of single-conductor and three-conductor cables and gives a large amount of data on dielectric loss measurements up to 40 kv. or 50 kv., both single-phase and three-phase current. The results of this work induced German engineers to use single-conductor cables on the Prussian State Railway as described by Pfannkuch in *E. T. Z.* of 1913.

Klein's careful tests on single-conductor cables at stresses at surface of conductor up to 60 kv. per cm. reported in *E. T. Z.* for 1913. show for the first time in published data, so far as we know, the effect of ionization, although the significance of this was not apparently known to Klein at the time.

In 1913, from the tests he made on four out of six cables, I found that ionization or internal corona began at from 25 kv. to 30 kv. per cm. in solid insulation or about 75 kv. to 90 kv. in air spaces. At that time (1913) I suggested a law of loss for evenly distributed air spaces including a term (based on the proof of Monash in 1907) varying as the square of the voltage and representing the inherent loss in the solid insulation as well as another term based on Peek's law of corona for the loss in the air spaces at voltages above the start of ionization or internal corona.

Wm. A. Del Mar: Messrs. Clark and Shanklin assign much importance to the tangential stresses pointed out by Höchstadter. These stresses occur in a three-conductor cable because at any instant the drop of potential between any conductor and the outside of the insulation surrounding it, is unequal along different radii from the center of the conductor, due to the different voltages between that conductor and the other conductors and sheath. This variation of potential along the surface of the insulation of the conductor gives rise to circumferential leakage currents and dielectric losses. All this is based on the assumption that the spaces between the insulated conductors are filled with a relatively imperfect insulating material, such as impregnated jute. If paper fillers are used there will be no well defined surface on which this phenomena can occur, and the limitation pointed out by Höchstadter will not directly apply.

The remarks by Messrs. Clark and Shanklin regarding the impracticability of grading cables are much to the point. It has long been known that it is commercially impracticable to accurately predetermine the specific capacitance of varnished cambric or rubber insulation.

The general proposition of using single-conductor cables in place of triplex cables offers an interesting field for speculation. I am of the opinion, however, that single-conductor systems will not come into extensive use because of the failure of triplex cables at 30,000 volts. I believe that future development will result in raising the voltage limit for triplex cables.

Like Mr. Fisher, I am rather surprised at the limiting stresses which the authors give for paper insulated cables and I can't help wondering whether a fallacy has not been introduced by basing these stresses upon the results of tests on concentric air films, as it is possible that air bubbles near the surface of the conductor will not break down as easily as flat films.

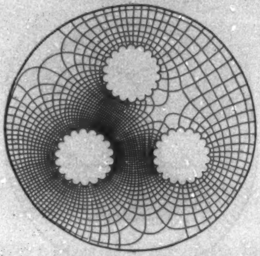

Fig. 1.—Equipotential Lines and Lines of Electric Stress in Round Conductor Cable

Referring to Mr. Atkinson's paper, I wish to state that for several months the Research Laboratory of the company with which I am connected has been carrying out a series of tests, similar to those recorded by the author, and using substantially the same methods. The primary purpose of the tests recorded in this discussion was to determine the best form of sector cable from the point of view of reducing di-

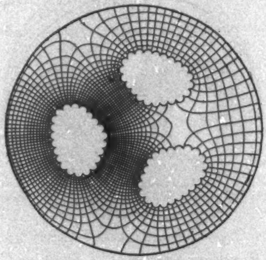

Fig. 2—Equipotential Lines and Lines of Electric Stress in Sector Cable

electric stresses and dielectric losses to a minimum, and increasing the heat dissipation to a maximum.

Mr. Atkinson states that tests in the electrolyte tank indicate that relatively slight change of shape of sector cable can influence the value of the maximum stress sufficiently to vary it from lower to higher than for round conductors. This agrees with our observation, and we therefore, made tests on various modified sectors for the purpose of ascertaining the best form of sector cable from the electrical point of view. A close approximation to the ideal form thus found, being easy to manufacture, subsequent tests were made upon conductors of this form.

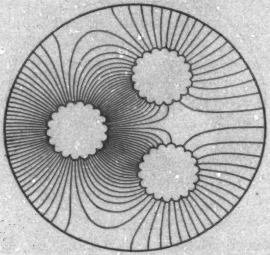

FIG. 3—LINES OF ELECTRIC STRESS IN A ROUND CONDUCTOR CABLE

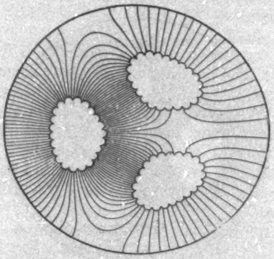

FIG. 4—LINES OF ELECTRIC STRESS IN A SECTOR CONDUCTOR CABLE.

Figs. 1 and 2 show the equipotential lines and stress lines in three conductor round and sector cables, respectively, and Figs. 3 and 4 show the stress lines without the equipotential lines. It will be noted that the maximum density of stress lines is about 15 per cent greater in the round than in the sector cable. Furthermore, the stress along the line joining the centers of round conductors, checks Mr. Atkinson's curve *A*, Fig. 17, very closely, while in the case of the sector cable, the corresponding curve would have only about one-half the sag.

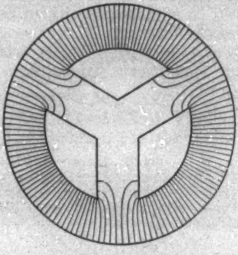

FIG. 5—LINES OF HEAT FLOW IN A THERMALLY IDEAL SECTOR CABLE

Mr. Atkinson mentions the application of the electrolyte method for the determination of lines of heat flow in cables. Fig. 5 shows the lines of heat flow for a thermally ideal sector cable. Such a cable is electrically impracticable, due to the high stresses that would occur at the sharp points. Fig. 6 shows corresponding lines for a round conductor cable and it is interesting to note the much greater average length of path that the heat must travel in flowing from conductor to sheath. Fig. 7 shows the heat flow from a sector cable, indicating that an electrically good sector cable is intermediate between a thermally ideal sector cable and a round-conductor cable. The relative heat conductance from conductors to sheath are as follows:

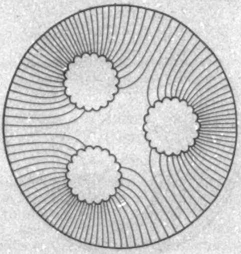

FIG. 6—LINES OF HEAT FLOW IN A ROUND CONDUCTOR CABLE

Thermally Ideal Sector Cable.............	100
Electrically Good Sector Cable............	110
Round-Conductor Cable (Stranded)........	120

Figs. 8 and 9, are similar to Figs. 6 and 7 except that isothermal lines are added. It should be noted that these curves apply only to cables wherein the dielectric loss is negligible and therefore, do not represent conditions in a 60-cycle high-voltage cable at normal operating temperature.

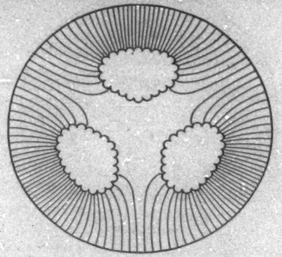

Fig. 7—Lines of Heat Flow in an Electrically Good Sector Cable

The method of test differed from that used by Mr. Atkinson in the employment of 300-cycle current in conjunction with a telephone receiver and resistance bridge. The ratio of the bridge resistance, when balance is obtained, is equal to the ratio of the potential drops between the movable contact needle and each of the two electrodes. A diagram of connections is shown in Fig. 10.

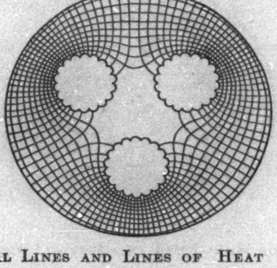

Fig. 8—Isothermal Lines and Lines of Heat Flow in a Round Conductor Cable

The tests recorded above are from a general research on dielectrics, which is now being carried out under my direction by my associates, Mr. C. F. Hanson and Mr. L. Cockaday.

Mr. Atkinson states that the dielectric losses in a cable may be taken to be proportional to the charging current. This is practically true for cables having conductors of similar form. It is not true when comparing round conductor with sector cables, as the effect of increase of charging current in the latter may be neutralized by the elimination of spots wherein dielectric stresses are excessive. As the dielectric loss, at any point, is proportional to the square of the dielectric stress at that point, it is obvious that making the stress more uniform will have a marked effect in decreasing the dielectric loss.

DETAILS OF TEST

1. Electrolyte = Hydrant water.
2. Depth of electrolyte = 1.5 in. (6 cm.).
3. Size of tank = 36 in. by 42 in. and 6 in. deep, (91½ cm. x 107 x 15.2 cm). Painted white with lines ruled 1 cm. apart on the bottom. Plate glass placed on bottom for insulation. The numbers on the lines could be read clearly through the electrolyte and glass.

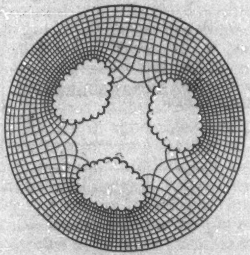

FIG. 9—ISOTHERMAL LINES AND LINES OF HEAT FLOW IN A SECTOR CABLE

4. A chart ruled to be a duplicate of the bottom of tank was used in plotting the readings taken with the exploring needle.
5. Size of model round conductor = 6.70 in. (17 cm.) diameter.
   Size of model cable = 28.0 in. (71 cm.) diameter.
6. Exploring needle = 0.4 mm. platinum wire sealed in end of glass tube.
7. Model conductor represents a 350,000 cir. mil cable, insulation

$$= \frac{7 \times 7}{32} \text{ in.}$$

8. R_1 and R_2 are resistance boxes, R_1 and R_2 = 10,000 ohms.
9. R and $2\,R$, a 3220-ohms rheostat.
10. $K\,R_1$ and $K\,R_2$, a 52-ohms rheostat.
11. The observer's stand and telephone receiver were kept at the same potential by varying $K\,R_1$ and $K\,R_2$. This balance was obtained by disconnecting telephone from exploring needle and connecting it on to the slider of the rheostat $K\,R_1 - R_2$ which was adjusted to obtain a minimum in the telephone receiver.

This prevented current from passing from telephone receiver through the observer, thereby eliminating any error due to the high-frequency current producing an error in balancing with the exploring needle.

12. Balance with needle point was obtained with 300-cycle current. Practical silence was obtained in the telephone on balance.
13. As a bridge method was used, readings are independent of variations of line voltage.

D. W. Roper: Referring to the paper by Messrs. Clark and Shanklin in which they suggest the use of a sectional sheath on two-conductor cables, this may be all right for normal operating conditions but I think it would be a little awkward for some of the abnormal conditions such as the failure of the cable. In the earlier years in Chicago, when the Fisk St. Station had just started and there was no problem regarding electrolysis, the lead sheaths connecting the transmission lines were not bonded together and it was discovered that when we had a transmission line burnout that the voltage of the lead sheath was raised to such an extent that there was arcing and burning of the lead sheath wherever there was actual contact with the manhole brackets or where the sheaths were in contact.

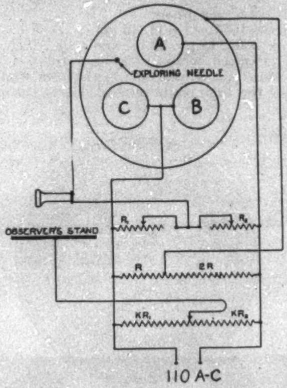

Fig 10—Bridge Connections for Finding Equipotential Lines

It would seem, therefore, very probable that in the case of a burnout of one section of a single-conductor cable with a sectional lead sheath, that this portion of the sheath, in spite of the fact that there was a ground connection, would be raised to such a point as to cause arcing or pitting or perhaps the jumping of the insulation between sections.

The paper speaks of hot spots, apparently referring to hot spots in the conduit due to local heating conditions, but slurs over the fact that there are also hot spots in the cable entirely independent of the hot spots in the conduit. In other words, although a cable appears to be of quite uniform construction throughout, as a matter of fact the construction is not uniform —far from it! So that the dielectric loss, particularly in the resinous compound cables, is far higher at some points than at others, so when you approach the limiting temperature at which the cable can be safely operated or if that temperature is exceeded, then breakdowns in the cable will occur due to the dielectric loss being higher in some points of the cable than in others although the surrounding conditions may be identical.

That might be thought to be a disadvantage but in some respects it might be considered a safety valve because experience has shown that when breakdowns occur at that point due to overloading of the cable, that the damage to the cable is entirely local and that there is no sign of charring or injuring in any way of the paper insulation at other points between the point of breakdown and the source of supply. It is entirely possible that as we get into the use of cables with lower dielectric loss that this question of the local hot spots may not be so important and if that actually results, then it is equivalent, you might say, to screwing down the safety valve that you have with the present compound cables.

The question of heating of the cables and the limiting of the load is an interesting one but as a matter of fact the company with whose operation I am familiar has never yet pulled a switch on a transmission line that was overloaded. The fact that the line is overloaded is an indication that some emergency has arisen so that it is more important than ever that the cable be maintained in service as long as possible and it is their universal practise on such occasions not to pull the switch but to allow the load to be carried until other arrangements can be made to carry the load or until the cable fails and the switch thereon opens automatically.

In the case of the dielectric loss, I think the shape of the curve is just as important as the value of the dielectric loss at some particular temperature. The thing that is desired is that the dielectric loss should not increase at or near the maximum loads which are likely to be carried in emergencies and that the resulting hot spots which result, will not cause breakdowns at too low loads.

With regard to the statement on page 959, I think that is made a little bit too rigid where it says, at the top of the page, "Eddy currents in stranded-cable conductors have been shown both theoretically and by actual measurement to be negligible." This is the case, I believe, for ordinary values of current but under exceptional conditions it is quite possible to get kinetic current loss, which is very important and serious.

We recently had a case of trouble in Chicago, an excess current for some eighteen minutes on a stranded-cable conductor which made up a reactance coil between sections of the bus. That was an eight hundred thousand circular mils, stranded-cable conductor, barrel wound in three layers. After the trouble was over it was found on examination that the top and bottom turns of the reactance coil was still bright but that the turns in the middle had been oxidized by the heating and there were places four inches away from the conductor in the middle of the coil where the varnish on the wood adjacent had been blistered by the heat.

Referring to the diagrams which have been passed around by Mr. Del Mar showing the isothermal lines and the relative heat conductivity of round and sector cables, it would be very interesting if he could add to his discussion some figures which would indicate the relative carrying capacity of round and sector-shaped conductors of the same size.

Philip Torchio: I am very glad to have a paper like the one of Mr. Clark and Mr. Shanklin on record, especially with the further comments which Mr. Shanklin has made that 88,000 volts operating voltage can be used. However, I would like to say a few words on the plan of the paper to prevent confusion and, perhaps, misconstruction of the intent of the writers. I believe that the writers have gone a little too far in showing the advantages of the single-conductor cables.

The comparison is made between a three-conductor cable versus the single-conductor cable, using three three-conductor cables compared with one circuit of three single-conductor cables. Also the comparison is made, I believe, for three-conductor cables, round conductor. If they had used the sector cable they would have had a smaller diameter; also, as the thermal characteristics of the sector cable would be better than those of the round conductor cable, these changes would have, to a large extent, modified the relative values, as arrived at in the paper, of the saving due to the use of single conductors.

In practical application, I would not consider the installation having three independent circuits, each consisting of a three-conductor cable as merely equal to a three-phase circuit consisting only of three single-conductor cables.

I think in the first layout with one cable out you still have two-thirds, with two cables out you still have one-third, but in the single-conductor cables if you lose one you lose the whole circuit.

The object in making these remarks is not in any way to detract from the value and the essence of this paper. The paper gives a limit of construction for three-phase cables of 30 kv., which, possibly, if the sector cable had been considered, Mr. Clark and Mr. Shanklin might have raised to 40 kv., but anyway we are limited in the three-phase cable while we have a much wider range in the single-conductor cable.

That is the essential and important value brought out by the paper. But we must not conclude that perhaps we can accomplish similar results at lower voltages. Although single-conductor cables are used for the lower voltages, as we have been using them quite generally in this country in central station work for wiring up generators to the switchboard, the single-conductor leaded cable has many disadvantages. If you keep the lead sheath open-circuited, high voltages are caused on the sheath and if you ground the sheath then you have large currents flowing in the lead sheath.

Another point is that when you use low-voltage cables where the currents are high, in case of short circuits, the mechanical stresses are enormous. As a matter of fact, in our own generating stations we provide frequent supports, say four feet apart, throughout the run of the cable unless it is in pipe.

Difficulties with these stresses would be serious in manholes if such cables, for general distribution, were not properly planned and properly protected against these stresses.

H. L. Wallau: I have a few figures here that may be of interest. They refer to a test made on two lengths of one million cir. mil conductor, 61-strand, low-tension cable, separated 26 in. (67 cm.) center to center. The cables were in a dry subway and each cable was 1,140 feet (347.4 m.) long. The far ends of the conductors were sweated together, the near ends of the sheaths permanently bonded, and the far ends of the sheaths cross connected through cables with bolted lugs so that the sheath circuit might be opened or closed.

One hundred and twenty volt, 60-cycle current was applied to the conductors. The current in the conductor was 650 amperes. With the sheath circuit closed the sheath current was 225 amperes; with the sheath circuit open the induced voltage was 95 volts. This shows that the induced current for distances of 1,000 ft. (304.8 m.) between bonds is apt to be material and add quite some to the heat generated in the cable.

In fact in our own station, where we have long (500 ft.) runs between generator terminals and switches, the use of leaded cable was found to be objectionable and a cambric, non-leaded, single-conductor cable used instead.

However, we did look into the question of grounding the sheaths of leaded cables at the middle of the run and inserting a limiting resistance in the ground connection, to prevent an abnormal flow of current in case of breakdown.

Such an installation was found would be satisfactory, but in the particular case in mind the resistance required would have been about 0.3 of an ohm and of large physical dimensions. There was no space available in the manholes, for a resistance of this size, so the plan was not carried out.

R. W. Atkinson: The paper by Messrs. Clark and Shanklin contains much of value, as is to be expected from investigators of their standing. It is, therefore, with some hesitation that I venture to point out where they have fallen into error in making some assumptions and in drawing conclusions from their data not borne out by careful investigation. Attention is drawn also to the importance of data already on record concerning matter for which formulas are developed.

From reference to Fig. 16 and Fig. 19, it seems that dissipation characteristics of duct systems have been given and assumed without regard either to the number or kind of cables involved. Fig. 16 is given as showing "representative heating characteristics in conduit lines." For example, in the right hand curve, 10 watts per foot of cable is considered to give 57 deg. cent. rise of temperature above the surrounding soil temperature. No distinction is made, as to the size or construction of the cable or its insulation, and no allowance is made for the fact that much higher temperature rises are found where cables are grouped. It is most unfortunate that data should appear in this form with the weight of the authority of these authors.

Furthermore in the application of these data, as given in Fig. 19 in the comparison of single-conductor and three-conductor cables, conclusions are drawn which are very seriously in error. In Fig. 19, it is stated that a final temperature of 82 deg. or a temperature rise of 57 deg., is attained with a loss per foot of cable of 10 watts, with either of the two described systems. One of these systems consists of three single-conductor cables, the other consists of three three-conductor cables. In order to determine the probable error of the assumption that the temperature rise of the two would be the same, and would be that assumed by the authors, calculation was made from our own data, as to the probable temperature rise, basing these calculations on fairly average duct conditions, and on fairly representative thermal resistivity of the insulating materials. On the basis of the relatively small amount of duct rise which there would be with only one cable instead of three, I calculate that the rise for the three-conductor cable would be 53 deg. and for the single-conductor cable 79 deg. With a greater relative rise of duct structure, there will be less relative difference between the three-conductor and the single-conductor cable, but it is immediately obvious that the particular single-conductor cable will rise to a very much higher temperature for the same amount of loss per unit length, than will the three-conductor cable. Furthermore, where three cables of either type are involved, it seems that the estimate of temperature rise for even the three-conductor cable is too low. Thus, the rating even for the latter is too high and that for the single-conductor cable is not only altogether too high, but is out of proportion to and not comparable to the rating given for the three-conductor cable.

In the last paragraph, beginning on page 951, it is apparently assumed by the authors that where there occurs a loss in the sheath equal to 28 per cent of the conductor loss, this can be fully compensated by an increase of conductor area of 28 per cent. This is erroneous as it will actually require about 39 per cent more copper, as presently will be explained. This is of comparatively little consequence, but on page 953, reference is made to an increase in copper of from 50 per cent to 100 per cent for two certain cables. Apparently these are the two cables on page 943 where about 50 per cent in one case is the increase in loss and about 100 per cent is the increase in loss in the other case. Now if there is 50 per cent loss in the sheath, that loss in the sheath is not going to be affected by changing the size of the conductors. Thus in order to have the same total loss as without the sheath loss, the loss in the conductor must be reduced by 50 per cent, that is its area increased by 100 per cent. Where the sheath loss is 100 per cent, the conductor loss would need be reduced to zero. It seems therefore, that large sheath losses become prohibitive at much lower values than indicated by the authors. The situation is not materially different for very small sheath losses, than is indicated by the authors.

In their sentence at the top of page 937, I believe the authors have underestimated the value of previous work that has been done, in determining the voltage or current, or loss produced in the sheath of single-conductor lead-covered cables. The formulas of Berg, which they mention, are approximate as said, but do give results which apply fairly well within the range considered in the paper now under consideration.

In 1909, in a paper presented at Frontenac by Mr. Fisher, a solution appears which gives results in agreement with the formulas developed by the present authors, within a fraction of one per cent. Of the two formulas which are necessary for this solution, one is identical with the corresponding formula given in the present paper. The other formula is very much shorter and simpler than the corresponding formula which is now given; in fact, it is as simple as the formula of Berg. The simplicity of the older formula is due to the fact that the calculation is made on mean instead of outside diameter, thus eliminating all that part of formula 6, page 962, contained in the bracket following the minus sign. With this omission and the substitution of mean for outside diameter formula 6 becomes identical with the older formula.

Besides the approximation mentioned by the present authors at the bottom of page 959, another approximation was used in deriving the older formula, the assumption that the sheath could be considered as the equivalent of a thin shell of the same resistance and of a diameter equal to the mean diameter of the actual sheath. This latter approximation is even closer than the other. On the whole there seems no warrant for using the long formula on page 962 instead of the older and simpler one.

Solution of these problems of voltage, or current induced in lead sheaths is greatly facilitated by a set of curves presented by the writer in the discussion of a paper by Mosman presented before the Institute in 1913.

A very interesting and important commercial application of single-conductor cables for carrying alternating current is the use by some operating companies, of two, three, or four such cables in a single duct on 2200-volt distribution. Such cables are used instead of multi-conductor cables on account of the greater facility of splicing while the conductors are "alive". It is interesting that the losses with any size of conductor usual for this construction are of small consequence and do not reduce the carrying capacity. That the loss is small is due to the very short spacing. Where the spacing is so short, neither the formulas given by Messrs. Clark and Shanklin, nor the older formulas mentioned above, are strictly applicable. They do serve, however, to show that the losses are comparatively small, and actual measurement of these losses confirms this.

W. A. Del Mar: Referring to the relative carrying capacity of round and sector cables, the curves of heat dissipation which I showed this morning indicate that the heat resistance from the conductor to the sheath of a sector cable is about 10 per cent less than that of a round-conductor cable. The heat resistance from the conductor to the air is made up of two important parts, that from the conductor to the sheath and that from the sheath to the air.

Mr. Atkinson, in a previous paper, has shown that the total heat resistance of a triplex cable is divided approximately in the proportion of two-thirds from conductor to sheath and one-third from sheath to air. Suppose we take the heat resistance from conductor to sheath as $66\,{}^{2}/_{3}$, and from that from sheath to air as 33⅓, a total of 100. If we decrease the heat resistance from conductor to sheath 10 per cent, making it 60, and if we increase the heat resistance from the sheath to air, by about 10 per cent, making it $36\,{}^{2}/_{3}$, which would correspond approximately to the lesser diameter of the sector cable, the total heat resistance would be reduced to about $96\,{}^{2}/_{3}$ per cent. Now the current causing a given temperature rise is inversely as the square root of the heat resistance, so that a simple calculation shows that a sector cable of the type described will have a carrying capacity of something like one to two per cent greater than that of a round-conductor cable. Substantially, they are equal.

E. B. Meyer: The paper by Messrs. Clark and Shanklin on high-voltage single-conductor cables for polyphase installations is a step in the right direction toward solving the problem of underground cable construction, where the transmission of large blocks of power at high voltage is under consideration.

The use of three-conductor cable as pointed out by the authors seems to be limited to about 30,000 volts on account of the properties of the insulating material and the unwieldy size of the cable itself.

Three-conductor cables as at present manufactured are limited in capacity to about 12,000 kv-a. and if the operating voltage is to be increased materially, the use of single-conductor cables appears to be the readiest solution unless some radical improvement in three-conductor cable construction can be effected.

On the other hand, opinion as to the desirability of concentrating heavy loads in a few large cables, is by no means unanimous and as I have some doubt as to the safety of such a course, I would be glad to hear some discussion on this point.

The question with which most central station companies are interested at this time is the problem of reducing failures in underground cables, as with the increase in capacity and extension of electrical systems, disturbances are more destructive, and it will be interesting to know whether by the use of single-conductor cable the maintenance cost per mile of circuit will be materially reduced.

I question whether the maintenance cost of a single-conductor installation would be as low as that of a three-conductor installation.

Probably the most serious objection to the single-conductor installation would be the necessity of splitting up the cable sheath into sections and installing grounded bonds. This would introduce troublesome complications and would probably add materially to the cost of cable maintenance.

I am heartily in sympathy with the authors comments regarding the methods of determining the maximum safe loading of a cable system. It appears that the strictly logical way to arrive at the proper loading of any cable is to determine by actual temperature survey what may be called the constants of each main duct line. With this information available the rating of cables based on maximum allowable operating temperatures is comparatively simple.

A large amount of data is gathered each year by operating companies relating to temperature rises of conduit systems under different conditions of installation and it would be of extreme value to the industry as a whole for such data to be made generally available. It is only in this way that the rule-of-thumb methods of rating cables can be discarded for a more logical method based on an accurate knowledge of conditions.

In Mr. Atkinson's paper on the dielectric field in three-conductor cables, I am pleased to note the author's statement that the maximum stress in sector cable is lower than for a corresponding round-conductor cable, the stress in the outer surface next to the sheath being in all cases much less than for round conductor. This seems to offset the erroneous theory which has been advanced by a number of engineers that sector cable is not desirable for transmission at voltages above 15,000.

It is to be regretted that time did not permit the author to include the subject of dielectric losses in three-phase cables, as this subject is a very live one at the present time. It is known that the magnitude of these losses and their variation with temperature has a very marked effect on the operating characteristics of the life of a cable, This subject is a most vital one and is receiving considerable attention by a number of central station engineers and while it appears that no fixed rules can be formulated at present, it is desirable that this subject be given careful consideration.

G. B. Shanklin: Mr. Del Mar's discussion of our paper is much to the point. I am in full accord with the sentiment expressed in his belief that future development will result in raising the voltage limit for multi-conductor, cable possibly as high as 40,000 volts. This goal may ultimately be reached, but it is a development that must be approached with caution. It is one of those developments whose factor of safety cannot be even approximately determined until it is actually tried out under operating conditions, at least with the knowledge we have at present.

For some time past our laboratory has been investigating dielectric losses in three-conductor cable, hoping to throw some light on the causes of the excess loss and deterioration in the central triangle of the cross section, a factor that plays a big part in limiting the voltage rating of this type of cable. Our work has consisted mainly of comparing actually measured 3 ϕ losses with those calculated from measurements with 1 ϕ voltage. These measurements were made on cables whose cross section were at uniform temperature. Now, the only difference between 3 ϕ and 1 ϕ stresses occurs in the central triangle and if the loss in this section was an appreciable part of the total then it would be expected that the 3 ϕ readings would give higher values than the 1 ϕ calculations. We have tested cable from the very best grades to those so poor in quality that they could hardly be classed as insulated cable but we have never found any difference between the measured and calculated 3 ϕ losses, provided the temperature conditions were exactly the same.

I have come to the conclusion that the generally accepted view of an inherent excess 3 ϕ loss in this central triangle is, in the sense as usually interpreted, not strictly true. The deterioration of this section under actual operating conditions is found only infrequently and in extremely poor grades of cable. This leads me to believe that it is primarily caused by an accumulative heating action that renders the central triangle much hotter than the rest of the cross section, and that without this accumulative heating action there is no more deterioration in the central triangle than in the rest of the cross section.

With the cross section at uniform temperature we have never found any indication of ionization starting in the central triangle at a voltage lower than that required to start it at the surface of the conductors, where the stress is greatest. It is probable that when the central triangle becomes excessively hot there is a tendency for the stresses to re-distribute themselves, and in doing so to concentrate at certain points, causing, ionization at a lower voltage than otherwise. This, of course would cause deterioration but it should be classed as an effect of the accumulative heating mentioned above and not as a cause.

It is consideration of these factors that encourages me to believe that higher voltage ratings of three-conductor cable will ultimately be reached. The main requirement will be the lowest possible dielectric losses, generally throughout the cable cross section, but particularly in the central triangle. The fact remains, though, that we are rapidly approaching the upper limit of voltage rating for three-conductor cable. Whether the final goal will be 35, 40 or even 45 kv. we must if we wish to go beyond this goal resort to single-conductor cable.

Messrs. Fisher, Del Mar, Atkinson and several other gentlemen have criticized the limiting stress of 19.5 kv. per cm. recommended in our paper. I will confess full responsibility for this recommendation. Mr. Clark has always been skeptical of this limit and agreed to its inclusion in the paper more for the sake of having a target to be shot at than anything else. He thinks it ultra-conservative in view of the fact that three-conductor cables have in some cases operated satisfactorily for years at maximum stresses as high as 28 kv. per cm.

Different types and makes of cable, like all other apparatus, have their own individual merits and peculiarities. It was a mistake, perhaps, to select an arbitrary value of maximum stress and I feel that an explanation is due. In the first place, it was not proposed for general application and I think it was made clear in the Institute paper, "Ionization of Occluded Gases in High-Tension Insulation," page 489, that it represented a *minimum* value.

We have tested many types of cable and have found that their limiting safe stress as far as ionization is concerned, has varied from 10 to 40 kv. per cm. The reasons for selecting 19.5 kv. per cm. (which is only an average of the range from 19 to 20 kv. per cm., as first recommended) were:

a. A start had to be made some where. It is difficult to form a judgment of performance without some criterion or basis to estimate from.

b. As far as we could judge our data seemed to indicate 19.5 kv. per cm. as a fairly good dividing line between acceptable and unacceptable grades of paper cable, that a cable showing ionization at an appreciably lower stress than this could hardly be considered as satisfactory.

c. When a cable is highly stressed we have not only ionization to consider but also high dielectric losses with consequent reduction in safe current carrying capacity and greater possibility of accumulative heating at "hot spots." This is the more pronounced the higher the voltage rating and certainly when we approach 40-kv. ratings we must pay more attention to this factor than we have in the past. Quite often it will be found more economical to operate a high-tension cable at a stress of, say, 19.5 kv. per cm. than at 28 kv. per cm. because of the greater carrying capacity and factor of safety. When the thickness of insulation is increased the question of heat dissipation introduces another factor. Obviously, the economical balance between dielectric and copper losses, heat dissipating ability of the cable, and possibility of ionization will be governed by the grade of insulation, the thermal conditions of the duct line and the relative costs of the materials used in the cable. No set rules can, therefore, be applied.

We agree with Mr. Roper in his criticism of sectionalized sheathing for single-conductor cable. He is right in stating that it would, "be a little awkward for some of the abnormal transient conditions, such as failure of the cable." Throughout the paper we have tried to make it clear that short-circuited continuous sheathing is much superior to sectionalized sheathing.

It is quite difficult to answer the many points in Mr. Atkinson's discussion. He has apparently misintrepreted and taken issue with practically every thing in our paper. Some of his criticisms are really helpful and add to the paper in throwing light on a very difficult problem. I will attempt to answer a few of the main issues.

Fig. 16 is given as showing representative heating characteristics in conduit lines. Mr. Atkinson does not think Fig. 16 is representative and says: "No distinction is made as to the size or construction of the cable or its insulation, and no allowance is made for the fact that much higher temperature rises are found where cables are grouped." We purposely avoided, for the sake of simplicity, any detailed discussion of the theory of heating in duct systems, but simply assumed the thermal conditions as such that Fig. 16 held for our particular problem. Incidentally, Fig. 16 is, as far as our experience shows, fairly representative of the heating conditions met with in the average duct system. I believe our assumptions would have been made clearer to Mr. Atkinson if we had represented the temperature in Fig. 16 as that of the duct air and then added on the temperature drop through the cable insulation but that would have involved complications that would have made our paper uselessly long.

The greater part of Mr. Atkinson's discussion is a detailed criticism of our assumption that the temperature drop through the insulation of the single- and three-conductor cables compared in Fig. 19 is approximately the same. Now, anyone who studies this problem carefully can see that it is impossible to deal with it theoretically unless certain approximations are accepted. To begin with it must be assumed that the cable cross sections are at uniform temperature, that is, the copper and lead sheath are at the same temperature. Obviously, this assumption gives approximately correct results only over that range of heating conditions where the cable takes up only a small part of the total temperature drop. In water-cooled or forced air-cooled duct systems or with buried cable it would not, of course, apply very well but in the ordinary still-air duct systems, and especially at "hot spots" where poor heat conductivity of the soil is encountered, it applies much better.

I am sure that our assumptions have not led to any such grave errors as Mr. Atkinson's discussion would seem to indicate. For instance, he states in reference to Fig. 19 and cables (a) and (c), "assuming a loss of 10 watts per duct foot for each of these two cables and fairly average duct conditions, I calculate that the rise for the three-conductor cable (a) would be 53 deg. cent. and for the single conductor cable (c) 79 deg. cent.

Now, let us see if his calculations are correct. Instead of assuming "fairly average duct conditions" let us assume that the total temperature drop is between copper and lead sheath, as would be the case if the cables were immersed in water. For the same heat dissipation the temperature drops of the two cables will be inversely proportional to their thermal conductances between copper and sheath. The thermal conductances are directly proportional to the electrostatic capacities so we need only to find the ratio between their electrostatic capacities. This is easily obtained from Formula **6** (bearing in mind that the total capacity between copper and sheath is 3 C_1), and the ordinary formula for concentric cylinders. The ratio found in this way is;

$$\frac{\text{thermal conductance of cable (a)}}{\text{thermal conductance of cable (c)}} = 1.40$$

In other words, the three-conductor cable has 40 per cent better heat conductance. With the cables immersed in water and dissipating equal quantities of heat let us assume the drop through the three-conductor cable as 53 deg. cent. The drop through the single-conductor cable will be, $53 \times 1.4 = 74.2$ deg. cent. This agrees so closely with Mr. Atkinson's value of 79 deg. cent. that it shows the fallacy of his assumption of "fairly average duct conditions."

Under average still-air duct conditions the drop from copper to sheath can be roughly assumed as 20 per cent of the total drop. Then, if the total drop of cable (a) is 53 deg. cent the drop between copper and sheath is 10.6 deg. cent. With cable (c) in a similar duct and dissipating an equal quantity of heat the total drop will be 57 deg. cent. and the drop between copper and sheath 14.6 deg. cent. The difference is only 4 deg. cent. and our assumptions, therefore, hold reasonably well.

Messrs. Torchio and Meyer both express the same thought in a different way in their comparison of the merits of single- and three-conductor cable. It is true that in the case of single-conductor cables the failure of one cable throws three times the carrying capacity out of commission than in the case of equivalent three-conductor cables. It is partly for this reason that single-conductor cable shows promise of the greatest usefulness only where there are such large blocks of power to be transmitted that the temporary loss of three cables instead of one will not prove serious.

R. W. Atkinson: This use of paper filler instead of jute has been mentioned. This, however, can only partly overcome the weakness of that portion of the ordinary three-conductor cable which is occupied by such filler. This is because paper is better than jute to an important extent, only for stresses normal or nearly normal to the surface of the paper. Most of the paper of the filler will be subject to stresses tangent or nearly tangent to its surface, and thus the important weakness of the jute filler is also shared by the paper filler.

Mr. Del Mar takes exception to the statement concerning the proportionality between charging current (volt-amperes) and dielectric loss. It is worthy of special comment that this proportionality exists only where the dielectrics are actually identical in properties. There may exist wide variations of the dielectric properties though the same grade of paper is saturated with compound of the same ingredients. It should further be noted that such a dielectric as paper will have different properties according as to whether the stress is perpendicular or parallel to the direction of the layers.

Also if the stresses in any specimen are beyond the range where the losses vary as the square of the voltage, then as indicated by Mr. Del Mar, that specimen cannot be compared directly with another specimen having lower stresses.

Study of the effect of these factors is facilitated by the data reported in the paper of the author. That is, effects of change of shape and dimensions may be differentiated from effects of differences in the properties of the material.

Mr. Roper has described an exceedingly interesting and important viewpoint concerning emergency overloading of cables. I refer to the fact that his company has never yet pulled a switch on a line because of overload. His arguments against doing that particular thing are very forceful and it may be recognized that the operating engineers have a real problem to solve to take care of such emergencies.

I wish to point out however that unless some provision is made, high-tension cables, as compared with most other electrical apparatus, are made to occupy an extremely hazardous position. Thus, a generator (or the turbine which runs it) sets for itself a very definite limitation of load which cannot be exceeded. To some extent most other apparatus does this in one way or another. Thus the low-tension cable does so by voltage drop; or its short-time overload capacity may be greater than that of its switches, unless these are very liberally rated. But a high-tension cable with very low dielectric losses will have no limitation short of destruction of the cable. Thus, if the policy be carried to the limit, we have, to use Mr. Roper's simile, a case of firing a boiler without a safety valve and with no regard to the steam gauge.

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PROBLEMS OF 220-KV. POWER TRANSMISSION

BY A. E. SILVER

ABSTRACT OF PAPER

The dependence of industrial progress upon an adequate supply of electrical power, together with the vital need for a rational policy of conservation of the country's fuel resources, points to a probable early demand for transmission of large blocks of power from distant energy sources—coal fields and water powers. Two hundred and twenty kv. is suggested as a logical voltage for such high-capacity, long-distance transmission, and the important problems introduced by large concentrations of power, high voltage and high service standards are discussed. The economic and technical considerations underlying design of a 220-kv. system are outlined, and general designs are developed for a typical 220-kv. transmission line.

The studies made establish confidence in the conclusion that 220-kv. transmission is feasible as an immediate commercial proposition. Established principles of design and present types of equipment, with proper adaptation to the new conditions, are applicable to 220-kv. service. While all essential problems seem assured of acceptable solution, attention is directed to certain points as to which further investigation and experimental research are needed to determine most effective designs.

It is hoped that the paper may in some measure aid in the working out of this advance in the art by promoting constructive discussion and stimulating the needed investigations.

INTRODUCTORY

ECONOMIC development of the country's power resources is fundamental to the industrial progress of our national life. The growing recognition by our economists, engineers and industrial leaders, and even by the public, of the significance of the power supply problem must soon result in an insistent demand upon the engineering profession for large and bold strides in the solution of the attendant engineering problems.

The vital dependence of industrial advancement upon an adequate power supply has been strikingly illustrated in our struggles to meet the industrial demands of the war emergency. The experiences of this trying period have resulted, also in a quickened understanding of the need for conservation of our natural power resources. The labor and commodity situation accentuates the demand for greater unit productiveness, possible only through greater use of power in industrial operations, and for prompt progress toward broadly economical methods of power supply. A major step toward conservation of fuel resources and general commercial economy is recognized to lie in gradual substitution of electric transmission of energy for railroad transportation of fuel to be used in local generation of power. Interconnection of electric power systems, both prior to and during the war, has demonstrated the marked benefits obtainable by taking advantage of diversity of loads and other attendant economies and the well established movement toward general and extensive interconnection is still gathering headway. The relation of the railroad electrification problem to fuel conservation is well recognized and for years has been consistently expounded by able engineers.

These influences lead directly toward large scale development, at the source, of our natural energy reservoirs, the coal fields and the potential water powers. An essential feature of such development will be the transmission electrically of great blocks of power, over increasingly great distances, to strategic centers of distribution in or near present and future industrial districts. This trend of the power industry has long been recognized, and important steps toward its realization are a certain and not distant eventuality. Further elaboration of the subject is unnecessary to the purposes of this paper.

THE FIELD FOR 220-KV. POWER TRANSMISSION

Visualization of the demands of this evolution points to the need of trunk electric transmission service of a capacity and range of greater magnitude than thus far developed or required. Increasing distances and increasing quantities of power require an increasing voltage for economic transmission. The quantities and distances involved in the broad field outlined in the preceding paragraph pass beyond the economic range of existing transmission voltages. The practical working out of this next step in the transmission art is a problem now definitely facing the engineer and manufacturer.

Scarcely more than ten years ago an operating voltage of 100 kv. was remarkable; today 130 kv. may be considered standardized and 150 kv. is in sufficiently extensive use to have established its reliability.

Two hundred and twenty kv. appears a logical choice for the next step in the transmission voltage schedule. Why 220 kv., it may be asked? Because, from an appraisal of the general situation, a voltage of this order is considered adequate for the immediately pending needs of the industry and commensurate with expected growth in transmission service demands for a considerable period. Its suitability to a variety of conditions and the probable extent of its use would assure it a place in the schedule of transmission voltages which commercial needs are rapidly standardizing. Furthermore, such a voltage, while representing a step beyond present usage sufficient to afford a distinct economic advantage, does not reach so far into uninvestigated fields but that the problems of development and design can be approached with full confidence of early commercial solution.

The particular numerical value of 220,000 is in accord with the well established practise of standardization in multiples of 11,000. In some instances, it may be an incidental convenience that this voltage is the double of the extensively used 110 kv.

An illustration of the advantage for long transmission distances of 220 kv. over the highest present system voltage, 150 kv., is given in Figs. 1 and 2. This comparison is based upon a transmitted load of 500,000 kw. The same relative advantages will obtain for larger loads, and, above a certain minimum, for smaller loads.

The field of 220 kv. is not broad. Its economic application is primarily to large blocks of power and long transmission distances. It is in no sense a panacea for transmission problems generally. It presumably will infringe to some extent upon the present fields of the lower transmission voltages, but will by no means tend to supersede their use, in fact it will considerably enlarge the field of usefulness of such secondary transmission voltages as 66 kv. and 110 kv. It is not a universally suitable medium for extensive interconnection of power systems.

Power from steam-electric stations in the coal fields or from large hydroelectric stations would advantageously be transmitted over 220-kv. lines to terminal substations at important load centers or at the hubs of secondary transmission networks serving industrial areas. The introduction of transmitted power, in amounts limited only by the load demands, will constitute a strong stimulus to expansion of these networks.

Power equalization between load centers might frequently best be accomplished through extension of these secondary transmission lines. Interconnection at 220 kv. would be expected only where the equalizing duty reaches a large magnitude, where there is no existing secondary transmission system suitable to serve as a basis for inter-connecting lines, or where interconnecting 220-kv. lines might function also as a supplementary or important reserve link in a main 220-kv. trunk transmission system.

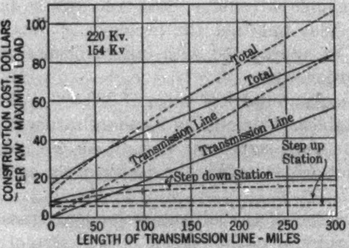


FIG. 1—ECONOMIC COMPARISON OF 154-KV. AND 220-KV. TRANSMISSION—CONSTRUCTION COST—SEVEN LINES 154 KV. AND FOUR LINES 220 KV.—500,000 KW. DELIVERED

Costs include those of lines, step-up and step-down substations and synchronous condensers.

Costs of line per mile 154 kv. $20,000, 220 kv. $23,500—all costs based on early 1919 prices.

Size of conductor 92,900 cm. steel—716,000 cm. aluminum.

Voltages high side of transformers, receiver end 150 kv. and 200 kv., sending end 170 kv. and 225 kv.

To a marked extent, and especially in the earlier stages of its introduction, power transmitted at 220 kv. will be high load-factor power. The initial investment in a 220-kv. system, including as essential elements the step-up and step-down stations, will be of such magnitude that there will be a strong inducement to utilize the investment as nearly continuously as practicable, thus reducing the unit transmission cost of energy supplied. The natural economic tendency in introducing power transmitted from distant energy sources will be to supply base load, leaving the peak loads to existing local generating stations. It is to be expected that high-voltage transmission from energy sources, fundamentally economic as it is believed to be, will in general for years contribute only a part of the power supply of any district. The relegation of existing stations to partial operation or reserve service will be gradual, and, even when transmitted power becomes the main reliance, presumably it will usually prove more economical to maintain local stations for reserve and short peak load service than to provide necessarily expensive transmission capacity for such short periods of actual use.

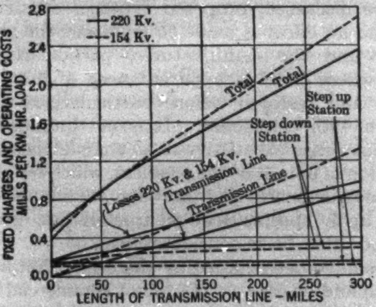


Fig. 2—Economic Comparison of 154-kv. and 220-kv. Transmission—Fixed Charges and Operating Costs—Seven Lines 154 kv. and Four Lines 220 kv.—500,000 kw. Delivered

Costs and losses include those of lines, step-up and step-down substations and synchronous condensers.

Costs of line per mile, 154 kv. $20,000, 220 kv. $23,500—all costs based on early 1919 prices.

Size of conductor 92,900 cm. steel—716,000 cm. aluminum.

Losses based on 0.95 load factor and 0.85 power factor delivered load.

Cost of energy 5 mils per kilowatt-hour.

Fixed charges and operating expenses of transmission lines 13 per cent, substations 15 per cent.

Voltages high side of transformers, receiver end 150 kv. and 200 kv., sending end 170 kv. and 225 kv.

A characteristic of the field of 220-kv. transmission which exercises a determining influence upon designs and costs is that it is a field of high service standards. A 220-kv. system, with the generating stations for which it would be the outlet, would represent a tremendous amount of power. The economic importance of reliability and continuity of this power, in view of the great volume of industrial enterprises and public utilities which would be dependent upon it, is of so high an order that new standards of care in design and conservatism in construction are imposed. The aim, and a not unrealizable aim, is to make high-voltage, large-capacity transmission for all practical purposes equal in dependability to local generation of power.

SCOPE OF PAPER

Based upon such a conception of the field of 220-kv. transmission, it is the aim of this paper to carry through an analysis of a typical transmission problem, sufficiently specific as to assumptions to afford a basis of designs, but not limited to any exact geographical location. Conditions underlying designs are analyzed, certain salient features of the design of lines and apparatus are discussed, proposed types of construction are considered and an effort is made to point out the problems of the designer and the manufacturer, particularly in those applications where there is apparent need of more thorough investigation and research before particularized conclusions can be drawn with confidence. It is not intended that the paper be in any sense exhaustive as to completeness of conclusions or solution of details of the problem, which for any specific case will involve extensive and thorough studies and investigations based upon specific load characteristics, geographical conditions and other basic considerations.

The studies have been guided by the desire that any recommendations should be capable of prompt commercial execution. In other words, it has been the aim to outline a type of construction built up essentially of established factory products in such way as to insure initially successful results. At the same time an attempt has been made to point out the short-comings which must be tolerated and the apparent opportunities for more efficient solutions.

It is the desire and hope that this tentative development of the problem in outline will afford a basis and incentive for full and constructive discussion by those interested in the advancement of the art of power transmission. It is hoped also that it may encourage designing engineers and manufacturers to undertake needed investigations into insufficiently explored fields as the foundation for developing designs for suitably improved apparatus and line materials. There would seem to be promising opportunities for distinct and beneficial departures from prevailing practises.

DESIGN FEATURES OF 220-KV. TRANSMISSION LINES

General Assumptions. Before considering the specific designs which are suggested for 220-kv. transmission lines, a brief statement will be made of the basic underlying assumptions as to loads to be transmitted, as to frequency and type of system to be adopted and as to climatic loadings and corona conditions to be assumed.

As to amount of load to be transmitted, it is assumed that even an initial 220-kv. system would be laid out on a basis of two or more main generating stations connected to load centers by a number of circuits. The load per 220-kv. circuit has been assumed to be from 100,000 kw. to 125,000 kw. A lower load per circuit than 100,000 kw. would entail a considerable sacrifice of the economy obtainable through use of 220 kv. No discussion is offered as to the maximum economic load per 220 kv. circuit, *i. e.*, as to the point above which additional circuits should be provided, since in any initial system the number of circuits would be determined from considerations rather of reliability insurance or load distribution than of maximum inherent economy. Where the studies involve a specific transmission distance, 250 miles has been assumed for purposes of illustration. The economic range of 220 kv. as to distance is very large.

The frequency of a 220-kv. trunk transmission system should be 60 cycles. For a general power distribution system furnishing lighting and power service in cities and industrial centers, the superiority of 60 cycles over 25 cycles has been well established, and for a transmission system delivering power to such a distribution system, the decision as to frequency unquestionably must follow the requirements of the load. In some districts, for which 220-kv. transmission may come up for consideration, it presumably will be found that both 60 cycles and 25 cycles are in use, possibly that 25 cycles in amount of load is still predominant. Even in these cases, however, the tendency in new development will be found away from 25 cycles, and it would be serious economic error to compromise so important an undertaking as a 220-kv. transmission system with a frequency approaching obsolescence.

High-capacity long-distance transmission will inevitably play a significant part in electrification of main line railroads. In instances where alternating-current electrification is an important factor in a project, there may be greater seeming inducements in favor of 25 cycles, but even in such cases it is believed that consideration of the general and industrial portions of the load, of the probable tendency of future growth and of the trend toward wide-spread interconnection will lead to a decision for 60 cycles.

From the standpoint of line performance alone, use of the lower frequency would afford better conditions of voltage regulation, but with synchronous condensers, which must be regarded as an integral part of a modern high capacity transmission system, the problem of satisfactory regulation at the higher frequency is not serious. Incidental considerations, none of them relatively important, are greater cost of 25-cycle equipment, lower reactance of a 25-cycle system and hence, heavier circuit breaker duty, and lower corona loss for 25 cycles.

The step to 220 kv. will by no means be the ultimate development in power transmission. Eventually a demand may be expected for transmission capacities and distances beyond the economic range of this voltage, and the advance then to be evolved may conceivably abandon 60 cycles for some very low frequency, for direct current or for some other revolutionary change in the transmission art. Such remote eventualities, however, do not affect the considerations governing the step to 220 kv.

Transformer connections, at all installations, should be grounded Y for the 220-kv. windings. The question of Y *vs.* delta connections has in the past evoked considerable discussion for each new undertaking, whatever the specific requirements of the system, and debate may arise in connection with projected 220-kv. transmission. Modern thought and experience shows a consistent tendency toward the use of grounded Y connections over the whole range of higher voltages. For a 220-kv. system, in addition to a distinct gain in dependability of operation, the requirements for line and equipment insulation, with their attendant effect upon the size and cost of equipment, particularly transformers, gives the grounded Y arrangement a marked advantage.

The simultaneous conditions of maximum climatic conductor loading used as a basis for line design have been assumed as follows:

a. Wind pressure of 8 lb. per sq. ft. of projected area, corresponding to an indicated wind velocity of 72 mi. per hr.;

b. Ice of 1½ inches radial thickness on all wires;

c. Temperature of 0 deg. fahr.

For checking clearances of conductors a maximum temperature of 120 deg. fahr. has been assumed.

These loadings are, of course, far in excess of those used as a basis for ordinary transmission line design. It is not believed that there is any reasonable likelihood of the line being subjected to such a simultaneous combination of conditions. From the standpoint of vertical tower loads; there is a real possibility of occasional ice loadings as heavy as or even heavier than assumed, and accordingly a reasonable margin of excess vertical strength is called for. Justification for these apparently extreme loading assumptions is found in the high service standards which, as noted, must in general apply in 220-kv. service. In view of the economic importance of continuity of service, it is considered that this basis is not unreasonable for a region where severe climatic conditions obtain, and in particular where heavy sleet is to be expected. Where climatic conditions are more mild, a lighter loading basis should naturally be assumed. In the planning of any particular project, this question of assumptions as to line loading, and the underlying economic conditions, should receive most careful consideration.

Corona formation and corona loss enter as a significant factor in the design of transmission lines at this high voltage. The conditions assumed for this study are:

a. Average elevation, 1000 ft. (normal barometer 28.9 in);

b. "Storm factor", or percentage of time during which conductors would be subject to increased corona losses due to rain, snow, sleet or fog, 12.5 per cent;

c. Conditions during "storm" periods, 28.4 in. barometer, average temperature 55 deg. fahr.

Corona conditions likewise will require special study for each particular installation, particularly the matter of "storm factor." The ordinary source of data will be Weather Bureau records. The published reports, however, while complete as to amount of precipitation and number of days of which precipitation occurs, do not give in summarized form the actual duration of storm conditions. A reasonably correct value of the factor for any district may be obtained from study, extending over a considerable period, of the original records of individual storms at several stations in the district. The dividing line between "storm" and "fair" conditions is, of course, not clearly defined, and hence preciseness of results is difficult to attain.

Conductor Materials and Types. Apparently the choice of conductor materials and types for 220 kv. service is limited, at least from the standpoint of immediate availability, to three alternatives,

1. Aluminum with steel core.
2. Copper with steel core.
3. All copper.

Aluminum without steel core does not possess sufficient mechanical strength for use on the span lengths in current use for high-voltage transmission. From the standpoint of mechanical strength, copper may be used either with or without steel core. A copper cable with a core of some other type than steel strand is a possibility; hemp cores have been used, but experience, while inconclusive, appears to indicate that there may be injurious chemical action; a semi-hollow copper cable with internal spacers of wood or metal has been suggested, but its feasibility has not been demonstrated and there seem to be no real benefits. Such a dead weight "filler" decreases the effective strength of the cable.

As to the satisfactory performance of steel cored cables, there appears to be but one point open to question; *i. e.*, possible electrolytic action between the galvanizing coat of the steel and the main conductors. Composite aluminum cables have been coming into increasing use during recent years, and from the experience gained, there is growing assurance of freedom from electrolytic action which would materially impair the durability of the cable. Aluminum and zinc are not far separated in the electro-chemical series, and aluminum is electro-positive to the zinc, so that there is less reason for anticipating trouble. There has been less experience with copper cable with galvanized steel core, but theoretically the conditions are somewhat less favorable, since copper is farther separated from and electro-negative with respect to zinc. It may be noted, however, that no injurious action has been observed in the extensive use of galvanized fittings with copper cable. Further experience with such composite copper cables is awaited with interest. In case injurious electrolytic action is established with galvanized cores, there appear to be a number of possible remedies, such as the use of copper plated steel cores. A sufficiently heavy copper plating would, however, be more expensive than galvanizing.

In general, for equal conductivity, the relative physical advantages of the three types of conductors may be summarized as follows. Certain features noted will later be discussed in more detail.

Aluminum-steel vs. copper-steel

1. Less corona loss, due to larger diameter
2. Skin effect presumably approximately equal
3. Greater area exposed to wind loading, hence, greater transverse tower strength and greater clearances required
4. Less tensile strength, hence more limitation upon height and spacing of towers
5. Less weight (unimportant)

All-copper vs. composite cable

1. More corona, greater than either of above
2. All material effective as a conductor
3. Skin effect more serious in larger sizes, owing to high priced material in the core
4. Less area exposed to wind loading
5. Less total tensile strength than either of above
6. Less weight than copper-steel
7. Homogenity of material, hence certain advantages in construction, more positive assurance of durability and higher scrap value

It will be seen that aluminum-steel, due to corona limitations, has a greater relative advantage for smaller line loads, which economically require smaller sizes of cable than for larger line loads.

For any particular case and any given magnitude and character of load to be transmitted, the choice of conductor type will depend upon a complex economic balance between the cost of materials involved, the losses due to resistance, corona, and skin effect, and the mechanical characteristics of the cables. Different loads may call for different types.

For the purpose of developing tower designs, a cable of 716,000 cir. mils of aluminum and 93,000 cir. mils of steel has been used in the studies which follow, the considerations upon which this selection was made will be discussed later.

Electrical Characteristics of Conductors. At such a high voltage as 220 kv., one of the primary considerations in selecting size and type of conductor is corona formation and corona loss in its relation to conductor diameter. The subject has been extensively treated by Peek and others before the Institute and the methods of calculation outlined by these authorities have been employed in the studies which follow. In these methods of analysis, it is necessary to assign values to the principal factors affecting corona. The physical and climatic conditions assumed have been stated earlier. The conductor arrangement has been taken as a flat horizontal configuration

TABLE I

CORONA LOSS DATA ON CONDUCTORS FOR 220-KV. CIRCUITS 20-FT. HORIZONTAL SPACING
(See Fig. 3)

	Aluminum-Steel				Copper	Copper-Steel			
Conductor Designation (See Table 2)	A	B	C	D	E	F	G	H	K
Dia. of cable—Ins.	0.952	1.036	1.092	1.196	0.814	0.857	0.904	0.946	1.004
Fair Weather Disruptive Critical kv. (e_0)									
Center wire	129	139	146	157	114	118	124	129	135
Outer wires	142	154	161	174	126	130	137	142	150
Storm Disruptive Critical kv. (e_0)									
Center wire	103	111	117	126	91	94	100	103	108
Outer wires	114	123	129	139	100	104	109	114	120
Total kw. Loss—3 Wires—250 Mi. (100,000-*kw. Circuit*) *Storm Factor* 12.5 *per cent*									
60 per cent load factor	464	162	66	16	1986	1253	758	493	254
75 per cent load factor	425	141	57	12	1835	1152	694	452	227
90 per cent load factor	378	119	47	8	1664	1032	620	404	195

with conductors separated 20 ft. Proper allowance has been made for the unbalanced disruptive critical voltages due to this horizontal configuration. The conductor irregularity factor, taking into account the effect of weathering of the conductor with age and the irregular surface resulting from stranding, has been assumed as 0.87.

In Table I are shown, for a number of conductor sizes, the fair weather and storm disruptive critical voltages and the total corona loss for 250 miles of circuit. Corona losses for various load factors are shown graphically in Fig. 3. In determining the values of line voltage to be used in corona calculations, effective r. m. s. values of load were obtained from hypothetical daily load curves prepared for various load factors, and the mean voltages corresponding were determined from typical line regulation curves (Figs. 4, 5 and 6). It is believed that this method of analysis affords a reasonably and sufficiently accurate estimate of corona losses.

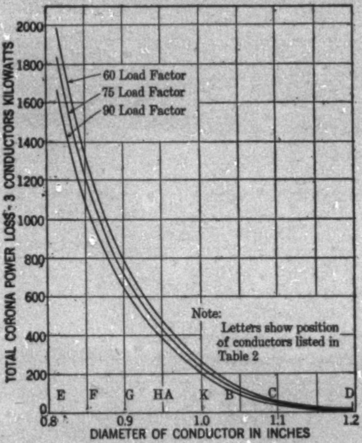
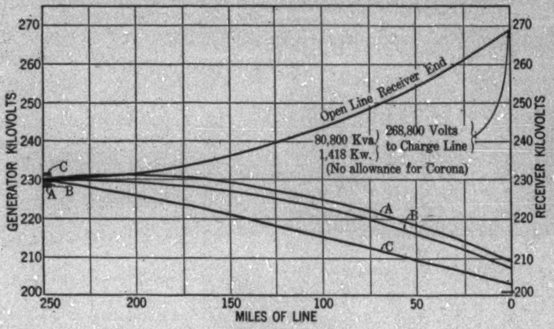


FIG. 3—220-KV. CORONA POWER LOSS FOR 250-MILE CIRCUIT (SEE TABLE I FOR EXPLANATORY DATA)
GENERATING VOLTAGE HELD CONSTANT AT 230 KV.

The electrical characteristics of composite cables, other than as regards corona, are somewhat more complicated of analysis than for a cable of homogeneous material, particularly as to the effects of the steel core on the internal inductance and effective resistance of the conductor. In the preparation of these studies, access has been had to data from certain experiments recently conducted in regard to these effects. It is understood that the investigators who have been carrying out these experiments anticipate presenting their findings in some detail to the engineering profession at an early date. It may be stated that this investigation indicates that in the practical



RESULTS

Curve	Rec'r kw.	Cond'r kv-a.	High voltage				Equiv. low voltage gen.		Losses (excl. corona loss)	
			Receiver		Generator					
			kv.	% p. f.	kv.	% p. f.	kv.	% p. f.	kw.	%
			250 mile line							
A	0	*49,800	209.0	* 0.18	230.0	† 4.61	227.9	† 4.69	2,646	..
B	50,000	† 7,500	207.2	*78.00	230.0	† 96.15	228.1	†97.28	2,780	5.6
C	100,000	†81,000	203.0	*98.72	230.0	*100.0	231.3	*99.67	11,927	11.9

NOTE: *Indicates lagging quantities.
†Indicates leading quantities.

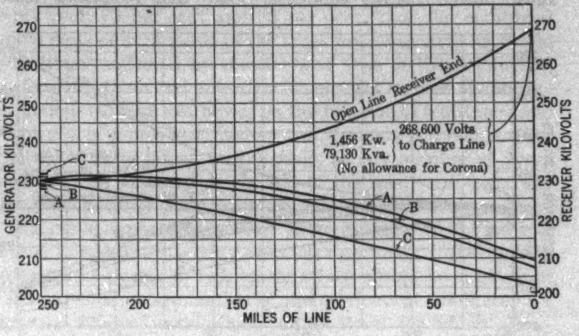
FIG. 4—220-KV. TRANSMISSION LINE CHARACTERISTICS—RELATIONS BETWEEN KW., KV., P. F., AND CONDENSER KV-A.—CONDUCTOR—716,000 CM. ALUMINUM—92,900 CM. STEEL

CALCULATION ASSUMPTIONS

Steel core aluminum conductor—716,000 c. m. aluminum and 92,900 c. m. steel—20 ft. horizontal spacing
Constant receiver volts—200,000 (low side receiver transformer, includes transformer drop).
Constant generated volts—230,000 (high side generator transformer)
Loads—delivered at 75 per cent power factor
Transformer bank of 50,000 kw. capacity with—resistance 0.5 per cent, reactance 12.0 per cent
Two banks of transformers in parallel at each end of line.
Losses include those of line, transformers and synchronous condensers

application of inductance and resistance formulas and calculations to transmission line studies, the effect of the steel core may be neglected.

Skin effect assumes appreciable proportions for the large sizes of conductor called for by 220-kv. transmission, particularly in case of the larger sizes of all-copper cable. The calculation of skin effect in composite cables is somewhat more complicated and burdensome than in the case of a homogeneous conductor.



RESULTS

Curve	Rec'r kw.	Cond'r Kv-a.	High voltage				Equiv. low voltage gen.		Losses (excl. corona loss)	
			Receiver		Generator					
			kv.	% p. f.	kv.	% p. f.	kv.	% p. f.	kw.	%
				250 mile line						
A	0	*49,000	209.0	* 0.18	230	† 5.48	227.9	† 5.56	2,704	..
B	50,000	† 7,600	207.2	*78.24	230	†96.89	228.6	†97.70	2,691	5.4
C	100,000	†81,600	202.9	*98.80	230	*99.97	231.7	*99.51	11,843	11.8

NOTE: *Indicates lagging quantities.
†Indicates leading quantities.

FIG. 5—220-KV. TRANSMISSION LINE CHARACTERISTICS—RELATION BETWEEN KW., KV., P. F. AND CONDENSER KV-A.—CONDUCTOR—450,000 CM. COPPER—308,200 CM. STEEL

CALCULATION ASSUMPTIONS

Steel core copper conductor—450,000 c. m. copper and 308,200 c. m. steel—20 ft. horizontal spacing.

Constant receiver volts—200,000 (Low side receiver transformer—includes transformer drop)

Constant generated volts—230,000 (high side generator transformer)

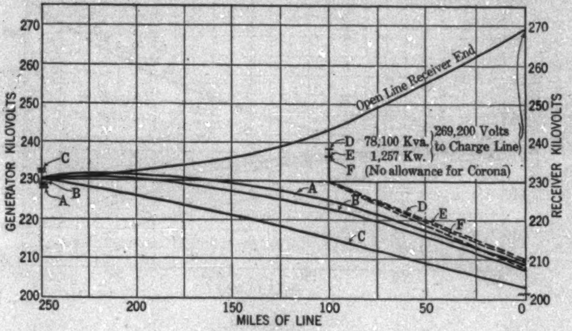
Loads—delivered at 75 per cent power factor.

Transformer bank of 50,000 kw. capacity with—resistance 0.5 per cent—reactance 12.0 per cent

Two banks of transformers in parallel at each end of line.

Losses include those of line, transformers and synchronous condensers

The operating characteristics of a 220-kv., 250-mile line are illustrated by Figs. 4, 5, and 6, which show for an aluminum-steel, a copper-steel and an all-copper conductor the relations between power transmitted, generator, receiver and line voltages, power factor, condenser load and resistance losses.



RESULTS

Curve	Rec'r kw.	Cond'r kv-a.	High voltage Receiver		High voltage Generator		Equiv. low Voltage gen.		Losses (excl. corona loss)	
			kv.	% p. f.	kv.	% p. f.	kv.	% p. f.	kw.	%
			250 mile line (solid line)							
A	0	*49,000	209.0	* 0.17	230	† 5.2	228.0	† 5.3	2,607	..
B	50,000	† 7,000	207.5	*77.8	230	†97.3	228.6	†98.1	2,168	4.3
C	100,000	†80,000	203.0	*98.5	230	*99.9	232.5	*99.3	10,562	10.6
			100 mile line (dotted line)							
D	0	*57,500	210.3	* 0.20	230	* 2.0	236.3	* 2.09	3,158	..
E	50,000	* 4,900	209.4	*68.5	230	*81.1	236.2	*79.1	1,478	3.0
F	100,000	†54,000	208.0	*91.5	230	*92.8	238.2	*90.0	6,242	6.2

NOTE *Indicates lagging quantities
†Indicates leading quantities.

FIG. 6—220-KV. TRANSMISSION LINE CHARACTERISTICS—RELATION BETWEEN KW., KV., P. F. AND CONDENSER KV-A.—CONDUCTOR—500,000 CM. COPPER

CALCULATION ASSUMPTIONS

500,000 cm. copper (no steel core—20 ft. horizontal spacing.
Constant receiver volts 200,000 (low side receiver transformer—includes transformer drop)
Constant generated volts 230,000 (high side generator transformer)
Loads—delivered at 75 per cent power factor.
Transformer bank 50,000 kw. with resistance 0.5 per cent, reactance 12.0 per cent
Two banks of transformers in parallel on each end of line.
Losses include those of line, transformers and synchronous condensers

Necessary explanatory details are shown in tabulations accompanying the curves. These studies show that effective voltage regulation of such long distance, high capacity lines may be obtained by means of large, but not impracticable, synchronous condenser installations.

A study of the relative economy of various sizes and types of conductors is shown in Fig. 7, with accompanying explanatory data in Table II. This study shows, for a load of 100,000 kw. per circuit delivered at 0.75 power factor and at load factors of 60 per cent, 75 per cent and 90 per cent, that the combined annual costs of such items of the transmission system as would be materially affected by the size and type of the conductor, *i. e.*, interest, taxes and amortization charges on cost of conductor, annual value of lost power on the line due to resistance and corona and of power absorbed by transformers and condensers. The curve falling lowest on the scale, at any number of years which may be assumed as the life of the line, represents the most economical of the conductors considered.

This curve is presented merely as an example of the general method followed in studying conductor economy. It does not represent the degree of refinement which would be warranted in making final determination of the economical conductor for an actual 220-kv. installation. Other items for which allowance should be made in such a study are the effect of conductor size and type upon cost of line structures and insulators, and the possible scrap value of the conductor. Amortization should preferably be calculated by the annuity or "sinking fund" method rather than by the simpler straight line method. It should be noted, however, that these additional refinements tend in some respects to offset one another, a feature which gives added justification for their omission from a preliminary study. Obviously, in any case great refinement in the technical assumptions is not called for until reasonably close values can be assigned to cost of conductor materials, for which the market will presumably be unstable for some time to come, to the equivalent costs of the power losses, and to the percentages to be employed for return on investment, taxes, etc.

For the purpose of developing the line designs which will be presented later, a cable was chosen consisting of 716,000 cir. mils of aluminum (54 strands of 0.1151-inch diameter) and 93,000 cir. mils of steel (7 strands of 0.1151-inch diameter). Particular attention, however, is called to the fact that these designs were started and this conductor chosen as a basis of study about a year ago, when prices were much higher than at present, and further that the choice was made upon the basis of a lower load factor than, in the light of further study, seems reasonable to assume as likely to obtain on the usual 220-kv.

transmission system. It is fully recognized that at present price levels and for a high load factor, the economical size of conductor would be larger than this, and that possibly the economic advantage would fall to a different type.

Mechanical Characteristics of Aluminum-Steel Conductor. The large and heavy cables required for 220-kv. transmission, and the heavy conductor loadings which the importance of 220-kv. service requires as a design basis, together with the unusual general precautions which must be taken to assure reliability, call for a new order of refinement in the mechanical features of line design. Careful study with a high degree of imaginative foresight is required to provide against unsafe loads being imposed upon insulators and towers or clearances being dangerously reduced as a result of unusual or unexpected con-

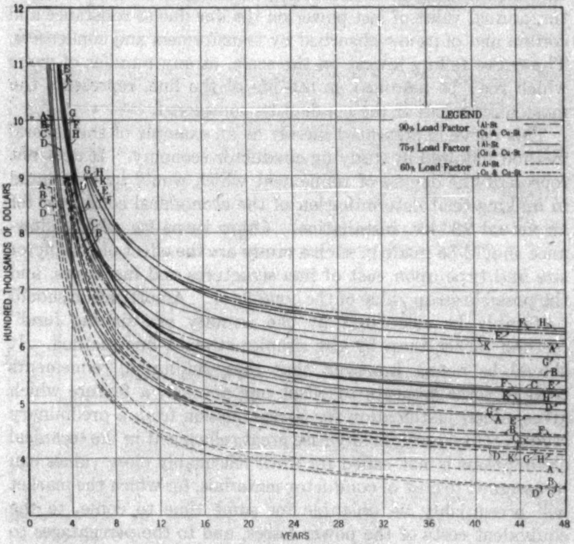


Fig. 7—Economic Comparison of 220 kv. Transmission Conductors—Annual Cost Curves—250 Mile, 100,000 kw. Single Circuit Line (See Table II for Explanatory Data)

tingencies, and against possible unpreventable failures becoming cumulative and occasioning extensive damage.

The mechanical characteristics of steel-core aluminum cables are complicated, and calculation of stresses and sags under varying loading and topographical conditions presents an involved and difficult problem. This is due to the fact that the co-efficients of expansion and moduli of elasticity are different for the two metals and consequently are not accurately determinable for the composite cables. The cable manufacturers, however, have investigated these characteristics and have developed practical, though laborious, methods of calculation of conductor stresses and sags. These methods have been followed and are considered in the main to give results of an accuracy satisfactory for design purposes.

TABLE II

DATA RELATING TO ECONOMIC COMPARISON OF 220 KV. TRANSMISSION CONDUCTORS

(See Fig. 7)

ANNUAL COST CURVES FOR 9 DIFFERENT CABLES AT 3 LOAD FACTORS

1. Annual cost curves are plotted, dollars as ordinates, years as abscissas, and show the annual cost for any period up to 48 years. This is explained as follows:

Annual Cost Includes
1. Depreciation expressed as first cost divided by number of years chosen.
2. Yearly interest and taxes taken as 8 per cent of first cost.
3. Annual value of Lost Power taken as 5 mills per kw-hr.

First cost includes only cost of finished Aluminum-Steel Cable at 44.9c. per lb. for aluminum and 12.2c. per lb., for steel f. o. b. factory. First cost of finished copper or copper-steel cable @ 27.7c. per lb. for copper and 12.2c. per lb. for steel f. o. b. factory. All other items of construction costs have been eliminated as not materially affecting the relative positions of the curves.

Lost power includes line $I^2 R$, transformer $I^2 R$, condenser loss and corona loss.

2. Cable Data.

Curve designation (See Fig. 7)	Kind of cable	First Cost	Circular Mils			Strands		Diam. of cable in.
			Alum. or copper	Steel	Total of cable	Alum. or copper	Steel	
A	Al.-St.	$1,175,700	605,000	78,000	683,500	54	7	0.952
B	Al.-St.	1,362,800	716,000	92,900	808,900	54	7	1.036
C	Al.-St.	1,513,500	795,000	103,100	898,100	54	7	1.092
D	Al.-St.	1,814,000	954,000	123,700	1,077,700	54	7	1.196
E	Copper	1,764,800	500,000	....	500,000	37	..	0.814
F	Cu.-St.	1,733,300	450,000	105,000	555,000	30	7	0.857
G	Cu.-St.	1,924,400	500,000	116,600	616,600	30	7	0.904
H	Cu.-St.	1,790,200	400,000	274,000	674,000	54	37	0.946
K	Cu.-St.	2,011,700	450,000	308,200	758,200	54	37	1.004

The basic data for calculations have been taken as:

	Aluminum	Steel
Elastic Limit (lb. per sq. in.)	14,000	130,000
Modulus of elasticity (lb. per sq. in.)	9×10^6	30×10^6
Co-efficient of expansion (per degree fahr)	12.8×10^{-6}	6.4×10^{-6}

Elastic limit of composite cable 17,300 lb.
(716,000 cir. mils aluminum, 93,000 cir. mils steel)

Within any limits thus far investigated, it appears economical under the design loading, to utilize the full strength of the conductor up to its elastic limit, *i. e.*, to keep sags to a minimum, thus enabling use of shorter towers or longer spans at the expense of stronger supporting structures at angles and dead-end points. With a new aluminum-steel cable, when the loading has been reached which will stress the steel core to its elastic limit, the aluminum will have passed its elastic limit and in consequence have been permanently stretched. The full working strength of the cable, 17,300 lb., will then have been developed, and it may then be said to have reached its final stretched condition, *i. e.*, to have received its "permanent set". As the heavy design loading is removed, the action of the aluminum strands will be to loosen infinitesimally on the steel core and to take no stress. In any subsequent applications of the design loading, the core will again be stressed to the elastic limit of the steel, and the aluminum will coincidentally reach its own elastic limit stress. At lighter loadings the aluminum will be a dead load upon the steel core, which will carry all of the stress.

This introduces an interesting feature in that the characteristics of the composite cable when *new* are distinctly different from its characteristics after it has received its "permanent set," and, for loadings less than the design loading, will follow different tension-sag curves. This relation is shown graphically in Fig. 8.

The problem of predetermining stringing sags for a suspension insulator line, in which there will be considerable differences in lengths of adjoining spans and in elevations of the ends of individual spans, involves considerations of unbalanced tower loads, abnormal insulator loads and reduced tower and ground clearances under varying conditions of conductor loading. It is necessary, initially, to assume a somewhat different mental attitude toward the problem of conductor stringing than that which largely has been customary. For the conditions of 220-kv. design, with heavy design loadings and large conductors under heavy tension, the problem assumes greater importance than in transmission line practise heretofore. In usual practise on suspension insulator lines, the conductor is strung to a certain predetermined tension, derived from tension-sag curves on the basis of the *average span* length between dead-end points, and the suspension insulators are then tied in in a vertical position. It is assumed that when the design loadings occur the conductor tensions and attendant loads upon insulators and towers will continue to maintain the condition of proper balance of stresses within reasonably close limits. Probably, with most present lines, the resulting unbalanced effects under design loading will not be serious.

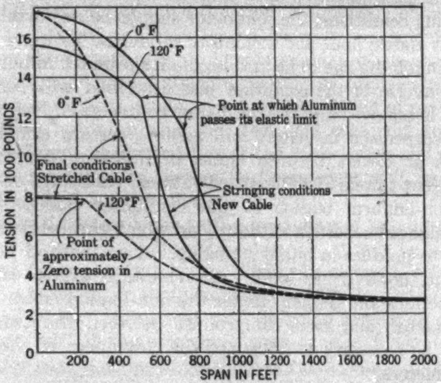


FIG. 8—TENSION—TEMPERATURE CURVES—DEAD ENDED SPANS—STRINGING AND FINAL CONDITIONS—CONDUCTOR—ALUMINUM 716,000 CM.—STEEL 92,900 CM.

Steel reinforced aluminum cable—716,000 c. m. aluminum—92,900 c. m. steel—elastic limit of cable and maximum allowable stress—17,300 lb. with 1½ inch ice and 8 lb. wind at 0 deg. fahr.

Solid line curves represent stringing tensions of *new* cable, without load, before first application of maximum loading.

Dotted line curves represent final tensions of *stretched* cable, without load, after application of maximum loading has given a permanent set to the aluminum.

nder the assumed 220-kv. design conditions, preliminary study has shown that such a method of stringing will result, under the extremely heavy design loading, in great longitudinal deflections of suspension insulators intended to operate purely as suspension units, and will consequently impose unsafe tensions upon these insulators, with attendant unbalancing of tower loads and possibly dangerous reduction in tower clearances, in addition to over-stressing and stretching the conductor itself.

The theoretically proper point of view is that, under design loading conditions, the conductor should be uniformly stressed to its elastic limit and insulators should be hanging normally. Assuming the line to be in this condition under maximum load, the inverse of the condition just described will occur as the load is removed and the conductor contracts in length, that is, the suspension insulators will assume certain definite deflections at towers between spans of different lengths and at adjacent towers at different elevations, resulting, of course, in a non-uniform tension in the conductor itself. It is this unbalanced condition without load which careful design should aim to produce in initial stringing. This method of stringing might, however, in some cases result in excessive insulator deflections and tensions under the non-loaded condition of the conductor, and some compromise between the two methods may be necessary to safeguard the clearances of the unloaded conductors.

A method of predetermining for stringing conditions the proper conductor tensions or sags in individual spans and the proper insulator deflections is accordingly a desirable refinement. Some approximate treatment of the subject, at least, will probably be necessary. It should be noted that the 220-kv. designs suggested in this paper contemplate the use of very long suspension insulator strings, and that this length of string tends to lessen the amount of abnormal insulator and tower stresses and reduction of tower clearances which may be set up. If, as is hoped, improvement in insulator design should enable the adoption of a shorter insulator, an accurate method of conductor stringing will become correspondingly more important.

A purely mathematical treatment of the problem appears complex in the extreme, although approximate methods of analysis seem to be feasible. The working out of such approximate methods offers an interesting and valuable subject for research. An exceedingly instructive experimental investigation could be made along these lines, either by means of a series of spans of a full-sized line capable of being artificially loaded, or by means of a miniature properly proportioned model. One troublesome feature of such an experimental determination of these phenomena would seem to be that of readily obtaining and controlling the necessary temperature range, or of compensating or correcting for temperature variation by some indirect means. Such an investigation would fulfill a most valuable function in supplying empirical constants to be used as a basis for mathematical treatment and in verifying the results of approximate methods of analysis.

The problem of actual stringing is further complicated by the use of a composite cable, owing to the shifting of tensions between the aluminum and the steel, and especially to the radically different characteristics of the cable before and after it has been stretched to receive its "permanent set", as previously described and as illustrated by Fig. 8. This difference in characteristics leads to the possibility of two different methods of stringing a composite cable, *i. e.*, it may be strung as received from the factory, or it may be stressed, prior to sagging and tying in, to its full strength, 17,300 lb., thus giving it nearly all of its "permanent set" (it would receive all of its "permanent set" if the stretching were done at 0 deg. fahr.) In the latter case a series of tension-sag curves based upon the "permanent set" condition of the cable would be used for stringing. In the case of stringing the new unstretched cable, it would be necessary to develop a special series of tension-sag curves, while the "permanent" tension-sag curves would be used in locating towers and checking clearances.

For a new cable, in the absence of special data, the division of stress between the aluminum and the steel is indeterminate. Such special data may, however, be obtained, presumably by experiment. It probably will be found that the conditions of the cable as it comes from the factory is sufficiently uniform to enable such special stringing curves to be used consistently. The application of methods predetermining insulator deflections, as discussed above, would involve even further complication if these special unstretched tension-sag curves were to be used for stringing.

From the operating standpoint, however, it would seem preferable to string the cable without preliminary stretching.

The design loading, with the large margin of safety which it is assumed to contain, will probably rarely be reached or approximated on the greater portion of the line. The smaller initial sags would then in practise, for the greater part, never be increased to the "permanent" sags by natural causes. Hence there would be obtained the advantage of smaller normal operating sags, with the unstretched aluminum acting as a reserve to increase them suitably when, or if, the design loading should occur.

Possibly a combination of the two methods might be worked out, whereby the cable would be stressed before tying in to some definite tension sufficient to insure giving the aluminum part of its "permanent" set. It would then be sagged in by unloaded tension-sag curves based upon the definite relation between stresses in the aluminum and the steel as established by this preliminary application of tension.

The practical importance of the theoretical considerations involved in stringing irregular spans supported by suspension insulators, and the extent to which refinements may and should be carried will in the last analysis be governed by consideration of the practical limits of field application, taking into account the many variable physical and personal factors. The intent is to point merely to the interesting and apparently effective possibilities of theoretical and experimental research, pending further analysis and study. No field stringing curves embodying the refinements suggested have yet been developed, even in approximate form. For purposes of clearance determinations, of tower design and of study of tower economics, the use of "average span" tensions and sags probably embodies sufficient accuracy, and has tentatively been used.

Adequate splices for aluminum-steel cables no longer are considered to present a problem. Satisfactory types have been developed and are in successful use on existing lines. In making these splices, the aluminum is cut back from the ends of the steel core, the core is then spliced by means of a soft steel sleeve, twisted on by means of special wrenches, and the sleeve and a considerable length of the aluminum strand at each end are then covered by a heavy aluminum sleeve which is solidly compressed on the conductors between the dies of a portable oil operated jack. Such splices are reported to develop the full strength of the cable, the splice itself having considerable excess strength.

Overhead Ground Wires. The line designs presented provide for two overhead ground cables of 5/8-inch diameter high-strength steel, 16,000 lb. elastic limit for the cable. The justification for the use of ground wires in 220-kv. transmission is debatable, but no well-formed conclusions seem possible except as a result of practical comparative experience. Their use on present high-voltage steel tower lines is nearly universal, remarkably so in view of the meagre and inconclusive character of the data as to the benefits derived. At 220 kv., with the high insulation provided and in view of the diminishing importance which, with increasing line voltage, it is believed can be attached to induced lighting disturbances, it is wholly conceivable that the protection afforded may be found to be of disproportionately small value. The cost of using ground wires is undoubtedly a large item, particularly in view of the extremely heavy design loadings assumed and the consequent tower stresses which they occasion.

In view, however, of the great importance of 220-kv. service, it has been deemed conservative, until further experience is obtainable, to make provision for the use of ground wires. The plan of installation would be to omit ground wires from a considerable portion of one line, in districts where lightning conditions were severe, while ground wires would be used on a parallel line. Comparative performance data would be a guide to subsequent procedure.

Types of Insulators Available. There is no type of insulator as yet developed which has thus far demonstrated its ability to give adequate, or even reasonably satisfactory results on high-voltage lines. This condition, however, applies nearly as much to the high voltages in current use, 110 kv. to 150 kv., as it does to 220 kv. It does not in any way affect the feasibility of 220-kv. transmission. It is confidently believed that 220-kv. line insulation can, with existing types of insulators, be made as safe and dependable as can the line insulation of present installations. In fact, where foresight in design and careful maintenance are employed, the present unsatisfactory insulator situation makes its effects evident far less in impairment of service reliability than in the high first cost, direct and indirect, of precautions against insulator failure and in high maintenance expense and operating inconvenience. The economic value of 220-kv. service is so high that greater expense and attention than at lower voltages are warranted in measures to guard service.

The probable advent in the near future of such voltages as 220 kv. should serve, nevertheless, to emphasize the economic necessity of developing line insulators to a point consistent with the other elements of the electric power generation and transmission situation. The baneful effects of the insulator situation are evident not merely in the expense and trouble occasioned, but in a certain degree of popular suspicion, in essentials unwarranted, of the general idea of high-voltage transmission.

It is believed, as has just been noted, that practicable 220-kv. insulators can be obtained from present established types. There are commercially available three such types, all based upon the principle of a series string of disks, in practise of about 10-inch diameter, *i. e.*, the standard cemented cap and pin type, the Hewlett type and the newer Jeffrey-Dewitt type. Numerous other designs for high-voltage insulators have been suggested, some of the series unit type and some in one piece. Certain of these designs appear to offer real promise, but none of them have been developed to the stage of commercial production, or even to a point where their service performance can be predicted.

The following studies and 220-kv. line designs have been based upon the standard 10-inch cemented cap and pin type units. This implies no disparagement of the other types mentioned; in fact, these appear in some respects to offer significant advantages. The standard type has been taken as a basis of study because it has been used by far the most extensively and for long periods, and, therefore, more data are available in regard to its characteristics. Any general designs of towers and fittings developed for these units could equally well be used for the other types. The studies which follow, in conjunction with the general record of experience and investigation, indicate that there are certain characteristics, largely inherent, of these standard disk insulators which render them far from ideal for service at extra high voltage with heavy conductors. In certain respects this would be true of any insulator built up of a large number of disks. It is hoped that manufacturers will soon be able to develop an insulator, which will be more suitable for extra high-voltage use and will offer assurance of greater strength and permanence, both electrically and mechanically.

Electrical Characteristics of Disk Insulators. The designs suggested make provision for a string of 15 standard units for regular suspension service on a 220-kv. grounded neutral line. This relatively large number of units affords a considerable margin for deterioration and, with proper care in maintenance, should assure a degree of reliability in service commensurate with the economic importance of 220-kv. service.

It should be noted that a string of 15 standard units, with the necessary connecting pieces and fittings, will be nearly nine feet long. Such a length of insulator obviously involves great expense in obtaining the necessary tower clearances and heights, and it is also the determining feature in fixing conductor separation. This serves again to call attention to the need of more efficient and suitable insulators. An improvement in insulator design which would justify shortening the string, in addition to improving the electrical characteristics of the insulator itself, would enable material saving in tower costs. A wholly new insulator, having no greater length than necessary to insure requisite air clearances from conductor to support, say four or five feet, would enable a correspondingly greater and a very significant saving in tower costs. Such an insulator at moderate price might readily open the door to a variety of new types and arrangements of supporting structures.

The primary electrical characteristics are those of arc-over and puncture. It is essential to reliable service that units be employed with the largest obtainable ratio of puncture voltage to arc-over voltage. The particular importance of this high ratio for 220 kv. will be evident in the light of certain data which will be presented later in regard to concentrations of electrical stress upon individual units of the string.

The 60-cycle arc-over characteristics of long strings of standard disk insulators are shown in Fig. 9, which gives curves of arc-over voltage for wet and dry conditions. These curves are based upon published test data for shorter strings, extended mainly on a theoretical basis with the assistance of such fragmentary test data as have been available.* It is pertinent, and of great interest, to note that the dry arc-over curve flattens out as the number of units in the string increases, approaching a condition where increasing the number of disks adds practically nothing to the dry arc-over voltage, while on the other hand the wet arc-over curve follows nearly a straight line characteristic and for long strings reaches higher values than the dry arc-over curve. These characteristics, as is well understood, are due in the case of dry arc-over, to the action of the charging current on the system of distributed series and shunt capacities which the insulator string constitutes, the resultant effect being a concentration of potential on certain units of the string. In the case of wet arc-over, the effect of the charging current is lost in the greater effect of the large leakage current.

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*Peek, A. I. E. E. TRANS. 1912, Vol. XXXI, p. 907,—"Electrical Characteristics of the Suspension Insulator."

The dry arc-over characteristic thus appears to constitute the controlling feature of insulator design, in so far at least as 60-cycle characteristics are determining. The fact that an insulator has a higher arc-over value under rain conditions is of no advantage, since this condition obtains for but a small fraction of the time, and in particular since heavy lightning discharges, with possibility of resultant normal frequency surges on the line, are more likely to occur just before a rain storm than during it.

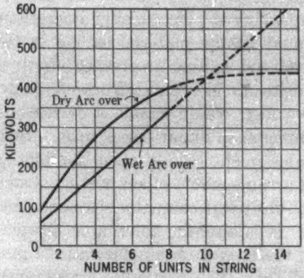

FIG. 9—TYPICAL 60-CYCLE ARC-OVER CHARACTERISTICS—SUSPENSION INSULATORS

The distribution of voltage stress over the units of a string of fifteen standard 10-inch disks and the concentration of stress on certain units, notably those nearest the conductor, is shown by curve *A* in Fig. 10, which is based in the main upon published test data.* At a line voltage of 220 kv., (127 kv. to ground), this concentration reaches a degree which not only interferes seriously with the efficient and economic use of the insulator units, but which brings the stress on the unit next to the conductor to a point higher with respect to its strength than the standards of practise set for 220-kv. service make desirable.

Two methods are recognized as offering relief from this excessive concentration. The first is the grading of the insulator units used in the string. This would be accomplished by making up the string of units of two or more distinct types, differing in size or diameter or in some other feature which would cause them to have different condenser capacities, those with the larger capacities being placed nearest to the conductor. The results of one form of such grading are shown in curve *B*, Fig. 10 which is based upon published test data.* It is evident that by such grading considerable and, for ordinary purposes, ample improvement in voltage distribution can be effected. From a practical standpoint, this expedient involves a certain amount of complication and expense in construction and of expense and inconvenience in operation due to the necessity of maintaining stocks of each of the different types of units

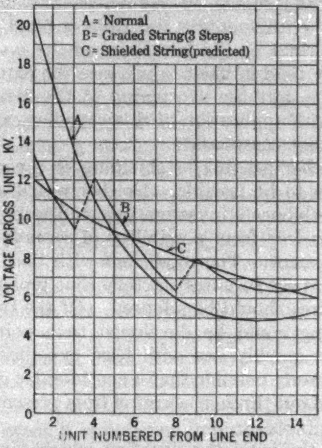

FIG. 10—TYPICAL 60-CYCLE VOLTAGE DISTRIBUTION—SUSPENSION INSULATORS—220 KV. GROUNDED NEUTRAL SYSTEM

and of insuring their proper use in maintenance replacements. This disadvantage, while of some consequence, cannot be regarded as prohibitive in view of the benefits which might be expected to accrue.

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*A. I. E. E. 1916 TRANS., Vol. XXXI, Part I, p.745, R. H. Marvin, "A New Method of Grading Suspension Insulators."

A second method of relieving this excessive concentration of stress consists in installing below or around the disk nearest the conductor suitably designed metallic shields or rings. The effect of such shields in improving the stress gradient may be even more marked than that of grading the insulator units. The stress distribution obtainable by this method has been predicted in curve *C* of Fig. 10, the values for which have been assumed from the fragmentary test data at hand. The use of such shields or rings at the lower end of the insulator string would obviously tend in itself to increase the requirements for tower clearances, but the shortening of the insulator string, which a successful application of this expedient would justify, might presumably compensate.

In general either grading or shielding or a combination of the two appears to be feasible. Neither would appear to require any very elaborate investigations and tests to determine effective designs free from possibility of secondary complications of any moment. The conditions with the 15-unit string of standard units are so unsatisfactory that probably some alleviating measures should be adopted. Of the two described, probably grading could be developed to a point ready for actual use most quickly and with least experimental investigation. It is wholly possible that a considerable grading effect might be worked out through selection from present commercial types of disks.

A feature of the insulator situation which complicates the question of voltage stress distribution and which will have some effect upon methods of carrying out remedial measures is the fact that in order to obtain adequate mechanical strength, as will be discussed later, two or three strings of standard disks must be used in parallel at suspension points, and proportionately more at tension points. So far as is known, no investigation has been made of the effect of parallel strings upon potential gradient.

The belief is widely entertained that arcing horns or rings or other discharge devices are of sufficient benefit to warrant their use. They would fulfill several functions, the first and primary function being to protect the insulator from the destructive heat of an arc. This function assumes particular importance on a large capacity, dead-grounded neutral system. For this purpose the devices should be so shaped and placed as to hold the arc securely away from the insulator string. Other functions are protection of the conductor from the possible burning by a high-power arc, and reduction of the likelihood of insulator puncture. It may be possible to combine in one device in some effective and economical manner the functions of a discharge horn or ring and of an electrostatic insulator shield.

The foregoing discussion of electrical stresses on insulators refers, as will be noted, to such stresses as may be produced at the normal line frequency of 60 cycles. Under conditions of high-frequency oscillations and of steep wave front phenomena generally, the insulator voltage stress characteristics will be quite different. In particular, the high-frequency voltage stress distribution over an insulator string is understood to follow approximately a straight line characteristic. The importance to be ascribed to high-frequency phenomena as a disturbing factor in 220-kv. operation is a problem of a highly speculative character. Such evidence, however, as is available, and the trend of theoretical opinion, tend toward the conclusion that high-frequency becomes of diminishing relative significance as the line voltage is increased, particularly when transformer neutrals are dead grounded. The suggestion has been offered that, with a line operating near the corona limit, high-voltage surges at high frequency, representing usually small amounts of energy, tend to dissipate themselves in corona, corona dissipation of energy being more rapid at high frequencies. In any event it is believed improbable that a line sufficiently well-insulated to withstand low-frequency high-power disturbances is likely to encounter trouble from high-frequency disturbances.

Mechanical Characteristics of Disk Insulators. The widespread dissatisfaction with the present insulator situation is believed to have arisen largely on mechanical grounds.

A very disturbing feature is the much discussed insulator deterioration. The progressive failure, after a relatively short period of years in service, of the cemented type disk insulator is well recognized and has been almost universally experienced. Existing high-voltage lines are facing the prospect of continuous and difficult tests and expensive maintenance, both attended by interference with operation. Whether the causes be also electrical, chemical or ceramic, the results are unquestionably serious. In this respect, however, the difficulties at 220-kv. would differ from those at low voltage only in the greater number of disks involved.

From the standpoint of deterioration, the non-cemented types of disk insulator, which avoid the effects of dissimilar expansion of the component materials, seem to offer and to be demonstrating marked advantages.

The working stress for standard cemented cap and pin type disks permitted by conservative practise is only about 2500 lb. With the large conductors and extremely heavy design loadings required for 220-kv. service, this low mechanical strength immediately presents itself as an embarrassingly serious limitation. The designs which have been worked out in this study call at normal suspension points for two strings in parallel with spans up to 700 ft. and for three strings in parallel with spans in excess of 700 ft. The complication and expense of hardware and the difficulties involved in clearances are obvious, not to speak of the direct cost of the insulators. If these insulators were to be used at dead-end points, tension assemblies of at least six strings would be required, at the unconservative design load of 2900 lb. per string. A failure in one string, by unbalancing the load distribution, would seriously jeopardize the whole assembly.

For service at dead-end points, however, the unsuitability of the standard disk is so pronounced that it is considered probable that resort would be made at once to an entirely different type of insulator.

Considerable promise is offered by a relatively new type of insulator, a wooden rod insulated with compound and enclosed in a suitably petticoated procelain shell. Any desired mechanical strength can readily be secured in an insulator of this type, so that the full dead-end tension would be carried on one unit. Probably two units would be used in series to obviate an excessively long porcelain tube. Such tension insulators are in limited use on 120-kv. and 150-kv. lines. They are a new development, however, and in the absence of long service demonstration there naturally arises a question as to their electrical permanence. Experiments with somewhat similar types of insulator have shown unfavorable results in the way of disintegration of the wooden rod under long applied high electrical stress. On the other hand, oil insulated wooden rods in circuit breakers have, with few exceptions, been found to be durable. Possibly some other material, less susceptible than wood to disintegration, might be employed for the rod. Any cracking of the porcelain shell or loosening of joints, which would permit escape of the insulating compound, would of course result in failure of the insulator. In the light of present knowledge, however, the lack of service trial appears no more serious than the known complication of huge assemblies of the standard type of disk insulator.

Summarizing the situation as to availability of present commercial disk insulators for 220-kv. service, it is evident that, in addition to known drawbacks, there exists considerable question as to fundamental characteristics which must to some extent be removed before confidence can be established in assumptions made and designs based upon these assumptions. Investigations should be conducted, with possible resultant developments, as to at least three features of the applicability of disk insulators to 220-kv. service, *i. e.:*

1. Tests to confirm or establish the dry and wet flashover characteristics, and accompanying potential gradients, of strings of ten to fifteen disk units, both singly and with several strings in parallel. These characteristics should be investigated both at 60 cycles and at high frequency;
2. Experiments to determine how grading or shielding or both can most effectively be applied to strings of present types of disks;
3. Continued investigation of insulator deterioration problem.

It is to be hoped that those concerned with the design and manufacture of insulators will be inclined to undertake investigations of this character. Of even greater ultimate importance, of course, are efforts looking to the development of a new type of insulator, more efficient electrically, with greater strength mechanically and of unquestionable permanence.

Clamps and Fittings. Reliable and satisfactory clamps are now on the market for composite cables of somewhat smaller sizes. It is not expected that any significant difficulty will be encountered in the design of clamps suitable for a cable of the size proposed. The clamps will presumably be of generally similar design to that of those now in use on some of the highest voltage lines in the country, of which illustration is given in Fig. 11.

In the design of all cable clamps for use at 220 kv., especial care would be required in avoiding of sharp projections, ridges or points which might afford opportunity for formation of corona or static discharges. Sleeve protectors should be used with all types of clamps. At the ends of the clamps, bells of sufficiently large radius should be provided to avoid any chance of conductor crystallization from continued vibration.

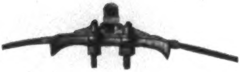
In the case of suspension clamps, attention should be given to obtaining long smooth clamping surfaces, and any corrugations should be of large radius. The clamp should hold the cable with sufficient strength to prevent slipping under normal conditions, and in case of conductor breakage slipping should not occur except at the tower adjacent to the break. With the great length of the suspension insulator string, this requirement should not be difficult to meet.

Semi-tension clamps will have an important field of use, since it is the aim to go to extremes in avoiding points where full dead-end tension will come upon insulators. This type of clamp will accordingly be used at smaller angle and stabilizing points. It should hold the conductor, under all conditions, with no appreciable slipping, a requirement which may involve separate clamping of the steel core. A somewhat similar angle or side tension clamp will be needed for use at points where the line makes a considerable horizontal angle.

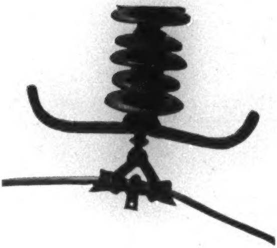
For tension clamps, the best present practise is to provide for separate clamping of the core, as illustrated in Fig. 11. For large cables worked to their elastic limits, as required for 220-kv. service, this method of clamping will be essential.

Equalizer yokes and connectors present more of a problem than for present lines, since, as noted earlier, even at suspension points two or three insulator strings in parallel would be used, while if disk insulators were to be used at tension points, not less than six parallel strings would be necessary. Whether the present patterns of cast yokes are satisfactory for such heavy duty service is questionable. Pressed steel or some design built up of structural shapes would appear preferable.

Jumpers at dead-end towers will obviously be long, more than 20 ft., and probably some form of jumper guide or anchor will be necessary. This has been satisfactorily accomplished in existing lines by means of auxiliary weights, auxiliary insulator strings or rigid guides of structural steel attached to the yokes.



1



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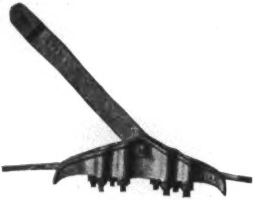

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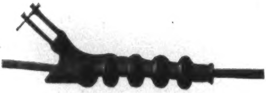
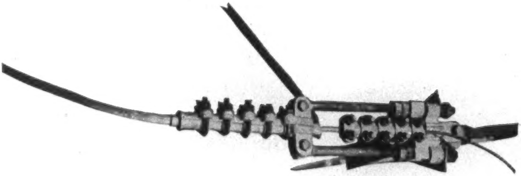


4



5

FIG. 11—CONDUCTOR CLAMPS [SILVER]

EXAMPLES OF PREVAILING PRACTISE

Nos. 1, 2 and 3........Suitable Type for Suspension
Nos. 3 and 4..........Suitable Types for Semi-Tension
No. 5................Suitable Type for Tension

Transmission Towers. Before proceeding to discussion of the problems of tower types and designs, it may be noted that for present purposes consideration has been confined to single circuit lines, primarily for the reason that at the start a 220-kv. system would be developed gradually, one or two circuits at a time, probably to a considerable extent over different rights-of-way. When, however, 220-kv. becomes established and conditions call for three or more parallel lines, or when networks have been established such that absolutely continuous service of any one line becomes relatively somewhat less important, then the question of using double-circuit structures will deserve careful investigation. The advantages will consist in a considerable saving in tower cost and some saving in right-of-way, against which would lie the relatively minor chance of failure of a structure from extraordinary causes involving two circuits instead of one.

The design of two circuit 220-kv. structures would open interesting possibilities. The large conductor separation, about 20 ft., employed in the accompanying designs, adopted largely as a result of the long insulator strings used, would severely handicap the conventional type of double circuit tower,—three conductors in a generally vertical plane on each side of the tower,—owing to the great height and great weight of steel required. A shorter insulator would tend to reduce this handicap. There are other feasible types of double-circuit tower for 220-kv. service, in some respects more promising, in particular a tower with three legs transverse to the line carrying all six conductors in a horizontal plane. This type makes the high transverse strength easy to obtain, and suggests the possibility of building two legs to carry one circuit initially and adding the third leg and the second circuit later. The design of a double-circuit tower would, of course, be determined primarily by economy of steel. A discussion of structures for multiple circuit lines is, however, beyond the scope of the present study.

The choice of structure material, whether wood, or steel, is primarily one of total economy, considering first cost, depreciation and maintenance. The long economic life which presumably a 220-kv. line would represent, causes durability of material to assume even greater importance than in present practise, so that for most localities, wood construction would not be economical, even assuming that the requisite strength for the heavy loads could be secured without resorting to unduly short spans.

As to type of structure to be employed, the conventional *rigid* tower has been adopted as the most available for construction in the immediate future. The rigid tower type of line has been thoroughly studied and tested by experience, and when properly designed, its performance record has been satisfactory, at least with the smaller conductors and lighter loadings thus far used. It is recognized, of course, that the very great stresses existing in such a line as is here proposed tends to disturb the balance of considerations which has deter-

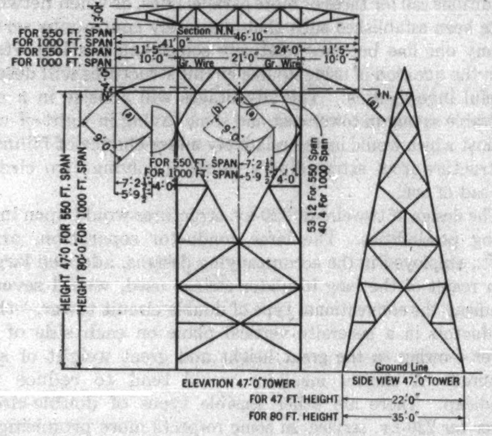

3'-0"
FOR 550 FT. SPAN
FOR 1000 FT. SPAN
FOR 550 FT. SPAN
FOR 1000 FT. SPAN
Section N.N.
46'-10"
41'0"
11'-5"
24'-0
11'-5"
10'-0"
21'-0"
10'-0"
Gr. Wire
Gr. Wire
5'-0
4'-7
N.
N.
(a)
(b)
(a)
(a)
9'-0
9'-0"
FOR 550 FT. SPAN 7'-2 1/2
FOR 1000 FT. SPAN 5'-9 1/2
7'-2 1/2
5'-9 1/2
4'-0
4'-0
53 12 For 550 Span
40 4 For 1000 Span
HEIGHT 47'-0 FOR 550 FT. SPAN
HEIGHT 80'-0" FOR 1000 FT. SPAN
Ground Line
ELEVATION 47'-0" TOWER
SIDE VIEW 47'-0" TOWER
FOR 47 FT. HEIGHT
22'-0"
FOR 80 FT. HEIGHT
35'-0"

FIG. 12—220-KV. STEEL TRANSMISSION TOWER—CLEARANCE DIAGRAM TYPE A (SUSPENSION)—FOR VERTICAL ANGLE 5 DEG. AND HORIZONTAL ANGLE 0 DEG. TO 2 DEG. (FOR DESIGN LOADING AND STRESSES SEE TABLE III.)

CONDITIONS CAUSING MAXIMUM SWING OF INSULATORS TAKEN FOR DETERMINING TOWER DIMENSIONS

Position	Span	Horizontal Angle Turned by Conductors	Wind	Ice	Temp.
a	550	——> 2 deg.	——> 8 lb.	0 in.	0 deg.
b	550	<—— 2 deg.	<—— 8 lb.	0 in.	0 deg.

TABLE III
DESIGN LOADING AND STRESSES

TYPE A TOWER (SUSPENSION)
VERTICAL ANGLE 5 DEG. AND HORIZONTAL ANGLE 0 DEG. TO 2 DEG.
CONDUCTOR, ALUMINUM 716,000 CIR. MIL.—STEEL 92,900 CIR. MIL.
GROUND WIRE 5/8 IN. STEEL STRAND SUSPENSION INSULATORS
WIND 8 LB. ICE 1½ IN. AT 0 DEG. FAHR.
(FOR TYPE A TOWER CLEARANCE DIAGRAM SEE FIG. 12)

Span (Normal and max. allowable with maximum angles)	550 ft.	700 ft.	800 ft.	1000 ft.
Height of cross arm above ground	47 ft.	57 ft.	63 ft.	80 ft.
Transverse Loading				
Wind on tower	750	850	1,000	1,250
Wind on 3 conds. with 1½ in. ice at 2.69 lb. per ft.	4,400	5,650	6,440	8,050
Wind on 2 gr. wires with 1½ in ice. at 2.42 lb. per ft	2,650	3,400	3,860	4,850
Pull, 3 conds. due to 2 deg. hor. angle	1,800	1,800	1,800	1,800
Pull, 2 gr. wires due to 2 deg. hor. angle	1,200	1,200	1,200	1,200
Total load	10,800	12,900	14,300	17,150
Test load—125 per cent	13,500	16,000	18,000	21,500

Torsional Loading

It is assumed that one conductor is broken and the pull of the conductor in the adjoining span is decreased due to the tendency of the insulator string to swing in the direction of the pull.

Assumed maximum load	8,000	9,000	10,000	11,000
Test load—125 per cent	10,000	11,300	12,500	13,800
Vertical Loading (At each conductor and ground wire support)				
Weight of conductor at 0.9 lb. per ft.	500	600	750	900
Weight of 1½ in. ice at 4.72 lb. per ft.	2,600	3,300	3,800	4,750
Due to 5 deg. vertical angle	1,500	1,500	1,500	1,500
Insulators and hardware	400	400	400	400
Men	200	200	200	200
Repair tackle	800	800	800	800
Total load at one support	6,000	6,800	7,450	8,550
Test load—125 per cent	7,500	8,500	9,300	10,700

Design Stresses

Above test loads include all factors of safety so that structural steel should be stressed as follows:

	Test load	Working load max. condition
In tension	30,000 lb. per sq. in.	24,000
In compression	$\left(30{,}000 - \frac{100\,L}{R}\right)$ lb. per sq. in.	$\left(24{,}000 - \frac{80\,L}{R}\right)$
Bolts in shear	25,000 lb. per sq. in.	20,000
Bolts in bearing	50,000 lb. per sq. in.	40,000

Values L/R to be used shall be:
a. For main members not greater than 120
b. For secondary members not greater than 200
c. For redundant members not greater than 250

Minimum thickness of metal ¼ in. All material to be galvanized.

Test loads as specified for transverse, torsional and vertical loading to be applied separately.

mined the present rigid tower type of line design, and further study, in conjunction with actual experience in 220-kv. construction, may later indicate that greater economy is obtainable with a different type of construction. For immediate purposes, however, it has been deemed that a conservative attitude should be adopted toward innovations not specifically called for by the conditions of 220-kv. service.

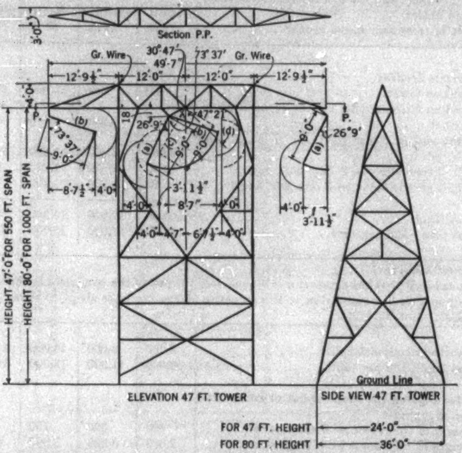

Section P.P.
3'-0"
Gr. Wire
30°47'
49°-7'
73°37'
Gr. Wire
12'-9½"
12'-0"
12'-0"
12'-9½"
4'-0"
P.
(b)
73°37'
9'-0"
18"
26°9'
(a)
(c)
(b)
47°2'
(d)
9'-0"
P.
26°9'
9'-0"
(a)
8'-7½"
4'-0"
3'-11½"
4'-0"
8'-7"
4'-0"
4'-0"
4'-7"
6'-7½"
4'-0"
4'-0"
3'-11½"
HEIGHT 47'-0" FOR 550 FT. SPAN
HEIGHT 80'-0" FOR 1000 FT. SPAN
ELEVATION 47 FT. TOWER
Ground Line
SIDE VIEW 47 FT. TOWER
FOR 47 FT. HEIGHT 24'-0"
FOR 80 FT. HEIGHT 36'-0"

Fig. 13—220-kv. Steel Transmission Tower—Clearance Diagram—Type B (Suspension)—For Vertical Angle 10 deg. and Horizontal Angle 2 deg. to 10 deg. (For Design Loading and Stresses See Table IV)

Conditions Causing Maximum Swing of Insulators Taken for Determining Tower Dimensions

Position	Span	Horizontal Angle Turned by Conductors	Wind	Ice	Temp.
a	1000	——> 2 deg. plus	<—— 8 lb.	0 in.	120 deg.
b	550	10 deg.	——> 8 lb.	0. in.	0 deg.
c	1000	0 deg.	<—— 8 lb.	0 in.	0 deg. or 120 deg.
d	550	2 deg. minus	——> 8 lb.	0 in.	0 deg.

It is growing practise in heavy line construction to provide a

TABLE IV
DESIGN LOADING AND STRESSES

Type B Tower (Suspension)
Vertical Angle 10 Deg. and Horizontal Angle 2 Deg. to 10 Deg.
Conductor, Aluminum 716,000 Cir. Mils. Steel 92,900 Cir. Mils.
Ground Wire $^5/_8$ in. Steel Strand Suspension or Semi-Tension Insulators
Wind 8 lb. Ice 1½ in. at 0 Deg. Fahr.
(For Type B Tower Clearance Diagram See Fig. 13)

Span (Normal)	550 ft.	700 ft.	800 ft.	1000 ft.
Span (maximum allowable with maximum angles)	700 ft.	900 ft.	1000 ft.	1200 ft.
Height of cross arm above ground	47 ft.	57 ft.	63 ft.	80 ft.
Transverse Loading				
Wind on tower	850	1,000	1,250	1,500
Wind on 3 conds. with 1½ in. ice at 2.69 lb. per ft.	5,600	7,250	8,050	9,700
Wind on 2 gr. wires with 1½ in. ice at 2.42 lb. per ft.	3,350	4,350	4,850	5,800
Pull 3 conds. due to 10 deg. horizontal angle	9,000	9,000	9,000	9,000
Pull 2 gr. wires due to 10 deg. horizontal angle.	6,000	6,000	6,000	6,000
Total load	24,800	27,600	29,150	32,000
Test load—135 per cent	33,500	37,300	39,400	43,200

Torsional Loading

It is assumed that two conductors are broken and that the pull of the conductors in the adjoining span causes unbalanced loading in the tower.

Load at any one conductor or ground wire support	17,300	17,300	17,300	17,300
Test load—135 per cent	23,400	23,400	23,400	23,400
Vertical Loading (At each conductor and ground wire support)				
Weight of conductor at 0.9 lb. per ft.	600	850	900	1,100
Weight of 1½ in. ice at 4.72 lb. per ft.	3,300	4,250	4,700	5,700
Due to 10 deg. vertical angle	3,000	3,000	3,000	3,000
Insulators and hardware	400	400	400	400
Men	200	200	200	200
Repair tackle	800	800	800	800
Total load at one support	8,300	9,500	10,000	11,200
Test load—135 per cent	11,200	12,800	13,500	15,100

Design Stresses (Same for Towers Types C. D. and E. Tables V, VI and VII)

Above test loads include all factors of safety so that structural steel should be stressed as follows:

	Test loads	Working load max. condition
In tension	30,000 lbs. per sq. in.	22,000
In compression	$\left(30,000 - \frac{100\,L}{R}\right)$ lbs. per sq. in.	$\left(22,000 - \frac{75\,L}{R}\right)$
Bolts in shear	25,000 lbs. per sq. in.	18,500
Bolts in bearing	50,000	37,000

Values of L/R to be used shall be:
a. For main members not greater than 120
b. For secondary members not greater than 200
c. For redundant members not greater than 250

Minimum thickness of metal $^1/_8$ in. All material to be galvanized.

Test loads as specified for transverse, torsional and vertical loading to be applied separately.

series of standard towers of different strengths, adapted to safe and economical use under varying conditions of span lengths and angles. The extremely heavy loading basis assumed for a 220-kv. line makes it feasible, and desirable from the standpoint of economy of steel, to carry this differentiation to a greater extent than has been customary heretofore. The tower design studies which follow are based upon a series of five

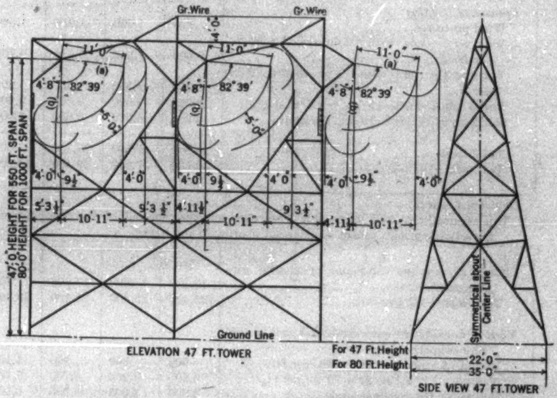

Gr.Wire
Gr.Wire
4'0"
11'0"
(a)
4'8"
82°39'
(b)
5'0"
4'0"
9½"
5'3½"
10'11"
9'3½"
4'11½"
47'0" HEIGHT FOR 550 FT. SPAN
80'0" HEIGHT FOR 1000 FT. SPAN
Ground Line
ELEVATION 47 FT. TOWER
Symmetrical about Center Line
For 47 Ft. Height
For 80 Ft. Height
22'0"
35'0"
SIDE VIEW 47 FT. TOWER

FIG. 14—220-KV. STEEL TRANSMISSION TOWER—CLEARANCE DIAGRAM—TYPE C (SIDE TENSION)—FOR VERTICAL ANGLE 15 DEG. AND HORIZONTAL ANGLE 10 DEG. TO 25 DEG. (FOR DESIGN LOADING AND STRESSES SEE TABLE V)

CONDITIONS CAUSING MAXIMUM SWING OF INSULATORS TAKEN FOR DETERMINING TOWER DIMENSIONS

Position	Span	Horizontal Angle Turned by Conductors	Wind	Ice	Temp.
a	550	——> 25 deg.	——> 8 lb.	0 in.	0 deg.
b	1000	10 deg.	<—— 8 lb.	0 in.	120 deg.

standard types of tower for use under the varying conditions presented by a 220-kv. line over a rolling terrain.

The determination of economic balance between height and weight of towers and normal length of span is an interesting and a fundamentally important problem. Studies of tower designs and of this economic balance for four different normal span lengths are shown in some detail in Figs. 12, 13, 14, 15 and

16, with accompanying data in Tables III, IV, V, VI and VII. The procedure followed in carrying out this study of economic span length will be described briefly. As a basis of study a typical section of profile, as shown in Fig. 17, (a section 23.3 miles long from an actual surveyed route), was selected as a fair example of average 220 kv. line location in rolling country,

TABLE V
DESIGN LOADING AND STRESSES

TYPE C TOWER (SIDE TENSION)
VERTICAL ANGLE 15 DEG. AND HORIZONTAL ANGLE 10 DEG TO 25 DEG.
CONDUCTOR. ALUMINUM 716,000 CIR. MIL.—STEEL 92,900 CIR. MIL.
GROUND WIRE $^5/_8$ in. STEEL STRAND
SUSPENSION OR SEMI-TENSION INSULATORS
WIND 8 LB.—ICE 1½ in. AT 0 DEG. FAHR.
(FOR TYPE C TOWER CLEARANCE DIAGRAM SEE FIG. 14)

Span (Normal)	550 ft.	700 ft.	800 ft.	1000 ft.
Span (max. allowable with maximum angles)	900 ft.	1100 ft.	1200 ft.	1400 ft.
Height of crossarm above ground	47 ft.	57 ft.	63 ft.	80 ft.
Transverse Loading				
Wind on tower	1,200	1,400	1,500	1,750
Wind on 3-conds. with 1½ in. ice at 2.69 lb. per ft.	7,250	8,875	9,700	11,300
Wind on 2-ground wires with 1½ in. ice at 2.42 lb. per ft.	4,350	5,325	5,800	6,750
Pull, 3 conductors due to 25 deg. horizontal angle	22,500	22,500	22,500	22,500
Pull, 2 ground wires due to 25 deg. horizontal angle	14,500	14,500	14,500	14,500
Total load	49,800	52,600	54,000	56,800
Test load, 135 per cent	67,200	71,000	72,900	76,700

Torsional Loading

It is assumed that two conductors and one ground wire are broken and that the pull of the unbroken conductor and ground wire in the adjoining span causes unbalanced loading on the tower.

Load at any one conductor support	17,300	17,300	17,300	17,300
Test load—135 per cent	23,400	23,400	23,400	23,400
Load at any one ground wire support	16,000	16,000	16,000	16,000
Test load—135 per cent	21,600	21,600	21,600	21,600
Vertical Loading (at each Conductor and Ground Wire Support)				
Weight of cond. at 0.9 lb. per ft.	810	990	1,080	1,260
Weight of 1½ in. ice at 4.72 lb. per ft.	4,250	5,200	5,670	6,600
Due to 15 deg. vertical angle	4,500	4,500	4,500	4,500
Insulator and hardware	1,200	1,200	1,200	1,200
Men	400	400	400	400
Repair tackle	1,200	1,200	1,200	1,200
Total load at one support	12,360	13,490	14,050	15,160
Test load—135 per cent	16,700	18,200	18,900	20,500

Design Stresses (Same as for Type B Tower, See Table 4) and the economic study was based upon this profile. For each of four normal span lengths; *i. e.*, 550 ft., 700 ft., 800 ft. and 1000 ft., four actual approximate line designs, based each upon

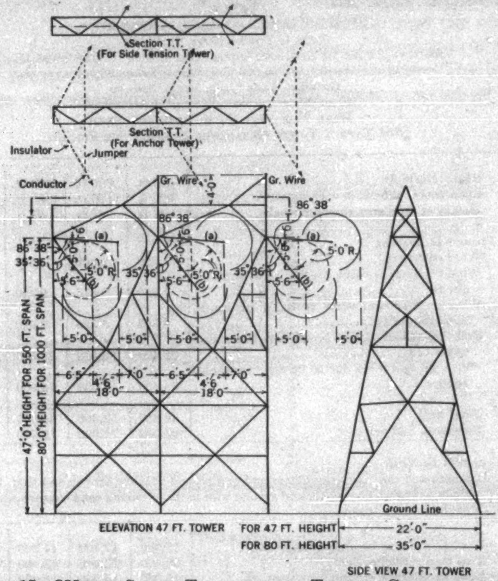

Section T.T.
(For Side Tension Tower)
Section T.T.
(For Anchor Tower)
Insulator
Jumper
Conductor
Gr. Wire
Gr. Wire
4'-0"
86°38'
35°36'
(a)
(b)
5'-0"R.
5'-6"
5'-0"
6'-5"
7'-0"
4'-6"
18'-0"
47'-0" HEIGHT FOR 550 FT. SPAN
80'-0" HEIGHT FOR 1000 FT. SPAN
ELEVATION 47 FT. TOWER
Ground Line
FOR 47 FT. HEIGHT 22'-0"
FOR 80 FT. HEIGHT 35'-0"
SIDE VIEW 47 FT. TOWER

FIG. 15—220-KV. STEEL TRANSMISSION TOWER—CLEARANCE DIAGRAM—TYPE D (SIDE TENSION AND ANCHOR)—VERTICAL ANGLE 25 DEG.—HORIZONTAL ANGLE (SIDE TENSION) 25 DEG. TO 60 DEG. (ANCHOR) 60 DEG. TO 90 DEG. (FOR DESIGN LOADING AND STRESSES SEE TABLE VI)

CONDITIONS CAUSING MAXIMUM SWING OF INSULATORS TAKEN FOR DETERMINING TOWER DIMENSIONS

Position	Span	Horizontal Angle Turned by Conductors	Wind	Ice	Temp.
a	550	→ 60 deg.	→ 8 lb.	0 in.	0 deg.
b	1000	25 deg.	← 8 lb.	1½ in.	32 deg.

its own set of specific tower locations, was carried out, and the cost of each line was estimated. These costs, as plotted in Fig. 18, show maximum economy with normal spans of 800 ft. with

TABLE VI
DESIGN LOADING AND STRESSES

Type D Tower (Side Tension and Anchor)
Vertical Angle 25 Deg.
Horizontal Angle (Side Tension) 25 Deg. to 60Deg., (Anchor) 60 Deg. to 90 Deg.
Conductor, Aluminum 716,000 Cir. Mil.—Steel 92,900 Cir. Mil.
Ground Wire $^{5}/_{8}$ in. Steel Strand
Suspension, Semi-Tension or Tension Insulators
Wind 8 Lb.—Ice 1½ in. at 0 Deg. Fahr.
(For Type D Tower Clearance Diagram See Fig. 15)

Span (Normal)	550 ft.	700 ft.	800 ft.	1000 ft
Span (maximum allowable with maximum angles)	1100 ft.	1300 ft.	1400ft.	1600 ft.
Height of gross arm above ground	47 ft.	57 ft.	63 ft.	80 ft.
Transverse Loading				
Wind on tower	1,200	1,400	1,500	1,750
Wind on 3 conductors with 1½ in. ice at 2.69 lb. per ft.	8,875	10,500	11,300	12,900
Wind on 2 ground wires with 1½in ice at 2.42 lb. per ft.	5,325	6,300	6,800	7,750
Pull 3 conductors due to 90 deg. horizontal angle	73,400	73,400	73,400	73,400
Pull 2 gr. wires due to 90 deg. horizontal angle.	45,300	45,300	45,300	45,300
Total load	134,100	136,900	138,300	141,100
Test load—135 per cent	181,000	184,800	186,700	190,500
Longitudinal Loading (Line terminal tower)				
Pull due to 3 conductors	51,900	51,900	51,900	51,900
Pull due to 2 ground wires	32,000	32,000	32,000	32,000
Total load	83,900	83,900	83,900	83,900
Test load—135 per cent	113,300	113,300	113,300	113,300

Torsional Loading

It is assumed that two conductors and two ground wires are broken and the pull of the conductors and ground wires in the adjoining span causes unbalanced loading in the tower.

Load at anyone conductor support	17,300	17,300	17,300	17,300
Test load—135 per cent of actual	23,400	23,400	23,400	23,400
Load at any one ground wire support	16,000	16,000	16,000	16,000
Test load—135 per cent of actual	21,600	21,600	21,600	21,600
Vertical Loading (At each conductor and ground wire support)				
Weight of conductors at 0.9 lb. per ft.	1,000	1,200	1,300	1,450
Weight of 1½ in. ice at 4.72 lb. per ft.	5,200	6,150	6,600	7,550
Due to 25 deg. vertical angle	7,300	7,300	7,300	7,300
Insulators and hardware	1,200	1,200	1,200	1,200
Men	400	400	400	400
Repair tackle	1,200	1,200	1,200	1,200
Total load at one support	16,300	17,450	18,000	19,100
Test load—135 per cent of actual load	22,000	23,600	24,300	25,800

Design Stresses (Same as for Type B Tower, See Table IV) the rigid type of tower assumed, while longer spans or shorter spans are more expensive. The 800 ft. normal span was selected as a basis for further studies.

The method of carrying out these line designs involved, first, determining the height of tower corresponding to the span length selected. To determine this height, a ground clearance of 25 ft. under design loading was assumed (corresponding with an 800-ft. span to about 35 ft. at 60 deg. fahr) and a sag-clearance template developed for the aluminum-steel conductor used. On level ground, for the 25 ft. clearance, this template gave the standard tower heights for the four normal span lengths under consideration as 47 ft., 57 ft., 63 ft. and 80 ft. For each height of tower, the five standard designs were developed in outline and, with the sag-clearance template, tower locations were spotted on the typical profile selected. With the locations determined, the required type of tower was selected from the

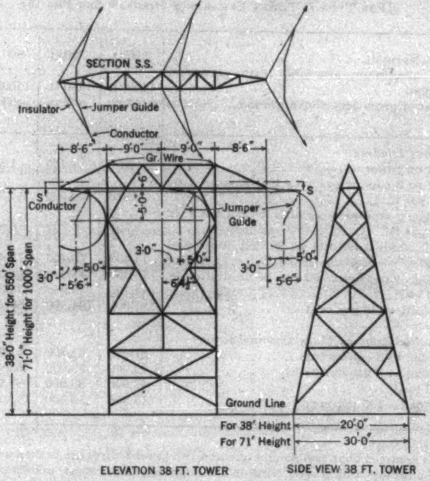

SECTION S.S.
Insulator
Jumper Guide
Conductor
8'6"
9'0"
9'0"
8'6"
Gr. Wire
S
S
Conductor
6"
5'0"
Jumper Guide
3'0"
5'0"
3'0"
5'0"
5'6"
3'0"
5'0"
6'4½"
5'6"
38'0" Height for 550' Span
71'0" Height for 1000' Span
Ground Line
For 38' Height
20'0"
For 71' Height
30'0"
ELEVATION 38 FT. TOWER
SIDE VIEW 38 FT. TOWER

FIG. 16—220-KV. STEEL TRANSMISSION TOWER—CLEARANCE DIAGRAM—TYPE E (ANCHOR)—VERTICAL ANGLE 0 DEG. TO 15 DEG. HORIZONTAL ANGLE (ANCHOR) 0 DEG. TO 60 DEG. (FOR DESIGN LOADING AND STRESSES SEE TABLE VII)

TABLE VII
DESIGN LOADING AND STRESSES

Type E Tower (Anchor)
Vertical Angle 0 Deg. to 15 Deg. and Horizontal Angle 0 Deg. to 60 Deg.
Conductor Aluminum 716,000 Cir. Mil.—Steel 92,900 Cir. Mil.
Ground Wire $^5/_8$ in. Steel Strand
Tension Insulators
Wind 8 Lb.—Ice 1½ in. at 0 Deg. Fahr.
(For Type E Tower Clearance Diagram See Fig. 16)

Span (normal)	550 ft.	700 ft.	800 ft.	1000 ft.
Span (maximum allowable with maximum angles)	1000 ft.	1200 ft.	1300 ft.	1500 ft.
Height of cross arm above ground	38 ft.	48 ft.	54 ft.	71 ft.
Transverse Loading				
Wind on tower	850	1,000	1,250	1,500
Wind on 3 conds. with 1½ in. ice at 2.69 lb. per ft.	8,070	9,700	10,500	12,100
Wind on 2 gr. wires with 1½ in ice at 2.42 lb. per ft.	4,830	5,800	6,300	7,300
Pull—3 conductors due to 60 deg. horizontal angle	51,900	51,900	51,900	51,900
Pull—2 gr. wires due to 60 deg. horizontal angle	32,000	32,000	32,000	32,000
Total load	97,650	100,400	101,950	104,800
Test load—135 per cent	131,800	135,500	137,600	141,500
Longitudinal Loading (Line terminal tower)				
Pull due to 3 conductors	51,900	51,900	51,900	51,900
Pull due to 2 ground wires	32,000	32,000	32,000	32,000
Total load	83,900	83,900	83,900	83,900
Test load—135 per cent	113,300	113,300	113,300	113,000

Torsional Laoding

It is assumed that two conductors and two ground wires are broken and the pull of the conductors and ground wires in the adjoining span causes unbalanced loading in the tower.

Load at any one conductor support	17,300	17,300	17,300	17,300
Test load—135 per cent	23,400	23,400	23,400	23,400
Load at any one ground wire support	16,000	16,000	16,000	16,000
Test load—135 per cent	21,600	21,600	21,600	21,600
Vertical Loading (at each conductor and ground wire support)				
Weight of conductors at 0.9 lb. per ft.	900	1,080	1,170	1,350
Weight of 1½ in. ice, at 4.72 lb. per ft.	4,700	5,670	6,136	7,080
Due to 15 deg.—vertical angle	4,500	4,500	4,500	4,500
Insulators and hardware	1,200	1,200	1,200	1,200
Men	400	400	400	400
Repair tackle	1,200	1,200	1,200	1,200
Total load at one support	12,900	14,050	14,600	15,700
Test load—135 per cent	17,400	19,000	19,700	21,200

Design Stresses (Same as for Type B Tower, See Table IV.)

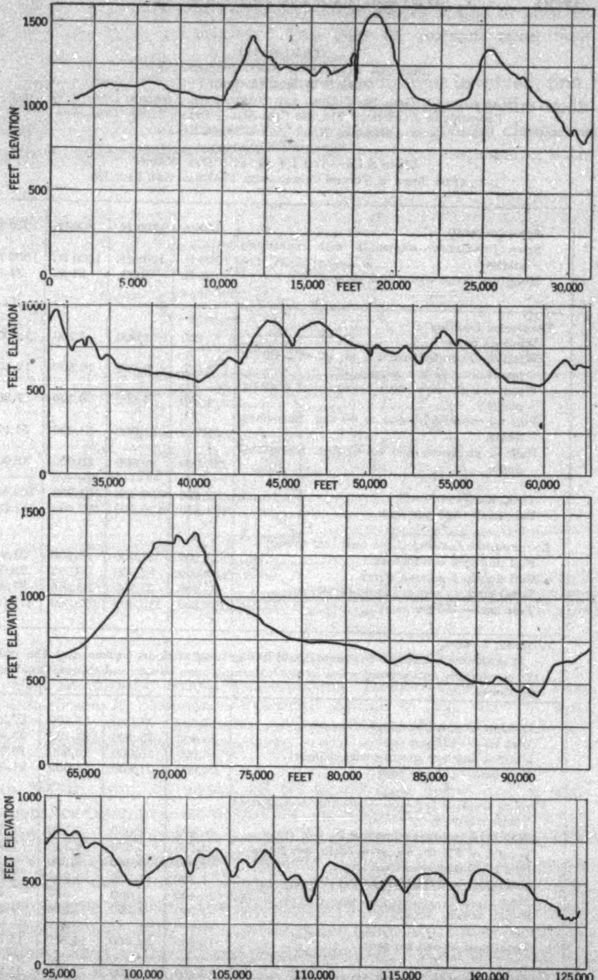


FIG. 17—SECTION OF PROFILE OF TYPICAL TRANSMISSION RIGHT OF WAY USED AS BASIS FOR 220-KV. LINE STUDIES TO DETERMINE ECONOMIC PAN

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220-KV. TRANSMISSION LINE STUDIES PRELIMINARY ESTIMATE TO DETERMINE ECONO

Article	550-foot span						700-foot span			
	Quan.	Unit		Total			Quan.	Unit		
		Used	Cost	Item	Div.	Group		Used	Cost	Item
1. TOWERS	1	mi 1	....	....	....	$10,455	1	mi. 1	....	....
A. *Cost f. o. b. Factory*	10.60	mi.	830	....	8,800	....	8.018	mi.	1,053	....
1. Type A	8.490	each	708	6,020	....	....	6.510	each	890	5,800
2. " B	1.770	"	1,130	2,006	....	....	1.120	"	1,450	1,625
3. " C	0.172	"	1,740	300	....	....	0.216	"	2,200	475
4. " D	0.084	"	3,600	300	....	....	0.086	"	3,950	340
5. " E	0.084	"	2,160	180	....	....	0.086	"	2,460	210
B. Freight	732	cwt.	0.28	....	205	....	703	cwt.	0.28	....
C. Handling and hauling	36.6	ton	6.00	....	220	....	35.1	ton	6.00	....
D. Assembling	36.6	"	10	....	365	....	35.1	"	9.00	....
E. *Excavating and Grading*	....	mi.	....	....	345	....	....	mi.	....	....
1. Type A	8.49	each	30	255	....	....	6.51	each	30	195
2. " B	1.77	"	40	70	....	....	1.12	"	40	45
3. " C	0.172	"	50	10	....	....	0.216	"	50	10
4. " D	0.084	"	60	5	....	....	0.086	"	60	6
5. " E	0.084	"	55	5	....	....	0.086	"	55	5
F. *Setting Footings*	....	mi.	....	....	170	....	....	mi.	....	....
1. Earth	10.35	twr.	10	105	....	....	7.63	twr.	10	76
2. Concrete	0.250	"	260	65	....	....	0.280	"	326	91
a– Type A	....	each	....	....	....	....	....	each	....	....
b- " B	....	"	....	....	....	....	....	"	....	....
c- " C	0.082	"	250	20 –	....	....	0.108	"	300	33 –
d- " D	0.084	"	300	25 –	....	....	0.086	"	350	30 –
e- " E	0.084	"	275	20 –	....	....	0.086	"	325	28 –
G. *Erecting*	....	mi.	....	....	295	....	....	mi.	....	....
1. Erecting	10.60	twr.	18	190	....	....	8.02	twr.	20	161
2. Backfilling	10.60	"	10	105	....	....	8.02	"	10	81
M. Miscellaneous	10.60	each	5	....	55	....	8.02	each	5	....
2. INSULATORS (Suspension)	1	mi. 1	....	....	....	$ 4,390	1	mi. 1	....	....
A. Cost f. o. b. factory	1,379	each	2.30	....	3,170	....	1,040	each	2.30	....
B. Suspension clamps	28	"	4.00	....	112	....	22	"	4.00	....
C. Semi-tension clamps	3	"	5.00	....	15	....	2	"	5.00	....
D. Tension clamps	6	"	6.00	....	36	....	2	"	6.00	....
E. Equalizer yokes	116	"	4.00	....	465	....	86	"	4.00	....
F. Ex. H. equalizer yokes	14	"	6.00	....	85	....	11	"	6.00	....
G. Links	65	"	1.00	....	65	....	47	"	1.00	....
H. Clevis eyes and shackles	259	"	0.50	....	130	....	194	"	0.50	....
I. Arcing horns	62	"	0.50	....	30	....	59	"	0.50	....
J. Freight	180	cwt.	0.40	....	72	....	127	cwt.	0.40	....
K Handling and hauling	9.0	ton	6.00	....	55	....	6.4	ton	6.00	....
L. Assembling and erecting	1,379	disc	0.10	....	140	....	1,040	disc	0.10	....
M. Miscellaneous	1,379	"	0.01	....	15	....	1,040	"	0.01	....
3. CONDUCTORS	1	mi. 1	....	....	....	$ 250	1	mi. 1	....	....
Stringing only	1	" "	250	....	250	....	1	" "	200	....
4. GROUND WIRES	1	mi. 1	....	....	....	$ 75	1	mi. 1	....	....
Stringing only	1	" "	75	....	75	....	1	" "	70	....
5. RIGHT OF WAY	1	mi. 1	....	....	....	$ 3,500	1	mi. 1	....	....
	10.60	twr.	330	....	3,500	....	8.02	twr..	420	....
6. TOTAL MAT'L. AND LABOR (*)	1	mi. 1	....	....	....	$18,670	1	mi. 1	....	....

TABLE VIII

NOMIC SPAN. SELF-SUSTAINING, HORIZONTAL ARRANGEMENT, SINGLE CIRCUIT. STEEL TOWERS

		800-foot span						1,000-foot span					
Total		Quan.	Unit		Total			Quan.	Unit		Total		
Div.	Group		Used	Cost	Item	Div.	Group		Used	Cost	Item	Div.	Group
....	$ 9,880	1	mi. 1	....	....	....	$ 9,895	1	mi. 1	....	....	....	$11,230
8,450	....	6.810	mi.	1,246	....	8,490	....	5.180	mi.	1,889	....	9,770	....
....	....	5.350	each	1,040	5,565	....	....	3.800	each	1,535	5,840	....	....
....	....	1.160	"	1,710	1,990	....	....	1.080	"	2,435	2,630	....	....
....	....	0.172	"	2,530	435	....	....	0.173	"	3,570	620	....	....
....	....	0.086	"	4,400	380	....	....	0.085	"	5,980	510	....	....
....	....	0.042	"	2,820	120	....	....	0.042	"	4,095	170	....	....
195	....	712	cwt.	0.28	....	200	....	813	cwt.	0.28	....	230	....
210	....	35.6	ton	6.00	....	215	....	40.7	ton	6.00	....	245	....
315	....	35.6	"	8.00	....	285	....	40.7	"	7.00	....	285	....
261	....	....	mi.	....	....	259	....	....	mi.	....	....	225	....
....	....	5.35	each	35	187	....	....	3.800	each	40	152	....	....
....	....	1.16	"	45	53	....	....	1.080	"	50	54	....	....
....	....	0.172	"	55	10	....	....	0.173	"	60	10	....	....
....	....	0.086	"	65	6	....	....	0.085	"	70	6	....	....
....	....	0.042	"	60	3	....	....	0.042	"	65	3	....	....
167	....	....	mi.	....	....	180	....	....	mi.	....	....	180	....
....	....	6.60	twr.	15	99	....	....	4.97	twr.	15	75	....	....
....	....	0.210	"	381	80	....	....	0.210	"	500	105	....	....
....	....	....	each	....	....	....	....	....	each	....	....	....	....
....	....	....	"	....	....	....	....	....	"	....	....	....	....
....	....	0.085	"	350	30 —	....	....	0.085	"	450	38 —	....	....
....	....	0.085	"	400	34 —	....	....	0.085	"	500	42 —	....	....
....	....	0.042	"	375	16 —	....	....	0.042	"	475	20 —	....	....
242	....	....	mi.	....	....	232	....	....	mi.	....	....	224	....
....	....	6.81	twr.	22	150	....	....	5.18	twr.	28	146	....	....
....	....	6.81	"	12	82	....	....	5.18	"	15	78	....	....
40	....	6.81	each	5	....	34	....	5.18	"	5	....	26	....
....	$ 3,300	1	mi. 1	....	....	....	$ 3,575	1	mi. 1	....	....	....	$ 2,870
2,400	....	1,152	each	2.30	....	2,651	....	920	each	2.30	....	2,117	....
88	....	18	"	4.00	....	72	....	13	"	4.00	....	52	....
10	....	2	"	5.00	....	10	....	2	"	5.00	....	10	....
12	....	2	"	6.00	....	12	....	2	"	6.00	....	12	....
345	....	45	"	4.00	....	180	....	42	"	4.00	....	168	....
65	....	42	"	6.00	....	252	....	32	"	6.00	....	192	....
48	....	45	"	1.00	....	45	....	42	"	1.00	....	42	....
97	....	202	"	0.50	....	101	....	161	"	0.50	....	81	....
30	....	51	"	0.50	....	26	....	29	"	0.50	....	15	....
51	....	140	cwt.	0.40	....	56	....	112	cwt.	0.40	....	45	....
40	....	7.0	ton	6.00	....	42	....	5.6	ton	6.00	....	34	....
104	....	1,152	disc	0.10	....	116	....	920	disc	0.10	....	92	....
10	....	1,152	"	0.01	....	12	....	920	"	0.01	....	10	....
....	$ 200	1	mi. 1	....	....	....	$ 175	1	mi. 1	....	....	....	$ 150
200	....	1	" "	175	....	175	....	1	" "	150	....	150	....
....	$ 70	1	mi. 1	....	....	....	$ 60	1	mi. 1	....	....	....	$ 50
70	....	1	" "	60	....	60	....	1	" "	50	....	50	....
....	$ 3,350	1	mi. 1	....	....	....	$ 3,250	1	mi. 1	....	....	....	$ 3,000
3,350	....	6.08	twr.	535	....	3,250	....	5.18	twr.	580	....	3,000	....
....	$16,800	1	mi. 1	....	....	....	$16,955	1	mi.1	....	....	....	$17,300

TOWER-DATA

Article	Unit	550-foot span					700-foot		
Type of tower	....	A	B	C	D	E	A	B	
Avg. span on typical profile (a)	ft.	485	526	700	700	700	655	701	6
Towers typical profile (b)	no.	197	41	4	2	2	151	26	
Towers per mile	no.	8.490	1.770	0.172	0.084	0.084	6.510	1.120	
Height of tower	ft.	47	47	47	47	38	57	57	
Weight of tower	lb.	5,900	9,400	14,500	30,000	18,000	7,400	12,100	18,0
Cost of steel per lb.	c.	12	12	12	12	12	12	12	
" " tower	$	708	1,130	1,740	3,600	2,160	890	1,450	2,2
Max. allow. hor. angle	0	2	10	25	90	60	2	10	
Max. allow. vert. angle	0	5	10	15	25	15	5	10	
Weight of towers per mile	lb.	50,100	16,600	2,500	2,500	1,500	48,200	13,500	4,0
" " " " " total	"	....	....	73,200	....	....	....	....	70,3

INSULATOR AND HARDWARE

Article	550-foot span								
Type of tower	A		B		C		D		E
Towers per mile	8.490		1.770		0.172		0.084		0.084
No. insul. strings	2		6		4		2 × 6		2 ×
Total number	per		per		per		per		per
	Tower	Mile	Tower	Mile	Tower	Mile	Tower	Mile	Tower
Insulators (c)	90	764	270	478	204	35	612	51	612
Susp. clamps	3	25	d 3	3	....	....	....	....	....
Semi-ten. clamps	....	....	d 3	3	....	....	....	....	....
Tension clamps	....	....	....	....	e 3	5	6	0.5	6
Equalizer yokes	6	51	30	53	12	2	60	5	60
Ex. H. equal. yoke	....	....	6	11	6	1	12	1	12
Links	....	....	30	53	12	2	60	5	60
Clevis Eyes	18	153	48	85	30	5	96	8	96
Arcing horns	6	34	12	21	8	1	24	2	24

Article	800-foot span								
Type of tower	A		B		C		D		E
Towers per mile	5.350		1.160		0.172		0.086		0.042
No. insul. strings	3		6		4		2 × 6		2 ×
Total number	per		per		per		per		per
	Tower	Mile	Tower	Mile	Tower	Mile	Tower	Mile	Tower
Insulators (c)	135	723	270	314	204	36	612	53	612
Susp. Clamps	3	16	d 3	1.8	....	....	....	....	....
Semi-ten. clamps	....	....	d 3	1.8	....	....	....	....	....
Tension clamps	....	....	....	....	e 3	0.5	6	0.5	0.6
Equalizer yokes	....	....	30	35	12	2.1	60	5.1	60
Ex. H. equal. yoke	f 6	32	6	7	6	1.1	12	1.0	12
Links	....	....	30	35	12	2.1	60	5.1	60
Clevis eyes	24	128	48	56	30	5.2	96	8.2	96
Arcing horns	6	32	12	14	8	1.4	24	2.0	24

NOTES:—(*) Part affected by span only. (a) The average span for the various types of towers is the resultant of applying the template for the normal span to 23.3 mi
an actual profile of a line 23.3 miles in length. (c) 15 discs per insulator string on Towers A and B; 17 discs per insulator string on Towers C, D and E.

ot span			800-foot span					1000-foot span				
C	D	E	A	B	C	D	E	A	B	C	D	E
620	620	620	787	739	743	743	743	1,015	1,060	657	657	657
5	2	2	124	27	4	2	1	88	25	4	2	1
0.216	0.086	0.086	5.350	1.160	0.172	0.086	0.042	3.800	1.080	0.173	0.085	0.042
57	57	48	63	63	63	63	54	80	80	80	80	71
8,000	33,000	20,500	8,760	14,250	21,050	36,700	23,500	12,810	20,250	29,810	49,780	34,120
12	12	12	12	12	12	12	12	12	12	12	12	12
2,200	3,950	2,460	1,040	1,710	2,528	4,405	2,821	1,536	2,435	3,572	5,980	4,095
25	90	60	2	10	25	90	60	2	10	25	90	60
15	25	15	5	10	15	25	15	5	10	15	25	15
4,000	2,800	1,800	46,880	16,530	3,620	3,160	990	48,690	21,880	5,160	4,236	1,434
0,300	....	....	....	....	71,180	....	....	....	....	81,300	....	....

RE DATA

		700-foot span										
E 084 × 6 per	Total	A 6.510 2 per		B 1.120 6 per		C 0.216 4 per		D 0.086 2 × 6 per		E 0.086 2 × 6 per		Total
Mile		Tower	Mile	Tower	Mile	Tower	Mile	Tower	Mile	Tower	Mile	
51	1,397	90	586	270	302	204	44	612	55	612	53	1,040
....	28	3	20	d 3	1.7	....	....	....	....	....	....	22
....	3	....	....	d 3	1.7	....	....	....	....	....	....	2
0.5	6	....	....	....	....	e 3	0.7	6	0.5	6	0.5	2
5	116	6	39	30	34	12	2.6	60	5.1	60	5.2	86
1	14	....	....	6	7	6	1.3	12	1.1	12	1.1	11
5	65	....	....	30	34	12	2.6	60	5.1	60	5.2	47
8	259	18	117	48	54	30	6.5	96	8.2	96	8.3	194
2	62	6	39	12	14	8	1.7	24	2.0	24	2.1	59

		1000-foot span										
E .042 × 6 per	Total	A 3.800 3 per		B 1.080 6 per		C 0.173 4 per		D 0.085 2 × 6 per		E 0.042 2 × 6 per		Total
Mile		Tower	Mile	Tower	Mile	Tower	Mile	Tower	Mile	Tower	Mile	
26	1,152	135	513	270	292	204	36	612	53	612	26	920
....	18	3	11	d 3	1.6	....	....	....	....	....	....	13
....	2	....	....	d 3	1.6	....	....	....	....	....	....	2
0.3	2	....	....	....	....	e 3	0.5	6	0.5	6	0.3	2
2.5	45	....	....	30	32.4	12	2.1	60	5.1	60	2.5	42
0.5	42	6	23	6	6.5	6	1.1	12	1.0	12	0.5	32
2.5	45	....	....	30	32.4	12	2.1	60	5.1	60	2.5	42
4.0	202	24	91	48	51.9	30	5.2	96	8.2	96	4.0	161
1.0	51	6	23	12	1.3	8	1.4	24	2.0	24	1.0	29

.3 miles of actual profile. (b) In determining the number and kind of towers, shown in this horizontal row, a template for given normal span was applied to
E. d-on ½ B Towers. e-special. f.3 way.

[Silver]

known angles and span lengths involved at each tower site. The results of this study are given in Table VIII, showing for each normal span length the numbers required of each type of tower and a comparison of line costs, exclusive of conductors and ground wires for which the costs would be constant.

It is believed that this method of analysis, *i. e.*, using an actual or typical profile, affords a more accurate and reliable basis of determining economic normal span lengths and tower heights than the simpler method of considering only straight lines on a level profile. Further refinements in the way of studying the effect of using a different height or more than one height for the heavier towers might be attempted, but it is questionable whether they would be warranted, in view of the many uncertainties involved, particularly in field work. It will be noted that this study of economic balance is carried out on the basis of the high prices obtained in early 1918. A lower price level would have some effect upon the results, as would a different size or type of conductor and a different type of insulator, particularly if it were stronger and shorter.

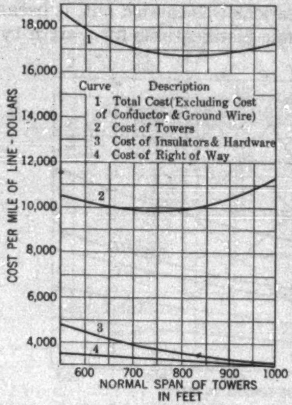


Fig. 18—220 kv. Transmission Line—Determination of Economic Span—Relation of Span Length to First Cost

In the tower designs study has been made of the general form, of the dimensions of the main members and of approximate weights, but the designs have not been carried into the details. No designs have been prepared of tower footings, side hill extensions, etc., which are distinctly contingent upon local conditions and which have no direct or material effect upon the general line designs. Figs. 12 to 16 show the general types of towers proposed, with details as to dimensions, and as to working and test loads. Estimated weights are shown in Fig. 19. The working loads were computed from the design loadings under the most extreme conditions of angles and span lengths for which the tower is to be used. In addition to the main margin of safety provided in the design loading, a further margin is introduced in the difference between working and test loads, *i. e.*, 25 per cent for suspension towers and 35 per cent for angle and dead-end towers. It may be noted that a still further margin of safety will result when a tower is employed in a location imposing less than its maximum designed working load, a condition which will obtain for the major percentage of the line.

After the study of tower locations was carried out, it became evident that there was economic justifications for a sixth type of tower, a special light weight type to be used primarily on short spans in tangents. This tower is illustrated in Fig. 20. An economic improvement in the design shown for this light weight type might be effected by shortening the tower to correspond to the smaller sags obtaining in short spans.

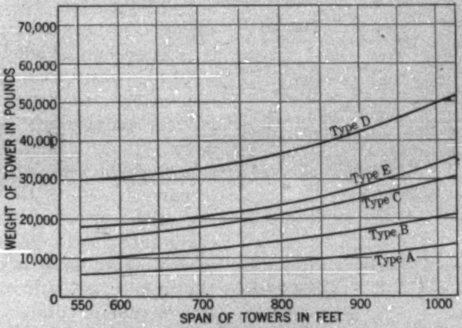


Fig. 19—220-kv. Single-Circuit Steel Transmission Towers— Estimated Weights

Type A—See Fig. 12; Type B—See Fig. 13; Type C—See Fig. 14; D—See Fig. 15; Type E—See Fig. 16

It is believed desirable that sample towers of each design should be subjected to thorough experimental tests before quantity production is started. Tower design involves approximations, and the heavy loads and consequent great weights of 220-kv. towers justify more than usual effort to obtain an economically consistent design. In the method of making such tests, more elaborate and refined methods than hitherto employed might be developed to advantage. It is desirable to ascertain not only the point and manner of failure under test, but also the simultaneous stress conditions existing in all important tower members.

Tower foundations will be largely a special problem for each locality. The conditions governing present practise will apply, the only new considerations being the unusually heavy loads involved and the strong emphasis which must be placed upon dependability. In general, for most soils, earth foundations would probably be adequate for straight suspension towers, while concrete foundations would be necessary for full tension and heavy angle towers and probably for semi-tension and

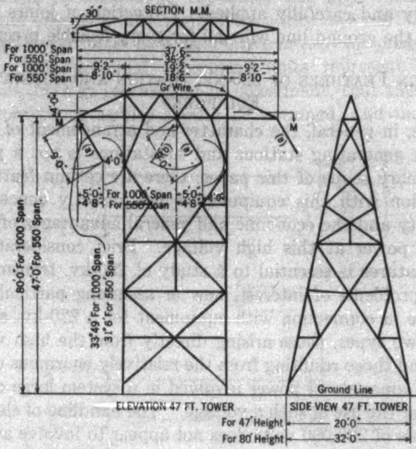

30°
SECTION M.M.
For 1000' Span
For 550' Span
For 1000' Span
For 550' Span
37'6"
36'2"
9'2"
19'2"
9'2"
8'10"
18'6"
8'10"
Gr Wire.
4'0"
M
M
(a)
(a)
(b)
9'0"
4'0"
9'0"
5'0"
For 1000' Span
5'0"
4'0"
4'8"
For 550' Span
4'8"
80'0" For 1000 Span
47'0" For 550 Span.
33°49' For 1000' Span
31°6' For 550' Span
ELEVATION 47 FT. TOWER
SIDE VIEW 47 FT. TOWER
Ground Line
For 47' Height
20'0"
For 80' Height
32'0"

Fig. 20—220 kv. Steel Transmission Towers—Clearance Diagram—Type AA (Suspension)—For Vertical Angle 5 deg. and Horizontal Angle 0 deg.

Conditions Causing Maximum Swing of Insulators Taken for Determining Tower Dimensions

Position	Span	Horizontal Angle Turned by Conductors	Wind	Ice	Temp.
a	1000	0 deg.	8 lb.	0 in.	0 or 120 deg.
b	1000	0 deg.	8 lb.	0 in.	0 or 120 deg.

light angle towers. Where concrete foundations are used, the tower should be carefully grounded.

Protection of towers from deterioration is of more than ordinary importance owing to the great investment represented by a 220-kv. line and the long economic life which it presumably will possess. Galvanizing would seem essential and it should be heavy and carefully applied. Protection of joints and of steel at the ground line will call for every feasible precaution.

DESIGN FEATURES OF 220-KV. STATION AND SUBSTATION EQUIPMENT

While, in general, the character and arrangement of equipment at generating stations and substations is not a part of the primary scope of this paper, there are certain features in connection with this equipment which directly concern the feasibility and the economic and general advantages of transmitting power at this high voltage. Brief consideration of these features is essential to a study of 220-kv. transmission.

The problems of interest, new or assuming particular importance in connection with equipment for a 220-kv. system, are of two types, those arising directly from the high voltage *per se* and those resulting from the relatively enormous capacities and amounts of power involved in a system large enough to call for the use of such a voltage. The handling of electrical potentials of 220,000 volts does not appear to involve any disturbing complications or uncertainties. In fact, the manufacturers do not recognize that any serious problem exists. Current design principles and materials now in ordinary use will be employed, the principal difference from present high voltage equipment being the greater amounts of insulation and the larger clearances required. The step to 220 kv. is relatively no greater than that previously taken from 66 kv. to 110 kv., or from 110 kv. to 154 kv. Certain of the manufacturers have already developed designs, and assert readiness to undertake commercial production of 220-kv. equipment on short notice.

The problems attendant upon the huge capacities which use of 220-kv. transmission makes possible and feasible,—generating stations of several hundred thousand kilowatts, interconnected systems of a half million or a million kilowatts,—are distinctly of a major order. From the purely physical standpoint, the principal problems are those of switching and protection from short-circuit stresses. Both of these, it is believed, can be handled satisfactorily. The more general questions of simplicity and reliability of operation and of efficiency and general economy attain a new order of importance and will probably lead to some significant departures from present practise in smaller scale systems.

Design for such large capacities and high voltage centers around one cardinal principle, *simplicity and intrinsic strength of equipment rather than flexibility and external protective measures.* The principal equipment units represent such high values, both in service and in investment, that the maximum of continuous service should be obtained, and more careful attention and large expense are fully justified in assuring virtual elimination of failures in service. The accompanying arrangement diagram of an assumed typical system, Fig. 21. indicates the extent to which the aim of simplicity is advocated in reduction of bus arrangements to a simple and even rudimentary form and in elimination of superfluous oil switches. In particular, attention may be called to the fact that no spare units, either generators or transformers, are provided, and none are contemplated. The investment in the large units will be so large that outage should be represented only by the irreducible minimum of apparatus troubles and maintenance, not by

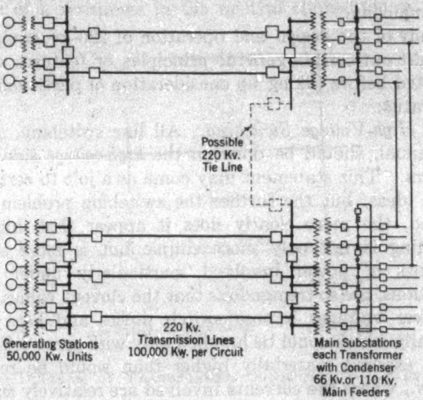

Possible
220 Kv.
Tie Line
220 Kv.
Transmission Lines
100,000 Kw. per Circuit
Generating Stations
50,000 Kw. Units
Main Substations
each Transformer
with Condenser
66 Kv. or 110 Kv.
Main Feeders

FIG. 21—TYPICAL ARRANGEMENT DIAGRAM—220-KV. SYSTEM

the nearly continuous idleness of spare equipment. This complete omission of reserve units of major apparatus is of course predicated upon adequate reserve capacity being available in local generating stations near the load centers, as has been discussed previously.

The obtaining of strength and high reliability of equipment is primarily a problem of construction economics rather than of system design, although the latter is an important element. Certain features of the problem are high reactance of generators and transformers, ample insulation of transformers and an efficient oil circulation system, high mechanical strength of oil switches, avoidance of low-voltage buses and low-voltage paralleling, and an effective and dependable system of relay sectionalizing.

Study of the design and operation of 220-kv. equipment has brought out certain general principles or features which may be noted before taking up consideration of particular pieces of apparatus:

1. *High-Voltage Switching.* All line switching, automatic or manual, should be done on the *high-voltage* side of transformers. This statement may come as a jolt to certain established ideas, but the further the switching problem has been studied, the more clearly does it appear that high-voltage switching is not only more simple but is more safe. The amounts of power involved, particularly under abnormal conditions, are so tremendous that the current values obtaining at lower voltages impose switch duties and heavy stresses generally which could be handled only with great difficulty and at an expense materially higher than would be required at 220-kv., where the currents involved are relatively small.

From the standpoint of automatic sectionalizing and continuity of service, high-tension switching is obviously desirable, since defective circuits in tripping out will leave the transformers in service to continue supplying load from parallel circuits or alternative routes.

2. *Transformer Connections.* All 220-kv. transformers, on the high-voltage side, should be connected in Y with the neutral grounded, as shown graphically in Fig. 22. The high potentials involved and the great lengths of line to be interconnected constitute conditions which might easily result in uncontrolable surges if the high-voltage circuits were isolated from ground. While grounding at one end of a line would materially alleviate

these conditions, more adequate stabilization of voltage, with consequent greater economy in equipment and security in operation, can be obtained by grounding at all points where transformers are located. It may be of interest to note that the voltage stresses on a 220-kv. grounded system (normal voltage to ground 127 kv.), will be less than for some certain existing isolated delta systems. There obtains also the general advantage of grounded Y operation that automatic sectionalizing is made more positive and reliable, a vital consideration on so large and important a transmission system.

The neutral should be grounded without resistance. The purpose of a resistance in the neutral connection is to limit current flow into a ground on a line or in apparatus. For lower voltages where excessive magnetic stresses would result, the expedient is frequently desirable, but for 220 kv., where the short-circuit current values are relatively small and provision for handling the resultant stresses relatively simple, any benefit which might be gained would not be justified by the expense and complication which the use of resistance would involve, and it would in any case be more than offset by the impairment of the function of the ground in stabilizing potentials.

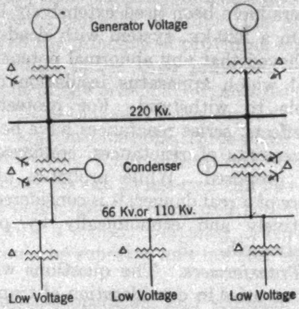


FIG. 22—DIAGRAM OF TRANSFORMER CONNECTIONS—220-KV. SYSTEM

3. *Protective Equipment.* No equipment of any character is contemplated for protection against over-voltages. As a fundamental principle it is believed that investment ca

applied more effectively and with greater ultimate economy in providing greater margins of safety in the apparatus against the stresses which are likely to be encountered, than in attempting to shield weaker apparatus by methods now known. For such a voltage as 220 kv., protective equipment would be proportionately more costly than in case of lower voltages and its efficacy more questionable.

High-voltage protective devices are of two general types, those intended to protect against abnormal potentials at normal or low frequencies, and those intended to protect against high-frequency surges or steep wave fronts which might give rise to localized high-potential stresses in the windings of transformers and in other apparatus. For protection against high potentials, electrolytic arresters and other so-called lightning arresters have been used extensively in systems of lower voltage. On a 220-kv. system with dead-grounded neutrals, it is not believed that any abnormal potentials are likely to be encountered which apparatus insulation cannot and should not be able to withstand. For protection against high-frequency effects, series reactances have been used and elaborate arrangements of reactances, condensers and resistances have been proposed. While probably high-frequency constitutes more of a real danger, it is considered that it can be met most effectively and economically by providing adequate insulation strength.

220-*kv. Transformers*. The questions which naturally will be the first to arise in consideration of transformers for a 220-kv. system are those of size of units and three-phase vs. single-phase units, the two questions being to some extent interdependent.

Considerations of simplicity in station arrangement and of economy in operation favor, in general, transformers of the largest size permitted by the conditions of each installation or by limitation of manufacture. Theoretically there seems no limit to the capacity of a transformer unit, but beyond a certain capacity the provision of strength to withstand the mechanical stresses of short-circuit current becomes a matter of such complication and expense that an economical limit will come into effect. At the present stage of the art, the manufacturers advocate that transformers be not attempted beyond 50,000 to 60,000 kv-a. for three-phase units, or 35,000 to 40,000 kv-a. for single-phase units. Transformer cores and windings for these capacities can be shipped assembled from the factories. Advance quotations and estimates from manufacturers show progressively lower unit costs up to these sizes. The economic advantage of large units depends not so much, however, upon lower unit costs as upon considerations of space, bus construction, number of switches, etc., all of which involve relatively less expense for larger units. The adverse difference in cost of handling facilities is not great.

As to choice between three-phase and single-phase units, from the standpoint of simplicity and cheapness of installation, three-phase units would be preferable. As to factory cost and efficiency, there appears on the basis of information at present available to be little difference, three-phase transformers being somewhat cheaper for same capacity of bank and somewhat more expensive for same capacity of unit. The great weight which should be given the principle of providing high intrinsic reliability of main units, rather than providing spare units, materially reduces one of the principal advantages claimed for single-phase transformers, that of the less capacity required in reserve units. For large installations, three-phase units offer advantages. For relatively small substations, it is believed that equally satisfactory service at less cost can be rendered by a bank of two three-phase units as compared with one bank of single-phase units with a spare. Certainly a bank of three three-phase units would be superior, and the cost should be no greater.

At generating stations, operating and economic considerations strongly favor the generator and its transformer equipment as a unit. If the generator capacity should exceed the practicable limit of three-phase transformer capacity, there apparently would be a close margin of choice between using a single-phase bank or two three-phase transformers operating as a bank and controlled by one circuit breaker.

In past high-voltage practise, probably one of the most troublesome features in operation has been the bushings. Experience indicates, however, that satisfactory bushing designs have now been developed and can readily be applied to 220-kv. service. Bushings will of course be identical and interchangeable for transformers and oil switches. Transformers will have but one high-voltage bushing per phase, the neutral ends of the winding not being insulated from the tank.

The saving in equipment costs which has been noted as one of the reasons for selecting grounded Y rather than delta for the 220-kv. transformer connections, is very marked in the case of transformers. Insulation is less throughout the whole transformer and extra insulation is required at only one end of the windings rather than at both, cores and tanks are correspondingly smaller, efficiency is higher, amount of oil required is less and bushings are lighter and shorter.

The large sizes of installations involved in a 220-kv. system, together with the necessity for careful attention to maintaining insulation strength and transformer reliability generally, will not improbably result in a radical change in the present method of handling and cooling transformer oil, *i. e.*, the abandonment of cooling of oil in each transformer and gravity circulation of oil through the windings in favor of forced oil circulation and external cooling. The idea of external cooling is by no means new, but whereas for the relatively small installations of present practise it appears to involve unjustified complication and expense, for installations of the size here contemplated this condition will probably be reversed. The apparent advantages, briefly summarized, are as follows:

a. Tanks smaller and cheaper due to elimination of cooling coils and smaller volume of oil in transformer.

b. Positive circulation of oil and possibility of more accurately directing it in its flow through the windings to the points needed and in amounts needed.

c. More effective cooling due to low temperature which may be obtained in incoming oil and more rapid circulation.

d. Oil kept absolutely dry and always at maximum insulation strength due to possibility of sealing oil system against the atmosphere, a most important advantage for extra high-voltage operation.

e. Oil may readily be filtered continuously to any extent desired.

f. System as a whole more reliable and probably cheaper.

Such an external transformer oil system might be developed to handle all oil in a common system, or to keep the oil for each transformer bank separate. The common system would be somewhat cheaper and simpler, and hence preferable if adequate precautionary measures could be worked out to prevent contamination of the entire oil system as a result of a failure in one transformer. The system in either case would include cooling devices located wherever desired, probably in a pond or stream, oil pumps, filters, a storage tank at a suitable elevation to provide circulating head, and at some point an expansion device which would maintain a reasonable pressure regulation under conditions of varying volume of the enclosed oil due to temperature changes. The entire oil system would be effectively sealed against the atmosphere. Vital parts of the oil system would presumably be installed in duplicate, the expense being small in proportion to total cost of the station.

A system of external cooling of oil may, at generating stations, open the way for a significant increase in overall efficiency. Transformer losses will amount to about 1.25 per cent of station output. Possibly half of this heat may be recovered from the oil by passing it, on its way to the final cooling coils, through some device installed in the steam system between the condensers and the economizers. While no studies have been made of the economic feasibility of such a scheme, the physical possibility opens an interesting problem.

220-*kv. Oil Circuit Breakers.* A system such as here contemplated, with huge generating stations, numbers of them interconnected, and large synchronous condenser capacity at substations, will necessarily involve unprecedented concentrations of energy at switching points, particularly in case of short circuits. The high duty required of circuit breakers to meet this condition is a vitally important problem in the design of such a system. It is, as has been noted, early evident in a study of the switching problem, that switching should be done wherever possible on the high-voltage side of transformers, since the high voltage can more easily be handled than the high currents at lower voltage. Two hundred and twenty-kv. short-circuit current values are not extreme, and in a 220-kv. circuit breaker the large clearances and switch openings necessarily involved by the high voltage contribute directly to giving the rupturing capacity required by the current to be handled.

Magnitudes of 220-kv. circuit-breaker duty will, of course, depend upon the size and arrangement of the 220-kv. system. With a system of the capacity and arrangement shown in Fig. 21, the duty of a circuit breaker at the bus of either the larger generating station or the larger substation would, according to approximate calculations, be from 1,000,000 kv-a. to 1,500,000 kv-a., depending upon the length of the 220-kv. lines. At the smaller generating station or substation the duty would be somewhat less. No difficulty should be involved in obtaining circuit breakers for this duty at 220-kv. With the gradual expansion which might be expected in such a power supply system, with other generating stations and substations added, extensive interconnections made in secondary networks and probably a considerable 220-kv. network established, there might be expected circuit breaker duties of 3,000,000 to 4,000,000 kv-a. Manufacturers foresee no insurmountable difficulty in designing 220-kv. circuit breakers for duties of this magnitude. The duty on such an extensive system might be reduced by adoption of some degree of sectionalized operation of the 220-kv. lines, but it is hoped that the need for such an expedient can be avoided.

Two types of circuit breakers have been offered by the manufacturers for 220-kv. high duty service. One type consists of a massive circuit breaker, each phase in one tank of heavy boiler plate, with two breaks in series for moderate duty and four breaks in series for heavy duty. The tank is built according to the principles of boiler design to withstand the internal pressure generated by the opening under oil of the rated kv-a. loads, as predetermined by test and calculation. The tank is not designed to withstand explosions in the space above the oil, this hazard being eliminated by provision for thorough ventilation or other expedients to prevent formation of explosive mixtures in this space.

Another type of 220-kv. circuit breaker consists of two breaks in series per tank, one such tank being used for conditions of moderate duty and two tanks in series for heavy duty. The four series breaks in the double-unit circuit breaker are operated simultaneously. These tanks likewise are designed to withstand the stresses generated under the oil under conditions of maximum duty, but in this case the full stresses are not permitted to come upon the tank. Each break is located in a carefully designed explosion chamber, of relatively small diameter and of any required strength, which confines the initial force of the explosion and serves both to reduce the pressures which can be set up in the main tank, and to utilize this force as an aid in extinguishing the arc. With this type of circuit breaker, one tank unit would probably be installed initially when the 220-kv. system was small, and a second unit added later when the growth of the system brought about power concentrations in excess of the rupturing capacity of the ingle unit.

It is believed that either of these types of circuit breaker will give satisfactory service on a 220-kv. system.

220-*kv. Air-Break Disconnectors.* Air-break disconnectors for 220-kv. service present a problem which, while distinctly of a minor character in relation to the general problems of 220-kv. construction, is troublesome, and for which no wholly satisfactory designs have thus far been offered. The difficulties attending development of such disconnectors do not concern feasibility, since a disconnector which will operate acceptably can be built after existing designs, but concern rather avoidance of unnecessary expense, space requirements, and general complication. The problem is one of making a firm high-capacity contact, without incurring hazard or undue inconvenience to operators or to service, under any sort of weather conditions, between two elements supported by long, heavy, cumbersome and somewhat fragile insulators.

These 220-kv. disconnectors would be used to disconnect an oil switch from or connect it to a live line or bus. It is not contemplated that such a switch need be able to break the charging current of a 220-kv. transformer, although a switch capable of this duty would be valuable if it could be developed. It might be used to take the place of a 220-kv. circuit breaker in some cases or to give added flexibility in cases where transformer circuit breakers were not provided.

In situations where disconnectors need not be operated with either pole alive, as for instance between transformers and transformer circuit breakers at the generating station in Fig. 21, the cumbersome 220-kv. disconnector would not be needed, and some form of simple readily removable link would be used.

Generating Station Arrangement. Simplicity should be the dominating principle in the electrical layout of the generating station. The generator and its transformer should be a unit and there should be no low-voltage bus and no low-voltage paralleling. For stations of the size contemplated, a bus at generator voltage would involve such tremendous duty on oil switches, such elaborate sectionalizing reactances and, in general, such expense and hazard to reliability of operation, all without any material benefit, that it unquestionably should be omitted.

Preferably, although not necessarily, a specific bank of generators should be assigned to each 220-kv. circuit, thus enabling segregation of circuits at the generating stations, if desired. The economic capacity of a 220-kv. circuit is roughly 100,000 kw. to 150,000 kw., so that usually this would mean two, or possibly three, generators per circuit. In case conditions governing generating station practise should favor larger generators, say of the order of 100,000-kw., and if satisfactory units of this capacity should be developed, simple station layout would be possible by using one generator to each 220-kv. circuit.

In case it were necessary for a 220-kv. generating station to deliver power also at a lower voltage, for instance 66 kv., it would still be desirable to avoid a bus at generator voltage. The preferable arrangement for such a station would be a bus at each line voltage, each fed by its own generators. These two busses might or might not, depending upon the particular conditions obtaining, be interconnected by transformers.

The omission of the low-voltage bus would not complicate the problem of auxiliary station service, since good practise already provides that such service shall be supplied from a special generator rather than from the station bus.

Synchronizing at 220-kv. can be effectively accomplished, it is believed, by an adaptation of the static synchroscope.

Main Substation Arrangement. Two hundred and twenty-kv. substation layout will be influenced largely by the particular local conditions of each installation. The usual type will probably serve to step down from 220 kv. to a secondary transmission voltage, such as 66 kv. or 110 kv. In such a case it will, in general, be necessary to have both a primary and a secondary bus system,—at 220 kv. a simple bus with sectionalizing circuit breakers, at secondary voltage, where greater flexibility would seem desirable, probably a ring bus. Owing to the high current values which would obtain at the secondary voltage, (it should be kept in mind that the large condenser capacity will aggravate short-circuit conditions), it appears advisable to provide sectionalizing reactances in this bus.

Synchronous condensers, of such capacity as may be demanded by length of 220-kv. line, amount, load factor and power factor of load, will be provided for each transformer bank. In the usual case, these condensers will be connected to a third winding in the main transformers. This method of connecting the condensers appears to be simpler, cheaper, and equally reliable as compared with connecting the condensers to separate transformers on the low-voltage bus, although the corrective effect would not apply to the secondary windings. Three-winding 220-kv. transformers, with the tertiary winding of 50 per cent of the capacity of the 220-kv. winding, would cost about 15 per cent more than two-winding transformers. The voltage of this tertiary winding and of the condenser would be as high as may be found practicable for satisfactory operation of this type of apparatus.

In cases where the secondary substation voltage were within range of generator operation, 11 kv., 13 kv. or 22 kv., the condensers might be connected directly to the secondary bus.

Relay System. The station and substation arrangements which have been proposed are predicated upon a relay system which may be depended upon for effective and consistent automatic operation, *i. e.*, for insuring that a faulty piece of apparatus, transmission line, or low-voltage feeder will be cut out correctly, promptly and in such a manner as to avoid interruption to the other elements of the system. Complete multiple operation of all 220-kv. apparatus, stations and lines is feasible only with a thoroughly dependable relay system. In view of the encouraging developments in the relay field during recent years, and of the fact that on such an important system much effort can be concentrated in working out a solution, it seems reasonable to assume that an adequate relay system will be available.

Complete 220-kv. multiple operation is essential to obtaining full economy and service reliability from the system, and any resort to sectionalized operation, due to lack of adequate relays, unreliability of circuit breakers or any other cause, should be looked upon as a distinct failure to develop the full possibilities of the system and as a temporary expedient to be dispensed with as soon as possible. With complete multiple operation, transmission lines are used to maximum efficiency, there is full flexibility in shifting of load between generators or stations, and, with dependable automatic disconnecting, trouble in any part of the system may be isolated with a minimum of disturbance to load.

CONSTRUCTION COST

The following estimates are intended to give an indication of the installation cost of construction carried out along the lines of the assumptions and recommendations in this paper. Local conditions, of course, may require a considerable variation in arrangement and cost, particularly of substation apparatus. These estimates are based on early 1919 prices, and apply to station capacities of 200,000 kw. and larger at 220 kv.

Step-Up Substation. Outdoor transforming and switching structures and equipment (220-kv. apparatus and connections only), installation and indirect expense.		$8 to $9 per kw.
Step-Down Substation. Outdoor transforming and switching structures and equipment (does not include low-tension feeder bus or feeder switching equipment) synchronous condensers of capacity for length of connected lines, building, control equipment, installation and indirect expense....		$15 to $20 per kw.
Transmission Lines. Single circuit steel towers suspension insulators.		
Towers........................	$8000 per mi.	
Insulators and Hardware........	2800 per mi.	
Conductors and Ground Wire....	5000 per mi.	
Special Structures...............	1000 per mi.	
Right-of-way..................	3000 per mi.	
Indirect expense...............	3700 per mi.	
Total....................		$23,500 per mi.
Total Cost 220-kv. Transmission.		
100 mi........................		$40 to $45 per kw.
200 mi........................		60 to 65 per kw.
300 mi........................		80 to 85 per kw.

CONCLUSIONS

1. It is generally recognized that the country's industrial advancement will require increasingly great amounts of electric power, particularly in view of the relation between use of power and unit productiveness of labor, and that this power must be supplied in accordance with a rational policy of conservation of our greatest national asset, our fuel resources. This points to the necessity of large scale transmission of power from distant points of generation at the energy sources, coal fields and water powers.

2. Two hundred and twenty kv. appears a logical choice for such large-capacity, long-distance transmission, which is clearly beyond the economic range of present transmission voltages. Two hundred and twenty kv. is high enough to meet pending

requirements in power supply evolution but not so far beyond existing practise as to involve question of its immediate feasibility.

3. Design of a 220-kv. system brings up important problems, new in character or significance. These problems are due rather more to the large amounts of power involved than to the high voltage.

4. Two hundred and twenty-kv. power supply will require unprecedentedly high service standards, load factors will be high, particularly at the start, and the frequency should be 60 cycles. For full economy the load for each 220-kv. circuit should be 100,000 kw. or higher.

5. The selection of type and size of conductor is an important economic problem involving a balance of many factors. Corona does not enter as an especially serious limitation owing to the large size of conductors otherwise required. Research is needed as to best method of stringing conductors.

6. Insulators of present commercial disk types can be made to give acceptable service, but there is need of an insulator more efficient electrically, stronger mechanically and of unquestioned durability. There is opportunity for valuable research and development work in this field.

7. The familiar single-circuit rigid type of steel tower appears to be most conservative for immediate use, owing to its reliability having been demonstrated by experience. The field offers promising possibility for development of other types of single-circuit and multiple-circuit towers. The economical span length (here taken as 800 ft. normal), may vary with the factors of each situation. A series of standard towers of graduated weights will be advisable.

8. The basic idea of 220-kv. station and substation design is simplicity and intrinsic strength of equipment, rather than flexibility and external protective measures.

9. Transformer connections should be Y on 220-kv. side with neutral dead grounded. Development of 220-kv., large capacity transformers presents no undue difficulties. Frequently the design might advantageously employ forced oil circulation with external cooling. Present types of bushings, suitably adapted, seem satisfactory for 220-kv. service.

10. The 220-kv. system should be laid out for multiple operation and switching should be done at 220-kv. A thoroughly dependable and carefully coordinated relay system is a basic requirement.

11. Development of circuit breakers presents no serious difficulty from standpoint of voltage, and it is believed that proposed designs will be adequate for the duty which will be demanded.

12. At generating stations each generator and its step-up transformer bank should be treated as a unit. There would be no low-voltage bus and no low-voltage circuit breaker.

13. At substations each transformer would be a unit with a synchronous condenser supplied from a tertiary winding.

14. The studies which have been made and the tentative designs which have been built up serve to establish confidence in the conclusion that 220-kv. transmission is immediately feasible as a commercial proposition.

15. It is hoped that public presentation of these 220-kv. studies may serve in some measure to facilitate the working out of this advance in the transmission art, and that in particular it may promote interchange of ideas, bring out constructive criticism and stimulate needed investigations.

The author wishes to offer acknowledgment and express appreciation of the assistance of his associates and of various engineers and manufacturers whose ideas have been sought and freely used in the preparation of this paper.

DISCUSSION IN "PROBLEMS OF 220-KV. POWER TRANSMISSION (SILVER), LAKE PLACID, N. Y., JUNE 27, 1919.

W. S. Murray: I think Mr. Silver has made a contribution apropos of the times and for it I have had in mind for some time a very concrete application.

The time is before you for very large things and it would seem to me that 220,000-volt transmission will offer an opportunity to exercise ourselves in that field.

The amount of waste due to improper generation of power, its transmission and application, is something enormous—far beyond, I think, the conception of many of you here present. I can epitomize such a statement by saying that I believe that a careful investigation will show that in the Boston-Washington territory, 450 miles long and say 100 miles inland, the amount of waste is about $300,000,000 per annum.

The plan of the super-power transmission to save that waste, is to establish on the coast, stations in which there will be units not smaller than 35,000 to 40,000 kw., each of these stations aggregating 50,000 to 1,000,000 kilowatts as a whole, and at the same time developing such hydroelectric points on rivers and steam stations at the mouths of mines as will economically justify themselves; all of this power so generated being transmitted to the super-power line to give an economy of between ten and eleven pounds of steam per kw-hr., the consumer securing it say somewhere between twelve or thirteen pounds.

The average rate for the generation of power throughout territory taking into account the steam power factory drive is somewhere between twenty-five and thirty pounds of steam, per kw-hr. Thus this rate would be cut in two.

In my experience with railroad electrification there is no question that the present economy of the electric engine over the steam, taking passenger, switching and freight, into consideration is in a ratio of two to one. In the case of steam driven factories it would be much greater than that.

The load factor of a steam railroad, say on the order of the New York Central or the New York, New Haven & Hartford, can be made to be 70 per cent. On the New Haven there are opportunties on its main line division of making it 75 per cent. As we all know, the load factor of the central station is good if it is 35 per cent to 40 per cent and so, in combination with the load factor of the railroads and that of industrial power and lighting, we might hope to get an average load factor of something like 50 per cent.

The idea of economy alone is good but the plan is even far more reaching when we consider that we must create very shortly some form of common carrier for power. I think that you will be surprised when I tell you that the cargo space used by coal shipment today on the Northeast Atlantic seaboard is 40 per cent and this means that our industrialism, which is expanding tremendously (all the earmarks of it are before us now), the necessary transportation to take care of it falling far behind, and therefore if we can take this enormous coal shipment off the rails and put it up overhead, we will take a tremendous burden off of the railroads.

There are two great things that enter the consideration of this matter. One is the question of the coal men who will only have to carry one pound of coal against two, if the economies I have suggested are correct. The answer is not to say to the coal man that his railroad is going to be knocked out of its dividends if it is a coal carrying road, but that his road is going to become the highway for new industries. New industries will spring up along that road and the space created by eliminating a part of the coal, will give space for the new industrial commodities which can be hauled at a higher rate.

In other words, it is a constructive and not a destructive policy.

The other most important question is the matter of the obsolescence of the plants already established. What are we going to do with them? The answer to that, I think, is not a difficult one. We are going to use those plants and we will use them this way: On peak load they will be a most valuable asset in the regional system. On light load they will be most valuable as condensers to take care of the idle or magnetizing current in order that the larger generating units may generate at maximum efficiency the real power. Thus instead of facing obsolescence their life will be extended in this new field.

Another question: What will be the corporate status of the present companies? The primaries of today will become the secondaries of tomorrow. The power will be generated in bulk and be transmitted to the present companies who will maintain their present entity as distributors and so far as franchise rights, etc., are concerned, will remain exactly as they are today.

If an investigation is made, and I trust will be, and we find, as I have stated that there are $300,000,000 being wasted in this zone, it seems to me that as we are now mining some 75,000,000 tons of coal anthracite, and we have only got 75 years of it left, and as we have jumped from 100,000,000 of bituminous coal mined per annum to 580,000,000 that every engineer must fight for this conservation plan.

It is very well to say that we have 3,000,000,000 tons of coal, which we have, in this country but where is it? It is not in a place where it can be economically transported. Much of it is away out in the Alaska fields. Of course we have the Pennsylvania, Illinois and Ohio fields but if you can see the acres that have been taken out of them you will see the force of the necessity of our economizing, I think, as much for ourselves as for the sake of posterity.

I want to leave a final thought with you in this statement:

When the matter of this investigation is presented to Congress an appropriation will have to be made to cover the work, and I think that we ought to all of us consider this not a local matter but a national one. It seems to me that this viewpoint is fair enough. The Senator from Ohio or the Senator from Nevada or even the Senator from California might get up and say "Well, why not make a super-power transmission in our territory? Why should we not vote for the Northwest Coast instead of the Atlantic seaboard?"

I think that that would be a very short-sighted attitude because the finishing shop of the United States is on the Northeast Atlantic seaboard for American industry. There is very nearly 80 per cent of the country's industry concentrated right here. Now, if we are going to maintain the American standard of wage and living while competing with the world —and this is the great angle of vision we should take now— that is where the American product must continue to be finished and placed on shipboard.

I was very happy indeed at the Boston Section to have that branch of the American Institute of Electrical Engineers, vote a resolution of approval of Secretary Lanes' proprosed appropriation to cover this investigation and as I said before, we can just visualize this great line constructed as described, into which is fed a kilowatt hour at an expenditure of ten pounds of steam keeping in mind for economies sake the wonderful contributing factors of load factor, to say nothing of breakdown service which will be established between plants. Surely this is a sound vision, it is certainly not impractical.

I don't believe I could leave a better slogan with you in closing than saying, that lately we have been spending billions for destruction for preservation, now let's spend billions for construction for conservation.

W. M. Dann (Pittsburgh): The transformers that Mr. Silver speaks of are certainly very interesting on account of their size and the high operating voltage. They are the largest units that have so far been proposed seriously, but I believe the manufacturers feel no hesitancy at all in building units that will be dependable and reliable in service.

On page 1091 referring to the choice between three-phase and single-phase units, it seems to me that all the arguments are in favor of single-phase units, particularly since the three-phase unit as large as 100,000 kv-a. seems to be impractical at the present time. For a bank of that size three 35,000-kv-a. single-phase units will be cheaper than two 50,000-kv-a. three-phase units. The switching would be simplified, the floor space required would be less and there would be some gain in efficiency.

The high intrinsic reliability which Mr. Silver refers to can almost be said to be characteristic of power transformers nowadays because they are actually responsible for very little real trouble. However, 100 per cent reliability is an ideal which we will have to admit has not yet been reached and it seems to me from the point of view of insurance that it would be unwise to figure on operating such a big and important system without spare transformer capacity. If this is conceded, the arguments in favor of single-phase units are even stronger.

The statement is made that for relatively small substations equally satisfactory service can be rendered at less cost by a bank of two three-phase units as compared with one bank of single-phase units with a spare. It seems to me that the arguments are in favor of single-phase units particularly since spare units are provided with the single-phase and not with the three-phase transformers.

Operating the generator and the transformer equipment as a unit will very likely meet with pretty general approval. Here again I believe that the choice of units is in favor of the single-phase transformer.

The suggestion that the future may see a radical change in the present method of handling and cooling transformer oil is certainly a very interesting one. Heretofore the forced cooling of transformer oil has not been very common and not very popular and the reason it has not been popular is the expense and complication and the lack of real necessity for the forced cooling system. It may be that in connection with transformers of this unusual size, experience may bring up some new points which will make it seem a little more desirable.

On page 1092 is given a list of apparent advantages for the forced cooling system and just to take the opposite side of the argument for the moment, I would like to submit a few points to go with this list:

(a) Tanks are smaller and cheaper due to eliminating the cooling coils. The saving in size is really not very great. It appears only in floor space because the height is practically fixed by the insulation and the length of the bushings for 220,000 volts. In other words, the height of the transformer is practically the same whether the cooling coils are internal or external. The saving in cost, if copper cooling coils are taken into consideration, amounts to from 5 per cent to 8 per cent. If iron cooling coils are taken into account, the saving is considerably less. This is not a net saving, for the cost of the external system with its pump and motor is greater than the internal cooling coils.

(b) Assuming that the ducts in the windings are free from restrictions and that there are no places where the oil does not flow naturally in response to the thermal head, there is nothing more positive in circulation or more positive in cooling than the natural flow of oil set up by the heat of the windings and it would seem that forced cooling should be unnecessary unless some new and compelling reasons are developed to justify it.

I question whether the rapid circulation of oil in a water-cooled transformer is fully appreciated, for it is really more rapid under the natural conditions than with forced cooling unless the external piping and pump are unreasonably large. The thermal head in a transformer forms one of the most efficient pumps. Anyone who has ever looked into a transformer when operating under load will remember the boiling of the oil at the surface due to the strong circulation.

(c) The temperature of the oil at the bottom of the case is dependent only on the size of the cooling plant, irrespective of whether it is internal or external, and low temperature of the ingoing oil is not specially characteristic of the forced cooling system.

(d) The oil must of course be kept absolutely dry, and if it is conceded that we can't get reliable cooling coils placed inside the transformer, then the present almost universal practise would have to be abandoned. It seems to me that we can count on reliable coils and perhaps the risk of getting water into the oil is not very much greater than the risk of getting air into the external system through the piping and pump, and that of course would be a very serious thing with these high voltages.

(e) Continuous filtering of the oil can be carried out whether the transformer is cooled by natural cooling or by forced cooling. It would be impracticable to filter the main flow of oil supplied to a transformer of this size because the volume is too great.

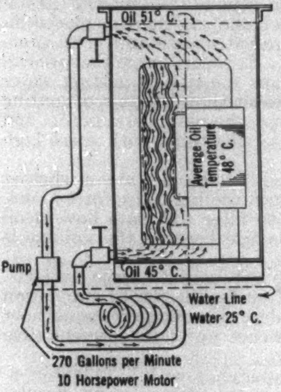
(f) It may be that when we get used to large systems, having these high-voltage transformers, the forced circulation of the oil may appear to be more desirable and perhaps cheaper than it has heretofore, but experience up to the present time really does not indicate it.

I happen to have had some experience with a 14,000-kv-a. three-phase transformer which is cooled with a forced circulation of the oil through an external system. This is quite a little smaller than a 35,000-kv-a. or 40,000-kv-a. single-phase unit but its test results are interesting and it seems as though they ought to be useful in considering the forced cooling of large transformers. This transformer is supplied with 270 gallons of oil per minute and is circulated by means of a pump driven by a 10-h.p. motor. The maximum temperature at the top of the oil is 51 deg. cent. and at the bottom, before it enters the case, 45 deg. making a difference of 6 deg. from bottom to top.

Now, if that same transformer were cooled in the more usual way with water circulated in internal cooling coils, it would require approximately 60 gallons of water per minute and a pump requiring not over 2 h.p. to drive it. The temperature of the oil at the top would be 51 deg. cent., the same as before, but at the bottom it would be 47 deg., a difference of only 4 deg. from bottom to top. The temperature of the water is 25 deg. cent. in both cases.

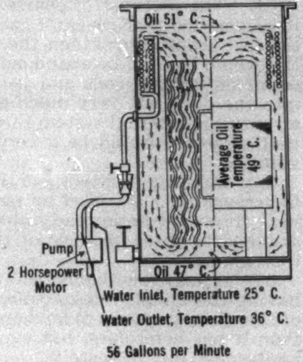
The interesting points here are that:

(1) The difference of temperature of the oil from bottom to top as a water-cooled unit is only two-thirds of what it is as a forced-cooled unit. This means that the circulation of the oil is about 50 per cent more rapid and 50 per cent greater in volume when the transformer is cooled with the natural circulation. This is a good illustration of the rapid flow of oil in a water-cooled transformer, which I said a moment ago perhaps is not fully realized. The thermal conditions tending to produce the rapid circulation of the oil in a water-cooled unit, are all present when it is operated as a forced-cooled unit, but the restriction to the flow of the oil in the piping and the external cooling coils is great enough to pull the flow down to about two-thirds of what it would be as a water-cooled unit. In other words, the efficient pumping effect of the thermal head plus the external pump are not sufficient to produce the flow that would result from the thermal head alone simply on account of the friction head of the external system and the only way to bring the circulation up to a par with the natural water-cooled circulation would be to spend an unreasonable amount of money on the external piping, pump and motor. Now this may seem surprising to some but when one stops to think that it requires very little pressure to make the oil flow up through the windings and to return it down the sides of the case and that it requires a very considerable pressure to circulate it


Oil 51° C.
Average Oil Temperature 48° C.
Pump
Oil 45° C.
Water Line
Water 25° C.
270 Gallons per Minute
10 Horsepower Motor

FORCE COOLED TRANSFORMER

Fig. 1


Oil 51° C.
Average Oil Temperature 49° C.
Pump
2 Horsepower Motor
Oil 47° C.
Water Inlet, Temperature 25° C.
Water Outlet, Temperature 36° C.
56 Gallons per Minute

WATER COOLED TRANSFORMER

Fig. 2

through the external piping and cooling coils, the results are not so surprising.

(2) The temperatures are more uniform under the natural cooling condition than with the forced cooling condition in this transformer. This follows as a natural result of the greater flow of oil.

(3) If enough money were spent in pump and motor to double the flow of oil that exists under the forced cooling conditions the gain in maximum oil temperature would be only 1.5 deg. over the natural water-cooled condition and the expense of this undertaking would be practically prohibitive.

Looking at this problem from the point of view of simplicity, it would seem to be a better proposition to go into the tank directly with the cooling medium and carry the heat away rather than to provide means for removing all of the oil with its heat units from the tank, carry it to some external point and there extract the heat and then return the oil to the tank.

Really, the big problem in cooling a large transformer is the problem of getting the heat out of the coils into the cooling stream of oil, and usually the cooling of the oil itself can very easily be taken care of without resorting to the forced cooling system.

After we have studied this problem of cooling unusually large transformers during the next year or two, perhaps the forced cooling system may show up to a little better advantage. I think it will be very interesting to watch the developments of the next few years.

Philip Torchio: I am representing in my employment, central stations. I want to say from the start that the central station as an industry don't care how the power is generated. Whether they generate it themselves or if they buy it from outside sources, it is the same to them. The central stations are purveyors of power to the public and they are not concerned about the question of selfish motives to have it generated locally under their direct control or from an outside entity, provided that equal cost can be secured and equal reliability of service can be given to the customers.

So, in commenting on the interpretation of Mr. Silver's paper in the broad generalization and application to the solution of the power problem of the nation, I want to put an emphatic denial on record that the central stations have any selfish motive in raising any question or any objection to such plan. In fact, the question of securing economical advantages of tie connections has been one of the characteristics of the central station industry.

In the particular field of activity of my people, we have probably been leaders in the interconnection of systems. We have had interconnection between the stations of the lighting companies and the railway companies since 1899 and they have been added thereto from year to year so that all the stations in that territory are now interconnected in one system.

In commenting on the general problem of power supply to the nation, as the time is short I want to refer you to a discussion I presented in January 1918, on Mr. Jackson's paper. In that discussion I brought out emphatically the essential necessity for the states east of the Rocky Mountains to develop to the highest degree the steam power plants. The question of utilization of water power is secondary. We have very little water power available east of the Rocky Mountains, except at one location, the Niagara and St. Lawrence River, and for the application of those powers I also made certain suggestions, which I think should be given serious consideration by the Congress Committee if such a Committee is appointed.

Now in our daily studies of supplying power to important centers or important industries, we are meeting daily with the question "Can you as safely transmit it ten miles away? If you have to cross a river, wouldn't it be better for us to have a station?" Now here comes a suggestion to go 250 or 300 miles. I don't think that the public is much concerned on the theoretical savings that can possibly be made by having the power delivered from a distant point, two, three or four hundred miles, versus the power generated locally, which may be a difference of one-tenth of a cent per kilowatt hour. They are more concerned that the power be reliable, continuous.

In 1911 I presented a paper to the Turin International Congress on the latest development of high-tension transmission which was then 120,000 volts. Being a delegate of the Association of Edison Companies, I transmitted to the Association the paper presented at the Congress, with a supplemental report, headed "What the Central Station Can Gain from the Latest Development of High-Voltage Transmission." While the report did not state specifically the location, still it covered the territory between Boston and New York, a part of the territory that Mr. Murray has described. I recommend that that paper be considered, because it gives the values of what you can gain and visualizes the relative advantages.

Returning to the point of reliability of service, Mr. Murray says the plan contemplates using the existing stations for peak loads. I assume that in a district like Greater New York the peak load would occur in the winter months, usually between November and February, and I imagine that during that period the stations would be under steam. I don't understand what Mr. Murray had in mind about the summer months. I imagine that he would use the transmitted power when the loads are light, as during the summer months, when the loads are normally about 60 or 70 per cent of the maximum winter load. It may interest you to know that on June 20th, 1919, a few days ago, at four o'clock, out of the clear sky, suddenly darkened, we got the highest load in the history of our company.

Now how could the customers be supplied from a peak station, shut down? It would be impossible. We will have to keep those stations under steam. Now when you keep stations under steam you have a great loss in banking fires and radiation, and those losses must be considered.

A valuable paper has been presented to the Smithsonian Institution by Mr. Pogue and another writer, emphazing the necessities of this power problem and the necessities of having power transmission lines the same as we have passenger and freight communication lines.

But I think one thing is overlooked, that we must use coal to generate our power. It makes very little difference in the saving of coal if we generate the power at the mine or if we generate it at tide-water or in Chicago. The only possible saving is of lightening the load on the railroads.

The point I have made in my quoted remarks is that by keeping up the development of the central stations as we are doing it, securing for our increment power the high efficiencies of which Mr. Murray speaks, and we concentrate all our combined efforts along the lines which the central stations are following, we will get out from coal all there is in it, the same as if we generated the power at the mine. Our goal must be to concentrate all efforts to prevent the small uneconomical plant from generating power. Those are the people that use the thirty, forty and fifty pounds of steam. We must also stop the trunk railroads from using from one hundred and twenty-five to one hundred and fifty millions of tons of coal a year at 5 per cent efficiency.

Now if we generate that power in our stations and we realize with these intercompany ties that I was proposing eight years ago, we will secure all the advantages that Mr. Murray points out, and we will give to the customer a more reliable service than if we tried to supply it with power over long distance transmission lines.

The time is short and I want only to emphasize this feature, that the central stations are not advocating from a selfish standpoint the higher development of their plants, but they are logically fulfilling a line of economic development. If we carry out that development we will reduce from one-half to two-thirds the freight for power coal on our railroads. Then the gradual electrification of trunk railroads will double their carrying capacity and the question of congestion of traffic will be solved in a logical way and in a saving way.

S. W. Mauger and **R. M. Spurck:** The importance of the subject and the advantages shown by Mr. Silver in the use of 220-kv. transmission, warrants some comments from the standpoint of the designing engineer and we are therefore presenting the following remarks on line insulators, switching devices and relays, confirming Mr. Silver's conclusions.

1. Although it is believed that the values shown in Fig. 9 for the dry flash-over voltages of insulator strings, is somewhat

low for strings of 8 units and more, the values which it is believed are more nearly correct, are not enough higher to neglect the additional advantages that can be obtained from grading. Such grading appears to be practically feasible. The use of shields, however, offers many mechanical difficulties that would make the results obtainable not worth the added complication.

Very little reduction in the flash-over voltage of strings of units is expected when such strings are combined in multiple. Attention is called to the probable mechanical difficulties involved in shielding strings of multiple units.

We believe that a great part of the insulator trouble noted by Mr. Silver, has been confined to insulators having cemented-on metal fittings. Practically no trouble has been experienced with the link type of insulator, which has no cement and whose design is such that the insulating section is maintained in compression.

The completion of the development of new designs of hardware promises further extension in the use of existing designs of link insulators which already have a service record, indicating that they are suitable for 220-kv. transmission with the moderate mechanical loads ordinarily required.

For higher mechanical loads, a similar insulator of much more rugged design and capable of handling the heaviest mechanical load estimated for the present 220-kv. transmission problems is being developed.

2. Eliminating low-tension switching is a big step in reducing the complication involved with such large units as are required for economical operation at 220-kv. and the saving will go a long way toward paying for the increased cost of the high-voltage switching apparatus.

This, of course, can only apply to generating stations, but in substations the voltage to which the transformers are stepped will be high enough to obviate the very heavy currents.

As Mr. Silver brings out, the handling of very heavy currents involves more difficulty aside from the question of space, than the handling of very high voltages.

3. Mr. Silver makes an apparently bold recommendation regarding protective equipment, but with the great saving in line conductor cost, we can possibly afford to spend a little of this saving in making our insulation proof against over voltage. This is at least worthy of careful consideration.

4. As far as oil circuit breakers and line disconnecting switches are concerned, we have found no design problems that cannot be worked out and the performance of the apparatus predicted with a fair degree of accuracy from designs and performance data on similar apparatus for lower voltages, namely, those voltages between 100 kv. and 155 kv.

In oil circuit breakers for 220-kv. in order to obtain the desired insulation, it is necessary to have exceedingly large tanks on account of the large breaks between contacts and large clearances between contacts and tank, and we believe therefore, that it is advisable to have a smaller and very strong arcing chamber which will relieve the tanks from stresses occasioned by opening large amounts of power. These explosion chambers also are very efficacious in breaking the arc because in their design the pressure of the gas is utilized to a good advantage in blowing fresh oil through the arc stream, thereby assisting in effecting a prompt extinction of the arc. Such breakers can be supplied with two or four breaks per phase, with four break design having all four breaks in one oil vessel or preferably two breaks in each of two oil vessels.

5. 220-kv. air break disconnectors must necessarily occupy considerable space, which of course involves difficulty in design and operation as well as expensive structures for supporting the switches. We cannot hope for any change in these conditions, but it is felt that the conditions can be met with entirely operative and rugged designs, even if necessary to open the exciting current of a transformer. It is not recommended, however, that this should be done except on the small capacity transformers. The following types have been suggested:

(a) Combination knife-blade arc-horn type with two blades in series operated simultaneously and mounted on rigid insulators. The blade opens upward.

(b) Underhung-blade type supported from link suspension insulators, the blade opening downward.

(c) Knife blade type with hinge end supported partly on insulating bushing of transformer or oil circuit breaker and partly on a separate rigid insulator which is revolved or otherwise moved to operate the switch and the other end supported by a rigid insulator on the tower structure. The blade opens downward.

The type mentioned above first is the only type we would recommend for opening with exciting current flowing. The second type with the proper mechanical design can probably be made to give the most reliable service from standpoint of insulation, because link insulators without cement can be used, thereby eliminating troubles attributed to cement.

6. It is gratifying to know that Mr. Silver recognizes the great progress that has been made in relay development. From the standpoint of relay application, there is no more reason to anticipate difficulty with a 220-kv. system than with one of lower voltage.

J. C. Parker: I take it that the comments by Mr. Torchio are not intended in disparagement of the very large social vision that Mr. Murray has given us of the implications of this paper on the 220-kv. transmission, rather that they are intended as a warning and a possible encouragement and incentive to the profession as a whole to tackle the large proposition of the super-power development as an integral matter, rather than as a specialized line of development.

I think it is very satisfactory to the electrical engineers that Mr. Murray has pointed out to us an opportunity for a social relation for our technical work. I think electrical engineers in the past have had occasion to feel that particularly the civil engineers have had a greater social relation in such things as matters of transportation and sanitation than have we electrical engineers.

The problem of coal conservation as affected by these super-voltages and super-power lines, it seems to me, divides itself naturally into two parts, one which is technical, the other which is more or less political. Those two parts are incidentally concerned with the things that are internal to the industry and peculiarly within the hands of the electrical engineers, and those things which are external. Whatever we may do in the way of technical improvement in getting higher voltage lines, lines capable of transmitting at high voltages and at high economies, will be utterly nullified, as pointed out by Mr. Torchio this morning, unless an inducement may be created for the ultimate user to take the power so economically generated and transmitted.

Mr. Torchio is quite right in the statement that if you wipe out the entire marginal cost of steam power production you have done no good toward getting economical coal usage, unless the ultimate consumer can find a selfish interest in such utilization. Now that does not mean that these higher voltage lines, these large projects necessarily fail to attain the end. It simply means that we must be spurred toward very large efforts in the direction of securing the one thing that is essential, continuity of service. With these very large systems, with large centers of population dependent on the service, continuity becomes preeminently the requirement, and much more so than even in our metropolitan systems of today. I don't think that that need discourage us. There is opportunity, there is hope for the right sort of development.

Mr. Silver has pointed out one line that must be pursued, that of simplicity. Much of the discouragement in our high-tension transmissions in the past has been due to the fact that we did not sharply differentiate between a transmission system and a distribution system. These super-power lines cannot be tapped for every small community. We will probably have intermediate voltage lines for distributing to the smaller communities.

I believe also that Mr. Murray pointed out the importance of a multiplicity of generating points on a relatively simple system. Where that sort of thing has been done with such voltages as 140 kv., continuity of service has been attained. On the other hand, the profession as a whole, if it is going to accomplish this very large social program, must devote to it a great deal of effort. We must make our utilization apparatus simpler and cheaper. We must make the means of distribution much more economical, with a marginal generation at as low as 0.2 of a cent in the pre-war days, the distribution companies were the thing that stood between cheap coal utilization and sales to the customers. There is a big problem for even the application engineer whereby he ties himself to this program of national conservation.

W. S. Murray: Mr. Torchio raised some very interesting and very valuable points, indeed, and I thought it possible that an impression might have been conveyed that in the study of this matter we rather considered we had plenty of hydraulic power in this region. Now I want to draw your attention to the fact that at first blush the study shows that there is a demand of some seventeen million horse power, of which ten is industrial and seven is required for the railroads.

Now the actual amount of hydroelectric development available for what you might call, economic application, is somewhere around a million, and so there is not a very great deal of hydroelectric power available as it stands. In other words, just as Mr. Torchio pointed out, the steam will carry the heavy end—93 per cent steam, 7 per cent hydroelectric. May be later as we extend the line up into Maine and South down to Richmond, we will strike into the Northern and Southern hydroelectric powers, and the percentage of water power will be increased, but it is primarily now a steam problem, a problem of getting the highest economies out of steam.

That was just the point I wanted to bring up for fear you thought perhaps we were thinking of the development of the hydroelectric feature.

Next is with regard to the point Mr. Torchio raised concerning a day that might come along in the middle of a low load season which would reach right up to the maximum of the high. We have got to look at this thing away out there and not right in front of us. We have got to keep our minds far ahead and with this super-power system installed one or two units quickly thrown into service would meet the conditions Mr. Torchio has described. It is just because of the lack of it today that we are at times embarrassed. His argument "against" is truly "for" when we view this matter broadly.

I agree that these great steam stations are preferably to be located in the large centers, such as New York, or Boston, or Philadelphia, etc. Mr. Torchio is absolutely right along those lines, but we must not lose sight when we have done that, of the fact that this tremendous super-power transmission at 220-kv. is the interlink between all those large centers, and provides a means to supply the very customers that Mr. Torchio pointed out we must reach and therefore, while entirely agreeing with him that his load centers are correct it should be remembered that the super-power transmission ties in those load centers and make them more effective, permitting us opportunity to create a high load factor for a regional instead of a city plant.

Now just remember the house. Then, after the house, the city. Then the district, then the territory, and now the region and that is all this proposition is; to establish a twelve pound line in this region. We are on the verge of large things, and it is within the vision of those large things and within the vision of practicability that we do the larger things in a more efficient way than the smaller.

J. C. Clark: The San Francisco Section has organized a local Committee on Railway Electrification, including in its membership a number of the best electrical engineers of the community. The Committee was organized about the first of April, 1919, and has been holding meetings every two weeks since that time. One object of the Committee is to gather all the data they can get in the way of actual figures of power consumed in railway operation, such things as maximum load swings, demands upon lines to carry trains at all times, and load factors. In this latter connection it is quite surprising and gratifying to hear Mr. Murray state that the load factor on his regional trunk, due to railroad operation, will run as high as 75 per cent. No such high load factor has been predicted by our people for the California mountain divisions.

Among other subjects which are being studied is the amount of potential water power in the Pacific Coast States, especially in California; the amount of fuel oil used by railroads; and more technical topics such as the practicability of dispatching trains with special attention to load factor.

The plan of the Committee is not to engage in political activity unless called upon. The Committee wish to be prepared so that they will have all the facts and figures at their disposal which they may need in order to assist any legislative body in studying the problems of conservation of fuel in their district. Fuel oil in California is going very fast.

Some of us do not look for the initiative in railway electrification to come from the railways. It is rather remarkable that the matter of railway electrification has been studied in the past almost exclusively from the railway standpoint. It is felt that we need not hope, certainly not in the near future, for any considerable move for railroad electrification from our railroads. One reason that has been apparent to some of us is that, in these days of regulation of such utilities as railroads, it is a growing practise to insure a reasonable return to the railroads, or any utility, on its investment.

Now, if the railroads are assured, say, 8 per cent, it perhaps removes one of the incentive toward more economical operation. If that return is assured in any case, it is not quite apparent why they should strive to improve their methods—at least not from the point of view of the fuel conservationist. It seems to many of us to be up to the public to insist upon railway electrification, and with the attitude of being prepared for a public movement for railway electrification, we are getting together all the data we can to enable us to be in a position to offer such assistance to legislative bodies as may be asked.

Mr. Silver expresses some apprehension about insulators. He has a good reason for this in view of the universally bad experience with the cemented type of insulator. Fortunately, however, there are some very good operating data, covering a period of perhaps ten years, on another type of insulator. I refer to the original suspension insulator known as the Hewlett. The failures in the cemented type of insulator have not been due primarily to electrical causes but to mechanical cracking and absorption of moisture. Electrical failures follow. The cracking may be caused by expansion of the cement or of tightly fitting metal parts. The presence of the cement greatly aids in the absorption of moisture by porous porcelain in contact with it. The absorption is apparently due to breathing. In the Hewlett type the metal parts are strung loosely through glazed cable ways. I have recently had occasion to examine many insulators of this type having a ten year service record with inappreciable loss. Electrical, mechanical, and porosity tests showed no depreciation. Some of the old original units that gave perfect service were poorly made compared to present standards. This speaks well for the design.

It may be necessary to grade the strings of insulators used on these high-voltage lines to lower the stress on the line unit.

In regard to the transformers, I believe decidedly, that all high-voltage neutrals should be grounded without resistance. With proper precautions there should be no difficulty with telephone operation. The connection will in general be Y-delta to prevent tripple frequency.

There are some advantages and many disadvantages in high-tension switching. The number of these switches should be reduced to a minimum.

As transmission voltages are increased, the number of times that lightning voltages exceeding operating voltage are induced on the line in a given season will decrease.

The question naturally arises whether it will pay to install lightning arresters. While it is probably true that a lightning arrester gap on a 220-kv. line would discharge only at infrequent intervals, great damage might be done if it were absent during these intervals. If it is assumed that the transient voltage is limited to the gap setting, it can be shown that about five times the insulation is necessary to make the apparatus equally safe without a lightning arrester. When the arrester is absent the voltage is limited by the line insulation. The turn insulation probably receives relatively greater transient stresses on very high-voltage lines than on moderate voltage lines. A lightning arrester is decidedly desirable if its cost is commensurate with the cost of other apparatus. Consideration must also be given to the enormous energy involved.

F. C. Hanker: The field of the 220-kv. system is apparently limited, as pointed out by Mr. Silver, to trunk line service where we have long distance transmission and heavy blocks of power. That would be the natural conclusion, based on present analysis of costs, the development of the necessary transforming and switching equipments involved in the line construction. It would naturally hold for pioneer installations but if we look back on the history of transmission voltage, we find parallel situations in the steps of 110-kv. to 154-kv-a. It has only been a few years since 110-kv. was discussed as a high voltage for transmission, while today it is so common that it may be considered almost in the class of distribution voltages.

The frequency of 60 cycles selected for the studies is in line with present tendencies toward the establishment of this frequency as the standard frequency of power supply in this country. A general analysis of the factors entering into a decision as to the best frequency to adopt in any specific case will show that the increase in 60-cycle systems is a natural one and is one that will undoubtedly continue at an accelerated rate.

The existing transmission systems that are operating at high voltages above 110-kv., have layouts that in general require regulation at one load center so that it is usually possible to adjust operating voltages at the generator station to correspond to the requirements at the load center and in this way keep the investment in synchronous condensers to a minimum. In the case of trunk line service where a mumber of generator stations and different load centers are involved, the maintenance of satisfactory voltage regulation at all points becomes more of a problem and it is usually more economical to supplement the synchronous condenser installation with synchronous boosters or the equivalent, such as induction regulators. This is particularly true where systems tie in with existing city systems that have standards established, making it necessary to maintain closer voltage regulation than is permissible on some of the pioneer lines such as exist in the West.

With the relatively high investment in lines and substation apparatus it is important that full capacity be available. This makes regulating apparatus at tap points desirable in order to reduce the circulating currents to a minimum. We have reviewed cases where parallel lines were involved in which it was desirable to provide independent regulation for each circuit. This was due to unequal loading at tap points that would have resulted in excessive circulating currents had the two lines been paralleled at both ends and regulation obtained by synchronous condenser equipment. It would also have cost very much more to control by the use of synchronous condensers than by boosters or equivalent regulating schemes. This condition would obtain on projects such as have been proposed for trunk lines along the Atlantic Coast tieing in the different existing power stations. To be of value it should be possible to transfer power in both directions, which would mean, in the case where synchronous condensers were used, installations at each end of capacity to give the full range of regulation. This condition can be better fulfilled by the use of sufficient synchronous condenser capacity to provide for operation at the most economical power factor and take care of the additional regulation by some other method.

With the installations under consideration, the capacity of the regulators that would be required would be larger than have been built of the usual type of induction regulator. There has been developed, however, a combination of the step type and induction type that would be feasible for the capacities involved. In this regulator the objectionable features of the straight step type regulator have been eliminated in that no switching is done with circuits having difference of potentials, and as a result, sparking is avoided. The short-circuit stresses that have been a limiting feature in the design of induction regulators are avoided, in that only a small percentage of the total short-circuit current is handled by the regulator.

The kv-a. required to charge the lines of the length that have been considered in the studies becomes of importance due to the effect on the generator excitation. A normal design of generator such as would be installed would have a short-circuit ratio of approximately unity, that is, with the excitation required to give normal voltage at no load, the sustained short-circuit current would be approximately full-load current. In the studies made, 80,000 kv-a. is given as required to charge the line so that it would only be necessary to supply 20 per cent external excitation to excite the full 100,000 kv-a. in generators to normal voltage. There will undoubtedly be times when only one generator will be available to supply a line, and under this condition, the charging kv-a. would exceed the capacity of generating apparatus. This would take the control of the voltage out of the hands of the operators and result in abnormal voltages on the transmission line.

This condition is not of importance when operating under the usual loads, but must be considered where high-voltage switching is involved.

In commenting on the design features of 220-kv-a. station and substation equipment the paper points out that the studies have developed that current design principles and materials now in ordinary use will be employed. This is encouraging after reviewing the analysis that has been made of the problems encountered in laying out the other parts of the system. While the margin on safe operating voltage is probably greater than is apparently the case with the line construction, it is recognized that new problems exist and the manufacturers appreciate the importance of these problems. They are not content to be satisfied with the present conditions but feel that just as careful analysis and research is fully warranted and should be undertaken to ensure the success of this higher voltage system.

The problems have been classified as of two types, those due to the high voltage of itself and those due to the enormous capacities involved. The insulation problems in the stations are concentrated where greater expense is justified in providing the margin in safety that is essential.

The conclusion that all line switching should be done on the high-voltage side will probably find opposition but if future investigation confirms this statement it will simplify the switching problem. In accepting the recommendation, it should be recognized that this method of operation will subject the transformers to higher stresses than in the case of low-tension switching. It is true that the potentials resulting from switching are of the same order as those the apparatus would have to stand from static disturbances or other transients that may occur on transmission lines, but they probably occur more frequently. On the 110-kv. and 154-kv. systems, we have heard of no serious results from the high-voltage switching, yet this point must be taken into consideration in laying out a system on the higher voltage. With the operating conditions that have been assumed it would be well to consider arrangements wherein the step-up transformers and the step-down transformers are considered as a unit and operated as such. This does not necessarily mean that the apparatus must be operated in parallel on the low-tension side at the generating station, as the layout can be so arranged as to provide for what is essentially unit operation.

In investigating breakdowns that have occurred on lower potential circuits, we have obtained results that indicated that the current was ruptured in the oil breaker at a rate several times the normal frequency. If this condition holds for low voltages with relatively short travel of the contacts and short arcs, it must be more severe at high potentials with correspondingly long arcs.

H. R. Summerhayes: Figs. 1 and 2 of Mr. Silver's paper show a comparison of the cost of the line construction, total operating cost for 154,000 volts and 220,000 volts, and at first it seems as if more of a gain should be made on 220,000 volts in spite of the increased cost of the towers, etc., which is incurred by the higher voltage, but in reality the comparison is between 170,000 and 220,000 because 170,000 as stated in the footnote is the voltage at the generating end of the line, just as 220,000 is the generating end voltage, and I think that the comparison is really between those two voltages and not between 150,000 and 200,000.

R. P. Jackson: Mr. Silver's statement that there is no type of insulator as yet developed that has thus far demonstrated its ability to give adequate, or even reasonably satisfactory results on high-voltage lines is an exceedingly severe indictment of the suspension insulator considering the amount of power now being actually carried by conductors hung on such insulators. "Reasonably satisfactory" is a relative term and if defined as being as satisfactory for example as the steel towers on which the insulators hang, the statement is doubtless correct.

In reading Mr. Silver's paper, it is obvious that he has some serious misgivings as to the insulators for the high-voltage high-power lines proposed, and with good reason. A link of nine to eleven ft. between the line and the tower appears altogether too long. Six feet would be possible as a pillar to support disconnecting switches. Further, a strain tower taking up the tension of the three lines in both directions apparently requires six strings, each of fifteen insulators for each conductor anchor. This means $6 \times 15 \times 6 = 540$ insulator disks per strain tower. This strikes one as a monstrosity.

In the first place it would seem to the writer that the use of 15 disks in series is unduly conservative on a solidly grounded neutral line that limits the voltage to 130,000. Ten, or at the most twelve, should be sufficient with say a 7 in. spacing. This would give a string about 6.5 to 7 ft. long and reduces the inequality in stress distribution.

When it comes to 2900 pounds as a maximum load on the ordinary disk insulator, Mr. Silver is not unduly conservative. Such an extraordinary proposition calling for such loads as Mr. Silver describes demands something larger and stronger in an insulator than anything now on the market.

The present types of insulators have been developed to meet certain prevailing conditions. In fact, some operating men state that for their use the usual cap and pin insulator is unnecessarily strong. It is undesirable to use a size and type of insulator developed for the low stress conditions of ordinary transmission lines for these new and much more severe conditions. It certainly ought to be possible to make up an insulator good for twice the stress of the ordinary unit, thus reducing the parallel strings for a conductor anchor from six to three.

Just which of the three types enumerated by Mr. Silver would lend itself most readily and most safely to increase in strength is somewhat debatable, but it would seem that the interlinked type in some form would offer somewhat the best starting point, judging from what can be readily obtained now in strain balls for guy wires.

It is to be hoped that no effort will be spared to do something in this direction before resorting to the use of such a multiplicity of insulator units as the present standard designs would appear to demand.

The writer would personally commend Mr. Silver's judgment in regard to lightning protection in retaining the overhead guard wires and eliminating the lightning arresters. As to something in the nature of choke coil protection against surges of steep wave front, one does not feel so sure. That largely depends on how well built are the high-tension windings of the transformers.

H. B. Dwight: The statement in Mr. Silver's paper that the frequency of 60 cycles should be chosen for an important generating and transmission system, even where there is a large present 25-cycle load, seems a logical and proper conclusion. As stated in the paper, the frequency of 60 cycles is superior to that of 25 cycles for the general supply of energy, and therefore any means which will tend toward the increased use of 60 cycles is a step toward standardization of the right kind.

It may be interesting to note some of the main items of extra expense entailed by choosing 60 cycles for the generating and transmission system where the greater part of the load is 25-cycle. Many residence districts and street lighting loads can be changed to 60 cycles at very small expense. Many 25-cycle transformers can be connected in two parallels and operated at 60 cycles at increased capacity. For power circuits, it would be possible to provide frequency-changer sets for a large part of the total system load.

The cost of the motors of the frequency-changer sets should not be charged to the fact that 60 cycles was chosen, because they take the place of the synchronous condensers which would have been needed for a system of this kind, as stated in Mr. Silver's paper. Although transmission is usually considered easier at 25 cycles than at 60 cycles, the advantage almost entirely disappears when synchronous condensers are used to eliminate voltage variation. Practically the same rating of condensers is needed for a 25-cycle line as for a 60-cycle line, for the same results in a certain case. Although there is less voltage drop to be overcome in the case of the lower frequency, there is less reactance to work on by the condensers.

It might be questioned whether the motor of a frequency-changer set would be as effective for voltage control as a synchronous condenser of the same rating. The answer is in the affirmative if the load power-factor is as low as 80 per cent, the motor of the set being operated at nearly unity power-factor at full load, and at a load power factor at no load, of such a value as not to endanger its stability of operation.

Therefore, if a large part of the load be supplied with 25-cycle energy through frequency-changers, it would not be necessary to scrap the 25-cycle apparatus in use, and yet new installations could be supplied with energy directly from the 60-cycle transformers and so could enjoy the advantages of better choice of motor speeds, better lighting and cheaper and more standard apparatus of all kinds.

Thus, at the time of building a new generating and transmission system, by the additional expense of the generating end of the frequency-changers, and a small duplication of distribution circuits, representing an extremely small percentage of the cost of the system, a 25, 30, or 50-cycle system could be put in the way of gradually becoming a 60-cycle system.

J. F. Peters: There are a number of factors that enter into the design of apparatus suitable for operating on a system as large and as high a voltage as the one proposed by Mr. Silver, that I wish to comment on briefly.

Referring to page 1086 under, Design Features, I note that the author states, "The handling of electrical potentials of 220,000 volts does not appear to involve any disturbing complications or uncertainties. In fact, the manufacturers do not recognize that any serious problem exists." Although that is true to a certain extent, I wish to point out that there are several factors which become of major importance in designs for high voltage, which for low voltage are of no consequence whatever.

In the design of low-voltage apparatus, it is not necessary to take into consideration the electrostatic field, whereas on high-voltage apparatus that becomes the most important factor in the design. Considerable work on the distribution of electrostatic field in transformers and methods of controlling that distribution has been done by Mr. Fortescue, some of the results of his work have been presented before the Institute.

Under high-voltage switching, I wish to call attention to the fact proven by experience that there are more or less high voltages produced by switching. The magnitude of these voltages and their relation to the applied voltage is not definitely known. There is a question in my mind as to the advisability of high-voltage switching on anything as high as 220,000 volts on account of that uncertainty.

Under transformer connections, I am pleased to note that the author has recognized the decided advantages that there are in the star connection. In addition to the advantages that he has stated, I might add that this connection results in transformers being smaller and more efficient on account of the lower voltage that is developed in the windings, consequently less insulation of inactive material.

With reference to limiting size of transformers, referred to on page 1090, from my experience on the design of large transformers, I have found that the mechanical stresses due to short-circuit currents are not serious for high voltage. The short-circuit stresses are not nearly as severe in transformers for high voltage as they are for the same size units on lower voltages. The reason for this is that the density of the leakage field is much less on the former. My experience indicates that the factor that limits the size of transformer units is transportation and handling facilities. Those limits are not only due to over-all dimensions, that is railway clearances, but for units of the size suggested by the author, they approach the limit of carrying capacity of even special cars.

On the top of page 1092 the author refers to the relative amount of insulation for the line and grounded ends of transformers. I do not believe that he means to infer that extra insulation is not necessary between turns on the grounded end of the transformer. It is true that the electrical stresses between turns on the grounded end are somewhat less than for the line end, but they are considerably greater than within the body of the winding. I believe that the insulation between turns should be correspondingly reinforced for both ends of the transformer.

F. F. Brand: I am glad to see that Mr. Silver has decided so definitely on the use of grounded "Y" connection. There is no doubt that for high-voltage transmission this connection which preserves a definite voltage relation of the lines to ground and by which the possible voltage oscillations set up by arcing grounds are greatly minimized, is the best connection.

This connection is made almost necessary by the decision to do all of the switching on the high-voltage side, since the tripping out of one phase, either by failure of switches or inability to close and open all three phases at the same time, does not produce a greatly unbalanced electrostatic condition.

The argument that the grounded Y connection increases the hazard due to short circuits, line failures, etc. damaging the apparatus by mechanical stresses, is not of great moment because on such high-voltage systems the reactance of the transformers and of the line must, of necessity, be so high that the current would be limited to a few times normal through any piece of apparatus.

The use of the grounded Y connection would permit appreciable savings in the design of transformers since less insulation is necessary to ground, and it is not necessary to insulate all points of the winding to ground to the maximum extent.

I believe in general that the insulation to ground in the high-voltage transmissions can be reduced, since the probability of over-voltage due to lightning, etc. becomes less and less as the voltage of the system increases.

I believe our standard Institute test of twice the line voltage plus 1000 volts, is unnecessarily high for the higher line voltages irrespective of whether the systems are isolated or grounded and I think that it would be much more logical to adopt a graded test value in which the factor of safety in test would be appreciably higher for the low-voltage units than for the higher-voltage units. Where grounded Y connections are used, the test should, I believe be still further reduced, at least in the higher voltage lines. If both transmitting and receiving neutrals are permanently and effectively grounded, it would appear that we could regard the transmission system essentially as three single-phase systems and could with safety, permit such test values as are ordinarily given for grounded single-phase systems.

I have taken the liberty of drawing up suggested curves of test voltage for both isolated and grounded systems which give a graded test depending on the line voltage. See Figs. 3 and 4.

The conclusion that the high test values now applied are not necessary for high-voltage apparatus is borne out by the fact that there are a number of installations in operation of old transformers in which the test value was only 1.5 times the line voltage although I believe that some, if not all of these systems are operated grounded Y, and, further, it has not been the practise in the past to insulate step-down transformers with a rated voltage lower than rated voltage of the step-up transformer to the same value, the insulation being based entirely on the rated voltage. It would appear that all apparatus on a given system or circuit should be insulated to the same value. As a matter of fact, under light load conditions the step-down transformers are frequently subjected to an even higher voltage than the step-up machines, and since these step-down transformers have withstood such conditions without failure, it would appear unnecessary to insulate a step-up machine to such a high value as is our present practise.

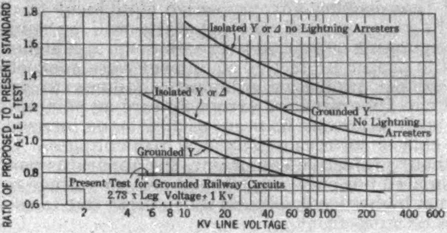


FIG. 3

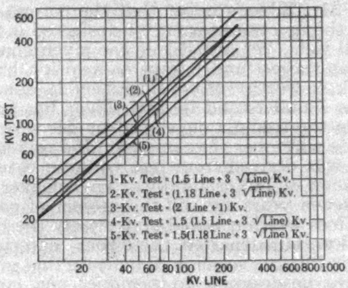


FIG. 4—KV. TEST

1. Ungrounded circuits, no lightning arresters.
2. Grounded *Y* circuits, no lightning arresters.
3. Standard A. I. E. E. Test.
4. Proposed Standard Test Ungrounded Circuit.
5. Proposed Standard Test Grounded *Y* Circuit

With Mr. Silver's conclusion to omit the use of protective equipment, I am not so much in sympathy. Lightning arrester equipment in general protects the apparatus against excessive voltages to ground. It is not as truly selective to frequency to the extent which we desire but it should be borne in mind that lacking any protective equipment to discharge over voltages to ground, the apparatus will be subjected to surges originating or applied to the windings at a higher voltage than if the proper protective equipment were connected.

With proper protective equipment, failures to ground on high-voltage apparatus are extremely rare as pointed out previously, such failures as do occur usually being internal failures between coils or turns caused by local high voltages. Undoubtedly without protective equipment, the apparatus would have to be insulated not only to withstand greater voltage to ground but against higher local voltages which occur due to oscillations or waves of steep front.

I do not feel that it would be safe to use apparatus without protective equipment unless the insulation was increased something in the order of 50 per cent over that otherwise required.

On the curves attached I have shown values which would be derived if the test values suggested were increased 50 per cent to take care of the cases without protective equipment.

C. F. Harding: Although Mr. Peek has pointed out in the discussion of Mr. Silver's paper that the corona loss on such a line, where the amount of energy transferred is large, will be rather a small item, yet Mr. Silver has given considerable attention to that subject and I want to mention briefly a point which I think is worthy of consideration in connection with that loss. As I understand it, Mr. Silver has based his calculations upon the formula developed by Mr. Peek, presented to the Institute a number of years ago. If I remember rightly, that formula was derived from test voltages extending up in the neighborhood of 140 or 150 kv. The speaker presented a paper about seven years ago, on the subject of corona loss on transmission lines which checked very closely the Peek data up to about 150 kv., but departed therefrom materially between 150 and 180, or 190-kv., seeming to indicate that above 150-kv. values, it might be desirable to introduce new constants into the formulas. These calculations in the neighborhood of 220 kv. may therefore be slightly in error, and it is hoped that in the near future some new data will be available in that range.

Also, very little has been done in connection with corona to ground. It was found in some investigations with which I have been familiar, that where the lower wire is in fairly close proximity to the ground, as must necessarily be the case with these long spans and wide spacings with the 9-ft. strings of the insulators, there is a relatively large loss between the lower wire and ground, due to corona. It may be necessary, therefore, in some of the future designs, even to go to the extreme measure of putting each conductor on a separate relatively light tower line, separating the conductors by means of separate towers.

So much has been said about the different types of conductors, copper, copper-clad and reinforced aluminum, it would seem well worth while in the future to study the possibilities of steel tubing, making the diameter of the conductor relatively large and keeping the tensile strength high, the resistivity of the metal used being of relatively small importance.

With regard to the question of bushings, although those have been mentioned in connection with transformers, little has been said about roof bushings or wall bushings, used out of doors. It probably will be necessary to use bushings with a much lower factor of safety upon such lines than those which are being used upon the 140 and 110-kv. lines. In connection with the new 600,000-volt laboratory installation at Purdue University, with which we hope to make some tests upon experimental lines and insulators in the very near future, we have had difficulty in getting our lines through the building with anything available at present in the way of bushings, even with a great reduction in the factor of safety. Of course, Mr. Silver's paper anticipates new designs available in the future, but I think the factor of safety will have to be cut down materially.

F. W. Peek: Regarding Mr. Harding's comments, my 1910 corona loss measurements were made up to almost 250 kv. The voltage range under discussion was thus actually covered. Very little would be gained by placing the conductors on single towers and separating them a considerable distance. The loss occurs due to high dielectric flux density at the surface of the conductor. The flux density depends upon the surface, or diameter of the wire, and the capacity. The capacity is not greatly reduced by increasing the spacing. The greatest reduction in flux density or stress can be made by increasing the diameter of the conductor.

I do not think that the bushing problem is serious. We have used bushings up to about 750 kv. without difficulty.

J. A. Koontz: High-tension power transmission at 220 kv. has been a problem confronting the western engineers for several years. While to date it is not a reality, I firmly believe that lines will be operating at such a voltage before many years. In fact, had it not been for the war and consequent difficulties in financing large hydroelectric developments in the past two years, I believe that such a system would now be operating on the west coast.

Mr. Silver's statement is quite true that such high voltage has a limited field and can be used economically only where large blocks of power must be transmitted over long distances. The trend of voltage increase in the past ten years has shown that with increased voltage it has been possible to get increased reliability. This is due largely to raising the factor of safety all along the line. Following this trend, and by carefully working out all the details, I believe the same high service standard could be incorporated in a 220-kv. transmission system and still show economy with improved reliability.

Some of the real problems to be solved where there may still be some doubt are those of corona, insulation and proper oil and disconnecting switches. There is one question concerning corona which I would like to ask Mr. Silver, and that is, if any tests have been made on large conductors in order of 1 in. in diameter to see if 0.87 is the proper conductor irregularity factor, as from some preliminary tests made at Stanford University on a 1 in. 37-strand aluminum conductor it would appear that 0.72 would be nearer the proper value than 0.87.

On pages 1049 to 1052, Figs. 3 to 6, are shown corona loss curves which I do not believe would represent operating conditions under good practise, as it does not seem to me logical to hold the generator voltage constant and permit of receiver voltage variations, but rather, hold a constant receiver voltage and increase the generator voltage when necessary to take care of maximum load conditions. In this manner, the customer will not be subjected to a line drop of both transmission and feeders, and the corona loss will be reduced.

Fortunately, the Pacific slope weather conditions are such that mechanical problems are greatly reduced, as in most cases we do not have to design more than 25 per cent of our line for sleet loading, and even where sleet is encountered we do not have to take care of such heavy load conditions as would be necessary in the Eastern climate.

I notice in the tower design that Mr. Silver has left only a 4-ft. minimum clearance from towers when insulators are deflected to their maximum condition. This does not seem to me ample, as from experience on high-voltage lines on this coast, flash-overs seem to be the principal service interruptions on high-voltage lines, and to eliminate this, it would seem necessary to maintain ample clearance, in fact, experience would indicate that four feet would be a minimum for 150,000 volts, as bringing the conductor close to the tower, intensifies the electrostatic field, and unless special precautions are taken the conductor may fire with corona near the steel tower members, which would tend to aggravate any flash-over conditions. This is a point which seems to me should be given serious consideration in any high-voltage line design.

The insulator problem is certainly very important and the exact method of taking care of same is one which will probably present as difficult a problem as any encountered. With a long string of insulators I believe grading of some sort will be essential so that insulator units in the long string have more nearly the same voltage impressed across each one.

The question of automatic operation on 220-kv. lines, should be given very serious thought before attempting same, as at the present most of the long high-tension lines are seldom interrupted by high-voltage switching. When such large blocks of power are essential with power problems of this magnitude, I believe the generating stations and substations should be given greater flexibility, as the failure of a piece of equipment would necessitate taking out such a large block of power that I think it would be difficult in general to pick up this power on the stand-by steam stations in order that there might be no serious interruptions to service, or, if the auxiliary steam stations were kept under condition to always take care of such a shift in load, it would mean unnecessary fuel expense. I have thought of the possibility of attempting to group the transformers and line as a unit, but have always been afraid that this would present operating difficulties regarding proper flexibility.

The question of omitting all arrester equipment, I believe is wise, as with very high voltage the insulation strength is such that the troubles that the ordinary lightning arrester would take care of are of little consequence, and I believe greater safety would result in omitting, rather than installing, any of the present arrester equipment.

H. G. MacDonald: The modern tendency toward large concentration of power has necessitated a revision of design and radical departures from previously well-established precedents. The forms of construction which had proved adequate during the period of small or moderate powers are entirely insufficient for modern requirements. Mechanical structures, form and location of contact elements, and speed of operation have all undergone modifications.

Best modern practise tends toward a construction built to withstand considerable internal pressures, as guarantees call for several repeated operations in quick succession in an attempt to locate and clear short circuits. It is well demonstrated that the maximum need of heavy construction is not due to the shocks of the actual circuit breaking operation, but to the formation of excessive amounts of gas, deterioration and disturbance of oil due to repeated operations, and the ultimate explosion of this gas. No device which has for its object simply the confining of the stresses due to circuit interruption without considering the incident stresses from the attendant phenomena will produce a breaker which can be considered safe.

Modern breakers of moderate interrupting ability will usually have elliptical tanks with bracing across the bottom, steel tops held by tie bolts secured to the bottom bracing, entirely enclosed operating mechanism, leaving a comparatively clear slightly crowned top, light moving elements permitting of high and rapid acceleration, and contacts placed well below the surface of the oil, and so shaped as to reduce voltage discharge and to deflect the arc and the attendant gases away from any insulating surfaces. Adequate vents will be located in such a manner as to facilitate the rapid escape of the gases by the creation of a draft of air through the chamber above the oil. The elliptical tank gives the maximum strength for a given space and in connection with the overhung steel top and tie rods, makes a construction sufficiently rigid for any service except the very heaviest where a round tank construction will be used.

A prime consideration in successful circuit interrupting is speed of opening. As the time during which the arc persists determines the amount of contact material vaporized, the amount of oil broken up, the amount of gas liberated, and consequently the pressure formed, it is obvious that quick opening is highly essential. The simple means of accomplishing this end is by highly accelerating the moving element. Where this element is light, this will be readily accomplished, but when the moving element becomes heavy, and a long travel is necessary to obtain the proper break distance, some auxiliary means must be used to quicken the contact separation. Quick break contacts have been added to certain designs of breaker by means of which rapid separation of the arcing contacts is obtained without unduly accelerating the entire moving member. Another means of introducing quickly into the circuit the necessary insulating distance is by using quadruple break contacts instead of the ordinary double break. By producing all four of these breaks simultaneously, it is obvious that twice the separation is obtained within a given time as compared to the double break. As a maximum resort, quadruple quick break contacts afford the most rapid separation obtainable under present designs. The quadruple break, when used in connection with the round tank construction accomplishes in a single circuit breaker what would require two sets of pole units in series in the case of the oblong flat-sided tank, and at no sacrifice of insulation clearances, and in combination with the maximum mechanical strength.

A circuit breaker for 220,000-volt service will inherently possess a large interrupting ability. The insulation requirements will very largely determine the physical proportions and if a consistent design from a purely mechanical standpoint is worked out, a structure of very considerable strength will result. An oil tank to retain the volume of oil required, and with fittings and covers sufficiently rigid to carry the operating means and the contact elements, will be of no mean proportions. By comparatively small additional reinforcement, and by the use of suitable material at vital points, a structure capable of interrupting very large amounts of power will be produced. As so much stress is being laid on continuity of service under all contingencies, the breaker design should be such as to insure this to the greatest possible extent. The only safe course is to make the whole breaker structure sufficiently strong to care for the maximum stresses which might occur under any conditions which might reasonably be anticipated. No existing method of ventilation will dispose of the gas with absolute certainty that no explosion will occur in the main body of the breaker. The explosion chamber used on certain designs of breaker does not eliminate the formation of gas. This gas will presently pass into the air chamber above the oil and must be disposed of. The arrangement of details, and the movement of the breaker parts present the possibility of the arc which originates in the top portion of this explosion cylinder (the cylinder being practically tight at the top) following the moving contact in its downward travel, expanding as it goes, and expelling all the oil before it. The cylinder is now filled with incandescent metallic vapor forming a conducting path for the full length of the cylinder. As the moving contact leaves the cylinder by a very small distance, the possibility of the formation of a large amount of gas and serious damage to the breaker seems to be not very remote. If, on the other hand, the breaker structure entire is made with a view to withstanding abnormal pressures, and the design provides for a rapid contact separation to a sufficient distance so as to preclude the possibility of the arc holding through the distance, the maximum safety and assurance of continuity of service is afforded.

L. B. Chubbuck: Referring particularly to the description on pages 1093 and 1094 of proposed 220-kv. oil circuit breakers, I note Mr. Silver's suggestion of breakers with rupturing capacity of 3,000,000 to 4,000,000 arc kv-a. We have furnished the Ontario Hydroelectric Power Company with a number of large 110-kv. breakers which have been subjected to very heavy short circuits and believe our experience may be of interest. Three Niagara stations are interconnected to give a combined generator capacity of nearly 200,000-kv-a. and while bus reactors are used, the main H. T. breakers have successfully handled short circuits up to nearly 1,000,000 arc kv-a. These breakers are of very heavy round tank construction with motor fans on each tank to prevent the collection of an explosive gas mixture in the expansion chamber.

We have found that older design breakers are not capable of handling such service, and on even much lighter service will smoke badly after one or two operations. As a result of serious trouble with such breakers used on too heavy service, the necessity for a large factor of safety in heavy capacity circuit breaker design cannot be emphasized too strongly.

On these proposed high-capacity, high-voltage power systems, out-door breakers are recommended, also as much sectionalizing, and as little switching at the generating station as possible.

J. N. Mahoney: Commenting on the problem of oil circuit breakers for high-voltage high-duty service, the cylindrical form of tank of the boiler drum type has particular advantages. This form is inherently adapted for the use of four breaks in series per pole or per phase.

When properly designed this form of tank will withstand explosion of hydrogen or hydro-carbon gas and air in the space above the oil. It is true however, that good design also includes provision for removing, displacing or neutralizing such gases and preventing their slow accumulation to any considerable pressure as an explosive mixture.

Because of the inherent possibilities, the tank design should be adequate to at least withstand an explosion of a mixture of hydrogen and air at atmospheric pressure with a reasonable factor of safety.

There should be no difficulty in meeting the insulation requirements with present well tried methods and materials. The only uncertainty is in predicting the probable voltage surges to be met when such a large system is disturbed by unusual conditions.

E. B. Meyer: The fundamental consideration underlying Mr. Silver's paper on 220-kv. power transmission is the economic utilization of our fuel and water power resources in a manner calculated to conserve them to a maximum extent.

During the period of the war the need of conservation, coupled with the necessity for a maximum useful expenditure of power, made this subject one of the very considerable import. The cessation of hostilities, however, has relieved the pressure somewhat, but the question of the practicability of high-voltage trunk lines is still one which will claim the attention of engineers and capitalists.

It is not my intention to go into the consideration of the technical features of the 220-kv. transmission project, believing that these problems can be solved as they are encountered.

The author points out that such a trunk line as he has in mind must be operated at a very high load factor, the loading per circuit being 100,000 kv. or higher.

With a transmission line having such a large capacity, continuity of service becomes at once the primary requisite and it is, therefore, necessary to take into consideration the fact that spare circuits must be erected to insure the service.

The cost of this insurance to the service, in the form of spare circuits, increases the fixed charges, per unit of energy delivered very materially, and it is doubtful whether a system operating less than three or four lines, delivering in the neighborhood of a half-million kv-a., could bear this expense and still prove to be an attractive financial investment.

The generation and distribution of an amount of power of the magnitude indicated as necessary to the financial success of such an enterprise would require exceedingly large expenditures of money and would have to be preceded by a thorough and radical reorganization of the engineering policies as at present contemplated by the central station industry.

In arguing for the erection of the large generating stations located in the coal fields, there are two basic considerations:

First: Savings in freight charges with the consequent release of railroad equipment for other uses.

Second: The opportunity of utilizing low grade fuel which would not be worth transporting.

There is some doubt as to the feasibility of carrying out a project of this nature, except on an exceedingly large scale, as the differential in favor of a plant at the distant energy source is quite small.

The erection of a plant in the coal mining district for instance would require that the following factors be very carefully investigated in arriving at comparative construction and operating costs:

1. Availability of cooling water and possible high cost of erecting dams, cooling ponds, etc.
2. Increased cost of boiler plant to provide grate and furnace space sufficient to burn low grade fuel.
3. Cost of transmission line right-of-way and cost of constructing and maintaining the line.
4. Line losses and transformer losses.
5. Labor costs and housing facilities for construction and operating force.

C. E. Howell: A study of Mr. Silver's paper appears to definitely bring out at least two apparent difficulties to be met in the construction and operation of the super-power transmission systems of the near future. These seem to me to be a matter of human inertia and commercial conservation rather than lack of ideas. These two points are:

First, the low mechanical strength of any high-voltage insulator now on the market;

Second, lack of methods of protecting a large portion of a transmission system from the effects of line trouble on a small section of it.

The practical application of any insulators on the market at the present time probably would necessitate the use of the usual disk with cap and pin, or the insulator known as the "Hewlett Disk." No other insulators have had sufficient application to eradicate apparent defects. These two types have an ultimate mechanical strength so low that it would undoubtedly be necessary to limit their ultimate load to approximately 3000 pounds per string of insulator units. To withstand the mechanical loads which will be impressed on insulators on high-voltage lines of the future, it would be necessary to use a large number of strings of insulators if the above types are employed. This would necessitate large expenditures for insulators, hardware, etc., as well as increasing the dimensions of the supporting structures and therefore their cost. To even imagine replacing a defective disk in a six string tension assembly with 17 disks per string on a line similar to the one described by the author of this paper, causes one to pause.

As a constructive criticism, it is suggested that the present types of insulator shapes be discarded for the moment and suspension units using comparatively large amounts of porcelain in compression to permit greater working loads per string of units, be employed. Small diameter insulators for low-voltage distribution lines and for guy anchors have been constructed on this principle with resulting mechanical strengths of comparatively enormous values, and it is believed that the same ideas may be extended to include the development of insulators for use on high-voltage large capacity lines. It is to be hoped that better ideas than this may be forthcoming soon, but the above suggestion should lead to a departure from the present practise of insulator construction and perhaps to an insulator worth perfecting.

The great transmission systems of the future will require better systems of protection from line trouble than those now employed. As systems and interconnected systems become larger and larger, difficulties of minimizing effects of scattered short circuits, etc., on the whole net work will multiply. The possibilities of obtaining switching equipment to successfully interrupt short circuits on individual circuits consisting perhaps of less than 5 per cent of a system, are good, but means for preventing this short or ground from demoralizing the remaining 95 per cent of a larger system are less easily perfected. Sooner or later service will demand some method of nearly instantaneously reducing the value of a short circuit to a reasonable figure (thus not effecting the remainder of the system), and later disconnecting the circuits in trouble. Most certainly a million kilowatt system with 220 kv. or even higher kv. transmission lines will not be permitted to lose say 800,000 kw. of its load because some small portion of the system is subjected to a "bump." The present-day method of interrupting a portion of the system in trouble and at the same time effecting the operation of the remainder for perhaps an hour or more, is analogous to applying brakes to a high-speed train in such a manner that the tracks become unservicable, thus disrupting the schedule of the remainder of the railroad indefinitely, although undoubtedly stopping the train in question.

It is suggested as a means of accomplishing the results which it will soon be necessary to obtain in operation, that switching equipment be so designed, constructed and arranged that it will be possible to nearly instantaneously insert in a circuit in difficulty resistance or reactance, or a combination thereof, before interrupting such circuits. This would relieve the remaining portion of the system of some of the effects of the short circuit. This scheme will, undoubtedly, be recognized as the idea which was intended to be employed in the reactance type of oil switch which was placed on the market several years ago, and which has had a more or less successful career. It is believed that either resistance or reactance, or a combination thereof, may be employed to perform the function suggested above, but that it will be necessary to liberally construct the switching equipment employing this feature, in order that mechanical difficulties may be prevented. This may necessitate employing two switches, or two separate switching elements in one switch, or a switch containing two separate elements and the reactance or resistance.

There are undoubtedly other and better ways of solving this problem but whatever they may be, it is to be hoped that better means of switching large capacity systems, than now employed, may soon be available.

T. B. Parker: In the design of steel towers for transmission lines, it has been found necessary to fit each type of tower to definite conditions, which shall as nearly as possible represent the requirements of actual operation. As in the paper under discussion, it is usual to specify three sets of design loadings; (1) a vertical load, representing the dead weight of structures and conductor and the effect of vertical angles; (2) a transverse, horizontal load, representing the direct over-turning effect of wind and horizontal angles; and, (3) a torsional load to provide for the effect of broken wires and unbalanced loading.

In view of the large savings that can be made by small changes in detail, it is desirable, when possible, to subject each tower type to actual tests. It is best to make test loads identical with design loads, and, for the sake of simplicity, to consider each set of loads as separately applied. This means that a tower will be designed for the maximum stress produced by any one of the three non-simultaneous sets of loadings.

Line towers, however, are normally subjected to loads which are not purely vertical or horizontal, but a combination of both. It is therefore necessary to know the effect of many different load combinations upon each type of tower, before choosing the proper type for each location. To determine these effects requires much study, while their neglect may result in danger to the line, or in loss of economy.

It is evident that separate design and test loads do not lend themselves to convenient use in line design, while combined test loads would needlessly complicate the procedure and make comparisons difficult. The solution would appear to be the use of separate design and test loads, with the addition of combined loads, each made up of definite proportions of the original vertical and horizontal loadings. Thus, a tower designed to carry non-simultaneously certain vertical and horizontal loads might also be required to support three-quarters of the horizontal load, together with one-half of the vertical. Two or three such specified combinations suffice to definitely establish the characteristics of a tower and to clearly define its limitations.

This method has the advantage of retaining the separate, non-simultaneous test loads, but extends the design work to cover mutually consistent combinations. Possibilities of overload are avoided, and greater economy secured by taking advantage of the maximum strength of towers under all conditions.

J. B. Crane: The paper presented by Mr. Silver is very important at this time as it points out one way in which large systems can be interconnected to secure the maximum advantages of diversified loads.

On page 1087 the author shows a typical diagram and it is suggested that it might be possible to cutout the 220-kv. bus at the main substation and to use the 66-kv. oil switches on the low-tension side of the transformers. This is, of course, on the assumption that the substations will be operated at 66 kv. or 110 kv. and that further transformation will be necessary in case it is decided to furnish low-tension power from the substations.

The use of 220 kv. for transmission presents some very interesting operating problems and on account of the present high cost of building transmission lines it is suggested that a careful study should be made to replace insulators with current on the line in order to obviate the expense of building and maintaining duplicate transmission lines for any single service.

The use of fifteen insulators in one string should allow a factor of safety large enough for the proper testing and renewal of defective insulators. The writer has had some experience with operation at 110 kv. where we have been using seven insulators on tension and eight insulators on strains or semi-strains. The practise is to test the insulators once a year (this will probably be shortened to once in six months) and where three defective insulators are found in one string the trouble is immediately reported and the line gang replaces same as soon as possible. Where less than three defective insulators are found the matter is reported on the regular daily report and the insulators are changed at the earliest suitable time. There was one case recorded where the line was operating with four defective insulators in one string leaving only three good insulators for protection of the line.

It is suggested that it would be possible to put two clamps in series on each wire and make suitable suspension hangings above each of these clamps so that an entire new string of insulators could be placed on a tower to take care of any defective strings and suitable fittings could be made for fastening the insulator assembly to the clamps and to the tower so that same could be done without interrupting the service.

The writer believes that an experimental line one-half mile long should be built and operated at 220 kv. as soon as possible in order to work out some of these operating details and to ascertain just what kind of trouble would be likely to be encountered in practise.

A. E. Silver: Several speakers have mentioned the interlinking type of insulator as being especially suitable for 220-kv. service. However, due consideration must be given to the difficulties of mechanical assembly under the heavy loading requirements of 220-kv. construction, which with present designs of this type of insulator would be even more serious than with the standard cap and pin type.

Mr. Hanker's point as to the possibility of over excitation of generators by the line charging currents, resulting in abnormal voltages on the transmission line, is well taken. In specific cases careful consideration should be given to eliminating danger from this source.

Referring to Mr. Summerhayes' point regarding Figs. 1 and 2, the titles are misleading, although it is noted in the foot notes accompanying these figures that generator voltages of 225 kv. and 170 kv. and corresponding receiver voltages of 200 kv. and 150 kv. are used in the comparison.

It seems to me that the essential thing now is not to endeavor to draw any definite conclusions but to come to a thorough realization that the problem of developing 220-kv. transmission is definitely confronting us and to present clearly to the engineering profession the conditions of the problem and then to all work together to bring out the best possible solution.

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THE EFFECT OF TRANSIENT VOLTAGES ON DIELECTRICS II

The Effect of Lightning Voltages on Arrester Gaps, Insulators and Bushings on Transmission Lines

BY F. W. PEEK, JR.

Abstract of Paper

This paper treats of some of the practical applications resulting from an investigation of the effect of lightning voltages on insulators, bushings and protective gaps.

There is a great difference in the relative lightning spark-over voltages of various gaps as well as a great difference in the settings imposed by operating conditions. Both of these factors must be considered in comparing the relative protective values.

A gap must be set so that the normal line voltage does not cause it to spark-over. Gaps are generally used out of doors. Rain lowers the 60-cycle spark-over voltage of all uncovered gaps and thus imposes a greatly increased setting and decreased protective value since the lightning spark-over voltage is not changed by rain.

The covered sphere gives the maximum protection. The protective value is constant under all conditions.

The sphere-horn, having electrodes of points, horns and spheres, gives very good protection over the whole range of frequency and wave front. The spheres discharge the very steep waves, the horns the moderate ones, and the points continuous high-frequency waves, of slanting front and static.

The protective value of selective gaps varies with the wave front. Its protective value is a minimum for very steep wave fronts and for waves of slanting front. Over a certain range its protective value is very good.

The relative protective values of various gaps for steep and slanting wave fronts and high frequency are shown graphically in Figs. 14, 15, 16 and 17. The relative protective values are approximately independent of the point on the 60-cycle wave at which the discharge occurs.

Data are given on the steepness of lightning waves actually occurring on transmission lines in practise.

Bushings and insulators with equal 60-cycle spark-over voltages may have entirely different lightning spark-over voltages. A bushing should be designed for a high lightning spark-over voltage.

The lightning wet spark-over voltage of a bushing or insulator is the same as the dry spark-over voltage.

IN 1915 I presented a paper covering an extensive investigation on the effects of transient or lightning voltages on air, oil and solid insulations, line insulators and the discharge voltages of various gaps.[1] An exact study was made possible by the development of the "impulse generator," which was also described in the paper referred to above, and as a result some very important fundamental relations were discovered.

It is the purpose of this paper to treat in more detail a few of the important practical applications which have been made of these relations within the last few years. The present discussion will be concerned principally with lightning arrester gaps, bushings and line insulators. It is hoped in particular, to make clear the advantages and disadvantages from a strictly practical standpoint of the various arrester gaps.

The fundamental relations referred to above, bearing on the present discussion, will be briefly reviewed:

When a 60-cycle voltage is slowly applied to a gap and gradually increased, spark-over will occur at some definite voltage. This is the minimum voltage that will cause sufficient ionization for the gap to discharge and it requires a relatively long time.

Lightning voltages, or voltages of relatively steep wave front start at zero or line voltage and increase at the very rapid rate of millions or billions of volts per second. When such voltages are applied across a gap or insulator, spark-over does not occur at the instant the minimum or 60-cycle voltage is reached, as considerable time is required at this voltage. When this voltage is reached the spark begins to form but is only completed after the rapidly rising voltage has reached some higher value. The "slower" the gap the higher the voltage will rise. In a uniform field, break-down takes place over a relatively short path, everywhere, at the same time. In the case of a non-uniform field represented, for instance, by the needle gap, corona forms around the electrodes before spark-over. A vast amount of air must be ionized. The condition is equivalent to putting the corona or arc resistance in series with an ever increasing capacity represented by the unbroken dielectric. Time is thus required to bring all of the space between the electrodes up to the break-down gradient and during this time, the lightning voltage rises higher and higher.

To summarize: (1) Two gaps or insulators with equal 60-cycle spark-over voltages may have entirely different lightning or impulse spark over voltages because of the time lag.

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1. "The Effect of Transient Voltages on Dielectrics," F. W. Peek, Jr., A. I. E. E., Vol. XXXIV, 1915, page 1857.

"Lightning," *General Electric Review*, July 1916.

(2) The time lag is the greatest in a non-uniform field or for electrodes where corona precedes spark-over; it is minimum for a uniform field.

(3) The time lag for any given electrodes and spacing is not constant, but depends upon the steepness of the wave or the rate at which the voltage is applied. The spark-over voltage increases and the time lag decreases with increasing steepness of wave front.

(4) Lightning or impulse spark-over voltages, unlike 60-cycle spark-over voltages, are not appreciably lowered by rain.

The above discussion means, of course, that certain gaps and insulators which have equal 60-cycle spark-over voltages may have entirely different lightning spark-over voltages. The ratio between the impulse and 60 cycle spark-over voltage was termed the impulse ratio. When there is no time lag the impulse ratio is unity; the greater the time lag, the higher the impulse ratio. Under certain conditions selective gaps may have an *apparent* impulse ratio of less than unity.

It is very important to utilize these principles in design; protective gaps should have an impulse ratio of unity or low lightning spark-over voltage, while insulators and insulation should have a high impulse ratio or high lightning spark-over and puncture voltage.

The practical application of these principles to various protective gaps will first be discussed.

PROTECTIVE GAPS

General. The lower the voltage at which a given arrester gap can be set the greater is its protective value. In practise, the setting must be such that the gap does not discharge under any normal operating condition. The 60-cycle spark-over voltage of a gap is very much decreased if the electrodes become wet. The decrease in voltage due to moisture differs greatly with the shape of electrodes. It is minimum for points and maximum for plane surfaces. The 60-cycle spark-over voltage of a gap may be affected by other surface conditions, but by far the greatest effect is that caused by moisture. See Table I. Practically all high-voltage arrester gaps are installed out of doors. These gaps must, therefore, be set so that the line voltage does not cause spark-over during a rain storm. This means that with any gap with "fast" electrodes the setting must be approximately doubled and the protective value thus reduced.

The wet and dry 60-cycle spark-over voltages of 6.25 cm. spheres is given in Fig. 1. That the lightning spark-over voltage is not appreciably changed by rain is shown in Fig. 2. In comparing the relative protective value of lightning arrester gaps it is, thus important to make the comparison on equal wet 60-cycle spark-over voltages or by the setting imposed by the operating conditions.

TABLE I

SPHERE GAP

The Approximate Effect of Rain, Ice, Dust, etc. on the 60-Cycle Spark-over Voltage of Sphere Gap.

Foreign material on sphere surface.	Voltage per cent of normal
Thin coating of dust...............	98
Coating of oil.....................	100
Heavy coating of oil and sand......	75 – 90
Thin coating of ice................	75 – 90
Thick coating of ice...............	75 – 80
Surface oxidized...................	100
Ordinary pitting...................	90 – 100
Rain 0.2 in. precipitation per min. Polished spheres................	40 – 50
Rain 0.2 in. precipitation per min. Pitted spheres..................	40 – 50

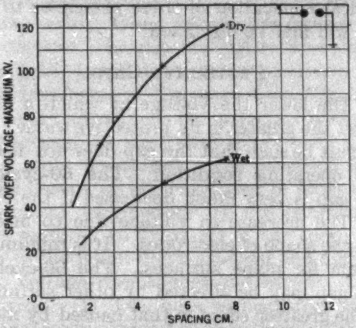

FIG. 1—SPHERES—WET AND DRY SPARK-OVER VOLTAGES

6.25 cm. spheres—06-cycles—one sphere grounded—0.2 in. rain—data Table II

Operating conditions other than rain or inherent properties of the arrester proper may make it necessary to increase the setting of certain types of gaps and not of others. Rain is, however, the chief factor in non-selective gaps. As an example of the effect of rain on the setting, assume a 66,000-volt line with grounded neutral. The voltage to ground is

$$\frac{66{,}000}{1.73} = 38{,}000$$

The arrester gap must be set at about 25 per cent above this or 47 kv. wet. Referring to Fig. 2, if the gap is protected from the weather the lightning spark-over voltage is 47 kv., wet or dry; if the gap is not protected from the weather the dry 60-cycle spark-over voltage must be 94 kv. in order to make the wet 60-cycle spark-over voltage 47 kv. and the apparatus is thus subjected to double the stress which would obtain if a covered gap were used. This follows because the lightning spark-over voltage approximately corresponds to the dry setting. There may be no gain in protection with a gap discharging at very low lightning voltages if in practise it must be set at a wide spacing to prevent line voltages from continually causing it to spark over.

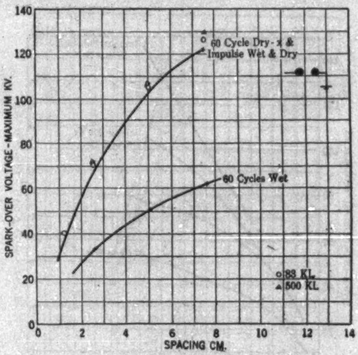

FIG. 2—SPHERE—WET AND DRY SPARK-OVER VOLTAGES

6.25 cm. spheres—60 cycles and impulses—0.2 in. rain—one sphere grounded—data Table II

The horn gap is not affected by the weather to as great an extent as the sphere gap. If a sphere and a horn are adjusted for equal wet 60-cycle spark-over voltages, the dry 60-cycle spark-over voltage of the horn will be lower. For low-frequency surges the horn would thus discharge at a lower voltage. For steep wave front lightning voltages however, the lag of the horn, which may easily have an impulse ratio of 2, will cause it to give inferior protection. See Fig. 3.

THE SPHERE GAP—THE SPHERE HORN

The sphere gap has an impulse ratio of unity. It thus offers equal protection for all sorts of transient voltages, and is without time lag when set at not greater than diameter spacing. When exposed to the weather, however, the setting must be high enough so that the line voltage will not spark-over during rain. See Figs. 1 and 2.

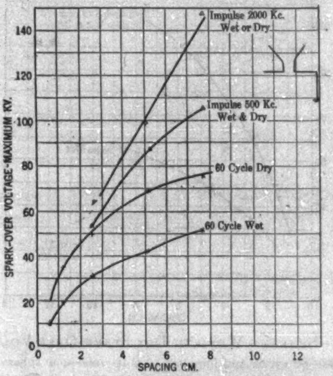

FIG. 3—HORNS—WET AND DRY SPARK-OVER VOLTAGES

60-cycle and impulse—wet and dry—data, Table IV

In the practical gap the sphere and horn were combined; the horn being used to assist in breaking the dynamic arc and for the gain in discharging low-frequency surges due to the smaller difference between the wet and dry spark-over voltages. The difference between the wet and dry spark-over voltage of points is less than with the horn. A point is sometimes added to further increase the protection at low-frequency surges.

This gap has proved very successful in its several years of practical use, very greatly increasing the protective value of arresters. See Fig. 4. Take for comparison equal wet settings of 50 kv. at 2000 kilocycles the spark-over voltage of the horn is 135, the sphere 100. For impulses below 500 kilocycles, the spark-over voltage of the horn is lower than the sphere. Thus, when a sphere horn is used the discharge takes place across the sphere for steep wave fronts and across the horn for low-frequency surges. The gain due to the sphere is greater at higher voltages and steeper wave fronts. The covered gap, shown by the dotted line is superior at all wave fronts.

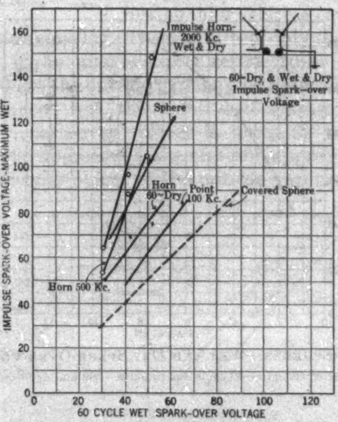

FIG. 4—SPHERE—HORN

Relative protective values of the component parts—data Tables II, IV, and V

The Covered Sphere. If a sphere gap is covered and shielded from the weather its protective value is greatly increased since the setting imposed by the condition that the normal line voltage must not discharge over the gap is cut in half. Such a gap, therefore, discharges lightning voltages of half the value of the uncovered sphere. This gap gives the highest degree of protection. It is not possible to use it with all types of arresters since a horn is often necessary to assist in breaking the dynamic arc. See Fig. 5. A properly designed hemisphere may also be used in this type of gap.

A gap not appreciably affected by the weather and still providing an arc breaking horn may be built as shown in Fig. 6. The way this accomplishes the desired results will be described later.

Since the gap requires two spheres in series, it is necessary to determine if such an arrangement has appreciable time lag and, therefore, high lightning-discharge voltages. Two 6.25-cm. spheres connected in series are shown in Fig. 7. If gap *A* is set at approximately 25 kv. and gap *B* at approximately 75 kv. the 60-cycle spark is not (25 + 75) = 100 kv. but is 75 kv.; the lightning or impulse spark-over is 91 kv. See Fig. 7.

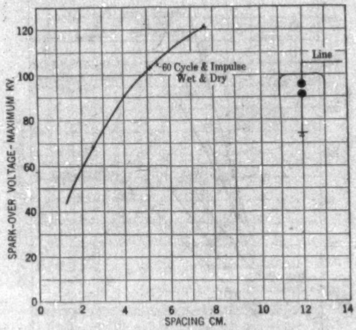

Fig. 5—Spheres—Wet and Dry Spark-Over Voltages

6.25 cm. spheres—covered gap—60 cycle and impulse—wet and dry—one sphere grounded—data Table V

The two gaps break down at 75 kv., 60 cycles, instead of the sum of the two or 100 kv. because the applied voltage does not divide evenly between them. The voltage reaches 25 kv. across the low-voltage gap and breaks that gap down before it reaches 75 kv. on the high-voltage gap. All of the stress is thus transferred to the high voltage gap and it breaks down as soon as a total voltage of 75 kv. is reached. If, now, capacities are adjusted across *A* and *B* so that the voltage divides in proportion to their relative break down voltages both gaps will break down simultaneously. The break down voltage will be equal to their sum. This is called the "balanced" gap. When two sphere gaps each without appreciable lag are placed in series it is found that unless the gaps are "balanced" there is considerable lag. This would be expected because one gap breaks down first and puts resistance in series with the capacity of the other similar to the corona in the needle gap. Balancing the gaps causes simultaneous spark-over and there is no appreciable lag. See Fig. 7.

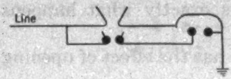

Fig. 6

If two gaps are placed in series as in Fig. 6 and balanced by properly adjusting their relative capacities there will be no appreciable lag. The rain affects only the outside gap. For example, if the outside gap is set at 10 kv. and the inside gap at 50 kv. the outside gap may be reduced to 5 kv. by rain. If balanced wet, the total wet spark-over is 55 kv. while the dry spark-over voltage is about 60 kv. This gap is thus without appreciable lag and not appreciably affected by rain. The only object of the outside gap is, of course, to transfer the dynamic arc to the horn where it rises and breaks.

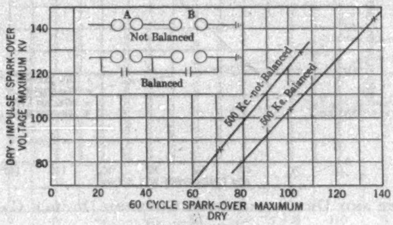

Fig. 7

Data Table III

The impulse and 60-cycle characteristics of this gap are shown in Fig. 8.

The advantage of the sphere gap is that it gives equal protection under voltages of all frequencies and wave front and is practically without lag.

Selective Gaps. Various forms of selective gaps have been proposed from time to time. Probably the most interesting and important of these is that investigated by Mr. Allcutt and shown in Fig. 9.[2] In this gap the division of voltage is not greatly affected at 60 cycles by the auxiliary electrode. The auxiliary electrode is held at mid-potential because it is connected at the mid-point between two equal condensers. The capacity current is too small at 60-cycles to cause any appreciable "drop" across the resistance. If the condenser circuit were opened on one side, the gap on that side would break down at about half voltage. This is exactly what happens under impulse.

For steep wave fronts the resistance has the effect of opening the condenser circuit on that side. See Fig. 10. The gap on that side breaks down. The voltage does not immediately disappear across the arc. The gap has lag for the same reason as the double unbalanced gap discussed above. Whether it is above or below the 60-cycle setting depends upon the impulse. The effect is similar to that which would result from a needle gap which could be set at, for instance, 100 kv. for 60-cycle operation and instantly and automatically reduced

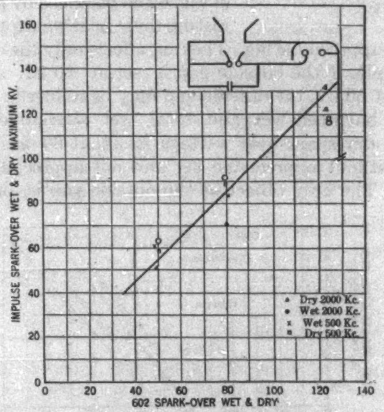

FIG. 8.—WET AND DRY SPARK-OVER COVERED DOUBLE GAP BALANCED

Data Table VI

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[2] "Lightning Arrester Spark Gaps," C. T. Allcutt, TRANS. A. I. E. E., 1918, Vol. XXXVII, Part II, p. 855.

to a 50 kv., 60-cycle setting whenever an impulse came on the line. For moderately steep wave fronts the spark-over voltage would be greater than 50 and less than 100 kv.; but for very steep wave fronts, the impulse ratio of the 50 kv. gap would be greater than (2), or the spark-over voltage would be greater than 100 kv. The impulse ratio of the selective gap is always greater than unity; the *"apparent" impulse ratio* is greater or less depending upon the steepness of the wave front. The reason this distinction is made is discussed elsewhere.[3] The apparent impulse ratio should be used in comparing protective values. This characteristic for the selective gap is shown in Fig. 11 and compared with a sphere gap for the same dry 60-cycle settings. The sphere gap spark-over voltage is practically constant for all wave fronts. The spark-over voltages are the same for 60 cycles. At moderate wave fronts the selective gap has about 5 to 20 per cent lower spark-over voltage than for spheres, while for steeper wave fronts the voltage is higher on the selective gap. The protective value of a gap, as already pointed out, depends not only on its lightning discharge voltage for a given 60-cycle setting, but also upon the setting which is imposed upon it by operating conditions. Fig. 11 shows the relative protective values of spheres and

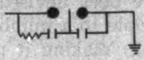

Fig. 9

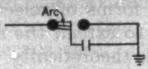

Fig. 10

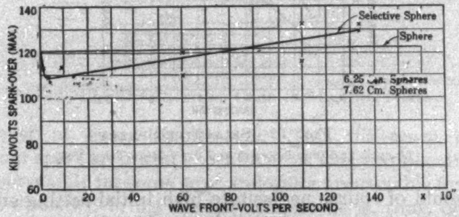

Fig. 11—Variation of Spark-Over Voltage with Wave Front—Sphere and Selective Sphere—
Data Table VII

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[3] "Lightning Arrester Spark Gaps," C. T. Allcutt, Trans. A. I. E. E., 1918, Vol. XXXVII, Part II, p. 855.

selective spheres assuming equal dry 60-cycle settings are possible. The settings must be such that the line voltage does not frequently spark-over and cause the destruction of the energy absorbing device under certain operating conditions. The effect of rain makes it necessary to set a non-shielded selective gap at about double the voltage that would be necessary in the protected gap. See Fig. 12.

Other forms of selective gaps have been devised and it is possible to extend the selective principle to a number of gaps in series, theoretically (neglecting lag) making it possible to discharge an impulse at a small fraction of line voltage. Such a gap would of course necessitate high initial setting and give very little protection against lightning impulses.

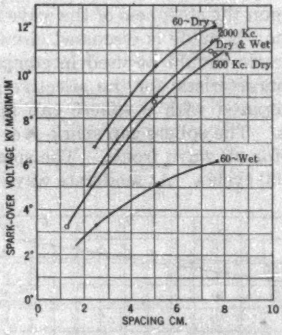

Fig. 12—Selective Sphere

60-cycle and impulse—wet and dry—Data Table VIII

The selective principle may also be readily applied to covered gaps if it is deemed advisable.

Relative Protective Values of the Gaps Already Discussed

The following comparison of the different gaps is the result of extensive research. The tests were made with the impulse generator. The methods of conducting the tests, the precautions, accuracy, etc. are the same as discussed in the former paper.[4] For convenience the connection diagrams are shown in Fig. 13—(a) is used when the impulse only is applied to the gap; (b) when the impulse is superposed on the 60-cycle wave.

There are three cases which require consideration.[5]

Case A. Where the impulse occurs at the zero point of the 60-cycle wave and is thus not affected thereby.

Case B. Where the impulse occurs at the maximum of the 60-cycle wave and is additive.

Case C. Where the impulse occurs at the maximum of the 60-cycle voltage wave, but in the opposite direction.

Case C is naturally the most dangerous case. Case A is equivalent to applying the impulse without 60-cycle voltage.

In making impulse tests it is found that there is a certain minimum impulse voltage that will spark-over the gap only occasionally and that the voltage must be increased to cause spark-over at every application. This difference between minimum spark-over and spark-over at every application may be inappreciable for some gaps and quite appreciable for others. This is discussed in my former paper. It is, thus, obviously necessary to record the method of making the tests. The data for all of the curves with description of method of making measurements will be found in the tables. Unless otherwise stated, the impulse voltage was increased until the gap sparked once in five applications. "Frequency" is used for convenience only. It means, unless otherwise stated, that an impulse approximating a single half sine wave of the frequency stated has been used. By applying a super-voltage or a voltage higher than the discharge voltage of the gap the steepness of

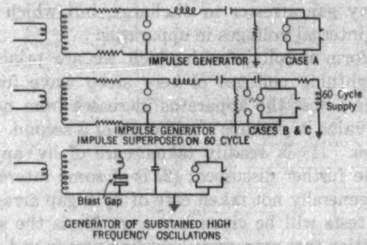
IMPULSE GENERATOR
CASE A
60 Cycle Supply
IMPULSE GENERATOR
CASES B & C
IMPULSE SUPERPOSED ON 60 CYCLE
Blast Gap
GENERATOR OF SUBSTAINED HIGH FREQUENCY OSCILLATIONS

FIG. 13—CONNECTIONS USED IN MAKING TESTS

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4. "The Effect of Transient Voltages on Dielectrics," A. I. E. E., F. W. Peek, Jr., 1915.

5. "Lightning Arrester Spark Gaps," C. T. Allcutt, A. I. E. E., May 1918. Discussion, F. W. Peek, Jr., A. I. E. E., June 1918.

the wave or rate of application is increased. For instance a super-voltage at 100 kilocycles might be steeper than a lower voltage at 2000 kilocycles. In making measurements by adjusting two gaps in parallel for equal spark-over, as is done in the case of super voltages, it is important to arrange the gaps symmetrically with equal length of leads; otherwise inconsistent results will be obtained. With steep waves and unsymmetrical leads it is possible to short-circuit one gap and still obtain sparks on the other.

The over-voltages that cause insulation failures in practise may be divided into three classes:

1. Gradual increase of voltage on the line due to static or low frequency surges.
2. Very high frequency oscillations of voltages generally too low for any gap arrester to discharge, but which may cause very high internal voltages in apparatus.
3. The form of voltage with which we are principally concerned—lightning voltages of very steep wave fronts where the voltage across the apparatus increases from normal to a very high value in perhaps a millionth of a second.

Condition. (1) is readily taken care of by any gap and need not be further discussed; (2) is of some interest but is a condition generally not taken care of by a gap arrester—some results of tests will be given however; (3) is the steep wave front condition that represents lightning proper and with which we are mostly concerned.

Relative Protective Value of the Horn, Sphere-horn, Selective Sphere Gap and Covered Gap.

Impulse Voltages of Steep Wave Front. The spark-over voltages of various types of gaps are plotted with equal wet 60-cycle settings in Fig. 14. Values are plotted for both wet and dry electrodes. The wave applied was a single half-cycle of a 2000 kilocycle wave with a 340-kv. maximum; that is, at super-voltage. The rate of application of voltage of the wave front was thus about 70×10^{11} volts per second. I believe that waves steeper than this occur on lines in practise. In fact, I first noted that there was a difference between the 60-cycle and lightning spark-over voltages of various electrodes by the existence of such waves on an operating line. The bushings on the line always "protected" the lightning arrester horns although the horns had a lower 60-cycle spark-over voltage. By measuring the impulse spark-over voltages of the bushing and the arrester gap in the laboratory it was found that the bushing protected the horn for a wave front at which the impulse ratio of the horn was over (2); this corresponds to a steeper wave than the one under immediate discussion.

It will be noted that the covered gaps give by far the best protection under this condition. For example, when all the gaps are set on the line at 100 kv., lightning voltage discharges respectively at 100 kv. on the covered gap, 115 kv. on the balanced covered gap, 225 kv. on the sphere of the sphere horn, 225 kv. on the selective sphere, and 320 kv. on a horn.

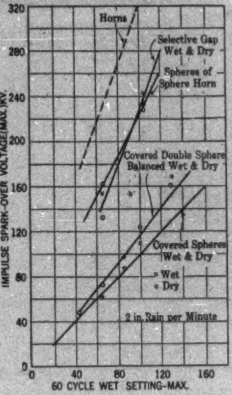

Fig. 14—Relative Protective Values of Horns, Sphere-Horns, Selective Spheres. Covered Spheres—Steep Wave Fronts

2000 kilocycles—340 kv.—impulse—data Tables IX, X and XI

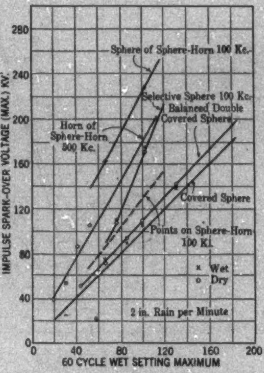

Fig. 15—Relative Protective Values of Sphere-Horn, Selective Spheres, Covered Sphere—Moderate Wave Fronts

Impulse—single half-cycle 100 kilocycles—non-grounded—data Tables IX, X and XI

Moderate Wave Fronts. A similar comparison is given in Fig. 15 for moderate wave fronts. The impulses being single half cycles of 100 kilocycle waves, the average fronts ranging from 0.5 to 1. $\times 10^{11}$ volts per second.

It will be noted that here, also, the covered spheres give the best protection. For example, at a 100-kv. line setting the impulse spark-over voltages are respectively 100 kv. for the covered sphere, 110 kv. for the balanced covered sphere, 170 kv. for the selective sphere, 178 kv. for a horn or the horn of the sphere horn, 130 kv. for points of the sphere-horn, and 222 kv. for the sphere. If these data are compared with that in Fig. 14 the value of the sphere horn combination is well illuslustrated. For the steep wave fronts the sphere affords the better protection, while for the moderate waves the horn affords the better protection and a still greater gain is made by adding points. This comes about, of course, due to the difference between the wet and dry setting.

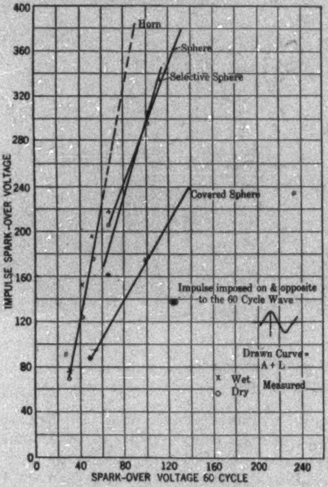

FIG. 16—RELATIVE PROTECTIVE VALUES OF VARIOUS GAPS

Impulses imposed on and opposite to the maximum of the 60-cycle wave—data Table XII

Ratio for Comparing the Relative Protective Value of Various Gaps. From the above discussion it is readily seen that in order to compare the relative protective value of various gaps two factors must be considered.

1. The increased 60-cycle setting imposed by operating conditions to prevent the gap from continuously discharging *due to rain or harmless surges.* Let the ratio of the actual operating setting to the normal setting be called α where the normal setting is the setting that just prevents the line voltage from arcing over under ideal conditions.

2. The impulse ratio (or apparent impulse ratio for the selective gap) for the wave under consideration. Let the impulse ratio be called β

The relative protective value of two gaps is then $\dfrac{\alpha_1\ \beta_1}{\alpha_2\ \beta_2}$

For example:—a gap must be set at 50 kv. (max.) to prevent the 60-cycle line voltage from causing it to spark-over under ideal conditions. The relative protective values of a horn and a covered sphere for the 2000 kilocycle wave are obtained as follows from Figs. 3 and 5.

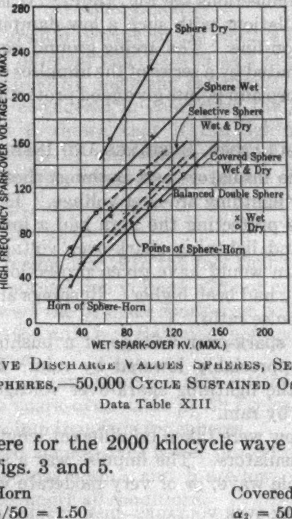

Fig. 17—Relative Discharge Values Spheres, Selective Spheres, Covered Spheres,—50,000 Cycle Sustained Oscillation—
Data Table XIII

Horn	Covered Sphere
$\alpha_1 = 75/50 = 1.50$	$\alpha_2 = 50/50 = 1$
$\beta_1 = 133/75 = 1.77$	$\beta_2 = 50/50 = 1$
$\alpha_1\ \beta_1 = 1.50 \times 1.77 = 2.65$	$\alpha_2 \beta_2 = 1$

$\alpha_1 \beta_1/\alpha_2\ \beta_2 = 2.65$

The horn permits the lightning voltage to rise to 2.65 times the value of the voltages permitted by the covered sphere.

Combination of Lightning and 60-cycle Voltages. The lightning spark-over voltage is a minimum when it occurs at the maximum of the 60-cycle wave and in an additive direction (case B). The lightning voltage is a maximum when it occurs at the maximum of the 60-cycle wave but in the opposite direction (Case C). The relative effects are approximately the same for all of the types of gaps discussed. If the lightning voltages for case A, Case B, Case C and the 60-cycle line voltage are called A, B, C and L respectively the lightning spark-over voltages are approximately:

Case A. A
Case B. $A - L$
Case C. $A + L$

Data for Case C are plotted in Fig. 16.

High Frequency Oscillations. The effect of sustained high frequency oscillations not very highly dampened is shown in Fig. 17. For connections see Fig. 13 (c). It is probably very rarely that oscillations with such a low damping factor occur on a transmission line. The arcing ground condition is more nearly approximated by a series of the impulses discussed above. Note that the horn and points give good protection for sustained oscillations.

LINE INSULATORS, BUSHINGS AND INSULATION

General. Line insulators and bushings should have a high impulse ratio or lightning arc-over voltage. The bushing mentioned above as protecting the horn had a low impulse ratio. The 60-cycle and lightning spark-over voltages were nearly equal. The horn would have given protection in this case if the impulse ratio had been higher. Bushings are now designed with a high impulse ratio.

The 60-cycle spark-over voltage of a bushing or insulator is often very appreciably lowered by rain. It is fortunate, however, that the lightning spark-over voltage is not appreciably changed by rain.

The data below were taken on different lengths of strings of Hewlett disk insulators. The impulse was a single half-cycle of a 200-kilocycle wave, or of very moderate wave front.

SPARK-OVER VOLTAGES—DISK INSULATORS
(One side Grounded—Dry)

No. of units	60-cycle spark-over	Impulse spark-over 200 kc.	Impulse ratio	String efficiency 60-cycle	String efficiency impulse 200 kc.
1	80	85	1.06	....	....
2	142	167	1.18	0.87	0.98
3	204	262	1.28	0.85	0.99
4	261	345	1.36	0.81	1.01
5	317	410	1.30	0.79	0.97
6	368	....	....	0.77	....

$$\text{String efficiency} = \frac{\text{Measured spark-over of string}}{\text{Number of units in string} \times \text{spark-over voltage of one unit}}$$

It is interesting to note that even with this moderately steep wave front the lightning spark-over voltage is approximately the product of the spark-over voltage of a single unit and the number of units in a string. The impulse voltage increases with increasing steepness of wave front.

The wet impulse spark-over voltage is approximately the same as the dry.

Impulse ratios of three or more have been obtained on bushings. More complete data on line insulators have been published elsewhere.[6]

CONCLUSIONS

1. There is a great difference in the relative lightning spark-over voltages of different gaps as well as a great difference in the settings imposed by operating conditions. Both of these factors must be considered in comparing the relative protective values. It is shown that if the lightning spark-over factor is represented by the impulse ratio β, and the setting factor by α, the relative protective value of two gaps are inversely as $\frac{\alpha_1\,\beta_1}{\alpha_2\,\beta_2}$.

2. β is due to time lag; α to the fact that the gap must be set so that the normal line voltage will not cause it to spark-over. Rain lowers the 60-cycle spark-over voltage of all gaps, and thus affects the ratio α. Rain does not lower the lightning spark-over voltage.

3. All uncovered gaps require a high setting factor, because rain lowers the spark-over value at 60 cycles. The effect is much less for points than it is for spheres.

4. The covered sphere gives better protection than any uncovered gap, because both α and β are low. The protective value is constant under all conditions.

5. The sphere-horn, having electrodes of points, horns, and spheres, gives very good protection over the whole range of frequency or wave front, due to the different values of α and β for its various electrodes. The spheres discharge the very steep waves, the horns the moderate ones, and the points continuous high frequency, waves of very slanting front and static.

6. The protective value of the selective gaps, as the name

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[6] "Factors Determining the Safe Spark-over voltage of Insulators and Bushings for High-Voltage Transmission Lines," F. W. Peek, Jr., *General Electric Review*, June 1916.

implies, varies with the wave front. Its protective value is a minimum for very steep wave fronts, and for waves of slanting front. Over a certain range its protective value is very good.

7. The relative protective values of various gaps are shown in Figs. 14, 15 and 16.

8. The relative protective values of various gaps are approximately independent of the point on the 60-cycle wave at which the discharge occurs, as shown in Figs. 14, 15 and 16.

9. Data are given on the steepness of lightning waves actually occurring on transmission lines in practise.

10. Bushings and insulators with equal 60-cycle spark-over voltages may have entirely different lightning spark-over voltages. A bushing or insulator should be designed for a high impulse ratio or lightning spark-over voltage.

11. The lightning wet spark-over voltage of a bushing or insulator is the same as the dry spark-over voltage.

APPENDIX

TABLE II

SPHERE GAP

Wet and dry 60-cycle and impulse spark-over.
6.25-cm. spheres—One grounded 2 in. rain per minute.

Spacing		Spark-over voltage (maximum)			
Inch	Cm.	60-cycle and impulse dry	60-cycle wet	Impulse 8? kc. wet	Impulse 500 kc. wet
0.5	1.27			40	
1.	2.54	68	33	71	72
2.	5.08	103	51	105	106
3.	7.62	121	61	126	130
Case B Impulse assisting and at maximum of 60-cyce wave.					
		Impulse dry 500 kc.			
1	2.54	52			
2	5.08	60.5			
3	7.62	86			
Case C Impulse opposing and at maximum of the 60-cycle wave.					
1	2.54	108			
2	5.08	148			
3	7.62	160			

No parallel gap—one discharge in five applications 60-cycle voltage ½ dry discharge voltage of the gap in cases B and C.

TABLE III
60-CYCLE AND IMPULSE SPARK-OVER OF TWO GAPS IN SERIES.

6.25-cm. spheres.
One sphere grounded.

Spacing gap				60 cycle spark-over				Impulse spark-over	Impulse ratio
A		B		A	B	Sum A and B	Measured A and B	500 kc.	
Inch	Cm.	Inch	Cm.						
		Gaps not balanced for simultaneous spark-over							
0.45	1.19	1.02	2.59	35	68	103	72	86	1.20
0.45	1.19	2.15	5.46	33.7	107	140	106	129	1.21
		Gaps balanced for simultaneous spark-over							
0.45	1.19	1.02	2.59	35	67	102	101	103	1.02
0.45	1.19	2.15	5.46	33.7	107	140	136	144	1.06

Impulse voltages applied and increased in values until one spark-over occurred in five applications. No. 60-cycle voltage on the gap.

TABLE IV
HORNS

Wet and dry 60-cycle and impulse spark-over
Voltage of horns (3/8 in.)
0.2 in. rain—one horn grounded.

Spacing		Spark-over voltage—maximum					
		Dry			Wet		
Inch	Cm.	60 cycle	500 kc.	2000 kc.	60 cycle	500 kc.	2000 kc.
0.25	0.63	19	....	....	9.2	....	....
0.5	1.27	35	39.7	44.5	19.7	....	....
1	2.54	50	53	63.5	31.0	....	45
2	5.08	69	88	96.5	41.5	....	99
3	7.62	74.5	105	148	52	117	148

No parallel gap.—One discharge in five applications. 2000 kc. (b) wave Table XIV.

TABLE V
COVERED SPHERES
Wet and Dry—60-cycle and Impulse Spark-over Voltage
6.25-cm. spheres—One sphere grounded.

Spacing		Spark-over Voltage—Maximum					
Inches	Cm.	60 cycles dry	500 kc. dry	2000 kc. dry	60 cycles wet	500 kc. wet	2000 kc. wet
Case A—No. 60-cycle voltage on the gap.							
0.50	1.27	....	....	....	....	....	....
1.00	2.54	68	68	68	68	68	68
2.0	5.08	103	103	103	103	103	103
3.0	7.62	121	121	121	121	121	121
Case B—Impulse assisting and at maximum of the 60 cycle voltage.							
1	2.54	....	52	....	....	....	....
2	5.08	....	60.5	....	....	....	....
3	7.62	....	86	....	....	....	....
Case C—Impulse opposed to and at maximum of the 60-cycle voltage.							
1	2.54	....	108	....	....	....	....
2	5.08	....	148	....	....	....	....
3	7.62	....	160	....	....	....	....

No parallel gap—One discharge in five applications—60 cycle voltage ½ dry discharge voltage of the gap applied in cases B and C. 2000 kc (b) wave Table XIV.

TABLE VI
Double Covered Sphere Gap—Balanced
Balanced Wet
(0.2 in. Rain—6.25-cm. spheres—One sphere grounded)

Total 60-cycle spark over max.	Gap setting Kv.max. dry		Impulse spark over total kv. max.	
			Wet	Wet
	A	B	500 kc.	2000 kc.
50	14	42	59	63
79	23.5	71	89	91
124	35	105	118	117
			Dry	Dry
48	14	42	61	51
80	23.5	71	83	71
123	35	105	132	122

Average measured voltages. 2000 kc. (b) wave Table XIV.

TABLE VII
SELECTIVE SPHERE-HORN

(6.25-cm. Spheres with Auxiliary Electrode). One side grounded.

Gap setting In.	Gap setting Cm.	60-cycle spark-over kv max.	60-cycle spark-over ½ gap kv. max.	Impulse spark-over total gap	Apparent impulse ratio	Impulse ratio
Single half cycle 83 kilocycles						
½	1.27	38	23	28	0.74	1.21
2	5.08	105	67	81	0.77	1.20
3	7.62	121	88	113	0.94	1.29
Single half cycle 500 kilocycles						
½	1.27	38	23	32	0.84	1.40
2	5.08	105	67	87	0.83	1.30
3	7.62	121	88	109	0.90	1.24
Single half cycle 2000 kilocycles (a) wave						
½	1.27	38	23	34	0.90	1.48
3	7.62	121	88	114	0.94	1.30
				Impulse spark-over total gap	Apparent impulse ratio	Wave front volts sec.
3	7.62	121	88	113	0.94	0.4×10^{11}
3	7.62	121	88	109	0.90	2.1×10^{11}
3	7.62	121	88	114	0.94	9.1×10^{11}
3	7.62	121	88	110–120*	.91–1.00	60×10^{11}
3	7.62	121	88	106–133*	0.88–1.10	110×10^{11}
3	7.62	121	88	130–132*	1.07–1.10	133×10^{11}

Above data for Case A—Impulse applied at zero of 60-cycle wave.
(*Range of equal spark-over.—Super Voltages, (b) wave Table XIV.)

TABLE VIII

SELECTIVE SPHERES

Wet and Dry 60-cycle and Impulse spark-over 6.25-cm sphere—One Grounded
0.2 in. Rain per minute

Case A							
Spacing		60-cycle spark-over Kv. max.		Impulse spark-over kv. max.			
				Dry		Wet	
Inches	Cm.	Dry	Wet	500 kc.	2000 kc.	500 kc.	2000 kc.
0.5	1.27	....	....	32	34	....	....
1	2.54	68	33	49	....	41	48.5
2	5.08	103	51	87	....	74	89
3	7.62	121	61	109	114	104	113
Case B—Impulse assisting and at the maximum of the 60-cycle wave.							
1	2.54	....	....	54	....	....	....
2	5.08	....	....	54	....	....	....
3	7.62	....	....	67	....	....	....
Case C—Impulse opposing and at max. of 60-cycle wave.							
1	2.54	....	....	97	....	....	....
2	5.08	....	....	115	....	....	....
3	7.62	....	....	147	....	....	....

Applied 60-cycle voltage ½ dry 60-cycle voltage of the gap and impulse one discharge in five applications. (2000 kc (a) wave Table XIV.)

TABLE IX

SELECTIVE SPHERES

12.5-cm. Spheres—Non-Grounded Impulses
Case—A

Setting		60-cycles spark-over Kv. (max.)		Impulse 2000 kc. 340 kv. (max.)		Spark-over 100 kc. single-half cycle.	
Inches	Cm.	Dry	Wet	Dry	Wet	Dry	Wet
2.80	7.10	162	66	133	153	107	109
5.34	13.5	227	102	233	242	170	174

Super Voltage Impulses measured at equal spark-over. 2000 kc. (b) wave.

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TABLE X

COVERED DOUBLE SPHERE GAP—BALANCED

7.6-cm. spheres—Non-grounded

Total 60-cycle spark-over kv. (max.)		Setting—60-cycles kv. (max.)				Impulse spark-over kv. max.			
						2000 kc. 340 kv.		100 kc.	
Dry	Wet	Outside dry	Outside wet	Inside dry	Sum wet	Dry	Wet	Dry	Wet
....	130	63.0	34	101	136	160	168	142	138
....	100	51.0	25.3	75	100.2	123	109	109	105
....	85	38.3	19.8	61	80.8	85.5	96.7	93	88
....	65.5	29.6	....	49.6	....	71	62	71	73
....	44	14.1	....	31.1	....	46.5	46.5	50.5	....

Super-voltage impulses measured at equal spark-over. (2000 kc (b) wave Table XIV.)

TABLE XI

SPHERES—IMPULSES

12.5-cm. Spheres—Non-grounded

Spacing		Spark-over kv. max.		Impulse Spark-over voltage kv. max.			
				(2000 kc.) 340 kv. max.		100 kc.	
Inch	Cm.	60-cycle Dry	60-cycle Wet	Dry	Wet	Dry	Wet
2.80	7.10	162	66	164	....	162	....
5.34	13.5	227	102	227	227	....	....

Super-voltage impulses measured at equal spark-over.
2000 kc (b) wave Table XIV.

TABLE XII

IMPULSE SPARK-OVER VOLTAGES WHEN THE 60-CYCLE LINE VOLTAGE IS ON THE GAP.—CASE C.

Spacing		60-cycle spark-over		Applied 60-cycle	Impulse spark-over kv. max. 2000 kc.					
Inches	Cm.	Kv. max.		Kv.	Case A No. 60-cycle		Sum A & L.		Case C measured impulse	
		Dry	Wet	L.	Dry	Wet	Dry	Wet	Dry	Wet
Selective spheres 12.5-cm.										
2.80	7.10	162	66	50	138	135	188	185	160	161
5.34	13.5	227	102	76.5	227	....	303	303	296	305
Spheres 12.5 cm.										
2.80	7.10	162	66	50	....	164	214	....	204	217
5.34	13.5	227	102	76.5	227	227	303	303	305	305
Covered Spheres 6.25-cm.										
0.70	1.8	50	50	37.5	50	50	87.5	87.5	87.5	87.5
1.58	4.0	100	100	75	100	100	175	175	175	175
Horns										
1	2.54	51	30.7	23	58.5	60	81.5	83	69	76
2	5.08	40.6	67.5	30.5	105	110	135.5	140.5	124	153
3	7.62	52	75	39	145	150	184	189	175	196

Case C—Impulse applied opposed to the maximum of the 60-cycle wave.
Applied 60-cycle kv. 75 per cent of wet 60-cycle spark-over.
Gaps not grounded. One discharge in five impulses.
2000 kc. (b) wave Table XIV.

TABLE XIII
SPARK-OVER VOLTAGES OF GAPS FOR SUSTAINED HIGH FREQUENCY OSCILLATIONS.

Gap-setting		Frequency of oscillations—Spark-over kv. max.									
Inches	Cm.	19,600		50,000		100,000		200,000		160,000 from high freq. coil	
		Dry	Wet	Dry	Wet	Dry	Wet	Dry	Wet	Dry	Wet
Selective Spheres 12.5-cm.											
2.80	7.10	112	105	100	95	77	90	77	92	108	125
5.34	13.5	200	143	131	123	126	127	120	122	170	186
Spheres 12.5 cm.											
2.80	7.10	162	106	162	126	163	106	162	118	163	....
5.34	13.5	227	173	227	164	12.27	173	227	169	227	....
Double gap—Balanced—Covered											
Setting Kv.-60 cycles Wet											
	100	....	98	....	105	....	102	93	100	112	112
	130	....	107	....	126	....	128	132	124	147	150
Covered Spheres											
....	50	50	50	....	....	....	50	50	50	50	50
....	100	100	100	....	....	....	100	100	100	100	100
Inches	Cm.	Horns									
1	2.54	....	....	58	33.5	....	....	....	....	....	....
2	5.08	....	....	82	54	....	....	....	....	....	....
3	7.62	....	....	97	67	....	....	....	....	....	....
Points											
1	2.54	....	....	28.5	28.5	....	....	....	....	....	....
2	5.08	....	....	46	46	....	....	....	....	....	....
3	7.62	....	....	64	64	....	....	....	....	....	....
4	10.2	....	....	74	74	....	....	....	....	....	....
5	12.7	....	....	88	88	....	....	....	....	....	....

TABLE XIV
IMPULSE GENERATOR CONSTANTS

Impulse	Wave	Capacity farads	Inductance henrys	Resistance
83	Single Half Cycle—Sine	1.25×10^{-9}	3.73×10^{-3}	1000
100	Single Half Cycle—Sine	1.25×10^{-9}	3.73×10^{-3}	2000
500	Single Half Cycle—Sine	0.62×10^{-9}	0.28×10^{-3}	750
2000	Single half Sine (a)	0.625×10^{-9}	3.34×10^{-5}	450
2300	(b) wave	1.25×10^{-9}	8.7×10^{-5}	3000

DISCUSSION ON "THE EFFECT OF TRANSIENT VOLTAGES ON DIELECTRICS II" (PEEK), LAKE PLACID, N. Y., JUNE 27, 1919.

Wm. A. Del Mar: Mr. Peek showed a series of curves giving the spark-over voltages for different gaps. I want to ask him whether it may be deduced from these curves that the Institute standard spark-gaps are correct with either dry or wet spark balls for voltages having a frequency of one million or more cycles per second.

F. W. Peek: Replying to Mr. Del Mar, the Institute standard sphere gap curves read correctly for either 60-cycle or transient voltages when the electrodes are dry. When the sphere surfaces are wet the gap sparks over at about half this value at 60 cycles, but for transients of the order of a single half cycle of a one-mlliion-cycle sine wave the spark over voltage is practically as given on the standard curves. There would, of course, be considerable error for continuously applied undamped high-frequency voltages.

C. T. Allcutt: Perhaps the most serious fault with Mr. Peek's paper is the presentation of a large amount of experimental data on discharge gaps, without giving any description whatever of the actual gap structures tested by him. He gives a diagrammatic representation of his ingenious balanced double-sphere gap and describes the theory of its operation, but his description of the gap structure itself is limited to the figures regarding the size of spheres employed. Whether the gap tested was a laboratory model or a commercially practicable gap such as might be applied to an electrolytic arrester has been left to the imagination of the reader. It is impossible for other experimenters to check Mr. Peek's results without further information concerning his apparatus.

Even more objectionable is his failure to describe the structure used by him in testing the type of selective gap now known commercially as the impulse gap. I wish particularly to call attention to the data given in Table VII. These data were given by Mr. Peek at the Atlantic City Convention last June, in his discussion of my paper on "Lightning Arrester Spark Gaps."[1] These data, which apparently contradicted my own, were presented by Mr. Peek without any description being given of the gap structure used in his experiments, the natural inference being that he had used a gap structure built exactly according to the description given in my paper. The fact that he presented these apparently contradictory data without giving sufficient information to enable the reason for the discrepancy to be pointed out was called to his attention in the closure to that discussion.[2] It is unfortunate that Mr. Peek has not taken advantage of this opportunity to correct the omission. Following are a few of the facts that should have been included in his paper and without which his data are of little value.

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[1] C. T. Allcutt, "Lightning Arrester Spark Gaps,", TRANSACTIONS, A. I. E. E. Volume XXXVII Part II, p. 833. F. W. Peek, Discussion, TRANSACTIONS A. I. E. E., Vol. XXXVII. Part II, p. 855.

[2] C. T. Allcutt, Discussion TRANS. A. I. E. E., Vol. XXXVII. Part II, p. 867.

(1) Nature and capacity of condensers employed.

(2) Nature and resistance of unbalancing resistors employed.

(3) Size, shape and position of auxiliary electrode.

(4) Position of the various conducting elements of the gap structure which determine the capacity between the various electrodes.

The same objection applies to the data given in Table IX, although in this case there is no direct implication that the gap tested was substantially the same as the one described by me last June. In fact, some of the data given in Table IX check fairly well with some results of tests that have been made in our laboratory on the standard commercial impulse gap structure so it is entirely possible that some of Mr. Peek's tests were made on such a gap. The difference existing between Mr. Peek's results and those given in the curves accompanying this discussion may then be due either to a gap structure differing materially from the one used in my tests, or, to improper adjustment of the gap electrodes and the fact that Mr. Peek's tests were made with non-grounded impulses. It is obvious that under service conditions one of the electrodes is at ground potential until after the discharge across the gap has taken place, so any tests made should be with one electrode grounded if service conditions are to be properly simulated.

The use of non-grounded impulses is also open to criticism in connection with the tests on the balanced, double-sphere gap recorded in Table X, although in this case it is not believed that the test results would be much changed by the use of grounded impulses.

As I do not believe that Mr. Peek's data on the selective gap in any way represent the characteristics that are obtained in practise with the commercial form of impulse gap, I am presenting here curves showing the discharge characteristics of a standard 44-kv. impulse gap. For comparison with Mr. Peek's data I have plotted the curves showing the impulse discharge voltage as a function of the 60-cycle discharge voltage under rain conditions. I heartily endorse Mr. Peek's contention that this is the only proper basis of comparison for gaps used out of doors. For some time past in our laboratory we have been comparing protective gaps on the basis of what we have termed "protection factor" instead of on the basis of "impulse ratio." Protection factor is defined tentatively as the ratio of the impulse discharge voltage to the 60-cycle discharge voltage under rain conditions. For in-door gaps the protection factor is the same as the impulse ratio. Mr. Peek's quantity is proportional to the protection factor.

Fig. 1 is a photograph of the gap tested. Fig. 2 is a schematic diagram of the electrical circuit of the gap. The capaci-

PLATE XLI.
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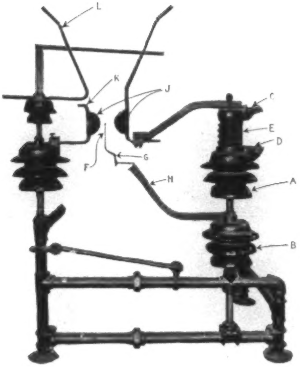
L
K
J
C
E
D
G
F
H
A
B

[ALLCUTT]

FIG. 1—44-KV. IMPULSE GAP.

tance for maintaining the auxiliary electrode at the proper potential is furnished by the pin type insulators (*a*) and (*b*). These insulators have an electrostatic capacity of about 2×10^{-11} farads. Due to the effect of capacity to ground, the auxiliary electrode is not maintained at a potential half way between the potentials of the two main electrodes by the condensers (*a*) and (*b*). It has been found by experiment that the proper position of the auxiliary electrode is such that its distance from the sphere horn "*m*" is six-tenths of the gap length. In this position it does not disturb the field when a 60-cycle voltage is applied across the gap. The unbalancing resistance is furnished by the resistors (*c*) and (*d*). These resistors have about 250,000 ohms resistance each. The resistors are about 12 in. (30 cm.) long and are of a special composition that retains its high resistivity under the most severe conditions of high voltage and high frequency. They are enclosed in porcelain tubes mounted on the porcelain pillar (*e*). The auxiliary electrode (*f*) is a pointed brass rod 0.0808 in. (0.2 cm.) in diameter held in a ⅜ in. (0.95 cm.) diameter brass support member (*g*), which is in turn mounted on an arm (*h*) of 1 in. (2.54 cm.) angle iron. The hemispheres (*j*) mounted on the horns are made of brass and are 12.5 cm. in diameter. In practise the charging resistance of the arrester is connected between the horn members (*k*) and (*l*). For the purpose of test, (*k*) and (*l*) were connected together and grounded to the frame of the gap structure.

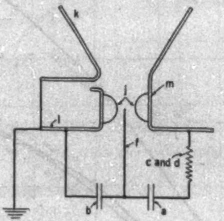

FIG. 2—DIAGRAM OF IMPULSE GAP

The impulse discharge voltage of the gap was determined by direct comparison with a 12.5 cm. sphere gap connected in parallel with it. The high-tension electrodes of the gap under test and of the sphere gap were connected together by a straight lead and the connection to the impulse generator was brought out from the middle point of this lead. The impulse generator itself was connected as shown in Mr. Peek's Fig. 3 (Case A). The following circuit constants were employed:

For 100 kilocycle impulse, $c = 5 \times 10^{-9}$ farads.
$L = 1.25 \times 10^{-3}$ henries and $R = 1000$ ohms.

For 420 kilocycle impulse, $C = 5 \times 10^{-9}$ farads.
$L = 8.3 \times 10^{-5}$ farads, $R = 320$ ohms.
For 3000 kilocycle super-impulse, $C = 5 \times 10^{-9}$ farads.
$L = 8.3 \times 10^{-5}$ henries and $R = 4800$ ohms.

The condenser employed consisted of a stack of one hundred impregnated paper condensers connected in series. Each condenser had a capacity of 0.5 microfarad. Single layer solenoids were used as inductances in the oscillating circuits and water tube resistances were employed. The test methods followed were those outlined in my papers.[3]

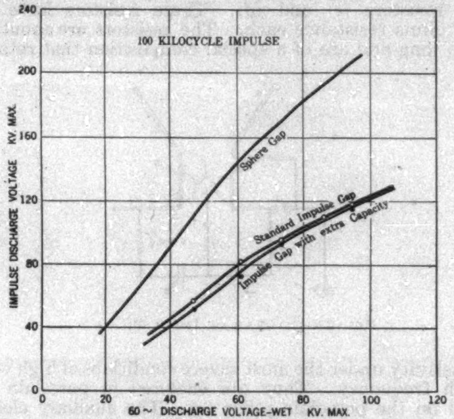

FIG. 3—IMPULSE GAP CHARACTERISTICS

Figs. 3, 4 and 5 give the results of tests made with impulse voltages of different steepness of front. The lowest curve, marked "Impulse gap with extra capacity," gives the results of tests made on the standard gap structure but with additional capacitance connected between the auxiliary electrode and each horn, bringing the total capacity on each side of the auxiliary electrode up to about 10^{-10} farads. This capacity is about the same as that employed in the experimental gap described in my 1918 paper and these results check quite well with the results published in that paper. It will be seen that the improvement due to the additional capacity is slight at 100 kilocycles and 420 kilocycles (Fig. 3 and 4) and, since it is believed that this represents the range of frequencies to be expected in practise,

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[3] "Lightning Arrester Spark Gaps," C. T. Allcutt, TRANSACTIONS A. I. E. E., Vol. XXXVII.

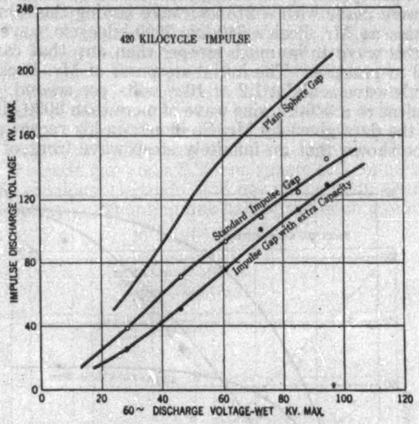

FIG. 4—IMPULSE GAP CHARACTERISTICS

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the added expense of the extra capacity was not believed justified in the commercial design.

Fig. 5 is given for the purpose of comparison with the tests

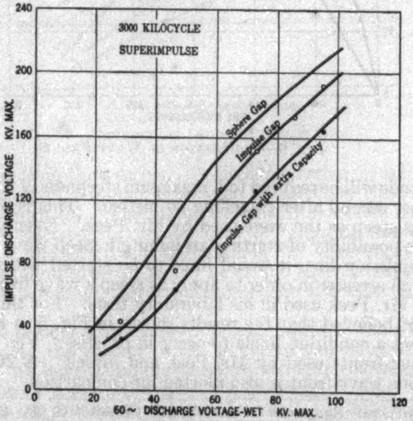

FIG. 5—IMPULSE GAP CHARACTERISTICS

at 2000 kilocycles, upon which Mr. Peek lays such stress. The tests were made with a 213-kv. wave having the same initial steepness as Mr. Peek's 340-kv. 2000-kilocycle wave. I believe this wave to be much steeper than any that can be expected in practise. The initial steepness of Mr. Peek's 2000-kilocycle wave is about 1.2×10^{-13} volts per second. This is equivalent to a 340-kv. sine wave of more than 5000 kilocycles. From the data given by Mr. Steinmetz in his recent paper[4] it may be shown that an infinitely steep wave front of 340-kv. magnitude will be reduced to a maximum steepness of 3.2×10^{-12} volts per second after travelling 30 meters. This is only one-third as steep as the wave used by Mr. Peek. Even if we admit the possibility of starting an infinitely steep wave front in a transmission line, it would have to be started within a few feet of an arrester in order to apply as steep a wave front to the gap as Mr. Peek used in his laboratory tests. For this reason it is not believed that the results shown in Fig. 5 in any way represent a condition liable to occur in practise. Fig. 6 shows the wave fronts used by Mr. Peek and myself. A 2000-kilocycle sine wave front is also plotted for comparison.

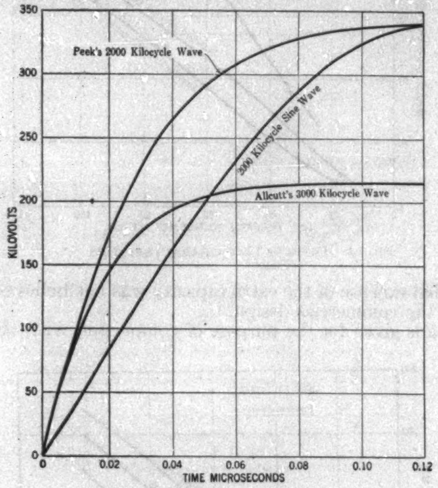

FIG. 6—COMPARISON OF WAVE FRONTS

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[4] "General Equations of the Electric Circuit," C. P. Steinmetz, TRANS. A. I. E. E., Vol. XXXVIII, p. 191.

Figs. 7 and 8 show the discharge characteristics of the selective gap in comparison with the covered or indoor sphere gap and Mr. Peek's balanced double-sphere gap. Fig. 7 shows a covered or indoor impulse gap as compared with a covered or indoor sphere gap. Fig. 8 gives the characteristics of a double balanced sphere gap in comparison with those of one of a number of forms of protected impulse gaps that are being experimented with. Fig. 9 shows diagrammatically the protected impulse gap tested. The gap structure shown in Fig. 1 was used for the main gap and a 6.25 cm. sphere gap connected between the auxiliary electrode and one of the main electrodes. The sphere gap (*g*) shunting one-half of the main gap may be protected from the weather. A discharge across (*g*) is followed immediately by a discharge between the auxiliary electrode (a) and the horn (*b*). The rising arc quickly transfers from the auxiliary electrode (*a*) to the horn (*c*), shorting out the arc across (*g*) and effectively extinguishing it. This construction has been found to secure a very quick and effective suppression of the arc across (*g*), this permitting the use of a relatively small shelter.

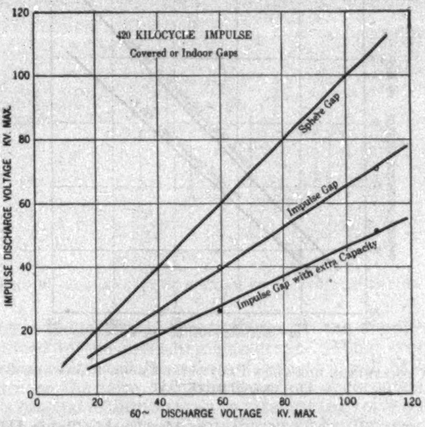

FIG. 7—COVERED OR INDOOR IMPULSE GAP

In the present state of development, however, there is some doubt as to the efficacy of the protected gap structures proposed. We have found that merely wetting the surfaces of the electrodes of a gap will reduce the discharge voltage to practically as low a value as will actual rainfall. It is obvious that a shelter which merely protects a gap from direct rainfall cannot prevent the formation of moisture on the electrodes in foggy weather, and unless the electrodes can be kept dry under all conditions the purpose of the shelter will be defeated.

In his discussion of the balanced double sphere gap Mr. Peek apparently loses sight of the fact that an *unbalanced* double sphere gap actually has a lower impulse discharge voltage than a balanced double sphere gap with the same gap setting. The higher impulse ratio of unbalanced gap is due to the fact that unbalancing the gap causes a greater reduction in the 60-cycle discharge voltage than in the impulse discharge voltage. This may be illustrated by reference to Mr. Peek's Table III. Mr. Peek finds that two gaps in series set at 0.45 cm. and 2.59 cm. respectively, will spark over on 60 cycles at 72 kv., unbalanced, and at 101 kv., balanced. On a 500-kilocycle impulse the same gaps will discharge at 86 kv., unbalanced, and 103 kv., balanced. It is obvious from these figures that the best protection would be obtained by having the gaps unbalanced for impulse voltages and balanced for 60-cycle voltages. This can be accomplished by connecting a high resistance or a reactance in series with one of the condensers used for balancing the voltage across the gaps, *i.e.*, by applying the selective principle of the impulse gap.

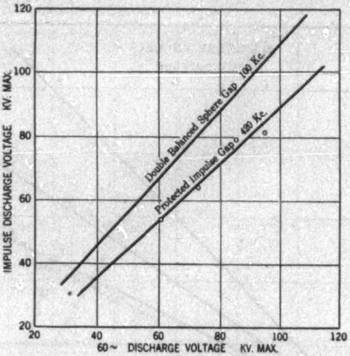

FIG. 8—COMPARISON BETWEEN PROTECTED IMPULSE GAP AND BALANCED DOUBLE SPHERE GAP

Following the line of reasoning given in his discussion at Atlantic City, Mr. Peek again maintains in his present paper that the impulse ratio of the selective gap is an *"apparent" impulse ratio.* We look in vain for a statement of his definition of true impulse ratio as applied to a selective gap. From the figures he gives in Table VII we may infer that Mr. Peek's definition of impulse ratio as applied to such a gap reads something like this: "The impulse ratio of a selective gap is the ratio of the impulse spark-over of the gap to the 60-cycle spark-over of a sphere gap having one-half of the gap setting of the selective gap." What is the reason for this arbitrary definition? Certainly not because the impulse gap may be regarded as consisting of two distinct gaps in series. The balanced double sphere gap also consists of two gaps in series and yet its impulse ratio is computed according to Mr. Peek's original definition of the term (See Table III). Suppose this same balanced double sphere gap were made into a selective gap by the simple expedient mentioned in the preceding paragraph. What would be the definition of its impulse ratio? I believe it will be found simpler to use the term "impulse ratio" in accordance with Mr. Peek's original definition than to attempt to evolve a new definition for every new type of gap that may be devised.

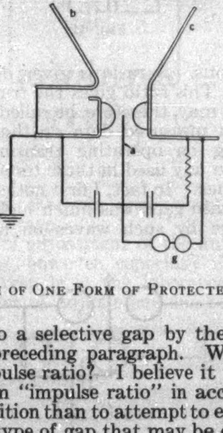

FIG. 9—DIAGRAM OF ONE FORM OF PROTECTED IMPULSE GAP

In discussing the theory of operation of the selective gap, Mr. Peek falls into the error of assuming that the discharge voltage from the auxiliary electrode to one of the main electrodes is one-half the discharge voltage of the whole gap. In the selective gaps which I have described and tested the discharge voltage of the half-gap was approximately *one-quarter* the discharge voltage of the whole gap, since the half-gap is, in effect, a needle gap while the whole gap is a sphere gap. Experimental data showing this fact are given in the discussion previously referred to.[5] If the half gap of the selective gap tested by Mr. Peek did have the characteristics that he ascribes to it, (in Table VII, for example), it must have been quite different in design from the one tested by me.

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[5] C. T. Allcutt, Discussion, TRANSACTIONS, A. I. E. E., Vol. XXXVII, page 868.

F. W. Peek, Jr.: My object in writing this paper was to put into convenient practical form, spark-over data for various transient and lightning voltages for different types of gaps. The gaps discussed range all the way from the needle gap and the horn to the large sphere. It is pointed out that the protective value of a gap depends not only upon the lightning spark over voltage, but also upon the setting imposed by operating conditions. A ratio is given dependent upon both of these factors. This ratio gives the true relative protective value of a gap and may, therefore, be called the protective ratio. I have also given measured data on the steepness of waves actually occurring on operating transmission lines. Much steeper waves than any used in these tests have been observed on transmission lines. In fact, I first noticed that the lightning spark-over of certain gaps was much higher than the normal spark-over voltage by such waves on a large transmission system.

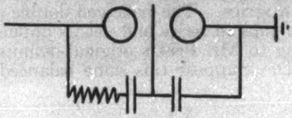

Fig. 10

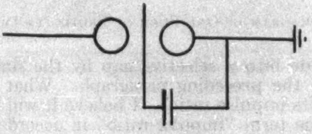

Fig. 11

I called attention to the importance of considering the effect of rain on the spark-over voltage in my discussion at Atlantic City last year. Complete data are now given for the effect of rain, etc. The relative protective values of the various gaps are given in Figs. 14, 15 and 16 in my paper.

Mr. Allcut has asked a number of questions with particular reference to the selective gap, which I will try to answer. I have always tried to give complete data as to methods and constants. Much of this will be found in my 1915 paper, my discussion at Atlantic City, and in the appendix to the present paper. I believe that all of the data necessary to duplicate the work are given.

Various types of condensers were used such as glass plates of very large capacity and high-capacity line insulators. The condensers used in the test had a capacity of the same order as those used by Mr. Allcut.

Water tube resistance was used. Anything above 200,000 ohms gives satisfactory results. The resistance should be as large as possible without giving an appreciable drop due to the 60-cycle capacity current.

The auxiliary electrode was placed between the spheres and set to give maximum 60-cycle spark-over and minimum lightning spark over voltage.

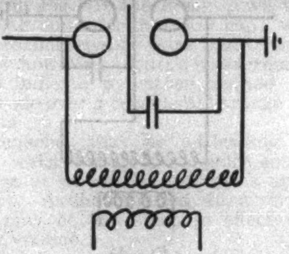

FIG. 12

It is true that careful adjustment of the auxiliary electrode is necessary. It is not, however, especially difficult to make this adjustment and keep it in the laboratory. The gap is also quickly thrown out of adjustment due to leakage by rain on the condensers. I have not found the exact size of the center electrode of the utmost importance. Tests were made on wires varying from 0.035 in. to 0.25 in. The smaller sizes were used on the 6.25-cm. spheres. A small enough electrode was used to give the best results. Some difficulty was had with the very small ones due to vibration caused by the 60-cycle voltages.

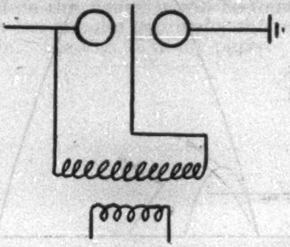

FIG. 13

The sphere part of the gap was always arranged so that distortion did not take place and the spark-over voltage thus corresponded to the standard curves.

Some of the tests were made as a matter of convenience without grounding one sphere. Check tests with one sphere grounded show that the relative protective value is the same for both cases. The method of making tests is always indicated on the data.

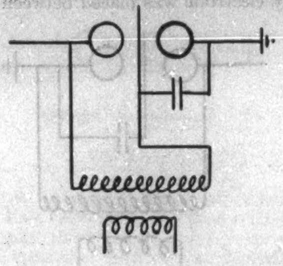

FIG. 14

Regarding Mr. Allcutt's question on the break down voltage of "one-half" the gap, in practise the lightning voltage is between line and ground or, in other words, is applied between the two spheres.

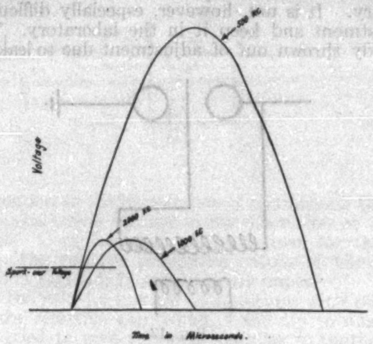

FIG. 15

At operating voltages the auxiliary electrode assumes the proper potential because of the two condensers. When transients are applied the resistance acts as an open circuit so that the potential of the auxiliary electrode is fixed by one condenser as in Fig. 11. In determining the break-down voltage of one-half the gap, therefore the voltage should be applied as in Fig. 12, and not as in Figs. 13 or 14.

The field due to the lightning voltage across the two spheres greatly decreases the stress on the center electrode. The break-down voltage is much higher for Fig. 12. When the test is made as in Fig. 12, the break down is approximately one-half the break down for the total gap; when made as in Fig. 13 it is approximately one-third that of the total gap.

I do not know how Mr. Allcutt's measurements were made, but his figures indicate by either method 13 or 14. The size of the electrode over a considerable range does not materially affect results.

The term frequency may be misleading. It is not the frequency with which we are principally concerned, but the rate or steepness at which the voltage is applied up to the breakdown point. As shown in Fig. 15, a very high voltage—500-kc. wave—may be less steep in effect than a 2000-kc. wave at a lower voltage.

I am interested to note that Mr. Allcutt now checks with me in the fact that the spark-over voltage of the selective gap increases with the steepness of the wave front. For instance, at 80 kv., in his Figs. 3, 4 and 5 the spark-over voltages are 100, 120 and 165 at respectively 100, 420 and 3000 kc. His former tests indicated that the effect of wave front was not material. Our data in general agree qualitatively.

It is of interest to note that the importance of the setting factor may be well illustrated in his Fig. 13. If the needle-gap curve is plotted on this figure, it will be found to give better protection than the impulse gap under this condition, due to the effect of rain. See Fig. 15 in my paper.

[illegible] are applied [illegible] so that the potential of the middle electrode is fixed by the connections, as in Fig. 11. In comparing the break-down voltage of one-half the gap, I believe the voltage should be applied as in Fig. 12, and not as in Fig. 13.

The [illegible] due to the [illegible] capacity across the two spheres greatly increases the stress on the center electrode. The break-down voltage is much lower for Fig. 13. When the test is made as in Fig. 12, the break down is approximately one-half the break down for the total gap; when made as in Fig. 13 it is approximately one-third that of the total gap.

I do not know how Mr. Allibone's measurements were made, but the figures indicate [illegible] method. If so, the size of the electrodes over a considerable range does not materially affect results.

The term "frequency" may be misleading. It is not the frequency with which waves are periodically repeated, but the rate or steepness at which the voltage is applied up to the break-down point. As shown in Fig. [illegible], in a very high voltage 500-kc. wave may be less steep in effect than a 200-kc. wave at a lower voltage.

I am interested to note that Mr. [illegible] admits and checks with me in the fact that the spark-over voltage of the sphere gap increases with the steepness of the wave front. For instance, in air, in his Figs. 3, 4 and 5 the spark-over voltages are 200, 120 and 103 kv. at respectively 700, 430 and 300 kc. The former tests indicated that the effect of wave front was not material. The data [illegible] agree qualitatively.

It is of interest to note that the importance of the safety factor may be well illustrated in the Fig. 15. If the insulator curve is plotted on this figure it will be found to give better protection than the impulse gap under this condition, due to the effect of rain. See Fig. 16 in the paper.

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the American Institute of Electrical Engineers,
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ORDER AND AMPLITUDE OF HARMONICS IN VOLTAGE WAVE FORMS WITH INDICATING INSTRUMENTS

BY LESLIE F. CURTIS

ABSTRACT OF PAPER

The author presents a method for the determination of the order and percentage of the various components of an alternating wave of e. m. f., using indicating meters and other inexpensive apparatus.

Two examples are given. Oscillograms are included to show interesting phenomena and to check the results of the calculations.

The value of so-called standards for the indication of wave form is questioned.

INTRODUCTION

MANY writers have discussed the relative merits of various combinations of resistance, inductance and capacitance when used as devices for indicating the deviation of alternating wave forms from the sine curve. They have also defined certain terms such as form factor[1], deviation factor, distortion factor[2], etc., applicable to the above devices when used with indicating meters as standards[3]. (They have, however, failed to show any method by which the order and amplitude of the various harmonics might be obtained by indicating meters.) Each of the proposed standards is also open to the objection that the result obtained by its use is entirely dependent upon the order of the harmonics in question and upon the size of the reactances used.

The maximum distortion, which, in practise, might result from the resonance of a certain harmonic in a long transmission line with distributed inductance and capacity, would be produced by such devices in test only by chance. It is, however, only this resonant condition for the higher harmonics that can cause disturbances which can be charged to generator wave form. In the absence of oscillograms, it is therefore necessary to produce resonance of the various harmonics generated within the machine before its worst behavior can be predicted.

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[1] See references at end of paper.

In making an oscillographic study of the charging currents for condensers of various sizes from the same source of alternating current it was found that the current wave forms varied greatly with the size of the condenser used. The ratio of the apparent reactance to the true reactance of the same condenser when supplied with a sine wave of voltage at the same frequency, as suggested by a previous writer[3], likewise depended upon the size of the condenser. It was also found that the resistance and inductance of the generator circuit had a decided effect on the indications. A method was then developed which gave a solution for the order and amplitude of the harmonics present in a given form from the readings obtained with inexpensive indicating meters and other apparatus.

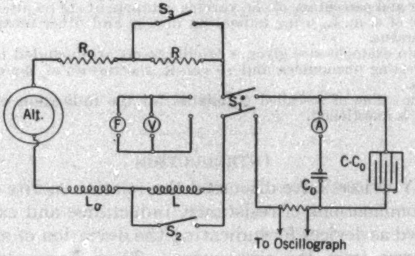

FIG. 1

It is the purpose of this paper to present this solution together with two examples of its use with oscillograms taken to show the various wave forms of current and voltage. The method outlined below gives no indication of the relative phase positions of the components and therefore no accurate indication of the maximum value of the resultant wave. In figuring the maxima of voltage or current waves the relative phase position is important only for the lower harmonics, the third and fifth. Thus for most purposes, the phase position of the components is not required and their order and percentage is a sufficient indication of the character of the wave.

METHOD

The method is that of obtaining resonant conditions for the *n*th harmonic in the generated voltage by several separate adjustments of inductance and capacitance in series with a certain amount of protective resistance. When resonance has been obtained, a second reading is taken with added series resistance.

The diagram of the connections used is shown in Fig. 1. The alternator supplying the wave to be tested is assumed to have an inductance L_0 and a resistance R_0. Resistance R and inductance L may be added to the circuit at will by means of switches S_2 and S_3. V and F indicate a voltmeter and frequency meter connected to the machine by throwing switch S_1 to the left. On throwing S_1 to the right an adjustable condenser $C - C_0$ is connected in the circuit. In parallel with this is a small fixed condenser C_0, the current in which may be measured by a small hot-wire meter A of negligible resistance. C is therefore the total capacitance.

For a given initial voltage and frequency the resonant points for the harmonics are indicated by a maximum deflection of meter A. If R_0 is not sufficient for protection of the condenser from abnormal voltages at resonance, more resistance may be added and considered as a part of the generator. If L_0 is not large enough to produce resonance of the lower harmonics with the amount of capacitance C available, more inductance may be added and considered as a part of the generator.

The calculation consists of two parts; the determination of the order of the harmonics present and the calculation of their amplitude. Table I gives a summary of the notation used in the calculations.

TABLE I—NOTATION

L	Known inductance.
R	Known resistance.
C_0	Known fixed capacitance.
$C - C_0$	Known adjustable capacitance.
C	Total capacitance.
C_1, etc.	Particular value of C.
ω	Angular velocity of fundamental wave.
n	Order of harmonic.
e_n	Effective *n*th harmonic in generated voltage.
P_n	Percentage of *n*th harmonic in terms of fundamental

E	Effective value of resultant generated voltage.
i_n	Effective value of *n*th harmonic in current.
I	Effective value of resultant current.
D	Deflection of hot-wire meter.
K	Calibration constant for A (See Fig. 13).
r_n	Resistance of circuit to *n*th harmonic.
x_n	Reactance of circuit to *n*th harmonic.

ORDER OF THE HARMONICS

The order of the harmonics present may be found from the values of capacitance, C_1 and C_2, necessary to produce resonance with and without the known inductance L in series.

In the first case

$$L_0\, n\, \omega + L\, n\, \omega = \frac{1}{C_1\, n\, \omega} \tag{1}$$

in the second

$$L_0\, n\, \omega = \frac{1}{C_2\, n\, \omega} \tag{2}$$

from which

$$n = 1/\omega \sqrt{1/L\,(1/C_1 - 1/C_2)} \tag{3}$$

If the result is not exactly a whole number it may be corrected as it is known that n must be an integer and probably odd. If another harmonic is present, its order may be determined in the same way from new adjustments of capacitance, C_3 and C_4, with and without the inductance L. An alternative method is to continue the curve upon which C_1 or C_2 produced resonance after one of the harmonics has been determined and take one additional reading of capacitance for the resonance of the other harmonic.

In this case

$$L_0\, \omega = \frac{1}{C_2\, n^2\, \omega} \tag{4}$$

and

$$L_0\, n'\, \omega = \frac{n'}{C_2\, n^2\, \omega} = \frac{1}{C_4\, n^1\, \omega} \tag{5}$$

from which

$$n' = n \sqrt{C_2/C_4} = n \sqrt{C_1/C_3} \tag{6}$$

AMPLITUDE OF THE HARMONICS

Having found the order of the harmonics present it is easy to determine their amplitude by taking additional readings of the current-squared meter with a known resistance inserted in the circuit and with the capacitance adjusted for the resonance of the harmonic in question.

In order to introduce no errors due to current in circuits in parallel with the capacitance and to necessitate no correction therefor, no voltmeter or other device should be connected when the readings of the hot-wire meter are taken.

For resonance of the nth harmonic

$$L_0\,\omega = \frac{1}{C_2\,n^2\,\omega} \tag{4}$$

and

$$i_n = e_n/r_n \tag{7}$$

$$i_1 = \frac{e_1}{\sqrt{r_1^2 + \left(\dfrac{1}{C_2\,n^2\,\omega} - \dfrac{1}{C_2\,\omega}\right)^2}} \tag{8}$$

$$i_3 = \frac{e_3}{\sqrt{r_3^2 + \left(\dfrac{3}{C_2\,n^2\,\omega} - \dfrac{1}{3\,C_2\,\omega}\right)^2}} \tag{9}$$

It will be noted that the resistance of the circuit is assumed to depend upon the frequency of the harmonic. In practise it is shown that the resistance increases with the frequency. In most practical cases the resistances r_1, r_3, etc., are so small compared to the other terms under the radical that they may be neglected.

Therefore

$$i_1 = \frac{e_1}{\dfrac{1}{C_2\,n^2\,\omega} - \dfrac{1}{C_2\,\omega}} = \frac{e_1}{x_1} \tag{10}$$

and

$$i_3 = \frac{e_3}{\dfrac{3}{C_2\,n^2\,\omega} - \dfrac{1}{3\,C_2\,\omega}} = \frac{e_1\,P_3}{x_3} \tag{11}$$

But

$$I = \sqrt{i_1^2 + i_3^2 + \ldots\ldots, i_n^2 + \ldots\ldots} \tag{12}$$

therefore

$$I = e_1 \sqrt{\frac{1}{x_1^2} + \frac{P_3^2}{x_3^2} + \dots \dots \frac{P_n^2}{r_n^2} + \dots \dots} \qquad (13)$$

The resultant voltage E is

$$\begin{aligned} E &= \sqrt{e_1^2 + e_3^2 + \dots \dots e_n^2 + \dots \dots} \\ &= e_1 \sqrt{1 + P_3^2 + \dots \dots P_n^2 + \dots \dots} \qquad (14) \end{aligned}$$

and when no harmonic exceeds 10 per cent of the fundamental, e_1 may be replaced by E with less than 1 per cent error.

Now

$$I^2 = \frac{K\,D\,C_2^2}{C_0^2} \qquad (15)$$

therefore

$$\frac{K\,D\,C_2^2}{E^2\,C_0^2} = \frac{1}{x_1^2} + \frac{P_3^2}{x_3^2} + \dots \dots \frac{P_n^2}{r_n^2} + \dots \dots \qquad (16)$$

When a resistance R is placed in series a new reading D^1 is obtained and the corresponding equation is

$$\frac{K\,D^1\,C_2^2}{E^2\,C_0^2} = \frac{1}{x_1^2} + \frac{P_3^2}{x_3^2} + \dots \dots \frac{P_n^2}{(r_n + R)^2} + \dots \dots \qquad (17)$$

therefore

$$P_n^2 = r_n^2 \left(\frac{K\,D\,C_2^2}{E^2\,C_0^2} - \frac{1}{x_1^2} - \frac{P_3^2}{x_3^2} - \dots \dots \right) \qquad (18)$$

and

$$P_n^2 = (r_n + R)^2 \left(\frac{K\,D^1\,C_2^2}{E^2\,C_0^2} - \frac{1}{x_1^2} - \frac{P_3^2}{x_3^2} - \dots \dots \right) \qquad (19)$$

Now usually all terms except those containing constants of the fundamental and the nth harmonic are negligible, therefore

$$P_n^2 = r_n^2 \left(\frac{K\,D\,C_2^2}{E^2\,C_0^2} - \frac{1}{x_1^2} \right) \qquad (20)$$

and

$$P_n^2 = (r_n + R)^2 \left(\frac{K\,D^1\,C_2^2}{E^2\,C_0^2} - \frac{1}{x_1^2} \right) \qquad (21)$$

from which numerical values of r_n and P_n may be obtained. If the other terms are appreciable, an approximate solution may be made by equations (**20**) and (**21**) for all of the harmonics and then corrected by using equations (**18**) and (**19**).

It is suggested that if a calibrated condenser is not available, that the adjustments be made for the current maxima and that the corresponding capacitance be measured on a sine wave before changing the condenser setting. If no adjustable condenser is available, a fixed condenser and calibrated inductance may be substituted. The equations should then be modified to use inductance as the abscissa of the resonance curves.

Example 1: The open circuit wave form of a delta-connected alternator, 1200 rev. per min., 220 volts, run at normal speed but at 100 volts, is shown by the oscillogram in Fig. 2. The resonance curves for this machine by the above method without and with the known inductance L in series are shown in Fig. 3.

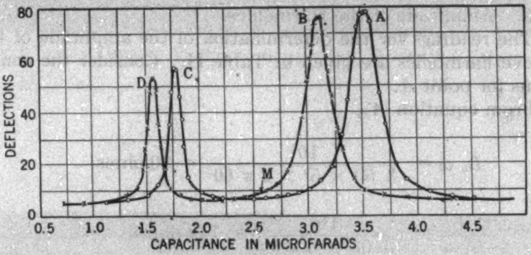

FIG. 3

The inductance L, when measured separately on a 180-cycle wave gave a reactance of 12.6 ohms, and therefore had an inductance of 0.0111 henry. During the test an additional inductance of unknown value was permanently connected in the circuit. C_0 was 0.72 microfarads.

a. Order of the Harmonics:

Maximum points A and B, in Fig. 3, corresponding to 3.53 and 3.11 microfarads, are evidently due to the same harmonic. The equation for the determination of its order is (**3**).

Substitute and $$n = \frac{1}{2\,\pi\,60}\sqrt{\frac{1}{.0111}\left(\frac{10^6}{3.11} - \frac{10^6}{3.53}\right)}$$

from which n equals 4.9 and therefore indicates the fifth harmonic.

Maximum points C and D, at 1.78 and 1.57 microfarads in Fig. 3 are due to the presence of another harmonic. A similar solution to that above gives a value of 6.9 for n and therefore indicates the seventh harmonic.

An alternative solution is, since we know that point A is due to the fifth harmonic,

$$n' = 5\sqrt{\frac{3.53}{1.78}} = 7.0$$

From these solutions it is therefore known that resonance points A and B are due to the fifth, and points C and D, to the seventh harmonic.

b. Amplitude of the Harmonics:

The readings for the determination of the amplitude of the above harmonics are given in Table II. Consider the conditions for point A.

From equation (**4**),

$$L_0\,\omega = \frac{10^6}{3.53 \times 5^2 \times 2\,\pi\,60} = 30 \text{ ohms}$$

$$\frac{1}{C_2\,\omega} = \frac{10^6}{3.53 \times 2\,\pi\,60} = 749 \text{ ohms}$$

$$x_1 = 749 - 30 = 719 \text{ ohms}$$

Substitute in equation (**20**) and

$$P_5^2 = r_5^2\left(\frac{0.01214 \times \overline{3.53}^2}{\overline{100}^2 \times \overline{0.72}^2} - \frac{1}{\overline{719}^2}\right)$$

Likewise for point A^1, when a resistance of 2.66 ohms is placed in series, substitute in equation (**21**) and

Solving

$$P_5^2 = (r_5 + 2.66)^2\left(\frac{0.00713 \times \overline{3.53}^2}{\overline{100}^2 \times \overline{0.72}^2} - \frac{1}{\overline{719}^2}\right)$$

Solve and r_5 equals 7.9 ohms and P_5 equals 4.1 per cent.

Points B and B^1 give similar results, 9.1 ohms and 4.1 per cent.

PLATE XLII.
A. I. E. E.
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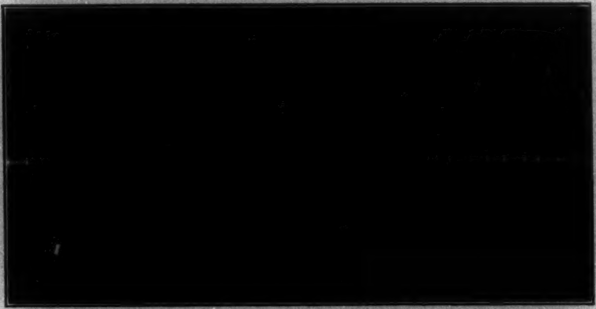
FIG. 2—OPEN CIRCUIT VOLTAGE—EXAMPLE 1 (EFFECTIVE VALUE, 100 VOLTS)



[CURTIS]

FIG. 4—CHARGING CURRENT—POINT A, EXAMPLE 1 (EFFECTIVE VALUE, 0.54 AMPERES)

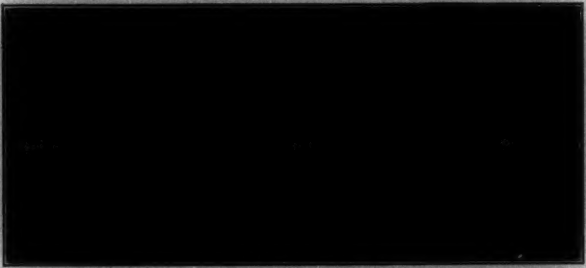


FIG. 5—CHARGING CURRENT—POINT C—EXAMPLE 1 (EFFECTIVE VALUE 0.24 AMPERE)

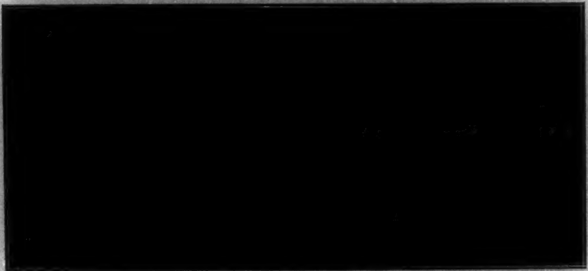


FIG. 6—CHARGING CURRENT—POINT M—EXAMPLE 1 (EFFECTIVE VALUE, 0.15 AMPERE)

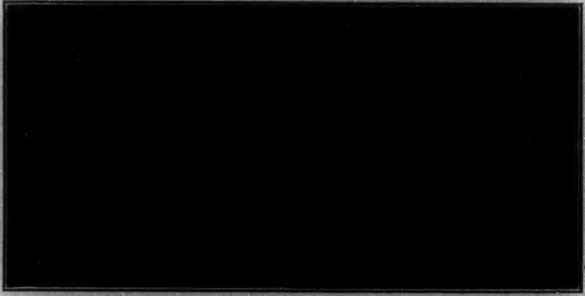


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FIG. 7—OPEN CIRCUIT VOLTAGE—EXAMPLE 2 (EFFECTIVE VALUE, 100 VOLTS)

Solving in the same way for the constants for the seventh harmonic from the values for points C and C^1 and r_7 equals 9.7 ohms and P_7 equals 1.7 per cent.

TABLE II
CRITICAL VALUES FOR EXAMPLE 1

Point	C	C_0	R	D	KD	n	I
A	3.53	0.72	0.00	78.0	0.01214	5	0.540
A^1	3.53	0.72	2.66	44.0	0.00713	5	0.415
B	3.11	0.72	0.00	76.0	0.01183	5	0.470
B^1	3.11	0.72	2.66	46.0	0.00745	5	0.373
C	1.78	0.72	0.00	57.0	0.0091	7	0.236
C^1	1.78	0.72	2.66	36.0	0.0059	7	0.190
D	1.57	0.72	0.00	53.0	0.00848	7	0.201
D^1	1.57	0.72	2.66	35.0	0.00574	7	0.165
M	2.62	0.72	0.00	10.0	0.0018	5 & 7	0.149

From points D and D^1, the corresponding values are 10.8 ohms and 2.1 per cent.

Oscillograms were taken to check the above results and to show the charging current for the various total capacitances. Fig. 2 is an oscillogram of the no-load voltage of the machine and is the true generated wave form of the machine as no capacitance was connected and the resistance was at least ten times as great as the reactance of the circuit to the seventh harmonic. An analysis of this wave gives 3.5 per cent of the fifth, and 1.6 per cent of the seventh harmonic, no others being appreciable.

Fig. 4 is the wave of charging current (0.540 ampere) for point A and shows the predominance of the fifth harmonic. Fig. 5 is a similar curve of charging current (0.236 ampere) for point C showing the seventh harmonic. Fig. 6 is the wave of charging current (0.149 ampere) for point M corresponding to 2.62 microfarads, Fig. 3 showing the reduction in the amplitude of the harmonics in current and the beats due to both fifth and seventh harmonics.

The resistance of the circuit used was measured with direct current and found to be 5.2 ohms for points A and C, and 6.2 ohms for points B and D. The solutions given above show that the resistance to the higher frequencies was very materially increased.

A summary of all of the above calculated data is presented in Table III.

Example 2: The machine used for this test was a synchronous motor, 1800 rev. per min., 110 volts, run as a generator at 100 volts. The open-circuit wave form of this machine is shown in Fig. 7, which upon analysis gives 3.5 per cent of the fifth, 1.4 per cent of the seventh, 7.0 per cent of the eleventh and 2.5 per cent of the thirteenth harmonics.

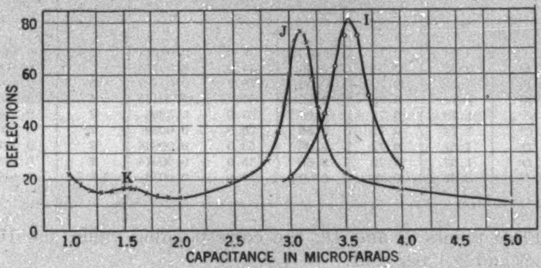

Fig. 8

Unfortunately the machine was fitted with carbon brushes, the resistance of which varied with the current density, so that no check could be made of the above percentages by the method described. It was, however, possible to solve for the order of the harmonics which are indicated in the resonance curves shown in Fig. 8 and Fig. 9. In Fig. 8 a large unknown inductance is connected in the circuit to make possible the resonance of the lower harmonics (points *I*, *J*, *K*) with small capacitances. This was omitted in taking the curves of Fig. 9.

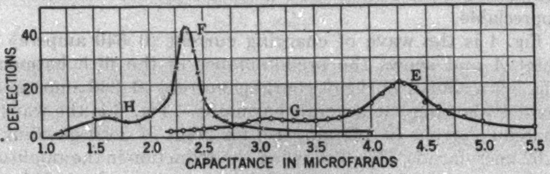

Fig. 9

In Fig. 8 points *I* and *J*, corresponding to 3.52 and 3.10 microfarads, show the resonance of a harmonic without and

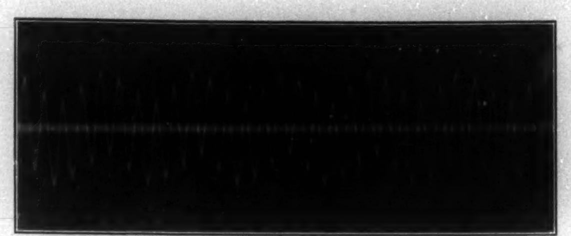

Fig. 10—Charging Current-Point J —Example 2 (Effective Value, 0.34 Ampere)

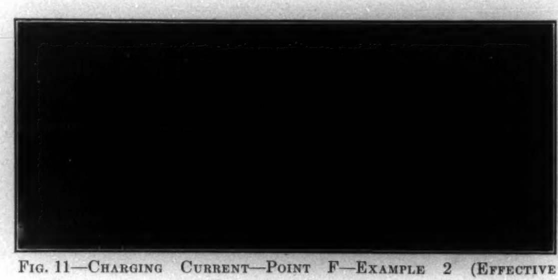

Fig. 11—Charging Current—Point F—Example 2 (Effective Value, 0.97 Amperes)

[CURTIS]

Fig. 12—Charging Current—Point H—Example 2 (Effective Value, 0.31 Amperes)

with the known inductance L in series. A calculation shows this harmonic to be the fifth. Point K, corresponding to 1.55 microfarads is on the same curve with point J (3.10 microfarads) and therefore indicates the seventh harmonic. An oscillogram of the charging current at point I (0.342 ampere) is shown in Fig. 10. In the above C_0 was 1.0 microfarad.

TABLE III
CALCULATED VALUES FOR EXAMPLE 1

Points	n	r_n	R	Mean P_n	P_n by Oscillograph
$A-A^1$	5	7.9	5.2	4.1	3.5
$B-B^1$	5	9.1	6.2		
$C-C^1$	7	9.7	5.2	1.9	1.6
$D-D^1$	7	10.8	6.2		
	1			100.0	100.0

TABLE IV
CRITICAL VALUES FOR EXAMPLE 2

Point	C	C_0	D	kD	n	I
E	4.27	0.20	21.0	0.0035	11	1.260
F	2.34	0.20	42.5	0.0069	11	0.971
G	3.10	0.20	7.5	0.0013	13	0.559
H	1.65	0.20	8.0	0.0014	14	0.309
I	3.52	1.00	81.0	0.01255	5	0.394
J	3.10	1.00	77.0	0.1020	5	0.340
K	1.55	1.00	16.5	0.0028	7	0.082

Similar solutions for points E and F, corresponding to 4.27 and 2.34 microfarads, and points G and H, corresponding to 3.10 and 1.65 microfarads, as shown in Fig. 9, give indications of the presence of the eleventh and thirteenth harmonics. An oscillogram of the charging current at point F (0.971 ampere) is shown in Fig. 11 and that for point H (0.309 ampere), in Fig. 12. In the above C was 0.2 microfarads. Fig. 12 is particularly interesting since it shows the beats due to the eleventh and thirteenth harmonics even when the thirteenth is in resonance, that is, the eleventh is not, in this case, negligible. This might also be surmised from the shape of the resonance curve near point H in Fig. 9.

A tabulation of the readings at the critical points in the above example is given as Table IV.

A calibration curve of the hot-wire meter is given in Fig. 13 so that further calculations may be made if desired.

CONCLUSIONS

It is practicable to test for the order and amplitude of the harmonics in any alternating wave of voltage by the use of inexpensive indicating meters, a known inductance and an adjustable condenser without knowing the resistance and inductance of the machine itself.

A small change in the capacitance or inductance of any of the so-called wave form standards may result in a great change in the current in the circuit, making such devices unreliable.

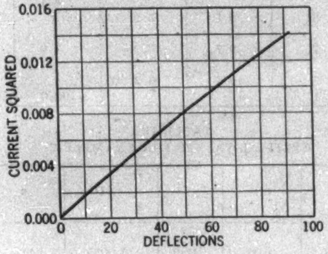

FIG. 13

The resistance of any circuit may change with the frequency of the harmonics present and this fact may introduce an error in making measurements with any of the so-called standards.

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DISCUSSION ON "ORDER AND AMPLITUDE OF HARMONICS IN VOLTAGE WAVE FORMS WITH INDICATING INSTRUMENTS" (CURTIS), LOS ANGELES CAL., SEPTEMBER 18, 1919.

N. S. Diamant: I will call attention to a few points which may be of interest from a theoretical and practical point of view. Considering the latter first, namely, the actual practical value of the method described by Mr. Curtis, I am afraid the method is not quite as useful, convenient and inexpensive as he seems to imply. When you consider the fact that you need *known* inductances, *known* resistances, *known* capacitances, some of which must be adjustable, it will easily be appreciated that the method is not so very simple or convenient; unless it should just happen that a given laboratory is fairly well equipped as to available inductances and capacitances, but does not happen to own an oscillograph. In any case, it would seem that one can safely venture that the practical value of the method is rather limited.

Now coming to the second consideration, namely, the theoretical and academic value of the paper, it is to be greatly regretted that the author does not make any reference whatsoever to the important contributions made on this subject years ago by men like Blondel, Pupin and a great many others. It is discouraging to find that the author gives references at the end of his paper, but really the references he gives have very little to do directly with the subject of the paper and refer more to the *standardization of wave form;* thus when Mr. Curtis states that "*they* have also defined certain terms such as form factor, deviation factor, etc." but "they have failed to show any method by which the *order* and *amplitude* of the *various harmonics* might be obtained by indicating meters," I am afraid he is not quite doing justice to the men who did considerable work along this line before and after the perfection of the oscillograph. The above remarks are not intended in any way to detract from the value of the paper, which I think is considerable from the point of view of an instructor of an electrical laboratory. Before closing may I ask Mr. Curtis what per cent variation he has in mind when he assumes that the resistance of the circuit under consideration depends upon the frequency of the harmonic. In radio work this might be so, but in ordinary electrical work it is not probably very bad practise to assume the resistance as independent of the frequency except in case of very heavy bars. When it is considered that in the method outlined by Mr. Curtis we are expected to use ordinary indicating meters and inductances and capacitances whose value, I suppose, will have to be determined by means of ordinary methods and further when it is considered that if the result which indicates the order of the harmonic does not happen to be an integer, the author just assumes it to be the nearest odd number, I say that when all these approximations are considered as permissible, the effect of frequency on resistance seems to be not a very important or rational refinement to be taken care of.

J. C. Albert: Mr. Curtis' paper presents an apparently simple and practical method for determining the order of harmonics in generator voltage waves, but his failure to obtain the amplitudes of the wave in Example (2) leads to several questions.

The author states that the variable resistance of the carbon brushes introduced unavoidable errors in the calculations of amplitude. It would be interesting to know how much the carbon brushes affect the results, and whether Mr. Curtis has made any experiments to determine how great this error might be. I would also like to ask whether there have been other experiments made on generators not having carbon brushes but having an appreciable eleventh and thirteenth harmonic.

In Example 1 in obtaining point A', a resistance of 2.66 ohms was added in series with the capacity and inductance. I assume this to be the resistance offered to the fifth harmonic, and it would be instructive to know how this is measured.

In deriving equations (**20**) and (**21**) the statement is made that, "Usually all terms except those containing constants of the fundamental and the nth harmonic are negligible," and concludes by stating that, "If the other terms are appreciable, an approximate solution may be made by equations (**20**) and (**21**) for all the harmonics and then corrected by using equations (**18**) and (**19**)."

As shown in the oscillogram Fig. 12, in which the thirteenth harmonic is in resonance, the eleventh has considerable effect. Now in the derivation of formulas (**16**) and (**17**) the resistance component of impedance is assumed negligible in all terms except the nth which is in resonance, and, therefore, has only resistance and no reactance.

It seems from Fig. 12 that in dealing with two successive harmonics of high order, as the eleventh and thirteenth, that even if resonance of the thirteenth is obtained the effect of resistance on the eleventh might be large compared to the effect of reactance on the same, and should enter into the calculation after the first approximation is obtained by equations (**20**) and (**21**) or, in other words, the impedance instead of the reactance only should form the denominator of the $(n-1)$th term of the series in formulas (**18**) and (**19**).

For example, the point marked "E" in Table IV for critical values, the author shows result of the eleventh harmonic obtained with the capacity of 4.47 microfarads. To balance this capacity, assuming the eleventh harmonic on a 60-cycle circuit, an inductance of approximately 0.013 henrys would be required. These values of capacitance and inductance combined show a reactance to the thirteenth harmonic of only a little over eight ohms. While the paper does not state the resistance used in obtaining this point, we find that the resistance in Example (1), point A, was 7.9 ohms, so that the reactance might be little greater than the resistance for this particular case.

Another point which might introduce error in the calculations, when two successive harmonics of high order are present, is that the hot-wire ammeter does not necessarily indicate that the nth harmonic is in resonance, but indicates that the impedance to the complex wave is a minimum near the point of resonance of the nth harmonic, or, as shown by the F. M. Mizushi (authors reference No. 2) that

$\omega = \sqrt{\sigma \div S\,L\,C}$

where σ is the integral distortion factor

and δ is the differential distortion factor

$w - 2\,\pi\,f\,n$ near point of resonance of nth harmonic then equation (4) would become.

$$2\pi f L_0 = \frac{\sigma}{2\,\delta\,\pi\,f\,n^2 C_2}$$

This solution for the unknown reactance of the generator in terms of C_2 involves two unknown quantities, the integral and differential distortion factors.

The author states in his conclusion that, "A small change in the capacitance or inductance of any of the so-called wave form standards may result in a great change in the current in the circuit, making such devices unreliable." Is not the same true of his method? Will not the amplitude for the different harmonics vary with the variation of the amount of inductance and capacitance used to obtain their resonance?

Mr. Price: From what little time I have had in reading over this paper here I don't think the method would be very good for anything that was not recurring. That is, for instance, where you take the oscillogram of a switch action, or short-circuit conditions; or something of like nature, you couldn't get conditions like that because you couldn't adjust your condensers or inductances rapidly enough to get that condition. You would have to have a recurring condition.

I would like to ask the author how difficult it is to obtain resonance with his method. How much time it takes. Whether this method could be adapted to taking curves on high-voltage circuits and the possible effect of the current potential transformer on the results obtained.

D. I. Cone: Perhaps the first proposal along this line was that given by Dr. Pupin in the 1894 TRANSACTIONS employing a somewhat similar method to the one presented in this paper. He employed an inductance and a variable condenser to determine by resonance the frequencies present in the voltage drop across a resistance. He placed this resistance either in series to measure current waves or in shunt, to measure potential waves. He used an electrostatic voltmeter to determine the resonance points by indicating the settings of capacitance to give maxima of voltage across the condenser.

The California Joint Committee on Inductive Interference made extensive use of a so-called "resonant shunt" consisting of a large inductance in series with a variable condenser and a telephone receiver. Each harmonic was resonated in turn by setting the capacitance for maximum current of the particular frequency in the telephone receiver. The amount of current for each harmonic was measured by comparing the sound in the telephone receiver to that from an auxiliary source of the same frequency. Knowing the impedances in the circuit, or arranging so that unknown ones were of negligible effect, fairly accurate determinations of frequency and size of harmonics could be made. Considerable apparatus was required for that work and an analysis of harmonics, ranging from the third harmonic up to perhaps the nineteenth, could be run through in about twenty minutes.

That particular method does not take care of the fundamental component at all, which could be obtained with the aid of an indicating meter.

The method given by Mr. Curtis makes use of relatively simple equipment and procedure for determining the more prominent harmonics. However, in the wave analysis of inductive interference work, harmonics of the order of one-half per cent may be important to determine.

The simplicity of the equipment and the method of solution which he employs for amplitude and frequency are the distinct contributions made by his method. He presents it with special reference to the open-circuit voltage of generators. However, it can be applied to the determination of the harmonics of any system and the limitation which he states on page 1183, of his paper, regarding the presence of a voltmeter or other devices, when the readings are taken, does not appear to be necessary; especially if such devices are normally in the circuit.

Now, referring to his introductory paragraph. The last sentence of his first paragraph reads: "Each of the proposed standards is also open to the objection that the result obtained by its use is entirely dependent upon the order of the harmonics in question and upon the size of the reactances used."

In the telephone interference factor meter which was described in the paper beginning page 261 of this volume, by Mr. Osborne, that objection is met in that the impedances within the meter are fairly large and the distinct effort is to sum up all the frequencies in the circuit as they normally exist; so that the objections here stated do not apply to that particular wave factor meter. It is recognized that such a meter has limitations. It serves but one definite purpose, and that is particularly true of the telephone interference factor meter.

The last sentence on the first page of the paper reads; "It is, however, only this resonant condition for the higher harmonics that can cause disturbances which can be charged to generator wave form."

I assume that in making that statement the question of possible damage to the power circuit itself was in the author's mind. Of course when we consider the question of harmonics in reference to interference to the communication circuits, the detrimental effect is felt whether a particular harmonic is in resonance or not. Of course, the detrimental effect is aggravated for harmonics in resonance.

Mr. Curtis points out that it is desirable to resonate the circuit for a given harmonic in order to develop what he calls its "worst behavior." However, he employs, and quite appropriately, a protective resistance to prevent the current in the meter becoming excessive. Hence it is not clear that the setup he employs actually develops the worst condition, although by studying the effect of the variation of the effective resistance in this circuit upon the behavior of the harmonic, one might get an idea of what its "worst behavior" would be.

H. A. Barre: It seems to me that there is in this paper a method of attack on two or three problems I have in mind. One problem is the finding of the cause of flash-over on the insulators of the Big Creek line. These flash-overs occur probably twelve or fifteen times a year. The current consists simply of the arcing over of the insulators and a heavy flow to ground. The effect on the system is not great and I do not think a single case of injury to apparatus has occurred. We seem to be unable to associate these flash-overs with any set of conditions. The line will be operating smoothly when apparently out of a clear sky will come a flash over and current flow to ground.

The attendants at the stations realize that this is the only trouble to be expected on the Big Creek lines, so when trouble arises they assume a flash-over and apply the cure.

This flash-over cure consists in lowering the voltage to a point, determined by experience. The arc goes out and immediately normal voltage is resumed, taking care not to over run. An automatic means of doing this work has been devised but has not proved as successful as the manual method. At the receiving station the operator also assumes a flash-over. In order to keep from boosting the voltage with the condensers, he opens the field thereby assisting the voltage drop. When voltage at receiving station becomes normal again, he assumes trouble is over, and resumes normal operation. In practically all cases he is right.

There is one interesting fact in connection with these flash-overs and that is their occurrence in a limited section of the line, the two middle quarters of the line,—very seldom close to the ends.

If this method of calculation can be applied to the investigation of this phenomena it will be of great assistance to us.

This method might also throw some light on the double frequency effect found at San Fernando. The circumstances were as follows: Test was being made on 36 miles of line, built for 15,000 volts and energized through a bank of star-star transformers with about 4000—25,000 volts in the legs of the wire. It was energized through a separate delta-delta transformer for the purpose of isolating the system. On closing the switch nothing apparently happened at first. After an appreciable time a disturbance occurred in the system, lasted several seconds, then died out. Once in a long while it recurred. The oscillograph showed that their transient current. or whatever it was, was in the form of a flow through the ground connection which, after passing the common point of the star of the transformers, divided into three currents and fed through the line.

The frequency of this current was measured at about 100 cycles, twice the fundamental frequency. The voltage between the cross and the legs of the wire increased to three or four times normal without, of course the voltage between wires increasing.

A practical application of this principle might show what is going on.

Leslie F. Curtis: In answer to Mr. Diamant let me say that the *known* inductances may be wound, the *known* resistances constructed from nichrome wire, and the *known* capacitances arranged from telephone condensers in any electrical repair shop.

It is true that the author was more concerned with the articles referring to *standardization of wave form* than to the work of Drs. Blondel and Pupin because he felt that the standards for wave form suggested within the last few years were faulty and that if the order and amplitude of the harmonic components of a wave are known, other standardization is unnecessary. The statement should have been made that "others have failed to show a *simple and readily applicable* method by which the order and amplitude of the various harmonics might be obtained with *ordinary indicating meters*."

The variations of the resistance with frequency for the machine tested are noted in Table III of the paper. It is there found that the resistance to the seventh harmonic, that is, seven times the fundamental frequency, or approximately 400 cycles per second, increases from 6.2 ohms to 10.8 ohms. I believe that in an investigation of a circuit of this sort that such a variation of resistance must be taken into account.

The values of the capacitance and inductance used were measured by ordinary indicating instruments from a source of alternating e.m.f. at 60 cycles, which was so arranged as to eliminate most of the harmonics,—in other words, a sine wave of e. m. f. was employed for a simple voltmeter, ammeter and wattmeter measurement.

Mr. Albert asks concerning the calculations for the amplitude in example (2). Even with the variable resistance of the carbon brushes in the circuit, it would have been possible to make calculations for the amplitude if several more readings had been taken—readings with small successive additions of resistance to the circuit. By the use of cross curves and further calculations the amplitude might have been obtained. I omitted such calculations because of their complexity.

Mr. Albert also refers to the effect of the 13th and 11th harmonics in producing an error in the measurement for amplitude. I believe that the paper shows that the order of the harmonic is obtained without an appreciable error, even when the harmonics are of nearly the same order, that is, as he stated, the nth and the $(M \pm 1)$th term. The effect of the resistance in the circuit may then be introduced in the proper equation; though I am not certain that it is justified. The added known resistance is a high resistance wire of such a cross section as to be independent of frequency. Therefore, the 2.66 ohms which Mr. Albert mentions holds for all frequencies.

Mr. Albert also mentions the equations presented by Mr. Mizushi showing that the impedance of the circuit is a minimum at some other frequency than $\frac{1}{n\omega\sqrt{LC}}$. The circuit used, however, is different. The calculation for the maximum current in the meter used in this paper shows that such maximum occurs at a point very close to the assumed resonance point. The current in the complete circuit is not necessarily maximum at the same instant.

Mr. Albert speaks of the variation of amplitude of the harmonics in the circuit with the amount of inductance and capacitance introduced. It is assumed by the author that the generated or initial value of the harmonics in e. m. f. remain constant. The magnitude of the harmonics in the current, of course, depend upon the inductance and capacitance.

Mr. Price asks concerning the time required to make a test of this sort. With the ordinary apparatus at hand in most testing departments, the variable or adjustable condenser is the only piece of apparatus which would offer any difficulty. One may be made from an extra good grade of telephone condensers and when provided with the proper switches for adjustment, a complete test might be made in possibly an hour. The measurements of inductance and resistance can be made by ordinary indicating instruments, they do not offer any particular difficulty.

Mr. Cone notes that the presence of the voltmeter should offer no difficulty. I should explain that the voltmeter indicated in the diagram of connections was used simply to indicate the constancy of voltage on open circuit. The voltmeter might be connected at all times and would offer no difficulty if the readings of the meter were taken only when the resonant circuits were open.

At the time my manuscript was prepared the telephone interference factor had not been suggested. Therefore, I did not intend to include a circuit giving the telephone interference factor among the objectionable standards.

Mr. Cone also notes that I assumed that damage might result, or rather that interference might result only from resonant condition of the harmonics. I intended to indicate that such interference would be a maximum during periods of resonance.

I believe I am unable to answer Mr. Barre's question concerning the trouble—the double frequency effect—at San Fernando. I hope some use may be made of the circuit in solving his problem.

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THEORY OF PROBABILITIES APPLIED TO FAILURES OF SUSPENSION INSULATORS

BY L. M. KLAUBER

Abstract of Paper

Nothwithstanding the recent improvements in porcelain insulators, failures are sufficiently common so that allowance must be made for them. A certain factor of safety is required, in the shape of extra insulation, to provide for the electrical unreliability of the insulators themselves aside from conditions of abnormal operating stresses.

There are a wide variety of operating conditions which affect the amount of over-insulation required, and after having found the minimum number of insulators per string required for any given operating conditions the author points out a method of determining the amount of extra insulation desirable from an insurance standpoint according to the law of probabilities. Equations are developed from which the probability of failure for any given case or the ratio between such probabilities for any pair of cases may be determined directly. A numerical example is also given which shows the development of the theory of minimum annual cost for combined mechanical and electrical failures.

AT THE present time there exists a diversity of views as to the primary cause of failure of suspension insulators of the disk type. Extensive investigations are being carried forward to determine the causes of failure in order that such causes may if possible be eliminated in manufacture. Unfortunately, however, while recent types have shown a notable improvement over those produced but a few years ago, failure is still sufficiently common to require allowance therefor in all specifications covering new lines. That is, it is not advisable in any construction to limit the number of insulators in a string to a quantity just sufficient to withstand the line pressure under service conditions, and thus cause failure if one member of the string fails. On the contrary, under present conditions, it is good practise to allow a certain proportion of extra insulators, a factor of safety required by the electrical unreliability of the insulators themselves, rather than by the necessities of abnormal operating stresses.

Obviously, the degree of over-insulation which should be indulged in depends on a variety of factors which cannot be directly expressed in mathematical terms. It is necessary to consider the first cost of the extra insulation and the fixed charges on such cost; also the counter-balancing expense involved in shut-downs, both prearranged and unforeseen. The requirements of continuity of service, and the availability of duplicate lines and stand-by power sources, have an important bearing on design. Also there is the factor of climatic conditions, the occurrence of lightning, temperature ranges, both daily and seasonal, humidity, the frequency of cleansing rains, the presence of salt fogs, wind-carried dust or the fumes of industrial plants. The nature of the line supports must be considered; we must know if the towers are of steel or wood, and if of wood, whether the support ends of the strings are electrically connected together. The transformer connections, whether with grounded or isolated neutral, must be noted. Mechanical stress is of importance. These, and many other conditions all affect greatly the degree of over-insulation necessary or advisable, and show why in some cases a 66-kv. line requires only three units per string, while under other conditions seven may be insufficient.

But having determined the minimum number of units per string required for a given voltage and set of operating conditions, it becomes interesting to investigate the value of additional units, purely from an insurance standpoint; and this investigation may be best performed by calculations of the chances of failure under the laws of probability. The physical condition under which the line operates will of course affect the rate of failure, but these conditions may be grouped and allowed for in the depreciation rate assumed.

In order to permit employment of the simpler laws of probability, it is necessary to assume that the successive failure of units in any string does not increase the liability to individual failure of the remaining members of that string until a minimum is reached which permits complete breakdown. At first glance this appears a most incorrect assumption, since, after the failure of one unit it is evident that there is an increased electrical strain upon the balance. Nevertheless, it is the belief of many investigators that the particular type of electrical failure which is today one of the great difficulties of transmission line operation, is in small degree hastened by increased electrical duty. In fact this opinion seems borne out by the relative frequency of occurrence of multiple failures which is usually found to be in the ratio expected under the laws of probability governing independent events. Furthermore, it has been the experience of some companies that insulator failures in lines out of service or in stocks held in the storeroom, have been in the same proportion as those experienced by units of the same lot in active use. So that while in some cases successive failures of members of the same string may not be considered independent events, in general we will not go far wrong in so considering them.

In consideration of the general case of insulator strings, consisting of any number of identical units, we may utilize the general expression of the probability of independent events happening or not happening:

$$P = p^n + n \,.\, p^{n-1} q + \frac{n\,(n-1)}{1\,.\,2} p^{n-2} q^2 \,.\,.\,.\,.\,.\,.\,.\,.\, + p^{n-n} q^n \qquad (1)$$

in which p is the probability of an event happening out of a single trial, $q = 1 - p$ is the probability that it will not happen and n is the number of trials under consideration. In this expression the first term is the probability that the event will happen n times consecutively or every trial, the second that it will happen $n - 1$ times out of n, the third $n - 2$ times out of n, etc. The probability of an event happening exactly r times out of n will be represented by the $(n - r + 1)$th term in the above equation, or

$$P_r = \frac{n\,!}{r\,!\,(n-r)\,!} p^r q^{n-r} \qquad (2)$$

and the probability of an event happening *at least* r times out of n trials is the summation of the terms up to and including the $(n - r + 1)$ th term.

$$\text{Thus:} \quad P_R = p^n + n \,.\, p^{n-1} q + \frac{n\,(n-1)}{1\,.\,2} p^{n-2} q^2 \,.\,.\,.\,.\,. + \frac{n\,!}{r\,!\,(n-r)\,!} p^r q^{n-r} \qquad (3)$$

Translating these equations to terms of suspension insulator failures, we may let n represent the number of units per string.

Likewise assume that if r units fail, a line failure will result, but that if less than r units fail the remaining insulation will be sufficient to "hold" the line and failure will not result. Then if $p =$ the probability of individual insulator failure (electrical) based on experimental data or experience with insulator depreciation, equation (**3**) gives directly the probability of line failure per string of insulators installed and the total probability of line failure will be found by multiplying the chances per string (P_R) by the number of strings (N) in the line or

$$P_N = N P_R \tag{4}$$

From equations (**3**) and (**4**) we may determine directly the probability of failure for any given case, or the ratio L between such probabilities for any pair of cases.

It will be noted that where p is small compared with q, as with depreciations of the order under consideration, only the $(n - r + 1)$ th term is of importance, the preceding terms being of such small value comparatively that to consider them is to assume an accuracy not warranted by the original data. We have, therefore, as a simplified equation of the chances of electrical line failure:

$$P_N = N \frac{n!}{r!(n-r)!} p^r q^{n-r} \tag{5}$$

Numerical examples are more easily solved, using this equation in the equivalent form:

$$P_N = N \frac{n!}{r!(n-r)!} \frac{(q/p)^{n-r}}{(1/p)^n} \tag{6}$$

as this will usually involve powers of integers rather than fractions.

The application of formula (**6**) to specific cases will often give interesting results. Fig. 1 indicates the range of example values of P_R plotted for three sets of values of $n - r$ and p Because of the minute values of P_R at the higher values of n the curves are plotted in terms of $\log 1/P_R$. Naturally where the safety factor $\frac{n}{n-r}$ is large $1/P_R$ attains values which are of little practical interest.

Fig. 2 is a diagram for the solution of the equation

$$P_R = \frac{n!}{r!(n-r)!} p^r q^{n-r}$$

From this diagram values of the probability of failure per string can be quickly determined, with reasonable exactitude. The solution is based on a straight line diagram deduced from this equation in the form:

$$\log 1/P_R = n\,[2 - \log (100 - s)\,]$$
$$+ r \log \frac{100 - s}{s} - \log \left[\frac{n\,!}{r\,!\,(n - r)\,!} \right]$$

where $s = 100\,p$ or the depreciation in per cent.

Of equal interest with the probability of failure in any specific case is the determination of the ratio L between the probabilities of failure in related cases and in particular the decrease in probability of failure (or insurance) derived from the addition of extra insulators. In determining the ratio between probabilities it will be sufficiently accurate to employ the approximate formula (**5**) and noting that in any assumed case having to do with a specific line, the quantity $n - r$, that is, the number of insulators at which failure will occur is constant, it may be readily shown that

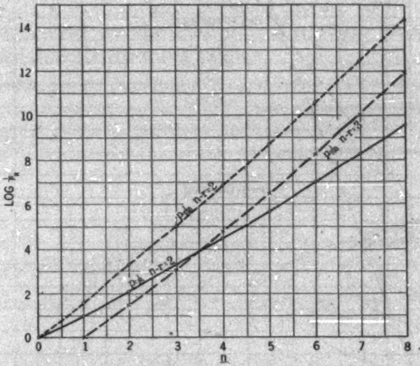

Fig. 1—Probability of Electrical Failure per Insulator String—Example Values of P_R

$$L = \frac{n_1\,(n_1 - 1)\,(n_1 - 2)\,.\,.\,.\,(n_2 + 1)}{r_1\,(r_1 - 1)\,(r_1 - 2)\,.\,.\,.\,(r_2 + 1)}\,p^{n_1 - n_2} \qquad (7)$$

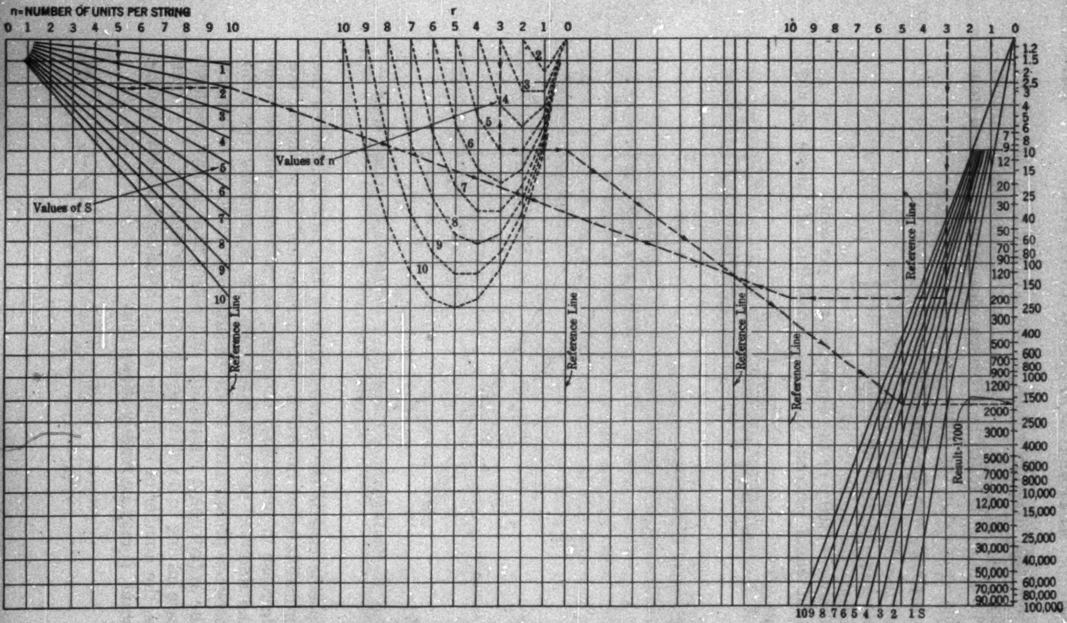

FIG. 2—DIAGRAM FOR GRAPHICAL SOLUTION OF $P_R = \frac{n!}{r!\,(n-r)!}\, p^r q^{n-r}$ = PROBABILITY OF FAILURE PER STRING PER YEAR

n = number of units per string; r = number of units per string that must fail to result in line failure; S = annual depreciation in per cent = $100p$.

Example worked out $n = 5$, $r = 3$, $S = 4\%$. Result $1/P = 1660$

in which the subscripts 1 and 2 indicate the two conditions assumed as to number of units per string.

It is evident that in most practical cases $n_1 - n_2 = 1$ or 2 since usually it will be desired to determine the reduction in probability of failure secured by increasing a string by one or two insulators. In Table I will be found values of

$$C_L = \frac{\dfrac{n_1\,!}{r_1\,!\,(n_1 - r_1)\,!}}{\dfrac{n_2\,!}{r_2\,!\,(n_2 - r_2)\,!}}$$

for $n_1 - n_2 = 1$ and varying values of $n - r$, and since when $n_1 - n_2 = 1$, $L = C_L\,p$, any problem having to do with the insurance derived by the addition of a single unit may be determined directly from the table.

TABLE I—VALUES OF COEFFICIENT C_L for $n_1 - n_2 = 1$ (1 insulator added to string)

$n - r$ Number at which break-down occurs	n_1 = Number of insulators in string before addition of one unit									
	1	2	3	4	5	6	7	8	9	10
1	2	1.5	1.33	1.25	1.2	1.17	1.14	1.12	1.11	1.1
2	..	3	2	1.67	1.5	1.4	1.33	1.29	1.25	1.22
3	..	..	4	2.5	2	1.75	1.6	1.5	1.43	1.37
4	..	..	..	5	3	2.33	2	1.8	1.67	1.57
5	..	..	..	..	6	3.5	2.67	2.25	2	1.83
6	..	..	..	..	..	7	4	3	2.5	2.2
7	..	..	..	..	..	..	8	4.5	3.33	2.75
8	..	..	..	..	..	..	..	9	5	3.67
9	..	..	..	..	..	..	..	..	10	5.5
10	..	..	..	..	..	..	..	..	..	11

Figure 3 shows some example curves plotted from the use of Table I.

In addition to these conclusions as to the insurance derived by adding units to strings of insulators in transmission lines, it may be noted with reference to dead-ends in higher voltage distribution lines, that where a single unit is adequate to normal service, but is subject to failure at the rates to be expected amongst suspension units, that is, of an order of magnitude of from one-quarter of one per cent to 4 per cent per annum, the single unit is no longer to be considered as economically sufficient; for the line reliability will then be only the relatively low reliability of the individual units multiplied by the number of units. With two units the probability of failure is greatly reduced and in a general way it may be said that with lines of sufficient voltage so that line failures almost invariably follow the failure of the principal insulating member at any point of support (that is in lines of 10 kv. and up where subsidiary insulators in the form of wood pins, cross-arms or poles will not withstand normal line voltage) it is never economical to use single suspension units as dead-ends. Even insulators of weaker design (electrically) are in such cases to be preferred in pairs to a single unit, provided that one unit of the weaker pair will withstand normal operating stresses in case of failure of the other. For instance, if the rates of depreciation are 4 per cent for the small units and 1 per cent for the large, the ratio of line failures in the two cases will be 6¼ to 1 against the single large unit.

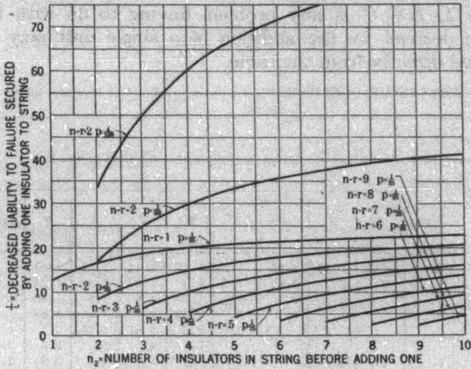

FIG. 3—INSURANCE AGAINST ELECTRICAL FAILURE BY ADDING ONE INSULATOR TO STRING—EXAMPLE VALUES OF *L*

These considerations likewise show why an insulator string composed of a number of virtually independent units is generally to be preferred to any form of support consisting of a single unit or group of interdependent units.

It may be said that it is difficult to determine $n - r$ or the number of insulators which is one less than will "hold" the line upon which the use of this formula rests. The ordinary 10-in. suspension insulator is usually rated as having a dry flash-over value of 90 kv. and a wet value of 70 kv.; in pairs the corresponding values are respectively 160 kv. and 110 kv. Yet we know from experience that two of these insulators would not stand up on a 66-kv. line under the most favorable conditions for more than a short period. Line surges, weather conditions, etc., all reduce $n - r$, and it is important in choosing a value for $n - r$ in these formulas that the quantity be understood to represent not the number of insulators at which break-down will occur instantly, but rather the number at which break down will occur under normal operating conditions within a short period subsequent to the failure of the $(n - r + 1)$th unit of a string. This break-down may always occur subsequent to surges which are to be periodically expected in any line and if these surges are sufficiently frequent to greatly outnumber the expected string failures, as they will be in almost every practical case, simultaneous failures will not be sufficiently frequent to render the formulas incorrect. The finding of numbers of shattered strings after a total failure is not generally an argument in favor of the occurrence of simultaneous failures, but rather these shattered strings represent previous unexplained momentary failures. Furthermore, the tendency of surges to produce simultaneous failures and thus reduce line outages, is to a certain extent balanced by the original assumption of successive unit failures being entirely independent events, which is not strictly true. In general, it may be said that owing to the difference between test and laboratory conditions, and further because of surges, which tend to give any line a true voltage as regards effects on insulators in excess of its rated voltage, we will not go far wrong in basing $n - r$ on half the wet flash-over rating given by manufacturers to their product. It may be noted from Table I that a variation of one in $n - r$ does not greatly affect the value of L in most practical cases.

As we increase the number of units and thus decrease the chances of electrical failure, we increase the chances of mechanical failure. Mechanical failures are of two general classes, those due to lack of strength or inherent defects in the insulator or its supports, and those due to external causes, such as the malicious use of firearms. Obviously, any mechanical failure means a line failure, particularly at dead-end points where mechanical failures of the first type are most likely to occur.

By mechanical failure is meant the complete breakage of a unit as a means of support. Mechanical failures of the several types which are the initiating causes of electrical break-down (as, for instance, those which follow cyclic temperature changes) but which do not in themselves cause the line to fall, must be considered the equivalent of electrical failures and must be allowed for in the choice of the value of p_e (the probability of electrical failure per unit), since they affect line failures under the same laws. But for true mechanical failures the chance of failure per string is the chance per insulator times the number in the string, or

$$P_m = n\, p_m{}^* \qquad \textbf{(8)}$$

or for the whole line

$$P_{Nm} = Nn\, p_m \qquad \textbf{(9)}$$

Since electrical and mechanical failures as above defined are independent and exclusive events, the probability of failure of both kinds is equal to the sum of the probabilities of failure of either kind, or

$$P_{Nt} = N\, n\, p_m + N \frac{n\,!}{r\,!\,(n-r)\,!}\, p_e{}^r\, q_e{}^{n-r} \qquad \textbf{(10)}$$

This equation may be most easily investigated for the value of n which will give a minimum of annual failures for any set of values of p_m, p_e and r by plotting, particularly as short cuts have been given for determining values of P_{Ne}. But in general we are not as much interested in the value of n which in any case will result in a minimum number of failures. For many values of p_e and p_m this will give impractical or impossible results. What we are interested in is the reduction of cost of service, including all elements of fixed charges, maintenance and depreciation.

Formula (**10**) may be applied to a practical problem in the following manner:

Assume a line containing 1000 insulator strings. Two good insulators in a string will result in a line failure; three will hold the line. It is required to determine whether four or five insulators per string will result in the most economical installation.

Let it be assumed that insulator units in the line as constructed will cost $3.00 each. The fifth unit is also estimated to cost $3.00, no additional cross-arm length or tower height being required for clearance owing to the extra 5½ in. of length of string. Fixed charges are taken at 12 per cent, this figure including functional depreciation of insulators, but being exclusive of physical depreciation, which is taken care of under maintenance. The cost of replacing an insulator unit which has failed electrically, but which has not resulted in a line failure, is estimated at $5.00, it being assumed that the line is gone over with suitable testing apparatus annually, all defective units being replaced at that time. The cost of field testing insulators is taken at 10c. each. The cost of an unexpected line failure, including location of the failure, replacement of the broken units, loss of business and stand-by plant operation, is taken at $150.00. Naturally, this amount is subject to wide variation, being made up of a number of items, some of which, like the value of continuity of service from a public policy standpoint, cannot be expressed directly in terms of money. In fact it is conceivable that in some cases the cost will be less, but in many cases greatly in excess of the above figure, particularly where service is dependent on a single line and the cost of stand-by energy (if available at all) is considerably in excess of the cost of energy secured over the transmission line. Assume an electrical failure of four per cent per annum, and a mechanical failure (as above defined) of one-fortieth of 1 per cent as an average for units both in suspension and in strain at dead-ends.

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*The subscripts m and e are used hereafter to distinguish between mechanical and electrical failures.

We now have the following tabulation of annual costs:

	Quantity			Amount	
	Four units	Five Units	Unit cost	Four units	Five units
Fixed charges	4000	5000	12% of 3.00	$1,440.00	$1,800.00
Total electrical failures (4%)	160	200	5.00	800.00	1,000.00
Electrical line failures (P_{Ne})	8.85	0.59	150.00	1,327.50	88.50
Mechanical line failures (P_{Nm})	1	1.25	150.00	150.00	187.50
Insulator testing	4000	5000	0.10	400.00	500.00
	Total annual cost			$4,117.50	$3,576.00

It will be noted that all of the above items of cost, with the exception of those due to electrical line failures, vary directly with the number of units in use, and may therefore be grouped. In the above schedule these items total 23.25 per cent of the first cost of the units installed, and the same result would have been secured by taking annual charges at that figure instead of 12 per cent. Summing up the terms of this nature, we have.

$$y = F_e A N n + B N \frac{n!}{r!(n-r)!} p_e^r q_e^{n-r} \qquad (11)$$

in which y is the total annual cost, F_e is the per cent covering fixed charges and items of maintenance varying directly with the number of units installed and determined as illustrated in the table above, A is the first cost per unit installed and B the cost per unexpected failure. Differentiating with respect to n and equating to 0 for a minimum we have:

$$F_e A + B \frac{n!}{r!(n-r)!} p_e^r q_e^{n-r} \left[(n-r)! \left(\frac{1}{n} + \frac{1}{n-1} \cdots\cdots\cdots + \frac{1}{n-r+1} \right) + \log_e q_e \right] = 0 \qquad (12)$$

This equation is too involved to be used conveniently in solving for n and it is more practical to determine the value of n which gives a minimum total cost in any case by using Fig. 2 in connection with equation (**11**) and solving by plotting or tabulation. For instance, in the example above discussed we have:

n	$F_e A N n$	$B N P_R$
3	$2,092.50	$16,595.00
4	2,790.00	1,327.50
5	3,487.50	88.50
6	4,185.00	5.30

A casual inspection shows that $n = 5$ gives a minimum annual cost in this case.

In the utilization of the formulas proposed, it is essential to differentiate between electrical depreciation failures and line condition failures. The first affects the selection of p_e, the second of $n - r$. The first is usually although not invariably characterized by failure by puncture, the latter by failure by flash-over. In some districts, notably along the California coast, climatic and atmospheric conditions are such that any line, regardless of the degree of over-insulation used will ultimately fail by flash-over, if not given continued attention. The gradual accumulation of salt and dust, particularly on the under side of the units, beyond the reach of the infrequent winter rains, will eventually result in flash-over. In other words, as time goes on, if the insulators are not artificially cleaned, r becomes zero and line failures result in great numbers without the occurrence of a single failure of the other type.

It is therefore necessary in considering a problem of this type to note the value which $n - r$ will reach before natural causes, such as a cleansing rain, or some artificial means is used to reduce this quantity to its original value.

On the other hand it should be emphasized that in this discussion the probability p_e of electrical failure is intended to cover those types of failures variously attributed by insulator specialists to porous porcelain, to expansion difficulties induced by cyclic temperature changes or to combinations of the two. In any lot of insulators under a specified set of conditions a certain proportion will become defective, due to these or similar causes, and the balance will remain intact. There is at any given time a reasonably sharp line between the good and the bad; that is to say, between those that will fail when moisture and line conditions are favorable to failure and those that will not. In general, insulators deteriorated in this manner under no circumstances will return to their original quality, although failure may not be instantaneous, and although testing by the several common methods (high-frequency, high-potential and megger) may indicate a variety of degrees of deterioration. But under conditions of incipient failure of the surface leakage type all the individual units will be found uniformly deteriorated (within fairly narrow limits), and in this type of deterioration of condition there is no sharp line between a "good" and a "bad" unit. Furthermore these units may at any time prior to damage by a power arc be returned to their original quality by cleaning. It is evident at once that while failures of the puncture type may be and probably are independent events, failures of the surface leakage type can under no circumstances be so considered. Under flash-over conditions individual units do not act independently.

Although nearly every insulator failure is finally a flash-over there is little practical difficulty in differentiating between the two types of failure. The flash-over due to deteriorated line conditions shows a heavy foreign deposit on the insulator surface. In almost every case the bottom member of the string will be shattered; often the top member also, but rarely an intermediate member. Unbroken units will before cleaning give a zero reading with a megger and a very low flash-over with a high potential test. Subsequent to cleaning, however, unless badly burned by the power arc, they will give as high a megger and flash-over test as when new. Lines subject to difficulties of this kind cannot be tested with a megger in the field even on a clear, dry day, unless each unit is wiped off before test.

In the other type of failure we have the fundamental condition that r units have failed by material depreciation (usually puncture) and the balance $(n - r)$ being unable to hold the line have subsequently flashed-over. The characteristic differentiating this class of failure from the other is the number of punctured units which occur at random throughout the string. Even if subsequently shattered by the power arc they may usually be readily distinguished from units shattered by the other type of failure.

In testing and renewing insulators on the line it is desirable that results be recorded in detail. The distribution of failures between strings, their location in different sections of the same line and the relative frequency of failure of units in corresponding positions in the string, all will be of interest in deducing the causes of failure. It is desirable to know whether bottom, top or intermediate units fail most frequently, and in what ratio; whether units in greater tension at dead-ends fail more frequently than those in suspension; whether units with bolts exposed to the direct rays of the sun fail more frequently than those on the same tower but with bolts toward the north and consequently protected from the sun by the porcelain shells. It is of interest to note whether failures follow hot spells, whether they are decreased by cleansing rains and other effects of the weather. If lines operating under similar conditions contain units differing in manufacture, it is of course important to determine separately the percentage of failures for each type. Also by noting the relative frequency of single and multiple failures, it is possible to determine whether failures follow the laws of probability for purely independent events, as above outlined, or whether successive unit failures are to a certain extent dependent events.

DISCUSSION ON "THEORY OF PROBABILITIES APPLIED TO FAILURES OF SUSPENSION INSULATORS" (KLAUBER), LOS ANGELES, CAL., SEPTEMBER 18, 1919.

W. D. A. Peaslee: It would be a very serious obstruction to the progress of insulator development if the viewpoint expressed by the author were to be adopted in general by the operating engineers charged with the care of high-voltage transmission lines.

In the first place, the author seems to take the mental attitude that the development of the high-tension insulator has reached its ultimate and that more admittedly poor insulator units are to be the solution as against fewer units of a better type with wider margins of safety in the individual unit.

His main contention seems to be expressed in the sentence, "It is not advisable in any construction to limit the number of insulators in a string to a quantity just sufficient to withstand line pressure under service conditions, and thus cause failure if one member of the string fails."

This statement pre-supposes the acceptance of the statement that when a string is *just sufficient* from an engineering standpoint, the failure of one unit of the string will cause the failure of the string. With a properly designed insulator, this is not the case.

In almost all cases, except where excessive salt, dust andf ogs are encountered, the determining factor in selecting the number of units of a string is the voltage stress on the end units. Too few engineers appreciate the actual numerical values of voltage strains on the end units and there is too little published data on the subject. In this connection, the writer will say that a paper is in preparation giving these distributions on strings up to ten units and under various conditions that it is hoped will be of interest to the engineering fraternity.

The end unit of a string must be operated at a voltage stress *below that at which corona appears.* I think all the men who have been studying this subject will agree on this point. If N is the number of units in the string to meet this condition, a string of N-1 units, caused by the failure of one unit will, it is true, probably operate with some slight corona on the end unit of the N-1 string; but with a properly designed unit, this will not be the case, and even under slight corona the unit will operate for an indefinite time if properly designed and made.

It would seem better to work towards the development of such a unit than to accept the present thin porcelain insulator as the ultimate and pay for extra unit insurance that is only a time function anyway.

It will be of interest generally, I think, to state here some progress that has been made, and show briefly what can be realized with an insulator of proper design and quality.

The function of an insulator is three-fold:

1. It must support the line mechanically.

2. It must insulate the line.
3. It must resist all possible temperature changes that may be met in practise, both as to range of temperature and time in which the range is encountered.

To support the line a simple mechanical strength is necessary, and for ordinary cases a unit with an ultimate mechanical strength of 10,000 pounds is considered ample. Every unit should be tested to 5000 pounds before the electrical tests are applied.

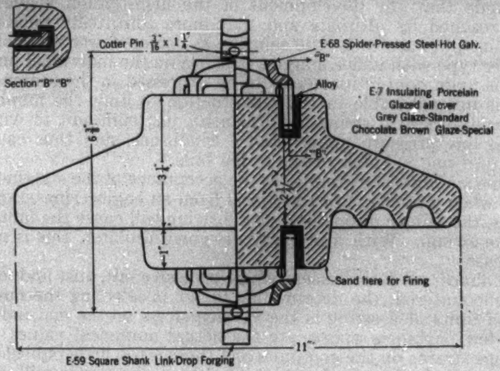
Section "B" "B"
Cotter Pin $\frac{3}{16}$" x $1\frac{1}{4}$"
E-68 Spider-Pressed Steel-Hot Galv.
"B"
Alloy
E-7 Insulating Porcelain
Glazed all over
Grey Glaze-Standard
Chocolate Brown Glaze-Special
"B"
6¾"
3¼"
2¼"
1"
Sand here for Firing
11"
E-59 Square Shank Link-Drop Forging

FIG. 1

In the matter of insulating the line, the problem becomes extremely complex. Irregular distribution of the electric stress on units of a string, lightning, transients, dust, etc., etc., are factors that must be taken into consideration. One of the most serious causes of failure of thin porcelain insulators is, the puncturing power of transients of extremely abrupt wave fronts. The writer has presented to the Institute two papers on this subject from a laboratory and operating experience. It may be said that the general answer to this series of factors is an ample thickness of homogeneous porcelain properly fired. A very few years ago the manufacture of such porcelain was admittedly impossible, but that is not the case today, as porcelain of almost any desired thickness can be made absolutely homogeneous and of the highest quality, free from porosity.

The temperature cycle resistance is solved by eliminating the cement cap and pin type of construction with its several materials of different coefficients of cubical expansion in rigid contact and its hydroscopic material for further trouble.

A further electrical requirement is a rational electrical design, avoiding zones of high dielectric flux concentration and producing a unit in which the dry flashover and first appearance of corona are as close together as possible. In a laboratory type, these may be made the same, but commercial considerations of manufacture make this ideal condition impossible of realization in quantity production.

Consider, then, an insulator of the design shown in Fig. 1 and of the following electrical and mechanical characteristics:

Mechanical Strength	10,000 pounds
(Each unit tested to 5,000 — before electrical test)	
Dry flash over 60 cycles	100,000 volts
Wet flash over 60 cycles	50,000 volts
Dry flash over 200,000 cycles	120,000 volts
First leakage corona	45,000 volts
Easily visible corona	60,000 volts
Puncture	250,000 to 300,000 volts

Thickness of porcelain between elecrodes 2¼ inches.

If, now, we use five of these units on 110,000 volts, (a line has been in successful operation for some time under these conditions) the stress on the end unit will be approximately 33,000 volts. This unit then operates with the following safety factors:

Safety Factor

Dry flash over 3.

Wet flash over 1.5.

(If the distribution were the same wet as dry), but as the distribution is better wet than dry, the safety factor as indicated by measurements taken under standard rain test conditions indicate that the safety factor due to reduction of stress on this unit reaches 2.1.

Puncture 7.5 to 9. Due to the very thick porcelain which reduces the flux concentration on the electrodes, the effect of transients is less and the thickness available for resisting them much greater than in the thin porcelain type.

Margin to Corona voltage	12,000 volts	Safety Factor
Dry flash over of string	340,000 volts	2.8
Wet flash over of string	220,000 volts	1.8

The alloy used in fastening the spider legs into the porcelain has the same coefficient of cubical expansion as the porcelain used in the insulator and this, with the spider support, makes an insulator that can be plunged from boiling to freezing water as often as desired without fracture or chipping.

Suppose now that a lightning stroke or rifle bullet cracks a unit. We find ourselves with four units supporting the line and the following conditions, remembering that total destruction of eight spider legs is necessary before the line will drop if a unit fails.

Voltage on end unit, approx	41,000 volts
	Safety Factor
Dry flash over	2.44
Wet flash over (as discussed before)	1.8
Puncture 6 to 7.5	
Margin to corona	4,000 volts
	Safety Factor
Dry flash over of string 280,000 volts	2.55
Wet flash over of string 185,000 volts	1.68

Under these conditions no corona would be visible though there would be a leakage corona. This string would operate indefinitely but is not the correct number of units for 110,000 volts.

Considerable has been said regarding the danger of using porcelain in tension and its supposedly excessive fatigue when in tension. Research laboratory tests now partially finished, indicate that there is no foundation in fact for these fears, as indeed there is none in logic. The results of these and other tests will be published at an early date.

It would seem then that the basic contention of Mr. Klauber's paper is wrong and that there is relief at present available for the condition he describes. We should look to improvement in insulators rather than to more units of inferior characteristics as a solution for the insulator situation.

I am very glad to see a recognition by one of the operating engineers of the fact that "any line, regardless of the degree of over-insulation used, will ultimately fail by flash over, if not given continued attention" along the California coast regions. This is a fact that I think some engineers have lost sight of in their enthusiasm for adding more units to a string to take care of this creepage condition.

C. O. Poole: Would say that as I view the situation, Mr. Klauber has endeavored, and I think has worked out a principle upon which we can calculate a certain factor of safety upon the ordinary insulators as they are used, and, by varying, it very probably can, with some uncertainty, be applied to different climatic conditions.

This subject of climatic conditions, I think, has a great deal to do naturally with the operation of insulators, as well as with any attempt that may be made to formulate a definite equation that might cover all conditions. As a matter of fact, I think it is impossible to make a definite equation that will cover all conditions with any degree of accuracy. However, I think Mr. Klauber has accomplished one thing and that is the fact that he would eliminate excessive investment for a given insulation on a line. We must, however, acknowledge that with the uncertainties of manufacture and the uncertainty of types of insulators, it throws an element of uncertainty into the whole proposition. While I feel that we will get some good out of the paper, yet I think it is open to betterment in this respect.

J. A. Lighthipe: The insulator game is progressing. Of course, every new insulator manufacturer that comes to the coast has finally developed the only perfect insulator. However, it usually takes some experience to get at the truth of the matter. The insulator game is one we have been watching very closely for the reason that all our service throughout Southern California and middle of California, is depending on the insulators doing their proper work.

Even with the tools that people have contrived to test these insulators, just as Mr. Barre said in the discussion of the preceding paper, out of a clear sky comes a flashover. Sometimes it is serious and interrupts service. I rather agree with Mr. Klauber in his idea. Three insulators will stand up one or two years; perhaps four will stand up six or seven years. That extra three or four years means a great deal to the operating company. What we have been working on as much as anything else is to try and get a careful record of the breakdowns of the insulators and, if possible, find out why they break down in that locality.

We all have our own theories. Perhaps this terrible catastrophe we are experiencing with insulators happened over the oil well sections. We all know that in our oil well sections, especially in the East, there is a constant flow of helium which, is an emanation from radium and, passing our insulators, may ionize them.

I thought my theory just as good as Barre's, but at the same time we are awaiting an insulator which will hold up and we are glad to say they are holding up a little better today, and without doubt a great deal better than those we had a few years ago. The unfortunate part of the test is that it takes two or three years to find out absolutely, whether they are good or bad. Any insulator is good for the first season; most of them last two seasons without any breakdown. But, with all your careful testing, it is expensive to root out the bad insulators. Every improvement that can be made in these insulators which will hold up the line another year means that much reduction in the operating expenses.

J. H. Anderton: From Mr. Barre's remarks regarding insulator flashovers on the Big Creek line, it would appear possible to tie Mr. Klauber's and Mr. Curtis's papers together.

If I am not mistaken, this question of flash-overs on the Big Creek lines was first noticed in 1915. It is perhaps difficult, if not impossible, to definitely explain it. I have, however, a theory which may apply to it as follows:

The length of the Big Creek lines is approximately 241 miles. At 50 cycles a quarter wave length of the fifth harmonic would be 187 miles or 54 miles from one end of the line, and I understand from Mr. Barre that these flash-overs occur at approximately this distance from the end of the line. The transformer connections at both ends of the line suppress the third harmonic and some others; they do not, however, suppress the fifth, which is often found in generators. The current in the fifth harmonic wave may be very small, in fact, such that the line offers no appreciable resistance to it and my theory is that a condition of local resonance is set up in the line due to the fifth harmonic which is, of course, superimposed upon the fundamental voltage and causes the flash-over. There are, of course, innumerable frequencies which could set up resonance conditions in the line such as lightning and similar disturbances, some of which may travel comparatively short distances before causing flash-overs. The recurrence, however, of flash-overs at or near a certain point in the line would usually indicate something inherent in the system.

This question of flash-overs naturally brings up the question of insulators. The selection of suitable insulator string involves the selection of a suitable factor of safety. The line may be so insulated that transient waves of voltage may be carried along the line to the power house and thus destroy apparatus. In general, it would appear preferable to select a factor of safety such as will give a reasonable degree of security against the break down of units in a string but will not prevent the flash over and consequently the dissipation of extreme high-voltage waves which might destroy transformers and generating equipment. In other words, it is preferable to have an occasional string of insulators destroyed than to have apparatus in the generating station destroyed.

H. A. Barre: There is a very pronounced family of harmonics in many systems. There is a very pronounced 11th and 13th and there was a strong third originating with the bank of compensators when the 50 and 60 cycles were tied together. That has been corrected to some extent by interconnection between this bank of compensators and the bank of transformers located in the same station, the compensators being star-star connected and the transformers star-delta. The 3rd harmonic current that circulates in neutralizing, at Eagle Rock, was measured at 1.8 amperes; that 1.8 amperes distributing itself through three large transformers in the ratio of 1000 star and 150,000 volts delta. This made the trouble negligible.

J. B. Fisken: On the last page of Mr. Klauber's paper, he makes this statement: "It is desirable to know whether bottom, top or intermediate units fail most frequently, and in what ratio." We have heard nothing on it and I am sure that I have nothing to offer.

I would like to know, from those who can tell us, something about the potential gradient through the string. That seems to me to have a direct application in the safety effect which is to be obtained by adding units.

There is something I do know about, however, and that is Mr. Klauber's next point: "Whether units in greater tension at dead-ends fail more frequently than those in suspension."

The Washington Water Power Co. does not have very many suspension units on its system. It has, however, a short line about 28½ miles, a steel tower line with suspension insulators, operated at 60 kv. Now that line of itself is short and does not introduce many difficulties, but that is not true of the rest of the system, consisting of about 600 miles of 60 kv. line, and it is liable to get any surge that may originate on other parts of the system. The line has been in operation for nine years and the company has yet to find, the first case of failure of the suspension units other than those caused by lightning and the small boy with a rifle.

On the other hand, some two years ago it had a great deal of trouble with the same units used as strains and it was necessary to replace, or rather it was decided to replace all of the strain insulators at that time. These were taken down without the particular position on the line or the particular position in the string of any particular unit being noted and a rough test was run on them, first with the megger and then by direct application of 90 kv. Thirty-three per cent failed under the test. The balance were put back into stock and so far as I know they have been used up and are on the lines today. I think that, perhaps, answers in part this question as to the relative difficulties on string and suspension insulators.

R. W. Shoemaker: We have made quite a number of tests on this very subject. You can take a string of ten insulators 5½-inch spacing, without arcing horns on high voltage, and, commencing with the insulator next the wire, the potential across number one unit will be about 20 per cent, or in other words, twice too much, the insulator third or fourth from the tower will have the least stress. With arcing horns the second insulator from the wire will have the highest stress and not the one next to the wire.

We have a device that is going to be used on our 150,000-volt transmission line now building by which the potential gradient in a string of insulators can be controlled and consists of a metallic disk that may be interposed between any two insulators and in metallic connection with the insulator hardware.

It is possible with this device to make the stress across each insulator in a string exactly equal if such a condition were desirable or necessary.

For example, a disk 12 inches in diameter interposed between the wire and the first insulator reduces the stress on that insulator from 20 per cent to 16 per cent, if the disk is 26 inches in diameter the stress will be 14 per cent. Of course this latter disk is too large for practical purposes, but by using two disks located between the wire and first insulator and between the first and second insulators (counting from the wire)the stress on these insulators may be kept to below 15 per cent.

We have taken a string of insulators and put potential on them until the hardware, without disks, about the wire began to fire, then putting the disks in place the string under the same potential as previously applied would be perfectly quiet even in a dark room.

Mr. Klauber's paper gives methods for figuring out the probability of certain conditions that may obtain using as basic data the percentage of failure. It seems to me that any results based on this method would be of little value as the "percentage of failure" is an extremely uncertain quantity and varies over a wide range with different types of insulators, conditions of service, etc., so that using this method no prediction of continuity of service could be made for a line without, first, several years' experience with that particular line on which to obtain necessary data.

E. F. Scattergood: In determining generating plant capacity we first figure out the demand or load requirements throughout the year and then give consideration to the question of reserved units, or per cent of reserve plant capacity giving consideration to the importance of the continuity of service as against the cost.

The subject matter of Mr. Klauber's paper relating to the number of units in a line insulator is, it seems to me, a direct parallel; first, an insulator unit, or string of units, has a certain working voltage capacity or rating as we call it, which takes into account the question of not over stressing the insulator under ordinary operating conditions. Then the question arises, shall we add a unit or two as reserve units? Giving reserve strength, rather than capacity in this instance, which may result in the prevention of service interruption by testing the units on the strings along the line every six months or so and replacing bad units before the string fails.

If an insulator deteriorates to the point of worthlessness in the interum, we have a reserve unit there ready to take up the work and avoid interruption of service the same as starting up the extra steam or hydroelectric unit, or extra per cent of plant capacity.

The objection is made that such added insulation throws a higher duty on the station arresters causes a greater menace to station equipment, which is true but the benefits may far outweigh the objections, which of course, would depend upon the circumstances, the length of the line and the importance of the distributing point at the end of the line.

On a short line the cost of an extra unit or two is quite nominal. The city of Los Angeles, in constructing its power transmission lines from its present large generating plant, 46 miles, by transmission from the city, and from the second large plant now building in approximately the same location, allowed extra insulation for the above purpose. We believe in it, especially in a short line. It adds to the duty of the lightning arrester, no doubt, but it does reduce the interruption of service on the line to a minimum, and that is important.

In considering the paper we must distinguish between insulators, or insulation, necessary to take care of the ordinary conditions of operation, allowing the proper factor of safety, and the proposition of adding one or two units in excess of that. We apply the principle generally in engineering and I see no reason why it should not be applied in line insulation provided we can protect the station equipment from the higher voltages, which are possible on the line through the use of lightning arresters.

L. C. Williams: Insulators, after having been in service for a period of years, will depreciate much more rapidly than those which have been in service for a period of one year. I would like to ask Mr. Klauber if he has taken that point into consideration?

From the manufacturer's standpoint it is quite possible to manufacture suspension insulators which will have much greater electrical and mechanical efficiency than those we have at present but it is also well known that those insulators will be prohibitive in cost from an operating standpoint.

Mr. Peaslee does not mention the unit costs of his insulators. Mr. Klauber takes the unit cost of $3.00 per unit installed on his line. If Mr. Peaslee's insulator being considerably heavier costs considerably more installed on the line, it should be taken into consideration that an economic comparison, even presupposing that four of his units will do the work electrically as against five of the ordinary cap and pin type, could not be accurately made without having the unit cost to base figures on, I cannot make the comparison myself but it is a point worth considering.

Now, as to Mr. Fisken's question of differential. There have been experiments made which give the curve practically the same as shown by Mr. Shoemaker's remarks. In a string of twelve insulators the unit nearest the line wire will take approximately twenty-six per cent of the duty without arcing horns. By adding arcing horns to the lower end of the string alone, that unit will take approximately ten per cent of the duty over a string of twelve and the next unit to the line will take approximately fourteen to sixteen per cent.

I have no data whatsoever on the failures with relation to the position on the string.

Mr. Peaslee mentions a line in successful operation at 100,000 volts with five units in a string of the type he advocates. One of the principal points of Mr. Klauber's paper is climatic conditions which have a decided bearing on his laws of probability. Unquestionably, under certain conditions, that type will operate satisfactorily on 100,000 volts, but on the Pacific Coast experience has shown that puncture value is not necessarily the governing factor, but leakage surface is much more so than the puncture value.

I think it possible the addition of extra units should be taken into consideration as insurance against the leakage over the strings rather than the puncture value of the units in the string.

L. M. Klauber: With reference to Mr. Peaslee's remarks, he would seem to think that I am a pessimist with regard to insulators. Of course, I know from experience and from discussing this matter with the representatives of nearly all manufacturers, that the manufacturers have indeed made great progress in the last few years in their product. If a person is a pessimist I think he is usually a pessimist between two and four o'clock in the morning when the insulators fail and the lines go out. We all know that they do go out. No one hopes more than I do that the manufacturers will make perfect insulators. Probably they are making perfect insulators today, but the last ones we got day before yesterday were not perfect; and we rather fear that when tomorrow comes and the insulators of today have failed we will again be told to overlook the past and think only of the perfection of the present.

With reference to Mr. Anderton's remarks on the subject of over voltage: this paper, being based as it is upon the determination of minimum annual cost, not only specifies what the minimum number of insulators in a string should be but also the maximum. It would seem that dependence on having insulators fail in order to protect station apparatus is a reflection on the manufacturers of lightning arresters. The great difficulty in depending on the insulators is not that they will not fail; they do that readily enough. But it is difficult and expensive to locate a failed string after breakdown and it would seem best to protect against high-voltage strains by other means especially designed for the purpose.

As to the ratio of failures in different members of the string and the effects of unequal stress distribution: our own results and extensive data given us by other companies would tend to show that there is virtually no difference in failures; in spite of the fact that the insulators close to the line are the more highly stressed, they fail no more rapidly than the others. Failure of the type under discussion is not primarily induced by electrical stress and may be almost as frequent amongst units in the storeroom as those on the line.

In reference to a remark made by Mr. Shoemaker as to the cost of an interruption, the particular problem worked out in the paper as an example must be taken as such. I did not mean to indicate that $150.00 would be an average cost of interruption. In using these formulas the values assigned the constants must be based on the particular conditions under which a line is to operate.

Presented at the Pacific Coast Convention of the American Institute of Electrical Engineers, Los Angeles, Cal., September 18-20, 1919.

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PREDETERMINATION OF SYNCHRONOUS PHASE-MODIFIER PERFORMANCE

BY HUBERT V. CARPENTER

ABSTRACT OF PAPER

The author reviews the method for showing the behavior of transmission lines first given by Perrine and Baum and then shows how it can be used in determining the effect of the use of a synchronous motor operating without load for improving the power factor. The effect of the losses in the motor are shown both in their effect on the line alone and on the line with the step-up and step-down transformers included.

The diagram given shows both the improvement in voltage regulation and the change in power factor due to the phase-modifier for any assumed condition of loading for the transmission line.

The errors of the method are discussed with methods for determining their magnitude, and the advantages of the graphical treatment pointed out.

RAPIDLY growing demands upon our transmission systems for more power and better service have made it imperative in many cases that increased capacity be provided without delay. Increasing the capacity means in most cases improved regulation, but frequently demands also a better efficiency of operation in order that the maximum possible fraction of the output of the generators may be delivered. This is usually thought to demand higher voltages or heavier lines or duplication of lines, all of which require time as well as money.

It is not perhaps fully appreciated that a synchronous motor operating upon the system may, if properly proportioned, improve the efficiency as well as the regulation, thus accomplishing both of the things desired.

While the theory of the synchronous phase-modifier has been fully treated before the Institute it is hoped in the following to make its general effect upon a transmission system a little more easily grasped.

In the consideration of transmission problems the writer feels that far too little use has been made of the remarkable diagram first proposed by Perrine and Baum, (TRANS. A. I. E. E. May 1900.)

The peculiarities of this treatment are that the voltage at the receiver is assumed constant and that at the generator solved for, and that the current is treated throughout as made up of three independent parts, charging current, I_c; load current, $I \cos \phi$; and reactive or wattless component delivered, $I \sin \phi$. The electromotive force required to force each of these components of current through the line is then figured, first for the resistance and then for the reactance of the line, thus giving the following components of e. m. f. required; $\frac{R\,I_c}{2}$; $\frac{X I_c}{2}$; $R\,I \cos \phi$; $X\,I \cos \phi$; $R\,I \sin \phi$; and $X\,I \sin \phi$. The charging current values are divided by 2 because I_c distributes itself over the line in a way assumed here to be uniform.

To make up the regulation diagram, Fig. 1, the e. m. f. at the receiver, E, is assumed as the vector of reference and the above values added to get the total voltage needed at the generator. In Figs. 1 and 2 $a\,b$ is $\frac{R\,I_c}{2}$, $b\,c$ is $\frac{X\,I_c}{2}$, $c\,g$ is $R\,I \cos \phi$, $g\,d$ is $X\,I \cos \phi$, $d\,e$ is $R\,I \sin \phi$ and $e\,f$ is $X\,I \sin \phi$, the last two being shown for 0.8 power factor, lagging. $\frac{R\,I_c}{2}$ and $\frac{X\,I_c}{2}$ are plotted first since they are closely constant and assumed to be exactly so here.

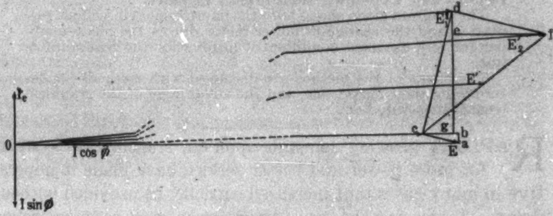

Fig. 1

The total volts required to force the working component of current through the line is $c\,d$, and is equal to $Z\,I \cos \phi$. The value of $c\,d$ will therefore vary directly with the kilowatts delivered since E is assumed to be held constant. A scale of kilowatts may be therefore used along $c\,d$ as in Figs. 2 and 3.

An ampere scale may also be added if desired. In the same way a scale of reactive kv-a. may be laid off along *d f* as shown.

The advantage of the method lies in the fact that the entire performance of a line for all conditions of load is shown by one solution, and further that the effect of any change of load is shown directly for any assumed case. The thing really most desired in studying a design for a line is to learn the effect which will be produced by the greatest sudden change in load that may be expected. Questions of this sort are answered directly by this diagram.

Fig. 2 shows that the diagram as there made up is really a chart in which distances along the axis, *c d*, represent power in

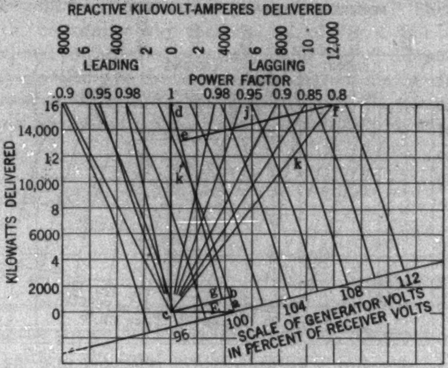

Fig. 2

kilowatts, while distances along the axis, *d f*, represent reactive kv-a. Thus, the point, *d*, Fig. 2, represents 16,000 kw. at unity power factor, and, from the scale below requires 100.8 per cent of the receiver voltage at the sending end. Also point, *j*, shows that an addition of 6000 reactive kv-a., requires the generator voltage to be increased to 106 per cent of *E*. Further, a load of 12,000 kw. at 0.8 power factor, lagging, can be handled without drop in the line if a pure condensance requiring 8250 kv-a. be connected to the system. See points *k* and *k'*.

The time thus spent in rehearsing the workings of the Perrine-Baum diagram is considered well spent for we are now ready to apply it directly to the problem of this paper.

In order to get reasonable accuracy in calculating the effect of an over-excited synchronous motor running on the line without load it is necessary to include the power losses in the machine itself.

To do this in a way best adapted to use with the regulation diagram just shown, the performance of the machine can be plotted in the form shown in Fig. 4. This represents the same current values as determined in the test for the "*V*" curves for the machine except that they are plotted according to their respective power factors and a scale of kw. one way and kv-a. the other used. In other words this plotting shows the active and the reactive components of the kv-a. taken by the machine for any total value, either over or under excited. The slight difference between leading and lagging losses shows the effect of the varying loss in the field windings for different excitations.

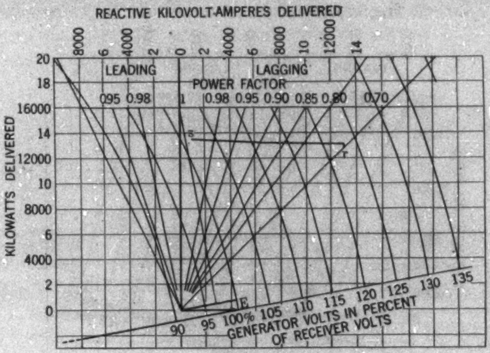

Fig. 3

It will be noted that the data for the machine are now plotted in the same form as is used in the Perrine-Baum chart, that is, with kw. for one axis and reactive kv-a. for the other, so that it can be drawn in directly to proper scale on the chart. This is done in Fig. 3. This figure shows, for example (see points *s* and *r*) that the addition of the phase-modifier working at full capacity of 12,000 kv-a. changes the load from 12,780 kw. at 0.7 power factor lagging, to 13,500 kw. at 0.99+ power factor lagging, and the line drop from 23 per cent to 2 per cent. It is evident that a condenser of one half this size would bring the line drop down to about 12 per cent and the power factor up to 0.88.

A study of the diagram shown and of the properties of the different sizes of line conductors will bring out the fact that a transmission line made up of very large conductors will respond more directly to the corrective action of a synchronous phase-modifier than one using small conductors. That is, if we substitute a conductor of one-half the cross-section of that used in the line figured above the resistance will be doubled but the reactance will be only about 6 per cent greater. This means that with the smaller wire the line, *c d*, of Fig. 1 will be much more inclined toward the horizontal and *d f* toward the vertical. This further tilting of the axes of the diagram makes the effect of a synchronous motor much less, so far as its usefulness in correcting regulation is concerned. It will still be effective in reducing the power lost in the line simply by its effect in reducing the value of the total current flowing in the line. This also applies to cases where the power line is made up of underground cable where the reactance is usually less than the resistance.

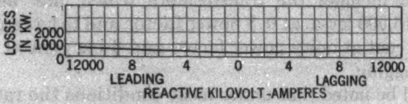

Fig. 4

Owners of small overhead lines may expect the same disappointing results in varying degrees depending on the material of the line, the size, and the spacing.

In all high-voltage systems where the use of phase-modifiers is most advantageous it is necessary to associate step-up and step-down transformers with the line whenever operated, and usually the transformer sets used with any line are the same from day to day. In all such cases there is no need of figuring the line behavior as a separate matter from that of the transformers which are really a part of the line. The resistance of a transformer, if taken over the ordinary range of working temperatures is only slightly variable, and its reactance, depending as it does on the relative positions of the windings and the dimensions of the non-ferrous portions of the leakage paths, is also quite constant and independent of changing load conditions. If then the same transformers are usually used with the line under consideration there is no reason why their resistance and reactance should not be included with the resistance and reactance of the line. Values for the low-tension coils must be changed to high-tension equivalents before adding them in.

In Fig. 3 is given the behavior of the same line as shown in Figs. 1 and 2 with the transformers included. The difference shows the futility of making an exact solution for the performance of a line alone which can never be used without its transformers. It also shows the faulty economy in trying to get a very good regulation in the line when the transformers will prevent it anyway.

Figs. 2 and 3 give the performance of a three-phase line 140 miles long with 300,000-cir. mil. stranded conductors spaced 12 ft., operating at 60 cycles and 110,000 volts between wires.

Fig. 3 includes the transformers which are assumed to handle 16,000 kv-a. at 0.8 power factor and to have a regulation of 1 per cent at unity power factor and 6 per cent at 0.8 power factor lagging.

It will be noted that under most conditions the ratio of *X* to *R* will be higher for the transformers than for the line. This may mean that a phase modifier will give good service in improving regulation where a study of the line alone would not make this apparent.

It seems that the reason more use has not been made of this method of studying transmission problems is based on the fact that it is considered inaccurate. The inaccuracy lies in the fact that the static charge which makes up the charging current is assumed to be uniformly distributed along the line, disregarding the variation in voltage, and also in the fact that the charging current is assumed for convenience to be just 90 degrees ahead of the receiver voltage. Inaccuracy is much more feared when the extent of the error is unknown. This difficulty is removed by the exact analytical methods available when needed, any of which may be applied to any one particular condition out of the variety which the chart represents and thus show the error for that point, in fact this is the way to learn to appreciate the graphical method which answers all questions with one solution. As a matter of fact it can be easily shown that the terms calculated in making up the Perrine-Baum diagram are, neglecting line leakage, nothing more or less than the first three terms of the following series which represents the behavior of a line to any required degree of exactness.

$$E' = E + I\,Z + \frac{Y\,Z\,E}{2} + \frac{Y\,Z^2\,I}{6} + \frac{Y^2\,Z^2\,E}{24} + \text{etc.}$$

For any of the lines now in use the error of the method outlined will be found to be well within all useful limits.

DISCUSSION ON "PREDETERMINATION OF SYNCHRONOUS PHASE-MODIFIER PERFORMANCE" (CARPENTER), LOS ANGELES, SEPTEMBER 20, 1919.

H. B. Dwight: A diagram drawn up to show the particular characteristics of an individual transmission line, as described by Mr. Carpenter is very useful in the design and operation of an electrical transmission system. An additional valuable feature is that the average power factor observed for various fractional loads may be plotted directly on the diagram, thus showing the generator voltage, or the phase-modifier kv-a., required at any load under usual working conditions.

The diagram described in the paper is based on the so-called "split-condenser method" of calculation, in which one-half of the capacitance of the entire line is assumed to be concentrated at the receiver end. It is stated in the paper that the diagram is not used to a greater extent because of the inaccuracy of this method. However, it is not necessary to use the "split condenser," or any approximate method. A diagram which is the same in its essentials as that described in the paper, but based on the complete hyperbolic transmission line theory, has been published by the writer.* This diagram is called a "circle diagram," and the circle corresponds exactly to any of the circles shown in Fig. 2 of Mr. Carpenter's paper, except that accurate transmission line equations are used.

The equations which have been published by the writer for drawing the diagram are as follows:

$$\text{Find } E' + jE'' = E\left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{2.3.4.} + \text{etc.}\right)$$

$$\text{and } R' + jX' = (R + jX)\ (1 + \frac{YZ}{2.3} + \frac{Y^2Z^2}{2.3.4.5} + \text{etc.})$$

Describe a circle with center (a', b') and radius c',

where $$a' = -\frac{E}{1000}\ \frac{E'R' + E''X'}{R'^2 + X'^2}\ \text{kv-a.}$$

$$b' = +\frac{E}{1000}\ \frac{E'X' - E''R'}{R'^2 + X'^2}\ \text{kv-a.}$$

and $$c' = +\frac{E}{1000}\ \frac{E_s}{\sqrt{R'^2 + X'^2}}\ \text{kv-a.}$$

where E is the receiver voltage, and E_s the supply voltage.

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*"The Calculation of Constant-Voltage Transmission Lines," by H. B. Dwight, *The Electric Journal*, Sept., 1914, and "Constant-Voltage Transmission," by H. B. Dwight, pages 78 and 99, John Wiley & Sons, Publishers, 1915.

The equations for a' and b' show that the position of the center is independent of the supply voltage. The equation for c' shows that the radius is directly proportional to the supply voltage. Therefore, after one circle has been drawn corresponding to a certain supply voltage, a series of circles may easily be drawn about the same center, with radii proportional to the supply voltages which the circles represent.

If the first equation is shortened, so as to omit the terms of the series after $\frac{YZ}{2}$, and if the second equation is shortened so that

$$R' + jX' = R + jX$$

the results are exactly the same as those of the "split-condenser" method. The length of some of the lines of the diagram will be changed by one or two per cent. This error is appreciable, and while it may seem a small amount for practical work, there is no reason why this mathematical error should be incorporated in a diagram which is drawn up to show the characteristics of a particular transmission line. It may be said that it is generally admitted that the "split-condenser" method is superseded by the complete hyperbolic theory, and that the small amount of additional work required for the latter method of calculation is worth while in any case where capacitance is worth considering at all.

M. O. Bosler: In the mechanical construction of the energy diagram the distance OE in Fig. 1 when applied to the kv-a. scale of Fig. 2 is $\frac{E^2}{Z} . 10^{-3}$ and the distance of the supply voltage circle from O, OF, is $\frac{E\ E_0}{Z}\ 10^{-3}$. The slope of the voltage line OE, Fig. 1 across the energy diagram Fig. 2 is $\frac{X}{R}$.

By keeping the line impedance Z constant and increasing the ratio of X to R, the power capacity of the line for a given E and E_0 is enormously increased when supplying power at a high or a leading power factor. When a line of this sort gets into trouble, the power factor drops to practically that of the line $\frac{R}{Z}$, and limits the effect of the disturbance by the excessive voltage drop that occurs at such a low power factor.

This point, which can be so clearly illustrated by means of this diagram, should be of importance when operating an inter-connected system of large power capacity.

J. A. Lighthipe: The operation is usually reversed. That is, you get a condenser, or you have a condenser, or you buy as large a one as you can, and then figure back and see how much you can improve the line with this condenser.

We are endeavoring to scatter our condensers as nearly to the center of our load as possible and we have something like five or six condensers scattered into various parts of our system. We have adopted for the small size, about 2000 kw., partly because it can be thrown on a line with a minimum amount of disturbance; in some cases we have put a second one at the same substation.

The method of figuring just what effect your condenser will have is what we really need and want, but to figure out a certain size condenser for a given size transmission line which is loaded constantly throughout the year, why we usually work backwards, that is, get the condenser and then figure out what good it will do. Then, if necessary, we put another condenser a few miles away or sometimes in the same power house.

The old condenser method brings up the early days in the Spokane Coeur d'Alene transmission. There we had one of the first regulators to hold the voltage constant at Spokane but up the other side of Coeur d'Alene in the mining section the regulation was poor. It was finally decided to put a condenser up there with a regulator. This was brought out in the discussions at the Monterey convention.

We are using a great many of them on our system, although most of them are regulated by hand and not by general regulators, except our main substation at Eagle Rock.

I have always figured in the rough that to about every 10,000-h. p. induction motor we should install at least a 2000-kv-a. condenser, as near as we could get to the load center, and it is working out pretty well.

P. M. Downing: My experience has been that synchronous condensers are nearly always afterthoughts. They are put in when voltage conditions force a change. They are limited in capacity and corrective effect and, therefore, it is a case of doing the best with what you have.

They have been generally used on our system in the north central part of the state for a number of years. It is hard to predetermine on that system just exactly what will happen because of a more or less complicated network of lines and the large number of power sources. Where installed however, they are equipped for automatically taking care of voltage and they do it successfully. The growth of the business has been such that they have not been given an opportunity to show just how they would work under ideal conditions.

J. H. Anderton: I note in Mr. Downing's remarks he refers to synchronous phase modifiers as afterthoughts. This may be true to a large extent on some systems, as for instance, the one with which Mr. Downing is identified, since with the gradual growth of such a system and the increased radius of transmission, phase modifiers are installed as required. However, this does not generally apply to phase modifiers. In most instances this apparatus is of such prime necessity as to be the determining factor in the economic possibility of a development. Phase modifiers modify the phase of the total load with respect to the generators without necessarily varying the amplitude of the voltage so that it becomes possible by their use to transmit power any distance with constant voltage at both ends of the line or at any point on a line. Such systems as the Big Creek and Great Western Power Company systems would be economically impossible without their use because of the impossible conditions of voltage regulation.

You are all more or less familiar with the usual computations of a transmission line and I have no doubt you all try to use the simplest methods of obtaining the results. For preliminary work I have found the so-called approximate method quite suitable. This consists of computations showing the various generator voltages required for given loads and constant receiver volts. These are then shown as points on an ordinate of generator voltage, an assumed condenser kv-a. is then taken as abscissa, and for the same receiver volts a second point for the generator voltage obtained; plotted this gives the familiar transmission line diagram. By reading off the diagram a suitable condition of generator and receiver volts with a given kv-a., condenser capacity may be selected. Since reading from a diagram usually involves some inaccuracy, the selection of the kv-a. condenser from these curves is perhaps not permissible. However, the condenser kv-a. may be obtained mathematically for each assumed load by a modification of the generator voltage equation, the mathematics for which does not involve anything beyond the quadratic form.

P. M. Downing: In explanation of my remarks, I might say that the lines I spoke of were those of the Pacific Gas and Electric Company, practically all of which are 150 miles and operated at 60,000 volts. Therefore we do not have the same condition to contend with that would be met on the higher voltage lines.

Leslie F. Curtis: Professor Carpenter is to be thanked for bringing to our attention the fact that a graphical representation of any computation is to be desired. Whether the method developed by Professor Carpenter or that suggested in the previous discussion is used, the graphical method shows the calculator a picture of what is taking place on his line. It, in any case, is useful in checking his computations.

Professor Carpenter also notes that it is of advantage to include the transformers in calculating a line. I think that some of us have not kept this in mind. The calculation of the line involves not only the conductors at the proper spacing, but also the transformers and all of the apparatus connected thereto.

J. F. Wilson: I might refer, briefly, to Mr. Dwight's discussion. Mr. Dwight's transmission line formulas involve only simple arithmetical calculations which will, I think, give practically the same results as does Professor Carpenter's graphical method.

Relative to the suggestion that Mr. Lighthipe made, about working backwards on the proposition, I think that is probably true, but as the application of the synchronous phase-modifier extends and becomes more accepted in its general application, I doubt if that proposition will hold because, as was brought ut here in the other discussions, it will certainly be advantageous, if not necessary, to determine before the line is built and put in operation what effect you are going to get from a given condenser, or rather, what size condensers you are going to require to produce a given per cent regulation, or required power factor.

H. L. Melvin (by letter): Several years ago the writer learned the use of the Perrine-Baum Chart and its adaptation to the predetermination of synchronous phase-modifier performance. Since that time I have had opportunity to use it quite extensively and to check its accuracy. It is sufficiently accurate for all practical purposes for transmission lines now in operation.

The chart is very easily and quickly constructed and once made for a line a vector picture of the performance of the line for any load is given at a glance. The charging current, resistance, reactance and impedance of the line must be determined and if standard cross section paper is used for the kilowatt and reactive kilovolt ampere scales it is necessary to determine the voltage scale. The line drop due to the magnetizing current for the receiver transformers must be calculated, and since the current lags practically 90 degrees, it can be subtracted from the drop caused by the line charging current, giving the vector a c (Fig. 2). It will be noted that if arcs of circles be drawn with *a* (Fig. 2) as a center they will be loci for constant current, kilovolt amperes and I^2R line loss (the loss due to the charging current neglected). On the scale of generator volts actual voltages may be used, or if the step-up transformers are included, low-tension bus voltages or even switchboard voltmeter readings. When superimposing the synchronous condenser characteristic, the point where the decrease in line loss is equal to the losses in the condenser may be determined. The use of synchronous condenser capacity beyond this point for the improvement of voltage regulation will decrease the efficiency of transmission. If a transformer bank is installed for the condenser only (the load current not passing through it) its regulation and losses can be included in the condenser characteristic curve (Fig. 4). When the maximum load change is known the minimum synchronous condenser capacity required, operating lagging and leading, may be determined and also the proper voltage to be held at the generator.

Practically all the information which the chart will give is needed in the analysis of the performance of a transmission line and it is really a remarkable solution that will give the results accurately.

Presented at the Pacific Coast Convention of the American Institute of Electrical Engineers, Los Angeles, Cal., September 18-20, 1919.

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CALIFORNIA 220,000-VOLT — 1100-MILE — 1,500,000-KW. TRANSMISSION BUS

BY R. W. SORENSEN, H. H. COX AND G. E. ARMSTRONG

ABSTRACT OF PAPER

This paper summarizes the power resources of California and the probable loads to be supplied within the next six or seven years. For the purpose of economically distributing the necessary power and supplying the load, a long high-voltage transmission line is proposed. As this line would interconnect a number of different companies, it assumes the nature of a bus bar. The authors show how the proposed line may link with some of the lines now in-service and enumerate the advantages of such interconnection. A comparison is made between the 240-mile Big Creek line now operating at 150,000 volts, 50 cycles, and the operation of this line at 220,000 volts, 60 cycles. Operating data on the Big Creek line are shown to indicate the character of the construction necessary for California conditions. Conclusions are drawn as to the particular features to be observed for successful operation of 220,000-volt lines.

INTRODUCTION

FUELS, particularly oil, must soon be used for isolated power only in places where electric power is not available, as in the propelling of air and ocean craft. In large power systems, especially in the West, the use must be limited to standby service, for peak loads, low water periods, and other emergencies.

POWER RESOURCES

California has available ample hydroelectric power to supply the industrial and agricultural demand for many years.

Small developments aggregating 325,000 kw. have been completed and many others of this type are available. There are also four large projects as indicated in Table I. which can readily be developed to a capacity of 1,500,000 kw. in the near future.

The data for the following tables of resources and loads, of the Northern part of the state, are taken from various reports which have been published and no attempt has been made to verify them.

TABLE I—LARGE POWER RESOURCES

	Now Developed and Under Construction	Proposed Developments 1926	Reasonable Future Development (Not Ultimate Capacity)
	kw.	kw.	kw.
Pitt River	None	200,000	500,000
Feather River	100,000	200,000	300,000
Big Creek	100,000	300,000	500,000
Colorado River	None	None	200,000
Total	200,000	700,000	1,500,000

Total 1926 hydroelectric power development including small projects is 1,025,000 kw.

LOAD DEMAND

The best available information indicates a demand in 1926 approximately as shown in Table II.

TABLE II.

1. Sacramento Valley, northern portion	70,000 kw.
2. Truckee River electrification	40,000 "
3. Sacramento Valley, southern portion	125,000 "
4. San Francisco Bay District	250,000 "
5. Fresno District	90,000 "
6. Bakersfield District, including Tehachapi electrification	125,000 "
7. Los Angeles District	300,000 "
8. Barstow and Needles District, including railroad electrification	40,000 "
Making a total of	1,040,000 "

In order to carry this load, approximately 500,000 kw. additional in hydroelectric capacity will be required.

A demand for power such as shown in Table II can be supplied most economically by power developed in large units. Large power units require transmission lines of the highest possible economic voltage.

*It has been shown that, for long transmission, 220,000 volts is economical under conditions which require a much more expensive construction than has proven adequate for the 150,000-volt lines of the Southern California Edison Company.

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*Silver, *Problems of 220-kv. Power Transmission*, page 1037.

California Transmission Bus

On this basis, a plan as shown on the map, Fig. 1, is proposed. In this plan the interconnection of all the California Power Companies has been assumed, as an economic necessity for its best utilization. Interconnections of limited capacity are not entirely satisfactory because they fail just at the time they are needed most to transfer from one system to another large blocks of power.

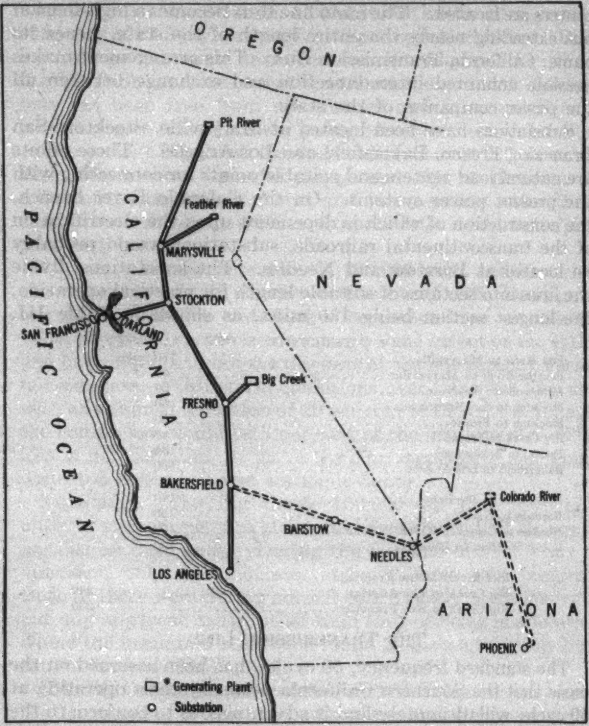
OREGON
Pit River
Feather River
CALIFORNIA
PACIFIC OCEAN
MARYSVILLE
NEVADA
STOCKTON
SAN FRANCISCO
OAKLAND
Big Creek
FRESNO
BAKERSFIELD
Colorado River
BARSTOW
NEEDLES
LOS ANGELES
ARIZONA
PHOENIX
Generating Plant
Substation

Fig. 1—California 1100-Mile—220 Kilovolt—Transmission Bus

The plan of the proposed scheme involves the construction of a two-circuit transmission system extending from Pitt River to Los Angeles, a distance of 570 miles. Branch lines of like voltage connect the three other power projects and the San Francisco load center to this main line on which the other load centers are located. The main line thus becomes a high-tension bus extending nearly the entire length of the state, hence its name; California Transmission Bus. This arrangement makes possible unlimited interconnection and exchange between all the power companies of the state.

Substations have been located at Marysville, Stockton, San Francisco, Fresno, Bakersfield and Los Angeles. These points are natural load centers and suitable points for connecting with the present power systems. On the Colorado River branch, the construction of which is dependent upon the electrification of the transcontinental railroads, substations would probably be located at Barstow and Needles. The substations divide the lines into sections of suitable length for practical operation, the longest section being 150 miles, as shown in Table III.

TABLE III.

Pitt River to Marysville	150	miles
Feather River to Marysville	60	"
Marysville to Stockton	90	"
Stockton to San Francisco	60	"
Stockton to Fresno	130	"
Big Creek to Fresno	40	"
Fresno to Bakersfield	100	"
Bakersfield to Los Angeles	100	"
Bakersfield to Barstow	110	"
Barstow to Needles	150	"
Needles to Colorado River	100	"
Colorado River to Phoenix	100	"
Pitt River to San Francisco	300	miles
Big Creek to Los Angeles	240	"
Big Creek to San Francisco	230	"

THE TRANSMISSION LINE

The standard frequency, 60 cycles, has been assumed on the basis that the Southern California power systems operating at 50 cycles will ultimately find it advantageous to conform to the A. I. E. E. standard. In the natural growth of the load as shown in Table II, 70 per cent of the 1926 load will be supplied by the 60-cycle systems. Interconnection of such large load centers or power sources through frequency changers, limits the exchange of power, is uneconomical, and increases tremendously the required operating vigilance.

The practicability of the high-voltage line has been well demonstrated by over five years of remarkably successful operation of the 150,000-volt lines of the Southern California Edison Company which, during this period have delivered from the Big Creek power houses over the 240-mile lines to the Los Angeles distribution system, 1,200,000,000 kw-hr. at an average efficiency of 87.5 per cent with a 45 per cent load factor. During this period there have been no interruptions for which the high voltage is responsible, and on the contrary, the system has been free from disturbance and interruption to a greater degree than the lower voltage lines in the same locality.

The present Big Creek lines can be operated at 220,000 volts, 60 cycles, without material change and this is proposed as a link of the transmission bus, and its operation under these conditions will be analyzed and applied to conditions of the proposed system.

Corona. As now operated, at 150,000 volts and 50 cycles, the voltage is only 80 per cent of the lowest critical voltage of any part of the line and there is no corona loss. At 220,000 volts, 60 cycles, corona loss occurs to some extent on the entire line but amounts to but 0.4 per cent of the line capacity during fair weather. With storm conditions over the entire line, and with an assumed reduction of 20 per cent in the critical voltage, the corona loss would be 8 per cent of the line capacity. This loss is not sufficient to make the line inoperative and would occur too rarely to be an economic factor.

Insulation. The Big Creek 150,000-volt lines have nine units in each suspension string and two eleven-unit strings in parallel on dead ends. During the five and one-half years of operation only two insulator string failures have occurred. Both of these were during normal conditions of operation without any apparent cause, other than that of being in a location where the insulators have been found to have a relatively high rate of deterioration.

The Big Creek line towers allow sufficient clearance to permit the lengthening of the nine unit suspension strings to eleven units, and to any desired number of units at dead ends. Table IV. shows safety factors for insulator strings, wet and dry.

The Big Creek line operated at 220,000 volts is at the critical corona voltage and any disturbances resulting in a higher voltage will quickly expend their energy in producing corona loss, which will permit a smaller safety factor to be used.

The curves in Fig. 2, showing arc-over voltage as reproduced from Mr. Silver's paper, *Problems of 220-kv. Power Transmission,* show no practical gain in dry arc-over voltage for strings of more than ten units, and with these facts in view it is proposed that for operating the Big Creek line at 220,000 volts, suspension strings have eleven units and dead end strings 12 units in series. Insulator testing crews have several times reported four and five defective units in a nine-unit suspension string, without any indication of trouble. The only apparent value of a longer string than that proposed would be a decreased probability of sufficient defective units in a string to cause breakdown. Developments of methods of grading insulator units and shielding insulator strings, will in all probability, materially change curves of Fig. 2.

TABLE IV. SAFETY FACTORS FOR INSULATOR STRINGS

	Wet arc-over		Dry arc-over	
	9 unit String	11 unit String	9 unit String	11 unit String
(87 kv. to ground) 150 kv........	4.3		4.8	
(127 kv. to ground) 220 kv.......		3.7	3.3	3.4

Present day method of insulator testing and maintenance would probably have prevented the two failures which have occurred on the Big Creek lines as previously mentioned. These methods applied to the lines operating at 220,000 volts and the use of the better types of insulators now available will insure successful operation.

Charging Current. Long high-voltage lines cannot be operated without synchronous condensers at the receiving station to regulate the voltage, and as a consequence the charging current, even at the standard 60-cycle frequency becomes a factor of no great importance as long as these synchronous condensers are connected to the line.

Without these condensers the line charging current must be furnished entirely by the generators, in which case the generators may become greatly overloaded and at the same time produce a very high voltage over which the operator has no control. To avoid this emergency a transmission line with its generators, transformers, and synchronous condenser must be considered as a unit and as such should be securely coupled together electrically at all times. This has been proved praccal in the case of the Big Creek system in which it is possible to start the 15,000 kv-a. condensers and bring them up to speed with the generators.

Line Capacity. The Big Creek lines as operated at 150,000 volts with 30,000 kv-a. condenser capacity per line at the receiver end are each good for 57,500 kw. at 85 per cent power factor and will have under these conditions a line drop of 11 per cent.

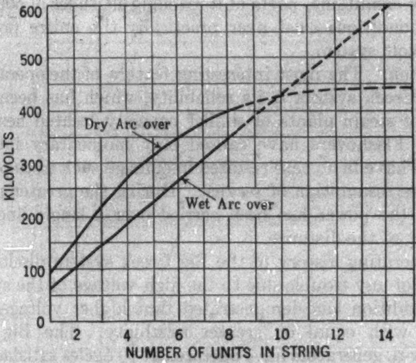

FIG. 2—TYPICAL 60-CYCLE ARC-OVER CHARACTERISTICS OF SUSPENSION INSULATORS

Operated at 220,000 volts these lines should each have a capacity of 125,000 kw. with an equal line drop when provided with the proper condenser capacity, which is approximately 75 per cent of the line capacity in kilowatts.

This is a fair indication of the conditions which will exist in the proposed system, the load centers of which are so distributed as to limit the actual average distance of transmission to about 200 miles. The economic gain in doubling the capacity of lines which cost approximately $6,000,000, the present cost of which would be at least 30 per cent more, would more than offset the cost of all necessary changes, including the adoption of the standard frequency.

Mechanical. The type of construction used on the Big Creek line has proven entirely adequate for California conditions. There have been only three mechanical failures all of which occurred shortly after the line went into service and were all due to defective line hardware. In one case the failure was due to faulty design. This fault was entirely corrected by redesigning the cable clamp so as to grip the steel core independently of the aluminum conductor. The other two were due to individual defects in parts. There have been no tower failures and no tower maintenance whatever has been required. Approximately 20 per cent of the Big Creek line is subject to ice and snow conditions, parts of it reaching altitudes of 5000 feet. Similar conditions exist over practically the entire proposed 220,000-volt system.

Operation. The most interesting feature of the operation of the Big Creek system is its reliability, which has been equal to that of steam plants of similar capacity located near load centers. Flashovers have caused only momentary interruptions and have in no case resulted in damage such as to prevent immediate resumption of service. During the greater part of the time the power has been carried over a single line for a large part of the distance.

The operating history of the Big Creek system discloses no evidence of any trouble due to the high voltage of the system, and in addition has demonstrated that higher voltages may be used with equal or greater reliability. The Big Creek 17,500 kv-a. generators have operated at 60 cycles satisfactorily and delivered full output at this frequency.

High-tension line switching and synchronizing has been carried on consistently throughout the operation of the Big Creek system without trouble, and should be possible on the 220,000-volt system. During times of switching, slight discharges, never followed by any energy current, occur on the arresters. Operating at corona voltage rather than at 80 per cent of the critical voltage it may be possible to absorb these disturbances without arresters.

Complete parallel operation of all lines must be adhered to in the proposed system. Satisfactory protective relay systems for dropping defective sections with little disturbance have been developed for present parallel transmission lines and there appear to be no obstacles to extending these to the higher voltages.

Generators. Curves of Figs. 3 and 4 show generator and line characteristics for 60-cycle, 220,000-volt systems. The full lines are the charging currents in amperes for different lengths of line plotted against per cent normal voltage. The broken lines are generator characteristics of various sizes of generators when connected to condensive loads with no field excitation. The point of intersection of the generator curves with the line charging current curve for any particular length of line determines the voltage to which the generator will build up when connected to that length of line with no field excitation. Fig. 3 is for generators with a short-circuit ratio of 1.0, while Fig. 4 is for those with a ratio of 1.5.

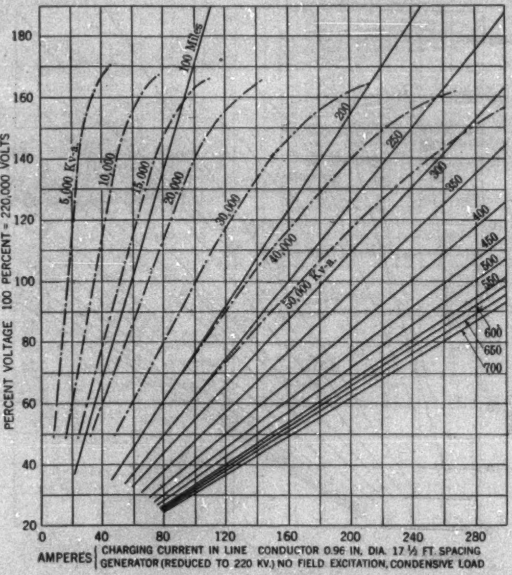

Fig. 3—Generator and Line Characteristics—60 Cycles—220,000-Volts—Generators with Short-Circuit Ratio 1.0

Fig. 3 shows that with 50,000 kv-a. of generating capacity connected to a line of 250 miles, the line can be charged without losing control of the voltage with generators of this design. By having synchronous condensers connected to the line at the receiving station generators of this capacity will bring up any length of line necessary to the successful operation of the proposed system. These curves show that generators for such a system should be designed with the highest short-circuit ratio

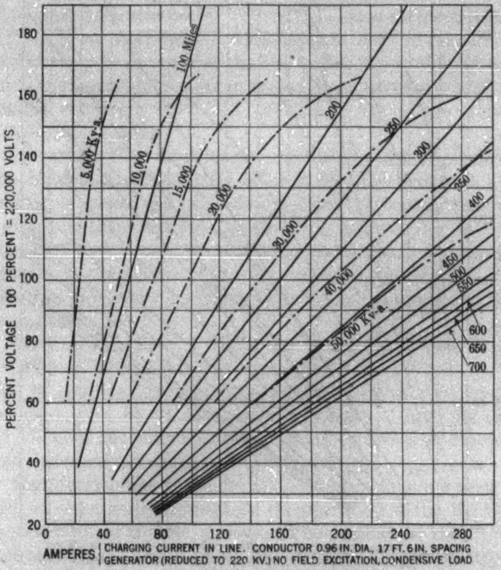

Fig. 4—Generator and Line* Characteristics—60 Cycles—220,000 Volts—Generators with Short-Circuit Ratio 1.5

that other conditions will permit in order to reduce to a minimum the tendency to become self exciting.

*Data for these curves were worked out in the laboratory of Throop College of Technology in 1915 and results verified by actual tests on the 17,500 kv-a. generators at Big Creek.

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*For further data and explanation of generator performance with large condensive loads see Newbury, *Electric Journal*, 1918.

Conclusions. Such a system as proposed is needed immediately; all engineering fundamentals essential to a solution of its problems are well understood and the Big Creek system can be used as a part of the project without material reconstruction.

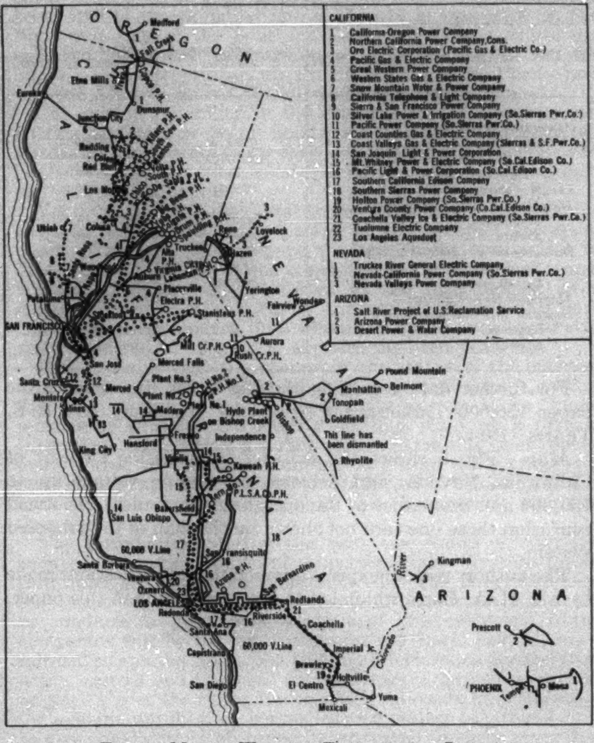
CALIFORNIA
1 California-Oregon Power Company
2 Northern California Power Company, Cons.
3 Oro Electric Corporation (Pacific Gas & Electric Co.)
4 Pacific Gas & Electric Company
5 Great Western Power Company
6 Western States Gas & Electric Company
7 Snow Mountain Water & Power Company
8 California Telephone & Light Company
9 Sierra & San Francisco Power Company
10 Silver Lake Power & Irrigation Company (So.Sierras Pwr.Co.)
11 Pacific Power Company (So.Sierras Pwr.Co.)
12 Coast Counties Gas & Electric Company
13 Coast Valleys Gas & Electric Company (Sierras & S.F.Pwr.Co.)
14 San Joaquin Light & Power Corporation
15 Mt.Whitney Power & Electric Company (So.Cal.Edison Co.)
16 Pacific Light & Power Corporation (So.Cal.Edison Co.)
17 Southern California Edison Company
18 Southern Sierras Power Company
19 Holton Power Company (So.Sierras Pwr.Co.)
20 Ventura County Power Company (Co.Cal.Edison Co.)
21 Coachella Valley Ice & Electric Company (So.Sierras Pwr.Co.)
22 Tuolumne Electric Company
23 Los Angeles Aqueduct
NEVADA
1 Truckee River General Electric Company
2 Nevada-California Power Company (So.Sierras Pwr.Co.)
3 Nevada Valleys Power Company
ARIZONA
1 Salt River Project of U.S.Reclamation Service
2 Arizona Power Company
3 Desert Power & Water Company
OREGON
Medford
Fall Creek
Copco P.H.
Etna Mills
Eureka
Dunsmuir
Junction City
Redding
Red Bluff
Los Molinos
Chico
De Sabla P.H.
Ukiah
Colusa
Reno
Lovelock
Truckee
Hazen
Auburn
Virginia City
Yerington
Placerville
Electra P.H.
Petaluma
Wonder
Fairview
Stanislaus P.H.
SAN FRANCISCO
Aurora
Mill Cr.P.H.
Rush Cr.P.H.
San Jose
Merced Falls
Santa Cruz
Monterey
Salinas
Merced
Plant No.3
Plant No.2
Madera
Hydo Plant on Bishop Creek
Bishop
Independence
Pound Mountain
Belmont
Manhattan
Tonopah
Goldfield
This line has been dismantled
Rhyolite
Fresno
Hanford
King City
Visalia
Kaweah P.H.
P.L.S.A.Co.P.H.
Bakersfield
San Luis Obispo
60,000 V.Line
San Francisquito
Santa Barbara
Ventura
Oxnard
Azusa P.H.
San Bernardino
LOS ANGELES
Redondo
Redlands
Riverside
Santa Ana
Coachella
Capistrano
60,000 V.Line
Imperial Jc.
Brawley
Holtville
El Centro
San Diego
Mexicali
Yuma
Colorado
River
Kingman
ARIZONA
Prescott
PHOENIX
Tempe
Mesa

FIG. 5—MAP OF WESTERN TRANSMISSION LINES

To supply this need, arrangements should be made without delay for a complete working out of all details of the proposed system, as otherwise in the future it may be necessary to do a large amount of reconstruction to bring together individually-designed systems, which is never a wholly satisfactory procedure.

COMPARATIVE DATA

	Big Creek lines at 150 kv.	220 kv. lines as proposed by Silver
Aluminum steel cable		
Diameter	0.95 in.	1.036 in.
Circular mils	683,000	808,900
Weight per foot	0.75 lb.	0.94 lb.
Length of average span	750 ft.	800 ft.
Weight of towers without footings		
Suspension	4300 lb.	9000 to 14,000 lbs.
Anchor	6450 lb.	24,000 lb.
Stringing tension at 80 deg. fahr.		
No ice allowance	4740 lb.	
Ice allowance	3130 lb.	
Maximum tension allowed	8500 lb.	17,300 lb.
Insulator strings to carry load		
Suspension	1	2 and 3
Anchor	2	6

For further data regarding the Big Creek line refer to the paper 150,000 *Transmission System* by Woodbury, A. I. E. E. TRANS., 1914.

Map. Fig. 5 shows the existing transmission systems of California, Nevada, and Arizona, varying in voltage up to 150,000 and the names of the operating companies. To avoid confusion these lines are not shown on the map of the proposed system.

The authors wish to express appreciation of suggestions made by Mr. H. A. Barre which led to the preparation of this paper.

DISCUSSION ON "CALIFORNIA 220,000-VOLT—1000-MILE—1,500,000-KW. TRANSMISSION BUS." (SORENSEN, COX, ARMSTRONG), LOS ANGELES, CAL., SEPTEMBER, 20, 1919.

Ralph Bennett: It is interesting to note that this paper can hardly be said to be in advance of actual construction since the lower half of the proposed line is already in operation in the Big Creek Line of the Southern California Edison, while the northern half of the transmission is under contemplation for immediate construction as the Pit River Line of the Pacific and Gas and Electric, and at the full contemplated voltage.

Our gain in reliability in the construction of power transmitting devises is no better illustrated than in the advance between the timid and temporary construction utilized in the latter 90's, and the substantial, permanent and well developed methods now at hand.

Yet it can hardly be said that there has been any considerable advance during this entire period in the abstract technical theory of the art. The change has been in the application of these abstract values to the concrete case of electrical transmission. This is well illustrated in the introduction as a matter of routine equipment of the use of synchronous condensers on these modern lines where the use of such apparatus was considered to be entirely experimental ten years ago.

It is probable that the change in the status of the business from the period of the latter 90's to the present date is greater than the change in its technical features. It is no longer possible to consider the electric lighting plant as a private enterprise. It is today a public utility extending over one or many states, and serving more territory than the average railroad system, and has come under state and Federal regulation more sharp and penetrating than the Government exercises over any other business.

The introduction during the past two years of inter-ties between the various physical properties of the Western power companies has long been foreshadowed. The paper proposes that these ties, now of a temporary and insufficient character, be rendered a permanent portion of the systems. Many companies will be fed by a common net work just as many railroad companies feed across their transfers, loads derived from many sources and intended for terminals as diversified as their origin. Under the ruling under which railroads have operated for a generation they are compelled as a matter of public policy to give equal care to all property entrusted to them regardless of its origin or its destination. They have become common carriers and far from rebelling from this condition they consider it to be an essential portion of their success.

The power business is rapidly developing all of the conditions which have surrounded railroad operation insofar as the interlinking of numerous independent companies is concerned.

It appears logical for conditions eventually so to shape themselves, that the power company transmissions will become common carriers, not merely as the result of mutual agreements between existing monopolistic companies, but by action of law. As a result, any producer of power, however small, will be assured that his production will be received on the transmission net work, and that he will receive therefor a price fixed by a state regulating body on a basis of a fair return on his investment.

It is, I think, obvious to anyone acquainted with the present power situation that this would result in the construction of numerous small plants capable of introducing a fair amount of power as an incidental to the handling of water, natural gas, oil, or other power sources.

There are numerous cases within the knowledge of every engineer in this vicinity where considerable blocks of power could be produced on a schedule which would not permit of sale to a local net work, even if such sale were permitted by the railway commission, but where the production cost of the power would be so low that the power could be produced to great advantage if it could be successfully wholesaled to a net work capable of absorbing it regardless of its fluctuations.

Almost every irrigation system in the state possesses drops capable of producing a more or less considerable amount of power.

Numerous desirable combined irrigation and power projects are neglected because it has not been possible in the past to develop a market for the power, although the power in connection with irrigation could be produced at a very low figure.

On the north end of the Pacific coast there are sources of power as a by-product from wood working plants; there are coal mines lying undeveloped because they are not favorably located with reference to a rail haul market; in the oil producing industry vast quantities of oil and natural gas are wasted because they are unsuited to market demands or are in a location such that they cannot be successfully delivered to distant markets, yet they could be sources of large amounts of power which could be readily handled over a transmission bus.

It is unnecessary and perhaps aside from the purpose of the paper to go into detail on these items. The one other matter in the same connection however is of interest. Railroad electrification has been much discussed of recent years but has been to an extent held back by the very considerable problem involved in the extremely irregular use of power which will occur on most of the western roads.

Taken in connection with an ordinary local transmission this makes a load so undesirable as to render the power rate abnormally high.

But in connection with a centralized power system handling one or two million kilowatts these fluctuations would be regularly supplied by the excess capacity of the net work.

The problems of engineering, of finance and of political expediency involved in the changes now taking place in the relations of the power companies to each other, and to the public, are too great to even touch in this discussion.

P. D. Jennings: This paper is especially valuable at this time owing to the rapid advance of fuel prices and the necessity for the conservation of coal and oil for the essential industries that can not avail themselves of hydroelectric central station energy.

I believe, as this paper slightly implies, that this high-tension bus scheme could economically be carried out to include all of the great hydroelectric developments west of the Mississippi River. Of course it is not supposed that there would be an exchange of energy, say, between eastern Montana and southern California, but it is a fact that power plants on this bus line would have the great advantage of the diversity of load demanded, by substations, at the load centers along the line.

The two important considerations to be given, of course, are its operation and financing. It would appear to me that the operation of such a bus net work would require a unified control; and that the load dispatching work would have to be divided up into districts, or regions, which in turn would be responsible to an operating board of control composed of regional operating directors. This method of operation, of course, would require a unit scheme of financing. The probable formation of a holding company whose stock would be purchased and allocated by some equitable plan among all of the great generating systems interested.

Of course such a scheme would probably have many very serious disadvantages, from the individual standpoint of some of the companies involved, as well as some of the Public Service Commissions.

But these difficulties I do not believe would be insurmountable owing to the fact that both the public and the companies would be materially benefited by helping to reduce the rapidly increasing operating costs.

C. O. Poole: To my mind this proposed plan is one of the most important projects presented to the engineering profession from a conservation point of view. I have been, for years, a firm believer in this scheme and can see many advantages that will accrue from it, such as making possible comparatively small hydroelectric developments that will be in reach of the proposed bus, that would otherwise be too remote or without market for the output; or there may be streams suitable for power development that do not have storage facilities to equalize the flow throughout the year, in which event it would necessitate a steam plant in order to carry the load during the non-run-off periods. Under such conditions the investment to make this development might not be justified to supply an individual system, while with the proposed bus such a development might be advantageously made, simply utilizing the output of the stream while the run-off period is on. This same condition might apply also to irrigation systems where there are drops in the canals and water supply streams.

One of the principle advantages of connecting all generating and distributing systems to such a bus would be the improvement of the load factor conditions by diversity of both the generating capacities and the demands of the many diversified conditions of the different systems. For this same reason there would be less necessity for over-installation on water power generating systems, thereby very materially decreasing the spare unit investment and with its consequent less over head expense and reduction in cost for generation. From recent statistics the average yearly load factor of electric systems in the United States is only 30 per cent, and, while California's load factor is materially better than this, it is subject to much improvement.

As pointed out by the authors of the paper, and as theoretically set forth by Mr. Silver's paper in this volume, page 1037, the voltage proposed is not an impossible one to operate, but simply a matter of insulation and clearances. From a transmission point of view I might suggest, however, that the eleven units in the insulator string, as suggested by the authors, might be somewhat inadequate for the insulation of the new line unless insulators of a more reliable nature can be obtained than have been made use of in the past.

On this subject, the experiments cited in Mr. Silver's paper upon the dry arc-over and wet arc-over tests, are of unusual interest and it would seem that the line insulation should be based upon the curve of the wet arc-over, rather than of the dry arc-over conditions, inasmuch as the real test of the insulation is during storm and fog conditions, and, while from the curve mentioned the eleven insulators in the string might approach the flat portion of the dry arc-over curve, yet there is plenty of justification for adding more units in the string when the wet arc-over line on the curve is considered. The operation of such an important line as proposed should carry with it larger factors of safety than an individual system might economically call for, and, therefore, extra insulation in such a system would be, in my estimation, fully justified.

Special interest is attached to the probable conditions of corona loss upon this proposed line. I have been of the opinion for some time that it might be possible so to proportion sections of the transmission system of the higher voltages that the corona loss might be a means of dissipating energy in the event of over-voltages from any cause, and in the present instance it would seem that the line approaches this condition. In the event of an unusual high-voltage surge, energy would be dissipated to such an extent that it would tend to hold down the rise in voltage and, therefore, would be a benefit and protection to the system. I think this a feature that should be very carefully considered and experiments conducted beforehand to determine to what extent the corona loss could be utilized for the purpose mentioned.

Aside from the purely engineering conditions involved in such an undertaking there is another feature, to my mind quite as important as the strictly technical side of the situation, and that is the method of handling the different companies' input to the bus and the output from the bus. With a dozen or more companies taking from and feeding to such a system, many problems are involved that would have to be satisfactorily worked out, both as to the protection of individual systems from overload conditions, and also protection of the bus from service interruption. These features are engineering problems that can doubtless be worked out by a comprehensive system of relay switches. Probably a still more difficult situation is involved in the method of measurement of the current to and from the different companies. This last question also involves the ownership of the transmission bus, as to whether all the distributing companies would have ownership in the bus proportional to the capacities of their plants, or whether each should have ownership in the bus proportional to the kilowatt hours used by the different companies, and each company share its proportion of maintenance and overhead costs of the system, or whether an independent company should own the bus, this independent company being composed of the contributing companies and each of the contributing companies sharing in the ownership of the bus proportional to the sizes of their plants, or as to their proportional use of the bus. These are all much involved questions the solution of which, it seems, should rest jointly between the engineers, the commercial department and the financiers of the institutions involved. Personally I have not, so far, worked out any practical solution to this problem.

To my mind this proposed transmission bus should not be treated as some mythical thing to be considered in the far distant future, but as something that is now needed, and immediate consideration should be given to working out of all details involved in carrying the project to a successful issue.

J. D. Ross: There seems to be no reason why 220,000 volts should not be used commercially, providing the proper synchronous apparatus controls the voltage and providing the insulation has a good factor of safety. The cost of such long lines will, of course, be commercially feasible only where there are large quantities of power to justify the expense. It is probable that before many years this voltage will be considerably exceeded and will be considered moderate.

The city of Seattle is at present embarking on an enterprise on the Skagit River one hundred miles distant from the city. On a 50 per cent load factor, there will ultimately be developed at this point one-half million horse power. On account of this very large quantity of power, it is desired to limit the number of lines as far as possible by using high voltage and for this reason six lines are intended and the voltage will be 160,000. Had the distance been two or three times as great, 220,000 volts might reasonably have been used.

Three of the coast cities besides Seattle own their own electric power systems; Los Angeles, Pasadena and Tacoma.

Some years ago I spoke to the men in charge of these systems of the hope that some day there would be a high-voltage line paralleling the coast which would allow one city to help the other but at that time the voltages possible were not adequate. From Seattle to Los Angeles is about 1500 miles and it is a notable fact that the line at present under discussion is 1100 miles in length. Such a line along the coast would be a benefit to many of the smaller towns especially, and could be participated in by companies and municipalities alike. Of course, probably no city or combination of cities would attempt such a line but the best and largest water powers are rapidly being used up and when it comes later to the proper conservation of all such resources, no doubt the states affected will take a hand and interconnection between states would make such a line an actual fact. One of the great troubles which prevents interconnection of systems at the present time is the dissimilarity of voltage and phase relation and one of the greatest works that the Institute could do would be to bring about a better standardization of voltages.

Leslie F. Curtis: Dr. Magnusson, in the November 15, 1918 issue of the *Journal of Electricity*, considered some of the phases of a similar type of bus for the state of Washington. He selected the state of Washington because he happened to be familiar with conditions there.

The principal points brought out by Dr. Magnusson in his paper are advantages to be obtained by the Federal regulation of such a bus system. I am not prepared to advocate either Federal or private control. I will simply refer you to his paper.

J. B. Fisken: Professor Curtis has brought out the point that I wanted brought out. Dr. Magnusson advocates a Federally owned bus and I think there are numerous objections to such a scheme. There would be difficulty connected with the utilities working on a Federally owned bus.

H. A. Barre: On this general matter of government control, I think the big question, or the big trouble, with government ownership in that sort of thing is that you don't want the umpire to play on either of the teams. If the government would get out of the way and let us do the job, we can do it.

We are coming more and more to recognize the fact that neither the government, or companies, or financiers, or any other element, will have much to say about the development of this project. The development will be according to the natural economic laws which are as unalterable as the multiplication tables. Improper financing, improper engineering and improper relations between the companies and the public, and the public and the organizations of the companies, can gum up the whole game; they can delay it; they can interfere with it, but they can no more stop the building of it than the building of a dam across a river can stop the water for there comes a time when it does run over.

I do not think that this is a good time to sell the United States short. I think there is enough common horse sense in the United States and enough economic pressure for the carrying out of such schemes and we will get the proper answer in spite of all these things that are trying to interfere.

I object to the paper as a whole because I do not like such an arrangement. I do not think we should worry about a 1100 mile bus in California. I think when this thing does come it will come very quickly. It is going to be tied in with the Colorado power for a starter; then south through New Mexico to Albuquerque, Needles, Bakersfield, Mount Shasta, and wind up, as has been said, in Butte.

The great part of the work is done. There are already a great many interconnections. Those interconnections have been the means of a great economic saving in the past two years. Just in our little corner of the job, the San Joaquin Valley would have been absolutely shut down this year if it had not been possible for an arrangement to be made between the Edison, the San Joaquin, the Mt. Whitney, the San Diego and the Southern Sierras Companies. It is not very far from being shut down now as far as the shortage of power is concerned, but the job is still running and the amount of interference to service has been extremely small, that is true in spite of the fact that it was necessary to buy current from San Diego. One of the great stumbling blocks, of course, is the 50-cycle system in the middle of a big 60-cycle territory.

The French government has done a very intelligent thing. They have standardized the whole country at 50 cycles and standardized the voltages in multiples of the square root of 3, as high as anybody wants to go. I do not think anybody would want to go higher than the 110,000 or 120,000 volts at the present time.

That is a line along which we could follow to some advantage throughout the United States through the action of the Institute.

Such a line is not going to be difficult to operate. It will be broken up into regions, as has been suggested, in which a group of plants will supply a group of territories, or a group of loads, and means provided for an interconnection between those groups.

The control of the voltage is one of the most serious troubles and that is going to come through an extension of our work with synchronous condensers, without any question. The real instructive thing that has come out of this interchange is the fact that it has been possible for companies whose interests were to a very considerable extent antagonistic to get together on a broad enough basis to help out each other with their troubles and not try to take all the money the other fellow has when they do it.

L. M. Klauber: I do not know how it should be done, but I hope it will be done soon.

I think that one point that Mr. Barre brought out is of considerable interest; this is that all of these things will be easier to do after they are done. They work themselves out beautifully. A lot of the problems that have been causing us sleepless nights (I refer to problems not of engineering but of operation and management) have really eliminated themselves. We sometimes wake up in the morning and find that the solutions which have bothered us for six months have already been accomplished and that the difficulties which worried us have failed to materialize when the time came.

You probably noticed a brief editorial in a recent copy of the *Journal of Electricity*, wherein it was stated that when the Pacific Fleet came into San Diego Bay, the Crane Valley reservoir of the San Joaquin Light and Power Company, distant many hundreds of miles and three companies from San Diego, had to be drawn on for an extra amount of energy. This, of course, is an interesting proposition. It would be somewhat more interesting, if true, which was not the case. As a matter of fact the shortage on that particular day was due to a defective condenser.

I think that the transmission bus, when the time comes, will work itself out without the difficulties in financing, in operation and in government supervision, which now appear almost insurmountable.

J. A. Lighthipe: I think Mr. Barre is perfectly right when he made the statement that while we are struggling and talking about this proposed bus, we will have it. Nothing is improbable and these great projects simply spring up when the need is there.

The great problem of this business is not only the diversity of the distribution, it is the great diversity of the supply. Our water sheds in California are very erratic and diversified. That problem within itself is a great one.

We have Railroad Commissioners who do not interfere with us. They pat us on the back and tell us to go ahead. As long as the State of California has a Public Service Commission, such as the Railroad Commissioners, with a modern man at the head of it, this matter of public control is not going to bother us.

It is a beautiful thing to talk Socialism and it is a beautiful thing to talk public ownership, but it does not get us anywhere. A man with the nerve will get hold of these projects and bring them to a proper solution. The State of California has reaped the benefit of the pioneers of the power situation and will continue to do so.

The question of voltage is an interesting one. When we had 10,000 volts we were afraid of it; we never dreamed of such a thing in our lives. When Southern California sprung to 33,000 volts even the manufacturers of apparatus shook their heads with uncertainty. I remember a man, in the early days who stated that 50,000 volts would be the ultimate we could hope for. That statement was made in the days when we knew nothing about all these modern troubles and the talk about a 50,000 voltage was thought impossible. However, we kept creeping up until we reached 60,000 volts and then 150,000 volts. We have also learned that the higher the voltage the less trouble we have. That is probably due to the fact that the engineers on the job have worked more thoroughly; that is, we know what we are running into and we build to meet that condition.

The future of the state is so great, where the price of oil used is ever increasing; where every bit of power in this state should be developed; where this power should be used and where we should start a campaign of conservation on what we have in the state.

P. M. Downing: It is very interesting to look at a map and see a 1500-mile bus extending from somewhere in British Columbia or the Northern part of Washington down to the Southern part of California or into Arizona, and from an engineering viewpoint we are able to convince ourselves that the problem is not one impossible of solution. In my opinion the real problem is not one of engineering but rather one of economics. There is no doubt but what in time the generation of power by the use of oil must be superseded almost entirely by water, and yet, at the same time, I do not know of any one particular state or district on this Coast that is going to have a sufficient surplus of hydro power to justify a high-voltage bus such as for transmitting energy from one state to another or from the system of one operating company to the system of another. There is no doubt in my mind but what in a very short time we will see lines operating at voltages as high or even higher than 220,000, but there is an economic limit to the distance that power can be transmitted. There is potential water power all along this coast from Alaska to Mexico that can be developed to meet the local demands. I look for a more general standardization of voltages and frequencies on the systems of the various operating companies and a more general interconnection of the various systems. If these interconnections can be looked upon as forming a bus, I feel that in time we may reasonably expect it to become a reality, but, obviously, interconnections of greater capacity that are necessary to carry the limited amount of energy that will be interchanged, cannot be justified.

When I say that the engineering problems can be more readily overcome than the commercial or economic ones I have in mind, particularly, an arrangement that was made a year or two ago between three of the companies in the northern part of the State of California. Those systems were tied together as a matter of economy and for the utilization of surplus power available.

When that tie-in was proposed it took the engineers but a short time to work out the engineering problems involved. It happened, however, that I also had something to do with drafting the final contract. It took us about three months to get the commercial details worked out. If there had been a larger number of companies involved, the problem would have been a correspondingly more difficult one. The physical or engineering details are simple, and can be taken care of very readily. So I say the whole problem is an economic one. Can any company, or any number of companies, or the State or Federal Government, afford to generate power up in the extreme northern part of this State and transmit it down to San Diego, when there is power nearer San Diego than that in the north? I do not look for a bus of great capacity to be built in the immediate future, except such as might result from the normal interconnection of lines of companies operating in contiguous territory. Negotiations are now under way looking for a connection between the lines of the Pacific Gas & Elec. Co. and those of the San Joaquin Light & Power Corporation. This connection will be made within a comparatively short time and when completed will give a connection from Oregon to San Diego. This connection is of limited capacity but is serving every purpose and in view of the amount of power that is available, a line of greater capacity cannot be justified. Interconnections between the various systems are very desirable and will continue to be made at whatever voltages the systems may operate, but until there is an apparent necessity for a bus I think we may reasonably conclude that the necessary capital to construct one will not be forthcoming.

J. B. Fisken: Mr. Downing has raised a question as to the advisability of this bus line. Personally, I think he is right and that this will solve itself in the interconnection of different systems without any great transfer of power over long distances. I do not ever expect to see the power from Spokane utilized in San Diego.

I see no reason, however, why Spokane power should not relieve some other power that could be transmitted a shorter distance.

This problem, I think, is largely a western one, but perhaps Mr. Stevens can tell us how they do these things in Pennsylvania.

J. F. Stevens. I would have given a great deal if, during the war, such a plan as is proposed in this paper had been in operation in the East, or even if there had been such limited inter-connections as you now have.

It happened that I was connected with one of the Government agencies which had to do with the power situation in Southeastern Pennsylvania. We were confronted with the fact that there was a distinct shortage of power in our district, not only a shortage of coal but also a shortage of generating capacity in our public service power plants. This meant that every pound of coal burned had to be burned at maximum efficiency and every generating plant operated at maximum capacity in order to secure the power necessary for the operation of our war industries.

The natural solution seemed to be a combination of the resources of the power companies operating in the district by tie-in connections which would render available their full generating capacity which then could be distributed to the various industries irrespective of their location. With this in view a conference was called and I found the power companies very willing to discuss the problem but without available funds with which to practically effect a tie-in. They were operating at from 25 to 60 cycles transmitting voltages from two thousand to sixty thousand so that the cost of installing frequency changers and transformers was considerable. The Government was prepared to spend unlimited millions in other directions but, while recognizing that power was essential to production, refused to appropriate any funds for the purpose of tying together the power plants of this district. We found ourselves in the usual position of a war-time bureau with authority to commandeer a tie-in but no authority to expend money for it or to compel the companies to spend money.

As a solution we had introduced into Congress a bill appropriating two hundred millions of dollars for the purpose of constructing power stations located at the mine mouth and erecting transmission lines to serve the district from Washington to New York. This bill failed of passage and, in view of the fact that it would probably have resulted in federal ownership and operation, I am not particularly sorry, for few of us in the East look with favor upon government ownership. The net result, however, has been to stimulate interest in power generation at the mine mouth in stations equipped with large units and transmitting over a very considerable radius. You are fortunate in having available water powers which are almost totally absent in Eastern Pennsylvania but we have as partial compensation large deposits of fuel coal of excellent quality.

Some experiments have been made and the results so far achieved have been exceedingly satisfactory, particularly in localities where there were a considerable number of small public service or municipal plants, and it bids fair to be a solution of the old municipal plant problem. It happens that back in the early days of electric lighting there was quite a mania for municipal ownership of lighting plants and almost all of them have fallen into sorry condition. In order to construct the plants the town would be bonded and no provision made for a sinking fund. They are, therefore, now in the position where they have an obsolete plant with a bond issue against it and no funds for the redemption of the bonds or for the reconstruction of the plant. I have in mind one such plant in Southern Ohio where the municipality was generating at a cost of from nine to twelve cents and selling at four and there are many more like it. They had no money with which to modernize their plant and even if they had the load was not sufficient to bring their generating cost down to a point where they could sell at a profit. The solution was found by establishing a generating plant at a mine not far distant and furnishing power to this town as one of its customers at a price which enabled them to sell to their citizens at a profit by transforming their generating station into a substation.

I firmly believe the time will come when all power in my district will be generated at the mine mouth, the various plants being tied in together on a common bus and distributed to various substations. You on the Pacific Coast have the advantage of water power and I think your plan most admirable. Even the discussion has been typical for when I came in the room the length of the transmission line was given at 570 miles and has now grown to 1500 miles but even the latter figure does not seem unreasonable in view of the existing situation. It is not, to my mind, the question of whether you have local power enough to meet the demands of each locality or whether you could expand your plants so as to secure it. It is the ability to interchange power, supply deficiencies and take care of break downs which is so very valuable. There have been many cases in the East where one large generator would go out in a public service plant which meant a shortage of power and light in that district which shortage could have been cared for had it been possible to tie-in with other power companies operating in the immediate neighborhood.

It does not seem to me that the plan here proposed presents any serious difficulties. The project naturally divides itself into generation, transmission and distribution. The various generating companies sell their power to the transmission companies which in turn would sell to the distributing companies in proportion to their demand. While there are some operative problems yet to be solved the principal problem is one of finance.

Relative to the operating end, I will state that in the East we have quite generally adopted a system of parallel transmission lines. As a rule we do not run two or more circuits on one pole line but mount each circuit on separately located pole lines half a mile or more apart. This practise has tended to save us from lightning troubles and break downs in the transmission circuits.

C. W. Koiner: I have been thinking of this paper along the line of economics. It appeals to me as practically suggesting the interconnections which saved us a great deal during the period of the war. If interconnection and tieing in, whether by as long a bus as this or a shorter one, is good in times of war it certainly ought to be good in time of peace.

The diversity factor which has been touched on by Mr. Lighthipe is the keystone of the light and power business. Without diversity we all would be in some other line of business because there would be no money in the power business under present conditions and circumstances of operation. There would be the gathering up of the crumbs, as it were, of the tieing in the plants that have water during some seasons of the year and none at others, enabling storage to be made, and all the other ways that would tend to mutually help out in times of stress.

I do not think of this paper as applying to the Pacific Coast particularly, but I am thinking of what Mr. Stevens touched on in the East during the war. The fact that the great coal mines lie close to the seaboard, would have made it a wonderful thing to have had the advantage of tying-in during the war. Why haul the coal and cinders and ashes to the seaboard and then cart them out of the city after making the buildings black, creating a lot of dirt producing and inefficient steam plants?

I think this principle was recognized ten or twelve years ago, or longer, but there are a great many things that enter into the problem, one of which has been touched on here and on which, I think, the Chairman invited discussion—government ownership and regulation.

It seems to me that it cannot be accomplished without regulation. Mr. Barre says that they try to rob one another and that they must have an umpire to prevent that kind of thing. The fellow who is able to take advantage, or figure the closest, will get the better of the other, figuratively speaking.

Now, to deny that the government has not the ability to even generate this power and deliver it to the distributor, or regulate it, is to deny that we are capable of efficiently living under democracy. Our government is supposed to be a democracy; therefore, if you cannot solve a problem by and for the people of this character, then we need a boss; we need somebody who will crack the whip over us and make us perform efficient service.

I am not one that believes that we are not capable of doing these things; I know we are. Some of the obstacles are the opposition of those who are entrenched. We may have stock in companies that have the water power cites, and we want more; we can make more money than if we let the government own it and run it for the people. Consequently, we as engineers are working for that side of the house and we naturally are opposed to government ownership and even regulation. But, some of the wiser heads, such as Mr. Phillip Cabot who wrote an excellent paper recently published in the *Electrical World*, recognizes that government regulation is here to stay and that even government ownership is coming on certain things. The water power belongs to all of the people; so does the coal. Somebody happened to see it first and, therefore, the coal is in the hands of private interests, yet the economic life of the country depends upon coal. Now, all monopolies should be controlled, or rather owned by the government.

I am not a Socialist, however; I am not making apologies for I believe in what I say. I believe that the water power of this country must be conserved or passed into the hands of private interests the same as the coal mines, and without regulation and proper control, the people of the state or of the various states, might not get the benefit. We must have regulation for the purpose of interchanging power between the states. It cannot be done without it.

When it came to winning the war we had to work as a government, all pulling together for the same thing. Now, in these problems we have to do the same thing and we might just as well face the problem in that way.

J. F. Stevens: The last speaker has touched upon the question of government ownership and I want to give you one example of the efficiency of government ownership in this line—something with which the Institute was very closely associated.

Just about the time of our entrance into the war, the Institute, seeing the seriousness of the power situation and recognizing that the defense of New York from attack by sea rested in Governors Island, called General Leonard Wood, then in command of the Island, into conference on the subject. We called attention to the fact that the Island was dependent upon the operation of its own power plant, government owned and operated, without which it would be impossible to use the search lights, ammunition hoists or even aim or fire the big guns, and that the failure of the plant, either through the act of an enemy or by accident, would leave New York defenseless from attack.

General Wood acknowledged that our statement of the situation was correct and asked for our solution. We told him we had a large amount of power in and around New York which we were prepared to tie together and deliver to him on Governors Island through submarine cables for use as an auxiliary to or a substitute for the Island plant. His reply was that the government had no appropriation with which to pay for such service. We then offered to raise the money necessary to pay for the tie lines and to buy and lay the cable. Then, to our surprise, he told us there was a regulation which prevented the landing of a cable for any purpose whatsoever on such a government reservation as Governors Island and that it would require an act of Congress to enable the Army to take advantage of our generous offer. Such an act was introduced into Congress but never has been passed. From this you can see the sort of efficiency we may expect from government ownership of utilities.

J. A. Lighthipe: Mr. Koiner said he was not a Socialist. I want to tell him that we all are. I want to say that when we want anything done in a hurry we always have to do it ourselves. I want to tell Mr. Koiner that the money he has accumulated during his lifetime does not belong to him—it belongs to Uncle Sam, and when he dies it goes to the government. Of course, the government does not want to take it all; it allows him some to will to his wife and children and the part the government gets is called inheritance tax.

I suppose Mr Koiner thinks he owns the money his employers pay to him. He does not do anything of the sort. Of course he uses some of it, but the government takes the rest and it is called Income Tax.

Now, this would be a wonderful thing to develop these water resources. It ought to be done as an economic measure, but Uncle Sam has not got any money and how are we going to solve the problem? The government will do this: We will let the individual people invest their money in a Public Utility Corporation, but we are not going to let them run it and bamboozle the country. What we are going to do is to put our own directors in there and tell them how their money is going to be spent; tell them just how much money they can borrow and criticize their method of construction. We are going to tell them how great dividends they are to pay. We call that the Railroad Commission.

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That is just what we are running into. If we are going to have any public enterprise in this country, we are not going to have Uncle Sam start it because he is a little too slow. We are going to have the individual enterprising pioneers of this country develop all this power and we are going to have the Railroad Commissioners regulate our rates, etc. This will be done because, although Uncle Sam has not the money to do it, and is never ready to give it to us, he will tell us to put our money in and he will see that it is handled properly and that we get a good interest on the investment. But there will be no watered stock and no high financing. Consequently, every move we make, whether it be the purchase of something; or the installation of a new steam plant; or the installation of various water powers, we make our plans and submit them to the Railroad Commission. They figure out whether we really need this plant, whether the cost would be excessive, and then tell us to go ahead. You then have the endorsement of the Railroad Commission.

Mr. Koiner stated that this thing can not go on without some control. I say we are under control. We cannot make a move in this state without the sanction of our Railroad Commissioners and we are fortunate to have a very liberal set of men at the head of that Commission. We welcome this regulation because we are infinitely better off than some of the other cities that own their plants, because we keep our books in a certain way, as they tell us to; we have a sinking fund, which is lacking in some of our city plants and we are allowed a depreciation which is properly agreed upon. In other words the whole thing is run by the Public Utilities, only we are running it under the direction of the state of California.

Lester S. Ready: I wish to say that I am optimistic regarding the installation of a transmission bus running throughout the State of California. From what we have seen during the last three or four years of the spirit of cooperation by the utilities during the period of the war we cannot help but realize that the problem is not an impossible one.

I believe that if it had been said five or six years ago that the various utilities in Southern California would have stood together during the period of short water supply and not taken advantage of each other the remark would have been considered almost ridiculous. Yet, during the past year the supply of electric power in the southern and central part of California has been divided between the companies, the company with additional supply rendering service to its competitor at reasonable rates. Steam-electric power produced in Los Angeles and San Diego has been used to make possible the supplying of adequate service in the San Joaquin Valley, where a material shortage occurred.

The big supply of hydroelectric power of the state is in the northern section. There is not a great deal of hydroelectric power developed or to be developed, in the south. The Southern California Edison Company has built its lines from Big Creek and there is still a great deal of undeveloped power on that stream which ultimately will be transmitted south of Tehachapi. It is doubtful whether the power supply of the development when fully completed will more than half supply the demands of Southern California, and power from the northern part of the state must be transmitted southward.

Glancing at a map of the state you will see that all of the transmission lines lead southward, and at present the general flow of power is in that direction. There may never be a great deal of power produced on the Pit or Feather Rivers delivered to Los Angeles, but these rivers will relieve the demand upon the plants south of them, so that these in turn may be relieved to supply the demands further south.

From a physical standpoint there are practically only two transmission systems in the state at the present time. From Merced north to the Oregon line there is a great network at present completely interconnected. All companies south of Merced form another great system fully interconnected, and it is not only possible but probable that within six months or a year there will be an interconnection between these two main systems.

The war has done a great deal of good toward the development of interconnections between the utilities in this state and its affect will be a lasting benefit to the state as a result. The district in the south is requiring the full capacity of all the power plants and were it not for interconnection there would be a great shortage of power in the San Joaquin Valley at the present time.

The Railroad Commission is ready to do its part in fostering the interconnections which should be made. It made special efforts during the war to assist in the full conservative use of the power supply of the state and I wish to state that we are very pleased with the way things have been carried out, and the electrical engineers have much to their credit. I am not fully advised of the basis by which Mr. Barre determined the rates which were charged between the companies during the emergency but am certain there has been no profiteering between the companies during this period.

George A. Damon: One great thing that we are doing right here in connection with this proposed high-tension bus line is to establish a "sense of direction." We may not, just at present, be certain of all of the details—or how the enterprise is to be accomplished but I am sure we all feel that we are on the way. The situation is something like the starting of a foot ball game: we all know where the goal is but we are not at all sure of what each play will be to take us across the goal line. We may be quite certain, however, that if we all start in the right direction and stay together by constantly co-operating, eventually, we will accomplish this very desirable advance in the art.

I think it is quite immaterial to the project, at this time, to consider whether the cost is to be financed "individually" or "collectively"—that is by private capital or by public funds. We all love the democracy which we retained at the cost of the great war and recognize that our next problem is to strike an equitable balance between individual interests and the common welfare. My own sense of direction seems to indicate that the economic pressure of the need of this great interconnecting bus will evolve its own system or method of financing and that, as electrical engineers, we need not stumble or hesitate over this difficulty.

From our own technical standpoint the great big important problem which we have not yet discussed is the question of frequency. As we all know we have in use in this part of the country both 50 cycles and 60 cycles. If I am properly informed about 25 per cent of our load is on a 50-cycle system. Under these circumstances we have yet to determine the cost and practicability of combining these two frequencies into one system. We are constantly and rapidly increasing the load on each of the two systems of distribution but here we are seriously considering the tying together of all sources of generation or supply by means of one great interconnecting tie line. Under these circumstances what do we propose to do eventually with these two frequencies? In my opinion this is the real electrical problem to which we should just at this time, address ourselves.

L. M. Klauber: I think that the question of frequency is somewhat like some of these economic problems—it will solve itself over night. The Edison Company has always claimed that 50 cycles is the proper frequency and that operation would be impossible at any other. However, last week when the engineering staff was not watching, the system speeded up to something like 59½ cycles and operated that way for about two minutes; they could as easily do it for two years.

L. J. Corbett: As has been stated, the electrical features involved in the line projected have not been discussed very much. It occurred to me after looking over this paper and listening to the remarks thus far, that some of these points have not been taken up because they are not of extreme importance from an electrical viewpoint. When you have a system as long as the one contemplated, with plants connected in at short distances apart, the actual effect on the system, at San Diego, for instance, due to connecting in the plant at Spokane, would not be of great importance, because the load at San Diego would be taken care of by the plants in the closer vicinity. Only in certain cases where for some reason an entire district would be cut out, would we have even the problem of a long distance transmission line arising. It may be recalled that in the case of a long line there are differences in wave form which result from the transmission line characteristics, but it is my opinion that these would be almost eliminated in such a system as outlined here.

As has been stated by one of the speakers, a 220,000-volt line is not necessary, as an interconnection of the ordinary existing systems, whether 30,000, 50,000, 60,000, 110,000 or 150,000 volts, can be made. The lines can be interconnected just as they are by using proper transformer substations. As for the different frequencies encountered, that feature would present a minor difficulty; it would be an economic question solely, as it would call merely for the additional cost of a frequency changer when connection is made to a plant of a different frequency. It seems to me that everything required can be done with a lower voltage line and with equipment already in use or available.

J. H. Anderton: All previous discussions on this paper have given practically no weight to the engineering problems involved in the design and operation of the proposed line. In fact, it has been stated here that the engineering problems are comparatively simple. This may be quite correct if the comparison be made with the financial side of the problem. I do not believe, however, that we should assume the engineering features as a negligible factor. For instance, I note from the figures given on page 1240 of the paper that the distance between Pit River and Los Angeles is 570 miles; to San Diego it would be approximately 700 miles. These distances are, of course, more or less approximate and while they approach the quarter-wave length of 60-cycle propagation, there will be no danger from this standpoint alone since a practical line has resistance and other constants which prevent the theoretical rise due to the quarter-wave length distance. However, with lines of any length and given constants, a point of highest receiver voltage and maximum charging kv-a. can be found. The converse is also true that with a given length certain constants will give the highest receiver voltage and maximum charging kv-a. With a line of the length under consideration, the use of normal line constants might reasonably give a receiver voltage of from 250 to 400 per cent of the normal generator voltage, assuming that the line were charged from one end. This latter assumption is, of course, impossible due to the kv-a. capacity which will be required to charge the line but indicates in a general way the necessity for a careful determination of all the constants. The complexity of the problem is very apparent from the fact that the line may be operated as a whole or in part and could be charged from various sources.

H. H. Cox: The idea in preparing this paper was that it is now time for some large development in California. California can use 100,000 or 200,000-kw. water power now and relieve the steam plants and shut off the use of fuel. You all know the price of fuel. Fuel means something more to us. In a pound of oil there is a whole lot of other things more valuable than the thermal unit. Therefore, the use of fuel is going to be turned in another direction to make these chemical products, giving us the use of the by-products.

Our confidence in the operation possibilities of the high-voltage line is absolute. We don't think there will be any difficulty at all. In answer to Mr. Anderton, regarding the distance of 750 miles, more or less, this system would never be operated as a transmission from Pit River to Los Angeles. You would have to get the individual sections together. That is, one line from Pit River to San Francisco and another from Big Creek to Los Angeles; then the two would be gotten together and power interchanged. It would be a physical impossibility to build up a line of this distance with generators because the quarter-wave length would act as a short circuit on the line.

In regard to the operation of the line, we state that complete parallel operation of all lines must be adhered to in the proposed system. Satisfactory protective relay systems for dropping defective sections with little disturbances have been developed for present parallel transmission lines and there appear to be no obstacles to extending these to the higher voltages. Such protective relay systems have been in use on 60-kv. systems and are operating satisfactorily.

This transmission bus, as I have said, will probably never transmit power from north to south, but it will be a means of caring for emergencies and transmitting surplus energy to take care of the diversity of the different loads.

The present interconnections have a very limited capacity, as you all know. One company may now have 10,000 kw. to spare and puts in an interconnection of that size. Tomorrow an emergency arises and the other company needs 20,000 kw. The interconnection would not carry it. In the beginning of our paper we assumed the interconnection of all the California power companies on an equitable cost basis. It is now time for a large development of power to take place in California.

G. E. Armstrong: I just overheard, from Mr. Downing, the remark that we will have a job to justify this line. It doesn't look that way to us. In the first place, Mr. Downing said that California had lots of power, I suppose ten years ago that the power we have now looked like quite a bit. When I first came with the Southern California Edison Company we had a peak of 30,000 or 40,000 kw. and we are now running over 175,000 kw. The increase is going on a percentage basis. When on that basis, and the percentage increasing every year, we are going to pick up load pretty fast.

Mr. Downing thinks he won't live to see this line an accomplished fact. I would like to call his attention to one point. For instance, the Southern California Edison Company, if this line isn't put in, is going to need an additional 150,000-volt line to Big Creek very shortly. The two lines we have now are sufficient for about 60,000 kv-a. each. The two Big Creek plants now have a capacity of 60,000 kv-a. The installation of another plant there will bring it up to 100,000 next year and we will need more for the next year. If the voltage is raised to 220,000 shortly, the capacity of each line would be about 125,000. This was the idea that we had in mind as a need of bringing this proposed line in operation.

The San Joaquin Light and Power Co. now needs a transmission line running north and south and having a voltage of at least 110,000; that is what they are figuring on. You can easily see that it would be a waste of money for them to build a line of that capacity when the Big Creek line at 220,000 volts could take care of both the Edison Company and the San Joaquin Light and Power Company.

There is one other point. The longer this proposition is put off, the harder it will be to accomplish it. If it is necessary for the Southern California Edison Company to build another 150,000-volt line to Big Creek, it would not be very favorable to changing to 60 cycles. This same thing would be true of any of the companies in the northern part of the state who find it necessary to extend their lines in the near future. But, if this bus line is considered now and worked out in a reasonable way, I don't see why the next lines could not be built having in view the interconnection of the systems, so they could be operated at 220,000 volts when the time comes.

Presented at a joint meeting of the American Institute of Electrical Engineers and the Institute of Radio Engineers, New York, October 1, 1919.

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TRANSOCEANIC RADIO COMMUNICATION

BY E. F. W. ALEXANDERSON

Abstract of Paper

The paper defines the state of the art of today which is the result of developments during the war. Transatlantic radio communication is at present maintained by five first class stations, two in America and three in Europe. These stations operate at wave-lengths between 12,500 and 17,000 meters. The range of wave-lengths suited for such traffic is rather limited, the desirable wave length being included between the limits of 10,000 to 20,000 meters. New developments indicate three methods for increasing the radio traffic without interference between the different messages. These methods are increase of the transmitting speed, closer spacing of wave lengths and directive reception. If these technical possibilities are intelligently used, the author predicts that radio communication will be equal to all demands that will be placed upon it.

The second part of the paper describes the radio transmitting system for the development of which the author is responsible. This system is represented by the naval radio station, New Brunswick, N. J. and comprises new means for generating modulating, and radiating the continuous wave energy. The generator is the high-frequency alternator with which the author's name has become associated. The modulating system is the "magnetic amplifier" which is described in a paper by the author before the Institute of Radio Engineers. The "multiple antenna" system of radiation is described in this paper for the first time. The general theory and figures for the increased radiation efficiency are given. The author also predicts that the multiple antenna will make possible directive radiation on a large scale.

IT has already become generally known that a new highway for world traffic has been opened up through the development of transatlantic radio communication. It is now a matter of history that radio was largely used for communication between the United States and armies in Europe and that the Great War was brought to a close by negotiations conducted by radio which led to the armistice. Now, we are ready for an international commerce of unprecedented scope, but lack adequate means for communication.

The recent achievements of radio technique have become common knowledge, and the world has now turned to this new method of communication clamouring that it step in and save the day. This is a condition which places a serious responsibility upon radio engineers. Fortunately, the technique has emerged from the cloud of mystery that used to surround it, and we are in position to treat the problem cooly and scientifically like any other problem in electrical engineering. However, it must not be inferred that the task is any easy one, if the radio technique is to fulfill all the hopes which are placed on it.

It has been demonstrated during the war period that transoceanic communication has become thoroughly reliable, every day in the year, and practically every hour in the day. Thus far, we can say, that the problem is solved. But a second question will be raised: What volume of traffic can be carried by the means at our disposal at the present time, and what is the relation of this radio traffic to the world traffic of to-day and to the world traffic of the future? The facts of the case are briefly the following:

Experience has shown that the wave lengths which are most suited for transoceanic communication lie between 12,000 and 17,000 meters. "This space in the ether" has already been taken up by five first class transmitting stations which, during the war period and up to the present time, have been in continuous service for transatlantic communication. Of these stations, two are in the United States, one in England, one in France and one in Germany. By extending the range of wave lengths down to 10,000 and up to 20,000 meters, and following the same system of intervals there would be room for about seven more stations or a total of twelve first class transmitting stations.

A first class station has such radiating power that its messages can be received in all parts of the world. This is one of the advantages of radio communication; but it implies that if such a station is to be used to full advantage it must have a "right of way" for its wave length over the whole world. Thus if we look at the matter pessimistically, without allowance for the improvements that further engineering developments are likely to bring, it would look as if the capacity of the world for first class radio stations would be about twelve. The rate of transmission at the present time from these stations is about twenty words a minute and it would thus be easy to figure out that the capacity of radio for handling any considerable portion of world communication would be totally inadequate.

The other side of the picture, which the radio engineer of the future must study carefully and closely, should indicate the technical possibilities for improving the situation. The tendency of present day developments points to the following means for expansion of radio traffic:

1. Increase in speed of transmission.
2. Improved selectivity based on the direction of the wave.
3. Improved selectivity making possible closer spacing of wave lengths.

As a basis of discussing the general situation it may be here stated as simple facts:

1. That signals have been transmitted and received at considerably more than 100 words a minute.
2. That signals have been received from Europe while an American high power station within comparatively short distance was radiating on the same wave length.
3. That is has proved practical to separate radio signals differing in wave length considerably less than 1 per cent.

Based on these facts, it is probable that the transmitting speed in the future will average 100 words a minute instead of 20 words a minute.

That the selectivity for direction of the waves will multiply by five the number of stations that may be operated on one wave length; and that the selectivity with reference to wave length will be improved so that the wave lengths of messages will be within 1 per cent of each other, instead of 7 per cent which is the spacing of the stations at present.

These prospects, taken in combination, give us an optimistic picture in which the possible capacity for transoceanic radio traffic of the world is 175 times as great as it is with the practise of to-day.

To claim that the traffic capacity could immediately be increased nearly 200 times, would be an exaggeration, because the different improvements which have been made may partly conflict with the execution of each other if they are to be used simultaneously. This optimistic picture is therefore to be regarded as a goal—perhaps never to be reached, but it points the way for almost unlimited possibilities for progress by continued engineering efforts.

In order to avail ourselves of these three improvements simultaneously, the transmitted wave must be a continuous wave which does not "spill over" with harmonics, decrements or variations, into the range of wave length assigned for other communication. When wave lengths are spaced 1 per cent apart, each wave must be sufficiently pure so as to have no objectionable components outside the limits of one-half of one per cent. While rules to this effect should be rigidly enforced it must be appreciated that there are certain fundamental limitations.

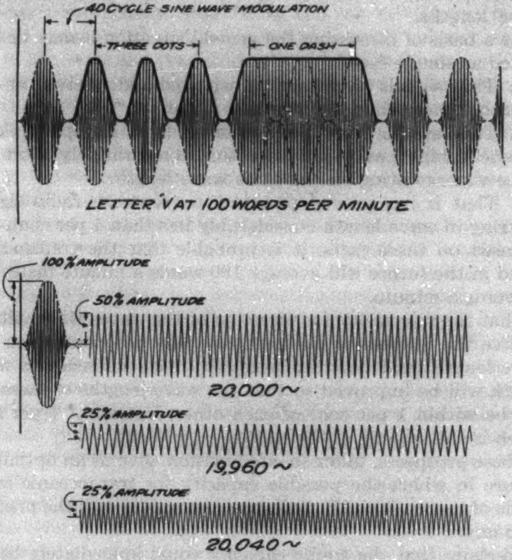

40 CYCLE SINE WAVE MODULATION
THREE DOTS
ONE DASH
LETTER "V" AT 100 WORDS PER MINUTE
100% AMPLITUDE
50% AMPLITUDE
20,000 ~
25% AMPLITUDE
19,960 ~
25% AMPLITUDE
20,040 ~

Fig. 1—Method of Resolving Sine Wave Modulation into Three Continuous Waves

High Speed Signaling

Modern transoceanic radio signaling is conducted by means of continuous waves. It must be appreciated, however, that signaling by an absolutely continuous wave would be impossible, because the making of dots and dashes introduces increments and decrements in the radiation. It will be shown that a repetition of increments and decrements can be resolved into a group of closely adjacent continuous waves. This agrees with the well known fact that the tuning of a wave with a decrement is known as a broad tuning. To illustrate this point we may take as a basis a signal at 100 words a minute (5 letters per word). If it is assumed that the increments and decrements in making the dots are not sharp interruptions but a continued variation by sine wave curves as indicated in Fig. 1, it is found by an analysis that such a wave may be resolved into a group of continuous wave components within the limits of 40 cycles above and below the average. This is the theoretical minimum width of the band of wave lengths which are necessary to transmit 100 words per minute. If the dots are defined by sharper interruptions the wave becomes still "broader" and it would not be unreasonable to say that the minimum practical band of wave lengths for 100 words per minute is 100 cycles on each side of the fundamental. This would make possible a spacing of the waves 1 per cent apart when using a wave length of approximately 15,000 meters. Messages at a higher rate of speed will occupy a correspondingly wider "space in ether."

As a conclusion from this analysis it may thus be said that an increase of the speed to 100 words a minute and increase of messages to a spacing 1 per cent apart may be accomplished simultaneously provided that waves are used of such a character and modulation that they contain no radiation except the one needed to accomplish the intended purpose.

DIRECTIVE RECEPTION

The second means for increasing radio traffic consists in improving the selectivity by taking advantage of the direction of the waves. The author's paper printed in the *Proceedings* of the Institute of Radio Engineers, August, 1919, described a receiver referred to as the "Barrage Receiver" which was used to demonstrate directive reception. There are several other types of receiving devices, developed by the United States Navy and other investigators, which accomplish substantially the same purpose. The broad principle underlying all these directional receiving devices is the one discovered by Bellini and Tosi and generally referred to under their names.

In the U. S. Navy's tests of the "Barrage Receiver", it was proven that it is possible to carry on simultaneous two-way communication on exactly the same wave length. If this method of directive reception is carried out consistently in a world system of communication, it may be assumed that transmitting stations operating on the same wave length may be located approximately as shown in the map in Fig. 2—one in Europe, one on the American east coast, one on the American west coast, one in the Far East and one in South America. The American receiving station for European signals in such a system should be located east of the American Atlantic Transmitting Station, and in line with the Pacific Transmitting Station. Thus messages from the two American transmitting stations could both be simultaneously neutralized in the American Receiving station by a "Barrage Receiver," while signals on the same wave length are received from Europe. Interference from the South American station may be neutralized by the use of a double barrage system, while the station in the Far East, though it may not be exactly in line with the two others would be sufficiently near the general direction so that, considering the great distance, it would not cause interference.

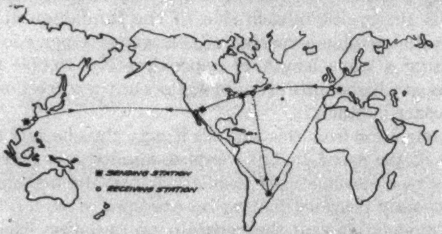
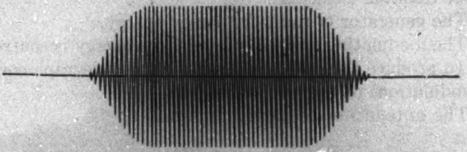


Fig. 2—Proposed Simultaneous Radio Transmissions on the Same Wave Length

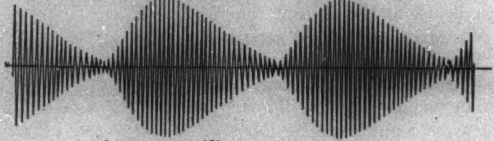
If this communication system is to be duplicated on a number of other wave lengths, the practical conclusion follows that the transmitting stations as well as receiving stations for each district should be grouped in centers, and these centers located relatively so as to make the directive neutralization as effective as possible. The neutralization of several transmitting stations simultaneously may not always work out as first designed. It has been shown by investigations of the Navy Department that the radio waves do not always follow straight lines and not always the same path. However, discrepancies from such origin may again be overcome by further extending the principle of neutralizing waves from several directions simultaneously.

CLOSER SPACING OF WAVE LENGTHS

The method of increasing traffic capacity by closer spacing of wave lengths has great possibilities. It has been shown that the selectivity with reference to wave length can be greatly increased by several successive tunings in either the radio, the audio or some intermediate circuit. It is thus entirely practical to receive either by ear or by photographic records, signals which are considerably less than 100 cycles apart. The theoretical limits for such selectivity in connection with high speed transmission are defined below. As illustrated in Fig. 1, a high speed message is not a single continuous wave, but a band of wave lengths—100 words per minute occupying a space of about 200 cycles. Therefore the same degree of selectivity is not to be expected with a high-speed signal if the interfering radiation is of considerable intensity.



WAVE TRAIN SHOWING ACTUAL NUMBER OF RADIO CYCLES IN A DOT OF THE TELEGRAPH CODE TRANSMITTED AT THE RATE OF 500 WORDS PER MINUTE AT 15,000 METERS WAVE LENGTH.



A SPARK WAVE AT 400 CYCLES SPARK FREQUENCY

FIG. 3

Speaking in terms more familiar to practical radio operators it may be said that high speed telegraph signals assume to some degree the objectionable characteristics of a spark signal. As an illustration of this, Fig. 3 shows the relative decrements in a continuous high-speed telegraph wave and a spark wave, showing that the increments and decrements of the continuous wave signal at 500 words per minute are about equal to a spark wave of 400 sparks per second. These illustrations apply to wave lengths of about 15,000 meters.

THE RADIO TRANSMITTING SYSTEM

Several types of radio transmitting systems are at present in use with a high degree of success. The descriptive matter in this paper will, however, be confined to the system for which the author is responsible as represented by the Naval Radio Station at New Brunswick, N. J.

Generally speaking, any radio transmitting system consists of three essential elements:

1. The generator of radio frequency energy.
2. The modulating system, whereby the energy is controlled so as to produce the dots and dashes of the telegraph code or the modulations of the human voice.
3. The antenna or radiating system.

GENERATING SYSTEM

There are four types of generating systems of high-frequency energy in use at the present time.

1. The spark or impulse generator.
2. The Poulsen-Arc generator.
3. The high-frequency alternator.
4. The vacuum tube oscillator.

The system which will be described, is of the type employing a high-frequency alternator. The installation of New Brunswick contains a 50-kw. alternator shown in Fig. 4 which was operated for sometime for experimental purposes with radio telephony at a wave length of 8000 meters, and later in transatlantic telegraph service at 9300 meters.

A larger equipment which has been in continuous service for the last year consists of a 200 kw. alternator shown in Fig. 5, 6 and 7. Fig. 5 shows the machine partly assembled, the rotor consisting of a solid steel disk. The spaces between the polar projections are filled with non-magnetic material so as to present a smooth surface and thereby reduce air friction to a minimum. The disk runs between the two laminated armatures which are cooled by water pipes as shown in the photograph. The armature winding which consists of wire wound back and forth in straight open slots, is divided in 64

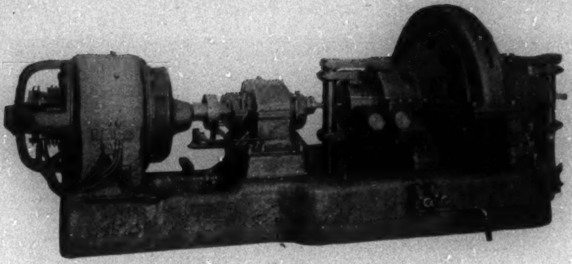


FIG. 4—50-KW. 50,000-CYCLE ALTERNATOR

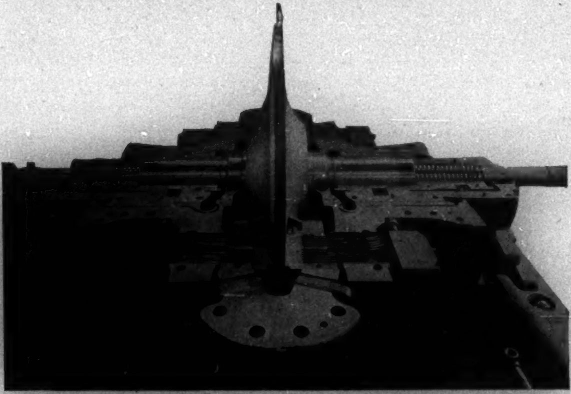
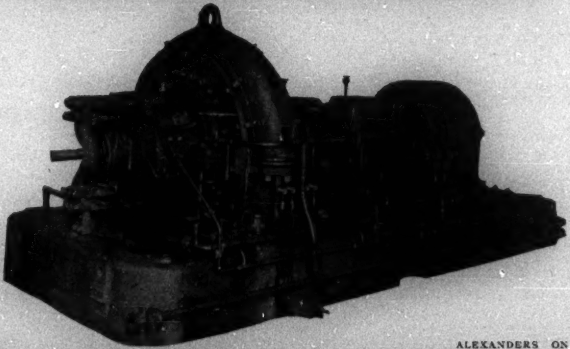


FIG. 5—200-KW. HIGH-FREQUENCY ALTERNATOR



ALEXANDERSON

FIG. 6—200-KW. HIGH-FREQUENCY ALTERNATOR

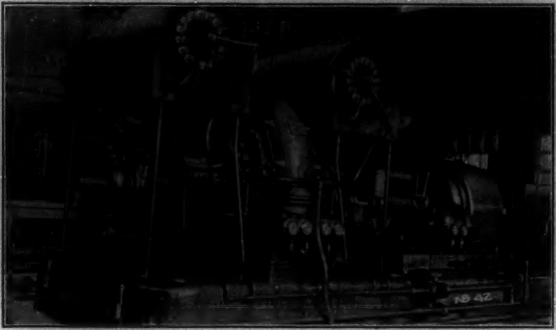


FIG. 7—200-KW. ALTERNATOR SET AS INSTALLED IN THE NAVAL RADIO STATION, NEW BRUNSWICK, NEW JERSEY

[ALEXANDERSON]

FIG. 9—COMBINED CURRENT AND VOLTAGE REGULATORS FOR HIGH-FREQUENCY ALTERNATOR CONTROL

sections, each section generating about 100 volts and 30 amperes. The current generated by these 64 windings is collected in the air core transformer mounted on the top of the machine, Fig. 7. This transformer has 64 independent primary windings corresponding to the armature windings. The single secondary winding of the transformer delivers the complete output of the alternator. This collecting transformer is thus to be considered as an integral part of the generating unit and for all purposes of calculation the characteristics of the generating unit, such as electromotive force and current, are given as delivered from this secondary winding. At full output the alternator delivers 100 amperes at an electromotive force of 2000 volts. It can thus be seen that the alternator is designed for a load resistance of 20 ohms. However, the same machine might be adapted for any other load resistance by selecting a different number of turns in the secondary of the collecting transformer. The reason why this particular machine is designed for a high voltage and low current will be given later in discussion of the new type of antenna with which it is used.

The 200-kw. alternator when operated at the New Brunswick wave length of 13,600 meters runs at a speed of 2170 rev. per min. It is driven by an induction motor through a gear of a ratio of 2.97:1 when the high-frequency alternator is used as a source of radiation, the wave length is determined directly by the rotative speed of the machine. Thus, obviously, it is important that the rotative speed should be as nearly absolutely constant as it is possible to make it. An important accessory of the alternator set is therefore the speed regulator. The 50-kw. alternator set shown in Fig. 3 is driven by a direct-current motor whereas, the 200-kw. set is driven by an induction motor of the slip ring type. The 50-kw. set was equipped with a direct-current motor because the problem of speed regulation of that type of motor is somewhat easier. Induction motors were, however, decided upon for the later types because alternating-current power is more easily available in most localities.

SPEED REGULATOR

The speed regulator consists of a speed determining element and a power controlling element. The speed determining element is a resonant high-frequency circuit fed by one of the 64 alternator windings which is set aside for that purpose. The oscillating energy of this high-frequency circuit is associated by magnetic couplings with a rectifying circuit in which the high-frequency energy is changed into direct current. This rectified current in turn, actuates the controlling magnet of a vibrating regulator of the type that is generally used for voltage regulation in power stations. When the driving motor is a direct-current motor it is easy to see how this vibrating regulator may be made to control the speed by regulating the voltage of the power supply to the motor. In order to accomplish the same object with an induction motor some new features have been introduced.

An ordinary induction motor is operated at constant potential. When the motor runs light it draws from the line a magnetizing current which is almost wattless. Thus it operates at a low power factor. When the motor is fully loaded it draws power at a high power factor—a motor of the type used having a power factor of 90 per cent.

When the New Brunswick station was adjusted for operation it was found that a wave length was desired which required the induction motor to work at 19 per cent slip. The rheostat in the secondary of the motor could easily be adjusted so that the motor would deliver the desired power with full load at 19 per cent slip. However, inasmuch, as the output of the alternator varies continually with the making of dots and dashes of the telegraph code, the motor is alternately loaded and not loaded. The tendency would therefore be for the motor to speed up during the intervals. If the potential of the power supply to an induction motor is varied, the motor torque varies by the square of the voltage. It is furthermore easy to show, by the theory of the induction motor, that if a motor consumes power at 90 per cent power factor at full load, and the load is reduced to ¼ by the reduction of voltage to ½, the power factor will remain 90 per cent. In fact it will always consume power at 90 per cent power factor regardless of its load if the voltage supply is adjusted accordingly, so long as the secondary resistance remains constant and the speed remains constant.

Thus it may be said that the standard method of operating an induction motor is at constant potential and variable power factor. The method of operating the driving motor of the radio set may on the other hand be characterized as variable potential and constant power factor.

The problem which thus presented itself was to find means for varying the applied voltage in accordance with action of the speed-determining element, and this has been done in the following way:

Between the motor and the power supply is introduced a choke coil with iron core, the permeability of which can be varied by saturation. The change in permeability is produced by a direct current which is controlled by a vibrating regulator. When the motor carries full load the iron core is saturated so that the choking effect is practically zero. At fractional load the choking effect is automatically adjusted by the regulator so that the motor delivers at all times the power required to hold constant speed. The motor itself operates at all times at its maximum efficiency and power factor, but the power factor of the current drawn from the line varies with the load. Thus when the motor operates at ¼ load the power factor of the line is 45 per cent while the power factor of the motor is 90 per cent. The circuits of the regulator are shown in Fig. 8 and the vibrator regulator in Fig. 9.

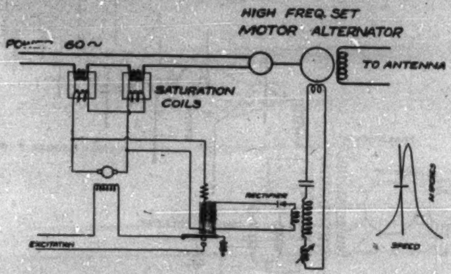


FIG. 8—SPEED REGULATOR—ALTERNATING CURRENT DRIVE

MODULATING SYSTEM

The method of controlling high-frequency energy involves an apparatus which has become known as the "magnetic amplifier." This device is described in a paper by the author in the Institute of Radio Engineers, January, 1916, and therefore needs to be referred to only briefly. The magnetic amplifier is a device which is physically of the nature of an oil cooled transformer. The iron core which is made of fine laminations, is designed in such a way that the magnetic permeability of the iron core can be varied by magnetic saturation. By a special combination of tuned circuits as shown in Fig. 10 it has become possible to separate the controlling current from the high-frequency current so that a comparatively weak current of a few amperes controls as many hundreds of amperes in the antenna. When the transmitting station is used for telegraphy the magnetic amplifier is controlled by the telegraph relays which are a part of the wire telegraph system. During the war service, the telegraph key was operated in the centralized operating room of the Naval Communication Department in Washington. When the station is used for telephony the controlling current is an amplified telephone current.

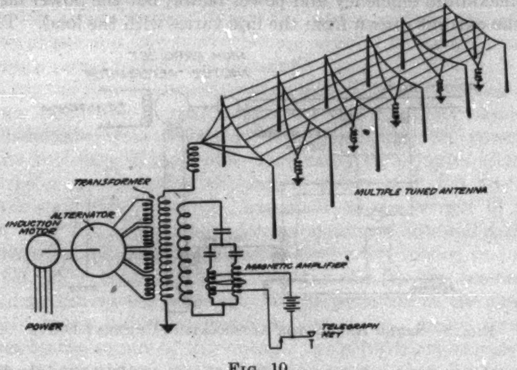
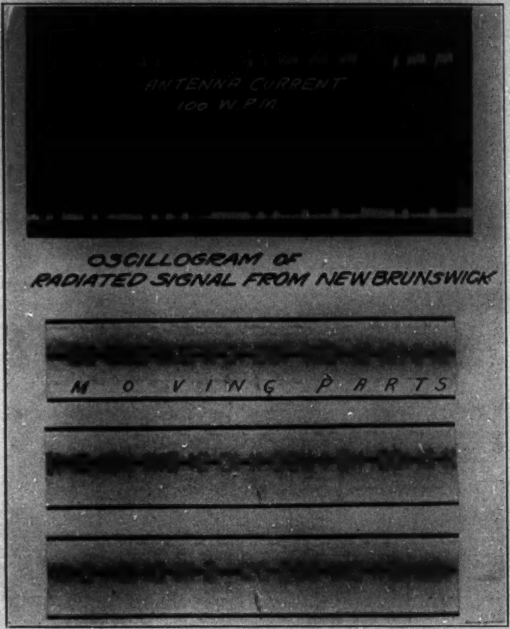


Fig. 10

While the magnetic amplifier has proven to be a very satisfactory and reliable controlling device for ordinary telegraphy its particular advantages are most prominent in high-speed telegraphic transmission and telephonic transmission, on account of its instantaneous magnetic action without any arcing contacts. Fig. 11 shows an oscillogram of radiation at 100 words per minute and a copy of a photographic record of reception at the same speed. Fig. 12 shows the telephone modulation of the antenna current when Secretary Daniels was speaking over the telephone line from Washington, controlling the output from the New Brunswick station, thereby transmitting his voice to President Wilson's ship at sea.



[ALEXANDERSON]

FIG. 11—PHOTOGRAPH OF RECEIVED SIGNAL FROM NEW BRUNSWICK AT 100 WORDS PER MINUTE

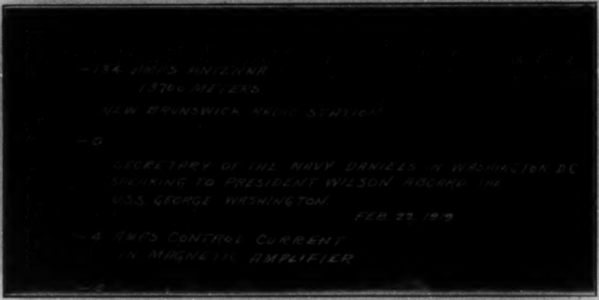


FIG. 12

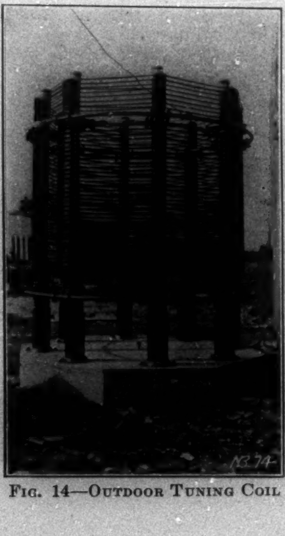


FIG. 14—OUTDOOR TUNING COIL

[ALEXANDERSON]

FIG. 15—PERSPECTIVE OF ANTENNA

THE MULTIPLE ANTENNA

The antenna of the New Brunswick station represents a new departure in the method of radiation. The old antenna structure was originally one of the horizontal Marconi antenna, 5000 ft. (1500 m.) long, 600 ft. (180 m.) wide supported on towers 400 ft. (120 m.) high. The original antenna had a resistance of 3.8 ohms.

The antenna as operated now has a resistance of 0.5 ohms distributed approximately as follows:

Radiation resistance	0.07 ohms
Tuning coils and insulation	0.10
Ground resistance	0.33
Total multiple resistance	0.5 ohms

The reduction in total resistance of the antenna is due to the reduction of the ground resistance. While the old antenna had one tuning coil located in one end, the new antenna has six tuning coils as shown in Fig. 10 and Fig. 13.

THEORY OF THE MULTIPLE ANTENNA

The theory of the multiple antenna can be explained in several ways. Without going into details, an explanation will be presented giving the point of view which has proved most useful for general discussion. For this purpose the multiple antenna may be considered as an aggregate of several antennas of the ordinary vertical type each having its own tuning coil. When regarded in this way, the multiple antenna in New Brunswick is equivalent to six independent radiators placed 1000 ft. apart. The ground resistance of each of the radiators is 2 ohms, the coil resistance 0.6 ohms and the radiation 0.07 ohms making a total resistance of 2.67 ohms. Each of these radiators has an aerial 1000 ft. long and 600 ft. wide, counted at an average height of 300 ft. A total resistance of about 2.67 ohms is, as a matter of fact, the resistance to be expected from an antenna of such dimensions and ordinary design. If one individual antenna such as described is operated with a radiation of 100 amperes, the energy consumption of the antenna would be 26.7 kw., and the radiation efficiency would be 0.07 ohm divided by 2.67 ohm which is 2.6 per cent. If it is desired to increase the radiation from 100 amperes to 600 amperes the energy consumption would be 36 times as great that is, 960 kw. There is, however, another way to produce a radiation equivalent to 600 amperes. If six separate antennas of the dimensions described were built and each operated with a radiation of 100 amperes, each antenna would emit a system of waves proportional to 100 amperes radiation. If all the waves emitted by the six antennas were in phase, the amplitude of the combined wave would be six times as great as the wave emitted by one antenna. Thus the amplitude of the combined wave would be equal to the amplitude of the wave emitted by one antenna when operated at 600 amperes. The energy consumption required for operating one antenna was 26.7 kw., thus it might off-hand be concluded that the energy consumption required for operating the six antennas simultaneously would be 160 kw. This conclusion is not exactly correct because there is an interaction between the radiating effects of the different antennas resulting in a somewhat higher energy consumption. This might be expressed as follows:

The radiation resistance which is 0.07 ohm is common for all six antennas, whereas the ground and coil resistance of 2.6 ohm belongs to the different antennas, individually. Thus the combined circuit of the multiple antenna can be represented by the common radiating resistance of 0.07 ohm connected in series with a group of six multiple resistance of 2.6 ohms each, so that the total current of 600 amperes flows through the 0.07 ohm radiation resistances which represent the ground and coil resistance of the individual antennas. Hence the total energy consumption of the combined antenna is found to be 180 kw. out of which 155 kw. is ground coil and insulation loss and 25 kw. radiation. The radiation efficiency of the multiple antenna is thus 14 per cent against the radiation efficiency of 2.6 per cent for the individual antenna. The above calculation is made for the present operating wave of 13,600 meters. If on the other hand the same calculation is made for the same length of 8000 meters, which has been found more efficient for telephonic transmission, it is found that the radiation efficiency is 30 per cent, the energy being distributed as follows:

Radiation resistance	0.2 ohms
Coil and insulation	0.07
Ground resistance	0.33
Total multiple resistance	0.6 ohms

For the sake of convenience in dealing with the radiation from a multiple antenna it has become the practise to indicate the total radiation from the multiple antenna as the sum of the currents measured in the six ground connections. The equivalent multiple resistance of the antenna is then determined by the equation I^2R = energy consumption, where I is the sum of the currents in the ground connection.

It has been assumed above that each of the individual antennas is operated so that the waves sent out by the same are not only of the same frequency, but exactly of the same phase. It remains to be shown how this is accomplished. Fig. 13 shows the relation between the antenna and the multiple tuning coils and the alternator. As shown by the diagram, the alternator is connected in series with one of the six tuning coils. Arrows on the diagram indicate further how the oscillating currents and the energy currents are distributed. Six independent oscillating circuits are formed, the current in each ground connection corresponding to the charging current of the corresponding section of the aerial. If the antenna had been shock excited so that it continued to oscillate in the way indicated, no current would flow in the horizontal wires between the different sections of the antenna. However, in order to maintain continued oscillations a flow of energy must take place from the alternator which is connected to one of

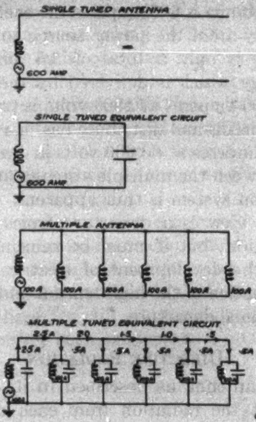

SINGLE TUNED ANTENNA
600 AMP
SINGLE TUNED EQUIVALENT CIRCUIT
600 AMP
MULTIPLE ANTENNA
100A
100A
100A
100A
100A
100A
MULTIPLE TUNED EQUIVALENT CIRCUIT
2.5A
2.0
1.5
1.0
.5
2.5A
.5A
.5A
.5A
.5A
.5A

FIG. 13

ol. 38
919

the six tuning coils. When the antenna is operated with 100 amperes in each of the ground connections, the energy consumption as shown above is 180 kw., that is, each oscillating circuit consumes 30 kw.

What actually takes place is the following: The tuning coil to which the alternator is connected transforms the energy of the alternator into a power supply at a potential of 60,000 volts, and each of the oscillating circuits draws energy from this power supply at that voltage. Thus the energy currents consumed by each oscillating circuit is only 0.5 amperes. It can thus be seen that while the total oscillating current of the antenna is 600 amperes the energy current which flows horizontally from the power source to the multiple oscillating circuits is only a total of 2.5 amperes. In other words, the energy which is delivered by the first tuning coil in the form of 100 amperes at 1800 volts is transformed by the first oscillating circuit and distributed as in a transmission line from which 0.5 amperes at 60,000 volts is drawn in five places. The analogy between the multiple antenna and a high-tension power distribution system is thus apparent.

This point of view is a departure from the conventional theory of radiation; but it must be remembered that there was a time in the development of electric power technique when the introduction of the high-tension multiple distribution system was a radical departure.

DIRECTIVE RADIATION

The multiple antenna as described in its simplest form is adjusted so that the radiation from each of the individual oscillators is in phase. If, however, the antenna dimensions are so chosen that the phase displacement of the traveling wave between the different radiators becomes an essential factor, it is possible to obtain directive radiation. The radiated wave then will not be a simple circular wave but an interference pattern which may be treated like the corresponding phenomena in light and sound waves. Furthermore the phase displacement of the oscillations of the individual radiators may be regulated by tuning. Thus a variety of interference patterns may be created and analysis of these possibilities shows that an efficient unidirectional radiation by such methods should be possible.

Methods for unidirectional radiation have been established through the well known work of Bellini and Tosi. Through the courtesy of Mr. Bouthillon of the French Post Office, results of tests made in France have been placed at the disposal of the author, which show conclusively directive radiation by the Bellini and Tosi antenna.

With the dimensions of antennas used up to the present time, efficient directive radiation has not been practical. It has, however, been proved by various tests that the system of a central power source and a distribution system of energy to a large number of multiple radiators places at our disposal means for constructing radiators of dimensions of one wave length or more. The New Brunswick antenna (1500 meters long) has a minimum wave length of 8000 meters as a single antenna, whereas it can be operated as a multiple antenna at 2000 meters wave length. A detailed analysis of the possibilities of multiple radiation would fall outside the scope of this paper but the author is in position to predict with confidence that directive radiation on a large scale will not only prove practical but that it will be the most efficient method of radiation.

To add directive radiation to the proposed program for increasing the capacity of radio traffic would perhaps be premature until it has been demonstrated on a large scale. However, it deserves mention in order to show that new principles which may be utilized for still greater expansion of the radio technique have not yet been exhausted.

Presented at a joint meeting of the American Institute of Electrical Engineers, and the Institute of Radio Engineers, New York, October 1, 1919.

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TELEPHONE REPEATERS

BY BANCROFT GHERARDI AND FRANK B. JEWETT

ABSTRACT OF PAPER

In this paper the authors have endeavored to set forth briefly but clearly the history of the research and development work which has led up to the final production of successful telephone repeaters. The various forms of amplifiers which have been suggested are described and their possibilities and limitations pointed out. The essential properties of repeater networks together with the necessary line conditions for successful repeater operation are described and illustrated. Tandem operation of repeaters is discussed as is also the use of repeaters in four-wire circuits. Illustrations are given showing a few of the more important repeater installations now in regular commercial service in the United States.

INTRODUCTION

DURING the past ten years developments in the design and construction of telephone amplifiers, or telephone repeaters as they are more universally called, and in the art of their application to all forms of telephone circuits have progressed to such an extent as to justify the presentation to the American Institute of Electrical Engineers of a comprehensive paper which shall cover not only the earlier efforts but the present state of development.

As indicated below, the idea of one or more repeaters inserted in a line for the purpose of reinforcing from some local source of energy the weakened current from the sending station is older than the telephone itself. In telephony innumerable attempts by a large number of investigators have gone on continuously almost from the inception of the telephone in an effort to extend the range of telephonic speech through the utilization of energy applied to the telephone line at a point or points between the transmitting and receiving stations. While the net result of all this work is, of course, the state of the art as we now have it, a survey of the earlier developments in the light of present-day knowledge discloses in striking fashion the fact that, unknown to the investigators, the early attempts were destined not to succeed where success was measured by the development of a practical device which would give satisfactory results on a regular two-way telephone circuit.

We know now that the final successful development and application of the telephone repeater had to wait not only for the slow accumulation of comprehensive knowledge concerning all the factors which govern the successful transmission of speech electrically over wires and of the intricate relation of circuits and apparatus to produce desired results with the maze of frequencies involved in speech, but also on developments in physical science which were in themselves quite foreign to the specific realm of telephony. For these reasons the work of the earlier investigators, ingenious though it was, was for years foredoomed to failure by factors over which they had no control and of which, in many cases, they had no knowledge even. In the light of what we now know, much of this early work appears almost impossibly successful and attests to the ingenuity and resourcefulness of the men who conducted it.

Another thing which the final successful development and extensive application of the telephone repeater indicates with striking clarity is the fact that a complete solution of the problem was made possible only by the existence of a great unified engineering and research department, such as that maintained by the Bell System. The elements involved and the multiplicity of detail to be worked out and correlated were far beyond the limitations of any single individual or any limited organization. The best that mathematics, physics, chemistry, engineering, manufacturing and operation could supply were needed for the solution. Further, if the solution were to be obtained in a reasonable time it was essential that the efforts of the experts in all of these fields must be directed in a cooperative attack on the main problem. With the growth of fundamental research groups of highly trained scientists in the engineering department of the Bell System the means for attacking the elusive repeater problem in comprehensive fashion became available and progress toward success became certain and rapid. The loose ends of discrete and desultory researches were gathered in, past and present work scrutinized and the attack directed from a foundation of certain knowledge not theretofore available.

The wonderful success which has attended the work and which has started what bids fair to be a revolution in the entire scheme of telephonic transmission should be a source of gratification to American electrical engineers, to whom more than to any others have been due the other wonderful telephone developments of the past.

In discussing the earlier work on telephone repeaters and that which led up to the first successful device, the authors have not attempted even to enumerate the large number of investigators, much less to allocate credit among them. The names mentioned are those of the men who stand out most conspicuously and the fact that many of them are employees of the Bell System is not remarkable, since it was the largest single agency consistently at work on all problems concerning telephone development. In a complex art which has been vigorously worked for nearly fifty years, it is inevitable that great numbers of patents should have been taken out on all conceivable kinds of devices and systems. Many of these are inherently worthless, while others contain elements of value, although not in themselves disclosing complete workable arrangements. While, therefore, no single invention has solved the repeater problem in its entirety, it is interesting to note that all of the inventions which make possible the successful telephone repeater of today are the work of Americans.

No paper dealing with the development and successful application of telephone repeaters would be complete without mention of the fact that throughout all the later years, during which progress has been certain and rapid, the principal credit for the broad engineering and commercial results obtained is due to Colonel J. J. Carty, past President of this Institute and for many years Chief Engineer and now Vice President of the American Telephone and Telegraph Company. To Colonel Carty more than to any one else is due the credit of having brought about that coordination of all of the elements mentioned above, which was vital to the ultimate success of the undertaking.

HISTORICAL

The foundation of the art of telephony by Alexander Graham Bell followed by almost fifty years that institution of the sister art of telegraphy which was based on the discoveries of Faraday, Ampere, Volta, and Henry. It was inevitable, therefore, that efforts to extend the range of telephonic communication should be influenced by earlier and successful efforts in telegraphic communication. As telegraphic operation was attempted between more widely separated stations it was found that the attenuated currents were insufficient to operate the recording mechanism. To obviate this limitation Morse introduced at the receiving station a local source of energy which supplied current to the recording mechanism but was controlled by the received signals through the medium of an electromagnetic relay.

In this invention was the principle necessary for an indefinite extension of the possible range of communication since the local source might supply current to another stretch of line. It is a curious bit of history that this application was not utilized for several years during which transmission over long distances was accomplished by manual instead of automatic repetition at the intermediate stations.

The delay in this natural development was due to the fact that the relay of Morse, although it was a repeater, was a one-way device since it was actuated only by currents arriving from one of the two telegraph stations on either side of it. It required the use of two such repeater elements and of a circuit arrangement to convert a one-way into a two-way repeater station. Of these circuit arrangements one of the earliest was that of Varley. During the next few years duplex repeaters received considerable attention from inventors. The differentiation, however, which seems so obvious to us today, between the repeater element and the circuit arrangement by which such elements may operate duplex, was infrequently if ever emphasized.

In the extension of the range of telephony by the use of repeaters the duplex feature was even more essential than in telegraphy. The problem was complicated not only by the inherited confusion between element and circuit but also by the greater importance of the third factor in such operation, namely, the lines on either side of the repeating station.

The necessity for distinguishing between these three factors of repeating element, repeater circuit, and lines was early recognized by the engineers of the Bell System. The exact and quantitative relationships between them have been adequately expressed for about a decade. In our discussion of the matter in hand we shall follow, therefore, the logical order which results from this differentiation and treat these factors separately in the above order. Before undertaking their more detailed exposition it may be well to vivify their individuality by the consideration of the concrete case illustrated in Figs. 1 to 5 inclusive.

A principle, which is familiarly known by its direct-current application in the Wheatstone bridge, has proved most fertile in the formation of duplex circuits. Thus in Fig. 1 we have an early method of duplex telegraphy, which is obviously reducible to the simple bridge circuit of Fig. 2. If *a* and *b* are equal then, provided also *R* is equal to *X*, the battery *B* will cause no difference of potential between *m* and *n* and hence no current through the galvanometer in the case of Fig. 2 and no resultant magnetism in the sounder of Fig. 1.

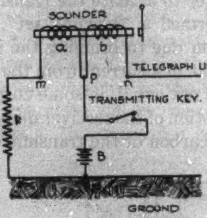

SOUNDER
a
b
TELEGRAPH LINE
m
p
n
TRANSMITTING KEY.
R
B
GROUND

FIG. 1

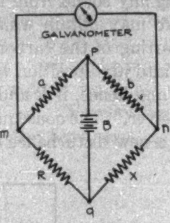

GALVANOMETER
p
a
b
m
B
n
R
X
q

FIG. 2.

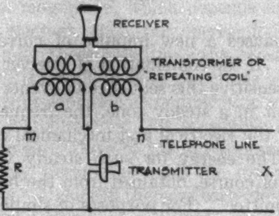

RECEIVER
TRANSFORMER OR "REPEATING COIL"
a
b
m
n
TELEPHONE LINE
R
TRANSMITTER
X

FIG. 3.

The same principle is applicable to the duplex operation of a telephone line as is seen in Fig. 3, where the sounder and the key of Fig. 1 are replaced respectively, by a receiver and a transmitter. If the resistance *R* is not equal to *X*, the impedance of the telephone line, then, the duplex set is not entirely "anti-side tone" since the operation of the transmitter causes some current in the receiver. The amount of current through the receiver and hence the motion of its diaphragm depends both upon the current from the transmitter and upon the amount of unbalance in the bridge which is occasioned by the inequality of *R* and *X*.

If we dispose the transmitter and the receiver as in Fig. 4, allowing any sound from the latter to actuate the transmitter, then we may obtain what is termed a "howling set". The mere jarring of the carbon button due to bringing the instruments into juxtaposition will result in a current from the transmitter and hence, if *R* and *X* are not equal, in some current in the receiver. The consequent motion of the receiver diaphragm causes a new disturbance of the carbon of the transmitter and this in turn causes a new impulse of current through the receiver itself. If the combination of receiver and transmitter is sufficiently sensitive this sequence of events may perpetuate itself and result in a steady tone, the frequency of which is determined by the electrical and mechanical characteristics of the system. The energy for such steady vibrations of the diaphragm is, of course, obtained from the battery associated with the transmitter. (For purposes of simplicity the battery is not shown in Figs. 3 and 4. It may be either a battery associated locally with the transmitter or one at the distant central office.)

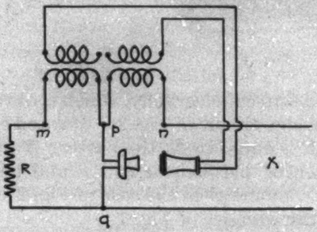


Fig. 4

From this illustration of the "howling set" the importance of line balance may be seen, for the combination of receiver and transmitter constitutes a repeater element. The element may more conveniently be symbolized as in Fig. 5 where the resistance, *R* of Fig. 4, has been replaced by a telephone line. In this last form of the bridge we may recognize the three factors mentioned above. The repeating element has input terminals *1, 2* and output terminals *3, 4*. It is associated with the lines on either side by the bridge circuit *m n q q* in such a manner as to repeat telephone conversations in either direction. Thus voice currents originating at a station on the east line are impressed on the terminals *1, 2* of the element through the action of the transformer. The resulting output from terminals *3, 4* is then transmitted equally to the east, where it is received as side tone by the speaker, and to the west, where it is usefully received by the distant auditor. Further details of this operation will be considered later. We may, however, note that if the unbalance between the lines east and west exceeds a certain magnitude, which depends upon the sensitiveness of the element, then the operation is that of the howling set. Under these conditions the repeater is said to "sing," for it will transmit in both directions a strong steady tone which renders telephonic communication impossible.

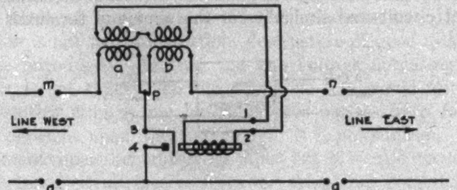


FIG. 5.

The difficulty introduced by this possibility of singing was not serious in the early days of the development of telephone repeaters and only became so with the invention of more sensitive elements. In fact in the very early days the difficulty was to obtain an element of sufficient sensitiveness. What sort of conditions must be met may be illustrated by the fact that the current at the transmitting terminal of a long line is only a matter of 2 or 3 milliamperes. In transmission over the line the current is constantly attenuated so that at the receiving terminal it may be only 3 or 4 hundredths of its original magnitude. Such small currents required greater precision of design than was appreciated by many of the early inventors, although as early as 1888 one of the Bell engineers, E. H. Lyons, had constructed a repeater which would amplify such small currents.

THE REPEATER ELEMENT

Before considering further these early designs we may well list the operating requirements for a successful repeater element. In so doing we shall not be concerned with the physical principles upon which the device operates. To all present intents and purposes we are concerned only with pairs of terminals *1, 2* and *3, 4* of an enclosed and unknown electrical network as illustrated in Fig. 6. If we subject such a hidden element to electrical measurement with alternating current we may obtain, for each frequency which we use, several characteristic ratios. Thus, from the ratio of the voltage impressed upon terminals *1, 2* to the resulting current we obtain the impedance of that circuit and similarly for the circuit of terminals *3, 4*.

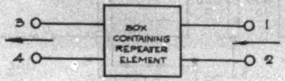


FIG. 6.

We obtain also a mutual impedance by taking the ratio of the voltage produced in the circuit *3, 4* to the current in the circuit *1, 2*. Similarly, there may be measured the mutual impedance in the opposite direction. In case one only of the mutual impedances is zero, that is, that no voltage is produced on one side when a current is flowing in the other, we speak of the device as having a "unilateral mutual impedance." Of the repeaters which we have found best adapted to telephony all are of this unilateral type. The requirements which must be met are identical whether the device is bilateral or unilateral but they may be more conveniently stated in terms of the unilateral type, of which the receiver-transmitter (or mechanical form) shown in Figs. 4 and 5 is a simple illustration. These requirements are:

1. The unilateral mutual impedance must be independent of the amplitude of the impressed current, within the range required for telephone transmission. In other words, there must be a linear relation between the output voltage and the input

current of the repeater, in which case the output is directly proportional to the input. If this condition is not met there will appear in the output one or more harmonics of the impressed alternating voltage.

1a. The input and output impedances of the element must be similarly independent of the impressed voltage or else distortion will occur.

2. The unilateral mutual impedance must be independent of the frequency of the impressed voltage within the range required for the transmission of the human voice. If this condition is not met the various overtones in such a complex sinusoidal current as is initiated by a voice will not be faithfully reproduced in their proper proportions.

2a. The input impedance of the element should be independent of the frequency of the impressed voltage or else it should be the same function of the frequency as is the impedance of the telephone line to which it is to be connected. If this condition is not met a distortion, destructive of good quality, may be introduced. In this case the voltage actuating the repeater is not that which the telephone line would impress upon another similar line but is greater or less by a factor which depends upon the frequency. It is not sufficient that the element repeat accurately its input but it is also requisite that its input circuit receive from the telephone line without distortion.

2b. The output impedance of the element should be similarly independent of the frequency or else properly related to the impedance of a telephone line so that the line may receive without distortion the output of the repeater.

3. The element must give amplification; that is, ignoring the energy sources which may be associated with or be parts of the element, the efficiency as measured by the ratio of output and input energies of telephone currents must be greater than unity. For example in the case of the mechanical element, which was pictured above, the energy is supplied by the battery associated with the carbon button. We are concerned at present only with its efficiency for transferring telephone variations from terminals *1*, *2* to terminals *3*, *4*. How much greater than 100 per cent this efficiency must be will depend, obviously, upon costs and other engineering factors. A repeating element adaptable to our telephone plant may well be expected to give an energy amplification such that telephone currents, which

have been attenuated by about twenty miles of standard cable, will be restored to their original values by its action. In telephone parlance "the element should give a gain of about twenty miles." Since by transmission through twenty miles of standard cable both the current and voltage would be attenuated to about one-eighth, the energy delivered to the repeater circuit would be only one sixty-fourth of the original energy. Of this only one-half reaches the receiving element for the rest is dissipated in the bridge. Hence there must be supplied to the line on the output side of the repeater circuit 128 times as much energy as is received. A glance at the circuit of Fig. 5 shows, however, that the output of the repeater itself is equally supplied to the outgoing and the incoming lines. The element must be capable, therefore, of supplying 256 times as much energy as it receives. In other words, the element should have a telephone efficiency of about twenty-six thousand per cent.

4. The sources of energy associated with the element, and by virtue of which amplification is possible, must have voltages which may safely and conveniently be utilized in the telephone plant. For example, a repeater whose source of energy had a voltage of greater than 250 would be unsatisfactory in a plant which normally employs voltages not greatly exceeding 100.

5. The element must be either insensitive to external electrical disturbances or easily shielded from them so that cross-talk from neighboring telephone circuits or apparatus may not actuate it.

5a. Conversely, the element must not be such as to cross-talk into adjacent circuits or apparatus.

6. The element must be constant in behavior, reliable and of fair life, and not such as to require frequent attention or laboratory conditions of operation. In other words the element must meet the conditions as to reliability of service which apply to other portions of the telephone plant.

7. In size, first cost, cost of energy and of maintenance, the element must conform to the economic conditions of an established plant.

Recognition of these conditions has guided our search for new elements during the past fruitful decade. Even before that time these requirements were recognized although they were not completely formulated. It is surprising how many independent inventors approached us during this time with devices formed by coils and magnets but involving no source of energy, or at least utilizing none, and hence patently incapable of producing amplification. Inherently such devices were static transformers, on the low voltage side of which there might be a greater current but obviously not a greater power than there was expended as input on the high voltage side. In some instances such devices were said to be intended for frequent use along a telephone line at fairly regular intervals. Such effect in improving telephone transmission as they might have would then be explainable by their function as inductances in the line; in other words, by the loading of the line. They thus offered imperfect substitutes for Dr. Pupin's method of loading.

Most of the earlier attempts at the design of telephone repeaters followed the recognized lines of telegraph practise or of electrical power engineering. The life of the telephone, however, has been almost contemporaneous with that of the newer physics which deals with discharges through rarefied gases and which has led to the discovery of the electron and of steady electron streams in vacua. General knowledge of the electronic structure of matter has grown by leaps and bounds in the last few years and coincident with this growth has been our practical development of electronic repeaters.

For purposes of classification, therefore, we may divide repeating elements into three groups depending upon whether their moving parts consist of (a) molecular ions, (b) electrons, or (c) molecular aggregates, that is, ponderable matter in the ordinary sense. The first group we might call "gaseous", the second "electronic," and the third, "electrodynamic." We may also classify according to the means by which the moving parts are actuated, as (a) electromagnetic if the operation is dependent upon a magnetic field which is altered by the received current, and (b) electrostatic if operation is dependent upon an electrical field which is established or altered by the received current.

Electrostatic devices involving moving members of mechanical size are too insensitive to serve as telephone repeaters. In the case of gaseous or electronic devices such a method of control is, however, entirely practicable and, indeed, for electronic systems marvelously effective. Our system of classification may, therefore, be simplified by the omission of one class of mechanical devices. It must, on the other hand, be further complicated by the introduction under gaseous repeaters of a type of device which has no cognate in the other two classes. Such devices are conductors which depend upon the passage of current through themselves to alter their own characteristics. Something of this behavior is familar to all engineers who have worked with carbon arcs and remember their instability. For our purpose we may designate this type of repeater as a "negative resistance device" and postpone the justification of the terminology to later discussion.

The classification of repeater elements in the order in which they will be discussed is, then, as follows:

I. Electrodynamic repeaters
    A. Receiver transmitter type
    B. Generator type
        1. Direct-current generator
        2. Alternating-current generator
        3. Asynchronous generator
II. Electronic Repeaters
    A. Electrostatic
    B. Electromagnetic
III. Gaseous Repeaters
    A. Electrostatic
    B. Electromagnetic
    C. Negative Resistance

I–A. The form of telephone repeater which most naturally suggested itself to the early engineers is the combination of a receiver and a microphone essentially in the manner depicted in connection with our earlier discussion of the problem presented to telephone engineers by the necessity of increasing the range of telephonic communication. In the early years of telephony no other means seemed available and prior to 1903 many unsuccessful attempts were made along this line. The parent Bell organization devoted the efforts of a large number of its best men to this problem, which was also a favorite with inventors outside of its organization. In the minds of the general scientific world further interest had been stimulated by the reported offer of a capitalist to pay one million dollars for a successful invention.

The solution was finally reached by one of the Bell engineers, H. E. Shreeve, who was assigned to the problem in 1903 after an earlier experience in the design of microphones and other telephone devices. Previous inventors had combined receivers and transmitters without recognizing that the normal functions of their diaphragms were those of sound collecting or emitting members and Shreeve omitted these unnecessary features.

Another difficulty in earlier designs had been the "packing" of the carbon of the microphone and one of his first studies had to do with its cause and elimination. He found it to be due to expansion which was occasioned by the heat liberated in the carbon chamber. In his first successful laboratory model a stretched steel strip was used as a connecting link between the receiver and the microphone and the microphone was designed so that the expansion of its parts under the influence of heat did not subject the granular carbon between the electrodes to increased pressure.

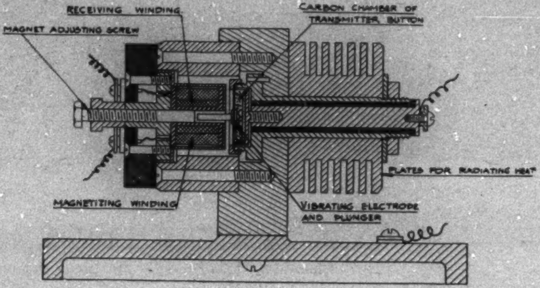

RECEIVING WINDING
MAGNET ADJUSTING SCREW
CARBON CHAMBER OF TRANSMITTER BUTTON
PLATES FOR RADIATING HEAT
MAGNETIZING WINDING
VIBRATING ELECTRODE AND PLUNGER

FIG. 7

With this form was obtained our first successful test of repeater operation over a telephone line. The device was tried in 1904 on a circuit between Amesbury, Massachusetts, and Boston and resulted in a greater intelligibility in the transmission of long lists of unrelated words than could be obtained over the circuit without the repeater.

The next step was to obtain more perfect quality reproduction and this was secured by making lighter moving parts and increasing the natural frequency of the moving system. The next model, shown in Fig. 7, was commercially operated on a circuit between New York and Chicago, from August 1904 to February, 1905. In this model the packing of the transmitter element was obviated by the thermostatic action of the vibrating disk, and the natural period of the moving parts was such as to emphasize the important frequencies of the voice. The element was associated with the line by a simple bridge circuit which had been patented by Edison in 1883. The transmission between New York and Chicago was found to be greatly improved.

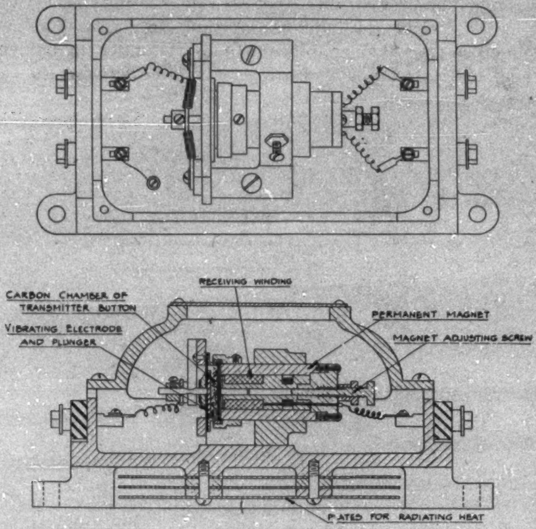

RECEIVING WINDING
CARBON CHAMBER OF TRANSMITTER BUTTON
PERMANENT MAGNET
VIBRATING ELECTRODE AND PLUNGER
MAGNET ADJUSTING SCREW
PLATES FOR RADIATING HEAT

FIG 8

In this installation an additional feature was incorporated to prevent amplification of the so-called "Morse thump," which occurs when a repeatered line is composited and carries superimposed telegraph signals. A repeating coil of low mutual inductance was inserted between the line and the receiving element. This coil was inefficient for currents of low frequency but was relatively efficient for those of voice frequency.

In spite of the success which was obtained with this first repeater installation, the problem was far from being solved. In fact, when an attempt was made to reproduce the model, it was found extremely difficult to obtain vibrating electrodes, which had the necessary thermostatic action in addition to other necessary characteristics. An attempt was, therefore, made to regulate the sensitiveness of the microphone by more positive means and the next model, Fig. 8, had the feature of a thermostat in the form of a zinc strip. This zinc strip was heated by a coil, which was in series with the microphone, and whenever the current through the microphone increased as the result of incipient packing it served to withdraw the rear electrode.

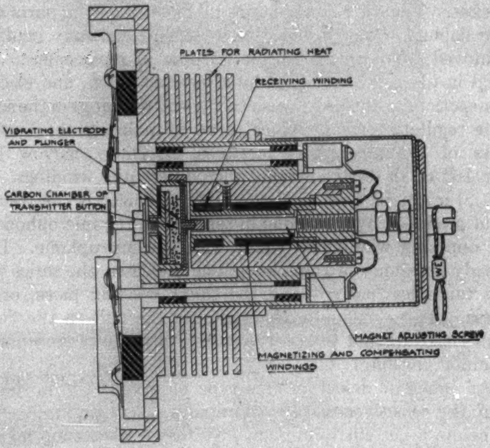

PLATES FOR RADIATING HEAT
RECEIVING WINDING
VIBRATING ELECTRODE AND PLUNGER
CARBON CHAMBER OF TRANSMITTER BUTTON
WE
MAGNET ADJUSTING SCREW
MAGNETIZING AND COMPENSATING WINDINGS

FIG. 9

This repeater, known as No. I-A, was reasonably satisfactory in service until the requirements became more exacting, particularly in the matter of tandem operation with the attendant requirements of improved quality reproduction. An intensive study was therefore undertaken, with the result that the natural frequency of the vibrating system was raised still higher, the weight of the moving parts was still further reduced and a still more sensitive form of regulator adopted.

Without going into two much detail in regard to intermediate steps which were made between the 1–A repeater and the so-called 3–A type, which is the present standard for instruments of this type, it will be sufficient to describe the features of this latest model and to point out the inherent limitations of this class of repeater.

The 3–A repeater, Fig. 9, is of the so-called "cartridge" type and consists of two main parts, the cartridge and the socket. The cartridge contains all of the working parts which are liable to become defective in service and may readily be removed from circuit for the purpose of replacement. It is held in place in the socket by a bayonet catch, and electrical connections are made to its contact posts by springs in the socket. The regulating means which serve to maintain the sensitiveness of the microphone between extremely narrow limits consists of an electro-magnet with differential windings. The initial magnetic field of the receiving element is the resultant field produced by a winding in series with the microphone and an opposing winding in shunt with the microphone. If the steady resistance of the microphone increases, the series winding takes less current and the shunt winding more, or vice versa. These magnetic changes alter the pull on the flexible disk electrode and thus serve to regulate the pressure on the granular carbon.

An inherent defect in this type of instrument is the fact that the sensitiveness falls off rapidly when the input energy is below a certain minimum. At the present time no completely satisfactory explanation for this phenomenon has been advanced. A defect of this character is obviously serious in the operating plant where circuit conditions and volumes of energy at any given point in a circuit fluctuate widely from time to time. Thus at one moment the repeater in a given circuit may be handling the energy from two persons speaking loudly, using telephones very near the terminals of the circuit, while at the next moment it may be called upon to function with the energy which it receives from telephones connected to long circuits, which in turn connect to the circuit with which it is associated. If the persons using the telephones in this latter case have weak voices, if they do not use the telephones properly by talking directly into the transmitter, or if the apparatus

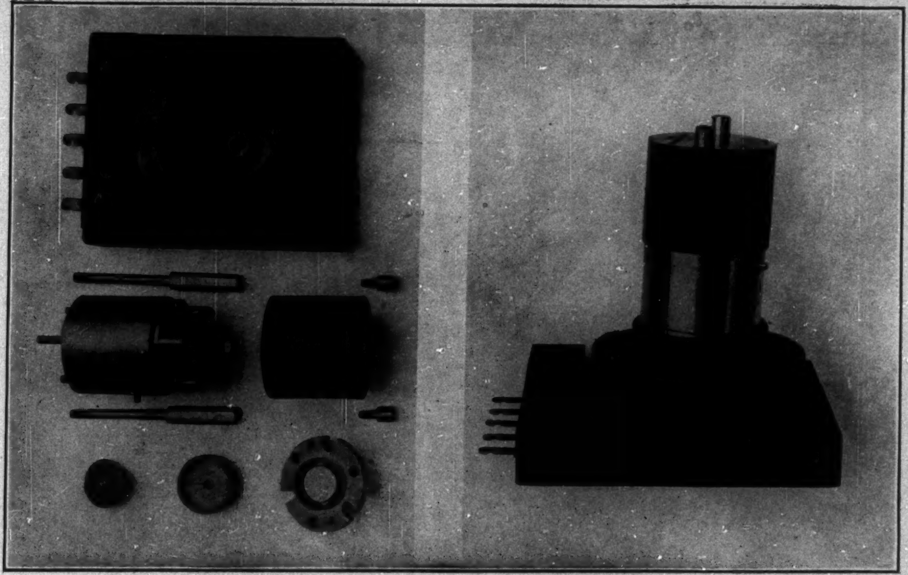


FIG. 9—MECHANICAL TYPE TELEPHONE REPEATER ELEMENT, SHOWING SOCKET, CARTRIDGE AND PRINCIPAL PARTS

[GHERARDI AND JEWETT]

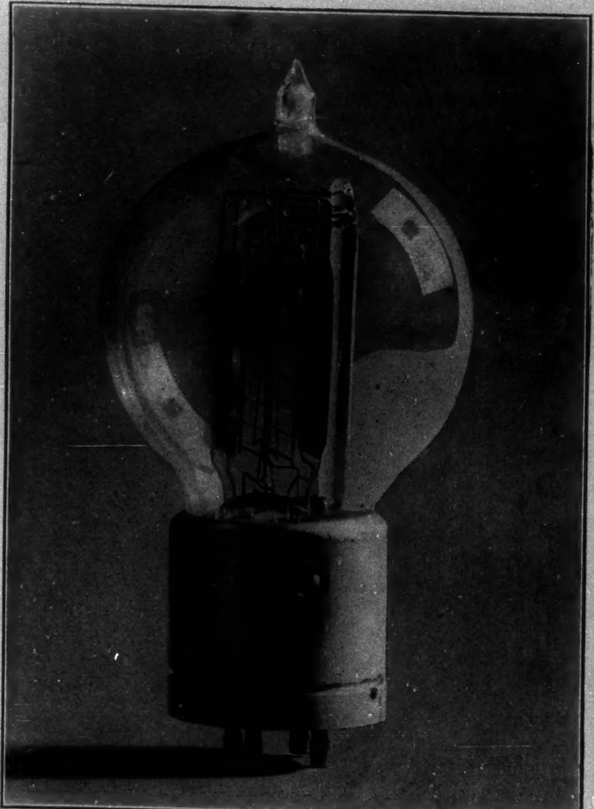


FIG. 13

[GHERARDI AND JEWETT]

or lines themselves are inefficient, it is clear that the energy at the repeater terminals may be many times less than that in the previous case. Further under those conditions maximum amplification is the more to be desired so that a repeater element whose performance fluctuates with the amount of energy which it receives and particularly one whose performance is worse when operating with the smallest energy is subject to a handicap when it comes to applying it generally in the telephone plant.

The Shreeve type of mechanical repeater is nevertheless distinctly serviceable and many units are in commercial service. The amplication is not, however, sufficiently independent either of the amplitude or of the frequency of the input to make them adaptable with best results to use in tandem, although they have been used to advantage on telephone lines the length of which did not require more than three repeater stations.

I–B. The application of the principles involved in direct- and alternating-current generators to the solution of the telephone repeater problem received the attention of some of the best minds in the art during the decade following 1900. During this time the problem was attacked both theoretically and in experimental design by H. S. Warren, M. I. Pupin, and others. The practise of power engineers suggested two typical methods.

(1) A generator of which the armature should constitute the output circuit of the element and the field winding the input circuit. Variations in the field current will result in corresponding variations of the armature voltage of such a machine. In order, however, that there should not be present in the armature voltage variations due to commutation, which would result in extraneous noise in the telephone, it is necessary that such a machine have a high frequency of commutation. If this frequency is well above that of the highest frequency which is essential to the faithful reproduction of the human voice, the noise currents due to commutation may be choked out by inductance or filtered out by a periodic structure like an artificial loaded line. When it is said that frequencies of 2500 cycles per second are required for good quality of telephone speech it is evident that for any practical design of armature and commutation the peripheral speed must approach the allowable mechanical limit,

Various unipolar designs were therefore considered in which no commutation is required. The principle involved may be seen by considering Fig. 10 which shows a form suggested in 1905 by G. A. Campbell. The field coil *M* is solenoidal and carries fixed disks, *w'*, which serve both as a magnetic core and as collectors for the radial e. m. f's. induced in similar disks, *w*, attached to the rotating shaft. In machines of this character where the field current is only of telephonic magnitude the flux is very small and the e. m. f. which may be obtained at any mechanically allowable speed of rotation is much below that requisite for a successful telephone repeater. In addition the eddy current and hysteresis losses are relatively high because of the frequencies involved and the efficiency is correspondingly low. The input impedance, being that of an electromagnet, is not independent of the frequency, as is desired, but instead is such as to discriminate against the higher frequencies of the voice wave.

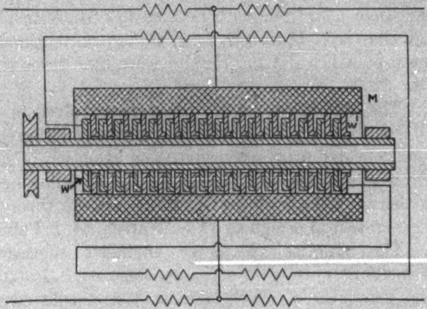


FIG. 10

Even if it were possible to build and operate such high-speed generators as repeaters of feeble currents, the type is deficient in so many of the characteristics required of a successful commercial repeater that the extent of its use is questionable. Now that simpler and practically perfect repeaters are available, it is doubtful whether the limits of physical possibility in this type of apparatus will ever be worked out.

(2) The practical difficulties of eddy current and hysteresis

losses, of limiting speeds and of small input, were also effective in preventing the development of a successful repeater along the lines previously followed in the power engineering of alternating currents.

(3) In 1900, Joseph Lyons disclosed to us his invention, a telephone repeater upon the principle developed and patented in power engineering by Hutin and Leblanc. The principle is perhaps better known as that of the asynchronous generator, which is an induction motor driven above synchronism by a local source of mechanical energy. The conditions under which such a device may serve to supply energy to a transmission line, across which it is connected, are well known to electrical engineers. It was Lyons' idea that the stator should be bridged across a telephone line and, through the interaction of the received current with that induced in the short-circuited rotor, should supply to the line additional energy of the same wave form as it received. Schematically the circuit is as shown in Fig. 11.

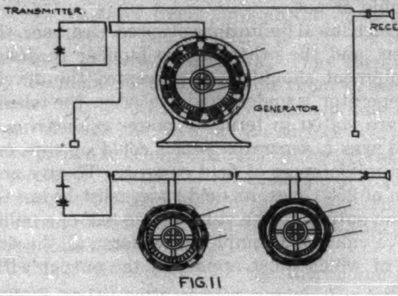


FIG. 11

In order, however, to supply energy at any telephone frequency the speed of the rotor must be at least as large as the quotient of the given frequency and the number of pairs of poles in the stator. Thus in the case of a 12-pole stator a frequency of 2400 cycles per second would require a speed of 400 revolutions per second. Not only are the speed requirements too high for practical operation but the machine would also fail to meet the second requirement for a repeater. The amplification for each frequency would depend upon the negative slip at that frequency and thus the lower frequencies of the voice current would receive more amplification than the higher ones. It was at one time suggested by Dr. Pupin that such distortion might be avoided by using several such asynchronous generators connected at different points of the line and each designed to amplify but a band of frequencies. Even had it been found possible to devise efficient machines to amplify the different groups of frequencies, it is doubtful if the arrangement would ever have proven satisfactory in an operating telephone plant, since at no point along the line between the first and last generator would the current have represented normal speech. This characteristic of the system would, of course, be a serious handicap to proper operation, since it would be almost impossible to tell whether the line were in trouble or not.

That the mechanical limitations of speed, clearance, thickness of lamination, and the other factors familiar to designers of dynamo equipment should have prevented the development of such a generator is understandable when the telephone requirements are stated in terms of power engineering. What was required was a generator which could operate on about one-half of a milliampere of field current. To give an energy amplification of 256 times, its field losses must be less than one per cent of the output. The value of one-half of a milliampere is the effective value of a complex wave form, all the sinusoidal components of which must appear in the output with equal amplification.

II. Electronic repeaters are those in which the moving parts are discrete electrons whose motions are unimpeded by molecular matter. Such devices, therefore, require a vacuum sufficiently high that the motions may be characteristic of free electrons. There must also be a means of supplying free electrons, that is of dislodging them from molecular matter of which they form constituents. The satisfactory method of accomplishing this is to "boil them out" of a piece of metal or metal oxide. Our present electronic repeaters are therefore "thermionic" devices.

The history of thermionics may be said to date from the publication in 1902–1903 of papers by O. W. Richardson in which he advanced a theory of the thermionic emission of electrons, which in all its essentials was accepted by the great majority of the scientific world and may now be regarded as definitely established. According to this theory (which is also a theory of metallic conduction) metals and conductors of electricity contain within their bounding surfaces immense numbers of free electrons which behave like the molecules of a perfect gas. When a metal is heated the kinetic energy of the electrons is increased and the number escaping across the boundary surfaces is likewise increased.

J. J. Thompson outlined the theory in his treatise on conduction of electricity through gases as follows:

"The emission of corpuscles from incandescent metals and carbon is readily explained by the view—for which we find confirmation in many other phenomena—that corpuscles are disseminated through metals and carbon not merely when these are incandescent, but at all temperatures, the corpuscles being so small are able to move freely through the metal and they may thus be supposed to behave like a perfect gas contained in a volume equal to that of the metal. The corpuscles are attracted by the metal so that to enable them to escape into the space surrounding it they must have sufficient kinetic energy to carry them through the layer at its surface where its attraction of the corpuscle is appreciable. If the average kinetic energy of a corpuscle, like that of the molecule of a gas is proportional to the absolute temperature, then, as the temperature increases, more and more of the corpuscles will be able to escape from the metal into the air outside."

In 1904 Wehnelt discovered that the thermionic emission from conductors coated with certain oxides was enormously greater than that of pure metals. Since that date the study of thermionic emission has proceeded most rapidly and the practical applications of the phenomena have more than kept pace during the last eight years.

Phenomena due to thermionic emission had, of course, been observed for years before a satisfactory theory was advanced. The first of these instances was the discovery by Edison in 1884 of an effect which is now known by his name. He found that if a metallic plate is introduced into the bulb of an incandescent lamp and if the plate is connected to the positive terminal of the filament then a current flows between the plate and the filament as may be observed by a galvanometer inserted as shown in Fig. 12. This conductivity of the space between the plate and filament is unilateral, for if the plate is connected to the other terminal of the battery no current flows. The explanation of the phenomenon resulted from work of Fleming in 1896 and J. J. Thomson in 1899 along lines earlier pursued by Elster and Geitel. Fleming studied the operation of a two-member vacuum tube device having a heated filament and cold plate, and Thomson demonstrated that the mechanism of current conduction between the filament and plate resulted from the emission of negative electrons from the filament.

Through such a tube a current may flow, since electrons may pass and will so do provided the plate is positive with respect to the filament and hence such as to attract the electrons emitted by the latter. The idea of using such a two-element device as a rectifier and hence detector of wireless waves was applied independently in 1905 by Wehnelt and by Fleming.

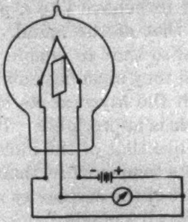


FIG. 12

For the next few years practical progress with thermionic devices was in the hands of the radio-engineers. In his American Institute of Electrical Engineers' paper of 1906 DeForest showed that the discharge between the hot cathode and the plate can be controlled by additional electrodes. In a patent issued the following year he shows inside the bulb such a controlling electrode to which he had given the form of a grid. With his discovery of a means for controlling the electron stream was born the most sensitive of repeating elements. Years of careful technical development were required, however, before the original idea had been perfected to the point where it met fully all the rigid requirements of application in a commercial telephone plant.

To understand the development which was required to adapt the first successful form of audion designed for radio-telegraph receiving to this use, we shall first consider in some detail the structure and physical principles of the device.

The "audion" consists of an evacuated vessel containing three electrodes, from one of which a thermionic emission of electrons is obtainable. Fig. 13 shows the characteristic structure. The filament is heated by the passage of a current as shown diagrammatically in Fig. 14. The other two electrodes are a plate and a grid. If a battery is connected to the filament and plate as in Fig. 14, so as to make the latter positive with respect to the former, then a current will flow in the circuit so formed. The electrons emitted at the filament are drawn across the intervening vacuum by the electrical field which the battery, *B*, in the plate circuit establishes. If an e. m. f. is now applied between the grid and the filament as by the source marked *V* in the figure, the field between plate and filament is altered and the current in the plate circuit is correspondingly altered. If the grid is made positive with respect to the filament more electrons are urged across the space between grid and filament. While some of this increase strikes the grid, and thus results in a current in that circuit, by far the greater number continue through the meshes of the grid to the plate. The result is an increased current in the plate circuit. Conversely, if the grid is made negative there results a decrease in the plate current. In this case, however, no current flows in the grid circuit because electrons can be drawn to an electrode only if it is positive with respect to the source of the electrons.

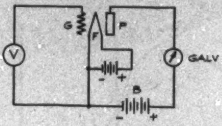


FIG. 14

The characteristic relation between the grid voltage, *V* and the plate current I_B, is that of Fig. 15. If the plate voltage is altered the form of the curve is not altered but the magnitude of the current is changed as illustrated in Fig. 16 which shows a family of such characteristics. It is evident from this figure that the number of negative volts which must be applied to the grid in order to reduce the plate current to zero is always the same fraction of the volts applied in the plate circuit.

Hence it appears that the current in the plate circuit may be altered either by altering the voltage there applied or by a much smaller alteration of the voltage applied to the grid circuit. The device thus gives a voltage amplification.

As long as the grid is kept negative no current can flow in that circuit and any alterations in its voltage are unaccompanied by any current variation and hence are entirely wattless. Such variations are accompanied by current variations in the plate circuit and result in an energy expenditure in that circuit. The telephone efficiency is thus seen to be practically infinite since an energy output may be obtained by a wattless variation of the input voltage.

To understand why the characteristic curve of Fig. 15 flattens out as the grid is made more positive we need only to remember that in the passage of a current through the plate-filament circuit two physical phenomena are involved. The first of these is the thermionic emission of electrons at the filament and the second is their enforced and directed transit between filament and plate. The emission takes place whether or not a voltage is impressed on the plate-filament circuit, and depends only upon the filament temperature. The process is much like evaporation in a closed vessel and soon reaches a statistical equilibrium where electrons are leaving and return to the filament at the same rate, that is number per second. Impressing a positive potential on the plate disturbs the equilibrium for it withdraws electrons from the tube. When this voltage is small it withdraws but a relatively small number per second; that is the current is small as in the lower portion of the characteristic. As the voltage is increased the current

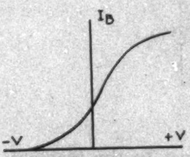


FIG.15.

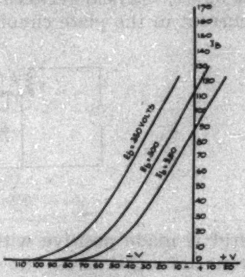


Fig. 16

increases but reaches a steady maximum value for a voltage sufficient to draw across the interspace as many electrons per second as the filament can emit at the operating temperature.

Although the number of electrons per second which pass between filament and plate and hence the current is so limited, the velocities with which the individual electrons move will continue to increase with the voltage. If there is a residual gas in the tube its molecules will be ionized by their collisions with the electrons provided the velocities of the latter are sufficiently high to disrupt the electronic systems of which these molecules are formed. Such ionization, however, increases the number of electrical carriers and hence the charge per second which may be transferred. The presence of gas in a tube is in general, therefore, evidenced by such a characteristic as the full line of Fig. 17.

The voltages involved in the detection of radio signals are much smaller than those met with in telephony so that no serious inconvenience arises from the presence of residual gas. As a consequence, in the production of his early successful audions for radio reception DeForest did not find it necessary to employ all of the known means for producing the highest possible vacuum.

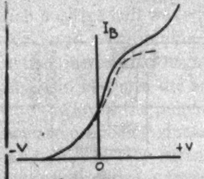


FIG. 17

In dealing with the very much larger energies which obtain under the conditions of telephonic repeater action, it is necessary to adopt methods of manufacture which will insure the removal to a high degree of all gaseous material in the tube, including that which may be occluded in the walls of the tube or the enclosed metallic parts. When such evacuation is accomplished all of the advantages of a pure electron discharge are retained, even for the higher voltages and currents which have to be dealt with in a telephone repeater. In other words, for a given plate voltage there is obtained a wider range of voltage which may be impressed upon the grid without departing from the true thermionic character displayed in Fig. 15.

In addition to securing the necessary conditions in this respect, a large amount of research work has been done by the engineers of the Bell System in producing suitable forms and proportions of tubes to give the best results with the various types of repeater circuits and in devising a type of filament which will insure a maximum of life and a minimum of power consumption for the amplifying tube and at the same time insure a uniform operating characteristic throughout the life of the tube. Further than this, the work has resulted in ability to produce in quantity tubes of exactly the same characteristics, a feature which is of the utmost importance in telephone repeater operation but which is of minor importance in radio work. Non-uniformity in the tubes would very seriously handicap the commercial operation of this type of amplifier since it would require the readjustment of apparatus each time it might become necessary to replace a worn-out or damaged tube.

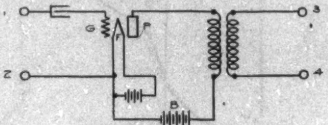


Fig. 18

Not only was the original form of audion deficient as a telephone repeater from the standpoint of its energy capabilities, but the conditions of its use in radio reception were inherently different from those which obtained in a telephone repeater circuit. We may express this fact by saying that for radio detection we make use of the ability of a device to distort a sinusoid, which is impressed upon it, while for telephonic purposes we require the very minimum of distortion.

Associated with the audion in DeForest's use was a condenser in the grid circuit as shown in Fig. 18. The input terminals were *1* and *2* of that figure and the output terminals were *3* and *4* of the repeating coil. If employed as the amplifier in a telephone repeater, the repeating coil serves to impress on the telephone line to which the device is connected the variations in the plate circuit current from the battery *B* which may be occasioned by any alterations in the potential applied to the input circuit. The condenser, however, serves as an electron trap if a sinusoidal e. m. f. is impressed on terminals *1*, *2*. Thus imagine that the first half wave of the sinusoid makes the grid negative with respect to the filament. This polarity tends to force electrons from the grid to the filament, but, since electrons are not emitted by the grid, no transfer takes place. Making the grid negative does, however, reduce the current through the plate circuit as we saw in connection with Fig. 15. Now, the succeeding half wave tends to force electrons in the opposite direction, that is from filament to grid, which is a possible direction of transfer in this unilateral circuit. The individual electrons so transferred to the grid cannot pass through the condenser and complete the journey back to the filament although others from the opposite plate of the condenser may be induced to make the rest of the journey. The result is that each positive half wave increases the number of electrons on the grid and its adjoining condenser plate and thus results in a further increase in negative potential. But the more negative the grid becomes the smaller is the current in the plate circuit and the lower the point on the characteristic curve about which we are producing sinusoidal variations.

Obviously, such an action is very satisfactory for detecting the presence in the input circuit of a train of sinusoidal waves of small amplitude since the effect is cumulative. Equally obviously, however, such action may absolutely prevent the passage through the system of a voice actuated train of waves since a word or two may make the grid so negative as to reduce practically to zero the plate current upon the variations of which the transmission of the wave depends.

The correction of the circuit arrangement by the elimination of this condenser, which aided distortion, was but a detail in the broader research which we have made on the application of thermionic devices. This work involved a study of appropriate circuits in which these tubes could be used, of thermionics, of methods of producing high vacua, of gages for indicating the degree of vacuum obtained, a study of various types of filament to secure efficiency and uniformity, and of the physical relations in three-element vacuum tubes of the geometrical form and separation of the electrodes. Some of these researches as for example those of Van der Bijl, W. Wilson, and others have already been published.

The net result of these researches has been our ability to design and construct tubes adapted to any of the purposes of the art of communication. In the final form in which we use a vacuum tube as a telephone repeater we obtain a characteristic relation between input voltage and output current which is free from distortion as far as we can recognize such effects. This is obtained partly by the design of the tube and partly by the proper adaption to it of its associated circuit.

In the first place we arrange that the repeater shall have a constant and finite input impedance by bridging between grid and filament a high resistance. To this resistance current is supplied by a step-up transformer as indicated in Fig. 19. The voltage effective in the grid-filament circuit is then the drop across the resistance. In the second place we generally arrange that the grid shall not be positive at any time in the cycle of the impressed voltage. This is accomplished by inserting a battery, *C*, in the grid circuit. In this way the tube acts as a device for amplifying the impressed voltage and introduces no distortion due to the unilateral conductivity of the grid circuit.

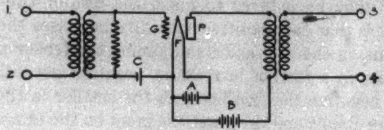


FIG. 19

The characteristic relation between grid voltage and plate current which is shown in the curves of Fig. 16 is obviously not that of a perfect repeater since there is not a linear relation between input voltage and output current. The voltages indicated on these curves are those of the battery in the plate circuit. The condition under which they were obtained is that of no external resistance in this battery circuit so that the voltages are really those which are effective between the plate and filament within the tube. The values of the current in these plots are due to the combined actions of this effective voltage and of the voltage impressed upon the grid circuit. For each of these curves the effective plate voltage is constant and the curve gives the relation between the input voltage and the output current. If, however, there is external resistance in the plate circuit, then the effective voltage between the plate and the filament will not be that of the battery, but will be less by the amount of the *IR* drop in the external circuit. An increase in the grid voltage will not, then produce as large an increase in plate current because such an increase as it would produce if the plate voltage was constant is partially neutralized by the decrease in the effective plate voltage which the *IR* drop occasions. The result is that by properly proportioning the external resistance of the plate-filament circuit we may obtain a characteristic relation between grid voltage and plate current which is essentially linear as shown by the full line curve of Fig. 20.

The requirements for a telephone repeater stated above are thus fully met in our construction of the audion and in the circuit arrangement for its operation. In later portions of this paper we shall meet illustrations of its perfection in the report of experiments and commercial operation.

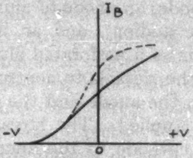


FIG. 20

In the case of the audion the control of the electron stream is electrostatic. It is possible, of course, to control such a stream by electromagnetic means since the stream is really a current. Such a control, is, however, inherently less efficient and sensitive than that of the voltage actuated device described. We shall, therefore, give no detailed discussion of such methods but will pass at once to the discussion of devices in which the presence of gas molecules results in an ionization and the consequent presence in the tube of positive carriers of molecular size as well as negative electrons.

III. The distinction between electronic and gaseous repeating elements is not a matter of the presence or absence of gas since an absolute vacuum is unobtainable. It is rather a matter of whether or not gas is an irrelevant and undesirable element or an indispensable one. In the vacuum tube of DeForest, gas was not indispensable as we have seen and such gas as was residual limited the range of voltages, throughout which the tube had the desired characteristics when used as a repeater.

III-A. In the repeating element of von Lieben which was invented in Europe somewhat later than the time the audion was produced here, gas is an essential constituent. This device has a Wehnelt cathode, that is a filament coated with oxides of the alkaline earths from which electrons are thermionically emitted. The function of the cathode is not, however, to provide all the carriers as in the case of the filament of the audion but rather to provide means for ionizing the gas present in the tube.

The tube is a three-element device and has in addition to the thermionic cathode, an anode of aluminum (corresponding to the plate of the audion), and an intermediate sieve-like electrode in the form of a perforated aluminum disk.

The function of the sieve is to control the discharge which takes place between the anode and the cathode, in case a potential is properly applied. The stream of electrons which proceeds from the cathode to the anode is impeded by the presence of gaseous molecules. The consequent collisions will result in the ionization of some of the gas molecules if the velocity is sufficiently high and this velocity depends upon the potential which is applied to the circuit. The ionization results in the presence of more electrons which are available for increasing the current and also in the presence of positive ions of molecular size which are similarly available but move in the opposite direction. The introduction of a potential between the filament and sieve permits of an external control of the motions of the electrons and positive ions, attracting the former and repelling the latter or vice versa.

Small changes in the potential applied to the sieve may result in relatively large changes in the ionization and hence in the current in the anode circuit. The device may, therefore, be used to give amplification but for reasons which will appear presently it is not adapted to telephonic work.

Since the tube contains carriers of both kinds a stream of one kind or the other will inevitably flow to the sieve and thus here will be a current in that circuit which will depend for its direction upon the polarity of the applied voltage and for its amount upon the conductivity of the gaseous space within the tube. If the cathode and sieve are used as the input circuit, corresponding to the filament-grid circuit of the audion, then it is evident that the impedance of this input circuit will vary with the impressed voltage. Such a variation will result in a serious distortion and is not in conformity with our requirements for a repeating element.

The presence of both kinds of carriers makes impossible the simple expedient which is so valuable in the case of the audion where the unilateral conductivity of the grid-filament circuit is utilized. Further the pressure of the gas within the tubes is subject to fluctuations which it is almost impossible to control. This results in large variation in amplification. In comparison with the electronic device the von Lieben tube is seriously deficient as a telephone repeater, and the electrostatic control of gaseous devices has not been found capable of as efficient and simple application to telephonic problems as is the electrostatic control of electronic devices.

III-B. The electromagnetic control of gaseous devices has likewise been investigated for telephonic purposes. The oppositely moving streams of positive and negative carriers will be deflected in the same direction by an electro-magnetic field. Provided that the motion of such a discharge stream may be made to alter properly the conductivity of either the circuit of the discharge itself or of some auxiliary circuit, a repeating element may be formed.

Some of the early work on such systems is that of Hewitt whose experiments on mercury vapor arcs are well known. The method which he proposed to employ may be seen from Fig. 21, where is shown a tube containing mercury vapor through which a current is passing from the battery B. A receiver is connected in series with the battery and the arc. The arc stream itself is controlled by the electromagnet which is excited by the transmitter. This arrangement, which is essentially a very insensitive unilateral repeating device, has never, so far as is known, been made to function as a telephone amplifier. In addition to its inefficiency, this arrangement is undesirable because of noise, distortion and variable amplification.

A later step in the development of an electromagnetically controlled mercury arc repeater was taken by Dr. H. D. Arnold. He utilized an auxiliary circuit for the output and so disposed it that its conductivity was altered in conformity to the motion of the arc stream. In the general idea of using a mercury arc stream and influencing it magnetically, Arnold of course, follows Hewitt.

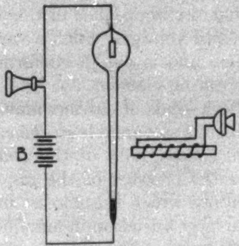


FIG. 21

The auxiliary circuit devised by Arnold is at right angles to the arc stream and includes three electrodes, one about the center of the stream and the other two approximately equally spaced on opposite sides as shown by a cross section of the tube in Fig. 22, the main arc stream being perpendicular to the plane of the diagram. The electromagnet is in the plane

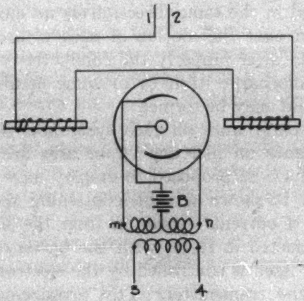


FIG. 22

of these electrodes and thus serves to deflect the main arc stream toward one or the other of the side electrodes depending upon the polarity of the magnet. The ions in the main arc stream are available carriers for current between these electrodes and the central electrode if a potential is maintained between them as by the battery, *B*.

In the normal condition, when the arc is undeflected, similar portions of the arc stream are included between the center electrode and each of the side electrodes. The conductivity of the two paths thus offered to the potential of the battery are then equal and there is no difference of potential between the terminals *m*, *n* of the coil, to the center tap of which the battery is connected. In so far as the conductivities of these two branches of this bridge circuit tend to vary as the result of variations in the ionizations of the arc stream no difference of potential will be produced between the terminals of the coil, since both conducting branches will tend to vary similarly due to a common cause. Fortuitous variations in the arc stream are, therefore, not repeated in the external circuit and the difficulty of arc noise is practically eliminated. When, however, the arc stream is deflected by the voice actuated electromagnet, the conductivities of the two branches are oppositely changed and a resultant current circulates.

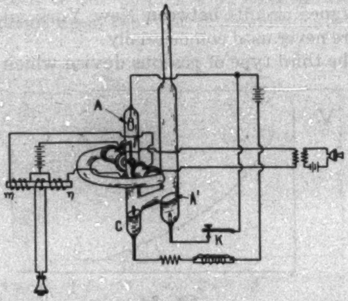


FIG. 23

The device as originally described for patent is shown in Fig. 23. An auxiliary anode, formed by a mercury electrode shown at *A′* is used for starting the arc stream. In this form the arc is struck by tilting the tube and allowing the mercury of the anode A' to run into contact with that of the main cathode C. As the tube is restored to its vertical position an arc is formed between A' and C, provided that the key K, has been closed. Opening the circuit at K allows the arc to form between C and A.

This gaseous device is capable of good amplification and is fairly free from distortion except such as may be introduced by the fact that impedance of the electromagnet may not be identical with that of a telephone line. It was tried out experimentally on telephone lines but has never been used for any length of time and then only under special engineering supervision. For example, units were installed on the important long distance circuits between New York and San Francisco but were never used commercially.

III-C. The third type of gaseous device which we have investigated as a possible repeater is the negative resistance type. A typical voltage-current characteristic for an arc stream is that of Fig. 24, where the ordinates give the impressed voltages and the abscissas the resulting current.

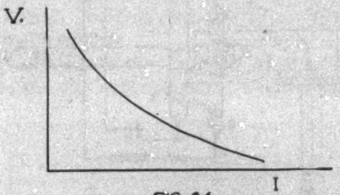


FIG. 24

For an ordinary resistance the slope of the relation of voltage and current is positive in the usual sense, since for any point on the curve dV/dI is positive, and represents a slope of less than 90 degrees measured counter-clockwise from the axis of I. For the arc, however, with its falling characteristic, the slope is negative. The resistance which the arc offers to a change in its condition is thus to be considered negative.

If an arc is connected in series with an ohmic resistance and a source of potential, as shown in Fig. 25, which is essentially that suggested at one time by Hewitt, who dealt with a mercury arc in an enclosed tube, the arc, owing to its falling characteristic, has the effect of making the total resistance in the circuit appear to be less when small current variations exist in the circuit than when steady conditions exist.

The reason for this is that any small increase in current is accompanied by a decrease in the voltage across the arc itself, thus leaving a still greater voltage across the ohmic resistance than would be obtained if the circuit consisted merely of elements obeying Ohm's law. If the source, V, is a transmitter and if there is in the circuit also a receiver, then it follows that the energy received by the latter may, under proper conditions, be greater than that delivered to the circuit by the transmitter. In other words there is a possible telephonic amplification.

The magnitude of the amplification with the Hewitt device obviously depends upon the relative magnitudes of the negative resistance and the external ohmic resistance. The external resistance may in part be that of a telephone line and for that reason this method has been considered as a means of producing telephonic amplification. The negative resistance of the arc can never do more than remove some of the effects of the external resistance. If it could do more, of course, the circuit would "sing." We have found in practise that the system can never increase the loudness of telephone speech unless the line resistance itself is sufficient to stabilize the arc. In practise the arc would have to be connected to various telephone lines, each having different characteristics, and in order that an arc which will amplify, when connected to a high resistance line, shall not "sing" when connected to a line of lower resistance, a certain amount of extra external resistance has to be added to the arc circuit, which resistance will be different for each different type

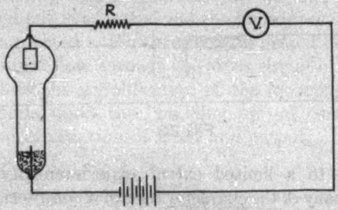


FIG. 25

of line. For practical telephony, therefore, this negative resistance device is not of service.

Even if the possible amplification were sufficient under actual conditions there would still remain the fact that the dynamic characteristic of the arc varies with the frequency of the impressed telephonic waves, giving prohibitive distortion and variable amplification. Further, in common with the other types of gaseous discharge apparatus which we have considered above, the negative resistance device would be too unsteady for practical use.

The two devices which we have found practicable under the exacting requirements of commercial telephone service are the receiver-transmitter element and the three electrode vacuum tube or audion. The electromagnetically controlled arc stream with auxiliary electrodes, which, as indicated above, has been used by us to a limited extent experimentally, and while possessing many of the characteristics of a commerical telephone repeater, is not in its present state of development adapted to general telephonic use. Of the three types, mechanical, electronic, and gaseous the electronic device with its thermionic emission is the most nearly perfect. In fact the amount by which it fails to meet all the requirements for a perfect repeater is so small as to be negligible except under the most rigorous conditions.

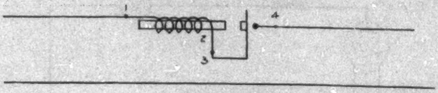


FIG 26

THE REPEATER CIRCUIT

The electrical network by means of which one or more repeater elements may be associated with the telephone lines is conveniently known as the "repeater circuit," as explained above. In two-way operation, such as telephony requires, there may be distinguished three general types of such circuits, of which the bridge or balanced circuit, described earlier in this paper, is the more usual type.

The other types do not depend upon balance for their successful operation. One of these is commonly called the "booster." Fig. 26 shows one form of series booster, in which the input and output of the element are connected in series with each other and with the line. A form of shunt booster is similarly indicated in Fig. 27. A large number of other forms of booster circuit are possible. Negative resistance devices are by their nature adapted to the booster scheme of connection.

In the case of booster circuits there is, for operation without singing, a limiting relationship between the amplification for which the element is adjusted and the impedance of the line in which it is connected. Part or all of the output currents from the element must flow through the input circuit of the element itself, and thus the amplification of the incoming telephonic current depends upon the "building up" of currents in the circuit by the interaction of input and output. It is obvious that if the lines do not offer a sufficient load such a circuit with its element may act as an oscillation generator, that is, there may be singing.

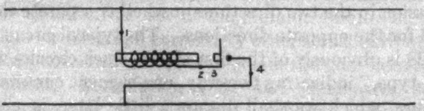


FIG. 27

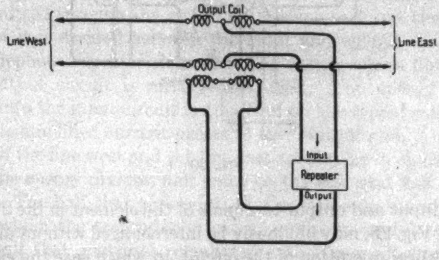


Fig. 28

The third type of repeater circuit is somewhat analogous to the single line repeater of telegraph practise. Incoming voice currents operate sensitive relays which switch the terminals of the element according to the direction from which the voice currents originate so that the circuit is equivalent at any instant to a one-way repeater circuit.

These types of circuit, which do not depend upon balance, have in general been found not so desirable for practical use as the bridge type of duplex circuit and will not be discussed in further detail.

Of the bridge type we may distinguish two classes depending upon whether a single repeater element is used to amplify transmission in the two directions or whether separate elements are used for the opposite directions. The typical circuit shown in Fig. 28 is obviously of the first class. Such circuits we may call "21-type," indicating two-way, one-element circuits. The other class, which we shall discuss a little later, is generally known as the "22-type", since it is a two-way, two-element circuit.

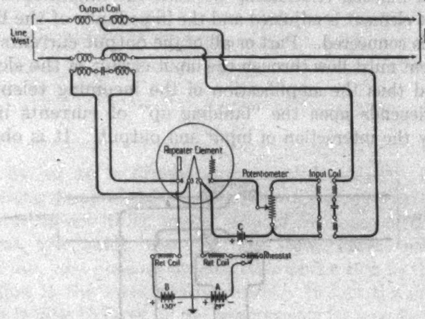


Fig. 29

The input and output terminals of the element in the 21 type circuit, Fig. 28, may obviously be interchanged without altering the physical operation of the circuit, in which case the element transmits to the two lines in parallel instead of in series. The choice between these two connections is influenced largely by the ease with which, through the medium of repeating coils, the input and output impedances of the element may be made similar to the impedances of the network to which they are connected.

A practical form of the 21 circuit is shown in Fig. 29 with its associated vacuum tube amplifier. The incoming energy is supplied through an input repeating coil, between which and the telephone lines is interposed a "filter," not shown in the figure. This filter, of which many satisfactory types have been invented by Dr. G. A. Campbell, is a physically periodic structure, consisting of similar networks in series, which is so designed as to discriminate against the passage of sinusoidal currents of frequencies above or below the necessary telephone range and thus prevents the element from receiving currents which arise from the simultaneous use of the metallic circuits of the telephone line for telegraph currents or other signal currents. The potentiometer is introduced to regulate the gain occasioned by the element. Retardation coils and condensers serve to prevent cross-talk into adjacent telephone circuits and to permit the use of a common battery to supply all the repeater sets at any repeater station. A rheostat is also introduced, as shown, to control the heating of the filament and thus its thermionic emission.

Before proceeding to describe the 22 type circuit we may well restate the physical operation of the 21 type in terms somewhat different from those used in our introductory discussion of the circuit. Let us follow the telephone current, assuming for the moment that it is traveling from west to east. The current that comes to the repeater from the west is partly transmitted through the circuit to the line east but the larger part of it is divided between the input and output circuits of the telephone repeater element. The energy which flows into the output circuit is merely dissipated. The energy which flows into the input circuit is amplified by the repeater element and the amplified current passes to the "output coil." At this point if the line west and the line east are similar in characteristics, the energy divides, half going to the east and half to the west. The half that goes west, that is, towards the direction from which the transmission came, serves no useful purpose. The half that goes east (combined with the small amount of energy that passed directly through the repeater) may be many times larger than the original energy coming from line west and is the useful amplified transmission.

When the lines west and east are identical in characteristics, it is evident by symmetry that there is no voltage set up across the input circuit, and there is, therefore, no possibility of a "singing" condition no matter how large amplifications are employed. This circuit therefore permits two-way transmission over similar lines.

In case, however, the line west and the line east are different in their impedance characteristics for any part of the telephone frequency range, then for such frequencies there will be voltages set up across the input circuit due to current flowing from the output circuit and a tendency towards singing and distortion. The conditions which hold when such unbalances exist will be discussed later.

The duplex bridge circuit, as we saw from our earliest discussion, is applicable as a terminal circuit whereby two-way transmission may be had over a single pair of wires. (See Fig. 3). In such operation the telephone line is balanced by the inclusion in the opposite arm of the bridge of an artificial line of similar impedance-frequency characteristic. At a repeater station we may, therefore, terminate both incoming lines by bridge circuits, balance each line by an artificial line, and operate both lines as a through circuit for two-way conversations provided we connect the input branch of each bridge to the output branch of the other. In connecting the input terminals of one bridge circuit to the output terminals of the other bridge circuit we may introduce a one-way amplifier as shown in Fig. 30.

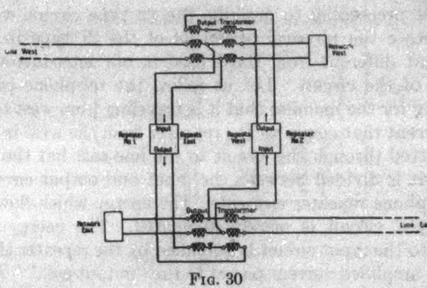


FIG. 30

The resulting circuit is the 22-type defined above, although its evolution did not follow the line which we have just followed for simplicity of discussion. It will be noted that this circuit makes use of a separate repeater element for transmission west to east from that used for transmission east to west, that it requires two "output transformers," and that each line, instead of being balanced by the line in the opposite direction, is balanced by an artificial line indicated as "network west" or "network east." Assuming again the transmission to be coming from the west, we trace the currents through the repeater circuit as follows: A small part of the energy goes through into the network west; the largest part, however, is divided between the input circuit of repeater No. 1 and the output circuit of repeater No. 2. That part which enters the output circuit of repeater No. 2 is dissipated without doing useful work. The part reaching the input of repeater No. 1 is amplified and the amplified energy flows through the output transformer connected to line east. The "network east" is made to have as nearly as possible the same impedance as the actual line for the full range of telephone frequencies. If the balance were perfect, there would be no voltage whatever set up across the input circuit of repeater No. 2, and therefore no possibility of setting up a singing condition whatever values of amplification were used.

Actually, however the balance can never be absolutely perfect, so that there will be a certain voltage set up across this input. This will, in turn, be amplified by repeater No. 2. These amplified currents will then flow through the output transformer connected to the line west. If the "network west" has exactly the same impedance as the line west, then the energy will equally divide between the two and there will be no voltage set up across the input circuit of repeater No. 1. This condition will then prevent any possibility of a singing condition being set up. Again, however, there will be in any practical case some unbalance so that a small voltage will be set up across this input circuit. If the two unbalances involved (*i. e.* line east against "network east" and line west against "network west") are sufficiently large, it is evident that the telephone currents have a closed path through the two repeaters and through these unbalances so that a singing condition may be set up.

This circuit is, however, very much more stable than the 21-type circuit. It will be noted that if either of the networks exactly balances its line, there may be any degree whatever of unbalance between the other network and its line without permitting singing. In general, the fact that the singing condition requires two unbalances simultaneously leads to a more stable condition.

There is another property of the 22-type circuit which is of great importance in connection with tandem operation of repeaters, that is, when two or more repeaters are used at different points in a line. As pointed out, transmission coming from the line west and reaching a 21-type repeater, as in Fig. 28, will result in an amplified current being sent out towards line east; but it will also result in an equally large current being sent back into line west. In the case of the 22-type, however, if the transmission comes in from line west and the balance between network east and line east is good, there will be very little energy sent back into line west. As will be pointed out in more detail below, under "Tandem Operation of Repeaters," this feature has a large advantage as the energy sent back into line west would react on any other repeater which was in circuit in that direction.

A 22-type circuit with vacuum tube repeater elements connected into it is shown in Fig. 31.

In connection with the presentation of the 22-type circuit it is interesting to note that this circuit, which is today the best known means of producing two-way telephone repeater operation with one-way amplifying devices, is the invention of Mr. W. L. Richards, one of the engineers of the Bell System, who devised and patented the arrangement in 1895—years before successful telephone repeaters were available for utilization in telephone systems.

BALANCING OF LINES

As pointed out above, the 21-type circuit depends for its stable operation on the balancing of the telephone line east against the telephone line west, and the 22-type circuit depends on the balancing of each of the telephone lines against an artificial line or "network." "Balancing" requires that the two lines or the line and its artificial line which are balanced against each other shall have closely the same impedance for the whole range of telephone frequencies.

Assuming that we have repeater elements which can give us sufficient amplification without appreciably affecting quality it is evident that the amount of amplification which we can get in any practical case depends on the degree to which this balance condition can be carried. This in turn, as will be more evident after reading the further discussion, depends on line conditions. It is then the line conditions, rather than the repeater element or its circuits which finally limit in every practical case the amplifications which can be obtained with a repeater.

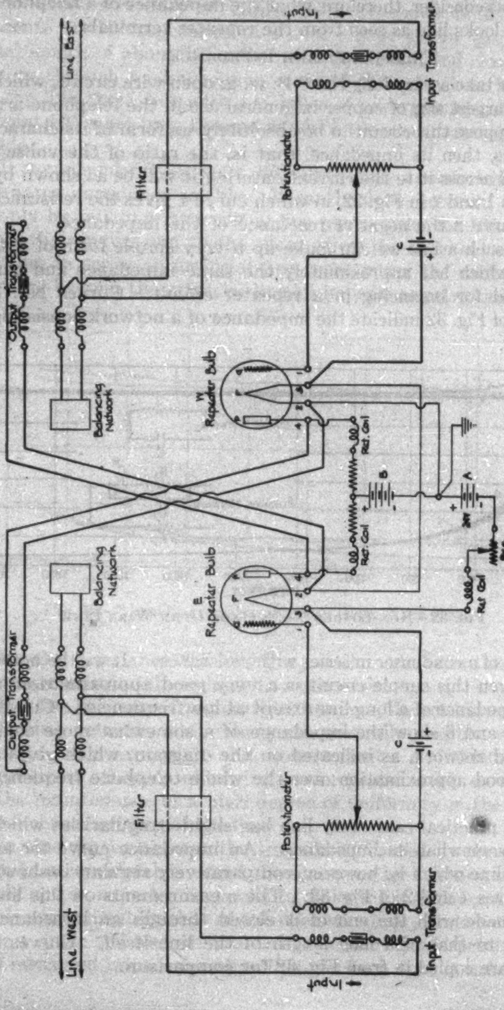

Input
Line East
Input Transformer
Filter
Potentiometer
C
Output Transformer
Balancing Network
W Repeater Bulb
Ret. Coil
B
A
Rheo
Ret Coil
E Repeater Bulb
Balancing Network
Output Transformer
C
Filter
Potentiometer
Input Transformer
Line West
Input

FIG. 31

Let us consider, therefore, what the impedance of a telephone circuit looks like as seen from the repeater terminals.

TELEPHONE LINE IMPEDANCE

If we take a very long No. 8 B. w. g. open wire circuit, which is the largest size of copper in general use in the telephone art, and suppose this circuit to be absolutely uniform in its characteristics, then its impedance, that is, the ratio of the voltage applied across it to the current entering it will be as shown by Curves 1 and 2 in Fig. 32, in which curve 1 gives the resistance and curve 2 the negative reactance of the impedance.

For such a line we can make up a very simple form of network which has approximately the same impedance and can be used for balancing in a repeater circuit. Curves No. 3 and 4 of Fig. 32 indicate the impedance of a network consisting merely of a condenser in series with resistance. It will be noted that even this simple circuit is a very good approximation of the impedance of a long line except at low frequencies. Curves Nos. 5 and 6 show the impedance of a somewhat more complicated network, as indicated on the diagram, which gives a very good approximation over the whole telephone frequency range.

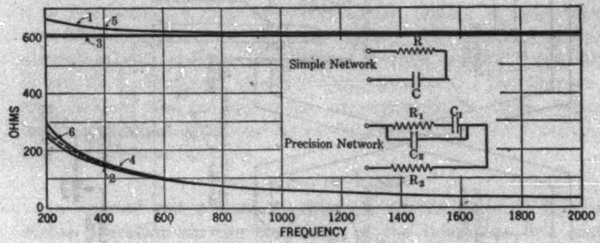


FIG. 32—NON-LOADED NO. 8 GAGE OPEN WIRE LINE

In a practical case every line has slight irregularities which affect somewhat its impedance. An impedance curve for an actual line which is, however, comparatively regular, is shown by curves 1 and 2 of Fig. 33. The measurements on this line were made with the end of it closed through an impedance similar to that of a long length of the line itself. Curves 5 and 6 are copied in from Fig. 32 for comparison.

It is difficult, however, to keep a long open wire circuit free from short stretches of cable. Fig. 34 shows both components of the impedance of a No. 8 gage circuit having a total length of about 165 miles with the distant end closed as before through an impedance equal to that of a long length of the line itself, but with 1¼ miles of underground cable at a point 105 miles from the end at which the measurements were made.

It would evidently be more difficult to make a network having the same impedance as this. It can be done, however, fairly simply by building an artificial line representing the 105 miles from the sending end to the place where the cable is located, putting in series an artificial line representing the cable, and putting beyond the artificial line an impedance network simulating the impedance of a long line, which might be in this case merely a resistance in series with a condenser.

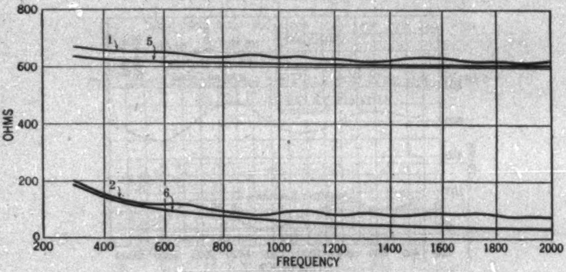


Fig. 33—Non-Loaded No. 8 Gage Open Wire Line

If a line has many irregularities, however, the setting up of a balancing network, while always theoretically possible, would become very expensive and inconvenient. A fundamental requirement, then, in obtaining good repeater action on a line is the maintenance of a high degree of uniformity in the electrical characteristics of the line.

In this particular case the irregularity caused by the cable can, to a considerable extent, be overcome by inserting in the cable one or more loading coils to make the impedance of the cable approximately that of the open wire line. In this actual case a single coil was cut into the cable near its mid-point, and the measured impedance after this had been done is indicated in Fig. 35. In this figure the curves after loading are shown in heavy lines and the old curves are indicated in lighter lines for comparison.

In the case of loaded lines the problem of maintaining sufficient regularity in the electrical characteristics to give a satisfactory impedance curve becomes considerably more difficult, due to variations in the inductances of the loading coils and in the distances between these coils. To illustrate this a number of impedance curves of loaded circuits will be given showing the effects of their irregularities on the impedance characteristics. In these curves, for simplicity, only the resistance component will be shown, as the reactive component is considerably smaller and goes through about the same irregularities as does the resistance curve.

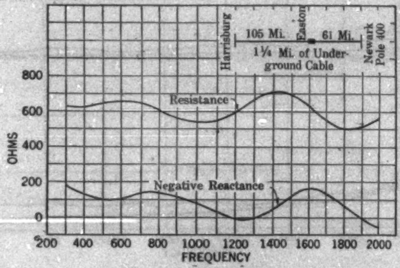


Fig. 34—Harrisburg to Pole 400 Newark, Wires 23/24 Coil out at Easton

Fig. 36 shows an impedance characteristic of an open wire line loaded at approximately 8-mile intervals with loading coils having approximately ¼ henry each. This circuit would give commercial service from a transmission standpoint. It will be noted, however, that its impedance characteristic is extremely irregular, and it would be difficult to obtain gains from a telephone repeater operated in conjunction with it.

Fig. 37 shows the resistance of a circuit of similar constants representing a less extreme condition of irregularity. This would have been considered, before the advent of repeaters, a reasonably good circuit so far as irregularity is concerned. These irregularities were due largely to comparatively small variations in spacing of loading coils and in the inductances of the loading coils. Loading coils have to be placed on a telephone circuit at particular points in the transposition layout of the circuit in order to prevent crosstalk between it and other circuits on the same lead. It is difficult, however, exactly to space transpositions in view of the variations in the route taken by a line, lengths of cable, junction points with other lines, etc.

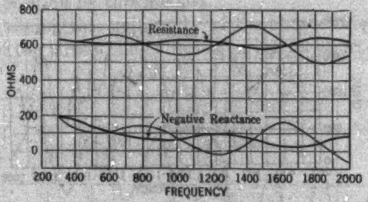


Fig. 35—Harrisburg to Pole 400 Newark, Wires 23/24 Coil in at Easton

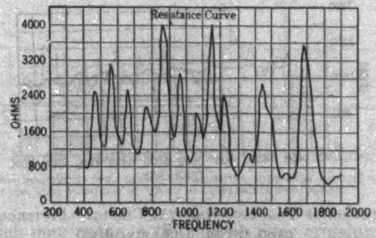


Fig. 36—Section of Transcontinental Circuit Pittsburgh to New York before "Clearing Up"

This led in the past to variations in the lengths between the successive loading points. Aerial loading coils are exposed to current surges due to lightning, and with some of the types of coils which have been put into use this has led to variations in their inductances. For any given frequency it may be that the effect of a large number of such irregularities are cumulative in raising the impedance of the circuit for that frequency, whereas with another frequency only slightly removed from the first, the effect of the irregularities may be to reduce the impedance. The result then with such impedance is indicated in Fig. 37 where the variation was due not to a few large irregularities, but to a large number of small irregularities.

This particular circuit was one of those entering into the transcontinental line, so that it was very carefully gone over, retransposed where necessary, the loading coils replaced by others having a particularly high degree of stability and uniformity and the spacings made very uniform. The result of this work is indicated in Fig. 38 in which the dark line shows the impedance after this work had been done and the light line is a repetition of Fig. 37 for comparison. The circuit will now permit of large gains with repeaters.

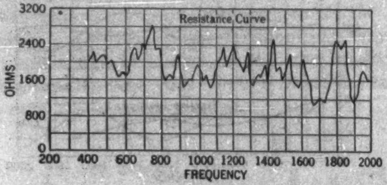


FIG. 37—SECTION OF TRANSCONTINENTAL CIRCUIT PITTSBURGH TO CHICAGO BEFORE "CLEARING UP"

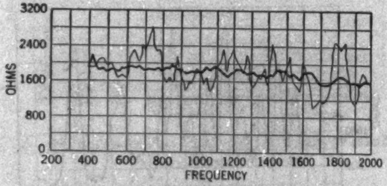


FIG. 38—SECTION OF TRANSCONTINENTAL CIRCUIT—PITTSBURGH TO CHICAGO AFTER "CLEARING UP"

In the cases where a large variation in impedance is due to one or more large irregularities, the impedance curves often offer a very interesting and convenient method of determining the point at which irregularity exists. For example: Fig. 39 shows the impedance of a circuit as measured from one end, the large wave-like variation being due to defective apparatus at some point in the circuit. By measuring the frequency range between successive "humps" on this "wave" it is possible to determine the distance from the sending end to the point of irregularity. In practical work many cases have arisen where omitted or defective coils or other irregularities have been located in position by this method.

In maintaining this uniformity of the characteristics of the line, one of the largest difficulties is caring for short lengths of cable which may occur in the line at river crossings, in going through towns, etc. It has sometimes been difficult to explain to those outside of the telephone field, but who are acquainted with the fact that by loading and telephone repeaters we can talk through long lengths of cable, why it is that we have so much difficulty with short lengths of cable which come into our open wire circuits, and why we are averse to cutting such short lengths into important long distance lines at villages and at crossings of railroads, power lines, etc. Considering, however, that a 500-ft. length of cable has as much capacity as three-quarters of a mile of open wire, it is evident that even a short length of cable has a very large effect on the operation of any

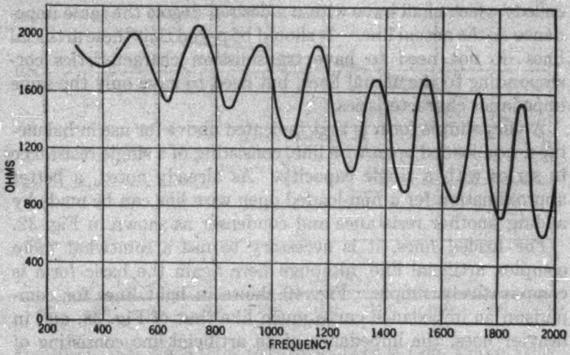


Fig. 39—Loaded No. 8 Gage Open Wire Line

Note: This irregularity was caused by defective apparatus located at a distance of about 15 sections from terminals at which measurements were made.

repeater on the line. In laying out a circuit, such short lengths of cable can be partially corrected for in the spacing of the loading coils. Changes made after a line is laid out are of course much more difficult to take care of.

In cable circuits the problem of maintaining uniformity is similar to that of open wire lines, requiring very accurate spacing of the loading coils and requiring loading coils of very uniform characteristics and of high electrical stability.

BALANCING ARTIFICIAL LINES

Assuming, then, in any case, that the lines have been made as uniform electrically as is economical, there is the problem of constructing artificial lines (or networks, as they are generally called) which shall have within a desired degree the same impedance as the actual line. It should be noted that these artificial lines do not need to have transmission characteristics corresponding to the actual lines, but need to have only the same impedance characteristics.

A very simple form is that indicated above for use in balancing a non-loaded open wire line, consisting of a single resistance in series with a single capacity. As already noted, a better approximation for a non-loaded open wire line can be made by adding another resistance and condenser as shown in Fig. 32.

For loaded lines, it is necessary to use a somewhat more complex artificial line, although here again the basic form is comparatively simple. Fig. 40 shows in light lines for comparison an impedance curve much like that of Fig. 38, and in heavier lines, the impedance of an artificial line consisting of two condensers, a resistance and two inductance coils arranged as indicated by the diagram in this figure. For completeness both the resistance and reactance of the impedance are shown in the figure.

In the case of a loaded line the impedance depends very markedly on the place in the loading section from which the circuit is measured. In practise lines are generally terminated either at mid-section or at mid-coil, since these points are symmetrical points in the loading and make for maximum flexibility in connecting circuits together. Most of the figures in this paper were taken at mid-coil.

It is evidently necessary that the balancing line should balance not only the actual line itself, but any apparatus between the line and the point at which the repeater is applied, so that all coils, composite sets or other devices used in series with the line must be, with the necessary precision, balanced in the artificial lines by corresponding coils, condensers, etc.

Tandem Operation of Repeaters

The above discussion has referred particularly to the operation of a single repeater in a circuit. Where several repeaters are used in tandem at different points in a circuit, the same considerations are involved. In addition there is the further consideration that not only can circulating currents be set up involving each repeater by itself, and which may lead to "howling," or to distortion in quality, but such circulating currents may follow paths involving any two or more of the repeaters operating in conjunction. As noted above, the 22 type circuit involving artificial balancing lines has a large advantage over the 21 type for tandem operation, due to the fact that transmission entering it from either direction does not cause a large amount of energy to be thrown back in the same direction, as is the case with the 21 type circuit. There is thus very much less interaction between the repeaters with the 22 type.

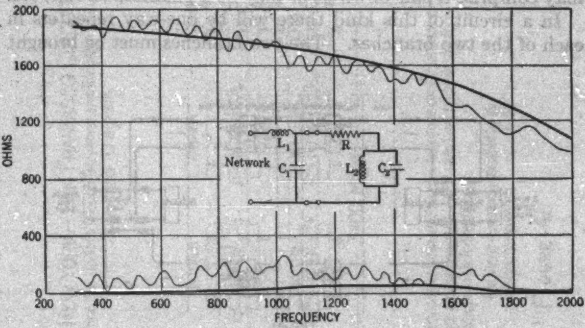


Fig. 40—Loaded No. 8 Gage Open Wire Line ½ Coil Termination

Effective tandem operation of repeaters requires, therefore, a high degree of electrical uniformity, and also requires that the reflection effects between the repeaters and their connected lines shall be reduced as much as possible, so that each repeater brings in as little irregularity to its adjacent repeaters as possible.

THE FOUR-WIRE CIRCUIT

In the above we have considered the application of repeaters in ordinary circuits in which the same pair of wires is used for transmission east as is used for transmission west. After the above discussion regarding balance, it is evident that there would be considerable advantage in transmission if separate pairs of wires were used for transmission east and for transmission west. A circuit of this type, designated as a "four-wire circuit," is shown diagrammatically in Fig. 41.

The term "four-wire" does not mean necessarily that four wires are used, but that different circuits are used for transmission east and transmission west. Each of these circuits may comprise a pair of wires, or may be a phantom circuit, etc.

In a circuit of this kind there will be one-way repeaters in each of the two branches. The two branches must be brought together at the ends for connection to two-wire lines, or to the subscribers' loops. There is, therefore, a possibility for circulating currents to be set up flowing over the two sides of the circuit in series. In order to minimize such circulating currents, artificial balancing lines are used at the points where the four wire connects to the two-wire in a manner similar to the arrangement of the 22-type circuit. It should be noticed that if the total amplification of both branches of the four-wire in series is less than the total transmission equivalent, the circuit cannot "sing." If the balance at the ends is good, the gains may be even greater than the transmission losses without producing singing. As in the case of two-wire operation, however, large distortion may be brought into the circuit by the

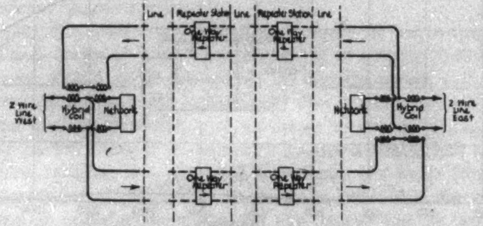
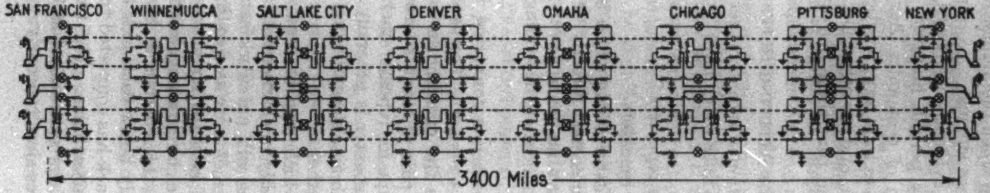

Line
Repeater Station
Line
Repeater Station
Line
One Way Repeater
One Way Repeater
One Way Repeater
One Way Repeater
2 Wire Line West
Hybrid Coil
Network
Network
Hybrid Coil
2 Wire Line East

FIG. 41.

NEW YORK – SAN FRANCISCO CIRCUIT
When Set Up for Through Telephone and Telegraph Circuits
Simplified Diagram


SAN FRANCISCO
WINNEMUCCA
SALT LAKE CITY
DENVER
OMAHA
CHICAGO
PITTSBURG
NEW YORK
3400 Miles

Line wire No. 8 B.W.G. copper
Inductance of side circuit loading coils 0.25 henry
" " phantom " " " 0.15 "
Spacing of loading coils 8 miles

Line equivalent = 56 miles of standard #19 ga. cable
Apparatus loss = 7 " " " " " "
Total equivalent = 63 " " " " " "
Total repeater gain = 42 " " " " " "
Resultant equivalent = 21 " " " " " "

This circuit provides for three simultaneous telephone conversations between New York and San Francisco. At the same time it provides for four telegraph messages in each direction.

Telephone repeaters indicated by ⊠
Telegraph apparatus " " ⊗

FIGURE 42

circulating currents even when the amplifications and balance are not such as to produce singing.

Circuits of this kind evidently use twice as many wires as ordinary circuits. They permit, however, the use of smaller wires. It is a question of economics then, as to whether a circuit of this kind proves in in any practical case. For long distance cable operation, and for other special cases, there are conditions where we find that these four-wire circuits have a field of use.

METHOD OF APPLYING REPEATERS

In the practical operation of repeaters there are two methods of connecting them into lines which have come into general use. These are:

(a) "Through line repeaters."
(b) "Cord circuit repeaters."

The through line repeater is one which is associated with a particular line and connected directly into the line. The operators, therefore, have no control over it. It is generally set to give a definite amplification, depending on the characteristics of the line.

The cord circuit repeater is connected into a cord circuit under the control of an operator. The two ends of the circuit generally appear before the operator as two plugs which she may connect into any pair of circuits terminating before her which are in proper shape for repeater operation. The amplification given by the repeater is generally under her control, the amplification in each case depending on the pair of circuits which is connected together.

Where the circuits which may be used with a cord circuit repeater vary considerably in characteristics, so that the same artificial balancing lines will not work satisfactorily with all of them, various arrangements are used so that proper balancing lines will be connected into the circuit corresponding to the actual lines to which the circuit is connected. The commonest arrangement for doing this consists in having a jack in the switchboard immediately below each jack to which a toll line connects. This extra jack has connected to it an artificial line corresponding to the actual line. The repeater circuit then terminates in double plugs so arranged as to pick up both the actual and artificial lines simultaneously.

PRACTICAL RESULTS OBTAINED

As already noted, the amount of gain which may be obtained in any practical case now depends almost entirely on the degree of electrical "uniformity" which it is economical to maintain in the telephone circuits to which the repeaters are connected, since we can obtain within practical limits as large amplifications in the repeater elements as we desire.

We have already indicated the large effect which this electrical uniformity requirement has had on the design of loading coils, and on practises with regard to their use. About every feature of open wire and cable construction and office equipment which affects the electrical characteristics of the circuits has been in some way affected by this repeater requirement. Not only in the design and construction, but also in the daily operation and maintenance of circuits with repeaters, this requirement has to be kept constantly in mind.

At the present time more than 1000 telephone repeaters are in service in the plant. We now will describe a number of typical repeater circuits and will indicate the results which are being obtained.

TRANSCONTINENTAL LINE

The circuit over which transmission is given between the cities along the eastern coast and the Pacific coast cities is shown diagrammatically in Fig. 42. Originally there were three repeaters used in this circuit, located at Pittsburgh, Omaha and Salt Lake City. Since that date, largely for flexibility in switching traffic, the number has been increased to six located at Pittsburgh, Chicago, Omaha, Denver, Salt Lake City and Winnemucca. The total gains were only slightly increased by the change. The circuit consists of No. 8 B. w. g. copper conductors loaded at eight-mile intervals with coils of one-fourth henry each. This gives a line equivalent of 56 miles of standard 19 gage cable. There is a further transmission loss due to composite and other apparatus of about 7 miles, the total equivalent being, therefore, about 63 miles. This is reduced by repeaters to a resultant equivalent of 21 miles, which is, therefore, about one-third of the equivalent without repeaters.

The drawing shows the important points on the line, indicates the positions of telephone and telegraph repeaters and indicates the compositing apparatus used in separating the telegraph and telephone currents.

BOSTON-WASHINGTON CABLE

The largest use of repeaters on cables has been in connection with the circuits between the various cities which are served through the underground cable route extending from Boston through Providence, Hartford, New Haven, New York, Philadelphia and Baltimore to Washington. The use of repeaters on small gage cables placed in this subway became of the greatest importance during the war in caring for the tremendously increased requirements for telephone facilities between these large cities along the Atlantic seaboard.

Most of these telephone repeaters are located in the large cities along the route of this cable. At two places, however, Princeton, N. J., and Elkton, Md., the repeater requirements were such as to justify the putting up of buildings especially for repeater operation. Fig. 43 shows an illustration of the building which was put up at Princeton. There are now about 200 repeaters working at Princeton. The ultimate capacity of the building will be over 500 repeaters.

Fig. 44 shows a typical group of repeater racks, each rack carrying two complete repeaters. Fig. 45 shows at the left, front and back views of some of these repeater racks. At the right it shows the face of a test board from which the different repeaters and associated circuits may be tested. Fig. 46 shows the power board where the operating currents and potentials applied to the repeaters are measured.

The repeaters shown in the above photographs are of the vacuum tube type. Fig. 47 shows a front view of an installation of the mechanical type repeaters at Providence, R. I. Fig. 48 shows a rear view of the repeater racks.

The longest cable circuits set up along this route are those from Boston to Washington, a total distance of about 455 miles. A typical layout for a circuit of this kind consists of loaded conductors, No. 10 B. & S. gage from Boston to New York, No. 13 gage from New York to Philadelphia and No. 10 gage from Philadelphia to Washington. This circuit has an equivalent of about 30 miles of No. 19 gage standard cable, which by means of repeaters at Hartford and Philadelphia is reduced to about 14.5 miles. It should be noted here that No. 10 gage conductors, are somewhat larger conductors than would now be placed for this business, and are being used because they were available, having been placed before the use of repeaters had been developed to their present effectiveness.

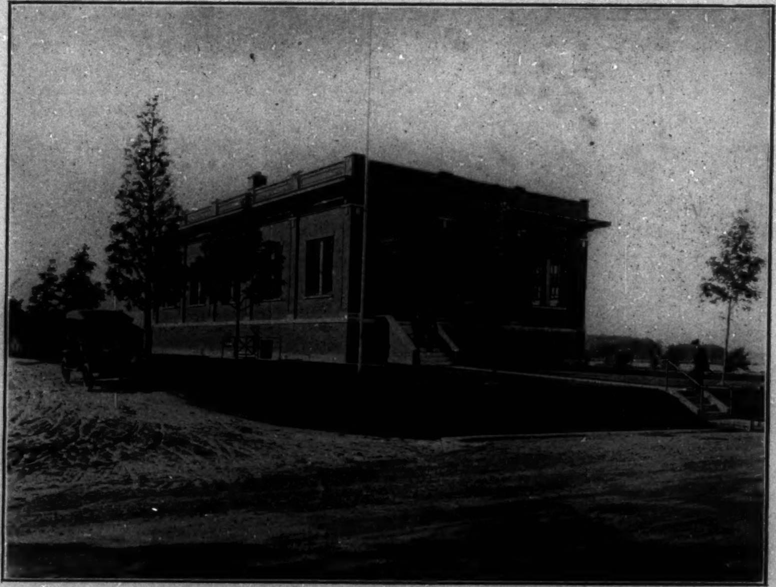


FIG. 43

[GHERARDI AND JEWETT[

PLATE LII
A. I. E. E.
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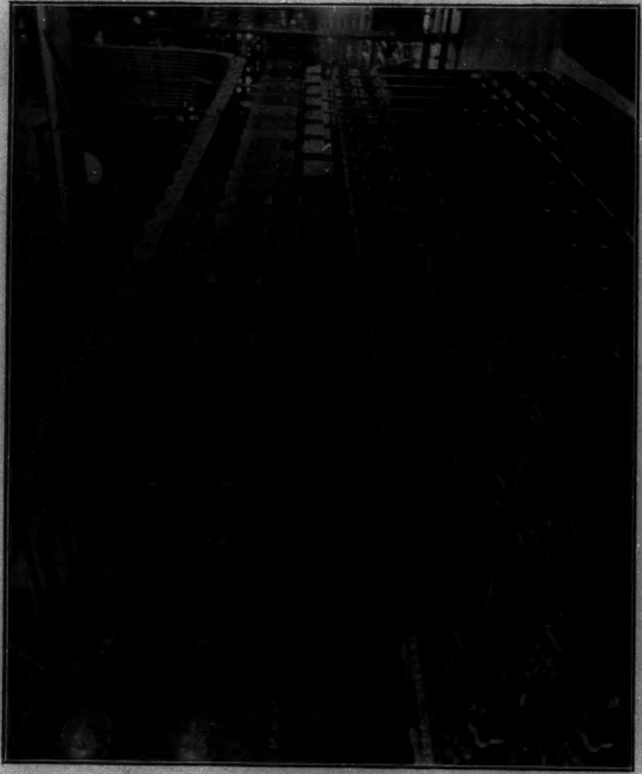
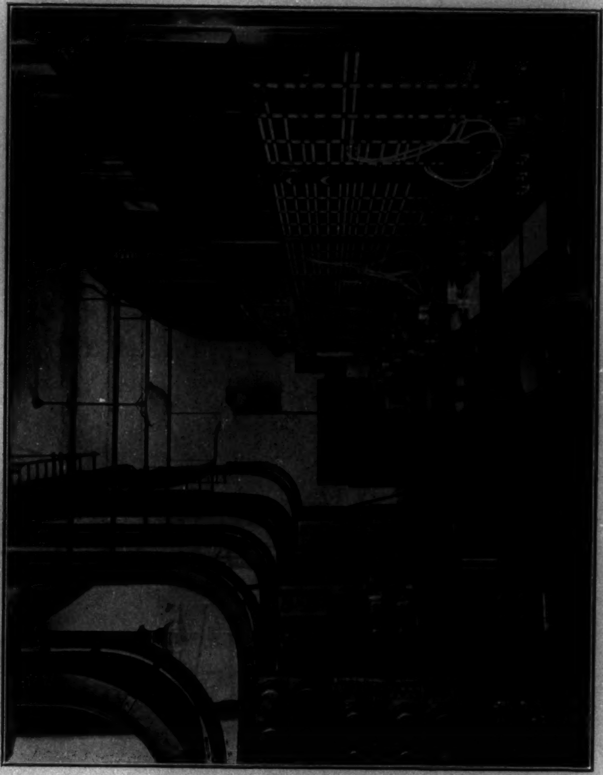


[GHERARDI AND JEWETT]

FIG. 44



[GHERARDI AND JEWETT]

FIG. 45

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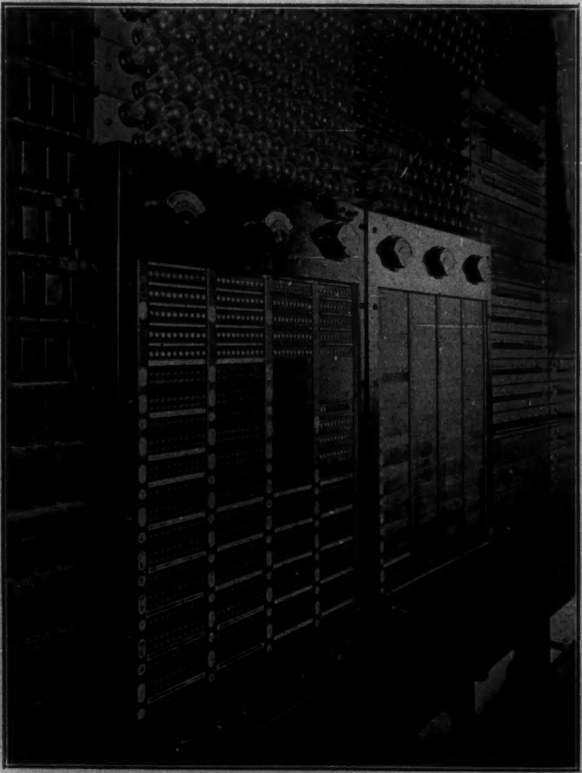

FIG. 46

[GHERARDI AND JEWETT]

PLATE LV.
A. I. E. E.
VOL. XXXVIII 1919

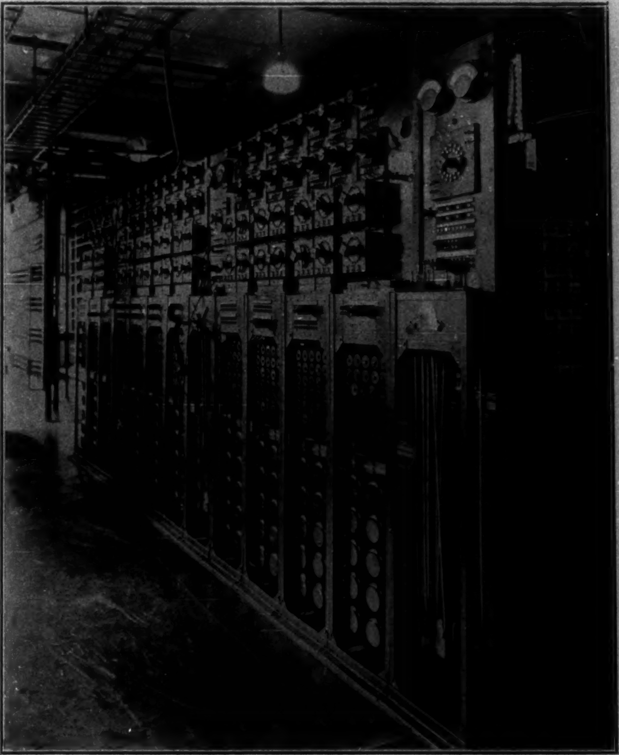

Fig. 47 [GHERARDI AND JEWETT]

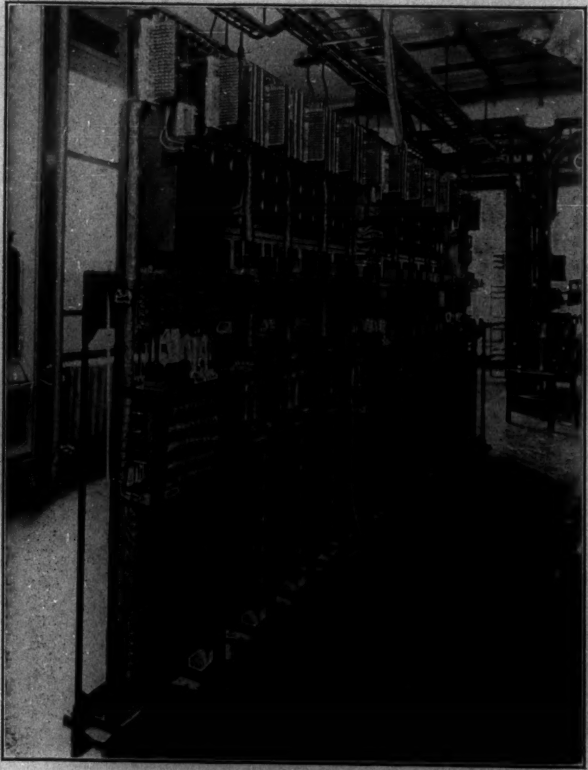

FIG. 48 [GHERARDI AND JEWETT]

The longest circuits electrically, that is, those having the greatest attenuation, which have been operated with repeaters along this route, are New York-Washington circuits, consisting of No. 19 gage cable loaded with coils of about one-fourth henry inductance, spaced at intervals of about 1.16 miles. The distance is about 225 miles. This gives a line equivalent of about 62 miles standard cable. By the use of four repeaters at Princeton, Philadelphia, Elkton, and Baltimore, the equivalent of this circuit is reduced to about 13 miles of standard cable.

The above circuits make use of the 22-type repeaters. Where there is but one repeater in a circuit, however, the 21-type gives, under many conditions, satisfactory service. For example, between New York and Philadelphia, a distance of about 90 miles, a group of circuits is being worked on No. 19 gage conductors, with a single 21-type repeater at Princeton. The circuit without repeaters has an equivalent of about 24 miles of standard No. 19 gage cable. The repeater reduces this to about 14 miles.

SUMMARY

In the foregoing pages we have endeavored to present a short history of telephone repeater development, a statement of the factors which control their successful operation, an idea of the possibilities and limitations of different types of repeaters and repeater circuits, and three or four typical examples of the present commercial line combinations in which telephone repeaters are vital links in the every-day telephone service of the public. These, we believe, will give the members of the Institute a fair picture of the state of the art as it now exists, both technically and commercially.

In concluding the paper it seems only necessary to add a word with regard to the further influence which the telephone repeater is likely to have on telephonic communication in the future.

As indicated in the earlier parts of the paper, the net result of the vast amount of work already done has been to produce amplifying devices capable of taking energy from a local source or sources and under the control of enfeebled telephonic currents from a distant transmitting station deliver back into the telephone line currents which are faithful reproductions of these enfeebled control currents but with many times their energy. Furthermore, the work already done has developed the methods which permit of utilizing the amplifying properties of these repeaters in the various kinds of telephone lines which go to make up the modern telephone network and methods of constructing and maintaining lines to secure maximum efficiency and economy from the use of repeaters. The tools thus at our disposal are of such a character as to permit of our forecasting the changes which their use is likely to produce in the evolutionary growth of the telephone plant. When combined with the benefits to be derived from the loading of cable circuits, the use of the practically perfect repeaters now available puts an entirely different aspect on many of the problems which have confronted the telephone engineer in the past. Not only will there be the opportunity to utilize smaller gage wires than has hitherto been possible but the economical field for underground and overhead toll cables will be greatly extended into the region which it has heretofore been possible to serve only with some form of open wire construction. Viewed from the standpoint of continuity of service alone, this is an advantage of almost inestimable value. Further, the range of reliable and commercially possible long distance telephony will be very greatly extended.

Numerous fields which have in the past been fruitful regions of experimentation have likewise been made obsolete by the development of the telephone repeater. Most noteworthy of these is that looking to the production of powerful or so-called loud speaking transmitters. Many investigators have in the past devoted much effort to the production of this type of equipment and for many years a large group of telephone engineers, notably those in Europe, looked upon the production of a practical loud speaking transmitter as a key to the solution of long distance telephony. Comprehensive studies by the engineers of the Bell System early convinced us that the solution did not lie in this direction and the success of the telephone repeater is a vindication of the adequacy of these early studies and of the consistent line of development which followed them.

In conclusion, the authors wish to express their appreciation to Mr. E. H. Colpitts, Assistant Chief Engineer of the Western Electric Company, Incorporated; Mr. O. B. Blackwell, Transmission Development Engineer of the American Telephone and Telegraph Company; Mr. A. B. Clark, of the American Telephone and Telegraph Company, and Mr. H. E.

Shreeve and Mr. John Mills of the Western Electric Company Incorporated, for the very great help they have rendered in the selection from a mass of information of the data for this paper and for their assistance in its preparation.

September 18, 1919

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PRINCIPLES OF RADIO TRANSMISSION AND RECEPTION WITH ANTENNA AND COIL AERIALS

BY J. H. DELLINGER

Abstract of Paper

Coil aerials are coming to replace the large antennas in radio work. The advantage of the coil aerial as a direction finder, interference preventer, reducer of strays and submarine aerial, make it important to know how effective such an aerial is as a transmitting and receiving device in comparison with the ordinary antenna. In this article the mathematical theory is presented and, as a result, the answer to this question is obtained. Experiments have verified the conclusions reached, and the formulas which are obtained are a valuable aid in the design of an aerial to fit any kind of radio station.

A great many questions and hazy ideas on the behavior of radio waves are cleared up by the study which was made and here presented.

It is found that the coil aerial is particularly desirable for communication on short wave lengths. A coil aerial is as powerful as an antenna only when its dimensions approach those of the antenna. For other reasons, however, a small coil aerial is in many cases as effective as a large antenna.

It is shown that an advantageous type of radio aerial is a condenser consisting of two large metal plates. This type of aerial has many of the advantages of the coil aerial.

The fundamental principles of design of aerials are given in this paper. On the basis of this work the actual functioning of any type of radio aerial can be determined either from measurements made upon the aerials or from actual transmission experiments.

The investigation has opened up a large and most interesting field for further research and progress in the utilization of radio waves.

I. INTRODUCTION

In a radio transmitting or receiving set, either the condenser or the inductance coil is made of large dimensions. It is then called the aerial, and effects the transfer of power between the radio circuits and the ether. The coil aerial has the inherent advantage of serving as a direction finder and interference preventer, but is less effective quantitatively as a transmitting or receiving device than the condenser type of aerial, commonly called the antenna. Both kinds of aerial are very simple in construction, consisting merely of one or more wires. An antenna consists of a wire or set of wires connected in parallel and constituting one plate of a condenser, the other plate being the ground beneath. The coil aerial is one or more turns of wire constituting a simple coil or loop. When an antenna is used its circuit is completed, in general, by placing an inductance coil in series with it and the ground; and when a coil aerial is used its circuit is completed by connecting a condenser across its terminals. The typical connections are shown in Figs. 1 and 2.

The antenna is used when it is desired to communicate over as great a range as possible or reduce the power of the apparatus as much as possible. The coil is used when directional properties are particularly important. The coil radiates and receives electric waves better in the direction of its plane than in the direction of its axis, whereas the performance of the antenna is much more nearly independent of direction of the waves. By arranging a coil so that it can rotate it makes an excellent direction finder. When thus used on a ship or an airplane, a coil aerial is sometimes called a radio compass. It has also been called a radio goniometer. By turning it so that its axis is parallel to the direction of propagation of the waves from some particular station, that wave is not received while waves from other directions are received. The coil may thus serve as an interference preventer. It is possible to attain some slight reduction of the effects of strays, commonly called static, by using combinations of coil aerials. Submarine communication is more successful with coil aerials than with antennas because the coil can be protected from the short-circuiting effect of the water while an antenna can not. The numerous advantages of the coil aerial make it highly im-

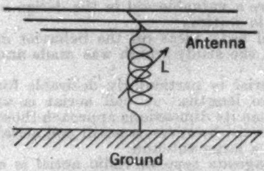

FIG. 1—SIMPLE ANTENNA CIRCUIT

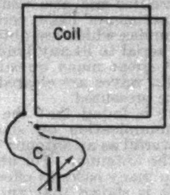

FIG. 2—SIMPLE COIL AERIAL CIRCUIT

portant to know the relative sensitiveness or power of transmission of the device in comparison with the antenna. This publication provides the answer to this question and sets forth the theory of radiation and reception and the action of antenna and coil aerials. The relative effectiveness of any coil and antenna is given by formula (**32**) to (**36**) in Sec. IV 1 below. The uses of the coil as a direction finder, interference preventer, reducer of strays, and submarine aerial, are not treated in this article.

The most important question considered is the practical one: How far can communication be maintained by the use of any specified antennas or coil aerials. Formulas are developed by which the current received in an antenna or coil is calculated in terms of the current in a transmitting antenna or coil, resistance of receiving aerial circuit, the distance, wave length, and dimensions of the aerials. The formulas have been found to be useful in the design of aerials and in the selection of an aerial for a particular kind of communication. They were worked out before there was any experimental information available to answer the question of the comparative quantitative value of the two kinds of aerials. Not much information on this has been obtained from experiment even yet (1919), but such experiments as have been made have substantiated the formulas. The work described in this paper was done in 1916 and 1917. The results were given in "Radio Transmission Formulas," a confidential paper of July, 1917, which was circulated in the Signal Corps and Navy. Publication was withheld during the war at the request of the Signal Corps. The formulas have also been given by the writer of the present paper on page 234 of "The Principles Underlying Radio Communication," 1918, Signal Corps Pamphlet No. 40, a book which can be purchased from the Superintendent of Documents, Washington, D. C.

Historical. The coil aerial and the condenser aerial (antenna) both date back to the first experimenter with the electric waves that make radio telegraphy possible. H. Hertz in 1888 used an open oscillator, which was the forerunner of the antenna, as his transmitting apparatus. For receiving he used a circle of wire, which was the first loop or coil aerial, and observed its directional properties.

The possibility of a loop or coil aerial as a transmitting device was discussed from the theoretical standpoint by G. F. Fitzgerald and later by J. A. Fleming, in *Electrician*, 59, pp. 936, 976, 1016; 1907. Fleming derived expressions for the radiated fields, using a curious theory in which the four sides of the coil were replaced by Hertzian doublets.

The use of a large loop or single-turn coil as an aerial in practical radio communication was described by G. Pickard in Proceedings of the Wireless Institute of America, 1, May 1, 1909. He discussed its properties both as a radiation and receiving aerial. He described its use as a direction finder, stating that he had determined directions with it to better than 1 degree.

In spite of this work and proposals by others, the antenna was used almost exclusively as the transmitting and receiving device until 1913. The use of the coil aerial received a great impetus by the publication of an article by F. Braun in *Jahrbuch der drahtlosen Telegraphie und Telephonie*, 8, p. 1, 1914; on the Use of Closed Circuits in Place of Open in Radio Telegraphy. He discussed the advantages of a coil aerial as a receiver and transmitter, both from the theoretical and the experimental standpoint.

Since 1913 there has been a great deal of development work done on coil aerials and they were extensively used in the war. The development of the coil aerial as a practical direction finder and receiving device was begun at the Bureau of Standards in 1915. Using electron tubes as the detecting apparatus, transatlantic signals were received on a coil inside a room. Experiments with the coil as a transmitting device were carried out at the Bureau in 1917. Among the very few published treatments of development and use of the coil aerial are those in Bucher's text-book, "Practical Wireless Telegraphy" 1917, p. 256; and "Radio Direction-Finding Apparatus" by A. S. Blatterman, *Electrical World*, 73, p. 464; 1919. Most of the descriptions to date have been confidential reports of the military services of various countries.

The theoretical discussions by Fleming and Braun are cumbersome and needlessly complicated and the results are not well adapted to practical use. The present paper presents an original treatment that is relatively very simple but none the less exact and leads to conclusions that apply directly to practical work. This paper also points out a number of misconceptions that have existed, and endeavors to clear up some of the controversial points on the radiation of waves and the functioning of aerials.

II. DERIVATIONS OF THEORETICAL FORMULAS

1. Radiation from an Antenna

Formula (8) below, giving the radiated magnetic field at a distance from an antenna, is a well-known formula. It has been given by various writers, and is the only one presented in this paper that requires any deep consideration of fundamental electromagnetic theory. The result is in fact implicit in Maxwell's classical treatise, "Electricity and Magnetism". The derivation given here is much more direct and brief than the others the author has seen, and is given only for that reason. The derivations of formula (10) and following ones are still simpler, and will be of more interest to most readers.

The units used in this paper are international electric units, the ordinary electric units based on the ohm, ampere, centimeter, and second. (See paper by the author on "International System of Electric and Magnetic Units", Scientific Paper of the Bureau of Standards No. 292). The unit of magnetic field intensity is the gilbert per cm., often called the cgs. unit. The only exception to the use of units of the international system is in certain of the practical formulas where lengths are expressed in meters or miles where so stated.

In the following discussion is calculated the magnetic field intensity produced by a flat-top antenna, having electric current of uniform value throughout the length of the vertical portion. Most antennas in practise approximate closely this condition.

The symbols used are:

i = instantaneous current
I_0 = maximum value of current
I = effective value of current
H_t = instantaneous value of magnetic field intensity
H_0 = maximum value of magnetic field intensity
H = effective value of magnetic field intensity
h = height of aerial
d = distance from sending aerial
ω = 2π times frequency of the current
t = time
λ = wave length
c = velocity of electric waves = 3×10^{10} cm. per second

Subscripts s = sending, r = receiving, a = antenna, c = coil.

In Fig. 3 the upper heavy line represents the flat top of the antenna, and the lower heavy line the grounding area. Suppose a current is flowing, having the instantaneous value i in the vertical portion. The magnetic field intensity at any point due to a varying current is different from that due to a steady current. Consequently the field cannot be calculated in the same way that the magnetic field intensity of a straight wire is ordinarily calculated. When the current is varying, the magnetic field intensity is calculated by the aid of a quantity called the vector potential in such a way that the variation with time is taken into account. The instantaneous value of the vector potential of current in the vertical conductor at a distance d in a plane perpendicular to the conductor, is

$$A = \frac{[i]\,h}{d} \tag{1}$$

where $[i]$ indicates that for any time t the value of i is taken for the instant $(t - d/c)$.

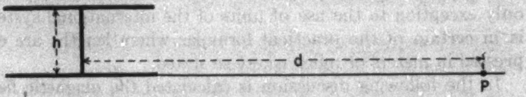

FIG. 3—CALCULATION OF MAGNETIC FIELD AT A DISTANCE FROM AN ANTENNA

Suppose the current in the antenna is a sine-wave alternating current,

$$i = I_0 \sin \omega t \tag{2}$$

$$\therefore [i] = I_0 \sin \omega\,(t - d/c)$$

$$A = \frac{h\,[i]}{d} = \frac{h\,I_0}{d} \sin \omega\,(t - d/c) \tag{3}$$

The magnetic field intensity is calculated from the vector potential by the general relation $H_t = 0.1$ curl A, which for this simple case of a straight conductor becomes

$$H_t = \frac{1}{10}\,\frac{\partial A}{\partial d} \tag{4}$$

the direction of H_t being perpendicular to the plane of h and d. From equation (3),

$$H_t = -\frac{h\,\omega\,I_0}{10\,c\,d} \cos \omega\,(t - d/c) - \frac{h\,I_0}{10\,d^2} \sin \omega\,(t - d/c) \tag{5}$$

This equation gives the magnetic field intensity at any point P at a distance d from the antenna. The second term represents the ordinary induction field associated with the current, while the first term is the radiation field. At a considerable distance the second term is negligible because the second power of d occurs in the denominator. The first term then represents the magnetic field radiated from an antenna at the distance d from the antenna. The distance d is measured along the earth's surface, because the waves follow the curvature of the earth's surface instead of proceeding straight out into space. For a considerable distance from the antenna, the maximum value of the magnetic field intensity during a cycle is therefore

$$H_0 = \frac{h\,\omega\,I_0}{10\,c\,d}$$

Expressing in terms of effective values,

$$H = \frac{h\,\omega\,I}{10\,c\,d} \tag{6}$$

Henceforth H means the radiated field unless it is specifically stated to be the total field. The last equation may be expressed in terms of wave length instead of ω by the relation

$$\frac{\omega}{c} = \frac{2\,\pi}{\lambda} \tag{7}$$

$$\therefore H = \frac{2\,\pi}{10}\;\frac{h\,I}{\lambda\,d}$$

Using the subscript s to indicate that it is the sending rather than the receiving antenna which is considered,

$$H = \frac{2\,\pi}{10}\;\frac{h_s\,I_s}{\lambda\,d} \tag{8}$$

This derivation follows the conceptions presented in the early pages of Lorentz, "The Theory of Electrons". It is equivalent to Hertz's intricate proof, but is more direct. The way in which the result is expressed here accords more closely with the physical ideas and with actual practise, being expressed in terms of current rather than electric charge, since it is current that is actually measured in an antenna and the current furthermore is generally uniform in the vertical portion of the antenna.

Formula (**8**) gives the radiated magnetic field from a sending antenna at a distance *d* along the earth's surface. The units are the gilbert per cm. for *H*, the ampere for *I*, and the centimeter for all lengths, as previously stated.

Undamped alternating current in the antenna was assumed. The same result, however, is obtained if the current is damped. At very great distances from the sending aerial, the magnetic field is less than that calculated by formula (**8**), because of absorption of the power of the wave as it travels along. This may be taken into account by multiplying the right-hand member of (**8**) by a correction factor F_1. The value of this factor for daytime transmission over the ocean, derived from the experiments of L. W. Austin, Scientific Paper of the *Bureau of Standards No.* 159; 1911, is

$$F_1 = \epsilon^{-0.000047\, d/\sqrt{\lambda}} \tag{9}$$

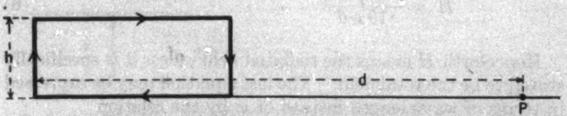

FIG. 4—CALCULATION OF MAGNETIC FIELD RADIATED FROM A COIL

for *d* and λ both in meters. This correction ordinarily needs to be applied only when the distance is greater than 100 kilometers.

2. RADIATION FROM A COIL

It was formerly the belief that a coil could not radiate, because the current up one side of the coil (Fig. 4) produces a field equal and opposite to that down the other side of the coil. This is erroneous because the two equal fields are not exactly opposite. The phase between the two departs from 180 deg. because of the finite time required for the field to be propagated from one side of the coil to the other. It is only along the axis of the coil that the calculated radiation is zero. The actual resultant field radiated from the coil may be deduced in either of two very simple ways, both of which are interesting from the physical standpoint. The first deals with the instantaneous values of the magnetic field, and the second with the effective values.

The following additional symbols are used:

l = horizontal length of coil aerial.
N = number of turns of wire of coil aerial.
θ = phase angle between values of field intensity a distance l apart in the wave.

First Deduction. Consider a rectangular coil of height h and horizontal length l. The magnetic field at a point P in the d direction is the resultant of the fields arising from current in the two vertical sides of the coil, the horizontal sides contributing nothing. The magnetic field at P due to any one of the vertical wires of the coil is calculated from equation (**5**) above. Neglecting the second terms, because d is large, the instantaneous values of the magnetic field (Fig. 4) at the distance d and $(d - l)$ respectively from the two vertical sides are

$$H_d = -\frac{h\,N\,\omega\,I_0}{10\,c\,d}\cos\omega\,(t - d/c)$$

$$H_{d-l} = +\frac{h\,N\,\omega\,I_0}{10\,c\,(d-l)}\cos\omega\left(t - \frac{d-l}{c}\right)$$

The resultant field H_t is the algebraic sum of these two, which becomes since $(d - l)$ is very nearly d when d is large,

$$H_t = -\frac{h\,N\,\omega\,I_0}{10\,c\,d}\,2\sin\omega\left(t - \frac{d - l/2}{c}\right)\sin\frac{\omega\,l}{2\,c}$$

The effective value of the resultant field is

$$H = \frac{2}{10}\,\frac{h\,N\,\omega\,I}{c\,d}\,\sin\frac{\omega\,l}{2\,c}$$

Using the relations

$$\frac{\omega}{c} = \frac{2\,\pi}{\lambda}$$

and $\sin\dfrac{\omega\,l}{2\,c} = \dfrac{\omega\,l}{2\,c}$,

the latter holding when the angle is small, *i. e.*, l small compared with the wave length,

$$H = \frac{4\,\pi^2}{10}\,\frac{h_s\,l_s\,N_s\,I_s}{\lambda^2\,d} \qquad (10)$$

This is the radiated magnetic field from a sending coil aerial at a distance d along the earth's surface, the direction of d being in the plane of the coil. The units are international units as stated under equation (**8**). The deduction assumes that the ground below the coil is not so good a conductor as to form an image of the coil. Thus the formula applies to a radiating coil in an airplane as well as to one at a ground or ship station.

The formula applies for either damped or undamped current I_s in the sending antenna. For very great distances the right-hand side of the formula must be multiplied by the distance correction factor F_1 given in (9), the same as for a radiating antenna.

Second Deduction. The radiated magnetic field due to one of the sides of the coil is $N_s H_1$, and from formula (**8**),

$$H_1 = \frac{2\pi}{10} \frac{h_s I_s}{\lambda d} \qquad \textbf{(11)}$$

If the two vertical sides of the coil coincide, their magnetic fields would be equal and opposite, as shown by the lines $O\,A$ and $O\,B$, Fig. 5. But since the two vertical sides are separated by the distance l, at any instant the field at P (Fig. 4) from the left side of the coil has traveled a distance l farther than the field from the right side. If then $N_s H_1$ is the field at P due to the right side, the field at P due to the left side is shifted in phase from the position $O\,B$ to $O\,C$ in Fig. 5, where the angle θ between them is the phase angle between the values of the field a distance l apart in the wave.

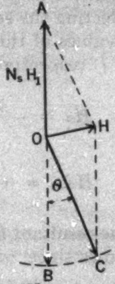

FIG. 5 — PHASE RELATIONS OF MAGNETIC FIELDS RADIATED FROM A COIL

The distance l is the same fraction of the wave length that the angle θ is of a complete cycle, 2π. That is,

$$\frac{\theta}{2\pi} = l_s / \lambda$$

or

$$\theta = 2\pi\ l/\lambda \qquad \textbf{(12)}$$

The resultant of $O\,A$ and $O\,C$ is their vector sum,

$$H = N_s H_1 \sqrt{2\,(1 - \cos\theta)} \qquad \textbf{(13)}$$

When θ is small, *i. e.*, l small compared with the wave length,

$$\frac{H}{N. H_1} = \sin\theta = \theta$$

$$\therefore H = N. H_1\,\theta \tag{14}$$

Thus the radiated magnetic field from a coil is equal to the field from one side of the coil multiplied by the phase angle θ corresponding to the distance l between the sides of the coil.

From (**14**) and (**11**),

$$H = \frac{2\,\pi}{10}\,\frac{h. N. I.}{\lambda\, d}\,\theta \tag{15}$$

This equation, together with (**12**), gives identically formula (**10**) obtained by the first deduction.

It was assumed in these deductions that the current was uniform throughout the coil. If the distributed capacity of the coil is appreciable the current in the coil will be different at different points. Thus the current in the middle may be greater than at the ends. This also may give rise to radiation from the coil, but is an entirely separate phenomenon from the phase angle between the two sides of the coil which has been discussed. This question of distributed capacity requires consideration particularly when coils are used having dimensions comparable with the wave length. The phenomenon is discussed further under "Antenna Effect" in Sec. VI 3 below.

3. RECEIVED CURRENT IN AN ANTENNA

The current flowing in the receiving aerial circuit when the field intensity of the wave traversing the aerial is known can be calculated in several ways. An electromagnetic wave in space has both an electric and a magnetic field intensity which are at right angles to each other and to the direction of propagation of the wave. The two field intensities are related to each other by

$$\mathcal{E} = 300.\, H \tag{16}$$

where $\mathcal{E}$ is in volts per cm. and H in gilberts per cm.

The following additional symbols are used in this and the following section:

$\mathcal{E}$ = electric field intensity
E = electromotive force in receiving aerial
R = resistance of receiving aerial circuit
ϕ = magnetic flux

First Deduction. The electromotive force, and thence the current produced in an antenna may be calculated from the principle that relative motion of a magnetic field and a conductor create an electromotive force in the conductor whose value is

$$E = 10^{-8}\, h_r\, Hc \tag{17}$$

when the directions of the field, the motion, and the conductor are mutually perpendicular, h_r being the length of the conductor and c the velocity of the relative motion. This then gives the e. m. f. in an antenna of height h_r, produced by electromagnetic wave having magnetic field intensity H and travelling with the velocity of c.

In ordinary practise, the reactance in series with the antenna is varied to produce resonance to the frequency of the incoming wave, so that

$$I_r = \frac{E}{R} \tag{18}$$

Inserting for c its value, 3×10^{10} in equation (**17**).

$$I_r = 300\ .\ \frac{h_r\, H}{R} \tag{19}$$

This is the current in amperes received in a flat-top antenna using the centimeter as the unit of length, with resistance of circuit in ohms, and the magnetic field intensity in gilberts per cm.

The received current is less than that given by the formula if the wave is damped, since an undamped alternating field was assumed in the discussion. For a damped field the e.m.f. acting on the aerial is similarly damped and equation (**18**) does not hold. Correct results are obtained by multiplying the right-hand side of formula (**19**) by the correction factor F_2 obtained as follows:

If the magnetic field intensity and hence the e. m. f. has the decrement δ', the effective current is not I_r, defined by (**18**), but another value which we shall call I_ρ. The value of I_ρ may be found by the aid of the generalized definition of decrement given in the author's paper, "The Measurement of Radio-Frequency Resistance, Phase Difference, and Decrement", *Proc. I. R. E.*, 7, p. 27; Feb. 1919.

For decrements smaller than about 0.2, the logarithmic decrement is one-half the ratio of the average energy dissipated per cycle to the average energy associated with the current at the maximum of the cycle.

Taking the average power as E^2/R, the average energy dissipated per cycle $= \frac{E^2}{f R}$. The average energy associated with the current at the maximum of each cycle $= L\, I_p^2$. The energy-ratio definition of decrement just given applies to the sum of the decrements acting, *viz.*, the decrement δ' of the e. m. f. and the decrement δ of the aerial circuit. The value of δ is $\frac{R}{2 f L}$. Applying the decrement definition

$$\delta' + \delta = \frac{\frac{E^2}{f R}}{2\, L\, I_p^2}$$

$$= \frac{E^2 R}{2 f L R^3 I_p^2}$$

$$= \frac{E^2 \delta}{R^2 I_p^2}$$

$$I_p^2 = \frac{E^2}{R^2} \, \frac{\delta}{\delta' + \delta}$$

From the relation, $E^2/R^2 = I_r^2$,

$$I_p = I_r \sqrt{\frac{\delta}{\delta' + \delta}} = I_r \sqrt{\frac{1}{1 + \frac{600.\, L\, \delta'}{R\, \lambda}}}$$

where L is in microhenries and λ is in meters.

This reduces to $I_p = I_r$ when δ' is small compared with δ. Thus in the particular case of an undamped wave, where $\delta' = 0$, no correction is needed.

Correct results are obtained from equation (**19**) for any damped wave by multiplying its right-hand member by the correction factor F_2, given by

$$F_2 = \sqrt{\frac{1}{1 + \frac{600\ .\, L\, \delta'}{R\, \lambda}}} \qquad \textbf{(20)}$$

where L is the inductance of the receiving aerial circuit in microhenries and λ is wave length in meters, and δ' is the logarithmic decrement of the damped wave that is being received.

Second Deduction. The same formula may be derived from entirely independent consideration of the electric field associated with the wave. The e. m. f. between two points in space is the product of the distance between them by the electric field intensity along the line joining them. Thus the e. m. f. produced in a flat-top antenna is $\mathcal{E}$ times the height, the direction of $\mathcal{E}$ being assumed to be vertical.

$$E = h_r \mathcal{E} \qquad \textbf{(21)}$$

Inserting the value of $\mathcal{E}$ from (**16**) and dividing by the resistance,

$$I_r = 300 \cdot \frac{h_r H}{R} \qquad \textbf{(22)}$$

This is identically the same formula obtained above from consideration of the magnetic field.

4. RECEIVED CURRENT IN A COIL

The current in a receiving coil aerial can be calculated in a number of different ways, all very simple and all giving the same result. The first conception which will be presented is simply that an e. m. f. is produced in the circuit by the time variation of magnetic flux through it.

The other modes of calculation involve the phase angle between the two vertical sides of the coil. The e. m. fs. acting in the two vertical sides are exactly equal and oppose each other in producing a current around the circuit when the plane of the coil is perpendicular to the direction of propagation of the wave. When the coil is turned in any other direction, however, the e. m. fs. in the two sides are not exactly opposite in phase because of the difference in time required for the field to be propagated to one side of the coil and to the other. The e. m. f. can be calculated either from the electric or the magnetic field, as in the discussion of received current in an antenna. The resultant e. m. f. can be found either from the algebraic sum of the instantaneous e. m. fs. in the two vertical sides or the vector sum of the effective e. m. fs. These two methods are used in the second and third deductions respectively, below.

The phase angle between the two sides of the coil is a very different thing from the phenomenon caused by the distributed capacity of the coil. It is assumed in the deductions given here that the current is uniform in all parts of the coil, which is not true when the distributed capacity is appreciable. Such capacity is large in coils of dimensions comparable with the wave length, and in such cases consideration must be given to the separate and additional phenomenon of distributed capacity.

First Deduction. Assuming that the dimensions of the coil are small compared with the wave length, the magnetic field intensity is practically uniform throughout the coil. When the plane of the coil is parallel to the direction of propagation of the wave, the e. m. f. induced in the coil is

$$E = 10^{-8}\,\omega\,\phi$$

$$\text{Now, } \phi = \mu\, h_r\, l_r\, N_r\, H$$

Since the permeability $\mu = 1$, and $I_r = E/R$ because in ordinary practise the condenser in series with the coil is adjusted to produce resonance with the frequency of the incoming wave,

$$I_r = 10^{-8}\,\frac{\omega\,\phi}{R} = 10^{-8}\,\frac{\omega\, h_r\, l_r\, N_r\, H}{R}$$

$$= 2\,\pi\, c\, 10^{-8}\,\frac{h_r\, l_r\, N_r\, H}{R\,\lambda}$$

$$I_r = 600\ .\ \pi\,\frac{h_r\, l_r\, N_r\, H}{R\,\lambda} \qquad (\mathbf{23})$$

This is the current received in a rectangular coil aerial of N turns, with its plane parallel to the propagation of the wave. The units are international units as stated under formula (**19**). No image is assumed in the ground, so the formula applies not only to a receiving coil at a ground or ship station but also to an airplane direction finder. The heights at which airplanes fly are such that the field of the wave is usually not much different from its value at the ground.

There are two correction factors that may need to be applied to this formula, both of which make the result smaller. If the wave is damped, the right-hand side of the formula should be multiplied by the decrement correction factor F_2, given by (**20**), the same as for a receiving antenna.

When the plane of coil is in some direction other than parallel to the direction of propagation of the wave, the right-hand side of formula (**23**) must be multiplied by the direction correction factor F_3 given by

$$F_3 = \cos \alpha \qquad (\mathbf{24})$$

where α is the angle between the direction of propagation of the wave and the plane of the coil.

Second Deduction. The e. m. f. produced in any one of the vertical wires of the coil is given by either equation (**17**) or (**21**) above, deduced from considerations of the action of the magnetic and the electric field intensity, respectively. Each of these equations reduces to

$$E_1 = 300 \,.\, h_r H \qquad (\mathbf{25})$$

The instantaneous e. m. f. in either of the two vertical sides of the coil is therefore

$$e' = 300\, h_r N_r H_0 \cos \omega t$$

The instantaneous e. m. f. in the other side of the coil is produced by the magnetic field existing in the wave a distance l away, when the plane of the coil is parallel to the direction of propagation of the wave. This e. m. f., e'', has the same direction in space but the opposite direction as far as producing current around the circuit is concerned.

$$e'' = -\,300 \,.\, h_r N_r H_0 \cos (t - l/c)$$

The resultant e. m. f. in the circuit is the algebraic sum of these two,

$$e = 300 \,.\, h_r N_r H_0 \, 2 \sin \omega \left(t - \frac{l}{2\,c} \right) \sin \frac{\omega\, l}{2\,c}$$

The effective value of the resultant e. m. f. is

$$E = 600 \,.\, h_r N_r H \sin \frac{\omega\, l}{2\,c}$$

Since when the angle is small, *i. e.*, l small compared with the wave length,

$$\sin \frac{\omega\, l}{2\,c} = \frac{\omega\, l}{2\,c} = \pi \,\frac{l}{\lambda}$$

$$E = 600 \,.\, \pi\, \frac{H_r l_r N_r H}{\lambda} \qquad (\mathbf{26})$$

Dividing by R, this gives the identical value of I_r obtained in (**23**) by the first mode of deduction.

Third Deduction. The e. m. f. produced in one of the sides of the coil is $N_r E_1$, where from either equation (**17**) or (**21**) above, *i. e.*, from consideration of either the magnetic or the electric field intensity, respectively,

$$E_1 = 300 \ . h_r H \tag{25}$$

If the two vertical sides of the coil coincided, the e. m. fs. produced in them would be equal and exactly neutralize each other, as shown by the lines $O\,A$ and $O\,B$, Fig. 6. But since the two vertical sides are separated by the distance l, at any instant the field acting on one side of the coil has traveled a distance l farther than that acting on the other side. If then $N_r E_1$ is the e. m. f. in one side of the coil, the e. m. f. in the other side is shifted in phase from the position $O\,B$ to the position $O\,C$ in Fig. 6, where the angle θ between them is the phase angle between the values of the field a distance l apart in the wave.

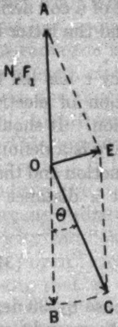

Fig. 6—Phase Relations of Electromotive Forces in Receiving Coil

The distance l is the same fraction of the wave length that the angle θ is of a complete cycle, $2\,\pi$, *i. e.*,

$$\frac{\theta}{2\,\pi} = \frac{l\,r}{\lambda} \tag{27}$$

The resultant of $O\,A$ and $O\,C$ is their vector sum

$$E = N_r E_1 \sqrt{2\,(1 - \cos\theta)} \tag{28}$$

When θ is small, *i. e.*, l small compared with λ

$$\frac{E}{N_r E_1} = \sin\theta = \theta$$

$$\therefore E = N_r E_1\,\theta \tag{29}$$

From (**25**),

$$E = 300\,h_r N_r H\,\theta \tag{30}$$

Thus the e. m. f. acting in the coil is equal to the e. m. f. in one side of the coil multiplied by the phase angle θ corresponding to the distance l between the sides of the coil.

Equations (**27**) and (**30**) combined give (**26**) and by dividing by R formula (**23**) is obtained.

III. DISCUSSION OF THEORY OF RADIATION AND RECEPTION

1. Distinction between Induction and Radiation

Certain fallacies which have appeared in text-books and discussions arise from insufficient understanding of the difference between an induction field and a radiation field. Such fallacies are:

a. An "open" circuit can radiate, while a "closed" circuit can not.

b. There is no radiation from a circuit at low frequencies.

c. Induction and radiation are the same phenomenon.

d. The action of an antenna differs from that of a coil aerial in that the former is due to electrostatic fields and the latter to magnetic fields.

These fallacies will now be discussed. Fallacy *c* has led to the supposition that the radiation and reception of electric waves can be taught in terms of transformer action. It should not be difficult to separate the two ideas, for there is a definite and clear distinction between the field due to induction and that due to radiation. The total magnetic field at a distance d from a radiating antenna is, from equation (**5**)

$$H = \frac{2\pi}{10}\,\frac{h_s I_s}{\lambda d} + \frac{j}{10}\,\frac{h_s I_s}{d^2} \qquad \textbf{(31)}$$

where j indicates that the two terms differ in phase by 90 deg. The first term represents the radiation field and the second term the induction field. The fact that one contains $\lambda\, d$ in the denominator while the other contains d^2 makes them radically different in nature. This gives the mathematical distinction between induction and radiation. The physical difference is discussed in Sec. 3 below.

The radiation field becomes relatively more important than the induction field as the distance d is increased or as the wave length is diminished (*i. e.*, the frequency increased). The question whether radiation or induction predominates in any given case can be settled by calculation from the formula. Thus, the two fields are equal at a distance

$$d = \frac{\lambda}{2\pi}$$

For points closer to the antenna than this the induction field predominates. For points farther away, the radiation field predominates and the induction field falls off rapidly with distance and becomes negligible.

Certain early experiments in wireless signalling used true induction, *e. g.*, the induction telegraphy of Preece and of Dolbear. When higher frequencies were used by later experimenters, signals of appreciable strength were received at distances of several wave lengths. These were genuine radiation signals, now commonly called radio.

2. IS RADIATION LIMITED TO HIGH FREQUENCIES?

The answer to this should be obvious from formulas (**8**) and (**10**). The radiated field does not become zero, no matter how great λ is. For alternating current of any frequency, no matter how low, radiation takes place from the circuit. To be sure, the radiation is greater the higher the frequency, so that high-frequency circuits are better radiators than low-frequency ones, and this is all the basis there is for the mistaken idea that only high-frequency circuits radiate.

This applies to radiation from a coil as well as from an antenna. It has sometimes been stated that a coil will not radiate, the statement being put in the form that only "open" circuits radiate. The statement is doubly faulty since electricity can flow only in closed circuits. The meaning intended by "open" circuit is a circuit containing a condenser of open form, that is, with two plates well separated. There are two misconceptions at the base of the belief that a "closed" circuit or one not containing a condenser would not radiate. In the first place, some have doubtless thought that waves would be started in the ether only by an electrostatic disturbance and thus could not be produced by a metallically closed circuit. Or, supposing it was understood that a magnetic disturbance in the ether would send out a wave just as readily as an electrostatic disturbance, it may have been thought that the radiation from one side of the circuit would be neutralized from that from the opposite side. As has already been shown in this paper, the two disturbances do not exactly neutralize each other, on account of the finite time of propagation from one side of the circuit to the other, and the resultant is what gives rise to the radiation from a metallically closed circuit.

3. EQUIVALENCE OF ELECTROSTATIC AND MAGNETIC FIELDS IN A WAVE

The physical distinctions between radiation and induction are: (a) the latter is fixed in space and the former moves through space with the velocity of light, and (b) in the case of radiation the magnetic field is always accompanied by an electrostatic field of value

$$\mathcal{E} = 300 \;.\; H \tag{16}$$

and vice versa, whereas in the case of induction there is no fixed relation. It is, of course, true that whenever magnetic induction varies an electrostatic field is produced, and similarly whenever electrostatic induction varies a magnetic field is produced. But it is only in a radiated wave that these variations take place in such a way that one can be calculated from the other by the fixed relation (**16**). When there is a fixed electrostatic field associated with a circuit which does not vary, the magnetic field associated with this electrostatic field is zero, and vice versa.

In a radiated wave, then, the electrostatic and magnetic field are no longer independent phenomena but are strictly equivalent. Indeed, they are but two aspects of the same thing. Perhaps this will be clearer from the analogy of a sound wave. In a mechanical apparatus, elastic action and inertia act independently in various parts of the apparatus. In a sound wave, however, the effects of elastic action and inertia are mutual parts of a single phenomenon, the sound wave.

In considering any effect of the electromagnetic wave, it is equally permissible to consider the electrostatic or the magnetic field associated with the wave. They are equivalent and lead to the same result. This has been amply demonstrated above in this paper. The current received in an antenna, calculated from the electrostatic field, was exactly the same as calculated from the magnetic field. The same agreement was found for the coil aerial. This disposes of the question whether the current produced in an antenna or a coil aerial is caused by the electrostatic or the magnetic field present in the wave, or both.

Complete discussions of electromagnetic waves are given in such treatises as Maxwell, "Electricity and Magnetism", 1873; Jeans, "Electricity and Magnetism", 1907; Lorentz, "The Theory of Electrons", 1909.

4. WHAT RADIATION IS

It has been shown that radiation differs from induction by a definitely calculable amount, that either kind of circuit radiates at any frequency, that there are both an electrostatic and a magnetic field present in every wave, having a constant ratio, and that any effect of the wave may be considered as due either to the electrostatic or the magnetic field of the wave.

Radiation is the moving disturbance of the ether, the energy associated with which does not return to the radiator.

This conception leads to more correct ideas than are current on the mechanism of radiation from an antenna; and permits explanation of the radiation from a coil aerial, which is not covered at all by the usual explanations of radiation in text-books. Such explanations have led to the impression that the radiation largely depends on the form of the electrostatic lines of force which are present at the edges of the radiator. It might thus be supposed that in a flat-top antenna or a condenser aerial the current in the central portions of the condenser was not effective in causing radiation while only that which spread into the surrounding space from the edges was effective. This appears incorrect. If it were correct, the builders of long flat-top antennas must have wasted a great deal of wire. All of the dielectric current sends a moving disturbance out into the ether. The portion of the energy associated with this disturbance that does not return to the radiator is that connected with the first term of equation (**31**). In this term the total antenna current appears. The radiation is the moving disturbance caused by the whole of the current which the antenna makes flow in the dielectric.

The ordinary treatment of the mechanism of radiation from an antenna is misleading also because it deals with radiation at the fundamental wave length. In practise, antennas are usually loaded. The radiation depends to no degree whatsoever on the value or location of any of the field lines attached to the aerial, but only on the variation of the lines. And *all* the lines when varying give rise to radiation. Thus the stationary field is given by the second term of formula (**31**), the first is the radiation term, and they are independent.

IV. COMPARISON FORMULAS

1. Derivation from Theoretical Formulas

Formulas are here derived to answer the practical question of how far a given coil will send or receive in comparison with a given antenna. The formulas also answer such questions as the length of a coil aerial required to give a particular ratio of performance of coil and antenna.

The ratio of the magnetic field radiated from a current coil to that from an antenna, for a given sending current, distance, wave length, and height, is obtained from equation (**13**). The ratio of the distance from a coil to that from an antenna, at which a given magnetic field is produced, is the same as the ratio of the magnetic field produced by a coil to that produced by an antenna at a given distance. Either ratio is therefore given by the following expression, which assumes the same current I_s, wave length λ, and height h_s, for the coil and the antenna,

$$d_c/d_a = N_s \sqrt{2\,(1 - \cos\theta)}$$

Inserting the value of θ and neglecting the subscript s,

$$d_c/d_a = N \sqrt{2\,(1 - \cos 2\,\pi\, l/\lambda)} \qquad \textbf{(32)}$$

When the length of the coil l is small compared to λ (*i. e.*, for most practical purposes, less than 0.1 λ), this simplifies to

$$d_c/d_a = 6.28\, N\, l/\lambda \qquad \textbf{(33)}$$

This could have been deduced directly from (**8**) and (**10**). The expression is similarly deduced for comparison of the distances obtained with a coil and an antenna of different heights,

$$d_c/d_a = 6.28\, N\, l/\lambda\ h_c/h_a \qquad \textbf{(34)}$$

The length of a coil required to give a particular ratio of performance to an antenna is given by solving these formulas for l. From (**32**),

$$l = \frac{\lambda}{2\,\pi} \cos^{-1}\left(1 - \frac{1}{2\,N^2}\left[\frac{d_c}{d_a}\right]^2\right) \qquad \textbf{(35)}$$

When the length of the coil is small compared to λ, the simpler formula suffices,

$$l = 0.16\, \lambda/N\, d_c/d_a \qquad \textbf{(36)}$$

The relative distances at which an antenna or a coil will receive a given wave are given by the same identical expressions that have just been deduced for sending aerials. Thus formula (**32**) may be deduced from (**28**) and (**33**) from (**19**) and (**23**). They give the ratio of the distance from the source at which a given e. m. f. will be produced in a coil aerial to that in an antenna, assuming the same height h_r and wave length λ for the coil and the antenna. They also give the ratio of the e. m. f. produced in a coil to that in an antenna for a given value of magnetic field intensity, or the ratio of currents when the resistances and other quantities are the same in coil and antenna. Equations (**34**), (**35**), and (**36**) similarly hold for receiving as well as sending aerials. For comparison of current in a coil and an antenna of different resistances as well as different heights

$$d_c/d_a = 6.28\, N\, l/\lambda \quad h_c/h_a \quad R_a/R_c \tag{37}$$

The relative distance of transmission between two coil aerials and between two antennas, for a given sending current, is similarly found from equations (**8**), (**10**), (**19**), and (**23**). The ratio of received current for coils and antennas the same distance apart is given by the same formula, which assumes the same sending current I_s and wave length λ for the pair of coils as for the pair of antennas.

$$\frac{d_{cc}}{d_{aa}} = 39.5\, \frac{l_s l_r N_s N_r}{\lambda^2} \quad \frac{(h_s h_r)\, c}{(h_s h_r)\, a} \quad \frac{R_a}{R_c} \tag{38}$$

All of these formulas assume that the decrement correction factor F_2 is the same for coil and antenna in all cases. If waves of different decrement are used, apply the factor F_2 as stated in connection with (**20**). If the plane of the coil considered is not parallel to the direction of propagation of the wave, apply the factor, cos α, as stated in connection with (**24**).

2. EXAMPLES OF COMPARISON OF COIL AND ANTENNA

What is the length of the coil, either as sender or receiver, equivalent to an antenna of the same height? The answer is given by (**36**). For $d_c/d_a = 1$,

$$l = 0.16\, \lambda/N \tag{39}$$

This is the correct length except for a single-turn coil. Then $N = 1$ the more exact formula (**35**) must be used. This gives, for the equivalent coil,

$$l = 1/6\, \lambda$$

Thus a single-turn coil of length 1/6 the wave length is equivalent to an antenna of the same height. For a coil of 8 turns, however, the length of the coil equivalent to an antenna of the same height is, from (**39**), 0.02 of the wave length.

When the length of the coil is small compared with the wave length, *i. e.*, as already stated, l less than about 0.1 λ, the performance ratio is given by (**33**). For a length greater than 0.1 λ, however, the more accurate formula (**32**) must be used.

Thus, when the length is exactly a sixth of a wave length, from (**32**),

$$d_c/d_a = N$$

Thus any coil of length 1/6 the wave length is equivalent to an antenna of N times the height of the coil. When the length of the coil is a quarter wave length, similarly

$$d_c/d_a = \sqrt{2}\,N$$

For a coil of length equal to half the wave length,

$$d_c/d_a = 2\,N$$

This is the maximum or best performance for a coil aerial. If the length is increased beyond a half wave length, the performance ratio decreases, and at $l = \lambda$ it is equal to O just the same as for $l = O$.

These values of the performance ratio of a coil aerial are obvious from Fig. 5 or 6.

These comparisons all apply to either transmitting or receiving aerials. They assume, however, in the case of a transmitting antenna or coil, that the same current flows; and, when applied to receiving aerials, that the resistance is the same in either coil or antenna. As a matter of fact, however, it is easy to secure a considerably lower resistance in a coil aerial circuit than in an antenna circuit. This is taken account of by the factor R_a/R_c as in (**37**) and (**38**). The difference in current in a transmitting coil and antenna is taken account of by multiplying the right-hand members of (**32**) and (**33**) by the ratio of the sending currents I_c/I_a. On this account a coil is sometimes a more effective radiating or receiving device than an antenna of considerably greater dimensions.

The comparison formulas and conclusions drawn from them are subject to the same errors as the transmission formulas, as discussed in Sec. V 2 below.

3. THE CONDENSER AERIAL

Since the dimensions of a coil aerial which would give the same performance as a given antenna are a length equal to $\dfrac{0.16}{N}$ times the wave length and a height equal to the antenna height, rather large structures are required. For example, a flat-top antenna 30 meters above the ground operating on a 600-meter wave is equivalent to a four-turn coil 24 meters long by 30 meters high. The dimensions of the equivalent coil are thus of the same order as the dimensions of the antenna.

It is possible to escape from the apparent necessity of large structures for effective radio transmission and reception in two ways. First, the coil aerial can easily be made to have a lower resistance than the antennas ordinarily used, and its size reduced in proportion to the reduction of resistance. This is mainly because the condenser used in the coil aerial circuit can be one having practically no resistance while the condenser consisting of antenna and ground has a large resistance. Thus by due attention to the minimizing of resistance in its circuit, the coil aerial may be of small dimensions and yet highly effective. The size may, of course, be reduced also in proportion as the number of turns is increased.

It is equally possible to avoid an aerial of large dimensions without having recourse to a coil aerial. The alternative is to use the antenna principle, but use a special construction of much lower height. At first sight it would appear that this would make a poorer antenna, since the effectiveness is proportional to the height, according to either (**8**) or (**19**). And this is true if the antenna is merely lowered a moderate amount. Such lowering increases the capacity only very slightly, not nearly in proportion to the decrease in height. In order to secure an appreciable gain it is necessary to have the height very small and use a special construction to reduce the resistance as much as possible. A good method is to replace the ordinary antenna-ground structure in which the antenna is one plate of a condenser and the ground the other plate, by an aerial consisting of two horizontal metal condenser plates. This may be called a "condenser aerial". The formulas derived for antennas apply to it.

Such an aerial has lower conductor resistance than the ordinary antenna, and since it has greater capacity a small inductance will be used in series with it which will also have smaller resistance and thus reduce the resistance of the circuit. Furthermore, the resistance of an antenna largely arises from the imperfect dielectrics, such as vegetation, buildings, and poor insulators, present in its field (as shown in Scientific Paper of the *Bureau of Standards* No. 269, by J. M. Miller), and the resistance from the grounding wires to ground. These can be eliminated in a condenser aerial. Finally, then, the resistance of the aerial circuit can be reduced to more than compensate for the reduction in height. This will result in a larger current I_r in formula (**19**), or in a larger H in formula (**8**) because of the increase of the sending current.

The advantage of the very low antenna has been observed in the experiments of the Kiebitz and others on so-called earth antennas. It is probable that still greater advantages would be obtained by the condenser aerial as here described. The special construction required to eliminate dielectric loss would involve making the lower plate considerably wider and longer than the upper plate, or else having both plates a considerable distance above the ground, and keeping the space between the plates free from poor dielectrics. An aerial consisting of a pair of metal plates elevated from the ground was used and described by Oliver Lodge in 1897, and again by Lodge and Muirhead in *Proc. Royal Soc.*, 82, p. 227; 1909, who found that it worked best without being grounded. The author is informed that the same sort of an aerial has recently been tried on airplanes, using the upper and lower planes as the condenser plates. Such an aerial would be ideal for airplanes if the space between could be kept free from poor dielectrics. If the plates of the condenser aerial have their length and width approximately equal, the aerial radiates in all directions. If a long narrow condenser is used it would probably be very directional, both as a transmitting and receiving device. Such a condenser might consist of a pair of parallel wires, which would be a considerable improvement on the ground antenna.

An example will make clear how the size of the condenser aerial compares with other aerials. It was found above that antenna 30 meters high was equivalent to a four-turn coil 24 meters long by 30 meters high, both operating on a 600-meter wave and with circuits of the same resistance. For the same wave length and with an inductance of 100 microhenries, in series, the capacity of a condenser aerial would need to be 0.00102 microfarad, which would be given by a pair of square plates 1 meter apart and 10.7 meters on a side. The height is thus reduced in the ratio of about 25 to 1, and the horizontal dimensions 3 to 1 in comparison with the coil aerial.

The aerial can be made as small as desired. If a given coil is to be used in series, the capacity of the aerial is maintained constant by reducing the distance between the plates when the area of the plate is reduced. The author made some interesting experiments with a small condenser aerial as a receiving device, used inside the laboratory with no ground connection. The plates consisted of copper netting. The top plate was 250 cm. square and the distance between them was 15 cm. The signals received, with either a crystal detector or electron tube, were roughly of the same intensity as those received with a simple coil aerial of the type and size ordinarily used as a direction finder.

The indication of absolute direction of propagation of the waves as well as line of propagation which has been developed by French and other workers, using combinations of ordinary antenna and coil aerials was observed in the experiments on the condenser aerial. An inductance coil of rather large dimensions used in series with the condenser acted as a receiving aerial. As this coil was rotated, the signal varied from maximum in one angular position to zero in a position 180 deg. from the first, instead of 90 deg. as occurs when a coil aerial is used independently of any antenna action. Apparently the action of the condenser aerial reinforced that of the coil in one position and neutralized it in the opposite position. When the connections to the coil were interchanged, the effect shifted 180 deg. Reversing the connections of the coil reverses the e. m. f. in the coil, *E*, in Fig. 6, just as a reversal of the direction of the wave would do, whereas the direction of the e. m. f. in the antenna or condenser aerial is unchanged. The reason why the condenser e. m. f. can neutralize the coil e. m. f. is probably that the capacity of the coil introduces different values of reactance to the two e. m. fs. Thus, when the circuit is tuned for one of these e. m. fs. the currents due to the two differ 90 deg. in phase. This phase angle may be shifted 180 deg. by a very slight variation of the reactance of the circuit. Because of this, systems for determining the absolute direction of radio waves require very delicate adjustment.

The ordinary laboratory type of condenser used in radio circuits does not function as a condenser aerial. This is because the interleaving of the plates results in the current in each portion of the dielectric being balanced by the current in a neighboring portion. This is discussed further below in Sec. VI 3 and illustrated in Fig. 17.

V. TRANSMISSION FORMULAS

1. STATEMENT OF FORMULAS

The current received in any aerial may be calculated in terms of the current in any transmitting aerial, either antenna or coil by the following four formulas. They are derived by combining equations (**8**), (**10**), (**19**), and (**23**). The symbols are as previously given, also stated in the Appendix below.

Antenna to antenna:

$$I_r = \frac{188 \; . h_s h_r I_s}{R \lambda d} \qquad (\mathbf{40})$$

Antenna to coil:

$$I_r = \frac{1184 \; . h_s h_r l_r N_r I_s}{R \lambda^2 d} \qquad (\mathbf{41})$$

Coil to antenna:

$$I_r = \frac{1184 \; . h_s l_s h_r N_s I_s}{R \lambda^2 d} \qquad (\mathbf{42})$$

Coil to coil:

$$I_r = \frac{7450 \; . h_s l_s h_r l_r N_s N_r I_s}{R \lambda^3 d} \qquad (\mathbf{43})$$

Formulas such as (**40**) have existed heretofore. The formulas here given generalize the antenna-to-antenna formula, so that calculations can be made for any kind of aerials.

The lengths in these formulas may be in any units, provided the same unit is used for all the lengths. The meter is usually the most convenient unit. If the heights and wave length are in meters and the distance *d* in miles, the four constants in the four formulas become respectively:

0.117
0.736
0.736
4.63

To calculate the distance at which a given current will be received, as when a particular receiving arrangement is specified, the formulas may be stated explicitly for *d*. I_r and *d* are interchanged in each formula. For example, the formula for antenna-to-coil (**41**) becomes

$$d = \frac{1184 \; . h_s h_r l_r N_r I_s}{R \lambda^2 I_r} \qquad (\mathbf{44})$$

All of these transmission formulas are for daytime transmission. Greater values are obtained at night, probably because the waves are reinforced by reflection from ionized layers of the upper atmosphere, which are broken up by sunlight in the daytime. The formulas are all subject to correction factors for distance and for decrement. If the distance is very great (in ordinary cases, over 100 kilometers), the right-hand side of the formula should be multiplied by the correction factor F_1. The value given below for F_1 is for transmission over sea water. Its value for transmission over land would be greater. If damped waves are used, the correction factor F_2 should be similarly applied. Furthermore, if the plane of the receiving coil is not parallel to the direction of propagagtion of the wave, the correction factor F_3 must be similarly applied to formulas (**41**) and (**43**) and related formulas such as (**44**). In formulas (**42**) and (**43**) the direction of the wave is taken to be that of the plane of the transmitting coil. The three correction factors are:

$$F_1 = \epsilon^{-0 \cdot 000047\, d/\sqrt{\lambda}} \tag{9}$$

$$F_2 = \sqrt{\frac{1}{1 + \frac{600 \;.\, L\,\delta'}{R\,\lambda}}} \tag{20}$$

$$F_3 = \cos\alpha \tag{24}$$

All of the correction factors make the resultant numerical values smaller.

2. Discussion of Transmission Formulas

The power of wave length in the denominator is different in the several formulas. Thus when a coil aerial is used for both transmitter and receiver the received current is inversely proportional to the cube of the wave length. Thus transmission between coils is better the shorter the wave length. This advantage of coils at short wave lengths applies only for short-distance transmission. When the distance is hundreds or thousands of kilometers, the increased absorption of the waves makes the correction factor F_1 so great that short waves are impractical, so for long distances the comparison favors the antenna rather than the coil. The coil compares most favorably with the antenna, then, for transmission over short distances with very short waves. This is subject to the proviso that current of the same order of magnitude can be gotten into a transmitting coil aerial as into an antenna, or that the resistance of a receiving coil is the same as that of a receiving antenna. Neither of the assumptions is wholly fulfilled, in practise, with the result that the difference of applicability of the two kinds of aerials at long and short wave lengths is less marked. For additional comparisons of antennas and coils and further discussion, see Sec. IV above.

Limitations of Formulas. The formulas can not be expected to give results of great accuracy, certainly not better than a few per cent, because of the ideal conditions assumed in their derivation. Thus it is assumed that no image of the aerial exists in the ground beneath it, that is, the ground is not perfect as a conductor. As a matter of fact, the ground varies greatly in conductivity; and while in most cases the currents induced in the ground below a transmitting or receiving aerial probably have very little effect, these currents may be appreciable in some cases. This is discussed further below under "Height of Aerial." On account of the uncertainty introduced by the ground, the formulas may apply better to airplane aerials than to those on ships or on land.

There are other sources of uncertainty in the application of these formulas. An antenna does not form a flat-plate condenser with the ground of such form that the curving of the field at the edges can be neglected. The simple method of calculating the radiated field is thus in doubt. Similarly, in the case of a radiating coil, the field from the top and bottom of the coil may have some effect at a distance, which has not here been taken into account. It is not certain with how great propriety the earth's surface can be taken to be equivalent to the equatorial plane of the radiating aerial. Frequently radio waves have a wave front that is tilted and not perpendicular to the earth's surface as assumed in the calculation of received current. Furthermore, the formulas assumed uniform current throughout the aerial, which sometimes does not hold because the antenna may have a vertical portion of appreciable capacity or the coil may have rather large distributed capacity. Calculations involving coil aerials are subject to the additional uncertainty arising from the capacity of the coil to ground or the surroundings so that it acts like an antenna as well as a coil. This is discussed under "Antenna Effect" in Sec. VI 3. Another difficulty discussed in the same Sec. is the effects of surrounding objects.

With these departures in the action of the aerials and the behavior of the waves from the conditions assumed, it is impossible to calculate received currents with great accuracy. It is almost surprising that the experimental results check the formulas as closely as shown in Sec. VI 2 below.

Height of Aerial. The value used for h is the length of the vertical side of a coil aerial, the distance from the surface of the ground to the flat top of an antenna, or the vertical distance between the two flat plates of a condenser aerial. In previous discussions it has been assumed that the ground beneath an antenna was a perfect conductor and thus the height of the radiator was twice the value of the h defined here. Experiment however corroborates the view here taken, which assumed that the radiating structure is independent of the earth, the waves becoming attached to the earth soon after leaving the antenna. In the present state of our knowledge the most satisfactory conception is that the radiating structure is the actual structure above the ground level. (Questions of the conductivity of the ground, presence of earth currents, etc., near the radiating aerial, are expressly not considered).

Austin's empirical formula[1] for antenna-to-antenna transmission is equation (**40**) with a constant twice as great, and quantities h_1 and h_2 used instead of h_s and h_r. These quantities h_1 and h_2 are the "height to the center of capacity" of the transmitting and receiving antenna, respectively. This height is not defined, but its value for any particular antenna is the value that is required to make experimental results fit the formula. Now, as has been stated, such experiments as have been performed agree in general with formula (**40**). For instance, see the first two examples in Sec. VI 2 below. It must follow since the constant in Austin's formula is twice as great as the constant in (**40**) that

$$h_1 h_2 = 1/2 \, h_s h_r$$

This may be satisfied by various values of h_1 and h_2. One set of values would be

$$h_1 = 0.5 \, h_s$$
$$h_2 = 1.0 \, h_r$$

Another would be

$$h_1 = 0.707 \, h_s$$
$$h_2 = 0.707 \, h_r$$

Austin's values for the height of various antennas thus deduced in such a way as to make them fit the experimental values observed, do in fact vary from half to full value of the actual antenna heights, and average around 0.7 the actual heights. It is much simpler and more direct to use the formula and the interpretation presented in this paper, bearing in mind that it is subject to the uncertainties introduced by the varying character of the ground.

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[1] Scientific Paper of the Bureau of Standards No. 226, Equation 5.

The idea that the ground is not a good enough conductor to form an image of a transmitting aerial, and that the waves become attached to the ground after leaving the aerial, is in harmony with the ideas of Lodge and Muirhead, already referred to. They found that they got better transmission by using what amounted to a condenser aerial, elevated from ground, with no ground connection. This conception conflicts with the commonly accepted view that Marconi's achievement was the connection of a radiating system to the ground. What then was Marconi's achievement? The best answer to this may be one stated to the author by Prof. A. E. Kennelly, *viz.*, the use of a large radiating system, arranged vertically.

VI. EXPERIMENTAL VERIFICATION OF FORMULAS

1. PRINCIPLES OF MEASUREMENT OF RECEIVED CURRENT AND VOLTAGE

The formulas presented in this paper not only make it possible to calculate approximately the field intensity produced or current received with given aerials, but also give the basis for determining what constants to select for the circuit of a particular aerial to secure the maximum effect. In other words, these formulas furnish the principles of design of aerial circuits. There are a great many points not obvious from mere inspection of the formulas, which are of importance equally in design and in the measurement of received signals. These will now be considered. While this discussion is limited to what takes place in receiving aerials, the same principles and treatment can readily be applied to transmitting aerials.

The received current or voltage can be measured in a number of different ways. It is important to know just what quantity is being considered or measured. Suppose an indicating instrument *G*, which may be a galvanometer or a telephone receiver, is connected to a rectifying device *D* in parallel with the condenser of the receiving circuit, as in Fig. 7, where either *L* is a coil aerial or else *C* is an antenna or condenser aerial. Does the indication of the instrument measure directly (a) the e.m.f. which the wave causes to act on the circuit $C\,L$, (b) the current in the circuit, or (c) the voltage across the condenser? The answer is, of course, none of these things. The system can, however, conveniently be calibrated in terms of the voltage across the condenser. This voltage V is related to the received current I_r by the relation

$$V = \frac{I_r}{\omega\,C} \tag{45}$$

and since I_r is related to the e. m. f. acting by

$$I_r = E/R, \tag{46}$$

the relation of V to E is

$$V = \frac{E}{R\,\omega\,C} \tag{47}$$

When a detecting apparatus like that of Fig. 7 is used, in which the deflections or signals depend directly on the voltage across the condenser, the results obtained with various receiving circuits will be entirely different from those obtained when the current in the circuit is directly measured, as in Fig. 8. Equations (**46**) and (**47**) show at once that the effects of varying the constants of the receiving circuit will be different, depending on whether it is E, I_r, or V that is being measured. These three quantities for a receiving antenna are, from equations (**17**), (**46**), and (**47**), for unit magnetic field intensity,

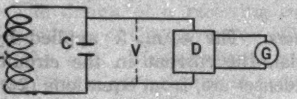

FIG. 7—AERIAL CIRCUIT WITH DETECTING APPARATUS ACROSS THE CONDENSER

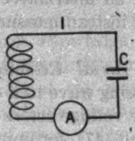

FIG. 8—AERIAL CIRCUIT WITH CURRENT MEASURING INSTRUMENT IN CIRCUIT

$$E_a = 300\ .\ h_r \tag{48}$$

$$I_a = 300\ .\ h_r/R \tag{49}$$

$$V_a = 300\ .\ \frac{h_r}{R\,\omega\,C} \tag{50}$$

The quantity h_r may be called the "e. m. f. reception factor" for an antenna, the e. m. f. in the receiving circuit is proportional to it. Similarly h_r/R may be called the "current reception factor" since it determines the received current. And $\frac{h_r}{R\,\omega\,C}$ or the equivalent $\frac{h_r\,L}{R\,\lambda}$ may be called the "voltage reception factor" of an antenna since it determines the voltage across the antenna.

The most favorable or optimum value of any of the variables that determine the antenna e. m. f., current, or voltage, can be determined either by direct experimental measurement of their values when actually receiving or by calculation from the reception factors. It is desired to learn simply what will produce the maximum E_a, I_a, or V_a. For example, it is obvious that E_a increases indefinitely as h_r increases, but more careful consideration is required to determine what will be the effect on the received current of increasing h_r. The reception factors furnish an alternative to direct reception measurements, requiring instead measurements upon the constants of the aerial circuit.

Coil Aerial Reception Factors. The e. m. f. applied by the passing wave to a coil aerial, the current in the circuit, and the voltage across the condenser are, from equations (**26**), (**46**), and (**47**), for unit magnetic field intensity,

$$E = 600\ .\ \pi\,\frac{a^2\,N}{\lambda} = \frac{1}{10^8}\,\frac{a^2\,N}{\sqrt{C\,L}} \qquad \textbf{(51)}$$

$$I = 600\ .\ \pi\,\frac{a^2\,N}{R\,\lambda} = \frac{1}{10^8}\,\frac{a^2\,N}{R\,\sqrt{C\,L}} \qquad \textbf{(52)}$$

$$V = 36\ .\ \pi^2\,10^{12}\,\frac{a^2\,N\,L}{R\,\lambda^2} = \frac{1}{10^8}\,\frac{a^2\,N}{R\,C} \qquad \textbf{(53)}$$

These equations assume the coil to be square having both height and length $= a$. For a coil that is not square, the formulas apply, replacing a by $\sqrt{h_r\,l_r}$. Two values are given for each reception factor; the first of the two is the more useful, since it is more common to consider the dependence of the reception on λ than on C.

$$\text{E. m. f. reception factor} = \frac{a^2\,N}{\lambda} \qquad \textbf{(51)}$$

$$\text{Current reception factor} = \frac{a^2\,N}{R\,\lambda} \qquad \textbf{(52)}$$

$$\text{Voltage reception factor} = \frac{a^2 N L}{R \lambda^2} \qquad \textbf{(53)}$$

Design of Receiving Coil Aerials. The response produced in a coil aerial circuit may be measured in a great variety of ways. In the first place, it may be considered either from the viewpoint of the e. m. f. acting, the current, or the voltage across the condenser. The first of these, the e. m. f., E, may be determined for any particular case from the e. m. f. reception factor in (**51**). The current I or voltage V may each be determined in four different ways: (a) by direct measurement with a suitable instrument, (b) by measurement of the quantities which make up the appropriate reception factor, (c) by measurement of signal strength in some such device as sketched in Fig. 7, which has been calibrated in I or V, (d) from a "signal intensity reception factor," which can be calculated for any signal measuring device when the law connecting the signal intensity with either I or V is known.

The design of a receiving coil requires knowledge of the dependence of the current or voltage upon the dimensions, etc., of the coil. Measurements made in all of the ways just enumerated give results in agreement with one another, provided due care is given to the avoidance of errors. The sources of error are numerous, as discussed in Sec. 3 below.

While direct measurement of the received current or voltage can be replaced by calculation from the reception factors, the fact remains that the design of an aerial requires experiments. This is because the quantity R in the reception factors cannot be obtained by calculation. It must be obtained by measurement for the particular coil and mode of connections employed.

Measurements upon receiving aerials to determine their constants and the best design for given conditions constitute a most interesting study. In later publications, the results of experiments will be published giving such data for typical coil aerials.

The capacity C in the formulas is the total capacity of the circuit, including the capacity of the coil, L is the pure inductance of the coil, and R is the actual resistance of the circuit. R includes the resistance of the conductors, effective resistance of the condenser and of the coil capacity, effective resistance of the detecting apparatus, and radiation resistance. All of these vary with frequency, and thus measurement of R at the frequency concerned is necessary. On account of the complexity of the quantities entering into the total R, its measurement is no easy matter. The capacity of the coil and other stray capacities may easily vitiate the measurement of R, C, or L. The effect of the detecting apparatus always requires most careful consideration. Even if D in Fig. 7 is an electron tube, it is necessary to consider the resistance which it introduces into the aerial circuit.

Dependence of Received Current and Voltage on Dimensions of Coil and Wave Length. Let R_c = resistance of coil and R_x = resistance external to coil;

$$R = R_x + R_c$$

$$\text{Current reception factor} = \frac{a^2 N}{(R_x + R_c)\,\lambda} \qquad \textbf{(54)}$$

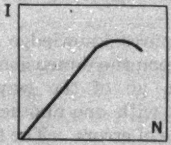

Fig. 9—Dependence of Received Current on Number of Turns

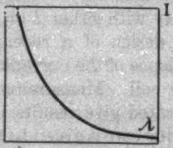

Fig. 10—Dependence of Received Current on Wave Length when External Resistance is Large

The variation of received current with number of turns, wave length, and size of coil is readily found by considering the the variation of the quantities in (**54**). In the following discussions the spacing between turns of wire, which affects resistance and inductance, is assumed constant.

a. Varying N, with λ and a constant. When R_x is large compared to R_c, we see from (**54**)

$$I \alpha N$$

When R_c is large compared to R_x, since $R_c \alpha N$, roughly,

$$I \alpha \text{ constant}$$

However, R_c increases somewhat faster than proportional to N as N is increased, because of the proximity of the added turns, and hence I decreases somewhat as N increases instead of being strictly constant.

R_x is likely to be large compared with R_c when N is very small, and hence for small N, the variation of I with N will be a straight line, as shown in Fig. 9. As N increases, R_c becomes large compared to R_x and the tendency of I to increase with N is reversed. As a result the curve of I has a maximum. The value of N at this point may be called the "optimum number of turns." Its value will be greater the greater the external resistance.

b. Varying λ with N and a constant. When R_x is large compared to R_c and does not vary appreciably with wave length

$$I \alpha 1/\lambda$$

This variation is shown in Fig. 10. When R_c is large compared to R_x, however, since $R_c \alpha \frac{1}{\sqrt{\lambda}}$, roughly,

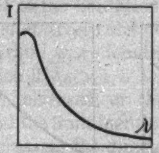

Fig. 11—Dependence of Received Current on Wave Length when Coil Resistance is Large

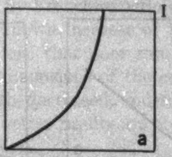

Fig. 12—Dependence of Received Current on Size of Coil when External Resistance is Large

$$I \alpha \frac{1}{\sqrt{\lambda}}$$

However, the effect of the adjacent turns increases R_c faster than stated as λ is diminished and hence I tends to approach a constant value for short wave lengths, as shown in Fig. 11. These conclusions may, however, be vitiated by the variation of R_x with λ.

c. Varying a, with N and λ constant. When R_x is large compared to R_c,

$$I \alpha a^2$$

This is shown in Fig. 12. When R_c is large compared to R_x, since $R_c \alpha a$,

$$I \alpha a,$$

giving the straight line in Fig. 13. From these two extreme cases it follows that an actual curve is likely to have a form that is a combination of these two, as shown in Fig. 14.

d. Varying a, with λ constant, allowing N to vary in such a way that length of wire is constant. The condition is that $N \alpha 1/a$,
When R_x is large compared to R_c,

$$I \alpha a$$

The curve of I is thus a straight line. When R_c is large compared to R_x, the same conclusion holds but only roughly. R_c increases slightly as a is decreased because of the proximity of the added turns, hence I increases a little faster than proportional to a. This is shown in Fig. 15.

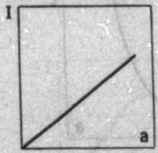

FIG. 13—DEPENDENCE OF RECEIVED CURRENT ON SIZE OF COIL WHEN COIL RESISTANCE IS LARGE

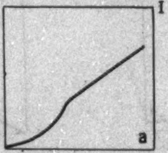

FIG. 14—DEPENDENCE OF RECEIVED CURRENT ON SIZE OF COIL IN TYPICAL CASE

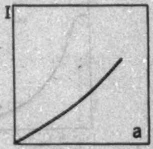

FIG. 15—DEPENDENCE OF RECEIVED CURRENT ON SIZE OF COIL WHEN LENGTH OF WIRE IS KEPT CONSTANT

The voltage reception factor differs from the current reception factor by L/λ.
Thus,

$$V \alpha \text{ Voltage reception factor } = \frac{a^2 N L}{(R_x + R_c)\lambda^2} \qquad (55)$$

It is thus a more complex problem to determine the variation of voltage with N, λ, and a, because the variation of L must be considered in addition. This may be done in each case, just as was done above for current, taking into account the relations:

$$L \alpha N^{2-} \qquad R_c \alpha N^{1+}$$

$$L \alpha \lambda^{0-} \qquad R_c \alpha \frac{1}{\lambda^{1+}}$$

$$L \alpha a^{1+} \qquad R_c \alpha a^1$$

where the + or − sign after the exponent indicates that the actual power is slightly greater or less than that given.

The relations obtained for voltage are similar to those obtained for current. There are some characteristic differences as, *e. g.*, with varying N the optimum number of turns comes out greater than for received current. Thus when the detecting apparatus depends essentially on the current the size of the coil should be as large as possible, whereas when it depends essentially on the voltage across the condenser the number of turns should be as large as possible.

On the whole, the received current or voltage or signal intensity is increased by increasing the dimensions (N and a), and by decreasing the wave length. These conclusions are subject to the limitation, discovered by French experimenters, and qualitatively obvious from the known increase of R near the natural wave length λ_0 of a circuit, that poor results are obtained if $\lambda_0 > 1/3\ \lambda$. Thus the dimensions of the coil can not be increased, or the wave length decreased, indefinitely. Beyond the limit mentioned, in fact, better results are obtained by decreasing the dimensions of the coil.

It is an interesting fact that these discussions apply not only to the design of a coil aerial for receiving signals but that they also solve the problem of design of wavemeters. The choice of constants of a wave meter coil for any particular case is settled by the formulas and ideas here presented. The considerations given for received current and voltage apply respectively to the design of wavemeters for measurements upon undamped or damped waves, *i. e.*, to the securing of minimum resistance and minimum decrement respectively.

2. EXAMPLES OF MEASUREMENTS

Measurements having as their object the verification of the transmission formulas were discussed in the preceding section. Any experiments which verify the transmission formulas may also be considered as checking the "theoretical formulas" and "comparison formulas". In fact experimental tests of the transmission formulas are the most rigorous test of the theory presented in this paper. All of the limitations and errors discussed in Sec. V 2 affect the transmission formulas while only a portion of them affect the theoretical and comparison formulas.

The complicated practical conditions of any experiment, the tilting of the wave front, the combination of antenna and coil effects discussed in the next section below, and other uncertainties make close agreement between theory and experiment unlikely. Agreement to 30 per cent should be considered as highly satisfactory verification of the essential correctness of the theory. On account of its field being more definitely localized, experiments with a coil aerial may be expected to yield greater accuracy than experiments with an antenna. The same advantage appertains to a condenser aerial. No quantitative data have been obtained with condenser aerials, to the author's knowledge; such experiments would be very desirable.

Experimental data obtained at the Bureau of Standards on radio transmission and reception are presented below. The agreement between the received current observed and the values calculated from the transmission formulas can be considered as very satisfactory. The author is informed that experiments made by the Signal Corps have led to a similar verification. In some of the Signal Corps experiments it was thought at one time that wide departures from the transmission formulas for coil aerials were observed, the received current for very short waves being much less than the transmission formulas indicated. When, however, the actual values of the resistance at wave lengths used were determined, the agreement was very good. Particular care must be given to proper measurement of the resistance of the receiving aerial circuit.

Antenna to Antenna. Experiments which supply a check on formula (**40**) have been published by Dr. Austin, chief of the Naval Radiotelegraphic Laboratory located at the Bureau of Standards.

For transmission between two antennas located on ships (*Jour. Wash. Acad. Sciences*, 1, p. 275; 1911), $h_s = 29.2$, $h_r = 29.2$, $I_s = 30.$, $R = 25.$, $\lambda = 1000.$, $d = 1000$. The lengths given in all these examples are in meters. Calculating from (**40**),

$$I_r \text{ calculated} = 0.19$$

$$I_r \text{ observed} = 0.21$$

For the Washington Navy Yard antenna transmitting to an antenna at the Bureau of Standards (*Bull. Bureau of Standards*, 11, p. 74; 1914), $h_s = 36.$, $h_r = 30.$, $I_s = 7.0$, $R = 70.$, $\lambda = 2800.$, $d = 10{,}000$. Calculating from (**40**) and (**20**)

$$I_r \text{ calculated} = 0.53 \times 10 - 3$$
$$I_r \text{ observed} = 0.55 \times 10 - 3$$

As already pointed out, the agreement of observed values with the transmission formulas indicates that it is proper to take as the antenna height the actual height from the ground to the flat top.

Antenna to Coil. A number of experiments to check the antenna-to-coil transmission formulas have been made at the Bureau of Standards radio laboratory since early in 1917. The first ones did not involve quantitative measurements but served to give a rough check on the formulas. The calculated value of current was compared with the current as estimated from the loudness of signal in a telephone receiver connected to various types of detecting devices. These signals were interpreted on the assumption that a fairly audible response is given by the currents indicated with the several types of detector.—

10^{-2} ampere, thermoelement.
10^{-4} ampere, crystal detector.
10^{-6} ampere, simple audion.
10^{-8} ampere, oscillating audion.

For the Arlington antenna received on a coil aerial at the Bureau of Standards, $h_s = 122.$, $h_r = 4.$, $l_r = 4.$, $N_r = 22.$, $I_s = 102.$, $R = 25.$, $\lambda = 3800.$, $d = 7800$. The received current calculated from (**41**) is 0.0018 ampere. The observed signal using crystal detector and telephone, was very loud, thus checking in a qualitative way the result calculated value.

Two similar qualitative experiments were made, transmitting from an antenna at the Bureau of Standards and receiving on a portable coil aerial. In one experiment, $h_s = 36.$, $h_r = l_r = 1.07$, $N_r = 11.$, $I_s = 0.5$, $R = 2.$, $\lambda = 850.$, $d = 16{,}000.$, whence calculated $I_r = 11. \times 10^{-6}$. In the other, $h_s = 12.$, $h_r = l_r = 1.07$, $N_r = 11.$, $I_s = 0.25$, $R = 2$, $\lambda = 600.$, $d = 11{,}000.$, whence calculated $I_r = 5.7 \times 10^{-6}$. In both cases the observed signal was loud with a simple audion, again giving a qualitative check on the formula.

A number of experiments have been made by Dr. Austin during 1918 and 1919, who has kindly placed the results at my disposal. A couple of typical ones will be given. For the Arlington antenna transmitting to a coil aerial at the Bureau of Standards, $h_s = 122.$, $h_r = 1.82$, $l_r = 1.29$, $N_r = 56.$, $I_s = 100.$, $R = 50.$, $\lambda = 6000.$, $d = 7800$.

I_r calculated $= 1.4 \times 10^{-4}$
I_r observed $= 2.1 \times 10^{-4}$

For the same antenna transmitting to a large coil suspended from masts outdoors, $h_s = 122.$, $h_r = 21.6$, $l_r = 24.4$, $N_r = 7.$ $I_s = 100.$, $R = 50.$, $\lambda = 10{,}000.$, $d = 7800.$, $\alpha = 42$ deg. From (**41**) and (**24**),

I_r calculated $= 1.0 \times 10^{-3}$
I_r observed $= 1.2 \times 10^{-3}$

A large number of transmission experiments from antenna to coils have been made by the radio laboratory of this Bureau in the early part of 1919. In a typical case, where $h_s = 21.$, $h_r = l_r = 1.44$, $N_r = 8.$, $I_s = 3.$, $R = 7.7$, $\lambda = 700.$, $d = 4800.$, $\delta' = 0.1$, $L = 541.$ microhenries. From (**41**) and (**20**),

I_r calculated $= 24. \times 10^{-6}$
I_r observed $= 28. \times 10^{-6}$

The fact that the observed current is larger than the calculated, in these and other cases, is probably due to the antenna effect, discussed in the next section. The coil structure has capacity, which makes it receive the wave by antenna action in addition to the coil action, thus increasing the current actually received.

Coil to Antenna. In an experiment made by Dr. Austin, with a large coil at the Bureau of Standards transmitting to the Arlington antenna, $h_s = 21.6$, $l_s = 24.4$, $h_r = 122.$, $N_s = 4.$,
$N_s = 7.$, $I_s = 1$, $R = 50.$, $\lambda = 2800.$, $d = 7800$, $\alpha = 42$ deg. From formula (**42**),

I_r calculated $= 1.3 \times 10^{-4}$
I_r observed $= 1.5 \times 10^{-4}$

Coil to Coil. The only data available to the author on the use of the coil aerial for both transmitting and receiving are from experiments made in 1917 by Messrs. Kolster, Willoughby, and Lowell, and these are only qualitative. For transmission from a coil at the Bureau of Standards to a portable coil 40. kilometers away, $h_s = l_s = 3.$, $h_r = l_r = 1.$,

$N_s = 4.$, $N_r = 15.$, $I_s = 10.$, $R = 1.$, $\lambda = 600.$, $d = 40{,}000$. The received current calculated from formula (**43**) is 4.6×10^{-6}. The observed signal was loud with a simple audion.

For transmission from a coil located at a lighthouse to a coil on a ship 48 kilometers away, $h_s = l_s = 3.05$, $h_r = l_r = 1.22$, $N_s = 3.$, $N_r = 10.$, $I_s = 10.$, $R = 2.$, $\lambda = 550.$, $d = 48{,}000$. The calculated I_r is 1.6×10^{-6}. The observed signal was audible on a simple audion. Comparing with the current sensibility of an audion stated above, it is seen that both of these results furnish a rough check on the formula.

3. DISCUSSION OF EXPERIMENTS

The agreement of the experiments with the theory is highly satisfactory in view of the simple conditions assumed in the theory. The complex practical conditions preclude the likelihood of agreement within a few per cent. The various limitations of the formulas arising from actual experimental conditions are discussed above in Sec. V 2.

One characteristic of the experiments with coil aerials is that the observed value of received current is in every case greater than the calculated value. This strongly suggests that the action of the coil structure involves something additional to the pure action as a coil. This would be expected also from theoretical considerations. The inevitable capacities between parts of the aerial circuit must introduce an action analogous to that of an antenna. When it is borne in mind that the coil action is really a second-order effect in comparison with the action of a system of antenna nature, it appears extremely likely that the stray capacities of any coil aerial circuit would introduce an antenna effect which would have to be considered in addition to the pure coil effect. Besides the reasons thus given by theory and by the excessive values of current observed in experiments with coil aerials, there are two other lines of evidence that the antenna effect is not negligible in coil aerials.

One of these additional lines of evidence is furnished by measurements of current in different parts of a coil aerial or the circuit thereof. If the capacities between parts are appreciable some of the current must leave the conductors and flow off into the dielectric; the current observed with an ammeter must therefore be different in different parts of the circuit. These differences are actually found. The fourth kind of evidence that the antenna effect is appreciable in coil aerials is furnished by considerations of radiation resistance, which will now be discussed. Following that, the antenna effect will be considered in more detail.

Radiation Resistance. It is possible to determine whether in a given system the antenna effect or coil effect predominates by measurements of radiation resistance. The radiation resistance has different values and follows different laws for antenna and coil.

Radiation resistance in general is defined by

$$P = R I^2 \qquad \textbf{(56)}$$

in which I is the current in the aerial used as a transmitting device, P is the power radiated, and R the radiation resistance. The study of radiation resistance is an important means of facilitating work on aerials. This may be seen from the simple fact that the magnitude of the radiation resistance gives at once the power radiated, and hence the effectiveness of a transmitting aerial or the range of communication can be judged without making transmission experiments. Field tests are thus in large part replaced by laboratory measurements. In addition to this, it is possible to discriminate between the antenna and coil effects.

The magnitude of the radiation resistance of a flat-top antenna, at wave lengths considerably greater than the fundamental, is given by the well-known expression

$$R_a = (39.7\, h/\lambda)^2 \qquad \textbf{(57)}$$

An approximate expression for the radiation resistance of a coil can be derived very simply, as follows. When a radiated field exists in any part of space, the relation of the power radiated through that portion of space to the magnetic field intensity there existing is

$$P' \propto H^2 \qquad \textbf{(58)}$$

for any given distance from the source, whatever the source may be. The total power radiated is proportional to the integral of P over any surface entirely surrounding the source. This integral will be of the same form for H_c, the field due to a coil, as for H_a, the field to an antenna, except for the effect of the variation of H_c in a plane around the radiating coil, which varies from zero to the value given in (**10**) for any given distance from the source. As a first approximation, this variation may be considered to make the integrated value of H_c one-half as great as it would be if H_c had the value given in (**10**) in all directions around the radiating coil.

$$\therefore \frac{P_c}{P_a} = \frac{1}{2}\,\frac{H_c^2}{H_a^2} \tag{59}$$

From (**8**) and (**10**), for a given distance from the source and a coil and antenna of same height with same current,

$$\frac{H_c}{H_a} = 2\,\pi\,\frac{l\,N}{\lambda} \tag{60}$$

From (**56**),

$$\frac{R_c}{R_a} = \frac{P_c}{P_a}$$

Hence from (**59**) and (**60**)

$$\frac{R_c}{R_a} = 2\,\pi^2\,\frac{l^2\,N^2}{\lambda^2}$$

Inserting the value of R_a from (**57**)

$$R_c = 31{,}100\,.\,\frac{h^2\,l^2\,N^2}{\lambda^4}$$

If the coil is a square one with $h = l = a$,

$$R_c = (13.3\,a/\lambda)^4\,N^2 \tag{61}$$

This approximate expression for radiation resistance of a coil gives at once the variation with size, number of turns, and wave length. *E. g.*, for a set of coils of varying size, in which the length of wire is kept constant, $R_c \,\alpha\, 1/N^2$. It shows that for a given ratio of size to wave length, $R_c \,\alpha\, N^2$. The principal point of interest is that R_c is inversely proportional to the fourth power of wave length.

Since the radiation resistance of an antenna is inversely proportional to the second power of wave length, and that of a coil inversely proportional to the fourth power, the radiation resistance furnishes a means of determining whether a given structure functions as a coil or as an antenna. Rough determinations of radiation resistance which were made upon a particular coil aerial showed a variation of observed radiation resistance inversely as the third power of the wave length, thus verifying the idea that the action is a combination of coil and antenna effect. The observed values however were all higher than the sum of the theoretical R_a and R_c. The measurement of radiation resistance is an extremely difficult operation, and satisfactory methods can not be said to have been developed as yet.

Antenna Effect. Since there are differences of potential between various parts of a coil, acting either as a transmitting or receiving aerial, there must be some dielectric current through the space around the coil and between the coil and ground. It follows that there must be some antenna action, proportional to the amount of this dielectric current and the length of path over which it flows, and this will produce a current additional to that produced by the coil action unless the coil structure happens to have an exact symmetry which causes the antenna effect in each part of the coil to be balanced by an antenna effect in some other part.

Fig. 16 shows the origin of the antenna effect. As in ordinary practise, the leads cause some part of the apparatus to be practically at ground potential, the shield of the condenser is shown connected to ground. An appreciable dielectric current flows from various parts of the conducting circuit to other parts and to ground. Typical paths of this dielectric current are shown by the dotted lines. The line *a b* suggests the dielectric current from the coil structure to ground, the lines *c d* and *e f* the dielectric current between turns of the coil, and the line *g h* the dielectric current between coil and leads. The flow of dielectric current between turns of the coil is in a horizontal direction when the coil is of prismatic form with the turns separated and all of the same area. This part of the antenna effect arises in a receiving coil of this form only when the wave front is more or less tilted from the vertical.

On account of the flow of current off through the dielectric from various parts of the circuit, ammeters placed at different places in the circuit would show different values of current to be flowing. In radio circuits it can not be assumed that the current is the same at all points around the conducting circuit, as was shown by the author in his investigation of high-frequency current measurement described in *Bureau of Standards* S. 206; 1913.

To the extent that these dielectric currents flow, the conductors of the circuit may be considered as an antenna system. Perhaps only the current typified by the line *a b* might be thought of as giving rise to an "antenna" effect, since the others do not flow to ground; still this part of the dielectric current does not differ from the others in nature or effect, and it seems hence advisable to use the suggestive term "antenna effect" to indicate all of the effects arising from the presence of currents in the dielectric.

It might be supposed that the same sort of an effect would be caused by the flow of dielectric current in the condenser of the coil aerial circuit. This is not true ordinarily because a condenser of the laboratory type is used, in which the condenser plates are interleaved. As shown in Fig. 17, the current in one direction in the dielectric is balanced by a current in the opposite direction in the neighboring part of the condenser. This is a non-radiating condenser; and is the analog of a non-inductive coil, which is also non-radiating. A condenser consisting of a single pair of plates would radiate, but is not ordinarily used because it would be much bulkier than the laboratory type of condenser. The condenser consisting of a single pair of plates would be in fact the "condenser aerial," which has been recommended by the author in Sec. IV 3 as worthy of serious consideration in radio practise.

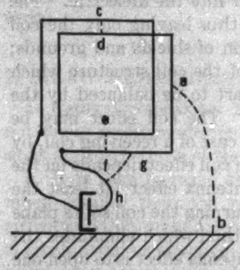

FIG. 16—PATHS OF DIELECTRIC CURRENTS WHICH CAUSE ANTENNA EFFECT IN COIL AERIAL

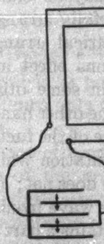

FIG. 17—DIRECTIONS OF FLOW OF DIELECTRIC CURRENTS IN LABORATORY TYPE OF CONDENSER

The effect of the distributed capacities of the aerial circuit must not be confused in any way with the phase angle between the fields existing at the two vertical sides of the coil aerial. The phase angle referred to is the seat of the action of the coil aerial as such. The dielectric currents flowing in the distributed or stray capacities of the circuit, however, give rise to the direct action as an antenna, not depending in any way on the separation between the two vertical sides of the coil. All of these remarks apply both to transmission and reception.

It is rather difficult to determine what fraction of the effect of a coil aerial is due to antenna action and what part to coil action. In many cases, doubtless, the antenna action predominates. It is possible, however, to separate the effects in any particular case by the several different methods. The antenna effect may be calculated, at least for parts of the circuit, by the aid of careful ammeter readings which show what amount of the current has flowed off into the dielectric. The antenna effect may be eliminated, thus leaving only the coil effect, by a carefully arranged system of shields and grounds; or, by a symmetrical arrangement of the coil structure which causes the antenna effect in each part to be balanced by the antenna effect in some other part. The coil effect may be eliminated, on the other hand, in the case of a receiving coil, by taking advantage of the fact that the coil effect depends on the direction of orientation while the antenna effect (at least the major part of it) does not; *i. e.*, by turning the coil so its plane is parallel to the wave front. A method which eliminates the coil effect and retains a part of the antenna effect is to open one of the coil leads, thus leaving the coil aerial connected to the circuit at one point, placing in series with it an inductance coil of very small dimensions but of the same inductance, the circuit being completed by the capacity of the coil aerial to ground.

Effects of Surroundings. Currents are induced in metal and other objects near a transmitting aerial, and sometimes are powerful enough to affect the radiation appreciably. The objects near a receiving aerial have currents produced in them by the passing wave. These currents in nearby objects, which may include the ground, induce e. m. f's. in the receiving aerial. It is to be noted that this effect of neighboring objects is caused by induction, and not radiation from them, which would be comparatively feeble.

The e. m. f. thus induced in a coil aerial from the surroundings is of the same or opposite phase as that caused by the wave. It differs in this respect from the e. m. f. due to the antenna effect discussed above. The antenna effect ordinarily produces an e. m. f. which is 90 deg. out of phase with the coil effects and does not vary with the orientation of the coil. The antenna effect can thus never balance out the coil effect, and it is impossible to reduce the received current to zero no matter how the coil is turned. The e. m. f. induced by the surroundings, however, depends upon the orientation of the coil. This e. m. f. will be reduced to zero by turning the coil at a different orientation from that at which the e. m. f. due to the wave is zero, unless the line between distorting object and the coil aerial is the direction of propagation of the wave. The result of this is that the total e. m. f. is reduced to zero at some orientation other than that obtained when the wave alone acts on the coil aerial. There is thus a distortion in the apparent direction of the wave, caused by objects surrounding the coil aerial.

VII. PRACTICAL CONCLUSIONS

1. RELATIVE EFFECTIVENESS OF ANTENNAS AND COIL AERIALS

a. Generally speaking, a coil aerial is as powerful a transmitting or receiving device as an antenna only when its dimensions approach those of the antenna.

b. It is easy to make the resistance of a coil aerial circuit much smaller than the resistance of the ordinary antenna circuit and thus make a small coil as effective as a large antenna. A small aerial as effective as a large antenna can however also be secured by the use of the antenna-like aerial called the condenser aerial. Heeding these principles and using amplifiers in receiving, radio aerials can in the future be much smaller than heretofore.

c. The relative effectiveness of a coil and antenna, in terms of the wave length, number of turns, etc., is given by formula (33) and the related formulas.

d. A coil aerial exhibits antenna action as well as coil action, because of capacities between its parts and surroundings. The antenna action sometimes overbalances the coil action.

e. The advantage of the coil aerial is greatest for short wave lengths. It is consequently likely to be well suited to airplane communication. The increasing advantage of the coil as a transmitting aerial, as the wave length is decreased, is subject to the proviso that the same current can be gotten into a coil as into an antenna. In fact, the whole practical problem is to get as much current as possible into the aerial.

f. The use of coil aerials at both receiving and transmitting ends of the communication is particularly suitable for short waves, since the received current in such a system is inversely proportional to the cube of the wave length.

2. PRINCIPAL FORMULAS

The units used are international electric units, the ordinary electric units based on the ohm, ampere, centimeter, and second; except where otherwise stated. The principal symbols are the following.

SYMBOLS

i = instantaneous current.
I_0 = maximum value of current.
I = effective value of current.
H_t = instantaneous value of magnetic field intensity.
H_0 = maximum value of magnetic field intensity.
H = effective value of magnetic field intensity.
h = height of aerial.
d = distance along earth's surface from sending aerial.
ω = 2π times frequency of the current.
t = time.
λ = wave length.
c = velocity of electric waves = 3×10^{10} cm. per second.
l = horizontal length of coil aerial.
N = number of turns of wire of coil aerial.
a = length of side of square coil.
θ = phase angle between values of field intensity a distance l apart in the wave.
$\mathcal{E}$ = electric field intensity.
E = electromotive force in receiving aerial.
δ' = logarithmic decrement of H or E.
ϕ = magnetic flux.
R = resistance of receiving aerial circuit.
C = capacity of receiving aerial circuit.
L = inductance of receiving aerial circuit.
α = angle between direction of propagation of wave and plane of coil.

Subscripts: s = sending, r = receiving, a = antenna, c = coil.

The following are the principal formulas presented in this paper:

Radiated Magnetic Field Intensity from an Antenna or Condenser Aerial.

$$H = \frac{2\,\pi}{10}\,\frac{h_s\,I_s}{\lambda\,d} \tag{8}$$

Radiated Magnetic Field Intensity from a Coil.

$$H = \frac{4\,\pi^2}{10}\,\frac{h_s\,l_s\,N_s\,I_s}{\lambda^2\,d} \tag{10}$$

Received Current in an Antenna or Condenser Aerial.

$$I_r = 300\,.\,\frac{h_r\,H}{R} \tag{19}$$

Received Current in a Coil.

$$I_r = 600\,.\,\pi\,\frac{h_r\,l_r\,N_r\,H}{R\,\lambda} \tag{23}$$

Distance Correction Factor.

$$F_1 = \epsilon^{-0.000047\,d\,/\sqrt{\lambda}} \tag{9}$$

Decrement Correction Factor.

$$F_2 = \sqrt{\frac{1}{1 + \dfrac{600\,.\,L\,\delta'}{R\,\lambda}}} \tag{20}$$

Direction Correction Factor.

$$F_3 = \cos\alpha \tag{24}$$

Antenna-to-Antenna Transmission.

$$I_r = \frac{188\,.\,h_s\,h_r\,I_s}{R\,\lambda\,d} \tag{40}$$

Antenna-to-Coil Transmission.

$$I_r = \frac{1184\,.\,h_s\,h_r\,l_r\,N_r\,I_s}{R\,\lambda^2\,d} \tag{41}$$

Coil-to-Antenna Transmission.

$$I_r = \frac{1184\,.\,h_s\,l_s\,h_r\,N_s\,I_s}{R\,\lambda^2\,d} \tag{42}$$

Coil-to-Coil Transmission.

$$I_r = \frac{7450 \,.\, h_s\, l_s\, h_r\, l_r\, N_s\, N_r\, I_s}{R\, \lambda^3\, d} \qquad (43)$$

The lengths in the four preceding formulas may be in any units. Meters are commonly used. Any of these formulas may be expressed in terms of d. *E. g.*,

Distance at Which a Given Current is Received in a Coil for a Given Transmitting Current in an Antenna.

$$d = \frac{1184 \,.\, h_s\, h_r\, l_r\, N_r\, I_s}{R\, \lambda^2\, I_r} \qquad (44)$$

Total Magnetic Field from an Antenna, Including Radiation and Induction.

$$H = \frac{2\,\pi}{10}\,\frac{h_s\, I_s}{\lambda\, d} + \frac{j}{10}\,\frac{h_s\, I_s}{d^2}\,. \qquad (31)$$

Relative Effectiveness of Coil and Antenna, for Same Height and Wave Length.

$$d_c/d_a = N\,\sqrt{2\,(1 - \cos 2\,\pi\, l/\lambda)} \qquad (32)$$

Ditto, l small compared to λ.

$$d_c/d_a = 6.28\, N\, l/\lambda \qquad (33)$$

Length of Coil Aerial Equivalent to Antenna of the Same Height.

$$l = 0.16\, \lambda/N \qquad (39)$$

Current in Aerial Circuit.

$$I_r = E/R \qquad (46)$$

Voltage across Condenser in Aerial Circuit.

$$V = \frac{E}{R\,\omega\, C} \qquad (47)$$

Coil Aerial Reception Factors.

$$\text{E. m. f. reception factor} = \frac{a^2\, N}{\lambda} \qquad (51)$$

$$\text{Current reception factor} = \frac{a^2\, N}{R\, \lambda} \qquad (52)$$

$$\text{Voltage reception factor} = \frac{a^2\, N\, L}{R\, \lambda^2} \qquad (53)$$

Radiation Resistance.

$$R_a = (39.7\,h/\lambda)^2 \tag{57}$$

$$R_c = (13.3\,a/\lambda)^4 N^2 \tag{61}$$

3. FUTURE RESEARCH NEEDED

The subject of research on electric waves can be considered as barely begun. The study presented in this article has revealed vast and most interesting problems awaiting solution which can be solved. The functioning of aerials, both in transmitting and receiving, can now be considered as roughly understood. Recent advances in radio measurements and technique open the way to experiments and progress which will bring about far-reaching control of electric waves. A few of the detailed problems which border on the subject matter of this paper and await solution will now be mentioned.

Theoretical Problems.

a. Develop a simple and straightforward derivation of the radiated field from a coil, without consideration of shape of the coil or dealing with the electrostatic field at all.

b. Work up an explanation of the mechanism of radiation that brings out clearly the relation of the radiation to the induction field and shows that all of the dielectric current is effective in causing radiation, which shall take the place of the usual explanation in terms of the snapping off of lines of force.

c. Determine the effects of the phase angle between different parts of the dielectric field in an antenna or condenser aerial, especially the long, low types.

d. Develop methods of measuring radiation resistance.

e. Work out laws of variation of voltage reception factor of coil aerials, and laws of variation of both current and voltage reception factors of antenna and condenser aerials. Similarly, develop accurate and useful transmission factors.

Experimental Problems.

a. Determine the relative effectiveness over a very wide range of sizes, wave lengths, etc., of the various types of aerials. Do this by: (1) direct measurements to verify transmission formulas (2) measurements of the factors that enter into the reception factors, (3) measurements of radiation resistance.

b. Make transmission experiments at very great distances over typical kinds of land, to obtain distance absorption factors.

c. Try out condenser aerials, comparing performance with transmission formulas. Build such aerials with minimum resistance. Demonstrate the non-radiating nature of the laboratory type of condenser, comparing it with condenser aerials.

d. Compare trailing wire, condenser aerial, and coil aerial, on airplane.

e. Find out how directive as transmitting devices coil and condenser aerials and "earth antennas" are; measure magnitude and direction of field at various distances from the aerial, at numerous wave lengths, etc.

f. Determine relative magnitudes of induction and radiation close to transmitting aerials. Determine also directions of fields, to secure complete knowledge of phenomena near radiating systems.

g. Measure currents in ground as well as the fields above the ground, to determine how wave attaches itself to the ground.

h. Study distributed capacities in coil aerial circuit by measuring current at different points in circuit.

i. Determine values of antenna effect, and develop means of controlling or eliminating it by shielding systems, etc.

j. Make quantitative investigation of receiving systems combining antenna and coil aerial. Measure phase of currents. Determine under what circumstances the indication of absolute direction is reversed when the tuning is slightly varied.

k. Determine effects of surrounding objects on currents in transmitting and receiving aerials. Measure magnitude and phase of currents in typical cases.

l. Develop methods of connecting generating apparatus to various types of aerials to get maximum current into the aerial, especially at short wave lengths.

VIII. SUMMARY

The advantages of the coil aerial as a direction finder, interference preventer, reducer of strays, and submarine aerial make it important to know how effective the coil aerial is, in comparison with the ordinary antenna, as a transmitting and receiving device. This article gives the answer. Simple formulas are worked out from fundamental electromagnetic theory, by which the performance of any aerial can be calculated. Experiments have verified the formulas, and show that they are a valuable aid in the choice and design of an aerial to fit any particular requirements.

The principal formulas are of three kinds: theoretical formulas, giving the magnetic field intensity at any distance from either kind of aerial and the current produced by a given field intensity in either kind of aerial; comparison formulas, giving the ratio of performance of antenna and coil aerial under various conditions; and transmission formulas, giving the current in any receiving aerial in terms of the current in the distant transmitting aerial.

The theory and nature of radiation are discussed, and applied to the elucidation of some current fallacies. There has been a vast haziness of ideas on these points. The distinction between induction fields and radiation fields is presented. It is shown that the receiving action in any kind of an aerial may be considered as arising either from the electrostatic or the magnetic field present in the wave. Such questions are discussed as the distinction between "open" and "closed" circuits. It is shown that a metallically closed circuit can radiate, and that radiation takes place at all frequencies, the amount of radiation being greater the higher the frequency.

The ratio of the range of communication obtainable with a coil aerial to that with an antenna is proportional to the number of turns and horizontal length of the coil and inversely as the wave length. The coil aerial is hence particularly suited to communication on short wave lengths. A coil aerial is quantitatively as powerful as an antenna only when its dimensions approach those of the antenna. However, it is easy to make the resistance of a coil aerial circuit much smaller than the resistance of the ordinary antenna circuit and thus make a small coil as effective as a large antenna.

A small aerial as effective as the ordinary antenna may be secured without recourse to the coil principle by using an aerial consisting of a condenser having two large parallel plates, arranged so that the dielectric of the condenser includes no ground. The circuit of such an aerial may be made to have a very low resistance. It appears likely that, with the use of either condenser or coil aerials together with sensitive amplifiers, radio aerials will in the future be much smaller than heretofore. These principles apply with particular advantage to airplane aerials.

A coil aerial usually functions by a combination of the pure coil action and antenna action. The latter arises from the stray capacities and capacities to ground which are inevitably present. The existence of these capacities may be shown by differences in ammeter readings at different points of the circuit. The antenna effect makes the actual received current in experiments with coil aerials larger than the values calculated from the transmission formulas. The observed values are also affected by currents in neighboring objects.

A formula for the radiation resistance of coil aerials is worked out. Comparison of experiment with this formula supplies additional evidence that the coil aerial operates by a combination of antenna and coil effects.

The fundamental principles of design of aerials are given. The various modes of measuring received current and voltage across the condenser are discussed. The relations of these two quantities to the electromotive force acting in the aerial must be carefully observed in calculations or design. Reception factors are derived, to which the received current or voltage are proportional. Experimental data on the functioning of aerials may be secured either from actual transmission experiments or from measurements of the quantities which enter into the reception factor.

This investigation has opened up a large and most interesting field for further research. Progress in the control and utilization of electric waves depends on the investigation of such theoretical and experimental problems as have been suggested in Sec. VII 3 herein.

DISCUSSION ON "PRINCIPLES OF RADIO TRANSMISSION AND RECEPTION WITH ANTENNA AND COIL AERIALS" (DELLINGER), NEW YORK, N. Y., OCTOBER 1, 1919.

F. W. Grover: In the derivation of the formula for the field of an antenna, the current is assumed to be uniform throughout the length of the antenna, and the field of this vertical current at a point P is found by using the vector potential. In evaluating the latter, the assumption is made that the point P is so far away, compared with the height of the antenna, that P is sensibly at the same distance from all points of this vertical portion.

Having obtained the equation (5) for the field due to this vertical current, it is then tacitly assumed that this is the whole field of the antenna. Since it is expressly emphasized in the latter portions (and rightly so, it will be admitted by every one) that the whole displacement current must be reckoned as contributing to the field, some explanation ought to be given as to why the displacement currents of the antenna are not taken into account in this demonstration. Thus far, I have not been able to give a satisfactory explanation, and am here setting forth the difficulties that have occurred to me, in the hope that I may be set right and that perhaps the objections of others may be forestalled.

First, it would seem necessary to make some assumption as to the form of the displacement lines. Here this is difficult, because it is assumed (1) that there is no image of the antenna, and (2) that the current in the vertical portion is uniform. These are of course, strictly speaking, incompatible conditions, since either the displacement currents must return from the flat top to the lower portion of the vertical wire which combats assumption (2) or else they must return by the earth which modifies statement (1). In any case, they must, on the whole, flow in the opposite direction to the current in the vertical portion. To some extent, then, it would seem as though the resultant effect of the antenna would have to be less than that of the vertical portion alone.

Suppose, to fix ideas, that the displacement currents flow vertically. Then, if we consider a filament of displacement current $C\,D$, it may be regarded as completing the circuit of an equal current in the vertical portion $B\,A$. Similarly, the symmetrically placed filament $E\,F$ forms a portion of a circuit $B\,A\,E\,F$. If each current in $B\,A$ produce a field H at a point P in the plane of rectangle, the effect of $B\,A$ totals $2\,H$, which may be represented by the vector $O\,F$ (Fig. 3). $E\,F$ produces a field H which is θ degrees different in phase from the vector opposite to $O\,F$. This may be represented by $O\,G$, just as in the demonstration for the coil. The currents $A\,D$ and $B\,E$ balance out in their effects at a distant point. In like manner, current $C\,D$ gives a field H of phase $O\,K$. The field due to currents $A\,C$ and $D\,B$ balance out at point P. The resultant must accordingly be $H' = 2\,H - 2\,H\cos\theta$ or

$$\begin{aligned} H' &= 2\,H\,(1 - \cos\theta) = 4\,H\sin^2\theta/2 \\ &= 4\,H\,\theta^2/4 = H\,\theta^2 \text{ when } \theta \text{ is small} \\ &= H\,.\,4\,\pi^2\,(l/\lambda)^2 \end{aligned}$$

which is much smaller than H.

This treatment may be extended to the whole antenna, each filament of displacement current being regarded as completing the circuit of an equal current in the vertical portion. It would seem, then, that the effect of the whole antenna, of the symmetrical construction here shown, must be much less than that of the vertical portion. In fact, whereas the radiation of a coil is a second order effect, that calculated above is a third order effect, being the difference of two coil effects.

Now, actually, the displacement lines of flow will not be straight lines but curved. Each element of a curved path of current may, however, be regarded as equivalent in its magnetic effect to the sum of a vertical and horizontal element (Fig. 4). Summing up for the whole line, the horizontal elements tend to balance out in their effects at a distant point, while the vertical elements sum up to give the effect of a vertical current between the antenna and the ground. In any case, where the antenna is symmetrical about the vertical portion, the above conclusion of a resultant field much smaller than that due to the vertical portion alone seems to be justified.

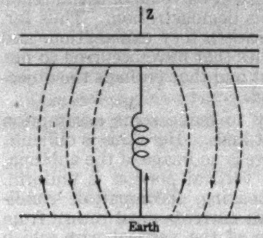

Fig. 1

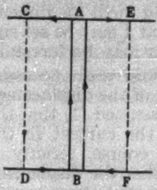

Fig. 2

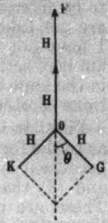

Fig. 3

Likewise the same ideas and conclusions seem to follow for an antenna consisting of a *single vertical wire.* For the displacement lines may be regarded as equivalent to vertical currents opposite in direction to the current in the wire. Any two current filaments symmetrically placed about the vertical wire may be combined with equal currents in the vertical wire and are equivalent to equal rectangles in the same plane as in the previous demonstration. If the point P is in their plane, the equation for the resultant field given above holds. Otherwise the factor cos α has to be applied to this, and the resultant would be very small in every case.

Applying the same ideas to an L antenna, it would seem that it should be equivalent to a coil of width somewhere between zero and the length of the antenna l. That is, such an antenna should be directive like a coil and its field should be a second order effect like that of a coil, *i. e.* less than that due to the vertical portion.

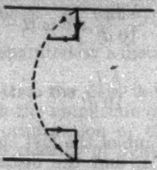

FIG. 4

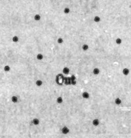

FIG. 5

In the case of the low condenser aerial we are not met by these difficulties, since we may regard the circuit as being completed through a coiled wire of negligible vertical portion and the coil is of such small dimensions that it may be negligible in its "coil effect." The current in the dielectric only would then need to be considered.

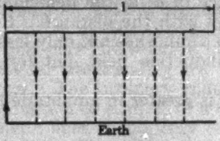

FIG. 6

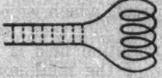

FIG. 8

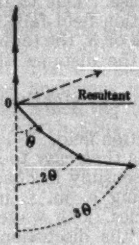

FIG. 7

Recapitulation. To agree, then with the formulas of this paper and with the experimental results, it would seem necessary to consider for coil effect only the current in the metallic portions. The condenser aerial may be treated by considering only the dielectric current. The antenna can be treated by considering either the current in the dielectric or the current in the vertical portion alone. Taken together they nearly cancel out. For unity, it would seem to be the dielectric current which should be chosen as in the condenser aerial. Finally, then, what is the reason why the whole circuit should not be considered?

DISCUSSION OF THE DISTINCTION BETWEEN RADIATION AND INDUCTION

The magnetic field due to a vertical current $i = I_0 \sin \omega t$ is, for a point P at a large distance d from the current,

$$H_t = - \frac{h \, \omega \, I_0}{10 \, c \, d} \cos \omega \, (t - d/c) - \frac{h \, I_0}{10 \, d^2} \sin \omega \, (t - d/c) \quad \mathbf{(5)}$$

$$= - \frac{2 \, \pi}{10} \, \frac{h \, I_0}{\lambda \, d} \cos \omega \, (t - d/c) - \frac{h \, I_0}{10 \, d^2} \sin \omega \, (t - d/c)$$

The negative signs simply take account of the fact that if the current is along the positive Z axis, and the distance d is measured along the positive X axis, the magnetic field must be along the $-Y$ axis. The first of these terms is called the radiation field and the second the induction field.

The time d/c is the interval which must elapse for a disturbance at the origin to travel the distance d to the point P. That is, action at P at any moment t depends upon the value of the current at a moment d/c seconds previous.

A comparison of the expressions for the two fields shows that the induction field is in phase with the current (excepting for its interval d/c) but that the radiation field is 90 degrees different in phase. Due to the presence of d^2 in the denominator the induction field falls off rapidly with the distance. For small values of λ, the radiation field diminishes not only less rapidly with the distance than the induction field, but for a given distance greater than $\frac{\lambda}{2 \pi}$, it is greater in amplitude than the induction field.

It is thus easy to see that the "induction field" is the usual field sensibly in phase with the current, which we consider as inducing e. m. f. in nearby circuits. Further, at low frequencies (long wave length) the radiation field is negligible. Thus the discussion in the paper of the mathematical distinction between the two terms is easily seen. The term "induction field" is, as just stated, appropriate, and the first term, which formulates the only portion of the field which is important at radio frequencies and distances is very properly called the "radiation field." Also, it is very important to emphasize the fact that both fields are always present, even though one of them may be entirely negligible. A further point which may be properly emphasized is that, since the time d/c required for the propagation of the field can never be exactly zero, it is never rigorously correct to speak of the magnetic field at a point as being strictly in phase with the current which produces it.

It is not so easy, however, to agree entirely with the statement in the paper of the *physical* difference of the two fields. It would seem to be misleading to speak of the induction field as "fixed in space" (last line of p. 1365). The presence of the quantity d/c in the sine factor of the induction field shows that this field must travel with the velocity of light, as well as the radiation field. At each point, the field oscillates, but for two points separated by a distance λ, the quantity $\frac{\omega d}{c}$ differs by 2π, so that the field is in the same phase at such points,—although the amplitude of oscillation is different.

This propagation of the induction component of the field with the velocity of light is also evident from a consideration of the induction of e. m. f. With regard to this the induction field has no monopoly. The induction of e. m. f. in a conductor by the radiated field is treated in equation (**17**), the usual idea of relative motion of conductor and field being employed, except that here the relative velocity is the velocity of light, a velocity far transcending any velocity attainable in electrical generators. The ordinary induction case, analogous to this, is the induction of e. m. f. in a coil through which the number of magnetic lines is suddenly changed by varying the current in the circuit or in an adjoining circuit. It is easy to show that the usual method of calculating the e. m. f. as equal to the time rate of change of the number of magnetic lines through the coil is entirely equivalent to that derived on the assumption of a relative motion between the coil and the field equal to the velocity of light. The following is a proof of this point.

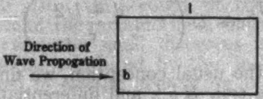

FIG. 9

Consider a rectangular circuit of height b and length l in the direction of propagation of the wave. The induction field has a value $\frac{h I_0}{10 d^2} \sin \omega (t - d/c)$ at points along the nearer side of the circuit, so that the induced e. m. f. in this side is by (**17**) (relative motion conception) $\frac{h b c I_0}{10 d^2} \sin \omega (t - d/c)$. The e. m. f. induced in the further side is likewise

$\frac{h\,b\,c\,I_0}{10\,(d+l)^2}\sin\omega\left(t-\frac{d+l}{c}\right)$. The resultant e. m. f. in the coil is the difference,

$$E=\frac{h\,b\,c\,I_0}{10}\left[l/d^2\sin\omega\,(t-d/c)-\frac{l}{(d+l)^2}\sin\omega\left(t-\frac{d+l}{c}\right)\right]$$

If l is small compared with d, this gives

$$E=-\frac{h\,b\,c\,I_0}{10\,d^2}\left[2\cos\omega\left(t-\frac{d+l/2}{c}\right)\sin\frac{\omega\,l}{2\,c}\right]$$

or, for l small compared with λ.

$$E=-\frac{h\,b\,\omega\,l\,I_0}{10\,d^2}\cos\omega\left(t-\frac{d+l/2}{c}\right)$$

According to the usual conception, we find the number of lines through the coil, $H\,b\,l$, and differentiate with respect to the time. For H we may, for values of l small compared with λ, consider the average phase as that of the center, and neglect the difference between d^2 and $(d+l/2)^2$.

Thus

$$N=\frac{h\,b\,l\,I_0}{10\,d^2}\sin\omega\left(t-\frac{d+l/2}{c}\right)$$

and

$$E=-\frac{d\,N}{d\,t}=-\frac{h\,b\,l\,\omega\,I_0}{10\,d^2}\cos\omega\left(t-\frac{d+l/2}{c}\right)$$

The two methods are in agreement, and for coils whose dimensions are small in comparison with the wave length, and whose distance from the source is small, we have our usual expression giving an e. m. f. 90 deg. behind the field (current) in phase.

Repeating the demonstration with the radiation field, we have from (**17**)

$$E=\frac{h\,\omega\,b\,I_0}{10\,c\,x}\,.\,c\left[\cos\omega\,(t-d/c)-\cos\omega\left(t-\frac{d+l}{c}\right)\right]$$

$$=\frac{2\,h\,\omega\,b\,I_0}{10\,x}\sin\frac{\omega\,l}{2\,c}\sin\omega\left(t-\frac{d+l/2}{c}\right)$$

$$= \frac{\omega^2 h\, b\, l\, I_0}{10\, c\, x} \sin \omega \left(t - \frac{d + l/2}{c} \right)$$

By the second method

$$N = \frac{h\, \omega\, I_0}{10 c\, x} \,.\, b\, l \,.\, \cos \omega \left(t - \frac{d + l/2}{c} \right)$$

$$E = - \frac{d N}{d t} = \frac{h\, \omega^2\, b\, l\, I_0}{10\, c\, x} \sin \omega \left(t - \frac{d + l/2}{c} \right)$$

In this case also the two methods lead to identical results.

The conclusion, therefore, seems to be certain, that both fields are propagated with the velocity of light, and that both, when they cut a conductor, give rise to an induced e. m. f. which induces it. Since the two fields differ in phase by 90 deg. the e. m. fs., will also. On account of the relatively insignificant magnitude of the radiated field, however, at the usual a-c. frequencies, and the small distances involved in usual apparatus, the e. m. f. induced by the induction field only need be considered in electric generators and transformers.

Another distinction, which is made in the paper, is that accompanying the radiation field, there is propagated an electrostatic field which bears a constant ratio to the magnetic, while with the induction field there is no such constant predetermined relation. From what has already gone before, it would seem that this constant relation between H and E must also be true of the induction field. The induced e. m. f. in a wire of length h has been shown to be equal to $c\, H\, h \,.\, 10^{-8}$ (equation **17**) and I have shown above that this equation is true for *both* components of the magnetic field. Now, in the present case, the field is supposed to be uniform along the wire, so that the electrostatic field, which is equal to the e. m. f. per cm., must be $c\, H \,.\, 10^{-8}$ or 300 H, in which H includes both components. The statement is made 1366 that this relation is true only in a radiated wave, but it must not be concluded herefrom that for H *only* the radiation field is to be taken. Both fields are radiated, and the induction field, however small it is, must, strictly speaking, be included in H in the above relation.

As to what is the difference in physical nature between the radiation and induction fields, there remains the point mentioned in the first paragraph of 1367, "The portion of the energy associated with this disturbance that does not return to the radiator is that connected with the first term of equation (**31**)" (*i. e.* the radiation field). This, of course, means that the energy associated with the induction field, although naturally it flows back and forth, must average zero. That is, the flow through any volume element in one direction over half the cycle is equal to the flow in the opposite direction during the remainder of the cycle.

Now this statement appears plausible, and from the usual a-c. idea of wattless current is what would be expected of the induction field. I have, however, been unable to *prove* it, and the demonstration given below would seem to lead to the idea that a portion of the energy associated with *each field* is returned, and a portion radiated, *i. e.* that there is no difference in nature, respecting net energy flow, between the two components of the field.

In the case of a simple making of a steady current I, energy is flowing out to all points of space during the rise of the current and finally after an infinite time, there is a distribution of energy through space of $\frac{H^2 \mu}{8 \pi}$ per cu. cm. which, since H decreases and becomes zero at infinite distance, is less in remote regions and greater near the circuit, but finite in sum and equal to $1/2\, L\, I^2$.

When the current is broken, this energy all eventually returns to the source. If the current is made to rise and fall periodically, however, it is easy to comprehend that some of the energy sent out would not have a chance to come back before the flow of energy again takes place away from its source, so that some of the energy would be permanently severed from the source. Since this would appear more likely to be the case, the higher the frequency, it is natural to suppose that this energy would be that propagated by the travelling of the *radiation* field rather than that connected with the moving induction field whose amplitude does not involve the wave length. In what follows, the idea of the Poynting vector is introduced to investigate this matter.

The Poynting theorem states that the instantaneous flow of power through any portion of space is equal to $1/4\, \pi$ times the integral of the normal component of the vector product of the electrostatic and magnetic fields, taken over the entire bounding surface of the space in question.

In the present instance, the electrostatic field is along the Z axis, the magnetic field along the $-Y$ axis and the flow of energy along the X axis. The flow of energy through an element of volume $d\, V = dx\, dy\, dz$ is equal to the excess of the amount through one $dy\, dz$ face over that through the opposite face. Since H and E are at right angles, the vector product is their simple product.

Taking a point (x, y, z) the power entering the face $dy\, dz$ is $\frac{E\, H}{4\, \pi}\, dy\, dz$. That over the opposite face is

$$\frac{\left(E + \frac{d\,E}{d\,x}\,d\,x\right)\left(H + \frac{d\,H}{d\,x}\,d\,x\right)}{4\,\pi}\,dy\,dz,$$ so that the net

power, is, in the limit, $\frac{1}{4\,\pi}\left(H\,\frac{d\,E}{d\,x} + E\,\frac{d\,H}{d\,x}\right)dy\,dx\,dz.$

Now we have shown that $E = c\,H$ so that the power is

$$\frac{1}{4\,\pi}\left(c\,H\,\frac{d\,H}{d\,x} + c\,H\,\frac{d\,H}{d\,x}\right)d\,V = \frac{2\,c}{4\,\pi}\,H\,\frac{d\,H}{d\,x}\,d\,V$$

If we place $H = H_r + H_i$, the power is

$$\frac{2\,c}{4\,\pi}\,(H_r + H_i)\left(\frac{d\,H_r}{d\,x} + \frac{d\,H_i}{d\,x}\right)d\,V$$

Now

$$H_r = \frac{h\,\omega\,I_0}{10\,c\,x}\cos\omega\,(t - x/c)$$

$$\frac{d\,H_r}{d\,x} = +\frac{h\,\omega^2\,I_0}{10\,c^2\,x}\sin\omega\,(t - x/c) - \frac{h\,\omega\,I_0}{10\,c\,x^3}\cos\omega\,(t - x/c)$$

$$H_i = \frac{h\,I_0}{10\,x^2}\sin\omega\,(t - x/c)$$

$$\frac{d\,H_i}{d\,x} = -\frac{\omega}{c}\,\frac{h\,I_0}{10\,x^2}\cos\omega\,(t - x/c) - \frac{2\,h\,I_0}{10\,x^3}\sin\omega\,(t - x/c)$$

The term $H_r\,\frac{d\,H_r}{d\,x}$ may be regarded as due to the radiation field only, that due to the induction field is $H_i\,\frac{d\,H_i}{d\,x}$, and the remaining terms

$$\left(H_i\,\frac{d\,H_r}{d\,x} + H_r\,\frac{d\,H_i}{d\,x}\right)d\,V$$

may be regarded as the power due to the interaction of the electrostatic field of one component magnetic field with the other component of the magnetic field.

$$H_r \frac{d H_r}{d x} = \frac{h^2 I_0^2}{100\, x^2} (\omega/c)^3 \cos \omega\, (t - x/c) \sin \omega\, (t - x/c) - \frac{h^2 I_0^2}{100\, x^3} (\omega/c)^2 \cos^2 \omega\, (t - x/c)$$

$$H_i \frac{d H_i}{d x} = - \frac{h^2 I_0^2}{100\, x^4} (\omega/c) \sin \omega\, (t - x/c) \cos \omega\, (t - x/c) - \frac{2\, h^2 I_0^2}{100\, x^5} \sin^2 \omega\, (t - x/c)$$

$$H_i \frac{d H_r}{d x} = \frac{h^2 I_0^2}{100\, x^3} \sin^2 \omega\, (t - x/c) - \frac{h^2 I_0^2}{100\, x^4} (\omega/c) \sin \omega\, (t - x/c) \cos \omega\, (t - x/c)$$

$$H_r \frac{d H_i}{d x} = - \frac{h^2 I_0^2}{100\, x^3} (\omega/c)^2 \cos^2 \omega\, (t - x/c) - \frac{2\, h^2 I_0^2}{100\, x^4} (\omega/c) \cos \omega\, (t - x/c) \sin \omega\, (t - x/c)$$

Now the terms in cos $(t - x/c)$ sin $(t - x/c)$ integrated over a whole cycle give zero, but the terms in $\cos^2 (t - x/c)$ and $\sin^2 (t - x/c)$ give a finite amount, since the integral of $\sin^2$ or $\cos^2$ over a cycle gives 1/2. It would appear, then, that the induction component as well as the radiation component contributes to the radiation of energy. At large distances and short wave lengths, however, the $H_i \frac{d H_i}{d x}$ term is the smallest of all. On the other hand the $H_r \frac{d H_i}{d x}$ term is equal to the $H_r \frac{d H_r}{d x}$ term.

In conclusion, then, there seems to be no essential difference in the nature of the two components of the magnetic fields, except in the way they vary with the frequency and the distance from the source.

A. Press: In the paper by Dr. Dellinger mention is made of a method which has been referred back to some work of Prof. Lorentz. Strangely enough the same general method was gone over quite independently by myself but I must say that it seems to me that the real value of the method of vector potentials suggested is rather approximative. In the case of a horizontal antenna which was the case considered in my own investigations the difficulty arose in imagining the manner in which the interlinked flux lines or rather the interlinked flux sheets could disentangle themselves from the conductors (or coils).

The true basis after all of estimating the value of H must be the Maxwellian equations of wave propagation. In my later work therefore, which by the way was gone into in my classes at the University of California about a year ago I gave the following derivation:

A round wire is assumed to carry a current

$$i = I_0 \sin p t$$

It is required to determine the type of magnetic field that is set up as a function of the wave length of the system. The individual conductors of a two-wire parallel system will be investigated with the origin at the center of a conductor. The required equations of condition are (for polar coordinates)

$$\frac{d B_\theta}{d t} = \frac{d E}{d \gamma}$$

$$\frac{d D_z}{d t} = 1/\gamma \, H_\theta + \frac{d H_\theta}{d \gamma}$$

The above two equations can be made to result in the following

$$1/\gamma^2 . \; \frac{d^2 D_z}{d t^2} = 1/\gamma \; \frac{d D_z}{d \gamma} + \frac{d^2 D_z}{d \gamma^2}$$

$$1/\gamma^2 . \; \frac{d^2 H_\theta}{d t^2} = \frac{d^2 H_\theta}{d \gamma^2} + 1/\gamma \frac{d H_\theta}{d \gamma} - 1/\gamma^2 . H_\theta$$

Writing the simpler of the two equations in the form

$$\frac{d^2 D}{d \gamma^2} + 1/\gamma \frac{d D}{d \gamma} = q^2 D$$

With

$$q^2 = 1/\gamma^2 . \frac{d^2}{d t^2}$$

a solution is found to be

$$D_z = -\frac{1}{2\pi\alpha}\cdot\frac{I_0\cos p\,t}{\gamma}\cdot\frac{J_0\left(\frac{2\pi\gamma}{\lambda}\right)}{J_1\left(\frac{2\pi\alpha}{\lambda}\right)}$$

where α is the radius of the wire and $p = 2\pi f$, which leads to the corresponding solution for H which is

$$H_\theta = \frac{I_0}{2\pi\alpha}\cdot\frac{J_1\left(\frac{2\pi\gamma}{\lambda}\right)}{J_1\left(\frac{2\pi\alpha}{\lambda}\right)}\cdot\sin pt$$

It is seen therefore that there is no out of phase term but rather that a stationary wave is set up. The above solution therefore does not offer any physical difficulties of picturing interlinked flux lines disentangling themselves from the system to produce radiant energy. In reality galvanic effects above described have been found not to produce any actual radiation, in the case of a vertical antenna. The whole problem of the vertical grounded antenna has been treated by the writer. The radiation factors are gone into in detail but from the standpoint of wave propagation wholly.

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THE VACUUM TUBE AS A GENERATOR OF ALTERNATING-CURRENT POWER

BY JOHN H. MORECROFT AND H. TRAP FRIIS

ABSTRACT OF PAPER

The first part of this article deals with the operation of the tube when separately excited, the variation of power with the amount of excitation, the load impedance, etc., and also gives an analysis of the forms and phases of voltages and currents in the different parts of the circuit.

The second part deals with the efficiency of the tube as a generator; the action is analyzed in detail and the conditions for maximum efficiency deduced, the theoretically deduced conclusions being substantiated by experimental data. Oscillograms are given to show the action of the tube under practically all the conditions which are likely to occur.

THE three-element vacuum tube utilizes the controlling effect of the grid potential on the plate current, that is, the electron flow from the hot filament to the cold (comparatively) plate. It may be used as a detector of high-frequency waves as used in radio communication, or as an amplifier of electrical signals of any frequency or as a generator of alternating-current power of practically any frequency desired, from perhaps one cycle per second to a hundred million cycles per second. This paper deals with its use as a generator; it seems that such a paper is well worth while because of the undoubtedly wide use which will be made of the tube as a convenient source of high-frequency power, especially for laboratory purposes, and radio telegraphy and telephony.

A three-electrode tube, such as the Type P pliotron, will give about 500 watts of high-frequency power at any frequency desired. In addition to the tube itself there is required a small continuous current generator of about 2500 volts and a set of suitable coils and condensers. Its only competitor as a piece of necessary laboratory apparatus is the high-frequency alternator; this is however no true competitor, its comparatively high cost, difficulty of speed control, limitation of frequency, decrease of output at frequencies differing widely from that for which it was built, attention required to maintain it against mechanical failure, make the vacuum tube superior unless several kilowatts of power are required; in this case the Poulsen arc must also be considered. Even then the flexibility of the tube circuit gives it a marked advantage over the machine. For the very high frequencies the tube is the only practical source of power.

Power output of the tube and its variation under different conditions. The relation between plate current, grid potential, and plate potential, is given by the equation

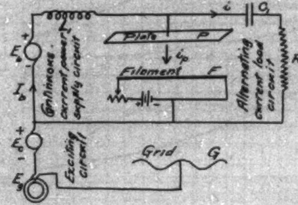
$$I_p = A\,(E_p + \mu_0\,E_g)^x \qquad (1)$$

in which E_p = plate potential, referred to filament,
E_g = grid potential,
I_p = plate current,
A = a constant, depending on the size and spacing of the parts of the tube.

The factor, μ_0, is called the theoretical voltage amplification factor of the tube; it really indicates the relative effectiveness of the grid and plate potentials in controlling the plate current. The exponent, x, has been given as 1.5 and 2 by various writers; it is however a variable, not a constant. Its value is nearly 2 for most tubes throughout a large variation in E_p and E_g, but for extreme values of either, x departs widely from this value. Equation (**1**) is based on the assumption that the plate current is not large enough to draw from the filament all of the electrons emitted from its hot surface, that is, I_p is less than the saturation current of the tube.

The study of the output, phase relations, etc., was carried out with a type P-10 pliotron, rated at 1000—2000 volts on the plate, filament current of 3.65 amperes, and a safe loss on the plate of 250 watts. The connections were as shown in Fig. 1; in reality the grid is placed between the plate and filament but in this diagram, as in the succeeding ones, we have placed it on the opposite side of the filament to gain clarity in the circuit diagrams. The continuous-current generator E_b furnishes about 1000 volts to the plate through a choke coil L_1, this being suitably large. The continuous-current generator E_c serves to maintain the grid at an average negative potential of any desired amount, and the alternator E_g furnishes the exciting potential to the grid. The condenser C_1 must have a comparatively low reactance if a resistance load is being studied; in series with this is the load resistance R, which absorbs the alternating-current power generated.

The alternating-current resistance of the plate circuit of the tube R_p is the factor which determines how large L_1 and C_1 must be; the value of R_p varies somewhat with the magnitude of the exciting voltage E_g. For large values of E_g it is equal to E_b divided by I_b, if the resistance of L_1 is low; if this resistance is appreciable the voltage E_p must be used instead of E_b. For small values of the exciting voltage E_g the value of R_p is lower than this, perhaps one half or one third. The values of L_1 and C_1 must be so chosen that for the frequency used the reactance of L_1 is large compared to R_p and that of C_1 is small compared to R_p. The suitable values of L_1 and C_1 are really determined by the value of the load resistance, R, but as this must be of about the same value as R_p if much power is to be delivered, R_p does indirectly fix their values.



i
C1
L1
Plate
P
ip
Filament
F
R1
+
Eb
-
Ib
Continuous current power supply circuit
Alternating current load circuit
+
Ec
-
Eg
Exciting circuit
Grid
G

FIG. 1

When the grid potential fluctuates the plate current must vary in magnitude according to the relation given in equation (1); this fluctuating current is equivalent to a continuous current with an alternating current superimposed; the alternating component will practically all flow through the R-C_1 circuit because of the high reactance of the coil L_1.

The first effect studied was the variation of output as the value of the load resistance R, was varied, the excitation being comparatively low; the results are shown in Fig. 2. The curve sheet shows the variation of the various quantities as the value of R was changed; it is seen that a maximum output occurred with R equal to 1000 ohms. For the conditions of this test the value of R_p was 1100 ohms, although the quotient of E_p and I_b gives very nearly 3000 ohms.

In Fig. 3 is shown the behavior of the circuit as the exciting voltage, E_g was varied; the connections were the same as those of Fig. 1. The alternating-current output increases with the square of the exciting voltage up to a certain value of E_g and then increases more slowly. For the lower values of E_g (less than 120 volts) the fluctuating plate current has an approximately sinusoidal form but for the higher values the plate current becomes a series of pulses.

The decrease in I_b in Fig. 2 and increase in Fig. 3 are both due to the fact that the relation between grid and plate potentials and plate current is not linear; if sufficient resistance is put in the load circuit, I_b remains nearly constant as the value of E_g is changed.

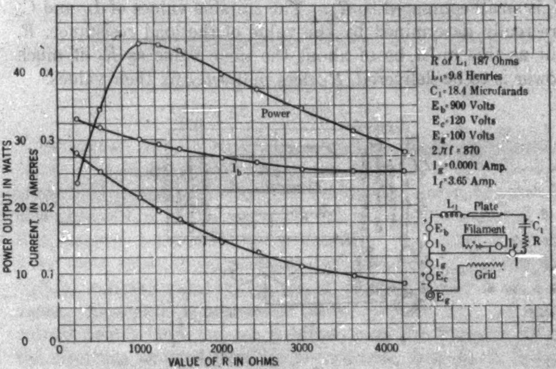


FIG. 2

In Fig. 3 is shown the grid current, as read on a continuous current ammeter; this reached a readable value when the effective value of E_g reached 95 volts. The maximum value of E_g is 134 volts and as E_c was 120 volts it is evident that the grid had to be positive as much as 14 volts before it took an appreciable current. The value of E_g at which I_g becomes readable depends to some extent on the value of the load resistance R.

Safe load of a vacuum tube. The two factors limiting the output of a vacuum tube are the safe filament current and the amount of power which can safely be dissipated from the plate (generally double). The heating of the plate is due to heat radiated from the filament and to the bombardment by the electrons leaving the filament, being accelerated by the high positive potential of the plate, and then being suddenly stopped when impinging on the plate. With a properly evacuated tube the amount of power which can be thus expended on the plate is sufficient to bring it to a dull red heat, this color being judged when the filament is incandescent; if the filament current is quickly reduced to zero the plate has quite a bright cherry red color.

When the tube is not generating any alternating-current power all of the input $E_b I_b$ is used in heating the plate but when alternating current is flowing in the load circuit some of the power supplied to the tube circuit is used up in R and the amount of power used on the plate is equal to $E_b I_b$ minus whatever power is used in the R-C_1 circuit. If the values of E_c and E_g are properly adjusted the amount of power used in R may be equal to or even greater than that used on the plate so the safe value of $E_b I_b$ may be as much as three times the safe plate rating without endangering the tube; if however the tube stops oscillating for any reason the input must be at once reduced or the tube will probably be spoiled.

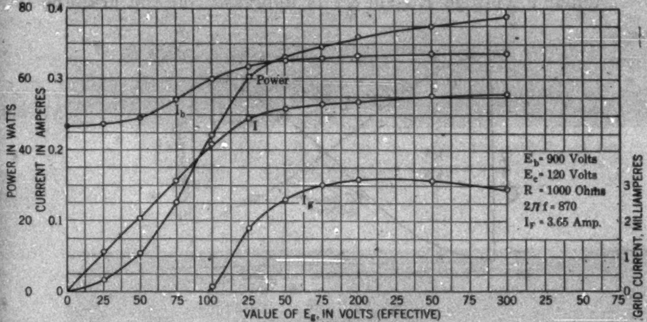


FIG. 3

An approximate idea of the behavior of the tube circuit as the exciting voltage is increased is given in Fig. 4; assuming a constant input, the output and losses follow the variation shown. The efficiency may reach quite high values when suitable adjustments are made. In the small sets used by the Signal Corps the tubes were adjusted for an efficiency of about 30 per cent but it has been found possible to so adjust one of these tubes in the laboratory as to get a fair output with an efficiency better than 70 per cent.

Phases and Forms of Currents and Voltages in a Tube Circuit. If the impedance of the power supply circuit (Fig. 1) is very high compared to that of the load circuit the current I_b is constant, and the equations of the alternating current circuit may be worked out as though the power supply circuit did not exist.

When a voltage E_{mg} sin ωt is impressed on the grid the changes produced in the plate current are the same as though a voltage $\mu_0 E_{mg}$ sin ωt had been introduced into the plate circuit. The changes produced in the plate current circuit by the grid excitation may be calculated on the assumption that the voltage $\mu_0 E_{mg}$ sin ωt was operating in the plate circuit. Thus a voltage E_g on the grid is replaced in our calculations by a suppositious voltage $\mu_0 E_g$ in the plate circuit; the current produced by this voltage can be at once calculated from the impedance of the circuit in which the alternating-current must flow, namely R_p, R, and C_1 in series.

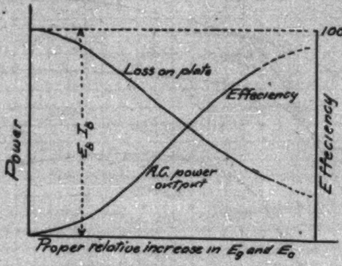
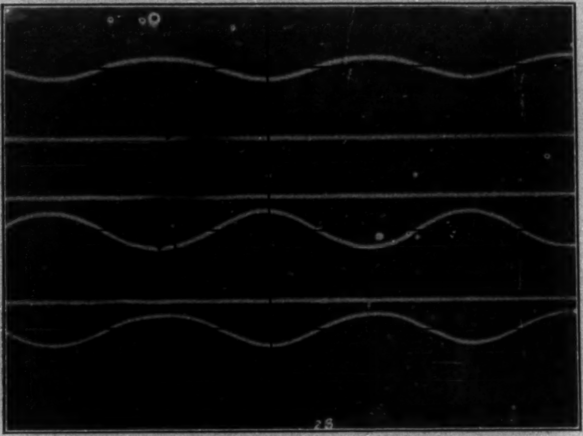
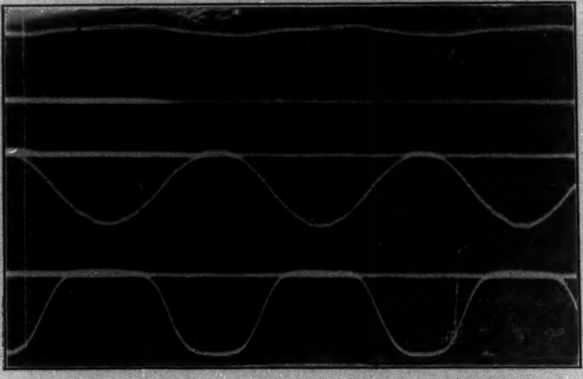


FIG. 4

The vibrators of an oscillograph were introduced in the circuit as shown by Fig. 5 and the polarities so adjusted that currents in the directions indicated by the arrows are positive; when a current is shown on its film below the zero line the

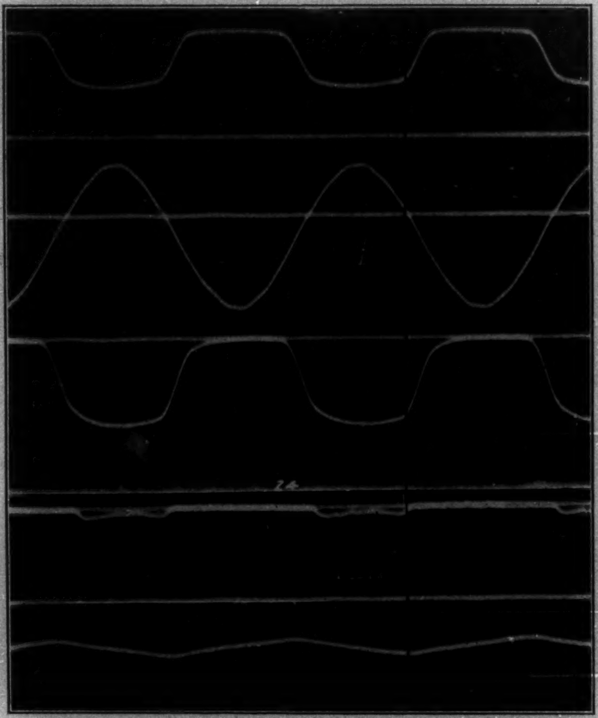


$E_b = 900 \quad I_b = 0.25 \quad E_c = 120 \quad E_g = 50$ Frequency = 140
Load R = 1000 ohms $C = 18.4\ \mu f$
Fig. 6



[MORECROFT & FRIIS]

$E_b = 900 \quad I_b = 0.34 \quad E_c = 120 \quad E_g = 100 \quad f = 140$
Load R = 100 ohms $C = 18.4\ \mu f$
Fig. 8



[MORECROFT & FRIIS]

$E_b = 900 \quad I_b = 0.34 \quad E_c = 120 \quad E_g = 300 \quad f = 140$

LOAD $R = 1000$ OHMS $C = 18.4\ \mu f$

FIG. 7

direction of the current was in the opposite direction to that shown by the arrow. The oscillograph used had only three vibrators; it was sometimes necessary to take several exposures to get all the quantities wanted. This was done by always running the film at the same speed and making one curve (generally i_p) common to all films; it was then possible to so place the films together for reproduction that all quantities were in their correct relative phases.

In the first circuit tested the total resistance in the alternating-current circuit (R_p plus R) was about 4000 ohms and the reactance of the condenser C_1 was 62 ohms; we should expect therefore that the current I_p and voltage E_g would be in phase and moreover if the grid excitation is kept low the alternating component of the plate current, I_p, should be of the same form as the grid voltage. Then as the voltage e_p is determined by the constant voltage E_b and the drop in R, this voltage must also have a sine wave fluctuation in such phase that when the plate current is a maximum the plate voltage is a minimum. Fig. 6 shows the form of plate voltage, plate current, and grid voltage, for the conditions noted below the film. It will be seen that both e_p and i_p are sinusoidal in form and that e_p is 180 degrees out of phase with e_g. The grid current was zero for the conditions given; it may be seen from the film that E_g was not sufficiently large to force the grid to a positive potential.

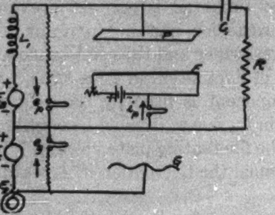


FIG. 5

In Fig. 7 are shown the various quantities for a much greater excitation than was used for Fig. 6. The plate current rises to a value of 0.66 amperes and decreases to zero; the current flowing in the alternating-current circuit must therefore have a maximum value of 0.33 amperes giving an $I R$ drop in R equal

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330 volts. The fluctuation in e_p as measured in the film is 340 volts. The grid takes current all the time the grid is at positive potential, but it is very small, due to the fact that the plate potential never falls to lower than 660 volts.

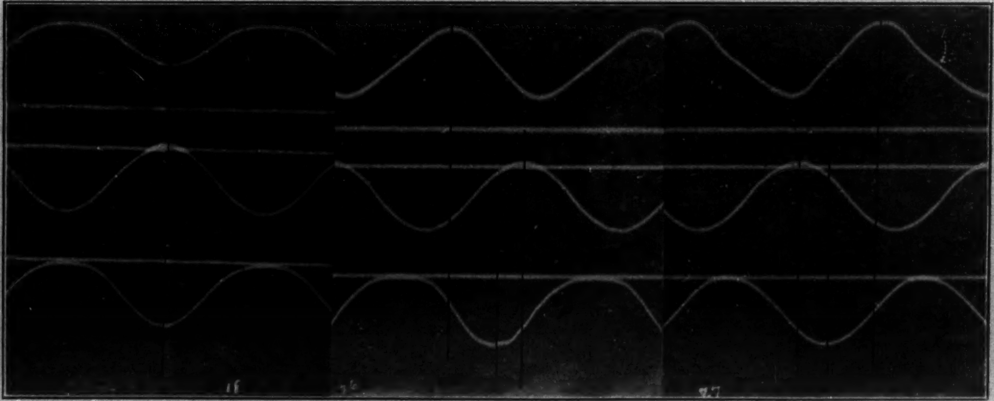
If a greater variation in the plate voltage is desired it can be brought about only by increasing the value of R; if this had been 3000 ohms instead of 1000, the plate potential would have fluctuated throughout a much wider range. As the normal maximum output (assuming sinusoidal variations of voltage and current, is obtained from a tube when both i_p and e_p vary from zero to twice their normal values it is evident that R should have been increased to 3000 ohms to get maximum output. But by referring to Fig. 2 it would seem that 1000 ohms was the proper value of R for maximum output. This discrepancy arises from the fact that the results for Fig. 2 were obtained with low grid excitation while the curves in Fig. 7 were obtained with an excitation three times as large. As previously stated, R_p varies with the excitation, and the maximum output is obtained when the load resistance and tube resistance are the same.

The amount of fluctuation in I_b is small; the value of the alternating component is fixed by the requirement that the reactance drop in L_1, due to it, must be equal to the alternating component of the fluctuating plate voltage. If the fluctuation in e_p were sinusoidal the fluctuation in I_b would also be of that form.

With lower values of load resistance the distortion in plate current occurs even with low excitation; this is shown in Fig. 8 for which R had a value of only 100 ohms. Although there was the same alternating current produced in the output circuit as for the case given in Fig. 7 the fluctuation in plate voltage is very small, due to the very low impedance of the load circuit.

The alternating current of the tube circuit may be analyzed by the laws of the ordinary circuit, in case we are interested only in the sine wave components of the quantities studied. The vector diagrams of the three possible kinds of load circuits are shown in Fig. 9; in these diagrams $\cos \phi$ gives the power factor of the load circuit itself and $\cos \theta$ gives the power factor of the whole alternating-current circuit, including the resistance of the tube as well as the load circuit resistance.

In Fig. 10 are shown the curves of e_p, e_g, and i_p for circuits corresponding to those assumed in Fig. 9; the three circuits as actually used had the same excitation and nearly the same



[MORECROFT & FRIIS]

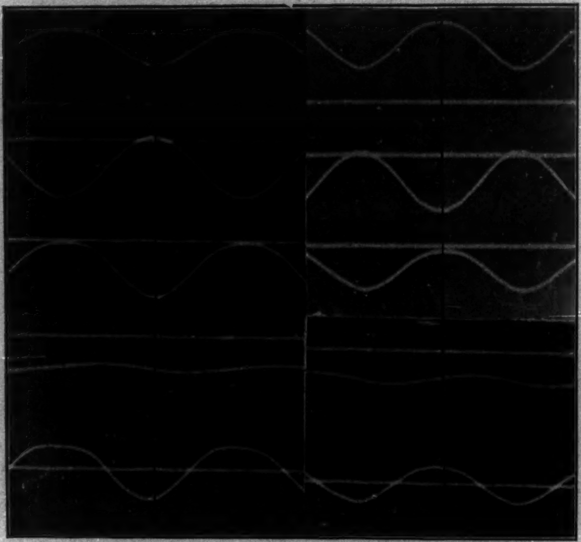
Resistance Load
$R = 1000$ ohms

Capacitive Load
$R = O \quad \frac{1}{\omega C} = 1150$ ohms

Inductive Load
$R = 30$ ohms $\quad \omega L = 1500$ ohms

For All Loads $E_p = 900 \quad E_c = 120 \quad E_g = 100$

Fig. 10



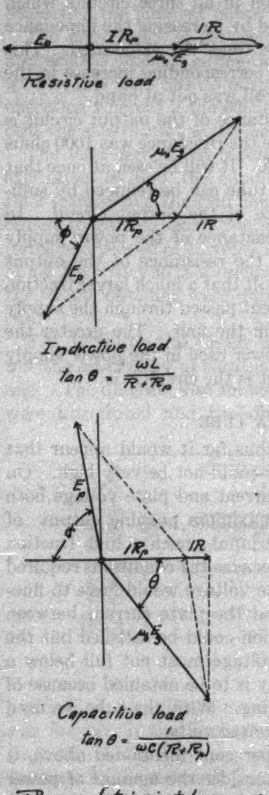
[MORECROFT & FRIIS]

$I_b = 0.295$ $R = 1000$ $I_b = 0.272$ $R = 2010$

$E_b = 900$ $E_c = 120$ $E_g = 100$ $f = 140$

FIG. 11

magnitude of impedance in the load circuit, the first being resistive, the second, capacitive and the third, inductive.


E_p
IR_p
IR
$\mu_0 E_g$
Resistive load
$\mu_0 E_g$
θ
ϕ
IR_p
IR
E_p
Inductive load
$\tan\theta = \frac{\omega L}{R+R_p}$
E_p
ϕ
IR_p
IR
θ
$\mu_0 E_g$
Capacitive load
$\tan\theta = \frac{1}{\omega C(R+R_p)}$

Phase relations in tube circuits

FIG. 9

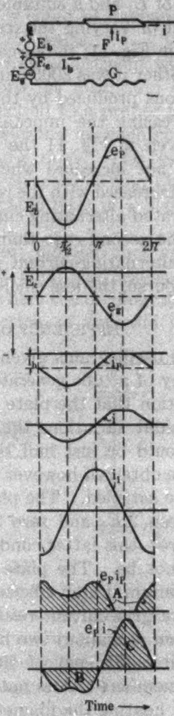

P
i
i_1
C_1
C_2
L
R
E_b
F
i_p
F_e
i_b
E_g
G
e_p
E_b
0
$\frac{\pi}{2}$
π
2π
E_c
e_g
i_b
i_p
i
i_1
$e_p i_p$
A
$e_p i$
C
B
Time →

FIG. 13

With resistive load the plate current (alternating component) is in phase with the voltage E_g, with the capacitive load it leads and with the inductive load it lags; the angles ϕ and θ

measured from the film, and the calculated values, agree within the precision of measurement.

There is considerable distortion in all three circuits which could have been nearly eliminated by increasing the impedance of the load circuit to about three times the value used. This would however, have required a corresponding increase in the inductance of L_1 and a suitable coil was not at hand.

The effect of varying the resistance of the output circuit is well shown in Fig. 11; in one case the resistance was 1000 ohms and in the other it was 2000 ohms. It will be seen at once that the distortions produced by the tube can be reduced by sufficiently increasing the impedance of the output circuit. In the case given in Fig. 11 the reactance of the power supply circuit was not increased when the resistance of the output circuit was increased with the result that a much larger fraction of the generated alternating current passed through the supply circuit in the second case than in the first. The greater the amount of alternating-current power used in the power supply circuit, of course, the less efficient is the circuit.

EFFICIENCY OF A TUBE

From the oscillograms given thus far it would appear that the efficiency of a tube generator could not be very high. On the assumption that the plate current and plate voltage both have sinusoidal variations the maximum possible output of the tube would be just half the input; such a high fraction could not be obtained however because the conditions required could not be satisfied. The plate voltage would have to fluctuate between $2 E_b$ and zero and the plate current between $2 I_b$ and zero; this latter condition could be satisfied but the first could not be. The plate voltage must not fall below a certain minimum if high efficiency is to be obtained because of the excessive grid current resulting. With the tube we used this minimum was about two hundred volts.

The possible efficiency of 50 per cent mentioned above, it must be remembered, does not consider the amount of power required for heating the filament nor the possible losses in the exciting circuit. If these losses are considered the possible efficiency would be much less, especially in the smaller tubes. We shall now show however that by using peculiar shaped plate currents it is possible to have an efficiency considerably in excess of 50 per cent; in fact the result of this study is that the efficiency of a tube may be pushed possibly as high as 90 per cent by proper design and proper operation, this value neglecting the filament power input.

The importance of getting a high efficiency will be at once appreciated when it is mentioned that a given tube (the one used in this test) has an output of about 200 watts in normal operation whereas if the efficiency could be increased to 90 per cent the safe output would be 2250 watts.

The tests carried out involved an adjustment with separate excitation to find the conditions for maximum output and then transferring the grid connection to a proper point of the circuit to get self excitation, recording for each condition the forms and phases of currents and e. m. fs. The tests were run at low frequency so that oscillograph records might be obtained; the results obtained were duplicated later in a high-frequency run.

Fig. 12 shows the circuit used; simpler ones may be used but the laboratory apparatus at hand was best suited to this one. The diagram also shows where the oscillograph vibrators were introduced and the direction of currents assumed as positive; if, on a film, a current is shown below its zero line, it was flowing in the opposite direction to that shown in the diagram. If the frequency of the exciting voltage E_g is chosen the same as the resonant frequency of the load circuit

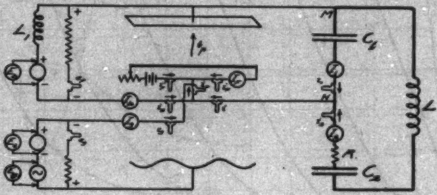


FIG. 12

$$\left(f = \frac{1}{2\pi \sqrt{L\left(\frac{C_1 C_2}{C_1 + C_2}\right)}} \right)$$

the impedance of this circuit between the two points M and N, where the tube is attached, will be resistive only, its magnitude being equal to $\frac{1}{\omega^2 C_1^2 R}$ ohms.

The quantities to be considered are shown conventionally in their phases in Fig. 13; the current i_1 which flows in the resonant load circuit may be several times as large as the current i, furnished by the tube. The two important things in this diagram are shown in the lower part of the figure, namely, the curves of $e_p\, i_p$ and of $e_p\, i$. These curves give the power loss on the plate and the power supplied by the tube to the load circuit, respectively. It is at once evident that

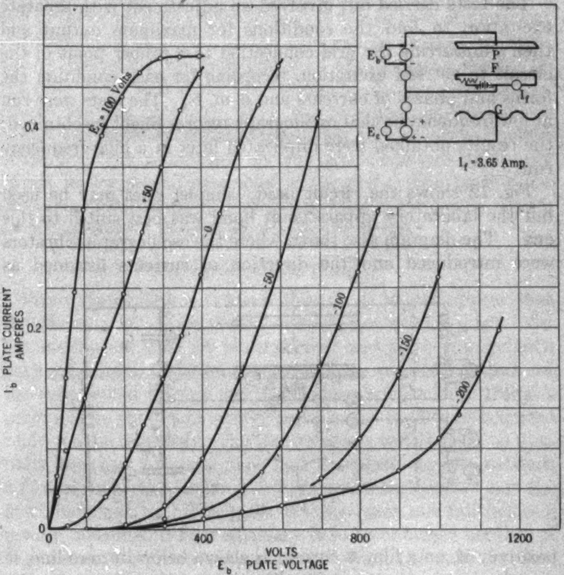


Fig. 14

$$\text{Energy loss on plate per cycle} = \int_0^{2\pi} e_p\, i_p\, dt = \text{Area } A$$

$$\text{Energy supplied to load circuit} = \int_0^{2\pi} e_p\, i\, dt = \text{Area C} - \text{Area } B$$

It is evidently desirable to make the latter as large as possible and the former as small as possible, if the tube circuit is to operate efficiently. Any ordinary scheme of analysis, using the relation given in equation (1) must fail because ther elation does not hold good for those values of e_p and e_g which are the most important ones in the cycle of operation, namely low e_p with positive e_g, and very high values of e_p with large negative e_g.

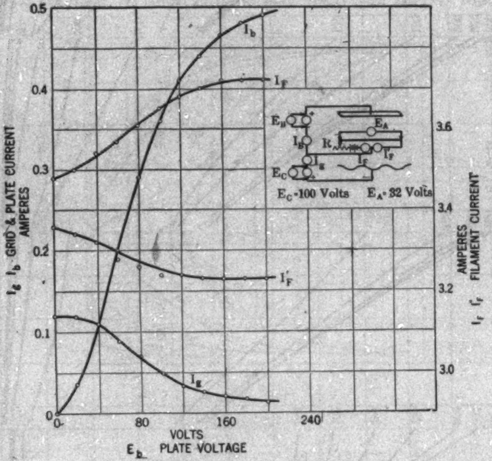


FIG. 15

The ordinary so called static characteristics of the tube used are given in Fig. 14; they are not of much service in predicting the behavior of the tube when the output is forced as high as possible. They did bring out the fact however that the filament ammeter, if a continuous current instrument, does not read correctly the filament current when the tube is generating alternating-current power. The ammeter indicated 3.65 amperes when getting the curves of Fig. 14 and the total emission for such a current is evidently about 0.5 amperes. Now when the tube was oscillating, the filament ammeter reading

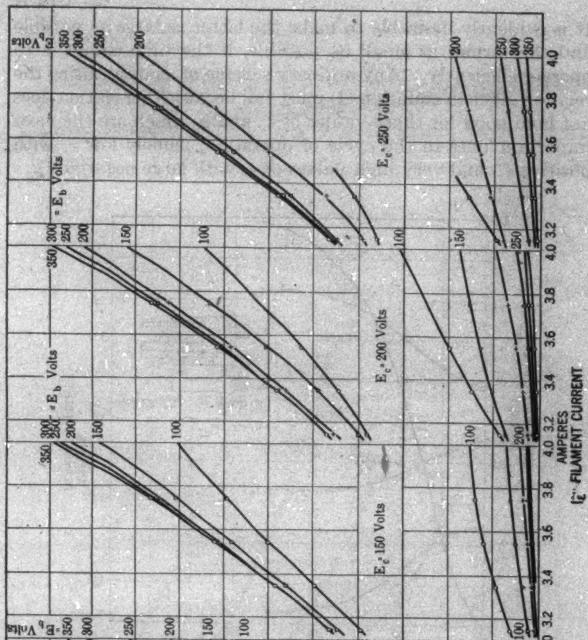

E_c = 100 Volts
E_c = 150 Volts
E_c = 200 Volts
E_c = 250 Volts
= E_b Volts
350
300
250
200
150
100
I_b PLATE CURRENT AMPERES
I_g GRID CURRENT AMPERES
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
3.2
3.4
3.6
3.8
4.0
AMPERES
I_f FILAMENT CURRENT

FIG. 16

3.65 amperes the total emission was about 0.8 amperes, showing that the filament temperature was much hotter than when not oscillating. Holding the voltage across the filament constant (approximately the condition when the tube is oscillating) the set of curves given in Fig. 15 were obtained. The grid was held at a positive potential of 100 volts and the plate voltage suitably varied. The electron current to the plate increases the filament current at one end and decreases it at the other; the relative increase and decrease will be determined largely by the resistance used in series with the filament battery. It can be seen that even with the larger filament current as great as 3.75 amperes the emission was only 0.5 ampere.

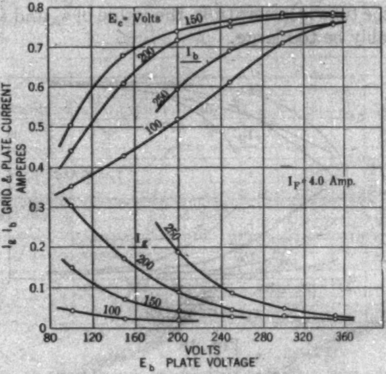


Fig. 17

From some preliminary oscillograph records we knew that in operation the total emission was about 0.8 ampere when the filament ammeter read 3.65 amperes. A brief test showed that the filament current required to give this much emission was 4.00 amperes but this seemed like an excessive current so we got the characteristics required from extrapolation. In Fig. 16 are shown a set of curves showing the variation of plate and grid currents for various filament currents and grid and plate potentials. From this set of curves the results given in Fig. 17 was obtained; as these are important curves they were verified for correctness of form by actually getting them for a lower filament current. These are given in Fig. 18, and are of just the same form as those of Fig. 17.

It is well to point out here that even if we had been able to get the curves of Fig. 17 with a filament current of 4.00 amperes they would not have given the proper values of i_p and i_g for the tube in operation. While getting these static characteristics the plate and grid get very hot, much hotter than when the tube is in operation as a generator. The emission from the filament is fixed by the filament temperature, and this in turn is fixed by the filament current and the temperature of the plate; if this is hotter when getting the static characteristics than when the tube is generating, the value of i_p and i_g obtained would probably be too large.

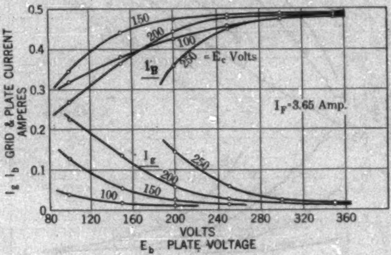
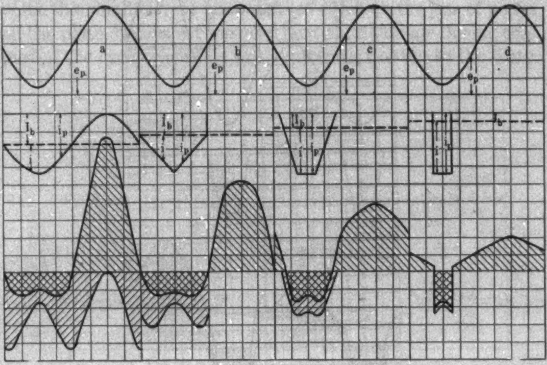


FIG. 18

The curves of Fig. 17, in connection with Fig. 13 enable us at once to give the minimum potential to which the plate should drop and the maximum positive potential for the grid. In order to make the area *A* Fig. 13 small the plate potential, at time $\pi/2$, should be as low as possible. This minimum will be controlled however by the other requirement that the area *C* should be large. If during the time when e_p is low i_p does not have its maximum possible value (saturation current) then the positive alteration of i will not be as large as it should be and if this is not large the power input to the load circuit, determined principally by the area of *C*, will be lower than its proper value.

As the average value of i must be zero, if its positive loop is to be as large as possible, and the area of *A* to be kept as small as possible the conditions should evidently be so adjusted that at minimum plate potential saturation current should flow, and this flow should last for a short time only. During the rest of the cycle the plate current should be zero.

Fig. 19 shows the calculated losses on the plate and input to the load circuit for four different forms of plate current, the plate voltage having the same form for each. It will be seen that both the losses and the output of the tube are greatest for the sinusoidal plate current, but the efficiency for this condition is only 39 per cent; as the form of plate current approaches a short pulse the efficiency increases, being 82 per cent for the form shown in curve (*d*). The trapezoidal form shown at (*c*) resembles very closely the form we used; the test actually gave about 60 per cent efficiency.



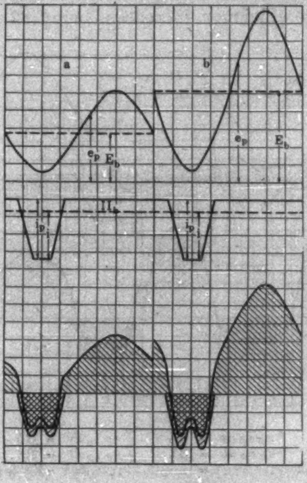
$\int e_p i_p dt = 72$	$\int e_p i_p dt = 35$	$\int e_p i_p dt = 35$	$\int e_p i_p dt = 7$
$\int e_p i_p dt = 47$	$\int e_p i_p dt = 31$	$\int e_p i_p dt = 28$	$\int e_p i_p dt = 23$
EFFICIENCY = 39%	EFFICIENCY = 47%	EFFICIENCY = 49%	EFFICIENCY = 77%

FIG. 19

All four curves are drawn with the maximum plate current the same, supposedly the saturation current for the filament current used; by carrying out other constructions it will be evident that any other condition would result in poorer operation.

By referring to Figs. 13 and 17 it may be seen that for the tube we have, the plate potential should not fall lower than 200 volts, that at this time the grid should have a positive potential of 150 volts. With greater or less grid potential, the plate potential being 200 volts, the plate current would be less than saturation value; with less plate potential the current (at time $\pi/2$) would be less than saturation value, and with greater voltage than 200 volts the loss on the plate would be greater than necessary.

It is to be noted that the efficiency will increase for all the cases given in Fig. 19 if the value of the power supply, E_b, is increased, providing that conditions are suitably changed to have the same minimum plate voltage as given in Fig. 19. This is shown by Fig. 20; the two cases given suppose the same form of plate current and same minimum value of plate voltage but in the second the voltage E_b is about twice as large as in the first case. It is seen that the loss on the plate is increased only 25 per cent whereas the input to the load circuit has been more than doubled. The higher the value of E_b the higher is



$\int_0 e_p i_p dt = 15$	$\int e_p i_p dt = 19$
$\int e_p i \, dt = 34.5$	$\int e_p i \, dt = 71.5$
EFFICIENCY = 70%	EFFICIENCY = 79%

FIG. 20

the efficiency, the limit being fixed by the safe voltage for the tube.

In the tube we used the efficiency did not rise as high as might be expected, due to fact that it took excessively high negative potential on the grid to bring the plate current to zero. The oscillograms showed this effect, so a static characteristic curve was taken to investigate this point; it is shown in Fig. 21. If equation (1) were valid for this tube a negative potential of 260 volts on the grid would have brought the plate current to zero, whereas it took about 1000 volts; although the plate current is small with a grid negative more than 300 volts, this small current has a marked effect on the loss of power on the plate, because of the very high plate voltage during that part of the cycle when this small current is flowing to the plate.

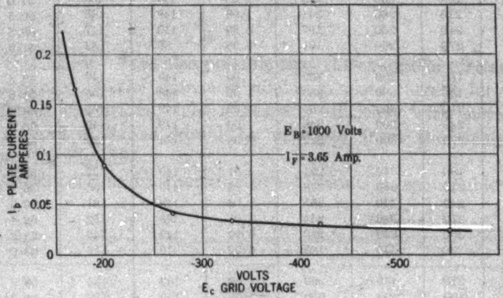


Fig. 21

Experimental proof of foregoing theory. To test the validity of the ideas presented above a series of runs was made with the tube, using the circuit given in Fig. 12 and the results therefrom are shown in Table I. The frequency was kept at the resonant value for the output circuit and each time a set of readings was taken the value of R was changed properly to maintain the current in the oscillating circuit constant. This was necessary in order to keep the form of the voltage, e_p, constant as the values of E_c and E_g were varied. While it was not thus pointed out in discussing the current forms of Figs. 19 and 20 the values of E_c and E_g are the factors which bring about the change of current form as the form of e_p is maintained constant. The form of current shown in (a) Fig. 19 was obtained with relatively low E_c and E_g, the value of each of these being increased for the succeeding diagrams of the figure.

TABLE I.

E_b = 1000 volts. $C_1 = 2_\mu F$. $C_2 = 3.91_\mu F$. $\sim$ = 138. L_1 = 9.8H. I_f = 3.65 Amp.

Run	E_c volts	E_g effective volts	Input watts	I_2 effective amps.	R ω	Output RI_2^2 watts	$\eta = \frac{\text{Output}}{\text{Input}}$ %
E	120	220	334	0.98	149	143	42.8
	150	220	298	1.00	149	149	50.0
	180	220	241	1.02	119	124	51.5
	210	220	186	1.00	89	89	47.8
	250	220	119	0.96	42	39	32.8
	150	260	302	1.00	149	149	49.3
	180	260	273	1.03	149	158	58.0
	210	260	241	1.99	149	146	60.5
	240	260	197	0.99	119	117	59.5
	270	260	161	0.96	89	82	51.0
	150	300	302	1.00	149	149	49.3
	180	300	283	1.02	149	155	54.8
	210	300	261	1.02	149	155	60.0
	240	300	246	1.00	149	149	60.8
	270	300	212	0.98	134	129	60.8
A	180	340	291	1.01	149	152	52.3
	210	340	278	1.03	149	158	56.8
	240	340	265	1.04	149	161	60.8
	270	340	244	1.01	149	152	62.4
B	300	340	229	0.99	149	146	63.8
	330	340	186	0.99	119	117	63.0
	270	400	260	1.02	149	155	59.7
	300	400	250	1.03	149	158	66.3
	330	400	235	1.02	149	155	66.0
	360	400	222	1.00	149	149	67.3
	390	400	197	0.96	134	124	63.0
	420	400	150	1.02	89	93	61.8
C	450	400	126	0.98	74	71	56.3
D	410	460	228	1.02	149	155	68
	420	500	245	1.05	149	164	67.0
	450	500	237	1.04	149	161	68.0
	480	500	222	1.01	149	152	68.6
	510	500	195	1.04	119	129	66.2
	540	500	176	1.02	104	108	61.5
	570	500	157	1.04	89	96	61.2

In Fig. 22 are shown the efficiency curves for the various runs of Table I and on the curve sheet are given the calculated values of the maximum positive grid potential for that condition in each run which gave maximum efficiency, as indicated at *a*, *b*, *c*, *d*, etc., For the comparatively low value of current in the oscillating circuit which obtained during these tests the form of plate voltage is somewhat different from a sine wave, and the variation of best grid potential may have been due to this cause. It is also possible that the change in efficiency was caused by the change in minimum plate current for the different excitations. The increase in efficiency with increase of E_g and E_c is as would be expected from the analysis given for Fig. 19.

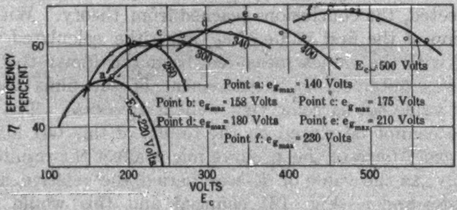


FIG. 22

A series of runs was then carried out (results given in Table II) to study the effect of varying the value of the minimum plate voltage, other conditions remaining the same; this was accomplished by varying *R*, thus cutting down the value of the oscillating current and hence the variation of voltage across the condenser C_1, Fig. 12. The variation of potential across this condenser, it will be noticed, is what controls the fluctuation of plate voltage.

TABLE II.

E_b = 1000 volts. $C_1 = 2\mu F$. $C_2 = 3.91\mu F$. $\sim$ = 138. $L_1 = 9.8H$. I_f = 3.65 Amp.

R_{xn}	e_p min volts	E_c volts	E_g effective volts	Input watts	I_2 effective amps.	R ω	Output $= RI_2^2$ watts	$\eta = \frac{\text{Output}}{\text{Input}}$ watts
A	30	270	300	134	1.12	37	46.5	34.7
	100	270	300	179	1.10	85	103	57.5
	160	270	300	204	1.02	117	122	59.8
	250	270	300	217	0.91	149	123	56.8
B	490	270	300	255	0.60	297	107	42.0

The value of minimum plate voltage can be calculated by subtracting from E_b the I_b resistance drop through L_1 (which was very small for most of our tests) and from this subtracting, the maximum value of the alternating potential drop across C_1. These calculations were made and the results are shown in the curve of Fig. 23; the results verify, better than might be expected, the conclusions reached from theory. With the exception of the first value of e_p (min.) the calculated values agreed with the values measured from the films; the value of 30 was obtained by measurement of the films the calculated value not agreeing very well in this case. Too much reliance cannot be placed on the results of this test however as the exact form of the e_p curve might affect the results considerably; as the value of R was varied the relative magnitudes of i and i_1 (Fig. 12) changed and this would affect the form of e_p.

For various of the runs given in Table I oscillograms were taken of some or all of the quantities involved. For the conditions of run *A* the curves of e_p, e_g, and i_p are given in Fig. 24. From this film, as from the succeeding ones, the first thing to be noticed is that the grid voltage and plate voltage are just 180 degrees out of phase, showing that the load circuit was resistive only. The maximum positive potential of the grid measures on the film 296 volts and the corresponding value of plate potential measures 220 volts. By reference to the curves of Fig. 17 it may be seen that for these respective voltages a large part of the electron current is drawn to the grid, resulting in the peculiar double humped curve of plate current. The maximum negative grid potential was 650 volts, but even this was not sufficient to make the plate current zero. Its values follow, exactly as can be measured, the values given by the curve of Fig. 21. The slight deformation occurring on the positive alternation of E_g is due to the pulse of current taken by the grid at this part of the cycle; the wave form of the alternator used for E_g was nearly a pure sine wave, as may be seen from some of the other films to be given, in which the grid took no current.

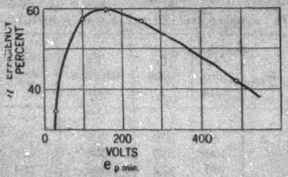
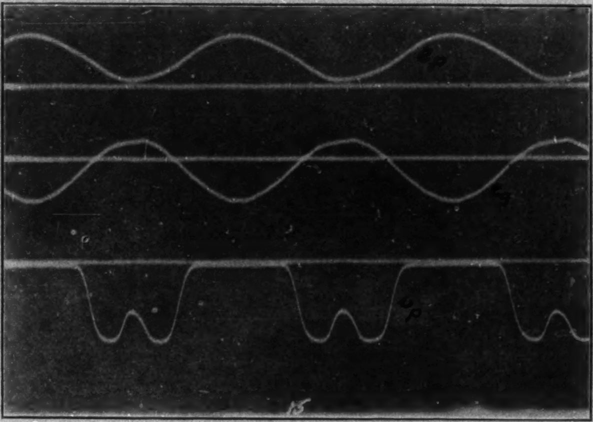
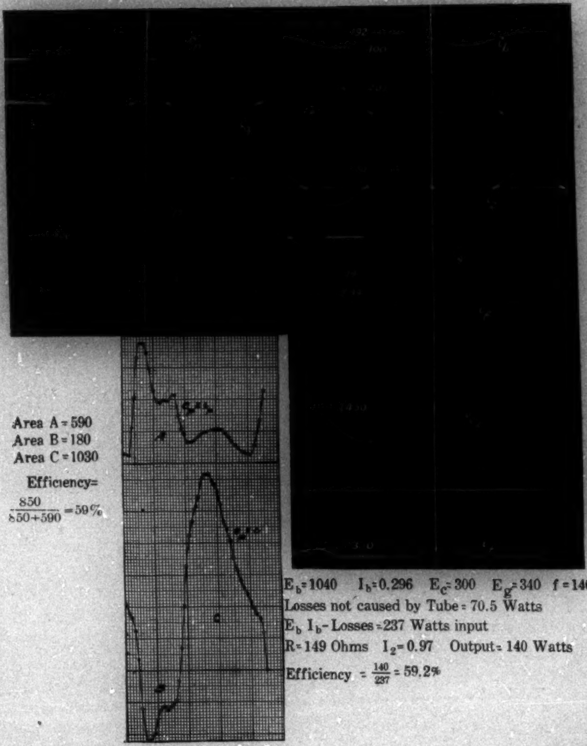


FIG. 23



[MORECROFT & FRIIS]

RUN A TABLE I
FIG. 24



Area A = 590
Area B = 180
Area C = 1030

Efficiency=

$\frac{850}{850+590} = 59\%$

$E_b = 1040$ $I_b = 0.296$ $E_c = 300$ $E_g = 340$ f = 140
Losses not caused by Tube = 70.5 Watts
E_b I_b - Losses = 237 Watts input
R = 149 Ohms $I_2 = 0.97$ Output = 140 Watts
Efficiency = $\frac{140}{237} = 59.2\%$

[MORECROFT & FRIIS]

FIG. 25

For run B a set of oscillograms was taken to show all of the quantities involved in the operation of the tube; it required five oscillograph records to get all the quantities wanted. These five films were combined to make the record shown in Fig. 25; in fitting the various films together care was taken to see that they had their proper respective phases. The white line drawn vertically through all the records gives a line of equi-phase.

This set of curves gives the complete story of the circuit and tube. The plate current is very nearly the form shown in Fig. 20, and the plate potential is nearly of the form shown in condition (a) of the same figure. The slight depression in the peak value of i_p is due to the grid taking some current, this depression coinciding in time with the peak of grid current. The form of the positive alternation of the i curve is not like those previously given, due to the fact that it has been assumed that I_p was constant whereas it actually had considerable fluctuation, as shown in the record. If the coil used for L_1 had more inductance this variation in I_b would be diminished; we had only 10 henries with a resistance of 189 ohms, the coil being air core. In practise an iron core coil of greater inductance would be used but we did not want to introduce any other sources of distortion than the tube itself.

The form of current in condenser C_1 differs from that in condenser C_2 because of the effect of i_1, which will practically all flow through C_1 for the circuit as arranged.

The grid current has just the form and magnitude predictable from Fig. 17; the amount of current taken by the grid in this test and the values of E_g and E_c used caused a loss of power on the grid (due to bombardment) of about 10 watts.

The two filament currents i_f and i_f' have forms which might be predicted from curves similar to those given in Fig. 15; in that end of the filament carrying the large current the continuous current ammeter measuring the current indicated only 3.65 amperes whereas the current actually went as high as 3.99 amperes when the plate was taking its maximum current. The exact amount of emission from the filament when the tube is acting as a generator cannot be predicted from the static characteristic; the temperature distribution in the filament which exists in the oscillating condition cannot be duplicated in a static test and it is this temperature distribution which determines the emission.

The drop across the condenser C_2 was taken to see whether or not it had the right magnitude and phase to serve for excitation of the grid when the tube was run self-exciting; the value of C_2 had been adjusted with this point in mind.

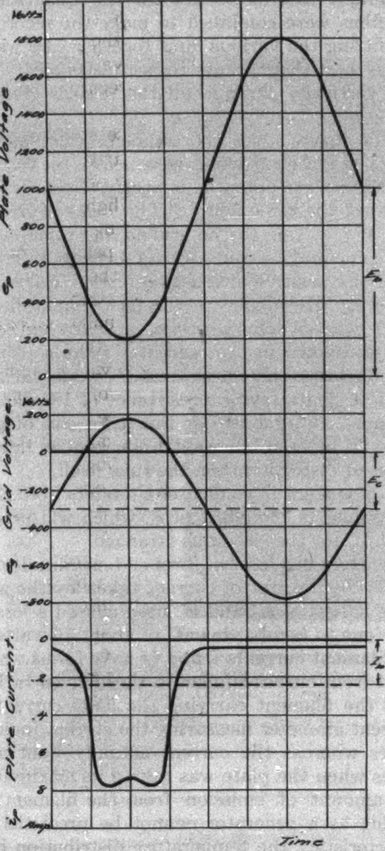


FIG. 26

The scheme of getting the efficiency indicated in Figs. 19 and 20 was tried on this record of e_p, i_p, and i, the power curves of $e_p\, i_p$ and $e_p\, i$ being shown in Fig. 25; the value obtained, 59 per cent, agrees within the precision of the test with that measured by the meters in the test. The value of 63.8 per cent given in Table I was the value obtained when the oscillograph circuits were not connected; the closing of the circuits changed the conditions enough to drop the efficiency to 59.5 per cent.

Fig. 26 shows the form of i_p which is predicted from Fig. 17 after the forms and magnitudes of e_p and e_g have been assumed; this form of i_p is very close to the actual form given in the oscillogram of Fig. 25.

The result of our tests and analysis have then shown that the efficiency of a tube as a generator can be accurately predicted from the three sets of curves given in Figs. 14, 17, and 21 after we have determined, from the curves of Fig. 17, what the best minimum plate potential is and also what the maximum positive potential of the grid should be.

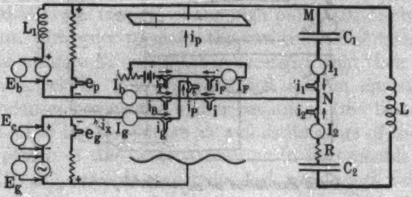
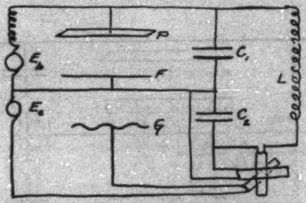


FIG. 27

To get a fair efficiency (60 per cent or better) the value of I_b should not be greater than 25 per cent of the saturation current of the tube; with the efficiency known and the safe radiation of power from the plate being known, the proper value of E_b is fixed.

Self Excited Tube. Using the circuit and constants used in getting the records of Fig. 25 an attempt was made to run the tube self exciting by changing the connections slightly as shown in Fig. 27. The choke coil L_2 serves to prevent the grid from being short-circuited to the filament (for the a-c. excitation) through the machine E_c. The voltage for excitation was obtained from the drop across the condenser C_2, the insulating condenser C_3 being necessary to prevent short-circuiting the machine E_b. With this connection the grid does not get quite as much excitation as shown by the curve e_{c2} in Fig. 25, because an appreciable part of this voltage is used in overcoming the reactance drop in C_3. (In this calculation the capacity of the grid circuit of the tube itself must be considered; in some of the type P tubes this capacity is as high as 200 $\mu\,\mu f$. when the load circuit has its proper impedance for maximum output.)

The circuit of Fig. 27 refused to act as it did for the separate excitation, giving a small output at a low efficiency; a more careful examination of the record in Fig. 25 gave the reason. The alternating components of e_g and e_p must be exactly 180 degrees out of phase if the maximum output and efficiency are to obtain, as becomes at once evident if the construction of Fig. 19 be carried out for any other than the 180 degree relation. Measurement of the film of Fig. 25 shows e_{c2} to be 33 degrees out of the 180 degree phase with e_p and that much phase displacement is sufficient to completely upset the conclusions so far reached. It was therefore necessary to change the relative phases of e_p and e_{c2}. A possible scheme is conventionally indicated in Fig. 28; a rotating field is produced by proper connection to the load circuit and a rotatable coil placed in this rotating field serves for the grid excitation. We had a simpler scheme at hand so did not try this one.



Grid excitation at adjustable phase

FIG. 28

The difference in phase in the voltages across C_1 and C_2 comes from the effect of the current i, present in C_1 to a greater extent than in C_2. By making the effect of this current small its disturbing effect may be reduced, and this can be done by

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increasing the values of C_1 and C_2, and decreasing the value of R, the value of L being properly reduced to maintain the same frequency. The increase in capacity will increase the value of the oscillatory current i_1 and as i remains constant its effect on the relative phases of e_{c1} and e_{c2} becomes proportionately less.

The arrangement of apparatus remaining as in Fig. 12 the constants were readjusted for efficient operation and a set of readings was obtained as follows; $E_b = 900$ volts, $E_c = 230$ volts, $E_g = 310$ volts, frequency $= 143$, $L_1 = 9.8$ henries, $I_b = 0.321$ amperes, $C_1 = 9.2$ microfarads, $C_2 = 18.4$ microfarad. The resistance of the load circuit was 7.80 ohms and the oscillatory current produced was 4.30 amperes giving an alternating-current output of 143 watts. The input to the tube circuit is obtained from the product $E_b I_b$ after certain losses, not chargeable to the tube circuit, have been deducted.

The condensers C_1 and C_2 each consisted of two condensers connected in series because of the high potentials occurring in the circuit. In order to make the two individual condensers divide the voltage E_b equally it is necessary that their insulation resistances be alike, a condition seldom encountered. That condenser having the higher resistance (the better one) will take all of the E_b voltage as well as its share of the alternating voltage of the circuit, resulting in its probable breakdown. To prevent this occurrence leak resistances were used across each of the condensers making up C_1 and C_2, the leaks each being 21,000 ohms making the leak resistance of C_1 and C_2 each 42,000 ohms. Subtracting the I^2R losses in these leaks as well as the $I^2 R$ loss in the choke coil L_1, gives the input to the tube circuit 229 watts; the efficiency was thus 62.7 per cent.

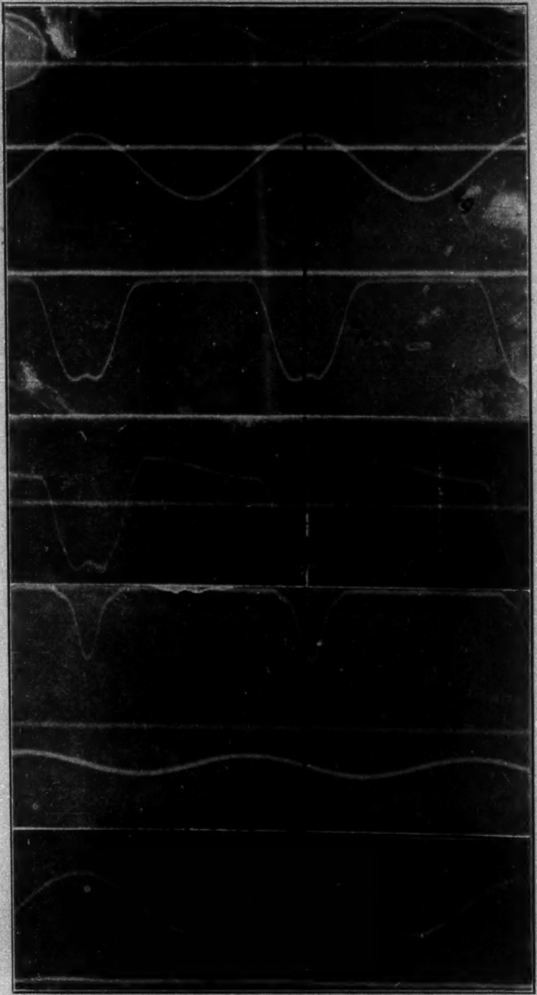
Oscillograms taken of the currents in this circuit are given in Fig. 29. It is evident that the values of E_g and E_c might well have been greater, resulting in a higher efficiency because of the resultant smaller minimum plate current. Although the plate current during the time $A - B$ (Fig. 29) is small, the plate voltage is large and so results in a high unnecessary loss on the plate.

The phase of E_{c_2} is now practically coincident with that of E_g and it should therefore serve as a source of excitation. The circuit did not give as much power however when made self exciting as it should, so the constants were changed slightly to get more power. As finally tested the self-exciting circuit had the constants and performance given herewith: $E_b = 1040$ volts, $I_b = 0.335$ amperes, $C_1 = 7.36$ microfarads, $C_2 = 13.8$ microfarads, $L = 0.201$ henry, $L_1 = 9.8$ henry, $L_2 = 9.0$ henry $E_c = 230$ volts, $R = 8.0$ ohms. The current produced in the oscillating circuit was 4.40 amperes resulting in an efficiency of 57 per cent.

Fig. 30 shows the currents and voltages in this self exciting circuit and it is at once evident why such a low efficiency was obtained; the minimum plate voltage instead of being 200 volts, as it should for this tube, was 300 volts. For this figure the curve of plate current included also the alternating part of the grid current, hence the absence of the depression at the peak value.

The current through the plate current vibrator reversed during part of the cycle, due to the fact that this vibrator carried in addition to the plate and grid currents, an alternating current which resulted from the voltage across the condenser C_2 acting through the reactance of coil L_2 and condenser C_3, Fig. 27. This current is shown as i_x in Fig. 30; when the plate current is corrected by this small amount it is seen that the plate current does not reverse, as we know it cannot with the conditions as they existed in this test.

Action of the Tube at High Frequency. It was desired to show that the action of the tube was just the same at high frequency as at the low frequencies used, so a circuit was arranged similar to that of Fig. 27, with smaller values of capacity and inductance. The choke coils L_1 and L_2 used in the previous tests would act as condensers of comparatively low reactance at the high frequency to be used, so they also had to be changed. The constants of the circuit used were: $E_b = 1000$ volts, $I_b = 0.285$ amperes $C_1 = 0.0144$ microfarad, $C_2 = 0.0284$ microfarad frequency $= 98{,}500$, $L_1 = 0.023$ henry, $L_2 = 0.016$ henry, $E_c = 240$ volts, $R = 6.16$ ohms (high frequency determination). There were no leaks used with the condensers in this circuit so that the product $E_b I_b$ after subtracting the $I^2 R$ loss on the choke coil L_1, gives the input. It is found to be 284 watts, and as the output to the load circuit was 160 watts the efficiency was 56.2 per cent which is in fair agreement with the results obtained at 166 cycles.



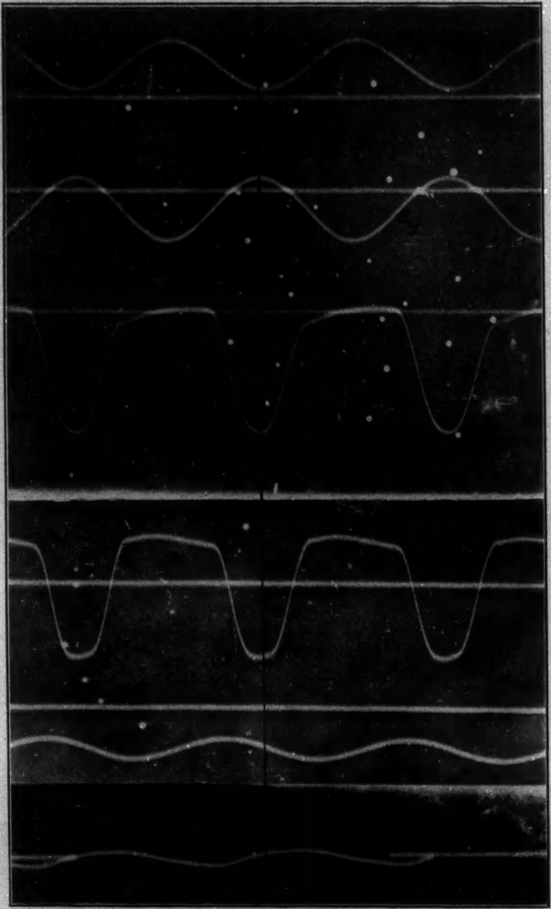
$E_b = 940$ $I_b = 0.310$ $E_c = 230$ $E_g = 310$

$R = 7.80$ $I_2 = 4.30$

INPUT = 229 OUTPUT = 143 EFFICIENCY = 62.5 PER CENT

FIG. 29

[MORECROFT & FRIIS]



[MORECROFT & FRIIS]

$E_b = 1040 \quad I_b = 0.335 \quad E_c = 230$

$R = 8.00 \quad I_2 = 4.40$

INPUT = 272 OUTPUT = 155 EFFICIENCY = 57 PER CENT

FIG. 30

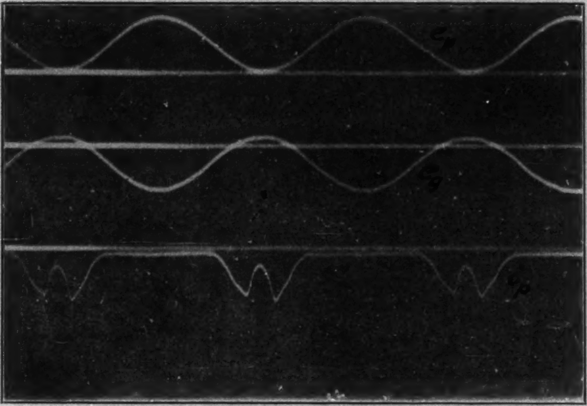
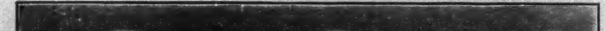


FIG. 31—RUN A TABLE II



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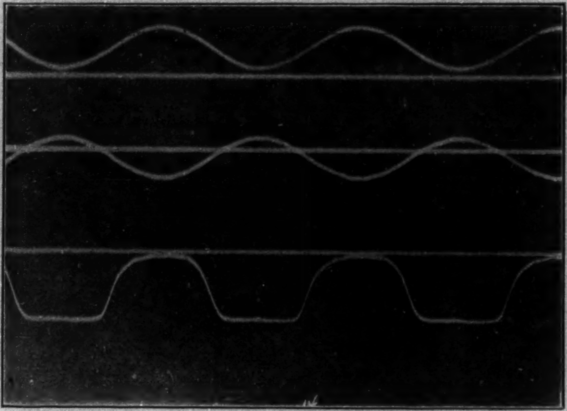
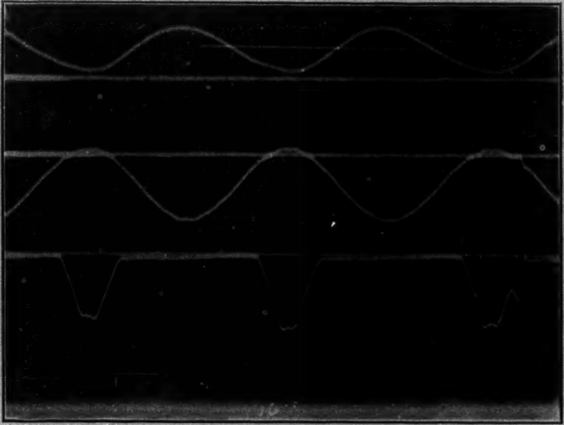
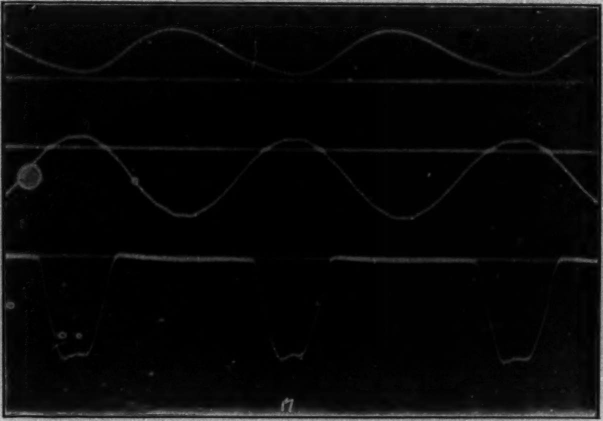


FIG. 33—RUN E TABLE I



[MORECROFT & FRIIS]

FIG. 34—RUN C TABLE I



[MORECROFT & FRIIS]

Fig. 35—Run D Table I

In Figs. 31 to 35 are shown some special oscillograms of the plate current, plate voltage, and grid voltage, all for the separately excited tube with the circuit shown in Fig. 12; the conditions of the circuit were as noted in Tables I and II.

The conditions obtaining when Fig. 35 was taken show the best adjustments for efficiency which we were able to get with the type P tube; the high efficiency was obtained without unduly decreasing the output. If this form of plate current could be maintained and the value of E_b be increased to 3000 volts the calculated efficiency becomes 85 per cent; this is probably as good as could be done with sine wave shapes of e_p and e_g but it seems as though, by suitably deforming both of them, the efficiency could be considerably increased over this figure.

Tests similar to those described in this paper were carried out using a much smaller tube, that styled by the Signal Corps VT-2. The results obtained with the large tube were duplicated almost exactly in so far as efficiency was concerned. Although the normal adjustments used with these tubes were such that the tube efficiency was about 30 per cent it was found possible to so adjust the values of E_c and E_g that the tube gave an output of 6.3 watts with an efficiency of 70 per cent the voltage used in the plate circuit being the rated value, namely 300 volts. It was found possible to get over 7 watts output with the plate loss considerably lower than its safe rated value; if the plate voltage had been increased to perhaps 400 volts the tube output might have been raised to 10 watts while still having the plate loss within its safe value.

These tests were all carried out with a separately excited tube; with the tube self excited the efficiency was not obtained higher than 61 per cent, with a plate voltage of 300. This run gave an output of 5.6 watts output with a current I_b of 0.0305 amperes; the frequency was 400,000 cycles, the value of R was 53 ohms, the oscillating current 0.325 amperes, C_1 and C_2 being 1360 and 770 micro-micro-farads, respectively. The value of E_c was 40 volts.

With the conditions of a self excited circuit adjusted for the best conditions as previously outlined difficulty may be encountered in starting the circuit to oscillate, a shock of some kind being generally required to start oscillations. Because of this possible difficulty it may be the best practise to run tubes separately excited, using one tube, (so adjusted that it oscillates readily) for exciting others. It may well be that with a high resistance in the oscillating circuit more output can be obtained from two tubes if one only is used as a generator, the other being used as exciter only. Certainly if more than two tubes are to be used it will be well to use one as exciter for supplying the grid voltage for the others.

Somewhat more manipulation is required with the separately excited tube than with the self excited one because two circuits (the resonant circuit of the exciter and the output circuit of the power tubes) must be tuned, but the probable increase in output will make it worth while.

Presented at a joint meeting of the American Institute of Electrical Engineers and the American Physical Society, Philadelphia, Pa., October 10, 1919.

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THE POSITIONS OF ATOMS IN METALS

BY A. W. HULL

ABSTRACT OF PAPER

When a narrow beam of X-rays passes through a fine powder of any crystalline material, it produces on a photographic plate placed just behind the powder a pattern of concentric circles. These circles are produced by the reflection of the X-rays from the planes of atoms in the crystal, and their diameters are a measure of the distances between these planes of atoms. By measuring the diameters of the circles the exact positions of the atoms can be determined. The results of this analysis are given for twenty common metals and several salts, with examples and brief description of the method, and a discussion of the results.

THE determination of the exact positions of atoms in solid bodies is the next to last of a series of discoveries, that have made atoms as real as the bricks of which houses are built.

The atom of 20 years ago was the "hypothetical smallest subdivision of matter." The atom of today is a real object of definite shape and size. We know what it is made of. We know its weight in grams. We can see its splash when it impinges on a plate of fluorescent material. We know its exact speed when it flies about as gas. And, lastly, we know its exact position when it forms part of a solid body.

A brief enumeration of these discoveries is a necessary introduction to the following discussion.

First came the discovery of *dancing molecules*. Heat had been considered a substance. The "Kinetic Theory of Gases" proved that it is a condition, *viz.*, the motion of the molecules, which fly about like frenzied bees, bumping against each other and the walls of their enclosure. Through this discovery, all the store of facts and laws about gases can be correlated by the single picture of these dancing molecules. We believe in these dancing molecules as firmly as in the law of gravitation. Whenever we think of gas we see dancing molecules.

The next discovery was J. J. Thomson's *streaming electrons*. Our text-books taught, and some still do, that electricity is not a fluid, though it behaves in many ways like one. Thomson proved that electricity is a fluid, that its atoms are the electrons which constitute the atoms of matter, and that it flows through wires just as water flows through pipes.

Next came the *weighing of the atom.* Faraday showed long ago how to determine the weight of an atom in terms of the charge it carries in electrolysis. There remained, therefore, only the measurement of this "unit charge" *viz.*, the charge of a single electron, by Millikan, to give the exact weight in grams of any atom that can be deposited electrolytically. As soon as the weight of any one atom is known, the weights of all the others can at once be calculated from the known relative atomic weights.

Then came the *counting of individual atoms.* This began with Sir William Crookes' "spinthariscope," and culminated in the beautiful experiments of Rutherford and Geiger, in which they counted one by one the helium atoms (the so-called "α particles") as they emerged from the surface of disintegrating radium; and then allowed them to pass, one by one, into a thin-walled glass tube, until enough had accumulated to form a gas whose pressure could be measured and spectrum analyzed.

These counting experiments led directly to the determination of the *composition of the atom.* J. J. Thomson had proved that every atom contains electrons. Rutherford proved that it also contains a positively charged kernel or nucleus, very small compared to the whole atom, but so dense that it contributes nearly the whole weight of the atom. The hypothetical atom thus became a concrete thing that can be visualized; a tiny, (but large enough to be studied) solar system, with nucleus sun and electron planets. The only respect in which one kind of atom differs from another is the magnitude of the positive charge of the nucleus, which determines how many electrons it can hold in its planetary system, and hence all its physical and chemical properties.

Finally came the discovery, by the Braggs, of the method of determining the positions of the atoms in solid bodies. The beautiful "point lattices" of the crystallographers were hypothetical. They enumerated possibilities, but could not point out the reality. The Bragg measurements of atomic distances give the actual arrangements. They are as accurate and reliable as those of the surveyor or astronomer. The only assumption made is that the arrangement of atoms is a regular one which repeats itself, and this assumption can be checked by experiment. The method consists simply in the measurement, by means of a special "measuring rod" which will be described, of the distance between atoms in three or more different directions. From these measurements a model can be constructed, which can then be checked by further measurements. The model must also agree with known physical properties of the substance, such as density, atomic weight, and crystal habit. A model which contains but one kind of atoms and satisfies all these tests may be regarded as very reliable. The reliability is still further increased by the fact that all the models investigated thus far have turned out to be very simple. In cases where there is more than one kind of atom, *i.e.*, compounds or alloys, an additional factor, *viz.*, the size and shape of the atoms, must be taken account of. There is one type of compound, containing only two kinds of atoms, whose structure is so simple that it cannot be misunderstood. Examples of this type will be included in the following discussion. Compounds containing more than two kinds of atoms have not yet been sufficiently studied to warrant their discussion, but there is every reason to believe that their analysis will be equally simple and reliable.

In the following pages, there will be given, first, a general survey of the results obtained, then a brief description of the method of measurement, and lastly, a more complete discussion of the individual models and some of their properties.

I have referred to the location of atoms as next to last in the series of atomic discoveries. For in order to complete the picture, one more discovery is necessary, *viz.*, the shape and size of the atom. An excellent beginning in this direction has already been made by Langmuir[1] whose theory of atomic structure predicts the shapes and relative sizes of all the atoms, and gives strong chemical evidence in favor of these predictions. The author hopes soon to be able to add the evidence of X-Ray measurements, which will determine not only the shape but the exact size of the atoms, that is, the positions of the electrons in the atoms.

2. *Summary of Results.* The most striking result of these investigations is the extreme simplicity of arrangement of atoms in common metals. Among the metals thus far ex-

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1. Langmuir, *J. Amer. Chem. Soc.* 41, 868, June, 1919.
2. Hendrick, *J. Chem. Met. Eng.* 21, 3, July, 1919.

amined only three types of atomic arrangement are found, and these are, with one exception, the three simplest geometrical arrangements known. The simplest arrangement of all is not found among metals, but is characteristic of salts, which are composed of equal numbers of positive and negative ions. This type and a fifth type, also very simple, which is characteristic of non-metallic elements, will be included in the discussion for the purpose of comparison.

The most common arrangement in metals is the *face centered cubic* arrangement, shown in Fig. 1. This is also the most important since most of the useful metals,—*e. g.*, aluminium, nickel, cobalt, copper, silver, platinum, gold,—have this arrangement of atoms. Perhaps it would be better to say that those substances are most useful as metals which have this arrangement, since, as will be shown later, their ductility is due largely to this arrangement.

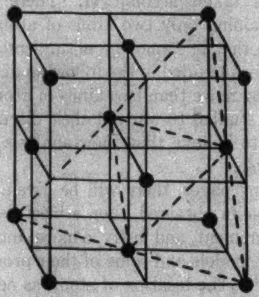

Fig. 1A—Face Centered Cubic Arrangement (Cubic Close Packing)

The face-centered cubic arrangement is obtained by dividing the space occupied by a single crystal or "grain" of metal up into a system of equal, closely packed cubes (Fig. 1A) and placing an atom at each cube corner and at the center of each cube face. All the atoms in this arrangement, both corner and face atoms, are similarly situated as regards symmetry and relation to neighbors. Each atom is surrounded by twelve others, all equidistant and exactly similarly situated for every atom. It is this high degree of symmetry, combined with the close packing, that makes substances of this type so ductile. This arrangement is known as "cubic close-packing" and is one of the two alternative arrangements that equal hard spheres assume (Fig. 1B) when pressed tightly together, with sufficient shaking to allow them to find their places. This suggests, and the other evidence at hand points to the same conclusion,[2] that the atoms of the substances which have this arrangement are fairly spherical in shape. The necessary shaking corresponds to the temperature required for "annealing." The only difference between the packing of balls and that of atoms of this kind is the ability of the atoms to hold on to each other after they have found their places.

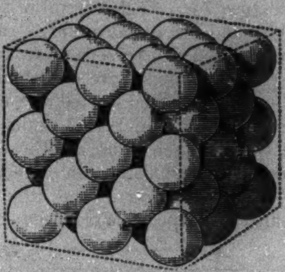

FIG. 1B—FACE CENTERED CUBIC ARRANGEMENT (CUBIC CLOSE PACKING)

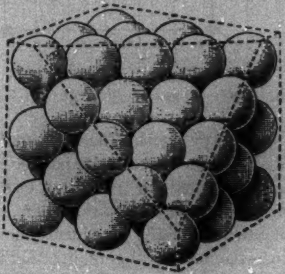

FIG. 1C—FACE CENTERED CUBIC ARRANGEMENT (CUBIC CLOSE PACKING)

There is one important exception to the rule that the most ductile, and therefore the most generally useful, substances are those whose atoms are in face-centered cubic arrangement, *viz.*, iron. The atoms of iron, and also of chronium, molybdenum, tungsten, and the alkali metals, are in *centered cubic* arrangement (Fig. 2). This arrangement is obtained by dividing the space occupied by a single crystal or grain into equal close-packed cubes, and placing an atom at each cube corner and each cube center. The two sets of atoms, the "corner-atoms" and the "center atoms," are interchangeable, so that if the system of lines in Fig. 2 had started with one of the center atoms, all the corner atoms in Fig. 2 would become center atoms and vice versa. Each atom, whether center or corner atom, is surrounded by eight others in perfect cubic arrangement about it, situated always in the same direction and at the same distances. Hence this arrangement has the same high degree of symmetry as the face-centered cubic arrangement. It is not, however, as closely packed. Smooth, hard spheres cannot be packed in centered cubic arrangement except by the use of constraints, and when so packed are in unstable equilibrium. A slight jar causes them to reassemble in one of the close-packed arrangements. (Fig. 1 and 3) It is evident, then, that the atoms of these elements are either not spherical or that they possess some special forces of attraction localized at cube corners (see Fig. 15).

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[2] cf. Langmuir, l. c. p. 878.

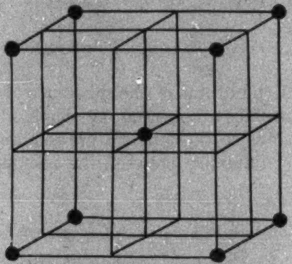

FIG. 3—CENTERED CUBIC ARRANGEMENT

The third type of arrangement found in metals is the *hexagonal close-packed* arrangement (Fig. 3). It is the second of the two alternative arrangements which equal hard spheres assume when closely packed by pressure and shaking. It is less symmetrical than the cubic close packed arrangement, but

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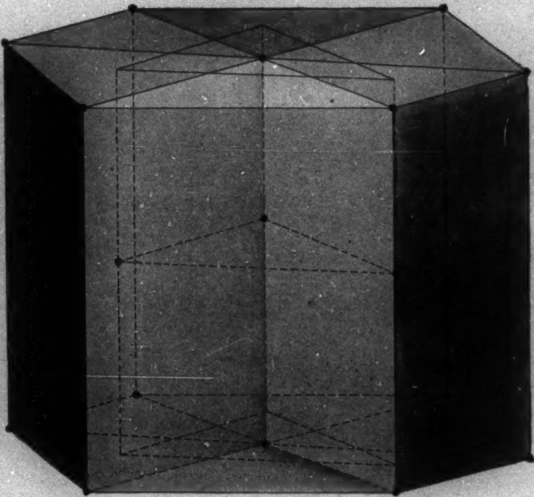

FIG. 2—HEXAGONAL CLOSE-PACKED ARRANGEMENT

[HULL]

FIG. 5—TETRAHEDRAL ARRANGEMENT

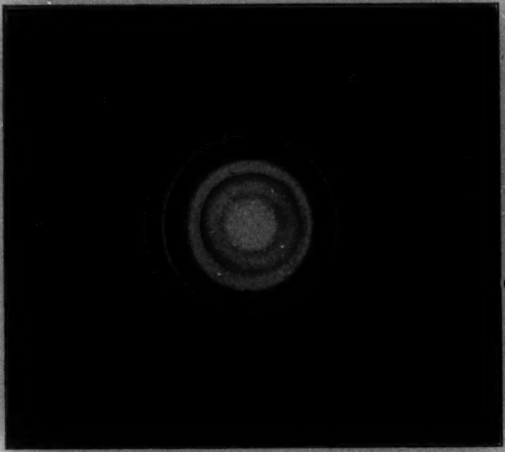

Fig. 7-a—Aluminum

[Hull]

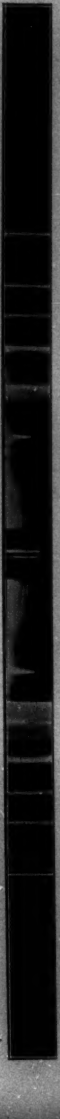

Fig. 7-b—Molybdenum

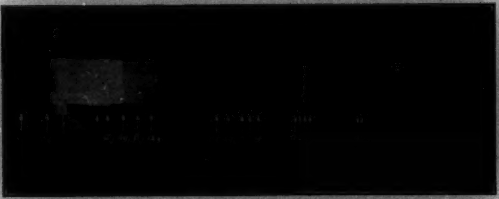

Fig. 8—X-ray Spectrum of Tungsten

equally close packed. The two are closely related, and each can be produced from the other by a simple gliding, as will be shown later. This is the arrangement taken by the atoms of magnesium, zinc, cadmium electrolytic cobalt, and probably to some extent by all cubic close-packed metals when strained (so as to cause gliding.) This arrangement is formed by dividing the space occupied by a single crystal of the substance into a series of equal closely packed right triangular prisms, the bases of which are equilateral triangles, and the altitudes equal to 1.633 times the length of the sides of the triangles. (Fig.1A.) An atom is located at each prism corner and at half of the prism centers. This arrangement is simpler and more symmetrical than it appears. Each atom is surrounded by twelve others, all equidistant and uniformly spaced about it in dodecahedra arrangement. These dodecahedra are of exactly the same dimensions as in the cubic close-packed arrangement, but are not quite regular, the upper half being rotated 60° from the position corresponding to a regular dodecahedron.

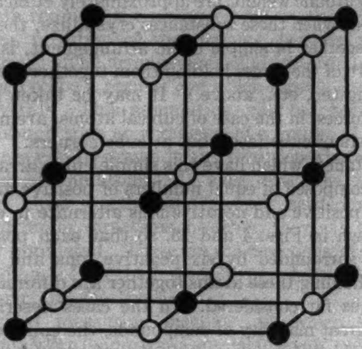

FIG. 4—SIMPLE CUBIC ARRANGEMENT

There are two other simple types of arrangement, which, though they do not occur among metals, are important for comparison, and their description will make clearer the distinguishing features of metals.

The first is the *simple cubic* arrangement (Fig. 4). It is formed by dividing the space occupied by a crystal into a series of equal, closely-packed cubes, and placing an atom at each cube corner. All the atoms are similarly situated, each being surrounded by six others, all equidistanct, in the direction of the cube faces. In spite of its great simplicity, it has only the same degree of symmetry as the two cubic arrangements already described. It is an extremely "loose packed" arrangement for hard spheres, and would be very unstable. A system of equal cubes, however, if packed in this manner would fill all space.

No elementary solid has yet been found whose atoms arrange themselves in this manner. This fact might be interpreted as evidence that none of the atoms are cubical in shape. There is strong evidence, however, that many of the atoms, especially those of low atomic weight, are approximately cubical in shape[3]. The fact that when these substances crystallize, their atoms do not pack together in simple cubic arrangement, is due rather to the nature of the forces holding them together (see discussion of iron, tungsten, etc., above.) It may be taken as evidence that these forces, in the case of cubical atoms, are not localized at the centers of cube faces, but at cube corners.

The substances which have this simple cubic arrangement of atoms are composed of equal numbers of positive and negative ions. The positive and negative ions alternate in every direction as shown in Figs. 4 and 16, so that each positive ion is completely surrounded by six negative ions and vice versa. The forces holding these atoms together are different from any of those thus far considered. In the cases described above, and in the great majority of compounds, the cohesion is due to the "stray fields" of the atoms. In these *ion compounds* it is due to the electrostatic attraction between the oppositely charged ions. This is stronger than the stray fields, and causes the atoms to pack together as closely as their shape will allow. The fact that they choose to pack in simple cubic arrangement is additional evidence that they are cubic in shape. No ion compounds of this kind, (*i. e.* containing equal numbers of positive and negative ions of approximately the same size) between spherical atoms have yet been examined, but it is to be expected that they will show one of the "close packed" arrangements (face-centered cubic or hexagonal) described above.

The fifth simple type of atomic arrangement is the *tetra-*

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[3] (See Langmuir, l. c. 41, p. 892 ff.)

hedral arrangement. (Fig. 5.) Each atom is surrounded by four others, arranged in a regular tetrahedron about it. It is not as symmetrical as the three cubic arrangements described above (Figs. 1, 2, and 4) for while each atom is at the center of a tetrahedron of neighboring atoms, half of these tetrahedra are positive and half negative; *i. e.*, upside down with respect to the first.

The only substances thus far found with the tetrahedral arrangement of atoms are diamond, silicon, and the "ion compound" $N\,H_4\,Cl$. There is strong chemical evidence that in each of these substances, the unit of structure, (*viz.* the *C* and *Si* atom, and the $N\,H_4$ ion) is really tetrahedral in shape.

The Measuring Machine. The determination of these atomic arrangements requires the measurement, in as many different directions as possible, of the distance between consecutive *planes of atoms*. The arrangement of atoms, whatever it may be, is assumed to be a regular one which repeats itself throughout the crystal[5]. This assumption can be checked by the result. Through such an arrangement a system of equidistant parallel planes can be drawn in any direction whatever so as to pass through all the atoms. In most directions, these planes will be very close together and sparsely settled with atoms. In a few particular directions, however, they will be far apart and densely populated These are the directions of easy cleavage and gliding. It is these densely populated planes whose distances apart are measured.

The original measuring machine, by which the pioneer measurements were made, was a special form of "spectrometer." It has been simply and charmingly described by its inventors in a book[6] worth reading. The measurements described in this paper were made with a modified form of Bragg machine. The original machine was applicable only to large, perfect crystals, required careful manipulation, and was subject to serious error unless the crystals were very perfect and the number of observations large. The author's modification is free from these errors, requires but one simple observation, and is applicable to all substances which are crystalline, *i. e.*, all in which there is any arrangement to measure.

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5. (If the arrangement is not regular, there is nothing to be measured. Such "amorphous" solids are very few in number, much fewer than was previously believed.)

6. (W. H. and W. L. Bragg, "X-Rays and Crystal Structure" G. Bell & Sons, London.)

The complete machine is shown in Fig. 6. It consists of a small transformer (or other source of high potential) capable of supplying 1 kw. at about 30,000 peak volts; a Coolidge X-Ray tube, *X*; a thin sheet of properly chosen material, *f*, serving as filter; a pair of slits, s_1 and s_2, in metal sheets, to limit the beam of X-rays; a tiny glass tube, *T*, containing the powdered substance to be measured; and a photographic plate or strip of film bent in arc of circle, *F*. The operation consists in filling the glass tube with a few milligrams of the substance to be analyzed, powdered as finely as possible; "loading" the photographic film holder; exposing over night to X-Rays at 30,000 peak volts and as many milliamperes as the tube will carry safely without watching, (a maximum exposure of 300 milliampere hours); and developing the film.

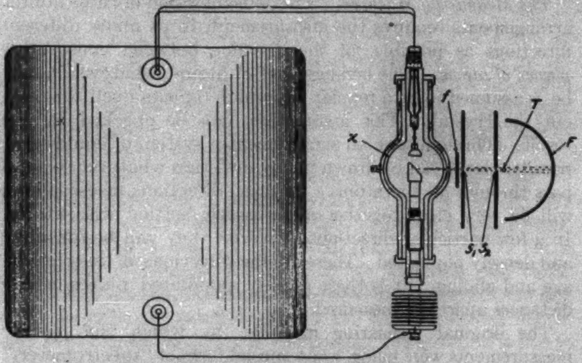

Fig. 6—Powder Photograph Apparatus

Typical photographs are shown in Fig. 7 A and 7 B. Fig. 7A is a photograph of aluminium filings, taken with a plate and very short slits, so that the trace of the direct beam in the center of the plate is a circular spot. Fig. 7B is a photograph of molybdenum powder, taken with circular film and slits as shown in Fig. 6[7]. In this case the traces of the circles on the film show as nearly straight lines. The circles and lines are due to the

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[7] The central line and first "reflected" line in Fig. 7B have been partially absorbed by a "stepped" filter, placed directly in front of the film, for the purpose of measuring the intensity of the lines.

"reflection" of the X-rays by the tiny crystals of molybdenum in the tube, as will be described later. The distances of these circles or lines from the central line on the film are nearly proportional, inversely, to the distances between the planes of atoms, and from them these atomic distances can be easily and quickly calculated. Some examples of calculation are given below.

The Measuring Rod. The measuring rod by which these atomic distances are measured is the wave-length of a particular X-ray.

The possibility of measuring the dimensions of any physical body depends, primarily, upon the possession of a measuring rod of length comparable with the dimensions to be measured. Thus, the discovery and calibration of wave-lengths of visible light opened up a whole new field of measurements, comparable in length with this new measuring rod, such as the thickness of films, imperfection of polished surfaces, displacement of vibrating membranes, increase in length due to thermal expansion, etc. In the same way, the discovery that X-rays are of the same nature as light, and the isolation and calibration of X-ray wave-lengths, opened up a vast new field of measurements of dimensions comparable with the wave-length of X-rays; *viz.*, atomic dimensions.

We are accustomed to think of the measurement of things too large or too small to see and touch as necessarily very rough and approximate. It is somewhat of a surprise, therefore, to note that the only measurements accurate enough to justify the use of eight-place logarithm tables are those of astronomy; that wave-lengths of light are measured to 1 part in 10,000,000; and that the wave-length of X-rays, and by means of it, the distances between atoms, can easily be measured to 1 part in 100,000.

The spectrum of X-rays is exactly like that of visible light, except that the wave-lengths are shorter. It consists (Fig. 8) of bright lines superimposed upon a continuous spectrum. The wave-lengths in the X-ray spectrum depend upon anode material and voltage (Figs. 9 and 10) in exactly the same way that the wave lengths in the visible spectrum depend upon incandescent material and temperature. And just as it is possible to obtain nearly monochromatic yellow light by putting salt in a flame under proper conditions, so by running an X-ray

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8. (For a more detailed description, see Hull, *Phys. Rev.* 10, 666, 1917.)

tube with proper anode at the right voltage, it is possible to produce a single wave length (line) of such great intensity that practically all the rest of the spectrum can be absorbed by a properly chosen filter, leaving nearly monochromatic X-rays.[8] Fig. 10. It is in this way that the monochromatic X-rays used in these measurements are produced. The measurements described in this paper were made with X-rays from a molybdenum target operated at 28,000 volts constant potential, and the filter was powdered crystal zircon, pressed, with a small amount of organic binder, into a sheet ¼ mm. thick. The resulting spectrum, before and after filtering, is shown in Fig. 10.

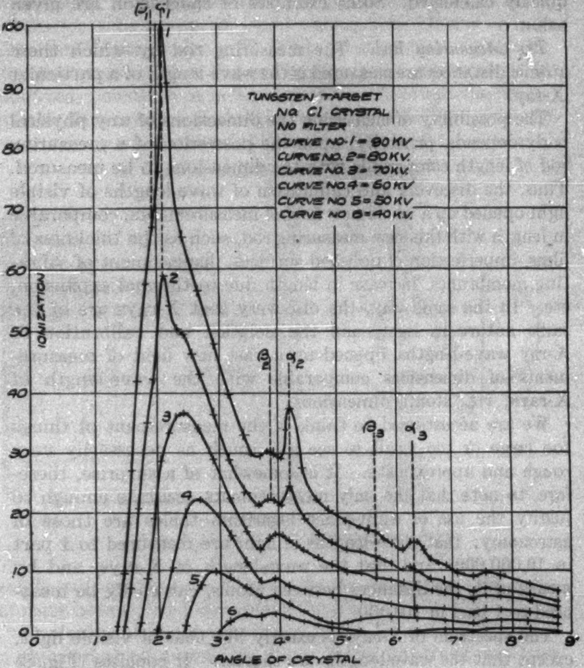

FIG. 9—EFFECT OF VOLTAGE ON X-RAY SPECTRUM OF TUNGSTEN

Calibration of Measuring Rod. The measurement of the wave-length of X-rays in centimeters was part of the pioneer work of the Braggs[9], and was accomplished in the same way as the measurement of visible wave-lengths, *viz.* by the use of a "grating" of known dimensions. It is interlocked with the determination of crystal structure, since the grating used was a crystal, and it was necessary, before using it, to determine its dimensions, *i. e.* the arrangement of atoms in it. The procedure was that of experiment and trial. Preliminary experiments indicated that the atoms of rock salt were in simple cubic arrangement, as shown in Fig. 4. Assuming this to be so, a rock salt crystal was used as a grating to measure tentatively the wave-lengths of X-rays. These wave-lengths were then used for further investigation of the arrangement of atoms in rock salt and other crystals, and were checked and corrected by successive trials.

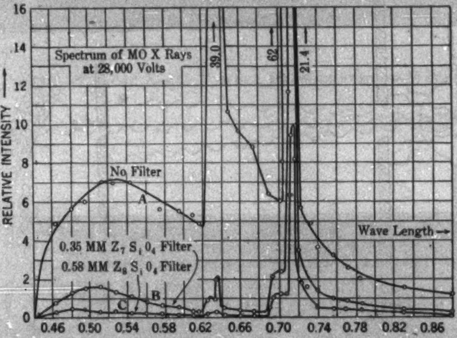

FIG. 10—EFFECT OF FILTERING ON MOLYBDENUM X-RAY SPECTRUM

The method of using the rock-salt grating is shown in Fig. 11. The crystal *C*, with its planes of atoms perpendicular to the paper, is placed in the path of a narrow beam of X-rays. Each plane of atoms acts like a mirror and reflects a small fraction of the rays. The reflection is a maximum when the reflected waves from all the planes (many million, except in the case of very long wave-lengths) are in phase, that is, when each is an exact wave-length or some whole number of wave-lengths behind the next. It is easy to show that this is true only when

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[9] X-Rays and Crystal Structure, p. 110).

$$n \lambda = 2 d \sin \theta \qquad (1)$$

where n is an integer, usually 1 or 2, λ the wave-length, d the distance between planes of atoms, and θ the angle of incidence (i in Fig. 11) of the rays on the crystal[11].

The determination of λ requires, therefore, the knowledge of n, d, and θ. θ can be observed, and the orders, n, counted. d must be determined initially from the physical properties of the crystal, *viz.* its density, atomic weights, and arrangement of atoms. This can best be explained by an example:

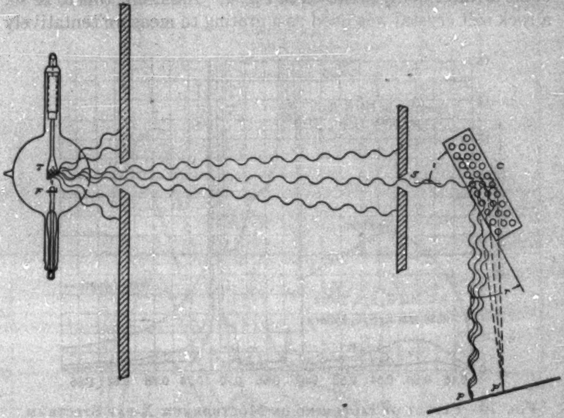

FIG. 11—X-RAY SPECTROMETER

The atoms of rock salt are in simple cubic arrangement, as shown in Fig. 4. There is one atom to each cube, as can be seen by displacing all the atoms in Fig. 4 in the direction of a cube diagonal to the cube centers. Half of the cubes contain sodium atoms and half chlorine atoms. The average weight per cube is the mean of the weights of one sodium and one

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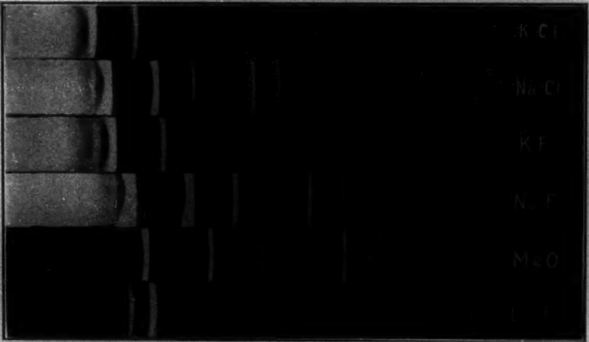
10. (It will be observed that in order to measure different wave-lengths the crystal must be rotated. This is the only essential difference between the crystal grating and the ordinary diffraction grating).

[HULL]

FIG. 12—TYPICAL X-RAY POWDER PHOTOGRAPHS OF METAL


KCl
NaCl
KF
NaF
MgO
LiF

[HULL]

FIG. 13—TYPICAL X-RAY POWDER PHOTOGRAPHS OF SALTS

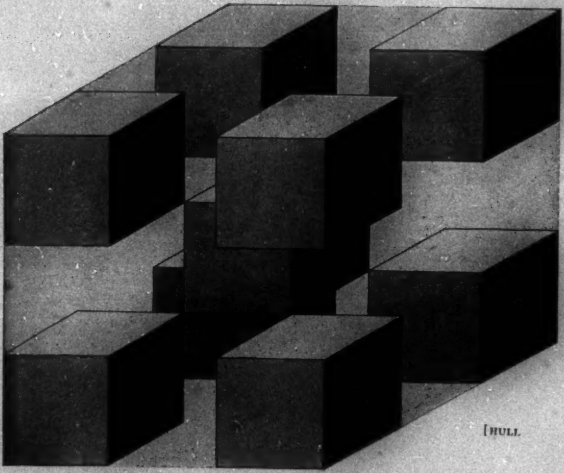


FIG. 15—PACKING OF ATOMS IN CENTERED CUBIC ARRANGEMENT

chlorine atom, *viz.* $1/2\ (38.00\times10^{-24}g + 58.50\times10^{-24}g) = 48.25\times10^{-24}g$. Hence the density, which is the weight per unit volume, must be equal to $\frac{48.25\times10^{-24}}{d^3}$ where d is the side of one of the small cubes (Fig. 4,) *i. e.*, the distance between planes of atoms parallel to the cube faces. This value of density must be the same as that obtained by measuring and weighing a large crystal, *viz.* 2.174. This gives

$$d = \sqrt[3]{\frac{48.25\times10^{-24}}{2.174}} = 2.814\times10^{-8}\,\text{cm.} = 2.814\,\text{Å}^{12}$$

The wave-length of the "α doublet" of molybdenum determined in this way is 0.712 Å[13]. This is the "measuring rod" with which the following measurements were made.

Interpretation of Powder Photographs. The X-ray wave length thus calibrated can now be used to measure atomic distances. If, in the arrangement shown in Fig. 11, the X-rays are made monochromatic by proper voltage and filtering, then as the crystal is rotated a series of intense reflections will be observed at angles whose sines are in the ratio 1 : 2 : 3 etc., corresponding to successive integral values of n in Eq. 1. If a new face is ground on the crystal at an angle to the first, and exposed to the rays in the same way, another similar series of reflections will be observed, at different angles, corresponding to the different distance (d, Eq. 1) between the planes parallel to this new face. The process of analyzing a crystal consists in observing these reflections from as many faces as possible, and calculating, from Eq. 1, the distance between the planes parallel to them. When a single crystal is used this requires many observations. The work is greatly simplified by using a powder, in which all possible orientations are represented at the same time by one or more of the tiny crystals, and photographing simultaneously the reflections from all these little crystals. This is the method sketched in Fig. 6, and gives patterns of lines like Figs. 7, 12 and 13. It might be expected

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12. (The Angstrom unit or Å (= 10^{-8} cm.) which is the standard unit for expressing wave-lengths of visible light, is even better suited to atomic and X-ray measurements. It will be used or assumed in all the following discussions).

13. (The best measurements of X-ray wave-lengths are those of Siegbahn, *Verh. Deut. Phys. Ges.* 13,300, 1917.

that the number of lines in these patterns would be infinite, since there is an infinite number of different possible planes in any crystal. The reflections from most of these planes, however, come at angles whose sines are greater than 1, (as is evident from equation (**1**) when d is small) and which therefore do not exist.[14]

The analysis of these photographs is very simple in the case of simple substances, like pure metals. It consists in finding, by successive trials, an arrangement of atoms whose planar spacings, beginning with the planes farthest apart and skipping none, exactly fit the observed pattern of lines. The calculation of the planar spacings for all the important planes is not difficult, and with simple substances but few trials are necessary.

The method of calculation can best be shown by an example. Potassium chloride (Fig. 13) gives the simplest pattern of lines yet observed. It consists of 6 regularly spaced lines, then a slight gap, then 7 more lines, (these may not all show in the reproduction), then another gap, etc. The best first guess is that a simple pattern like this corresponds to a simple cubic arrangement. The equation of solid geometry for the distance between planes in a simple cubic arrangement is

$$d = \frac{d_0}{\sqrt{h^2 + k^2 + l^2}} \qquad \textbf{(2)}$$

where h, k, and l are any whole numbers, and d_0 is the side of the cube. The numbers h, k, l, are called the "indices" of the planes. They are the reciprocals of the intercepts of the planes on the X, Y and Z axes respectively.

The largest distance is $\frac{d_0}{\sqrt{1}}$, corresponding to the "indices" 1, 0, 0. This gives the line nearest the center (see equation **1**). The next line is $\frac{d_0}{\sqrt{2}}$, given by the planes whose indices are 1, 1, 0. Then follow $\frac{d_0}{\sqrt{3}}$ (1, 1, 1), $\frac{d_0}{\sqrt{4}}$ (2, 0, 0), $\frac{d_0}{\sqrt{5}}$ (2, 1, 0), and $\frac{d_0}{\sqrt{6}}$ (2, 1, 1). The next line, corresponding to $\frac{d_0}{\sqrt{7}}$, should be lacking, as there are no three numbers the sum of whose squares is 7. Fig. 13 shows that it is actually lacking. Then follow, in regular order, $\frac{d_0}{\sqrt{8}}$ (2, 2, 0), $\frac{d_0}{\sqrt{9}}$ (2, 2, 1 and 3, 0, 0), $\frac{d_0}{\sqrt{10}}$ (3, 1, 0), $\frac{d_0}{\sqrt{11}}$ (3, 1, 1), $\frac{d_0}{\sqrt{12}}$ (2, 2, 2), $\frac{d_0}{\sqrt{13}}$ (3, 2, 0) and $\frac{d_0}{\sqrt{14}}$ (3, 2, 1). Then should come another gap, corresponding to $\frac{d_0}{\sqrt{15}}$, which cannot be formed from the squares of three integers. These regularities can be better seen in some of the other patterns in Fig. 13, which are exactly similar to that of *K Cl*, except that there are extra lines just in front of the first, third and fifth regular lines due to the difference in weight and size of the two atoms of which these salts are composed. The calculation of the positions of these extra lines is also simple, but will not be given here.

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[14] (This means physically that there is no angle of incidence large enough to make the reflection from one plane a whole wave-length behind that from the one in front.)

Returning to *K Cl*, the positions of the lines calculated from the above series of inverse square roots are found to coincide exactly with the observed pattern. This proves that the first guess was correct, that the atoms of *K Cl* are in simple cubic arrangement. The result can be independently checked, however, by the density. For the average mass in each of the small cubes, divided by the volume of the cube, must equal the known density of the substance. The value of d_0 calculated from equation 1 is 3.13 Å. The average weight of potassium and chlorine atoms is $\frac{1}{2}(64.5 + 58.5) \times 10^{-24}\,g = 61.5 \times 10^{-24}\,g$. This gives for the density $\frac{61.5 \times 10^{-24}}{(3.13 \times 10^{-8})^3} = 2.01$ which checks with the standard value within experimental error[15].

One more example may be briefly cited, *viz.* molybdenum. Its pattern is shown in Fig. 10. It is found that the series of lines which corresponds to a simple cubic arrangement does not fit this pattern. We therefore try a centered cubic arrangement. The distance between planes in this arrangement is calculated in the same way as for the simple cubic arrangement, except that half the lines, *viz.* those due to planes the sum of whose indices are odd, are lacking[16]. Hence the relative spacings, beginning with the largest, should be proportional to $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{4}}, \frac{1}{\sqrt{6}}$, etc. The series of lines calculated from these values is found to fit the pattern of Fig. 10 exactly. The length d_0 of the side of the small cube is 3.15 Å. As check, the density of molybdenum should be equal to the weight of two molybdenum atoms divided by the volume of the cube, *i. e.*

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[15] (These X-ray determinations of density are, in general, much more accurate than the standard ones).

$$\frac{318 \times 10^{-24}}{(3.15 \times 10^{-8})^3} = 10.0 \text{ which is the correct value.}$$

DISCUSSION OF RESULTS

The results of the measurements that have been made thus far are summarized in the following table. The list includes most of the metals that are easily obtainable in pure form, a few simple salts, as examples, and the non-metallic elements carbon (diamond) and silicon. The structure of graphite is less simple, and will not be discussed here. It apparently contains a mixture of forms, produced by gliding, like cobalt.

The case of cobalt is exceptional. A finely powdered sample produced by rapid electrolysis showed a mixture of cubic and hexagonal close-packing in nearly equal ratio. After annealing in hydrogen at 600 deg. this sample showed only the cubic form. Another sample, composed of filings from pure cast metal, showed slight traces of hexagonal packing, due presumably to straining. It is probable that the other cubic close-packed metals will behave in a similar manner, but this question has not been studied.

PHYSICAL PROPERTIES

Many of the physical properties of these substances are evident from an inspection of their atomic models. Thus it is clear why metals with face-centered cubic arrangement are soft and ductile. For in the planes parallel to the octahedral faces (the bevelled face in Fig. 1C) the atoms are so closely packed that one plane can slip over the one beneath it without

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16. (For details and theory see Hull, *Phys. Rev.* 10,673, 1917).

appreciable elevation, and without the atoms getting away from each other's attractive influence. There are 4 sets of planes parallel to which this easy gliding can take place, *viz.* those parallel to the 4 pairs of octahedral faces, and in each of these planes there are three equally easy directions. There is no direction, therefore, in which a shearing force can be applied to a crystal of this kind which would not be nearly parallel to one of these directions of easy gliding.

TABLE I.

Substance	Arrangement of atoms	Length of side of elementary cube in Ångstroms	Distance between nearest atoms (between centers) in Ångstroms
Aluminum	Face-Centered Cubic	4.05	2.86
Cobalt	(Cubic close-packed)	3.57	2.52
Nickel	"	3.54	2.50
Copper	"	3.60	2.54
Rhodium	"	3.82	2.70
Silver	"	4.06	2.87
Platinum	"	4.02	2.85
Gold	"	4.08	2.88
Lead	"	4.92	3.48
Lithium	Centered Cubic	3.50	3.03
Sodium	"	4.30	3.72
Chromium	"	2.91	2.52
Iron	"	2.86	2.48
Molybdenum	"	3.15	2.73
Tungsten	"	3.15	2.73
Magnesium	Hexagonal (close-packed)	..	3.22
Zinc	"	..	2.84
Cadmium	"	..	3.16
Cobalt	"	..	2.53
Diamond	Tetrahedral	3.56	1.54
Silicon	"	5.43	2.35
Lithium Fluoride	Simple Cubic	2.01	2.01
Sodium Fluoride	"	2.81	2.81
Sodium Chloride	"	2.81	2.81
Potassium Fluoride	"	2.69	2.69
Potassium Chloride	"	3.13	3.13
Potassium Iodide	"	3.55	3.55
Magnesium Oxide	"	2.11	2.11

If the shear takes place in such uniform manner that each plane moves over the one beneath it by the same amount, the original arrangement is reproduced and no strain results. If only the alternate planes move, however, the arrangement becomes hexagonal close packing. This is evident from Figs.

14A and 14B, which give the arrangement of atoms in successive close-packed planes (dotted and heavy lines resp.) in the cubic and hexagonal arrangements respectively. In the cubic arrangement there are three different positions of the planes, and these alternate 1, 2, 3, 1, 2, 3, etc. In the hexagonal arrangement there are only two positions, which alternate, 1, 2, 1, 2, etc.

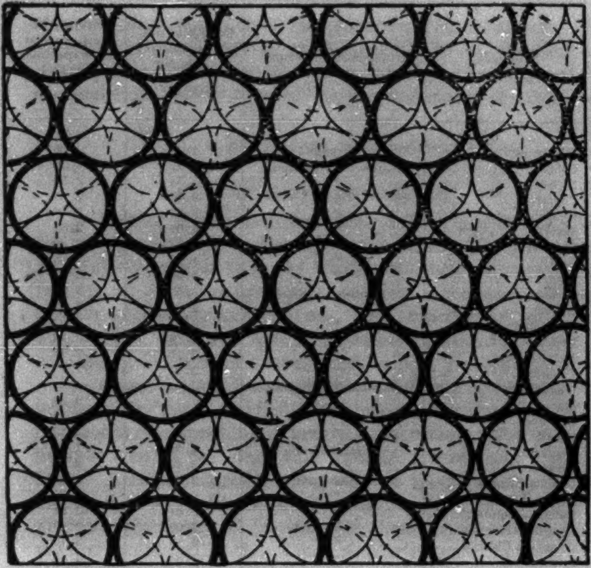

FIG. 14A—ARRANGEMENT OF ATOMS IN SUCCESSIVE PLANES IN CUBIC CLOSE PACKING

In the hexagonal close packed arrangement there is only one set of planes parallel to which this easy gliding can take place. Hence a strained metal becomes less ductile to the extent to which its arrangement becomes hexagonal, and it is evident that mechanical working should produce hardening.

The centered cubic metals should be somewhat less ductile than the face-centered. For the atoms hold on to each other at only 8 points (Fig. 15) instead of 12, and are more likely to move out of each other's influence during gliding.

In the tetrahedral arrangement gliding is impossible. The atoms touch at only 4 points, and in jumping from one stable position to the next would entirely lose hold of each other. This firm interlocking also accounts for the hardness of these substances.

In the case of the simple cubic salts (Fig. 16) gliding is impossible for a different reason. The atoms are held together by electrostatic attraction, each ion being surrounded by six oppositely charged ions. The process of gliding would bring ions of like charge opposite each other, with resulting repulsion and cleavage.

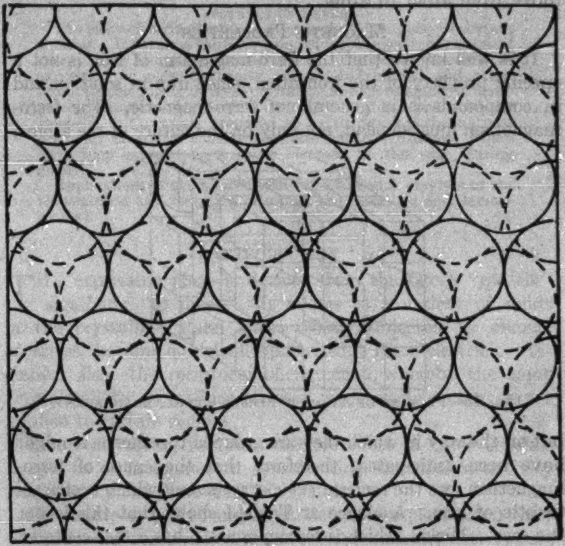

FIG. 14B—ARRANGEMENT OF ATOMS IN SUCCESSIVE PLANES IN HEXAGONAL CLOSE PACKING

The electrical conductivity of metals depends on the ability of electrons to move between the atoms. A discussion of this without a better knowledge of the shape and size of atoms would be premature. It can be seen at once, however, why "ion salts" and crystals like diamond are non-conductors. In each of these arrangements the electrons in the atoms are in complete groups of eight, which is such a stable arrangement[17] that large forces (corresponding to the dielectric strength of the substance) are required to remove them. The atoms of metals, on the other hand, have extra electrons which cannot find places in these stable shells, and are therefore "free" to move from atom to atom.

MAGNETIC PROPERTIES

It is well known that the ferro-magnetism of iron is not a specific property of the iron atom, since iron in solution and in compounds is in general not ferro-magnetic. The ferro-magnetism must depend, not only on the nature of the atoms, but on the way in which they are grouped together. It might have been anticipated, therefore, that the cause of ferro-magnetism was the *centered cubic arrangement* which is characteristic of iron. A glance at Table I shows that this is not the case. Nickel, which is ferro-magnetic, has a face-centered cubic arrangement, like copper. Cobalt is sometimes like copper, sometimes like magnesium. Neither is like iron. Chromium, on the other hand, which is not ferromagnetic, has a centered cubic arrangement like iron. Manganese has not yet been obtained sufficiently pure to determine its arrangement. It is evident, therefore, that while the centered cubic arrangement may be favorable to ferro-magnetism, and may make iron more magnetic than cobalt and nickel, it is not the principal or even an essential factor.

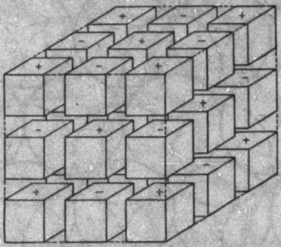

FIG. 16—PACKING OF ATOMS IN SIMPLE CUBIC ARRANGEMENT

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[17] Langmuir, l. c. p. 873.

Presented at a Joint Meeting of the American Institute of Electrical Engineers and the American Physical Society, Philadelphia, Pa., October 9, 1919.

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THE PIEZO ELECTRIC EFFECT IN THE COMPOSITE ROCHELLE SALT CRYSTAL

BY A. MCL. NICOLSON

ABSTRACT OF PAPER

The piezo-electric effect is an electro-elastic property of certain crystals. It involves the conversion of mechanical into electrical energy, and also the converse effect. The paper presents an exposition of piezo-electricity and related optical and other properties belonging to these crystals. Special reference is made to the comparatively large piezo-electric effects produced by Rochelle salt crystals prepared so as to develop the natural *composite* structure usually associated with substances which have been exposed to shearing stresses, in this case during growth.

Applications of the piezo-electric effect in such crystals to the transmission and reception of sound, are described and demonstrated.

INTRODUCTION

THE expression piezo is derived from the Greek "piezein" signifying "to press." It relates to a variety of solids in the crystalline state, which when subjected to change of stress, become electrically polarized. Piezo-electricity, includes, also, the reciprocal phenomenon, whereby the same crystals dilate or produce stresses when electric charges are applied to certain regions.

Before discussing the piezo-electric effect in these crystals it will be of assistance to consider first the optical activity exhibited by the same class of crystals.

For example, quartz and Rochelle salt are piezo-electrically active for the same reason that they are optically active, viz., on account of their individual structural asymmetry.

OPTICAL PROPERTIES

It has been known for a century that certain[1] substances either in the crystalline state or in solution possess the power of rotating the plane of polarization of polarized light. This property in the case of the crystal[2] as also of the solution[3] was

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1. Biot. Société Philomathique, Dec. 1815.
2. Herschel, Roy. Soc. 1820.
3. Pasteur, Soc. Chim., Paris. 1860.

shown to be due to structural asymmetry. In crystallography this asymmetry is called hemihedrism and refers either to an asymmetric arrangement of perfectly regular and superposable molecules or to an arrangement of molecules which are themselves asymmetric. For example, quartz and other minerals, so far as we know, do not possess *molecular* asymmetry; yet, on account of the right or left handed spiral or other unsymmetrical *structure* prevailing with respect to a principal axis, the optical plane of polarization is correspondingly rotated. When, however, the mineral structure or crystal is destroyed, the optical activity vanishes. On the other hand, the asymmetry of the active organic crystals and solutions resides in the molecule, and although optical activity is not always strongly manifest in the organic crystal, it is generally strong in the solution.[4] Each asymmetric molecule takes part in the rotation. Inasmuch as the molecule may offer a dextro- or laevo-rotation of light, there must necessarily be an asymmetric configuration or "stereo-isomerism." This is generally described as a molecular structure having that kind of asymmetry that is not superposable with its reflected image, possessing neither a center nor a plane of symmetry. In stereo-chemistry these so-called "enantiomorphic" isomers are of three kinds: right forms, left forms, and a mixture of both types which is optically inactive.

A crystal may be built up of one or other of the two *active* isomers—of dextro or laevo asymmetry. The corresponding hemihedral crystal is identified by certain simple marks—such as a particular face which slopes one way in one isomer, and the opposite way in the other, and by other indications. These two classes of crystal differ only in their geometry—as, for example, the right hand differs from the left hand, or as a *right* asymmetric tetrahedron differs from a *left* asymmetric tetrahedron. Chemically, their composition and properties are identical.[5]

Let us consider some other properties in these optical isomers. The crystal *form* is only one of many ways in which the symmetrical or asymmetrical arrangement of the molecules may be manifested. We know that certain physical properties determined by the arrangement of *atoms* and *molecules* within the molecule, according as this is symmetrical or asymmetrical, may disclose themselves in the crystal;[6] as for example, by selective action of solvents on faces, which are attacked in different ways, by unequal development of the faces, by electrical polarization due to temperature and stresses, etc. If we operate by various physical contrivances on certain faces or edges of a *hemihedral* crystal, then we find a *non-uniformity* in the resulting reaction. The etched figures on the prism will be unsymmetrical, or different in character on the basal planes. Changes in temperature and variations in applied stress will develop so-called analogous and antilogous electric poles on the crystal, and impressed electric charges will produce dilatation or deformation.

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[4] L. Pasteur, loc. cit., Nernst, Theoretical Chemistry, p. 88, 1911. and J. M. Jaeger: Lectures on the Principle of Symmetry, ch. 7., 1917.

[5] The identity persists so far as they are brought together with bodies which belong to the *symmetrical* class of compounds. But if the isomers are associated with other bodies of the *asymmetrical* class of compounds then their properties differ.

PIEZO-ELECTRICITY

The same mechanism in asymmetric crystals which operates on polarized light to rotate the plane of polarization also causes a liberation of electricity when the crystal is elastically deformed, or a dilatation when electric potentials are applied. This mechanism is found only in the hemihedral or hemimorphous crystal possessing either the asymmetric arrangement of the *atoms* in the organic molecule such as the tartrates, sugar, camphor, etc., or the asymmetric arrangement of the mineral *molecules* such as quartz, tourmaline, boracite, etc.

H. and P. Curie[7] discovered the effect and proved that it was due directly to stresses, or changes in the applied stresses, rather than to changes in temperature which, under the name of "pyro-electricity," had been known previously as a thermal-electric property of these asymmetric crystals; and the name "piezo[8]-electrique" was applied. Examples of piezo-electric crystals of the mineral type are shown in the accompanying Figs. 1, 2 and 3,—respectively tourmaline, boracite, and quartz. Tourmaline is hemimorphic hemihedral characterized by having uniterminal crystallographic poles, associated with different faces of the crystal, only alternate ones of which are developed. Boracite is a hemihedral tetra-hedron and represents a cube having four alternate corners truncated. It will be noted that in both crystals there are so-called "sharp" and "blunt" ends or corners.* If the crystals be pressed between sharp and blunt poles, definite electrification of these poles results. If the pressure be released a reversal of the electric charge takes place.

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[6] W. H. and W. L. Bragg, X-Ray and Crystal Structure, p. 146., J. M. Jaeger, loc. cit., p. 256.

[7] Compt. Rend. 91, pp. 294 and 383, 1880.

[8] Hankel, Abh. saechs. Ges. d. Wiss. 12, 1881.

Fig. 3 shows a well-known application of the piezo-electric effect for measuring small charges. A blade of quartz cut from the crystal as shown, and coated on its sides with tinfoil, or silvered, liberates electricity, the quantity of which depends directly on the weight which is suspended from it. An electrification, with the object, for example, of balancing a charge on an electroscope, may thus be accurately "weighted out." This method is used in the measurement of ionization currents.

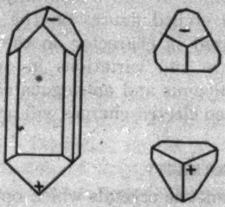

Fig. 1—Tourmaline

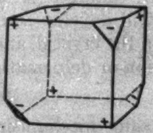

Fig. 2—Boracite

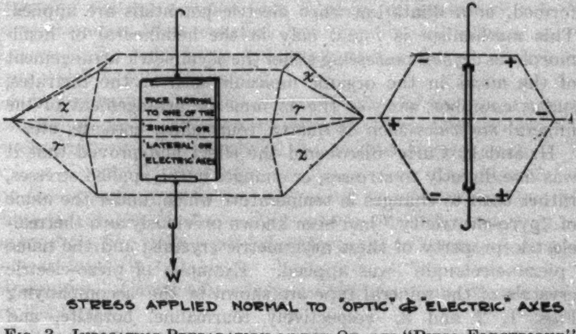
FACE NORMAL TO ONE OF THE 'BINARY' OR 'LATERAL' OR 'ELECTRIC' AXES
x
r
r
x
W
−
+
+
−
−
+
STRESS APPLIED NORMAL TO "OPTIC" & "ELECTRIC" AXES

Fig. 3—Indicating Preparation of the Quartz "Piezo-Electrique"

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*See S. P. Thompson, Elementary Lessons, in Electricity and Magnetism, Lesson VII.

The relation between pressure and electric charge for the quartz "piezo-electrique" as given by Curie is:

$$Q = K \frac{L}{l} P$$

where Q = charge in e. s. u.
K = piezo-electric constant = 0.0677 e. s. u. per kg.
L = length of coating (normal to crystallographic axis)
l = thickness between coatings
P = weight attached in kilograms.

Amongst many mineral and organic crystals investigated by the Curies and others,[9] the crystal of Rochelle salt was found to have the largest piezo-electric constant, approximately, 10 e. s. u. per kg. Apparently no other crystal has yet been found to approach the piezo activity of Rochelle salt, parcularly if the crystal is carefully chosen and specially prepared.

In 1917 the Research Laboratories of the American Telephone and Telegraph and the Western Electric Companies commenced an inquiry into application of the piezo-electric effect. As an outcome of some of the experimental work performed, it was found that Rochelle salt was susceptible of greatly increased piezo-electric activity. An absolute electric charge of 200 e. s. u. per kilogram pressure has been obtained, resulting in potentials as high as 500 volts and alternating currents measurable through a thermocouple. Acoustic tones from a crystal may be heard at a distance of several hundred feet.

Briefly, the increased efficiency is brought about by the following conditions:

1. Selection of particular habit of growth.
2. Desiccation.
3. Development of the crystal into a "composite" polar structure.
4. Application of static compression.
5. Use of electric poles normal to each other.
6. Application of torque.

CRYSTALLOGRAPHY OF "COMPOSITE" ROCHELLE SALT

Rochelle salt[10] crystals are grown from perfect nuclei possessing definite form. The nuclei or seed crystals are immersed in a saturated solution[11] of the salt under identical conditions of temperature. The crystals may be grown by the application of temperature gradients to saturated solutions of the salt or by concentration brought about by its evaporation. The former producing a specific type under conditions of rapid cooling, is the method preferred. The crystal may be grown in the mother liquor by suspending from a clean thread, by flotation on mercury, or by being laid on a glass plate. The two last are the methods practised in our laboratory.

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[9] See F. Pockels, Winkelmann's Handb. d. Phys., Bd. IV., p. 783.

[10] The chemical formula for sodium potassium tartrate is $NaKC_4H_4O_6 . 4H_2O$.

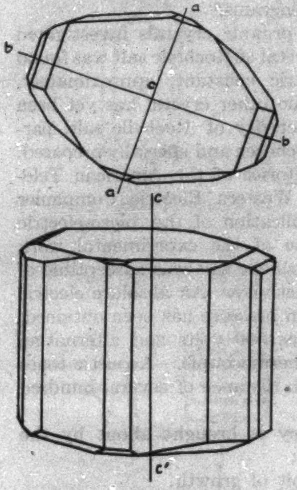

FIG. 4—CYRSTAL GROWN BY SUSPENSION FROM THREAD OR SUPPORTED ON MERCURY

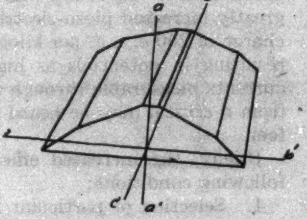

FIG. 5—CRYSTAL GROWN HORIZONTALLY ON A PLATE OR ON MERCURY

Fig. 4 indicates the general appearance of the Rochelle salt[12] crystal. It belongs to the rhombic system is hemihedral and enantiomorphic. It may be possible to classify crystal surfaces into two systems of surfaces or zones which are normal to each other. One system of zones will be parallel with the principal or crystallographic axis *c* and will therefore engirdle the crystal. The other system of zones will comprise the two basal planes parallel with the *a* and *b* axes. In Fig. 4 the zones are shown, respectively, as vertical and horizontal surfaces. In practise, the crystals are grown with the *c* and *b* axes in a horizontal position as is indicated in Fig. 5. This growth forms a particular habit, becoming dominant along the *c* and *b* axes while development along the *a* axis upwards is partially suppressed on account of the supersaturation and consequent gradation in density of the liquor.[13] In obtaining seed crystals, generally from a previous cropping, we select those in which growth along the *b* axis is fully developed (which happens when the seed nucleus grows with its *c* and *b* axes horizontal). These seeds are square, or nearly so, signifying that the growth along the *b* axis is about as great as, or greater than, that along the *c* axis.[14]

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[11] A density of 1.33 at 50 deg. cent. may be conveniently used and the seed crystal, previously warmed to the same temperature as the liquor, should be applied between 38 deg. cent. and 35 deg. cent.

[12] This drawing was made by Prof. H. P. Whitlock of the Amer. Mus. of Nat. Hist., New York.

On cooling rapidly,[15] the seed crystal will increase in size from a few grams weight to 50 or 500 grams, according to the volume and density of liquor used. An average size of crystal weighs 100 grams and its axial lengths approximate 65mm. x 65mm. x 25mm. The illustration, Fig. 6, shows a group of crystals of the habit displayed in Fig. 5.

Crystals thus rapidly grown develop internal stresses producing strain regions symmetrical with the principal axis. The crystal acquires a composite structure[17] closely related to the surface zones referred to and to the electric poles to be developed as now explained.

At each end of the seed-nucleus and along its *c* axis, there appears a pyramid, not always very pronounced in the nucleus itself, which forms a polar terminal,—see illustration of crystal

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[13] Lamethérie,—La Genese de la Science des Cristaux, H. Metzger, p. 187, 1918.

[14] The thickness of the crystal along its *a* axis is generally 0.4 *b*; this dimension is a function of the density and head of mother liquor.

[15] Growth occurs principally during the first 12 hours, although it may be continued several days during the condition of supersaturation.

[17] Mineralogists have termed this an "hour-glass" marking. The phenomenon is probably due to shearing strains set up by relatively greater contractions of the outer crystal envelops during cooling.

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No. 612 in Fig. 7. The pyramids consist of stratifications normal to the *c* axis and to the rest of the crystal structure, the stratification of which ordinarily is parallel with that axis. The electrical performance of the crystal suggests that the crystal molecules throughout the pyramidal regions during growth are subject to forces which turn them, in planes containing the principal axis, through a right angle. This is indicated by the fact that the crystal, after subsequent treatment tending to render more pronounced these pyramid terminations, develops its electric poles[18] in accordance with the two systems of zones previously described. The poles are accordingly at right angles to each other. This signifies that the pyramids are electrically (+) when the rest of the crystal structure is (−) and vice versa. The effect becomes very pronounced when the crystal is subsequently dried in alcohol and baked in an oven.[19]

POLARITY AND CRYSTAL DRESSING

The raw untreated crystal possesses a large number of local electric poles which are variable in their piezo-electric effect. The desiccated crystal is much stronger[20] in effect and its electric poles are readily found in the case of the composite crystal to reside as described on the crystal surfaces with corresponding signs, respectively, in the "vertical" and "horizontal" zones.

In order to prepare the crystal for piezo-electric use its exposed parts are painted over with special varnish, after which waxed tinfoil electrodes are pressed on the crystal as shown in Fig. 8.

SENSITIVE REGIONS OF CRYSTAL

Referring again to Fig. 8, it will be noticed that both basal planes of the crystal are filed slightly concave in the central or polar regions. The object of this is to render salient the four diagonal "horns" or ends of the basal planes, which are found to be the mechanically sensitive regions of the crystal, that is to say, the extreme outer edges parallel with the *b* axis. For the purpose of obtaining an efficient piezo-electric

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[18] Plus and minus corresponding with the conventional analogous and antilogous poles.

[19] 90 per cent alcohol, 24 hours; 100 per cent alcohol 6 hours, oven at 40 deg. cent. several days. This treatment reduces the weight 3 per cent.

[20] Aging and compression also improved them slowly.

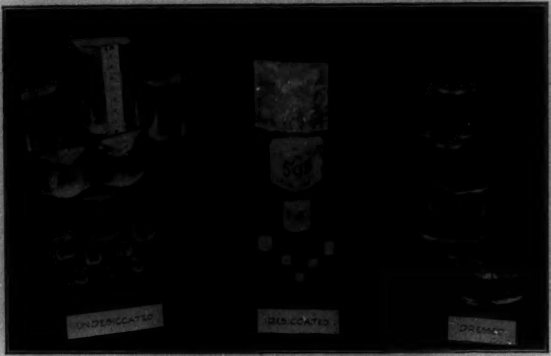
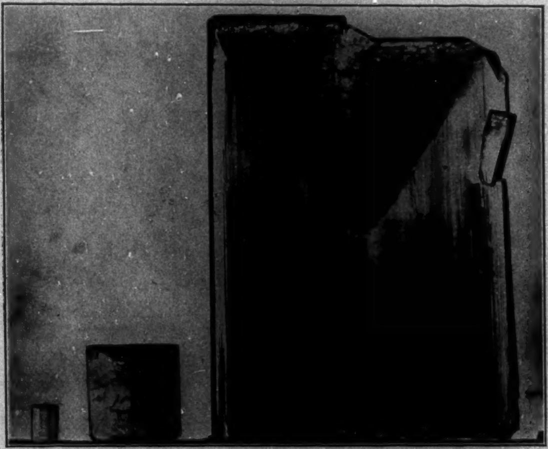

UNDESICCATED
DESICCATED
DRESSED

Fig. 6—Group of Rochelle Salt Crystals—Undesiccated, Desiccated and Dressed



[NICOLSON]

Fig. 7.

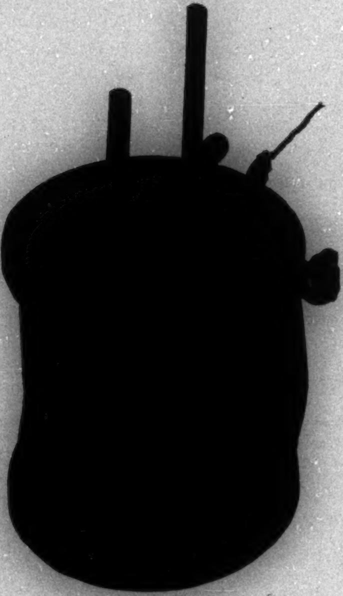


FIG. 20 [NICOLSON]

action at least two of these four diagonal corners at one crystallographic pole of the crystal should be perfect and flawless, apart from the composite structure already described.

APPLIED STRESSES

Analysis of the direction of applied stresses to the sensitive regions described has shown that, for the composite structure, a twisting couple about the principal axis excites the greatest electrification of the crystal electrodes. This may be proved in a variety of ways both for small and large piezo-electric effects. One method is to prepare a crystal by dropping its poles into a pool of melted fusible metal,[21] and after the crystal has recovered from a temporary paralysis due to the heating by the metal, connections should be made between the *girdle* pole and the metal poles with high impedance receivers, which will cause sounds produced by rubbing of the metal poles to be audible. It will be found that strokes applied so as to rotate the crystal about its *c* axis will sound much louder in the receivers than strokes applied in other directions. Similarly the electric charge measured by twisting the crystal

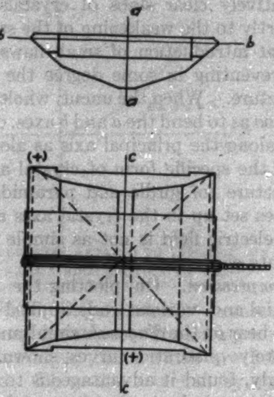


FIG. 8—DRESSING OF CRYSTAL FOR THE GIRDLE ORIENTATION

is much greater than the charges produced by applying axial or diagonal forces, not only for very reduced torques and stresses but for magnitudes of stress approaching saturation in the case of the axial and diagonal forces. In the case of torsion there exists the advantage of the couple, since the force is applied at a given radius from the polar axis.

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[21] Wood's alloy fuses at 71 deg. cent. and Litowitz's at 55 deg. cent. The crystal should be introduced just before the metal sets into a thin plate.

(*a*) *The Case of a Slab Cut from a Crystal.* It may be mentioned here that a plate cut from a homogeneous[22] crystal, with its electric or *a* axis properly normal to the *c* and *b* axis, is almost insensitive to torsion, while the application of "diagonal" stresses, in accordance with Voigt's[23] crystallographic classification, does not in our practise appear to be as successful even when comparatively clear slabs of crystals are used. This may be due partly to the weakening of the structure in slicing, with consequent introduction of small flaws, and also to the difficulty of preventing in some degree the formation of the composite structure. When the uncut, whole crystal is twisted about its *c* axis so as to bend the *a* and *b* axes, component strains are developed along the principal axis as along the diagonals; but in view of the specific form of crystal and corresponding composite structure (of girdle and pyramids) presented, the resulting stresses set up in the crystal axes are very complex. Obviously the electric field is not as simple as in the case of two parallel plate electrodes.

(*b*) *Axial Compression.* On plotting the relation between electric potential and various loads applied to the composite crystal, so as to bear on the *diagonals* simultaneously, we obtain the approximately quadratic curves shown in Fig. 9. We have, accordingly, found it advantageous to apply permanent static compression and obtain thereby greatly increased effect. In view of the fact that the pressure is applied to the crystal at the limited areas of the top and bottom corners, in the basal planes, the crystallographic axis being vertical, it has been found that over a large range of size of crystal, namely between about 30 and 500 grams, the value of the applied static pressure for maximum change in slope corresponds to an absolute force of approximately 15 kg. At this point in the curve a variation of applied axial force of 1 kg. will produce in a good crystal a difference of potential of 8 volts. Beyond this point, as shown in Fig. 10[24], the curve becomes less steep. The static pressure may be applied permanently to the crystal without apparent fatigue.

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[22] A crystal will be practically isomorphic throughout when permitted to grow at a very slow and uniform rate.

[23] W. Voigt and E. Riecke, Wied. Ann., 45, 1892.

For experimental purposes the crystal is inserted in an appliance we call a "spring compressor" which compresses the crystal to the necessary degree and at the same time forms a convenient method for electrically connecting the polar electrodes, previously described. The spring compressors comprise a pair of aluminium disks which are held together by stout steel springs. These are attached by eye-rivets to the lower plate, while thumb screws tighten the springs after a crystal has been inserted. The number of turns of the thumb

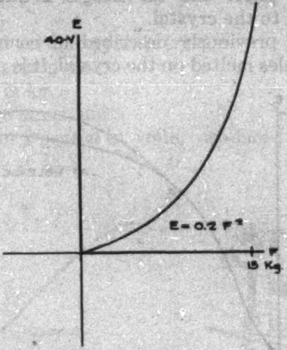

FIG. 9—AXIAL COMPRESSION APPLIED TO "HORNS" OF CRYSTAL (AVERAGE CROSS SECTION 10 CM.^{2})

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[24] The curves in Fig. 10 show slight hysteresis or fatigue near saturation values of the potential. It will be observed that the deflections at *release* of load are much greater than those for compression, probably for the reason that the action may be effected more rapidly and consequently with less loss of charge. The cycle of operations taken for obtaining the curve is as follows: a load is applied to the crystal so as to compress it; this gives a deflection on the electrometer of a value, say *b*; then the crystal is short-circuited for a moment so as to give zero reading again (shown at *c*). On releasing the load the greater deflection *d* is produced (which is in the opposite sense when the instrument is polarized, as indicated in the upper curve). An adaptation of the Riehle machine was used for large values of compression.

screw may serve conveniently as a gage of the degree of compression after calibration.

Fig. 12 shows the spring compressor with the dressed crystal properly balanced within. Any part of the metal structure will readily convey jars and vibrations to the crystal. If this crystal transmitter be laid on a large sheet of paper and receivers be applied to the electrodes, it is quite easy to hear gentle rubbing of the paper and the tick of a watch—transmitted along the paper to the crystal.

Torsion. As previously described in connection with the fusible metal poles melted on the crystal, it is probable that the compressed crystal receives and responds to components of forces in both axial and twisting senses. In order to analyze the relative motions of the crystal it was simple to reverse the crystal action by applying alternating potentials at different frequencies to the crystal electrodes and discover from the various modes of vibration of the crystal their relative magnitudes. A special microphone was applied to a crystal pole in various positions with respect to the crystallographic axis. The microphone, being in a local battery circuit operating a thermocouple, was actuated by the dilating crystal when the constant alternating potentials of various frequencies were

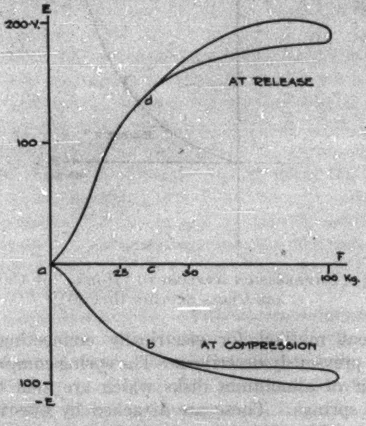

Fig. 10—Applied Axial Stress

applied. The resulting measurements showed conclusively that the crystal twisted or moved the microphone button to the greatest degree when the axis of the microphone was attached tangentially to one of the crystal poles. The motions applied to the microphone were much smaller when they were diagonal or axial in character.

The relation between piezo-electric charge and torsion is linear, see Fig. 11, and is given by

$$Q = K F L$$

where $K = 100$ for several crystals tried
$L =$ lever arm in cm.
$F =$ force in kg.
$Q =$ charge in e.s. units.

The constant for *potential* in volts, produced under similar conditions, is: $K' = 12$.

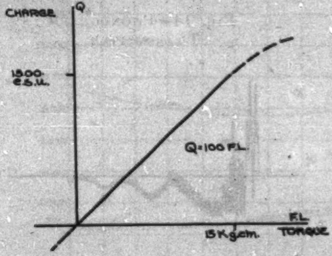

Fig. 11—Electric Charge Obtained when Torsion is Applied to Crystal

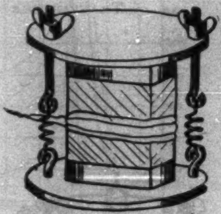

Fig. 12—Assembly

A right-hand torque determines the same electrification that is produced by subjecting the crystal to tension. If steady potentials be applied to the crystal the torsion is given, in one case, by

10^{-5} radian per volt (crystal 7 cm. long)

while expansion parallel with the crystallographic axis (c) is 2×10^{-9} cm. per cm. of length, per volt and is independent of applied pressure.

(*d*) *Alternating Effects.* If we apply rapidly alternating stresses to a crystal it becomes a generator of electrical oscillations, and we may use it as a detector of vibrations of sound, agitating it, for example, by a watch tick, a moving phonograph record, or by speech. If the crystal be tapped, an oscillogram shows that such an impact generates trains of electrical oscillations peculiar to the resonant frequencies of the crystal. Fig. 16 indicates trains of oscillations of 300 and 2000 cycles. A crystal used on a phonograph record will generate several volts, with sufficient power to operate a very large number of telephone[25] receivers.

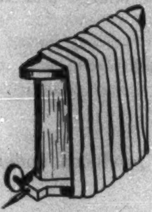

FIG. 13—PHONOGRAPH TRANSMITTER

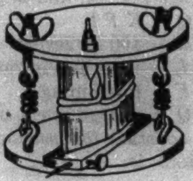

FIG. 14—PHONOGRAPH TRANSMITTER

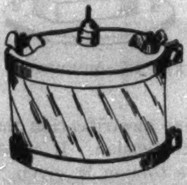

FIG. 15—CRYSTAL TRANSMITTER OR RECEIVER

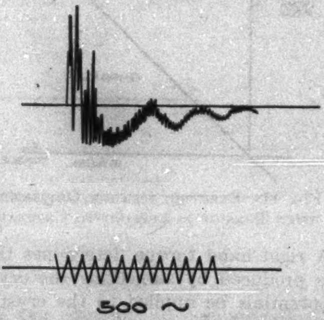

FIG. 16—EFFECT OF TAPPING A CRYSTAL INDICATING IMPACT OSCILLATIONS OF 330 CYCLES AND 2000 CYCLES

APPLICATIONS

Again, to operate effectively, the crystal applied to the phonograph is subjected to torsional vibrations. Figs. 13 and 14 indicate the housings employed. By means of the usual needle, vibrations are imparted to a light plate under the crystal. In Fig. 13 elastic bands are shown compressing the plate against the crystal.

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[25] As many as 200 receivers of 12,000 ohms each have been operated from one crystal.

Sound Transmitter and Receiver. Since, with these composite crystals, a given force produces the greater piezo effect when it is applied in such a way as to twist the crystal about its principal axis, and conversely, an applied electrical force produces the maximum mechanical response in the form of twisting motion, it is apparent that whether we are interested in producing electrical or mechanical results from the crystal, the diaphragms should be so attached as to make maximum use of the torsional effect.

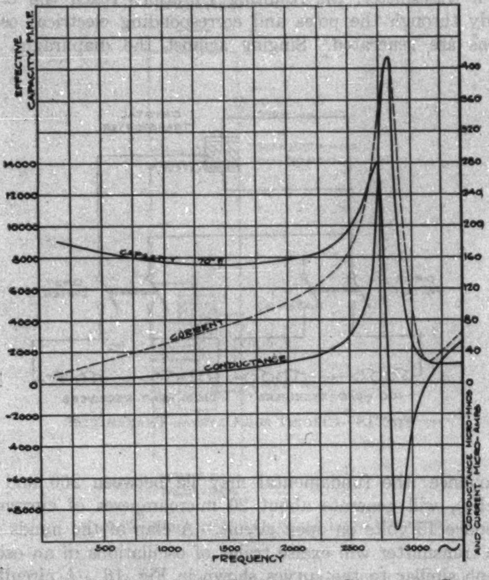

Fig. 17—Capacity, Current and Conductance Curves for Crystals

One way is to apply a cylindrical diaphragm so as to surround the crystal, and to attach the cylinder by means of rings to the compressor plates which hold the crystal. By screwing one of the rings over one of the compressor plates the cylindrical diaphragm itself becomes twisted into diagonal corrugations stretched tightly across the crystal poles. This construction is shown in Figs. 15 and 20. The "diaphragm" should be made of a strong light material like gold beaters' skin or bond paper. When aerial tones strike the diaphragm and actuate the crystal as a transmitter, the resulting vibrations reach the crystal body through the poles and corresponding electrical oscillations are generated. Singing against the diaphragm, near resonance (the fundamental may lie between 200 and 600 cycles), will generate about 20 microamperes of current or produce 15 volts on open circuit. A clap of the hands near the transmitter will excite trains of oscillations in an oscillograph similar to the curves shown in Fig. 16. A circuit for the crystal transmitter is indicated in Fig. 18.

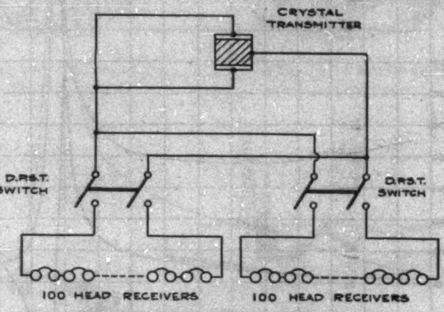

FIG. 18—CIRCUIT FOR CRYSTAL TRANSMITTER

If the crystal receiver, as described, is used with a microphone transmitter and the potential is stepped up to the crystal with a transformer as is shown in Fig. 19, very strong acoustic effects may be obtained as a "loud speaking" crystal receiver.

Crystal Transmitter and Receiver in a Line. With the aid of the vacuum tube amplifier, good transmission of speech and

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music may be obtained by using the piezo crystal at both ends of a line as sole transmitting and receiving apparatus.

Some Electrical Constants. The crystal may be considered as a leaky condenser having a shunt resistance in excess of 100 megohms. The ohmic resistance is several times greater from the analogous to the antilogous pole than it is in the reverse sense. There is however, no evidence of current rectification.

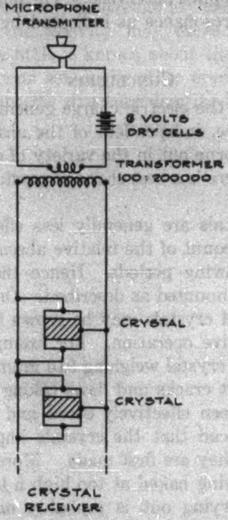

Fig. 19—Circuit for Crystal Loud Speaker

The capacity varies with size and may be as great as 10^{-8} F. in a good crystal. It is usually 10^{-9} F. Its impedance at acoustic frequencies varies from 100,000 ohms to 300,000 ohms. Fig. 17 indicates that these properties depend on the frequency of the applied e. m. f. From these curves[26] the so called "motional impedance" of the crystal may be found in the same manner as the motional impedance of an ordinary telephone receiver[27] which, as is well known, gives the resonance characteristics of the diaphragm. In the case of the crystal the dilatations reach certain maxima where the applied frequency of the alternating potentials (of constant amplitude) coincides with an elastic natural frequency of the crystal. At this frequency, the apparent capacity passes through the greatest change in value—from maximum to zero or negative values, depending on whether the crystal response, as a generator, is in phase or out of phase with the applied potentials. The impedance decreases to a minimum at resonance as indicated by the conductance curve.

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[26] Made by E. T. Hoch, Physical Lab., Western Electric Co.

CONCLUSION

Curie's law that the electric charge generated by a crystal is, for a given force, independent of the absolute dimensions seems to be well borne out in the variety of crystals discussed in this paper. There is a best shape of crystal which we have already considered.

Very small crystals are generally less effectively rendered "composite" on account of the relative absence of the stresses during shorter growing periods. Hence they are generally less efficient when mounted as described. On the other hand, properly articulated crystals may be grown far larger than is necessary for effective operation. For example, Fig. 7 shows an illustration of a crystal weighing 910 grams. It is difficult however, to prevent cracks and flaws taking place before the large crystal has been effectively cured and hardened.

It has been noticed that the crystals improve with time, particularly when they are first made. Moreover, when they are paralyzed by being baked at too high a temperature they slowly recover. Drying out is probably one cause of this improvement and also realignment or recrystallization of disturbed portions of the crystal molecules.

It may be said that the crystals, after several months' use, or non-use, reach a very steady operating condition in which their activity seems to be permanent, especially for alternating effects where weather conditions have negligible action on sensitive apparatus associated with the crystal. As regards general efficiency and comparison with known apparatus like the carbon transmitter and the electromagnetic receiver, it should be said that the microphone is more sensitive than the crystal transmitter. The microphone with its associated local battery gives out more energy than it receives and hence constitutes an amplifier; while the crystal at present translates only that portion of the energy applied to it which affects its mechanically sensitive regions.

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[27] A. E. Kennelly, Proc. Amer. Acad. Sc., 1912 p. 114.

The efficiency of the crystal receiver compares favorably with the electromagnetic receiver for equivalent resonance conditions. It is not improbable that other crystals of organic constitution, *i. e.*, of *molecular* asymmetry, may be found whose piezo electric activity may exceed that of Rochelle salt. So far very little is known about the electrical and mechanical orientations set up in other crystals capable of developing the composite structure described in this paper.

DISCUSSION ON "THE POSITIONS OF ATOMS IN METALS" (HULL)' "PIEZO ELECTRIC EFFECT" (NICOLSON) AND "THE OSCILLATING VACUUM TUBE AS A GENERATOR OF ELECTRIC POWER" (MORECROFT AND FRIIS), PHILADELPHIA, PA., OCTOBER 10, 1919.

John B. Taylor: Referring to "The Arrangement of Atoms in Metals. I do not know how the author bridged the gap between quartz and those things which we know are regular crystals built by regular structure, and common carbon metals of electrical engineering, such as copper, iron, etc. There are crystals in iron, and if it is polished a little it will show the crystals at 1/100 diameters, etc. This is true of the other metals. We also have a copper wire soft drawn without much crystal structure, and so I think the author might leave a little clearer idea in our minds regarding these things, and perhaps some others, if he went over once this gap between the regular structure metals, of which the bona fide crystals are the best examples, and a metal which is annealed and worked with its own crystal structure. I would like to ask if Dr. Hull has any insight into the arrangement of the atoms or whether there is anything you might call arrangement.

Referring to the paper on "Piezo Electric Effect", I would ask if we can be given more numerical information, that is, what are the limits in pressure—the chemical pressure and electrical pressure. Can we supply several hundred thousand volts to one of these crystals and break it up by mechanical force? In general, how far can we go, and what other metals, other than the Rochelle salt may be available for this work? Quartz has been mentioned, but apparently it has some advantages and disadvantages over the Rochelle salt.

Has Dr. Hull, in examining the structure of these metals, examined them under conditions of stress? For instance, if he takes the structure of Rochelle salt and stresses it by mechanical means, which is the same in this case, does he find a different arrangement by the atoms moving up, which can be shown on the photographic film?

A. W. Hull: I will answer Mr. Taylor's question about the relation between metals and substances commonly known as crystalline. In the materials we have investigated so far, the only substance that is not almost completely crystalline is glass. Metals, even ductile metals like copper, seem to be perfectly crystalline. There are cases in which alloying interferes with the perfection of crystal structure. Those have not yet been examined. Except for that, metals are fully as well crystalized and as perfectly crystalline as so-called crystalline substances like quartz. Metals may, however, be distorted by gliding. In the case of quartz, as in rock salt, which I specially mentioned, such gliding is impossible. It always results in cleavage.

John B. Taylor: The structure of metals permits no cleavage plane?

A. W. Hull: Yes, they have a cleavage plane, but the metals cleave much less easily. The planes are able to glide over each other.

John B. Taylor: After the gliding, the arrangement of the atoms still remains the same?

A. W. Hull: They still remain in close contact and the crystal is fairly perfect.

In answer to the second question, very little work has been done in that direction and I have nothing to offer in connection with metals under stress.

A. M. Nicolson: As regards sound emitters of quartz and Rochelle salt, if due consideration is made of the differing modes of vibration and forms of housing, a rough comparison would indicate that Rochelle salt crystals operate more effectively at 500 volts than crystals of quartz at 10,000 volts.

If high alternating potentials be applied to a Rochelle salt crystal it will eventually become warm by dielectric hysteresis, or otherwise, and cease temporarily to exhibit the piezo-electric effect. It may be operated continuously, however, with a total energy dissipation, of about one-half watt. This represents a current of a few milliamperes through a crystal of average capacity of 1000 micro-microfarads.

John B. Taylor: You speak of it as capacity rather than resistance.

A. M. Nicolson: The direct-current resistance of the crystal is very high, in fact, hundreds of megohms when desiccated. Energy dissipated in the crystal dielectric is principally due to hysteresis and mechanical friction. This is represented electrically as *alternating-current resistance,* expressible either as shunt or equivalent series resistance. The magnitudes are respectively of the order of 300,000 ohms and 100,000 ohms.

E. A. Eckhardt: Are clear crystals better than cloudy crystals?

A. M. Nicolson: *Clear* or slowly grown crystals do not generally display the composite structure, or hour-glass marking, which we have made use of. The cloudiness develops on the surfaces during desiccation, otherwise the composite crystal should be quite clear throughout, except where the polar planes of cleavage definitely articulate the crystal into its composite nature. We have found that crystals having a development of this composite structure, with its corresponding orthogonal electric poles, give piezo-electric effects which exceed the effects with the slowly grown crystals offering the usual parallel electric poles and corresponding straight electric field. It should be said, however, that advantage was taken of the fact that the composite crystal could be obtained by rapid growth, while the so-called clear crystals are difficult to develop and entail several weeks' growth under thermostatic control. But such carefully grown crystals as we were able to develop do not seem to offer the same electrical and mechanical qualities which are readily obtainable in the composite type of crystal.

J. P. Wintringham: It might be well to give a few minutes to the essential crystallographic elements. Crystals are divided into classes by the more or less regularity and symmetry of their geometrical forms. All other qualities, mechanical, optical, electric, etc., are governed by the degree of symmetry or regularity. There are only three unit fundamental forms, the cube whose symbol is 100, and its faces are called the faces of a cube or if taken in pairs pinacoids. The three figures refer to the three geometric axes and the one permuted through the three places 100, 010 and 001 indicates three of the faces, minus one gives the other three faces $\bar{1}00$, $0\bar{1}0$ and $00\bar{1}$ or total six of the cube.

The second form is the dodecahedron (110) with 12 sides as the permutation of the symbol shows. Its faces taken in sets parallel to one axis are called prisms of which the most important is the vertical prism.

The third form is the octahedron (111) with eight permutations, one for each side. To the unit fundamental forms may be added four more when some other figure is taken in place of the one. All other of the 1000 or so forms are made by combinations of two or more fundamental forms, or and this is the important point in this connection, part of the faces of a fundamental form may not appear and we may have only half or a quarter of them ½(100), ½(110) &c., the brackets implying a full set of faces.

It is evident that you cannot make a complete figure with only half of the faces of a cube but the octahedron works out all right.

It may be described as a four sided pyramid on top with an inverted four sided pyramid below base to base. The vertical or *c* axis goes from the top apex to the bottom one. Two opposite top faces may be extended together with the two alternate lower faces to make a figure with only four faces, the tetrahedron. The *c* vertical axis remaining in its place goes from the middle of the top edge to the middle of the edge below. It is evident that a great deal of the former symmetry has disappeared. This is just the asymmetry that is to be noted in Mr. Nicolson's Rochelle salt crystals. It may be said to be made up of three pair of pinacoid (or cube) faces with a number of vertical prism faces. None of these is significant. What is significant is a small face just bevelling the top front edge called $o = 1\bar{1}1$ which should appear on the left or another $v = 211$ which would appear on the right. They are the faces of a tetrahedrons or bisphenoids half forms.

I have only found them in his two drawings, Fig. 4, in the top plan the long and narrow faces that are not cut by *a* and *b* axes, in the lower drawing the *c* axis seems to go through one of them.

Crystals are divided into 32 classes of which 20 might show piezo-electric effect.

A. W. Hull: I anticipate it would be perfectly easy to show the results of stress. The relative displacement of atoms will be accurately registered, but as they are exceedingly small it would require very fine measurements to show them.

Irving Langmuir: From the outside dimensions of the change of shape, etc., of crystals under the influence of an electrical field or mechanical stress, you could not calculate the changes in the orbs, the arrangement of the atoms must still be the same, and even without distortion you could calculate them? What about the changes in the distance between the atoms?

J. E. Schrader: Dr. Hull states he has not made any experiments on metals under stress, containing crystals such as mica or quartz. Cannot we get some information about polarization and absorption?

A. W. Hull: I would like to qualify somewhat the answer I gave to that problem. I have not the slightest doubt but that we can get excellent information regarding strains from X-ray analysis. The strains we actually produce are of such magnitude that we can measure them mechanically on the outside of the crystal, and the ease of measuring them inside is greater. By means of the photographs I showed, it is easy to measure the distance between atoms to one part in fifty thousand.

The measurement of elastic coefficients, temperature coefficients of expansion, and densities are extremely easy by this method. The relative ease of the measurement of density in a substance by X-rays as compared with the ordinary method is so remarkable that it is worthy of attention. Density measurements to one-tenth of a per cent can be made in about four hours,—that does not mean four hours of personal work—the tube runs by itself and the analysis requires less than an hour.

D. MacL. Therrell (by letter): Simplified and reduced to its lowest terms the paper by Morecroft and Friis covers the use of the vacuum tube, pliotron, or audion for the purpose of generating an alternating current from a variable direct current.

Some years ago, while conducting a series of experiments in an effort to improve the constants of the telephone transmitter circuit, and avoid the evils of core saturation and transformer or induction coil losses, I discovered a means for deriving an alternating current from a direct variable current, which is identical with the use of the pliotron vacuum tube or audion, as described by Messrs. Morecroft and Friis.

By bridging a low resistance coil of high inductance across the terminals of the variable resistance element, or transmitter electrodes, and placing the load, or primary winding of the induction coil, in series with a condenser of suitable capacity in shunt around these points, or terminals, whose potential varied as a function of the resistance of the transmitter electrodes; I found that an alternating current corresponding exactly with the impressed voice component of the variable direct current was generated in the derived or shunt circuit through the primary coil by the charging and discharging of the condenser in circuit.

The discovery was later used and disclosed by me for the purpose of generating alternating currents in the primary or primaries of a transmitter circuit so that resonance or partial resonance might be obtained for the essential or desired frequencies to be transmitted.

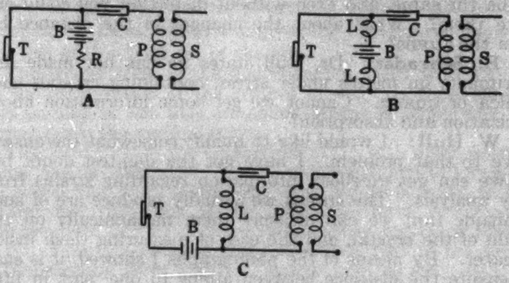

Fig. 1—Therrell "Autostatic Generator"

Both the discovery, of what I term the "Auto Static Generator," and this particular application which permits of tuning for the sinusoidal component of the variable direct current was a distinct and novel step in the art.

The importance of this device was clearly foreseen and fundamentally covered and claimed in U. S. patents issued to the writer for the "Art of Telephony."

This circuit arrangement was also described by me in an article in the *Electrical World,* page 1344, June 30, 1906. It was also described at length in a serial article by J. J. Goodwin, covering my discoveries, in the *American Telephone Journal,* page 67, August 4, 1906.

In the same issue the Editor, Franklin H. Reed, commented editorially in part as follows: "The device by which the primary current is made alternating in character in order that the required condition of resonance may be obtained, represents a distinct and novel step in the art."

In the patents above referred to the circuit coupling or auto static generator is shown under three equivalents as indicated at (*A*) (*B*) and (*C*) of Fig. 1, herewith, which it will be seen is identical with the vacuum tube or audion arrangement of Messrs. Morecroft and Friis as described in connection with Fig. 1 of their paper.

A simplified but exact equivalent of Fig. 1, is herewith shown as Fig. 2, in the usual symbols. This circuit is now seen to be identical with the "Auto Static Generator" arrangement disclosed by me in 1904, as will be most apparent to any one comparing the figures herewith.

The filament and plate in Fig. 2 are equivalent to the microphone in Fig. 1. The alternating-current generator and the grid are equivalent to the acoustical means for varying the resistance of the microphone in Fig. 1. The rest of the circuit is identical in every element and function and detail.

In this connection it is also pointed out that my "Auto Static Generator" circuit is used in the Goldschmidt alternator for wireless and radio telephony as recently disclosed in "The Principles Underlying Radio Communication," issued by the Bureau of Standards, pamphlet No. 40, page 162.

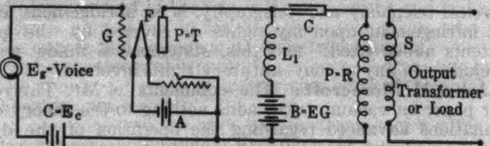

FIG. 2—AUDION ADAPTED TO THERRELL "AUTOSTATIC GENERATOR"

Among the claims granted to me on the "Auto Static" device, in the patents above mentioned are the following:

> "The method of producing an alternating current in the primary coil which consists in placing the primary coil and a capacity in a derived or shunt circuit between points whose potential varies as a function of the transmitter diaphragm, whereby an alternating current is caused to flow through the said primary coil due to charging and discharging of the said capacity as the potential varies across the terminals of the said derived shunt circuit."

Likewise the apparatus case covers:

> "Means for generating alternating currents in a circuit derived from the transmitter circuit and containing the primary coils of a plurality of transformers."

Also the generic claim for:

> *"Means for generating alternating currents in a branch of the transmitter circuit containing the primary coil."*

Likewise the following generic claim covering the tuning of the transmitter or primary circuit for any frequency to be transmitted:

> "The step in the art of Electrical transmission and reproduction of sound, which consists in making the terminal transmitter circuit, or circuits, wholly or partially resonant for the essential frequencies to be transmitted, thereby increasing the efficiency of transformation and the energy transferred to the secondary circuit."

When it is considered that the vacuum tube, with its input excitation of any kind, is equivalent to the ordinary telephonic transmitter, the scope of these claims is readily obvious and is clearly seen to cover broadly every use of the Audion, Pliotron or vacuum tube for the purpose of generating an alternating current or of preventing a direct current from circulating through the windings of the output transformer.

Of course I am aware that the right to a patent on improvements involving the use of prior devices or inventions, does not necessarily carry with it the right to proceed under such a patent free from domination thereof by patents on the prior devices or inventions.

I am, therefore, now merely calling attention, in the interest of historical accuracy, to the fact that the use of the vacuum tube, pliotron, or audion, or other device, for the purpose of deriving, or generating, an alternating current from a variable direct current, or its equivalent, in telephony, wireless telephony and telegraphy, is an abridgement of and an infringement upon my rights as covered by the generic patents above cited; and this, statement is made without prejudice to any of may statutory rights involved.

John H. Morecroft: The comments of Mr. Therrell on our paper on vacuum tubes adds nothing to the theory or explanations advanced regarding the operation of the device, and is of interest only to those engaged in patent litigation.

The question which Mr. Taylor asked, regarding the possible mechanical strains due to piezo electric forces, was not answered by Dr. Nicolson; I believe he has made no determination of this phase of the question. The piezo electric effect in quartz, which I have investigated to some extent, gives one a very vivid idea of the tremendous strains which the action may call into play.

If a rectangular piece of quartz, in the form of a thin slab, is properly cut from a good crystal, it may be made to actually pull itself into two pieces by these piezo electric forces. The faces of the slab should be parallel to one of the electric axes of the crystal, and the long edges should be perpendicular to the optical axis of the crystal. A typical piece, for example, might be one eighth inch thick, two inches long, and about one inch wide.

Pieces of tinfoil, nearly the size of the faces of the slab, are then shellaced to the two faces, and allowed to dry. If this quartz slab is considered as a rod, its fundamental period of vibration (longitudinal) may be calculated to be about 50,000 cycles per second. That is, if the slab were struck a sharp blow on one end it would sustain longitudinal vibrations of this frequency. The two pieces of tinfoil are connected to a high frequency source of electric power, the frequency of which may be accurately adjusted; a large vacuum tube oscillator is the most suitable source of power. If the frequency of the alternating e. m. f. is now varied until it is exactly the same as the natural period of the slab *the slab will actually pull itself into two or more pieces* after the e. m. f. has been applied for not more than a second or so; in general the slab will break itself into two nearly equal pieces.

It might well be conceived that the rupture of the quartz was simply a case of dielectric breakdown, due to excessive potential gradient through the quartz; that such is not the case is evident when the voltage required for rupture is considered. A piece one eighth inch thick may be ruptured by about three thousand volts; the dielectric strength of the quartz is perhaps ten times this amount. Moreover if the same voltage is impressed on the quartz, at a frequency differing from the critical one by as little as one-half per cent the slab will "stand up" indefinitely.

In general this test is best carried out with the test piece submerged in oil, preferably a light one, such as kerosene. The voltage required in the test will cause corona and leakage if the test is carried out in air, and these two effects tend to heat the quartz and so reduce its dielectric strength.

I have been successful in rupturing pieces as short as one-half inch, the required frequency in this case being of the order of 200,000 cycles per second.

When one considers the high tensile strength of a piece of clear quartz crystal it is evident that these piezo electric actions do bring into play very large mechanical strains indeed.

Presented at a joint meeting of the American Institute of Electrical Engineers and the American Physical Society, Philadelphia, Pa., October 10, 1919.

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A REPORT ON ELECTROMAGNETIC INDUCTION[1]

BY S. J. BARNETT

ABSTRACT OF PAPER

This report discusses briefly the chief fundamental results obtained from the days of Faraday to the present time in studying the electromotive forces ordinarily referred to the domain of electromagnetic induction.

Self-induction is first taken up, and the phenomena of self-induction are treated as essentially identical with the phenomena of inertia in dynamics, according to the idea of Maxwell and the idea originally accepted by Faraday. The only recent fundamental progress has been in studying the inertia of free electrons and other ions, and experiments on this subject are referred to.

The motional electromotive force, developed when matter moves in a magnetic field, is next considered, and is derived from Ampère's law on the electron theory. Especial attention is devoted to the motional intensity, and the resulting electric displacement, in insulators, of which nothing has been known until recent years.

The induced electromotive force in fixed conductors and insulators arising from the motion or alteration of other systems is next considered, and is expressed both in terms of magnetic flux and in terms of the general vector potential, which refers the phenomena back to the motion of electrons without the magnetic field as intermediary. The relations between the induced and motional electromotive forces are discussed, as well as the relation of the electric displacement produced in certain cases to the hypothesis of the fixed ether.

The report closes with a treatment of unipolar induction in both so-called open and closed circuits, including brief descriptions of some of the principal experiments, a discussion of the theories involved, and their application to the unipolar generator.

1. It is unnecessary to emphasize the great importance of this subject alike to the physicist and to the engineer. The world of the physicist is rapidly becoming fundamentally an electrical world, in which a vast part is played by electromagnetic induction; and without electromagnetic induction the world of the electrical engineer could scarcely be considered even to exist.

The general field of electromagnetic induction has become very broad, and it will be necessary to restrict this discussion to fundamental matters, and from among them to select for consideration only electromotive forces and electric displacements, and these only in stationary or quasi-stationary systems. Important related subjects such as the transfer of energy in the electromagnetic field according to Poynting and Heaviside must be excluded. Even the principle of relativity can scarcely be more than mentioned.

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[1] **For references see end of paper.**

To a very remarkable degree our knowledge of the fundamental facts and principles of electromagnetic induction is due to Michael Faraday. Most of the progress made by others has consisted in increasing the precision of his measurements and in developing ideas which originated in less perfect form with him.

2. The principal experiments of Faraday and his contemporaries in the field of electromagnetic induction may conveniently be grouped in four divisions, as follows:

I. Experiments in which the agent producing the magnetic field remains fixed to the earth and the conductor or insulator under examination moves. The electric intensity and electromotive force in the body due to the motion have been called the *motional* intensity and e. m. f. by Heaviside.

II. Experiments in which the body under examination remains fixed to the earth, and the agent producing the magnetic field moves or its electric current changes.[2] The electric intensity and e. m. f. produced under these circumstances will be called the *induced* intensity and e. m. f., although these terms are often applied also to the motional quantities.

According to the Einstein principle of relativity, no discrimination can be made between the motional and induced intensities, as only the relative motion of two bodies can be determined.

III. Experiments on unipolar induction, strictly belonging to groups I and II, but to be considered separately for convenience.[3]

IV. Experiments on self-induction.

3. The last group, on self-induction, will be considered first. When a steady electromotive force is applied to a circuit the current increases from zero until a steady value is reached when the counter e. m. f. of resistance just balances the applied e. m. f. and the electricity therefore moves uniformly. If the applied e. m. f. is made to disappear without otherwise altering the circuit the counter e. m. f. of resistance gradually reduces the current to zero or brings the electricity to rest.

Referring to the phenomenon of self-induction, Faraday says "The first thought that arises in the mind is that the electricity circulates with something like *momentum* or *inertia* in the wire." This idea, however, he soon gave up, but for inadequate reasons, as shown by Maxwell, whose treatment of the subject is based entirely on the idea of inertia. To set electricity into motion an e. m. f. is required just as a force is required to accelerate matter; and to bring electricity in motion to rest an opposing e. m. f. must be applied, just as an opposing force is required to reduce the velocity of a moving body. In strictness it is quite as improper to speak of an "electromotive force of self-induction" as of a "force of inertia" in dynamics. Failure to realize this has given rise to a great amount of confusion in electrical literature.

The fundamental self-induction relation in electric circuits is that of Faraday and Neumann: To alter the current in an electric circuit, and therewith the corresponding magnetic flux, an e. m. f. equal to the rate of increase of the flux must act in the circuit.

While a vast amount of progress has been made in measuring the quantities involved in self-induction and in studying the relations of other phenomena to those of self-induction, it appears that the only really fundamental progress made since the early days has come from studying the behavior of free electrons and other ions accelerated by electric and magnetic fields. Well-known experiments by J. J. Thomson, Kaufmann, Bucherer, Hupka and others have yielded important information on the constitution of the electron and of matter; but it is unnecessary to do more than mention these experiments here.

Along with these experiments should be mentioned the experiments suggested by Maxwell[4] on the electric displacement produced in conductors by their mere acceleration. If a conductor is accelerated, since some of the electrons or other ions are free, an electric displacement would, in general, be expected to result.

Such effects in electrolytes have been observed by Colley,[5] Des Coudres,[6] and others; and after having been looked for in metals by Maxwell, E. F. Nichols,[7] and Lebedew[8] have been observed and carefully measured in a beautiful investigation by Tolman and Stewart.[9] The behavior of the electricity in a circular coil of wire which has been rotating about its axis and is quickly brought to rest as in these experiments, is very similar to that in a coil which has been carrying a steady current when the applied e.m.f. is reduced to zero without altering the resistance.

4. Faraday's experiments of group I. on conductors prove that when a rigid circuit moves in a fixed magnetic field there is produced around it an e. m. f. equal to the rate of diminution of magnetic flux through it, or the net rate at which it cuts across magnetic flux, with proper respect to signs. The two statements are equivalent, since tubes of magnetic induction are always closed. The second form is the more fundamental, as will be evident from what follows.

Similarly, the motional e. m. f. along any part of the circuit, or any line in any conductor moving in a magnetic field, is equal to the rate at which it cuts across magnetic flux.

On the electron and ionic theories of matter this is accounted for by the Ampèrian force

$$F = e\,[v\,H] \tag{1}$$[10]

which acts on any electrified particle of the matter with charge e moving with velocity v in a field of strength H. For this gives a motional intensity, or force per unit charge,

$$f = F/e = [v\,H] \tag{2}$$

The integral of this intensity along any path in the conductor gives the result stated above.

Every experiment made since Faraday's time has confirmed this result. The most precise experiments are probably those which have been made in determining resistances in absolute measure by the method of Lorenz.[11]

In certain cases where different magnetic fields are superposed care must be taken to use for H in (2) the intensity of the field due to the fixed circuits or magnets with reference to which V is determined. An interesting illustration is that of an insulated wire inside a coaxial iron tube moving together with the wire across an (originally) uniform magnetic field due to fixed circuits or magnets. The e. m. f. developed in the wire is precisely the same as if the tube were absent, although the (resultant) field within the tube is greatly weakened by its presence. Since the tube moves with the wire the field due to its magnetization is not effective.

5. If non-conducting matter also is electrically constituted and moves in a magnetic field, each moving electrified constitu-

ent must, as in the case of a conductor, be acted upon by a force in accordance with (**1**), and thus by the motional intensity f given by (**2**).

Similarly, if the ether is electrically constituted, and if it moves in a magnetic field, it also will be the seat of a motional intensity given by (**2**).

The motional intensity will act to produce opposite displacements of the positive and negative constitutents of insulators as well as of conductors. This will, in general, give rise to an electric field, and the intensity E of this field will act on all electrical media which the field contains, whether at rest or in motion.

If both the ether and the matter which it permeates move together, as they do on the theory of Hertz,[12] both intensities act on both substances. But if the ether remains fixed, and the matter which it permeates moves through it, as on the theory of Lorentz[13], the intensity f acts on the matter only, while the intensity E acts on both ether and matter.

The motional intensity f, being equal to $[v\,H]$, is entirely independent of the nature of the moving substance provided only it has an electric constitution.

6. Referring to the matter of motional displacement, or electric displacement produced by the motional intensity, and to the displacement produced by the induced intensity, in insulators, Faraday says[14], "I have long thought there must be a particular condition of such bodies corresponding to the state which causes currents in metals and other conductors; and considering that the bodies are insulators one would expect that state to be one of tension. I have by rotating non-conducting bodies near magnetic poles and poles near them, and also by causing powerful electric currents to be suddenly formed and to cease around and about insulators in various directions, endeavored to make some such state sensible, but have not succeeded. Nevertheless, as any such state must be of exceeding low intensity, because of the feeble intensity of the currents which are used to induce it, it may well be that the state may exist, and may be discoverable by some more expert experimentalist, though I have not been able to make it sensible."

7. An imaginary experiment on this subject and unipolar induction combined was described by Poincaré in 1900[15]. But the first actual experiments after Faraday appear to be those of

Blondlot in 1901 on air[16]. In 1902 I began an investigation which included experiments on ebonite, sulphur, and rosin, but the work was not completed until 1908[17]. In the meantime H. A. Wilson[18] published an investigation on ebonite.

8. The essential theory of all these experiments can be given very simply. Consider an insulator in the form of a hollow circular cylinder with dielectric constant K rotating uniformly about its axis in a uniform magnetic field with intensity parallel to this axis. Suppose the inner and outer surfaces of the cylinder covered with conducting armatures of negligible thickness.

Let the number of revolutions per second be denoted by n, the magnetic flux across a right section of the space between the cylinders by φ, the capacity of the cylindrical condenser so far as it depends on the field between the armatures by $K\,C$, the capacity dependent on the field between the outer armature (together with any fixed conductors attached thereto) and the surrounding conductors, connected by a brush to the inner armature, by C'.

Let J denote the dielectric constant of that portion of the dielectric which moves. If the matter alone rotates while the ether remains fixed, $J = K - 1$. If the ether also moves, $J = K$. If the ether rotates alone, $J = 1$. It is clearly immaterial whether the armatures, which cut no magnetic flux, remain fixed or move.

When the cylinder is set into rotation the motional intensity f and motional e. m. f. $\Psi = n\,\varphi$ will act radially, let us say outwardly for definiteness, on the portion of the dielectric which moves. The electric separation produced in this part of the dielectric will give rise to an electric field whose intensity will be directed radially inward within the insulating cylinder and in which there will be a potential difference V directed from the outer armature to the inner armature, and from the outer armature to the surrounding conductors.

The total charge on the outer conductor and its connections, if any, will be zero, since it is completely insulated from the inner conductor and its connections. This charge consists of three parts superposed:

1. A charge $q_1 = -\,n\,\varphi\,J\,C$ (3)

on the inner surface;

2. A charge $q_2 = K\,C\,V$ (4)

on the inner surface;

3. A charge $q_3 = C' V$ (5)
on the outer surface and any conductors connected therewith. Since

$$q_1 + q_2 + q_3 = 0, \text{ we get, if } q_2 + q_3 = q_4,$$
$$q_4 = (K C + C') V = n \varphi J C = - q_1 \qquad (6)$$

In Wilson's experiments and my own, hollow insulating cylinders were used, and the experimental processes (considerably different in the two investigations) were equivalent to measuring q_4, n, φ, and C and therefrom calculating J, which was found equal to $K - 1$ within the limits of the experimental error.

In Blondlot's experiments a parallel plate condenser (a special case of a part of a hollow cylinder) was used, and a swift stream of air was sent between the plates while they were connected together by a wire at rest, thus making V zero and leaving the motional e. m. f. Ψ to act alone. The connection between the plates was broken when the velocity of the air stream was a maximum, and the quantity q_1 directly measured. It was found to be zero, thus showing that $J = K - 1$, since K for air and ether together is practically unity.

All these investigations thus confirm the theory of Lorentz, according to which the ether is at rest. The results would be identical if there were no ether.

9. The field inside the rotating cylinder is of great interest, but can be discussed here for a few cases only. The solid cylinder rotating about its axis and the parallel plate moving parallel to one surface are particular cases of the hollow cylinder.

(1) First suppose the ether to be at rest and suppose the thin armatures either not existent or else disconnected from all other conductors.

In this case the total electric displacement D at all points of a cylinder of any radius $r \geqq r_1$ (radius of inner surface) $\leqq r_2$ (radius of outer surface) coaxial with the surface of the insulating cylinder is

$$O = D = \frac{1}{4 \pi} \{ (K - 1) f + K E \}$$

$$= \frac{1}{4 \pi} \{K E + (K - 1) \omega H r\} \qquad (7)$$

where $\omega = 2\pi n$. Hence

$$E = -\frac{K-1}{K} f = -\frac{K-1}{K} \omega H r \qquad (8)$$

There is no intensity of either kind inside the inner cylinder or outside the outer cylinder.

(2) Next suppose the armatures present and connected by a wire at rest. This connection reduces the voltage $\int E\,dr$ to O, but the field intensity E does not vanish except over one cylinder of radius r_0 to be determined. If it did, the charge on the outer armature would be greater than the opposite charge on the inner, and the insulator itself would be internally charged. For f increases proportionally with r, and D would now therefore increase in the same way. The voltage through the insulator being zero, the direction of the field intensity E, always radial, changes in sign at the cylinder of radius r_0. Within this cylinder E has the same direction as f, so as to increase the total displacement; while outside this cylinder it has the opposite direction, so as to decrease the displacement which would exist were f alone acting. In this way the electric flux across all cylinders which can be drawn between the armatures is kept the same and equal to that across the cylinder of radius r_0. At the cylinder of radius r the field intensity is

$$E = -\frac{K-1}{K} \frac{\omega H}{r} (r_0^2 - r^2) \qquad (9)$$

while r_0 is given by the expression[19]

$$\left(\frac{r_2^2 - r_1^2}{2 \log \frac{r_2}{r_1}} \right)^{\frac{1}{2}}$$

(3) If the wire moves with the armatures a field intensity due to the potential difference V from the outer to the inner armature equal to the e. m. f. $n\varphi = \frac{\omega H}{2} (r_2^2 - r_1^2)$ developed in the wire will be superposed on the intensity given by (**9**), with the result that the displacement within the rotating condenser will be exactly the same as if it contained ether only, at rest, and were charged to the potential difference V[20].

(4)[11] If the ether moves with the matter in case (1), we must substitute K for $K - 1$, and we get in place of (**7**) and (**8**) the relations

$$O = D = \frac{1}{4\,\pi} K\,(f + E) \tag{10}$$

and
$$E = -f = -\,\omega\,H\,r \tag{11}$$

Neither matter nor ether is displaced, either within or without the cylinder.

(5)[12] If the ether moves with the matter in case (2), equation (**9**) becomes

$$E = -\frac{\omega\,H}{r}\,(r_0^2 - r^2) \tag{12}$$

while r_0 remains unchanged.

(6)[18] If the wire moves with the ether and the matter of case (4) [or if the ether moves with the matter and the wire of case (3)], there is no displacement anywhere, exactly as in case (4), where the wire was absent[21].

10. In the second group of experiments Faraday found that if a conducting circuit was kept fixed and the magnetic flux through it altered in any way by the motion of adjacent circuits or magnets, an e. m. f. was developed around it equal to the rate of decrease of magnetic flux through it, precisely as in the experiments of the first group for the same relative motion of the material systems. In this case the tubes of magnetic induction, which are always closed, must be conceived to move across the circuit, and the intensity induced therein at any point may be conceived as given by the expression

$$E = [H\,v] \tag{13}$$

where v is the velocity of the tubes. The electromotive force around the circuit is the line integral of this expression.

Furthermore, if the conducting circuit is kept fixed, as before, and the magnetic flux through it changed in any way by the alteration of adjacent magnets or circuits, or currents in these circuits, an e. m. f. is developed and is equal, again, to the rate of decrease of magnetic flux through it. In this case also we may conceive the (closed) tubes of magnetic induction to move across the circuit, producing therein at any point an intensity given by (**13**).

If the circuit is not conducting, but is merely a path drawn in an insulator or in the free ether, the electromotive force around it and the electric intensity at any point of it are given by the same expressions as before. The displacement, the effect of the intensity, of course differs with the conditions.

There is never any essential difficulty in determining the induced e. m. f., or line integral of the induced intensity, around a complete circuit from the Faraday-Neumann law; but the application of equation (**13**) to the determination of the intensity at a point is often impossible. For, except in very special cases, we do not know the velocities of the tubes of induction. Indeed, in certain cases the very conception of the motion of tubes of induction appears to many to be without meaning.

Consistently with experiment, however, the theory has been put in such a form by Maxwell, Clausius, and Lorentz, as to give for the induced intensity at a point, an expression which depends only on the motion of the electrons which produce the magnetic field and the time.

If e_1 denotes the (fixed) charge of a certain electron, r_1 its distance from the point P at which the induced intensity is to be determined, and $\mathbf{v}_1$ its velocity, then at the time t the vector potential at P due to this electron is by definition,

$$A_1 = \frac{e_1 \mathbf{v}_1}{r_1}$$

the (changing) quantities $\mathbf{v}_1$ and r_1 being the velocity and distance at a time earlier than the time t by the interval required for an electric wave to travel from the electron to P. The vector potential A at P due to all the moving electrons is

the vector sum of all the minute vector potentials $A_1 = \frac{h_1 \mathbf{v}_1}{r_1}$

$A = \frac{e_2 \mathbf{v}_2}{r_2}$, etc. That is $A = \text{vector } \Sigma\, A_1 = \text{vector } \Sigma \frac{e_1 \mathbf{v}_1}{r_1}$

In terms of this vector potential A the expression for the induced intensity is simply

$$E = - \frac{d A}{dt} \qquad \textbf{(14)}$$

and the electromotive force around the circuit is the line integral of this expression; Even in cases where it is not possible to determine the velocities of the electrons and thus calculate A, it is possible to determine whether A, and thus also $\frac{dA}{dt}$, changes when certain variations in conditions occur, and thus to obtain useful information with respect to E.

11. In many cases the magnetic field in which a body is moving is changing with the time, and we have the motional and induced intensities superposed. Similarly, they are superposed when the body under experimentation and the agent producing the field are both in motion.

If the system referred to as fixed is, instead of being fixed to the earth, fixed to a ship or car in uniform motion, there is no difference in observed results, according to experiments hitherto made. It is understood that the observer and instruments are on board, though this is often a matter of no consequence.

It will be instructive to consider some simple experiments, in part made familiar long ago by Faraday, with a circular cylindrical magnet A and an adjacent coaxial circular coil of wire B with a remote galvanometer connected into the circuit by long twisted leads.

(1) If A is fixed to the earth and B moves along the axis, there will be a motional e. m. f. in B and a certain electric charge q will traverse the galvanometer for a given change of magnetic flux through B. (2) If now B is fixed to the earth, and A is displaced, the relative motion of A and B being the same as before, there will be an induced e. m. f. in B exactly equal to the motional e. m. f. in the previous case, and the same charge q will traverse the galvanometer.

It is of no consequence whether the galvanometer is attached to A, or to B, or to the earth. The indications will be the same, and will appear the same to observers on A, or in B, or on the earth.

Suppose now that the region in which the experiments are performed is permeated by the ether, and for simplicity suppose this fixed relatively to the earth.

When experiment (1) is performed, the only electric intensity and displacement produced will be the motional intensity and displacement in the coil.

When (2) is performed, there will be the same displacement in the coil, due to an induced intensity equal to the previous motional intensity; and at the same time the field about the coil and magnet will be filled with circular lines of induced intensity all coaxial with the magnet and due to the motion of its tubes of magnetic induction. This induced intensity will produce electric displacement in the ether, the lines of displacement also being coaxial with the magnet.

The galvanometer shows no difference between the two cases, but the difference really exists.

If there is no ether, no electric displacement, aside from the current in the coil, will occur in either case.

If an experiment takes place in which relative motion between *A* and *B* occurs in an isolated region, and observers are placed on *A* and *B*, each observer is likely to assume that he is fixed and that the other is moving. The observer on *A* will conclude that there is a motional e. m. f. in *B*; while the observer on *B* will observe the same e. m. f. but will call it *induced*. A third observer, fixed to neither *A* nor *B*, will consider the e. m. f. to be partly induced and partly motional.

12. Faraday, as we have seen, attempted to study the displacement produced in insulators by the induced intensity, but without success. Except for a qualitative experiment of Hertz's[22] made with high-frequency oscillations and lying outside the scope of this discussion, no successful experiments on the displacement in material media appear to have been made. Kuehne[23] has, however, shown that a finite electrified body in a region traversed by moving tubes of magnetic induction is acted upon by a force consistent with equation (**13**).

13. Recently, in an investigation presented to the Physical Society, I tried to find out whether, in a certain special case, the induced intensity produces electric displacement in the ether.[24] But I expressed to the society an uncertainty which has persisted and grown, as to the validity of one of the assumptions underlying the method. The experiment will be briefly considered.

14. Suppose an air condenser, with horizontal plates *A B*, short-circuited by a wire *C* and placed in a uniform magnetic field whose lines of induction are parallel to the plates. Suppose that the condenser is screened (an absolutely essential precaution) by being placed symmetrically within a metallic box whose faces are parallel and perpendicular to the plates, and that the upper plate is connected to the top of the box by a wire *F*.

If the box and condenser move together in the magnetic field parallel to the plates and perpendicular to the lines of induction, the plates will become charged, but the charge on the lower face of each plate will be equal and opposite to the charge on the upper face of the same plate. If during the motion the lower plate is removed and tested for total charge, it will thus be found uncharged.

15. If the box and condenser remain fixed and the magnet is moved, the charges will be exactly as before on the hypothesis that there is no ether. If the ether exists, however, there will be no electric displacement anywhere within the box, and the plates will be without charge on either side. The test of the lower plate for total charge will thus give the same result as before, and the experiment will not discriminate between the two hypotheses.

16. Imagine the magnetic field to be sharply divided into two parts by the plane passing through the upper surface of the lower plate *B*, the complete field throughout the region occupied by the box remaining uniform, but the tubes of induction of the lower part remaining fixed like the condenser, while the tubes of the upper part move when the magnet moves. In the attempt to accomplish this end I used two similar magnets, symmetrically placed with respect to the plate *B*, the lower one fixed to the condenser.

If this arrangement secures the end desired, and the condenser remains fixed while the upper magnet moves, the result will be different according to the hypothesis made with regard to the ether.

If there is no ether the upper surface of the lower plate of the condenser will become charged, there being now no intensity or displacement beneath the lower plate. If the ether exists, the plate will be uncharged.

Experiments made in this way have shown that the lower plate of the condenser does not become charged when the upper magnet moves.

If, however, the assumption made above with reference to the division of the magnetic field and the application of equation (**13**) under these circumstances are not justified, and the principle of superposition holds in such a way that the two magnets independently produce their individual effects, the lower magnet will produce no effect (at the critical time) and the upper magnet will act exactly like the single magnet of Sections 14 and 15 (but with a non-uniform field, which does not affect the theory essentially), producing no (total) charge on the lower plate on either hypothesis. This is the result to be expected from the application of equation (**14**). In this case the experiments simply confirm Section 15.[25]

17. We come now to the subject of unipolar induction.[26] In one of Faraday's experiments a symmetrical magnet rotating uniformly about its axis was touched by a wire *A C B*, including the coil of a galvanometer, at two points *A* and *B*, one near the axis at one end, the other on the cylindrical surface near the center. An e. m. f. was developed in the circuit equal to that which would be developed if the magnet remained fixed and the conductor *A C B* moved, the relative motion being the same as before.

In another experiment a copper disk was mounted coaxially close to one end of the magnet and a galvanometer *C* connected to two points *A* and *B* near the center and edge of the disk. When the disk rotated, an e. m. f. equal to that which would be produced by the same relative motion of *A C B*, was developed as before, whether the magnet rotated or not. But no current traversed the galvanometer when the disk remained fixed.

Similar results are obtained when the magnet is replaced by an electromagnet or by an electric coil wound on a non-magnetic core.

18. Faraday considered his experiments to prove that the tubes of magnetic induction remain fixed and that the magnet and disk move through them, the e.m.f. thus being a pure motional e.m.f. with the disk or the magnet itself as its seat.

It was pointed out by Tolver Preston,[27] however, that the results are equally well explained by the assumption that the tubes of magnetic induction share the rotary motion of the magnet and produce an induced e. m. f. by cutting the conductor *A C B*.

19. Faraday's experiments have been repeated with various modifications a great many times, but the problem of the seat of the e.m.f. was brought no nearer to a solution until the experiments of E. Lecher were published[28] in 1895. Lecher made three beautiful experiments, two of which, at least, are quite difficult to explain except on the assumption that the tubes of magnetic induction remain fixed, or that the seat of the e. m. f. is in the moving part of the circuit.

In one of these experiments, for which alone we have space, two similar electromagnets were mounted coaxially and near together, with the current in both coils in the same direction. A galvanometer was connected in the usual way to two points on one of the magnets. It was found that the current depended entirely on the motion of this magnet, being the same whether the other magnet rotated in either direction or remained at rest. On the assumption that the seat of the e. m. f. is the moving magnet the explanation of this result is quite obvious. Improbable assumptions are necessary to explain it on the other hypothesis. By making sufficient assumptions however, this is possible; and Lecher did not consider his experiments *entirely* conclusive.

20. Many students of physics have thought that this problem could be solved by experiments on so-called open circuits—experiments in which static charges, or forces on bodies with such charges, would be looked for instead of permanent galvanometer currents. The list includes Weber, Tolver Preston, Hertz, Lodge, Lecher, and others. But the theories on which such experiments and proposed experiments have been based appear to be fallacious.

21. As a particular case, consider a long cylindrical air condenser placed in a uniform magnetic field with lines of induction parallel to its axis and short-circuited by disks closing the ends. If this condenser were rotated about its axis it would become charged by the motional e.m.f. in the disks, the charge of the inner armature, the one assumed to be tested, being equal to this e. m. f. multiplied by the capacity.

If, however, the condenser and wire remain fixed, and the agent producing the magnetic field rotates about the axis, the relative motion being the same as before, the result to be expected depends on the hypothesis made with respect to the ether.

First suppose that the ether does not exist. If the tubes of induction do not move, the condenser will remain uncharged. If the tubes of induction move with the agent, an e. m. f. will be induced in the disks producing a potential difference in the opposite direction through the condenser and charging it as when the condenser moved.

If, on the other hand, the ether exists, there will again be no charge in case the tubes of induction remain fixed. If the tubes of induction move, however, there will be superposed on the charge due to the potential difference developed in the corresponding case for no ether an equal and opposite charge due to the e. m. f. induced in the ether between the armatures. There will, as before, be no change on the inner surface of the inner armature. Thus it will remain uncharged, just as if the tubes of induction had remained fixed.

Thus if the armature should become charged in such an experiment it would prove that the ether as an electrical medium does not exist, and that the tubes of induction move.

But if the armature should not become charged, the experiment would prove only that either (1) the ether does not exist and the tubes of induction do not move, or (2) the ether exists.

22. In 1912 I completed an extended series of experiments of this kind, not with the hope (which I originally had) of solving the problem of unipolar induction, but to get what information they might yield on the relations between electromagnetic induction and relative motion.[29]

The experiments show that the condenser does not become charged, the experimental error being a small fraction of the charge the condenser would receive if it rotated while the agent producing the field remained fixed.

This is the first case in electromagnetic induction in which it has been proved that the observed effect does not depend entirely on the relativity of the motion of the essential material systems involved.

23. If the condenser and short-circuiting disks are made to rotate together with the agent producing the field, there will be superposed on the (zero) charge of the condenser in my experiments a charge equal to that it would take if the agent producing the field remained fixed and the condenser rotated. Rotating the agent alone produces no charge; rotating the condenser and disks alone produces a charge. When both are rotated together the effects must be superposed.

This corollary of my experiments has been confirmed in each of two recent investigations; one by Kennard,[30] who had done earlier work in this field; and one by Pegram.[31]

In their experiments the apparatus was essentially similar to my own, but the condenser and coil were both made to rotate. The condenser was found to become charged in accordance with the theory.

In this last rotation experiment we have a positive effect of electromagnetic induction without any relative motion between the essential material parts of the system. In my own rotation experiments we have a zero effect with a positive relative motion. Neither group of experiments, however, is inconsistent with the principle of relativity of Einstein, which relates to translatory motion only, or with the theories of Maxwell and Lorentz.

These experiments do not seem to me to solve the old problem of unipolar induction. As shown above (Section 21) they prove *either* (1) that the ether does not exist and that the tubes of induction do not move, *or* (2) that the ether exists. If we had evidence that the ether does not exist, the experiments would prove that the tubes of induction remain fixed.

24. The *simplest* assumption which will account for all the unipolar phenomena is Faraday's assumption that the tubes of induction are fixed and that the e. m. f. is a pure motional e. m. f. in the moving conductors. The electron theory, moreover, supports this assumption in the simplest case to which it can be applied,[31] viz. that of a thin solenoid carrying a direct current and rotating about its axis. When the solenoid is rotated there is simply superposed on the conduction current density at any point of the solenoid a convection current density of identical nature and zero magnitude, since the positive and negative charges per unit volume of the conductor are equal. And this could not, apparently, affect in any way the state of the magnetic field.

For practical purposes, however, it is of no consequence which assumption is made. On the simpler hypothesis, the calculation of the e. m. f. may be made from equation (**2**); on the other, from equation (**13**); and on either hypothesis, or any combination of the two, it may be calculated from the rate of change of flux through the circuit from the brush A in contact with the cylindrical surface of the magnet, through the galvanometer to the brush B in contact with the center of the end surface of the magnet, from B to a point C fixed on the magnet in the same transverse circle with A, and from C along the arc of the circle to the brush A. With the uniform growth of the arc $A\,C$, which contributes nothing to the e. m. f. and may encircle the cylinder any number of times, the flux through the circuit changes at the same uniform rate on any hypothesis.

25. This arrangement, essentially the unipolar or homopolar generator, in which the rotation takes place about the axis of a symmetrical field and one part of the system cuts the flux

uniformly and always in the same direction, is clearly the only one in which a steady direct current can be electromagnetically produced by rotation without commutation. With rotation about any other axis the e. m. f. will fluctuate or alternate periodically.

Department of Terrestrial Magnetism
Carnegie Institution of Washington.

FOOTNOTES

1. For recent general discussions of electromotive forces of induction see F. Emde, *Elektrotechnik und Maschinenbau* 26, 1908, pp. 997, 1023, 1074, 1119, and 27, 1909, p. 783; also de Baillehache, *Bull. Soc. Int. des Électriciens* 10, 1910, pp. 89, 288.

2. For a brief account of the work of some of the early investigators see Brillouin, *Propagation de l'Électricité*, pp. 159-167, 1904.

3. Much of the work prior to 1895 is described by E. Lecher, *Wied. Ann.* 54, 1895, p. 276.

4. Treatise, §§576, 577.

5. *Wied. Ann.* 17, 1882, p. 55.

6. *Wied. Ann.* 49, 1893, p. 284; 57, 1896, p. 232.

7. *Phys. Zeit.* 7, 1906, p. 640.

8. *Ann. d. Phys.* 39, 1912, p. 840.

9. *Phys. Rev.*, 8, 1916, p. 97; 9, 1917, p. 164.

10. The expression $[v\,H]$ signifies the *vector product* of the vectors v and H. Its magnitude is equal to the product $v\,H \sin \theta$, when θ ($<180°$) is the angle between the directions of the two vectors; and its direction is the direction in which a right handed screw would be translated if it were rotated through the angle θ from a position in which the slot in its head has the direction of v to that in which it has the direction of H. It is thus perpendicular to both v and H.

11. *Pogg. Ann.* 149, 1873, p. 251. The most recent and precise work on this subject is that of F. E. Smith, *Roy. Soc. Phil. Trans.* A. 214, 1914, p. 27. For some recent experiments on the motional e. m. f. in conductors, consistent, like all others, with equation (2), see C. Hering, TRANS. A. I. E. E. 27, 1908, p. 1341; and A. Blondel, C. R. 159, 1914, p. 674.

12. Electric Waves, p. 241 (*Wied. Ann.* 41, 1890, p. 369). See further section 9, (4), (5) and (6). The existence of $E = -f$, evidently necessary, since f diverges, to keep D ($=0$ in equation (11)) from divergence, and the insulator thus free from internal charge, appears impossible to account for and thus evidence against justification of the assumption that the ether alone, ($K=1$) or the ether and matter together, ($K>1$) can move. It is thus evidence against the validity of Hertz's theory.

13. *Archives Neerlandaises* 25, 1892, p. 363.

14. Exp. Res. § 1728.

15. *L'Éclairage Électrique* 23, 1900, p. 41.

16. *J. de Phys.*, Jan. 1902.

17. *Phys. Rev.* 27, 1908, p. 425.

18. *Roy. Soc. Phil. Trans.* A., Sept., 1904.

19. Barnett, l. c., pp. 431–433.

20. Attention was first called in print to this result by E. H. Kennard (*Phys. Zeit.*, 13, 1912, p. 1156), though I had previously made use of it.

21. The capacity of the wire is here neglected. The wire itself, of course, becomes charged and there is displacement in the dielectric about it.

22. *Electric Waves*, p. 95.

23. *Phil. Mag.*, Apr. 1910.

24. *Phys. Rev.* 12, 1918, p. 95.

25. As another illustration of the great difficulty involved in getting information about the ether from experiments in electromagnetic induction it is instructive to consider the following: A long circular solenoid with an iron core is traversed by an alternating current, so that tubes of magnetic induction periodically move into and out from the core, producing circular lines of electric intensity centered on the axis. A long narrow, and thin air condenser is mounted near the solenoid with its greatest length and the planes of its armatures parallel to the axis. The armatures are connected by a wire which includes a detecting instrument. If there is no ether, the condenser receives alternating charges on the *inner* surfaces of its armatures; if the ether exists, the condenser receives the same charges, but on the *outer* surfaces of the armatures, the detecting instrument thus indicates the same current on both hypotheses. Essentially the same difficulty would arise in an experiment similar to that of Section 15, but made without an electric screen.

26. In addition to Lecher, l. c., see Barnett, *Phys. Rev.* 35, 1912, p. 323; G. W. O. Howe, *Electrician* 76, 1915, pp. 169, 210, 323, 355; and Pegram, *Phys. Rev.* 10, 1917, p. 591.

27. *Phil. Mag.* 19, 1885, pp. 131 and 215.

28. Lecher, l. c.

29. *Phys. Rev.* 35, 1912, p. 323.

30. *Phil. Mag.*, Feb. 1917.

31. Pegram, l. c.

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PRESENT LIMITS OF SPEED AND POWER OF SINGLE-SHAFT STEAM TURBINES

BY J. F. JOHNSON

Abstract of Paper

This paper will be restricted to a discussion of some of the factors which influence limits as applying particularly to turbines of the reaction type. With the employment of high vacua the limit of power will be determined largely by the area obtainable through the last stage.

Limiting factors include:

1. Chosen maximum values of steam speed through the blades in order to keep the leaving losses within permissible limits. For highest efficiency the steam speed should be about 25 per cent greater than the blade speed, but in the last stages it is sometimes made 100 per cent greater as a compromise between efficiency and cost. Similarly the outlet angle of the blades is increased from 20 deg. to 35 deg.
2. Physical characteristics of materials employed and chosen limits to which these may be safely stressed. By varying the form of rotor construction stresses in it may usually be kept within necessary limits up to the point at which limiting stresses in the blades or blade fastenings are reached. For any given rotative speed and blade angle the steam capacity is directly proportional to the stress at the base of the blades regardless of the diameter and blade height.
3. Capacity limits of manufacturing facilities, increased bracing and clearances necessary to insure requisite rigidity and reliability, and capacity loss due to outages for inspection and repair.

Fig. 6 shows maximum capacity at various speeds which are physically possible without exceeding present limits of stresses. It is valuable chiefly as showing the physical relation between speed and capacity with given limiting stress values.

THE recent rapid development of steam turbine-driven electric power-generating units of large capacity has prompted your Society to undertake a study of the present limits of speed and power for such units. This paper will be restricted to a discussion of some of the factors which determine or influence such limits as applying particularly to turbines of the reaction type.

With the employment of high vacua, such as is the present universal practise, the limit of power of a turbine operating at a given speed will be determined largely by the area obtainable through the last stage for the final expansion and passage of the steam prior to its entering the condenser. The significance of this will be apparent when attention is called to the fact that whereas a pound of steam, when entering the first stage, has a volume of less than 2½ cu. ft., when passing through the last stage it has a volume of approximately 395 cu. ft. when expanded to 28½ in. vacuum, and 585 cu. ft. when expanded to 29 in.; a ratio in the latter case of 1 to 234.

Consequently, in any discussion of limits of power, it will be necessary to assume conditions of pressure and superheat of the steam entering the turbine, the vacuum to which the steam is to be expanded in the blading, and the efficiency or rate of steam flow per unit of power. For these conditions 250 lb. gage pressure with 200 deg. fahr. superheat and 29-in. vacuum referred to a barometer of 30 in., and efficiencies as are commonly obtainable with them, will be used.

Limiting factors may be divided into three classes: First; Theoretical, including limiting steam velocities and effect on efficiency of velocity remaining in steam after leaving the last stage, and the area through the blades as affected by blade angle. Second; Physical, including methods of construction, materials, stresses, factor of safety against rupture, reliability factor, and limitations of transportation facilities. Third; Economic, including limits beyond which it may be physically possible, but economically inadvisable, to go, such as effect of size of structure or of character of materials employed on cost, and time required to make inspection and repairs.

THEORETICAL LIMITS

In this class there are but few limitations as affecting capacity at a given speed because with materials of infinite strength and rigidity available it would be possible to build units of infinite capacity; but for a given diameter and blade height the capacity will be limited by chosen maximum values of steam speed through the blades, in order to keep the leaving losses, or available energy in the steam discharged to the condenser, within permissible limits. Throughout the entire turbine, with the exception of the last few stages, steam speeds only about 25 per cent in excess of the corresponding blade speeds are employed in order to secure maximum efficiency. In the latter stages, however, the volumes become so great that a compromise between maximum theoretical efficiency and physical dimensions becomes advisable by increasing the steam speed sometimes to approximately 100 per cent in excess of the blade speed. The steam after being discharged from the last stage, therefore, still contains a small portion of available energy the recovery of which would involve disproportionate expense.

For example, if the pressure drop in the last stage is such as to render available for work thirty heat units which will produce an equivalent velocity of 1225 ft. per sec., and if the blade speed is such that the steam after leaving it still has a velocity of 600 ft. per sec., which is the equivalent of 7.2 B. t. u., this 7.2 B. t. u. will be totally lost, whereas probably 80 per cent of it, (or 6 B. t. u.), might be recovered were it practicable to use an additional stage of proper proportions. This would improve the total efficiency of the turbine approximately 1½ per cent. Higher blade speeds will tend to improve the efficiency by reducing leaving losses, but generally not as effectively as would larger blade areas with lower steam velocities and correspondingly increased number of stages.

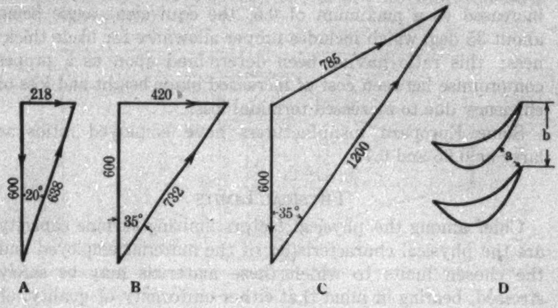

FIG. 1

Having fixed the height of a row of blades, the area of the steam space is dependent upon the angle formed between the center line of the row of blades and the outlet portion of the blade. The smaller this angle is the smaller will be the area and *vice versa*. On the other hand the smaller this angle the higher the efficiency because of the lesser absolute velocity left in the steam discharged to the condenser.

In Fig. 1A is shown a relation between steam speed and blade speed to give highest efficiency. The blade speed is 600 ft. per sec., steam speed 638 ft. per sec., blade angle 20 deg. and steam, speed after leaving blades, 218 ft. per sec. in direction at right angles to direction of rotation. Fig. 1B shows a similar condition giving maximum obtainable efficiency employing a 35 deg. blade angle and Fig. 1C a condition in which the steam speed is 100 per cent greater than the blade speed.

The leaving losses are, in Fig. 1A, 218 ft. per sec., Fig. 1B, 420 ft. per sec., and Fig. 1C, 785 ft. per sec., which is the equivalent of 0.95, 3.5, and 12.3 B. t. u. respectively.

The steam area is ordinarily expressed as a ratio of the perpendicular distance between blades, to the pitch of the blades, as a/b (see Fig. 1D). Highest actual efficiency is obtained by keeping this ratio between 0.25 and 0.3, and this is done in all stages except the last few in high vacuum machines where it is increased to a maximum of 0.5, the equivalent angle being about 35 deg. which includes proper allowance for blade thickness; this ratio having been determined upon as a proper compromise between cost of increased blade height and loss of efficiency due to increased terminal loss.

Some European manufacturers have employed ratios as large as 0.65 and 0.7.

PHYSICAL LIMITS

Chief among the physical factors limiting turbine capacity are the physical characteristics of the material employed and the chosen limits to which these materials may be safely stressed, bearing in mind that either uniformity of quality, or factor of safety sufficient to cover all possible variations, together with inaccuracies in calculation and irregularities of operation, must be provided for. While alloy steels possessing exceptionally high physical characteristics are procurable, their high qualities depend on relatively sensitive metallurgical processes which in the opinion of some engineers cannot as yet be carried out by regular workmen as a manufacturing process with a sufficient degree of reliability to justify their use, and that until this can be done, conservatism demands adherence to the lower strength, lesser sensitive materials. Such materials may, with suitable forms of construction, be safely stressed under the maximum test condition to within a few thousand pounds of their true elastic limits.

When the construction of the rotor is not limited to any one special form, the design may be varied so as to take full advantage of the low speeds in the high and intermediate stages, (where low speeds must be used in order to secure high efficiency), by employing a drum the thickness of which may be varied to keep the stresses within desired limits; while in low pressure stages where the stresses are highest, either disks carried on a shaft, or solid disks suitably held together, may be employed. With the solid disk construction the stresses may be kept within any reasonable limits up to speeds at which the design becomes too massive and expensive.

The steel regularly used by the Westinghouse company for turbine rotors conforms to the following characteristic specifications:

Test rings taken as close as possible to the point of maximum stress must show the following characteristics with standard two-in. specimens:

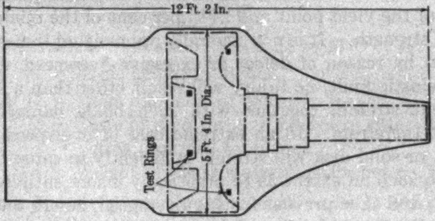

Fig. 2

Tensile strength 65,000 to 70,000 lb. per sq. in.
True elastic limit 22,000 to 25,000 lb. per sq. in.
Elongation 15 per cent to 18 per cent.
Reduction of area 20 per cent to 25 per cent.

The steel must be of best quality, having approximately 0.25 per cent carbon, 0.50 to 0.60 per cent manganese, 0.25 per cent silicon, and not over 0.025 per cent sulphur or of phosphorus.

The material is obtained ordinarily in the form of castings, though occasionally as forgings. The specifications in either case are the same. Especially in the larger sizes the forgings have been difficult to obtain, excessively expensive, and no more uniform or reliable in quality than the castings.

Fig. 2 shows the form of casting for a rotor end. It is cast vertically with the small end down, and after casting, is allowed to cool very slowly in the sand. After removal, the entire upper portion which constitutes the riser, is cut off and the casting then thoroughly annealed by being heated slowly and evenly to a temperature of about 1650 deg. fahr. and allowed to cool very slowly. It is then rough machined to within about ¼ in. of finished surface, after which it is put in a furnace and heated to about 1100 deg. fahr. and allowed to cool slowly to remove any possible internal stresses set up by reason of the metal removed in machining. It is then finish machined and given no further treatment of any kind.

The limit of stress to which this material is subjected is 20,000 lb. per sq. in. when operating at a speed 20 per cent in excess of the normal operating speed. The stress at normal speed is therefore $\frac{20,000}{(1.20)^2} = 13,900$ lb. This stress is 63 per cent of the minimum allowable true elastic limit, about 46 per cent of the yield point and 21½ per cent of the minimum ultimate strength. It is not generally appreciated that should the stress, by reason of defect or excessive overspeed, exceed the true elastic limit, no injury will result other than a slight permanent stretch, together with such blade damage as may result therefrom. In an extreme case of overspeed the rotor drum or solid disk will stretch sufficiently to cause blading to rub to such an extent as to practically insure entirely destroying it, and thus prevent further overspeed, before the ultimate strength and elongation of the material is reached.

If the rotor design can be so modified as to always keep the stresses within necessary limits, then the stress at the base of the blades, or in blade fastenings, determines the maximum capacity obtainable with a given speed.

There exists two interesting relations between the stress at the base of blades, steam passage area through the blades, and rotative speed. *For any given rotative speed and blade angle, the steam capacity or steam area through the blades is directly proportional to the stress at the base of the blades, regardless of the diameter and blade height selected.* This stress can only be modified by unevenly varying the cross sectional area of the blades, such for example, as thickening the blade near the base. *Also for any given stress the area through the blades will vary inversely as the square of the speed, i. e.,* if at a speed of 1800 rev. per min. a given stress and area are obtained, then at 900 rev. per min. the area will be increased four times if the stress is kept constant.

These relations will be readily seen from the following formulas:

$$\text{Area through blades} = \pi \times M\,D \times H \times R.$$

$$\text{If } R \text{ constant} = \text{Constant} \times M\,D \times H$$

$$\text{And stress at base of blades} = \frac{W\,V^2}{G\,r}$$

$$= \text{for steel} \frac{0.284 \times H \times \left(\frac{\text{rev. per min.} \times M\,D}{229}\right)^2}{32.16 \times \frac{M\,D}{24}}$$

$$= \frac{0.284 \times 24 \times M\,D \times H \times (\text{rev. per min.})^2}{32.16 \times (229)^2}$$

If rev. per min. is constant = constant × $M\,D \times H$.

Where H = Blade height in inches.

$M\,D$ = Mean diameter of blades in inches.

R = Ratio of area through blades to total annular area occupied by blades.

0.284 = Weight of blade material in lb. per cu. in.

The area and stress are therefore each equal to a constant times the product of mean diameter and blade height, and when the stress is constant this product will vary inversely as the square of the rev. per min.

The ratio of blade height to rotor diameter is, therefore, not a factor in determining physical limit of capacity, but only in determining efficiency, cost, and to some extent, reliability of the turbine.

Blading used in impulse stages and in low pressure reaction stages in which stresses exceed 15,000 lb. per sq. in. at 20 per cent overspeed is made of a 5 per cent electric furnace nickel steel in which the carbon sulphur and phosphorus are kept very low. It is really a nickel.iron having a very fine close structure.

Its physical and chemical characteristics are as follows:

Tensile strength	65,000 lb. per sq. in. minimum
True elastic limit	35,000 lb per sq. in. minimum
Elongation in 2 in.	30 per cent minimum
Reduction of area	60 per cent minimum

Carbon, not over	0.08 per cent
Silicon, not over	0.10 per cent
Phosphorus, not over	0.025 per cent
Sulphur, not over	0.04 per cent
Manganese	0.40 to 0.50 per cent
Nickel	4.5 to 5.5 per cent

This material is annealed by heating to 1425 deg. fahr. and cooled in open air after rolling into sections required for forming into various blade shapes, and is given no further heat treatment.

The maximum stress at 20 per cent overspeed to which this material is subjected is 25,000 lb. The corresponding stress at normal speed is, therefore, 17,350 lb., this being 49 per cent of the true elastic limit, and 26½ per cent of minimum ultimate strength.

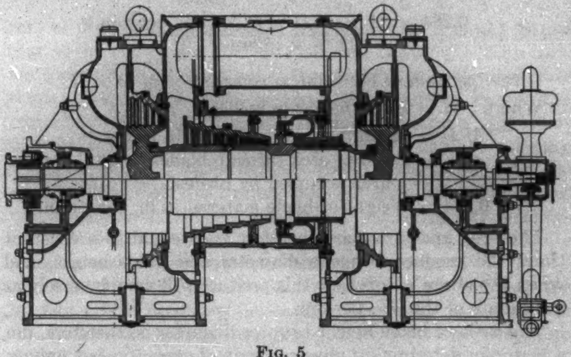

Fig. 5

For the lower stress reaction blading, a copper tin and phosphor bronze is employed, consisting of:

Copper	97 to 98 per cent
Tin	2 to 3 per cent
Phosphorus	0.03 to 0.07 per cent

Satisfactory methods of blade fastening involve no problems unless allowable stresses in the blades are very materially higher than those in the blade carrying element.

Fig. 3 shows a method used where the stress at the base of the blade does not exceed 17,500 lb., and Fig. 4 where stresses between 17,500 lb. and 25,000 lb. are involved. At 25,000 lb.

PLATE LXXIV.
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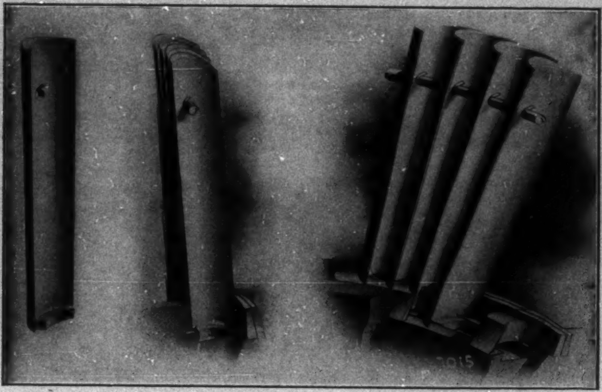

FIG. 3.

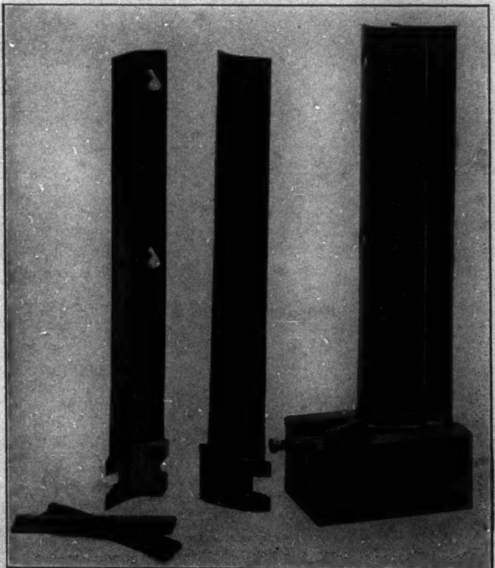

FIG. 4

[JOHNSON]

stress at the base of the blade the tensile stress in reduced section of base is 11,800 lb., and the shearing stress in the blade and blade carrying element is 10,000 lb. The compound side wedges are employed to insure an absolutely tight fit of the blade in the groove so as to protect the reduced section of the blade against any stresses other than direct tension.

Increased capacity without decrease of rotative speed or increase of stresses may be obtained by employing multiple low pressure stages. This well known and popular expedient possesses the merit of permitting high-vacuum turbines to be built t speeds and capacities up to approximately the present limits of generator construction, without exceeding moderate diameters blade lengths, and stresses.

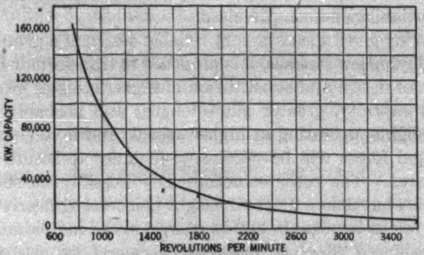

Fig. 6—Limits of Capacity of Steam Turbines with Double-Flow Low-Pressure Stages

Having steel blades of uniform cross-section stressed to 25,000 lb. per sq. in. at base of blades at 20 per cent overspeed and maximum efficiency at 80 per cent of rating—250 lb. steam pressure—200 deg. superheat—29-in. vacuum referred to a 30-in. barometer—with steam velocity through blades of 1225 ft. per sec. at a volume of 585 cu. ft. per lb.

Fig. 5 shows a cross sectional view of a turbine typical of this type.

Fig. 6 is a curve showing approximate maximum capacities at various speeds which are physically possible, employing double flow construction without exceeding the limits of stresses previously given. For equal capacities employing single flow construction the stresses would have to be doubled. The points marked (x) at 3600, 1800 and 1500 rev. per min. represent capacities which have already been built. This curve must not be interpreted as indicating suggested practicable present or ultimate limiting capacities of turbines, but merely as showing a physical relation between speed and capacity with given limiting stress values and operating conditions.

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An important limit of size and capacity now being approached is that imposed by transportation facilities. Stationary elements may be readily sectionalized as required and assembled after shipment. Ways may be devised also for partial dismantling of rotor elements, although diameter will be one of the limiting factors, and this cannot be reduced beyond the point of omission of blading. The low-pressure rotors of the 30,000-kw. Interborough Rapid Transit Company turbines were shipped with the last stage blade carrying elements removed.

While the physical dimensions and capacities of turbines are being constantly increased it is essential that the reliability factor be not decreased. The employment of special materials and higher stresses does not usually permit increased capacity or efficiency without a corresponding increase in weight and cost, unless reliability be compromised.

If, having given a satisfactory reliable design of a given capacity employing low stresses, it is proposed to transform it by modification of design and substitution of higher stresses into a unit of larger capacity, greater blade lengths and probably greater blade weights, operating at higher speeds, and involving higher centrifugal forces will be necessary, in order to secure the required area. The rotor structure may possibly be shortened somewhat, but unless its total weight is increased nearly in proportion to the increase in stored energy in the individual blade; the unbalanced effect or disturbance caused by one or more blades breaking (which must be recognized as an inevitable occurrence in any turbine) will be greater, imposing higher stresses in the rotor shaft, bearings, and bearing supports; and a greater factor of strength will be required to withstand these stresses. The greater blade weight and higher speed will also require increasing the mass of the casing in order to prevent the blades from permanently injuring and possibly breaking through it if they should fail. The endurance factor of a turbine when operating under imperfect or abnormal conditions will be higher in proportion to the ratio of stator mass to rotor mass, and of rotor mass to blade mass. The incorporation in the design of a turbine of features which increase the endurance factor will appreciably increase its cost but will also (to a very much greater extent) increase its value to the user.

ECONOMIC LIMITS

In the study of economic limits of turbines of large capacity, consideration must be given to the fact that as yet such units are not required in sufficient quantity to warrant equipping and operating shops for their exclusive manufacture, and that they must, therefore, be produced by largely the same processes and equipment as are used for smaller sizes which are built in greater quantities.

As sizes becomes larger, a greater proportion of special equipment and processes becomes necessary, resulting in increased rates of cost unless accompanied by very material increase in quantity of production. Under present conditions this economic limit of capacity agrees closely with the physical limit of 1500 rev. per min. units.

In the larger low-speed structures the physical proportions become such that using ordinary steel and cast iron, to which we are limited by the metallurgical art, the distortions due to temperature changes and elastic properties of the materials are such that increased clearances and bracing have to be employed, in order to maintain equal reliability and rigidity to a degree which causes the cost per kilowatt for a given efficiency to increase with increasing capacity. Further development of the allied arts and increased demand for larger units wil tend to reduce the influence of this limitation factor.

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Another factor tending to limit capacity of single units is the generating capacity loss resulting from suspension of service for inspection or repairs. For example, if a 30,000-kw. unit must be kept out of service ten days for a certain inspection or repair, a 60,000-kw. unit would have to be kept out probably fourteen days for a similar purpose because of the greater time required to handle the larger structure. Therefore, if two 30,000-kw. units were used and each held out of service ten days, the outage loss would be only five-sevenths as great as if a single 60,000-kw. unit were kept out fourteen days.

In order to avoid the limitations or undesirable characteristics just referred to, a number of turbine units of capacities varying from 30,000 to 60,000 kw. have been built in which the turbines have been divided into two or three separate compounded elements, each driving its own generator, and each capable of operating alone on high pressure steam in emergencies.

It is believed that units of this type will continue to be employed for the larger capacities because of the advantages not obtainable in single cylinder types, which will justify their somewhat greater cost.

Presented at the 355th meeting of the American Institute of Electrical Engineers, New York, November 14, 1919.

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PRESENT LIMITS OF SPEED AND POWER OF SINGLE SHAFT CURTIS STEAM TURBINES

BY ESKIL BERG

Abstract of Paper

This paper starts by showing that the limit of a single-unit turbo generator does not lie in the generator but is confined to the steam turbine, and that the last wheel of the turbine is the limiting feature.

The author therefore takes the last wheel of an 1800-rev. per min. turbine, giving dimension stresses, kind of material used etc., and then designs two turbines, one having 23 stages and the other 13 stages, both machines using this last wheel, and shows that a turbine can be built having its most economical point at 21,000 kw., under steam conditions given in the paper.

Under this condition the last stage absorbs 11.5 per cent of the total adiabatic available energy, and the wheel efficiency is 66.25 per cent. As the load increases, the work done in this stage also increases, so that at 36,000 kw. the energy is practically doubled, but with a sacrifice in efficiency of about 18 per cent, which naturally lowers the efficiency of the turbine as a whole.

A 5000-kw., five-stage, 3600-rev. per min. turbine load curve is also given and discussed, and the author claims that if the construction could be made similar to the large turbine, keeping the same stresses and number of stages, a turbine of this capacity could be built as efficient as the larger machine at 20,000-kw.

THE purpose of this paper is to discuss the limitations of the types of turbines used in the more important work now done by the General Electric Company, and to show the relative results which can be accomplished with such designs under different conditions of load. Different mechanical arrangements will make practicable different characteristics, and the conditions shown only apply to machines of these particular designs. The design of turbines must be governed by many compromises and it would not be possible to give a correct idea as to the whole range of possibilities.

For purposes of illustration, two machines have been selected, one representative of the largest size built for 1800 rev. per min. and one of the largest size now built for 3600 rev. per min. The figures given relate to the turbine alone, and do not include generators. The generators used with such turbines might be designed for various power factors or overload capacities, and would thus be of different efficiencies.

The turbines in question are both of the single-flow type and may be considered representative of capacity limitations of that type, since their construction is that which is adapted to the highest capacity. The single-unit turbine and generator is naturally preferable over the tandem and compound type on account of simplicity, lightness, and efficiency. There are however certain definite limitations in the size for a given speed that these units can be built with material available at the present time. The limitation in the size of a unit for a given speed does not depend upon the generator, but is entirely dependent upon the turbine.

Fig. 1 shows the load curve of the large 1800-rev. per min. machine above mentioned, which is designed to operate with 250 lb. steam pressure, 28.5 inches of vacuum, and dry steam. This turbine has 23 stages, all wheels being of the single-bucket type. The first stage wheel has a pitch diameter of 35 inches, which increases with each successive stage until the last wheel, which has a pitch diameter of 88 inches. This curve shows that for 1800 rev. per min., a turbine can be designed for a vacuum of 28.5 inches, giving its best efficiency at 21,000 kw., this point being marked on the curve by 1, and the water rate at other points being given in proportion to this point. This curve also shows that with a sacrifice of 5 per cent in efficiency, an output of 36,000 kw., can be delivered to the turbine shaft. Above a load of 21,000 kw., live steam is bypassed to the eighth stage shell, the effect of which is shown by the break in the curve.

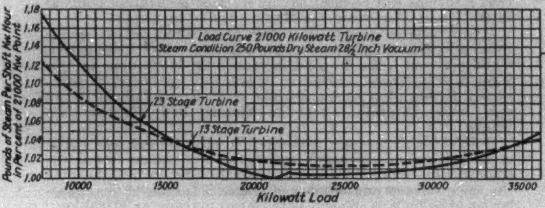

FIG. 1

The dotted line gives the load curve of a similar turbine in which the first eleven stages are replaced by one two-bucket stage. This turbine is designed with multiple valve steam admission. Such a modification may be desirable for the purpose of simplification, or where better efficiency at very light loads is important, as in the case of propulsion of warships.

Fig. 2 gives the vacuum curve of the 1800-rev. per min., 23-stage machine at 21,000-kw. and 36,000-kw. load on the turbine shaft. It will be noticed that the improvement in economy at a load of 21,000 kw., is 2.5 per cent between 28.5 and 29 in., whereas with a load of 36,000 kw., due to congestion of the steam passages, the gain is only 0.55 per cent.

Many large size turbines are now designed for a vacuum of

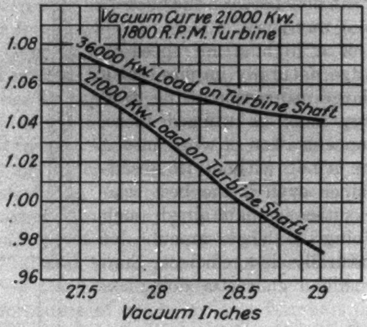

Fig. 2

29 in. The volume of one pound of steam at this vacuum is 652 cu. ft., almost 50 per cent greater than at 28.5 in., and about twice as much as at 28 in., which calls for a corresponding increase in the area of the steam passage in the last stage wheel. If this area is made too small, the steam when leaving the last row of buckets must have a high velocity, which gives a large loss of energy, this loss being in proportion to the square of the velocity. It is therefore important that this area be made as large as possible, and that the exit angle of the blades be made small enough to give good extraction.

Efficient action can only be accomplished by using a bucket speed that bears a proper relation to the steam velocity. Consequently to get the largest capacity we must not only use long buckets, but must move them at a very high speed. In order to obtain good bucket action, the buckets should not be more than about one fourth as long as the pitch diameter of the wheel. If they are made longer than this, poor bucket action with consequent loss in efficiency will result, due to the great difference in peripheral speed between the base and the tip of the bucket, the design being made correct for the middle point or pitch line. The flare also becomes excessive, so that the space between the buckets at the tip will be so large that steam can flow between the buckets without doing any work.

The use of a high steam speed in this last stage naturally implies that a relatively large proportion of the total steam energy must be utilized there, and such concentration of work into a single stage is not without its disadvantages, since even if the best relation of velocities is maintained, such a stage doing a large amount of work is naturally less efficient than a stage of similar character doing less work.

Since for the reasons given above, the design of the last stage in such a turbine constitutes the most important limitation, we will consider some details of the design used in this case. Fig. 3 represents the last wheel of the large 23-stage 1800-rev. per min., turbine which has been mentioned. This turbine is designed to operate at 250 lb. steam pressure, 28.5-in. vacuum, and dry steam. The pitch diameter of the wheel is 88 in., length of buckets 22 in. and bucket angles 60 deg. entrance and 40 deg. exit. The wheel is subject to the following stresses at normal speed:

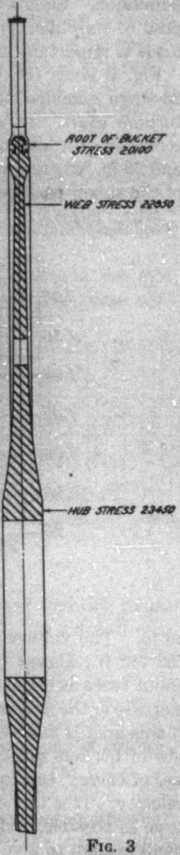

Fig. 3

Stresses in hub, 23,450 lb. per sq. in.
Stresses in web, 22,950 lb. per sq. in.
Stresses in bucket, 20,100 lb. per sq. in.

Elastic limit, (limit of proportionality) of material, 55,000 lb. per sq. in. The material of the wheel and bucket is quenched and tempered 3 per cent nickel steel.

The wheel is stiff enough to avoid vibration effects, and in the absence of such effects the centrifugal strains afford ample factors of safety, even if we assume considerable irregularities and imperfections of metal structure.

CAUSE OF WHEEL BREAKAGES

In some of the first large machines of the type here discussed, very serious trouble has developed through the formation of cracks in the forged wheels. Such cracks have caused wheels to break in three important installations. The cracks which have formed in these wheels have started at holes in the wheel provided either for balancing steam pressures on the two sides of the wheel, or for the attachment of balance weights. The occurrence of these accidents naturally gave rise to much alarm, uncertainty, and difference of opinion as to the cause of the trouble. Calculation showed that the wheels which broke were less stressed than many which were made from weaker metal and had operated for long periods of time. Holes in a centrifugally stressed wheel greatly increase the fibre stress in the vicinity of the hole itself, but such conditions had not caused the formation of cracks in large numbers of wheels in which such localized high stresses existed. Many evidences have now shown that the trouble with these wheels has not resulted from stresses in excess of those which had been previously found to be practicable, but has been caused by fluttering and vibrations of the wheels, which had become possible through the lightness and thinness of their construction. Such vibration gives a periodic character to the stresses normally imposed and so gives rise to the formation of fatigue cracks.

In machines of this type relatively light and narrow buckets have been used, and the wheels have been proportioned with a view to ample centrifugal strength, but with maximum economy of space and weight, consequently these wheels have had much less lateral stiffness than wheels used in turbines of previous types. To overcome such troubles as have developed, it is simply necessary to make the wheels stiffer and to put in holes in parts of the wheel near the hub where a suitable reinforcement of thickness can be provided which both stiffens the wheel and reduces stresses near the holes. Very slight changes of this kind make a great difference in the vibrating characteristics of such wheels, and the proportions used are such that they can easily be brought to the same standards of safety in these respects as have long prevailed in wheels of heavier construction in machines having less numbers of stages.

Experience has shown that in the absence of a tendency to form fatigue cracks through vibration, overspeed in such turbine wheels involves relatively little danger as compared with other types of high-speed machinery. Experimentally and in actual service, wheels have been stretched to a considerable degree of enlargement without the formation of any cracks, and such stretching is a normal condition if the cracks do not exist.

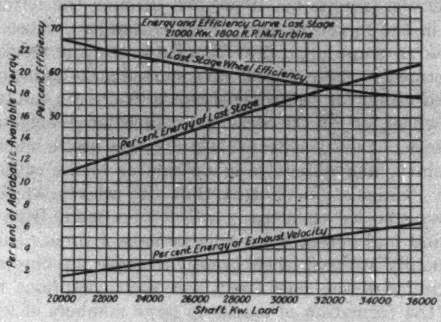

Fig. 4

It has been discovered that such fluttering or vibration of the web of wheels has not only been responsible for the formation of fatigue cracks in the wheels themselves, but has also caused loosening and breakage of buckets. The cure for these difficulties is to use stiffer wheels, and such wheels can carry stiffer buckets, so that the whole structure is incapable of vibration of any amplitude through such forces and periods as arise from the conditions of operation.

Fig. 4 gives the energy and efficiency curves of this last stage. It will be noticed that at the most efficient point, (21,000 kw.), this stage absorbs 11.5 per cent of the total adiabatic available energy, and that the wheel efficiency is 66.25 per cent. The energy represented by the exhaust velocity which is all wasted in the condenser is 1.5 per cent of the total energy. As the load increases on the turbine shaft, the energy in this stage also increases, decreasing its efficiency until at 36,000 kw. the energy in the last stage is 20.9 per cent of the total energy. The wheel efficiency however has been reduced to 54.2 per cent and the energy represented by the exhaust velocity has been increased to 6.4 per cent. This great amount of work in the last stage at such poor efficiency naturally lowers the efficiency of the whole turbine, and in this case the efficiency at 36,000 kw. is 5 per cent lower than at 21,000 kw.

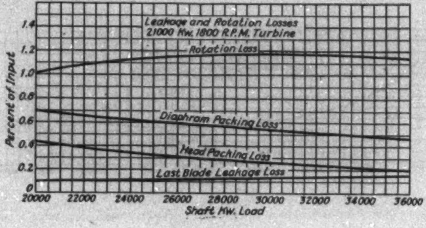

Fig. 5

Fig. 5 shows leakage and rotation losses of the same turbine in per cent of input.

From the above it will be seen that for 1800 rev. per min., a turbine can be designed efficiently for 21,000 kw. which, with a sacrifice of efficiency can deliver 36,000 kw.

Fig. 6 gives a load curve of the smaller 3600-rev. per min. turbine. The water rates are here given in reference to that of the larger machine, the load of 5000 kw. corresponding to that of 20,000 kw. on the 1800-rev. per min. turbine. This turbine has only five stages, one two-bucket wheel in the first stage, the other four stages having single-bucket wheels. The first wheel has a pitch diameter of 35.5 in., and the remaining four wheels a pitch diameter of 51 in. The bucket height of the last wheel is 9.125 in., the turbine being designed for a maximum of 6250 kw.

The reason for such a discrepancy in the number of stages calls for explanation. As the output of a turbine, keeping approximately the same stresses, goes up inversely as the square of the rev. per min., if the same number of stages could be used and clearance and all dimensions proportionately reduced, a 5000-kw. machine at 3600 rev. per min. could be made nearly as efficient as a 20,000-kw. machine at 1800 rev. per min., and developments of smaller multi-stage machines at our Lynn Works have already been made which approximate such possibilities. Constructions however which are practicable on a large scale are not practicable on a small scale, and consequently there are difficulties in getting the space economy in small high-speed machines which would be necessary for accomplishing the result stated. One of the difficulties has lain in the construction of diaphragms, the casting in of nozzle partitions being easy in a large diaphragm and very difficult on a small one. We are working upon types of diaphragms and other parts which may make possible the development of multi-stage high speed machines which afford improved degrees of economy.

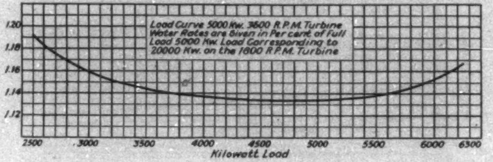

FIG. 6

If a 10,000-kw. turbine is designed for 1800 rev. per min., the only change necessary would be to make the nozzle and bucket heights about half the height of those in the 20,000-kw. unit. This reduction in height of buckets and nozzles would affect the weight, size, and cost of the turbine very little as compared with the 20,000 kw. unit. In regard to economy, the lower bucket heights would reduce the rotation loss somewhat but far from 50 per cent. The diaphragm packing loss, head packing losses, and bearing losses would be practically the same as on the 20,000-kw. unit, so that while a turbine designed for 10,000 kw. would be more economical than the large turbine running at half load, the difference would be small, being only about 6 per cent.

It will be seen from this paper that for a given speed there is one particular size of turbine which can be designed to be most economical as to steam consumption, weight, space, and price per kilowatt. Even if a size smaller than this is required, it would in many instances pay for the central station to install the larger unit, even though it would have to run at reduced load for some time before the station load increased sufficiently to utilize the full capacity.

Presented at the 355th meeting of the American Institute of Electrical Engineers, New York, November 14, 1919.

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PRESENT LIMITS OF SPEED AND OUTPUT OF SINGLE-SHAFT TURBO GENERATORS

BY F. D. NEWBURY

ABSTRACT OF PAPER

Output is determined broadly by rotor or stator dimensions. With speeds of 1200 rev. per min. and lower, the stator is the limiting member, while with higher speeds, the rotor is the limiting member.

The most effective rotor diameter is not necessarily the largest diameter. To obtain maximum output at a given speed the rotor proportions must be chosen to properly balance mechanical stresses, rotor ampere turns and flux. American design practise has established 400 ft. per sec. as an upper limit of rotor peripheral speed.

The maximum length of core is determined by such factors as ventilation, bearing temperatures, critical speed and limits to weight imposed by forging and transportation facilities.

Fig. 4 shows present limits to kv-a. rating at speeds from 3600 to 900 rev. per min. These limiting values are given as indicating present boundaries to knowledge and experience, rather than as real physical or other limits that cannot be exceeded.

Mechanical forces due to short-circuit current, and damage caused by armature winding failures, are no greater in the very large generators indicated by Fig. 4 than in present day 20,000 and 30,000 kv-a. units.

No opinion is expressed as to the wisdom of installing very large single-shaft units. If operating engineers desire units of 50,000 to 100,000 kw., there is no question but that such generators can be conservatively designed and constructed.

REDUCED to the simplest terms, maximum output at any speed is attained when slot space is provided for the maximum possible ampere turns (in either stator or rotor), and core cross section is provided for the maximum possible flux. These conditions require the most effective rotor diameter (or stator bore) and the maximum rotor and stator core length. All factors that limit rotor diameter (or stator bore) or core length have a possible bearing on limiting outputs.

The most effective rotor diameter for a given speed is not necessarily the largest diameter. For maximum output, the rotor should have maximum space for winding and maximum tooth and core section for flux. Obviously, these requirements are antagonistic and the actual design is a balance between slot area and tooth and polar area. Again, as the diameter is increased (with a given speed) there is more room for both winding and flux, but with increase in diameter each pound of copper exerts an increasing centrifugal force and the ratio of slot area to tooth cross section must be decreased in order to keep within desired stresses. Beyond a certain peripheral velocity, ampere turns must be decreased, in spite of the increase in available space, and the most effective diameter has been passed. It is seen, therefore, that in order to obtain maximum output at a given speed, the rotor proportions must be chosen to properly balance mechanical stresses, rotor ampere-turns and flux.

Turbo generator design has settled down to one type of rotor, so far as form is concerned. This is the so-called radial slot type, in which the ends of the winding project beyond the core body. This construction is shown in Fig. 1, indicating the slot arrangement. This type of design requires solid rings of very good material for holding these projecting coil ends. The hoop-stress in the coil retaining rings is an important limit to output, and is, in fact, a more important limit than the tooth stress in the main rotor body.

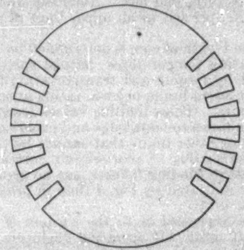

Fig. 1

In a large-diameter low-speed turbo generator (1200 rev. per min. and below) it is generally possible to employ a larger rotor diameter and more rotor ampere turns than can be properly balanced by stator ampere turns. The density of stator ampere turns is limited by the ability to dissipate heat (with permissible temperature differences) and by the permissible concentration of ampere turns in a single slot. Obviously, there is no such rigid limit to weight or depth of copper in a single slot in the stator as is imposed by centrifugal stresses in the rotor, but there is a limit to the depth of stator slots determined by the rapid increase in eddy current losses with deep slots and by the ability to construct and insulate long coils having a very large ratio of depth to width.

Thus, in certain cases the rotor is the limiting member and in other cases the stator is the limiting member. In general, the rotor first reaches its limit in ratings of 1500 rev. per min. and higher speeds, and the stator first reaches its limiting output in ratings of 1200 rev. per min. and lower, considering commercial frequencies.

It is apparent that unless the ratio of rotor ampere turns to stator ampere turns is a fixed design relation, all generators could be designed for the maximum possible output as determined by the stator, and the rotor need never be the limit. As a matter of fact, there is a tendency in this direction, the restraining fact being that as the field is weakened, relatively to the armature, the increase in field current and exciting voltage as the load is increased becomes greater. A reasonable limit to increase in excitation with load is desirable from the standpoint of voltage regulator operation. Regulators can readily handle a range of one to two or one to two-and-a-half, and large generators are proportioned to meet this ratio of no-load to full-load excitation. Voltage regulation has ceased to be a limit to output. These field and armature proportions result in regulations of roughly 25 per cent at 100 per cent power factor, and 40 per cent at 80 per cent power factor. Obviously, such regulations could not be tolerated if regulation were a factor in operation.

A fundamental difficulty in laying down definite limiting outputs is the difficulty in arbitrarily stating limiting stresses. The two principal stresses in the radial slot rotor are the tooth stresses in the main rotor body and the hoop stress in the coil retaining rings. Soft carbon steel is employed for the main rotor body, and a good quality ductile alloy steel (usually chrome-nickel or chrome-vanadium) is used for the coil retaining rings. Turbo generators are designed for a maximum speed 20 per cent above the running speed. At this over-speed, the tooth stresses should be approximately one-fourth the ultimate strength of the carbon steel and the coil-retaining ring stress should be approximately one-third the ultimate strength. This results in working stresses, in both cases, approximately half the yield point. It is important that the material be ductile; carbon steel with proper working, can readily be obtained with 22 per cent elongation and 35 per cent reduction in area and the alloy steel should have 22 per cent elongation and 50 per cent reduction. These figures refer to standard two-inch test pieces under tension.

American design practise has established 400 ft. per sec. as an upper limit of peripheral speed for maximum ampere turns and output for rotational speeds of 1500 rev. per min. and higher. This, of course, assumes existing rotor materials and factors of safety.

It is apparent that having increased the rotor diameter to the most effective value, output will be proportional to the length of the rotor and stator cores, and maximum output will be secured when the length is increased to its limiting value. This limit to length is even more a matter of opinion and judgment than is the limit to rotor or stator diameter. It is determined mainly by cooling air requirements, by bearing proportions, by limits to weight imposed by transportation facilities and the ability to secure forgings of necessary diameter and weight.

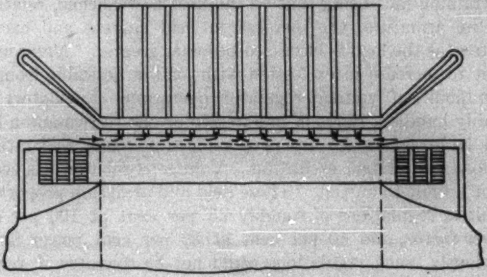

Fig. 2

Ventilation. The generator losses, and consequently the required volume of cooling air increase almost in proportion to the core length. In the simple radial or air gap system of ventilation, shown diagrammatically in Fig. 2, all of the cooling air must pass through the air gap entering the annular openings between stator and rotor at the two ends of the generator. The radial dimension of the airgap is constant with constant rotor diameter and consequently the volume of cooling air can only be increased as the core length is increased by increasing the air pressure.

Also, as core length is increased, the diameter of the shaft extension of the rotor body must be increased and the fan intake becomes restricted. In the axial system of ventilation, illustrated in Fig. 3, the stator ventilation is taken care of independently of the air gap and the requirements as to cooling air becomes less important from the standpoint of limiting output. But with either system of ventilation, designers are already finding it necessary to devise more complicated systems in order to take care of 3600 and 1800 rev. per min. ratings now in prospect.

Rotor Deflection. As the core length is increased, necessitating an increased distance between bearings, the rotor deflection increases. This increases the reversing stress in the shaft material at the journals and reduces the value of critical speed. As the core length is increased, the journal and bearing sizes must be increased in order to keep the shaft stress and the critical speed within desired limits and a limit may be imposed by bearing losses and temperatures.

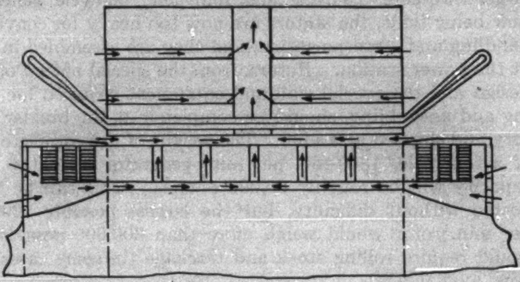

Fig. 3

Winding Temperatures. The limiting ratings given in this paper are based on 150 deg. total rotor winding temperature and from 125 deg. to 150 deg. total stator winding temperatures. It is not probable that ratings will be increased by increasing these temperature limits. Higher temperatures would be of most value in connection with the rotor winding since the rotor limits rating in the two- and four-pole designs commonly used. But temperatures higher than 150 deg. result in relatively little gain in rotor ampere-turns on account of the rapid increase in resistance of the winding. If the rotor winding temperature rise is assumed proportional to the loss, an increase in operating temperature from 150 deg. to 250 deg. (an increase in measured rise from 100 deg. to 200 deg.) results in an increase in ampere turns of only 25 per cent.* Thus doubling the temperature rise and rotor loss results in a gain of only 25 per cent in output.

Temperatures much above 150 deg. in connection with very long rotors are not considered favorably on account of the danger of trouble from "creeping" of the winding caused by linear expansion.

Transportation facilities may impose a limit to size in the case of six- and eight-pole 60-cycle generators. With the larger two-pole, 25-cycle and four-pole, 60-cycle generators now being built, the stators are now too heavy for convenient handling and transportation, and they are assembled in place at the power station. Rotors, from the special nature of their design and the special skill and equipment required for winding and assembling, should be completed at the builder's factory and shipped as a unit. The weight of the complete rotor of a four-pole, 1800-rev. per min. generator of 40,000 kv-a. capacity will be roughly 90,000 pounds. This can be transported without difficulty, but the largest possible 1200-rev. per min. rotor would weigh more than 200,000 pounds, and would require rolling stock and trackage (in some cases) not now available.

Another general limitation to output that applies to the larger diameter rotors is that imposed by the *forging facilities* of the country. At the present time it is not possible to obtain forgings of suitable physical characteristics weighing more than 50 to 60 tons nor much larger than 50 inches (assuming a minimum amount of working down from a 72-in. ingot). This limits the rotor, made from a *single* forging, to an output of roughly 50,000 kv-a. at 1500 rev. per min., and a proportionately decreasing kv-a. at lower speeds, assuming a solid rotor. By adopting the rotor construction involving two or three inch plates and up-set flanged shaft ends, the limiting diameter may be increased sufficiently for the largest 1500- and 1200-rev. per min. outputs, shown by Fig. 4. The design of 1200- and 900-rev. per min. generators for maximum output will be governed by questions of forging and shipping facilities, rather than by more strictly design matters.

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*A discussion of this point is contained in the author's paper, "Rational Temperature Guarantees for large A-C. Generators", TRANSACTIONS for 1916, Part I, Page 1497.

Fig. 4 shows in curve form limiting generator capacities at various speeds. At 1500 rev. per min. and higher, the capacity is determined by the rotor and is inversely proportional to the rev. per min. squared. At lower speeds, the capacity is limited by the stator and falls somewhat below the corresponding rotor limiting capacity as indicated by the dotted extension of the rotor curve. This curve is actually based on constant core length, when, as a matter of fact, the length can, with reason, be increased as the diameter is increased. The curve ratings, however, represent maximum lengths of core so far employed, and material extensions in core length involve questions of linear expansion that must be very carefully considered. This limiting capacity curve represents capacities that can be obtained with existing commercial materials and without radical changes in stresses and bearing proportions. The curve does not represent limits that may not be exceeded in the future. It is, more properly, an indication of present boundaries—boundaries that will be extended as our knowledge and experience are increased. It represents also the present judgment of designers, a judgment influenced greatly by the economic and operating advantages of still larger ratings.

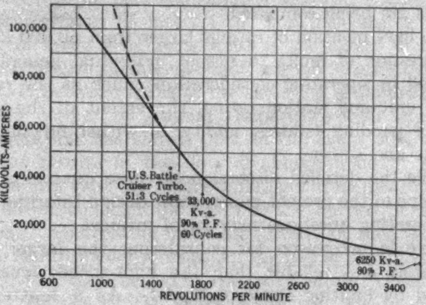


Fig. 4

The capacities shown by the curve are somewhat in advance of accomplished results. Ratings of several turbo generators that are the largest that have been placed in operation or are under construction by the Westinghouse Electric and Manufacturing Company have been added to Fig. 4 with self-explanatory comment.

As previously explained, the limiting capacities given for speeds below 1500 rev. per min. can only be attained by exceeding present transportation facilities if present design types are adhered to.

Incidentally it is interesting to note the advantage in limits gained by the use of 50 cycles as compared with 60 cycles. An increase of nearly 50 per cent in rating is made possible by the 20 per cent decrease in two-pole and four-pole speeds. This is of interest mainly when European and American maximum ratings are being compared.

The bare mention of ratings larger than 50,000 kv-a. raises the question of limits to size of individual generating units imposed by operating considerations, such as the relation between unit and station rating, the extent of the damage in case of winding failures, ability to withstand sudden short circuits and so on.

While the detailed discussion of these questions is beyond the scope of the present paper, some design information affecting operating questions may be of interest.

There is no reason for considering the larger low-speed generators less reliable than the high-speed generators indicated by limiting curve of Fig. 4. As a matter of fact, the lower speed ratings can usually be designed, both in stresses and in electrical factors, with more margin.

Mechanical forces developed by short circuits are determined by the short-circuit ampere turns of the armature winding per inch of armature circumference—and, to a limited extent, by the density of the magnetic field set up by the rotor winding. Both the distribution of ampere turns and the density of the airgap magnetic field are substantially constant for all limiting ratings of a given frequency. While the forces developed in a 25-cycle generator will be greater than in a 60-cycle generator—due mainly to lower reactance and the resulting greater values of ampere turns—all 60-cycle ratings indicated by the limiting curve will have substantially equal forces developed on sudden short circuit. The stresses in the coil ends will be determined by these forces and by those factors determining the rigidity of the winding. Except possibly in the maximum size 1500-rev. per min., 25-cycle generator, with its long coil-end extension, there need be no material difference in the rigidity of the winding. Thus a 50,000-kv-a., 25-cycle, 1500-rev. per min. generator represents the most difficult design from the standpoint of short-circuit stresses. However, such a generator would not differ materially in short-circuit stress conditions from 30,000-kv-a., two-pole generators that have been in successful operation for three years. It can be stated with confidence that the danger of winding failure due to sudden short circuit, with generators of the indicated limiting outputs, will be no greater than 20,000- and 30,000-kv-a. generators that have been placed in operation in large numbers during the past six years.

Another question of interest to those responsible for the operation of large generating units is the extent of damage to winding in case of internal short circuits caused by failure of insulation between turns of the same coil or failure of insulation from copper to ground. Experience with large units now in operation has shown that, in the event of a winding failure that results in the flow of abnormal power current, the chances are that the entire winding will be destroyed and that a hole of considerable size may be burned in the core laminations. Generating units are already of such size that a winding failure usually results in the loss of output from the unit for several months. The results of failures in still larger generators will be of the same degree and will be no more serious except, of course, in that loss in kv-a. output will be greater. In this connection, it is pertinent to point out that the fusing of metal and other local effects of an internal generator failure is a function of station capacity rather than of individual unit capacity. The volume of metal fused at the point of failure is determined largely by the impedance of the generating circuits feeding into the vault. Therefore, the only difference between a failure in one of two 30,000-kv-a. units and a failure in a single 60,000-kv-a. unit is in the impedance of the leads buses, and other connecting circuits between the two 30,000-kv-a. units. The two 30,000-kv-a. unit installation has the obvious advantages that protective reactance may be installed between the units, and the trouble is usually confined to half the station capacity.

The author has not intended to express an opinion as to the wisdom of installing very large single-shaft units. The only purpose has been to point out from the design standpoint the feasibility of certain ratings. Whether it is desirable or even wise to install very large units—above 50,000-kv-a.—will depend very largely on the growth and size of generating stations. When stations double in size—when stations of 300,000 kv-a. and 500,000 kv-a. become typical of American practise—there will undoubtedly be a demand of considerable volume for units of 50,000 kw., 75,000 kw., and 100,000 kw., and if single-shaft units are justified from the turbine standpoint, there is no question but that such generators can be conservatively designed and constructed.

Discussion on "Present Limits of Speed and Power of Single Shaft Steam Turbines" (Johnson), "Present Limits of Speed and Power of Single-Shaft Curtis Steam Turbines" (Berg), and "Present Limits of Speed and Output of Single-Shaft Turbo Generators" (Newbury), New York, N. Y., November 14, 1919.

Philip Torchio: It was a little over twenty-four years ago that The New York Edison Company started the operation of the first steam turbine used in a central station in this country, not a stone's throw from this building, at the West 39th Street steam station. A week ago the same company put in operation a 35,000-kw., 1500-revolution single-flow machine, which is the largest machine designed for that speed. The machine of twenty-four years ago was a 300 h. p. and the present a 50,000 h. p.

A few years after the first experiment on this turbine, built by the French De Laval Company, and operated for a few years with more or less success, the principal American electrical manufacturers started to build steam turbines. The first American built units were about 500 kw., but soon ascended to 5000 and 7500, and in 1906 units of 12,000 were built; in 1908, the largest was 14,000; 1912, 20,000; 1913, 30,000; and in the last year units up to 45,000 kw., 60 cycle, 1200 revolutions, were built.

I have not the statistics available, but probably two-thirds or three-quarters of the central station generating equipment today consists of turbo units. When we consider that the central stations furnish not more than one-sixth of the total power used in the country, aggregating considerably over 100,000,000 h. p., and when we further consider such facts as were brought out by Mr. W. S. Murray, that by central station power generation, of which 93 per cent by steam and 7 per cent by water, savings in the order of $150,000,000 a year and more can be made on the northeastern seaboard section of this country alone, we are forcibly impressed with the importance of steam turbine design.

B. A. Behrend: In 1906 I stated in *Cassier's Magazine* that: "The power station of the future will contain as many 20,000-kw. units as the power station of today contains 5000 kw. units." I venture now, twelve years later, to go on record that the power station of the future will *not* contain the 75,000 and 100,000 kw. units, of which we have heard a great deal in the papers presented this evening.

The course of evolution is never always in one direction. The time must come when the limit is reached, and the downward process must be commenced. I desire to discuss briefly the reasons for the limitations of size from a slightly different angle from that adopted by the speakers.

Beginning with mechanical stresses, as we encounter them in the electrical machinery or in rotating machinery like steam turbines, it must be emphasized that these are not static stresses, but essentially dynamic stresses.

Stresses in bridge members are both static stresses and dynamic stresses, according as the load of the bridge is a static load or a dynamic load. A live load, or a moving load, produces variable or dynamic stresses. In the rotating elements of the steam turbine, and of the electric generator connected thereto, the stresses are usually vibratory stresses changing both in magnitude and in sign. The shaft is bent in such a manner that at one period of the motion it is bent in one direction, and at another period it is bent in the opposite direction, and therefore the material and its molecular structure are subjected to alternating stresses. The same applies to other parts. The ancient rules of the mechanics as they are taught unfortunately even at this day do not apply to the problems we are facing. We must view the theory of elasticity from the point of view of vibratory stresses. Mr. Berg has told us that the disks in the steam turbines may have perfectly safe static stresses, and yet they may be entirely unsafe because of vibration set up in the disk. The maximum stress in a disk, subject to rapid rotation, exists in the center. In any ordinary test bar cut out of a plate, the test bar being shaped as test bars usually are, if a hole is made in it, the maximum stress at the inner surface of this hole is in general twice the ordinary stress which appears in the section. Suppose we call this ordinary stress 10,000 lb., the maximum stress as it appears on the inner surface will be approximately 20,000 lb., but this stress will be a serious stress only if the ductility of the material is small. If the ductility of the material is high, then the stress will be distributed, and the material will flow and rupture will not occur as a result of the initial excess stress caused by the hole in the test bar.

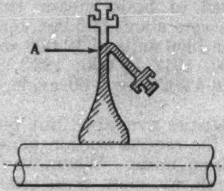


Fig. 1

If two test bars are made of equal and high ductility, one containing a small hole one-thirty-second inch diameter, and the other having no test hole at all, both bars will rupture the same load. The same is not true with bars made of hard steel lacking ductility.

I see little or no mention made of the fact that hard materials must be avoided. The ordinary tempered steel referred to for thin disks is very hard steel, and I doubt if such steel in the form of a disk could be bent flat on itself, into the position Fig. 1, without showing any seams or rupture whatsoever at the point *A*.

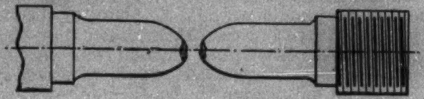
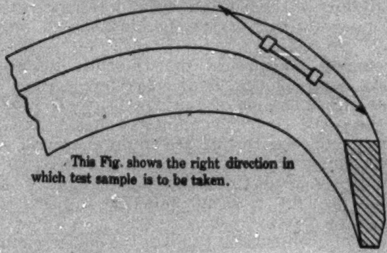


FIG. 2

The elastic limit is, after all, only one criterion of the quality of material. Material for these disks must be ductile and must bend flat on itself, cold, without rupture. If it does show rupture, you may be fairly secure in your assumption that that material is unsuited for the use to which it is put, namely, resistance to stresses reversing in magnitude and sign. All fractures with which I have been familiar have been due to vibratory stresses. I know of no failure of any part of a machine occurring under a pure static stress. There may be such failures, but they are unknown to me.



This Fig. shows the right direction in which test sample is to be taken.

FIG. 3

Reference to the limitation of capacity being due to the turbine only is, in my opinion, incorrect.

The rotating end bells are stressed to such a point that it became necessary to make them as thick as possible so as to reduce these stresses to a point where a satisfactory material can be obtained. The stresses run up to 30,000 lb. per sq. in. under normal conditions, and the ultimate stress of the material is approximately 100,000 lb., with a ductility measured by an elongation of 20 per cent and a reduction of area of 40 per cent in a 2 in. sample. The test bar in breaking must show a fracture like Fig. 2., and making these end bells is no small task, as they are subject to tangential stresses, so that the work of forging has to be done in such a manner that the material is homogeneous tangentially, Fig. 3. A test bar taken radially would show a very greatly reduced "strength" in the sense in which I am using that term, though the elastic limit might be the same, yet the ductility would be less, and therefore the "strength" would be less, so that you see that the term "strength" now means something different from what it meant thirty years ago, as it now includes ductility, whereas twenty years ago it meant nothing but the elastic limit, which is something quite different. These end bells, if thickened too much will make it impossible to put the rotor into the stationary element and therefore a limitation of construction lies in the length of the air gap.

These machines have not been in operation long enough for us to come to a definite conclusion as to the life of the structure. Fatigue may not become apparent in ten years, and yet later fractures may occur, due to fatigue. You may say, as Professor Scott likes to say, "You are pessimistic, as usual." I do not say this is going to happen. I say that vibratory stresses are dangerous, and that we have not had sufficient experience in regard to vibratory stresses to understand their effects thoroughly. It is to be hoped that the societies interested in testing materials will make tests with vibratory stresses, as our mills will not now accept specifications based on vibratory tests.

These remarks tend to show that there are limitations in the design of the electric generator and therefore the assertion made tonight that the limitation of units is entirely a question of the design of the steam turbine cannot be upheld.

To sum up my remarks, we are confronted with great engineering problems the ignoring of which will do us no good, but the bold and courageous facing of which, with all the knowledge that can be brought to bear upon the subject from every possible angle, will lead to success.

The course of evolution, as I said at the outset, cannot always be upward. The size of the machine, as far as cost is concerned, decreases to a certain point, and beyond that point it increases. Another point of importance is the increase in the armature current in machines of 50,000 kv-a. or over, which makes the problem of reducing the heating of the stator coils a difficult one. The loss which is troublesome to the designer is the loss produced in the stator coils as the result of eddy currents. That trouble increases with the increase in the cross sectional area of the coils. It may become necessary to increase the standard voltages now in use in order to obtain smaller conductors in which it is possible to reduce the eddy current losses to such an extent as to make the machines safe as regards coil heating.

W. L. R. Emmet: I have always thought it a strange condition that construction so radically different as the two types of turbines present in these papers should co-exist, but an analysis and examination of these papers, gives a good explanation of it. Both machines described show the evidence of highly scientific study, and the justification for either might be expected to be found in the results which they have produced.

These papers do not give direct comparisons. Mr. Berg's paper simply seeks to show the limitations imposed by rating up of machines. It gives a condition which corresponds to the highest economy obtainable with a machine of that type, and then shows the consequences of modifications of design or of increase of rating. Mr. Johnson's paper shows the general characteristics of limit of design, and gives a very good analysis of all the strains and conditions existing in the turbine, and a statement covering the whole field rather more particularly than Mr. Berg, who simply speaks of two specific turbines.

The inherent differences between the impulse and the reaction turbine might be stated as follows, as I understand them—of course, I understand the impulse turbine very thoroughly, whereas all of the limitations of the reaction turbine I do not understand, although I have studied it to some extent, generally speaking, in certain ways the less work you do in a single operation in a turbine, the more perfect the operation; that is, if you put a small amount of work into a stage of the impulse turbine, you can get rather more perfect nozzle action, and a rather better performance. The Parsons type turbine gives this condition, and in that respect is very good. It has, however, the disadvantage that it is more affected by leakage, and that the action of the stationary parts on account of leakage and other conditions, is presumably less definite, whereas in the impulse turbine, the leakage is very small, and the action of the stationary part, by suitable proportioning and using ample space, can be made very efficient. Thus by a good proportioning of the stationary part, quite high steam velocities may be justified, although in this type, also, the use of high steam velocities and the concentration of a large amount of work in a single stage, is not without its relative disadvantages.

Something has been said concerning these papers relating to limits of capacity, and I am rather inclined to agree with Mr. Behrend that there is no particular reason for building very large turbines; that is, there has been a good deal of demand for very large turbines, but the justification for them is not very apparent to me. With turbines of the type we build, such as described in Mr. Berg's paper, there is a distinct disadvantage in going to large sizes involving low speeds. On account of the great diameter of the wheels, there is a waste of space involved and practical difficulties in diaphragm construction and the physical structure of the stator becomes excessively large and is rather objectionable. The large double-unit machine which has been built by the Westinghouse Company in which the low-pressure element operates at a low speed is a splendid machine, but these machines are quite large and expensive, and there seems to be some question as to whether two entirely independent units, each one complete in itself, may not be about as good a solution of the problem, as far as the user is concerned. The effects of congestion in the low-pressure stages which is a thing which chiefly governs the rating and cost of turbines is a very important matter, but it is extremely hard to get at any numerical comparisons, and people must judge about the relative values from experience or from the guarantees which are offered, and by experience in use.

I can see a certain disadvantage in running very high bucket speeds in a machine of the reaction type, on account of concentration of a large amount of work in a single row of blades, discrepancies in the areas of successive blades, and such things as that in the low-pressure end, but I cannot say, quantitatively, what these limitations may be, so I really do not know very well what the economic limitations are of such turbines as some of those which have been described. In our machines, I do know very well what these limitations are, and Mr. Berg's paper has tried to make the whole matter perfectly clear.

If I have understood or observed the practise of Mr. Parsons himself, he has not gone in for very high bucket speeds in the reaction turbines. The construction now used by the Westinghouse Company from the mechanical standpoint seems to be conservative and good, but what its efficient limitations may be, I do not very well know. I cannot altogether agree with Mr. Behrend's generalizations on the subject of vibrating and static strains. If we could not calculate upon or predict the nature of vibrating strains, we could not build anything because almost everything is subject to motion. However, there are pretty definite laws which govern the question as to whether things do or do not vibrate. One of the essentials of sympathetic vibration is the rather perfect state of elasticity, and many structures which are used in motion, are very far from that perfection of elasticity, which will enable them to vibrate synchronously with any periodic force. As an example of that, I might refer to an armature mounted on a shaft—the shaft may be regarded as an elastic structure loaded; but in point of fact what it is is an elastic structure, the shaft upon which a very inelastic load of material is attached; that is the laminations are upon it, and these laminations make it relatively inelastic. In the same way these rings or end bells described here are loaded by masses of coils giving very high weight, in proportion to the elastic strain on the ring, and also this mass on it is, in itself, giving a condition particularly favorable to dead running, so I would really, without hesitation, put a strain on such rings to the highest possible degree, without any fear of their being broken by vibration.

As to this question of periodic motion, it is the cause in my opinion of the breaking of certain wheels in the General Electric turbines, and some of the possibilities of that situation were beautifully illustrated the other day in Schenectady. We made up out of rubber two or three wheels some rather thick with heavy edges, and some thinner with light edges. These edges were loaded with little staples hooked through them in a manner equivalent to the load imposed by turbine buckets on wheels, and the wheels marked out like a checkerboard. Holes were put in the wheels, they were then revolved on a shaft, and the shaft was made to actuate a high potential spark at one point of its revolution, so that the result of this, when running in the dark, was that this wheel apparently stood still, with an exact and perfect definition of its markings. The dry skin on the surface of the rubber was even visible upon close examination, and as the wheel loaded up, you could see the skin on the surface of the rubber crack, and draw into various forms, indicating the various distribution of strains around the holes, and the lines we drew on the wheels were distorted.

This gave us a beautiful illustration of the nature of centrifugal strains in such a wheel. That is what we did it for, and did not expect to show anything else. But in the case of the lighter wheel when we came to run it up to a given speed, we began to see something else, and that something was this—that the edge of the stationary wheel slowly began to work around like a snake, crawling slowly through the grass, that is, it would have three or four points in it that would go worming around, and as you speeded up the wheel, the snake would travel faster, but not in proportion to the increase in the speed of the wheel, showing the state of elastic distension in this thing had some tendency to equalize itself through a periodic action, which was not an accident.

With the stiffer wheels we had, although they were all very limber, made of rubber, and just a little bit thicker at the edge, we could not produce that effect. We could not run them at a speed so they would stay on the shaft, and at the same time be subject to this motion.

Generally speaking, all tendencies to vibration, and particularly all tendencies to injury through vibration, diminish very rapidly as the object is made stiffer, because the local strain produced by a certain amplitude of vibration is simply a question of the degree of elongation of the fibres on the surface. With small amplitude of motion, the degree of elongation is very minute, and consequently the strain imposed is very little. Movements of such character have unquestionably been the principal cause of the very serious troubles which have recently been experienced with wheels and buckets of General Electric turbines of the newer types.

The lightening of General Electric turbine wheels which has made such trouble possible has been incident to the use of a very light method of bucket arrangement which made the rim of much less weight, and made a great lightening of the web possible from considerations of centrifugal force alone. Lack of stiffness in the rim as well as in the web conduce to this tendency to motion. These rubber wheels showed that, because it was only with the light rim that such motion was produced.

Mr. Berg stated in his paper that the generators can be made up to the limits of the possibilities of the turbine. That applies, of course, to a turbine of a type which he speaks of, but if you made a double-flow turbine of the same speed, you would soon get beyond the capacity of the generator, and I personally do, not think that there is very much practical advantage in trying to build a turbine up to the capacity of the generator in that way. A double flow turbine of the same type could be made of twice the capacity, but it would not be any more efficient, and would be quite complicated and large, and you might find it necessary to have two generators, and probably would, and I think it is better to keep to a reasonable size, and have independent turbine units.

W. J. Foster: Mr. Newbury has very kindly asked me to tell you of the largest generators which have been built by the General Electric Company, and I will begin by telling you that about five years ago there were three 30,000-kw., 25-cycle generators put into operation. These were 25-cycle generators at 1500 rev. per min. Since that time we have put into service several 35,000-kw., 25-cycle generators, and one of approximately 39,000-kv-a. at 90 per cent power factor, now operating for one year.

In the 60-cycle line, we have several 6250-kv-a. 80 per cent power factor, 3600-rev., and one 7500-kv-a., 6000-kw., 80 per cent power factor, that has been in operation about one year, and we are building 3600-rev. per. min. generators up to 9375-kv-a. at 80 per cent power factor.

In the 1800 rev. per min. we have quite a number of 31,250-kv-a., 80 per cent power factor and some 33,333-kv-a. at 90 per cent power factor, and we are building 40,000-kv-a. at a power factor of about 85 per cent.

In the 1200 rev. per min. we have had a 50,000-kv-a. in operation about one year.

Mr. Newbury's curve, which I think is an excellent one, and is scientifically correct, is based apparently on 80 per cent power factor. He makes a statement which agrees with my experience, that the limit is reached in the rotor, and not in the stator, with these high-speed generators. In the curve Mr. Newbury draws, if we remove the power factor restriction, and put it on a unity power factor basis, the capacity can be raised about 20 per cent. In my experience, there is no trouble with such temperatures as Mr. Newbury mentions in at once going to 20 per cent higher kv-a. output on that curve for any given speed. In other words, it has been my experience to design the generators for lower temperatures. They are in the 105 deg. and not in the 150 deg. class, and that is also true in the case of practically all of the 60-cycle generators which have been built by the General Electric Company up to date.

The question as to how lower temperatures may be obtained in the stator is one which Mr. Behrend touched on. I hardly know why he considers the matter of eddy currents so formidable. It may be impossible to design a machine without some eddy currents, but there is no need of having them in anything like such quantities as to imperil the life of the machine or to cause excessively high temperatures.

There is a point with reference to the voltage—when it comes to the very large sizes—the further up we go in capacity, the higher the voltage must necessarily become. If we should build 90,000- or 100,000-kv-a. generators, it would be difficult to build them in less than the highest voltages at which we are now building generators. I think it is possible with 1200 rev. at the present moment, by new equipment, but not making such a radical change as we made six or eight years ago, when we stepped up to 30,000 kv-a., to build 90,000-kv-a. 1200-rev. machines.

In Mr. Newbury's paper reference is made to the ventilation, —axial and radial. I claim that the radial ventilation presents greater possibilities in furnishing larger surfaces for the cooling media to remove heat from the parts where it is generated, from the iron and from the windings. The coils are exposed at short intervals, and thus large areas of them exposed to the cooling air. In addition large surfaces of the core itself are exposed so that heat removal is far more effective than in anything I can conceive of as attainable in axial ventilation. The holes in the core that are put there for the air to pass axially, cannot be right in the teeth, to any great extent. They cannot be very close behind the teeth, except at a considerable loss in efficiency. The area of the walls of the holes must be small in comparison with the area which surrounds the numerous small sections in the radial ventilation.

I remember in one of our 31,250-kv-a., 11,000-volt machines, where a careful analysis was made of temperature drops from the inside of the coils, the temperature drop from inside the coil to the outer surface of the insulation or through the insulation, was between 30 and 31 deg. The drop from the coil surface through the short space of laminations was 4 deg., and from that surface to the air was 12 deg., even at the low rate of flow we had at that time.

A statement was made, I think, in the paper, that in the case of radial ventilation, the area of entrance at the two ends of the machine is limited. That is true. But we are discussing now what has been done, and what is possible, and I wish to point out that several years ago, the first General Electric machine embodying the multiple flow of air in a strictly radially ventilated core was installed. These machines have been tested out with the ventilation both ways—the air passing in simply at the two ends of the machine, and compared with air passing in with multiple paths, and the result has been a decidedly lower temperature with the air in multiple paths. That idea can be carried further, and so the possibilities of running at lower temperatures are many.

I do not see at all that it is necessary to have the stator as a limiting feature in the low-speed machines. If I were drawing a curve for Mr. Newbury, a practise curve, I should be inclined to put the heavy line on the other side of the dotted line, but I agree with him that in the lower speed machines, the chances for lower temperatures in the rotor are better, inasmuch as with the larger diameters it will be possible to ventilate the rotor to a certain extent which is not practicable in very small diameters.

Mr. Newbury makes a reference to periodicity, and he speaks of the advantage of 50 cycles over 60 cycles. I agree with him.

Alexander M. Gray: As to the theoretical limits of single-shaft turbo alternators discussed by Mr. Newbury, such alternators have been built with an output of 45,000 kv-a. at 1800 rev. per min. and one naturally wonders what is the maximum output that may be obtained at this speed.

The rotor diameter is not limited by the disk stress but rather by the hoop stress in the end connection retaining rings. In the case of a 51-in. machine rotating at 1800 rev. per min. the peripheral velocity at 24,000 ft. per min., the stress in a coil supporting ring of this diameter due entirely to its own weight is 15,000 lb. per sq. in., and this ring must also support the end connections.

The maximum rotor excitation. For a given stress in the rotor teeth there is a certain slot depth which gives the largest slot area. The number of ampere conductors that can be placed in this slot is limited by the internal temperature and a considerable portion of the temperature drop is through the rotor slot insulation. This drop can be kept reasonably low by the use of a large number of rotor slots, so that there is a large surface through which the heat may pass. It has been shown elsewhere that with a tooth stress of 14,000 lb. per sq. in. and 100 deg. cent. temperature rise above the entering air, the maximum excitation is about 1000 ampere turns per pole per inch of rotor diameter, or 51,000 ampere turns per pole, for a 51-in. machine.

The maximum stator excitation: In the stator m. m. f. with full-load current is equal to the maximum rotor excitation, then the demagnetizing effect is such that on low power factor the generated e. m. f. is practically zero, and the regulation of the machine exceedingly poor. Machines are now built with the armature ampere turns per pole as large as 0.6 (maximum rotor excitation). In the case of the 51-in. machine, this value is 30,000 ampere turns per pole.

The generated electromotive force can readily be determined if the air gap flux density is known. The flux density is limited by that in the stator teeth and if we assume the stator tooth density to be 100,000 lines per sq. in., and the ratio of tooth to slot equal to 1.1, reasonable value for 13,000-volt insulation, then with a peripheral velocity of 24,000 ft. per min. the voltage induced in the conductor is 1.88 volts per in. of iron, and 65 per cent of this value per inch of machine. This gives along with the figures for armature excitation a value of about 450 kv-a. per in. of iron.

The axial length of rotor. This is limited by the alternating bending stresses in the journal, it being assumed that the bearing holds the journal horizontally. With 19-in. water-cooled bearings the rubbing velocity is 9000 ft. per min., and with such bearings the length of a 51-in. rotor between bearing faces is 310 in., if the bending stress is limited to 5000 lb. per sq. in. This stress, be it noted, is alternating.

With a length between bearing faces of 310 in., the effective rotor length will be about 230 in., and the output about 70,000 kv-a.

Summary. The maximum output of an 1800-rev. per min. single-shaft, turbo-alternator, is about	70,000 kv-a.
The rotor diameter	51 in.
The peripheral velocity of rotor	24,000 ft. per min.
The temperature rise of rotor conductors	100 deg. cent.
The armature m. m. f. at full load equals 0.6 of the maximum rotor excitation.	
The flux density in stator teeth	100,000 lines per sq. in.
The ratio of tooth to slot	1.1
The bearing diameter	19 in.
The rubbing velocity of bearing	9,000 ft. per min.
The axial length of the rotor and stator conductors will be about	240 in.

I do not suggest that the machine such as the above should be built. There will be undoubtedly manufacturing troubles due to the length of coils, and also trouble due to the movement between conductor and insulation, because of the increase of length with increase of temperature. It is possible to reduce the length of the machine and the various factors in the above summary are open to attack.

R. B. Williamson: I desire to discuss Mr. Newbury's paper on "Present Limits of Speed and Output of Single-Shaft Turbo-Generators." The various elements that go to limit the possible output of a turbo generator operating at a given speed have been well brought out by Mr. Newbury in the present paper. In this type of generator the mechanical considerations are equal to if not of greater importance than the electrical features and as the two are more or less antagonistic the design as a whole must be a compromise. In the higher speed machines the maximum possible output is at present limited by the rotor while in large slow-speed generator the stator becomes the limiting factor.

So far as the rotor body is concerned this is made of carbon steel and as pointed out, the peripheral speed of 400 ft. per sec. is as high as it is advisable to run on the higher speed machines. With lower rotational speed the peripheral velocity might, if necessary, be allowed to go higher than this so far as tooth stresses are concerned, because of the lower angular velocity which is of the most importance in determining the centrifugal stresses. It should also be remembered that higher peripheral speeds result in greatly increased skin friction loss. If rotor bodies could be made of alloy steel, the stresses might be pushed higher and larger limiting output thereby secured but this kind of steel when used in large masses is very liable to develop cracks and at present is not reliable for this purpose. Attempts to use this steel for large shafts in other classes of machinery have shown this to be the case. For the coil supporting rings at the ends of the rotor it is necessary to use alloy steel but here the cross-section is comparatively small and the metal is very thoroughly worked during the process of forging and expanding the ring from a solid piece. Test bars from these end rings almost invariably show uniform physical qualities. As indicated in the paper, these end rings may have an influence on the maximum possible output. Referring to Figs. 2 and 3, of the paper it will be noted that the rotor coils project straight out beyond the core. Unless the wedge used for retaining the coils in the rotor slot is made very deep, which is undesirable because of the loss of valuable copper space in the rotor slots, it follows that the outer diameters of the end rings will be greater than the outside diameter of the rotor body as shown in Figs. 1 and 2. The inside diameter of the stator laminations must be slightly larger than the outer diameter of the end rings so that the rotor can be slid into the stator, thus the minimum air gap may be fixed by the end rings which thereby indirectly affect the ampere turns to be supplied by the rotor and the possible limiting output for a given speed.

Referring to the radial system of ventilation, Fig. 2, the amount of air that can be passed through a stator is limited if the air is passed through the air gap because the cross section of the gap is necessarily limited. However, it is entirely practicable to use the radial system by introducing the air at the back of the laminations between the latter and the stator yoke or casing and blowing it radially inwards against the rotor. The air then reverses its direction of flow and passed out radially through the adjacent parts of the stator core. By this means any quantity of air required can be handled readily by providing a suitable number of parallel paths, and the air inlets at the back of the laminations can be made of ample area to keep the air velocities within reasonable limits. This arrangement also has the advantage of introducing cool air into the center of the machine where the parts normally attain highest temperature.

This method of ventilation has worked out successfully in machines having a ratio of stator length to inside diameter of 3 to 1 and the results obtained indicate that much longer stators could be ventilated in this way and still maintain the central portions within the allowable temperature limits.

Referring to the curve shown in Fig. 4 the ouptuts indicated are, as stated by the author, somewhat in advance of anything so far accomplished. For example at 3600 rev. per min. a possible output of 10,000 kv-a. is shown. Machines have been in operation for some time that have carried loads as high as 7500-8000 kv-a. at 3600 rev. per min. in regular commercial service. Of course these generators have long rotors, but we are of the opinion that an ultimate output of 10,000 kv-a. at 3600 rev. per min. could be obtained as indicated by Mr. Newbury without going beyond the limits of present materials provided such output should be considered desirable for the steam end of the unit.

Francis Hodgkinson: The limit of capacity of a steam turbine is exceedingly hard to discuss because of its elasticity. Why should not one of the turbines which have been described have their diameters slightly increased, and be provided with slightly greater length of blades when the turbine would have a corresponding increase in its capacity. To what extent such things may be done, and what should be the limits of stress, is largely determined by the judgment of the designer. This applies to the calculated stresses of centrifugal force, etc., but in addition to these there are the obscure vibratory stresses to which Mr. Behrend refers.

It is difficult to draw definite lines to which materials may be stressed when combined with indefinite vibratory stresses and temperature strains, the result of the inner and outer parts of the revolving limits being at different temperatures on a change of load or operating conditions. To do so is, after all, to determine what risks shall be taken. The designer is sometimes in an unfortunate position, in that commercial pressure is brought upon him to increase speeds and dimensions beyond what he considers best judgment, which at times is hard to resist.

Turbines have increased in capacity of late years in the manner Mr. Torchio has recited, and as far as my association with them has been concerned, I have always felt that every increase of size should be accompanied by an increase in the degree of reliability.

Mr. Behrend, in his discussion, touched on some real truths. Of course, he referred principally to the type of turbine built by competitors of the company I represent, but I venture to say that any of the turbine disks he described would stand the destructive bending tests which Mr. Behrend pointed out as desirable. I believe they would pass such a bending test perfectly well, but such material, ductile and strong though it may be, would still be unable to resist continued vibratory stresses, should they produce strains slightly above the elastic limit in any fibers. I do not say that such vibrations necessarily exist, but should they do so and any fibers be stressed beyond the elastic limit, the end will be disastrous.

The views of engineers have undergone some change of late years in that the physical characteristics of reduction of area is considered one of the most important features, for it is this physical characteristic which, when structures are locally strained beyond the elastic limit, produces a strengthening of that part and permits an evening up of the stresses throughout the structure.

Mr. Emmet referred to a power station in which single units were employed as compared with cross-compound units. I have personally favored these cross-compound units for the reason of simplicity. A turbine, working with high pressures at one end, and high vacuum at the other, calls for skill in arranging the distribution of the metal of the turbine structure so that expansion and contraction shall be uniform and no distortion result. Any simplification of the structure is, therefore, an advantage. By cutting the steam cycle in two, and so similarly dividing the temperature range, there is a direct simplification. Many people regard the cross-compound principle as a redundancy, because there are twice as many bearings. Bearings do not give any trouble. That does not mean anything, but the simplicity of the turbine structure means a great deal. There is a further advantage that different speeds may be employed for each of the elements, selecting that speed which is most appropriate for the steam volumes in the high-and low-pressure elements respectively. For example; for 60-cycle work the high-pressure turbines may operate at 1800 rev. per min., the lows at 1200; similarly for 25-cycle service the high may operate at 1500 rev. per min., the low at 750, although in certain cases two low-pressure turbines have been employed, all three elements operating at 1500 rev. per min. By means of such construction high stresses may be avoided and ordinary commercial materials employed.

B. G. Fernald: In Mr. Johnson's paper the importance of the reliability factor in turbine design is strongly emphasized.

The calculated stresses in his turbine rotor when operating at 20 per cent over normal speed is given at 20,000 lb. per sq. in. or when operating at normal speed at 13,900 lb. per sq. in.

The elastic limit of the material used, as determined by test rings taken from the billet close to the point of maximum stress is from 22,000 to 25,000 lb. per sq. in.

The material used is ordinary carbon steel and is not subjected to any heat treatment other than thorough annealing.

In Mr. Berg's paper the calculated stresses at normal speed in the rotor wheels of his turbine design are given as 23,450 lb. per sq. in. in the hub and 22,950 lb. per sq. in. in the web and he uses a nickel steel disk quenched and tempered and claims an elastic limit of 55,000 lb. per sq. in. No information is given as to the method of obtaining the test specimens or their location with respect to the finished wheel.

Now, on the face of it the element of safety would seem to be greater with the heat treated alloy steel disks of the G. E. design than with the annealed disks of the Westinghouse design, since the margin between the calculated stress and the elastic limit of the material is greater.

The reverse, however, would seem to be the case since Mr. Berg's report mentions several failures of their design and Mr. Johnson reports no failure of the Westinghouse design. From my own knowledge, I would not pretend to offer a definite opinion on the cause of the accident to the disk wheels or to question the accuracy of Mr. Berg's diagnosis of the trouble, but I submit that his explanation does not take into consideration the fact that the failure of the disk occurred where the calculated stresses were lowest but where the unknown internal stresses would probably be highest if a disk of substantially the same form shown in Fig. 3 were quenched by immersion.

I wonder if any one has had such experience as will enable him to tell us that we can heat treat a disk and know when we get through that the disk is free from internal shrinkage stresses?

Recently the Emergency Fleet Corporation placed a contract for eight twin-screw troop ships, each screw being driven by a cross-compound single-reduction geared turbine of 6,000 s. h. p. or 12,000 s. h. p. total for the ship.

The element of reliability was, of course, of extreme importance on a troop ship and overshadowed all other design stresses. However, before the vessels were under construction and the final contracts for the turbine let, the war was over, and it was decided to convert the vessels into passenger steamships.

While reliability was still of the utmost importance and we wanted, if possible, to secure turbine units which would be free from the troubles which had been so prevalent on the smaller cargo vessels, it was necessary that the design take into consideration high economy as well as reliability. It was realized that the turbine designers task in satisfying the dual considerations of reliability and economy would be simplified by using material permitting high unit stresses.

Very little exact information about the character of alloy steel which we would receive from the mills was available to us, but, we did know from general experience during the war that the mills were not making extra good material and it was difficult and unreasonable to attempt to force them to produce the large quantity required and hold them to close limits of quality. Furthermore, our inspection facilities were inadequate to competently handle such an exacting task and our experience had not warranted us in dispensing with all inspection so it was finally decided to limit the designers of the turbines to the use of ordinary commercial carbon steel with no heat treatment other than thorough annealing and all calculated stresses had to be based on and kept well within the elastic limit of such material.

We realized that we were open to criticism for being ultra-conservative, but, we could not secure information warranting us in taking any other course and we were particularly uncertain about the possibility of successfully heat treating alloy steel disks for turbine wheels.

While the information would be too late for use in the connection just mentioned, I would ask Mr. Berg to give us some information about the heat treatment of the disks of his design and the test methods used for determining that the elastic limit mentioned existed in the disk itself. It is recognized, of course, that it is not difficult to obtain by heat treatment an elastic limit of 55,000 lb. in a small bar of uniform section such as is used for test pieces.

B. A. Behrend: As to the points brought up by Mr. Emmet and Mr. Hodgkinson, I may point out that vibratory stresses need not be stresses due to resonance. Vibratory stresses are all stresses which change in direction. If resonance occurs, there will be more rapid destruction in a shorter time interval. Where there is no resonance, the time period is lengthened out before fatigue occurs.

Comfort A. Adams: Mr. Behrend's remarks in regard to the fatigue of metals are very interesting. It may cheer him to know that a very elaborate fatigue phenomena research is now being conducted at the University of Illinois by Prof. H. F. Moore under the auspices of the Engineering Division of the National Research Council. More than $20,000 per year is to be expended for at least two years, of which $15,000 per year is contributed by the Engineering Foundation.

Just a word in regard to the electrical end of this problem. Mr. Newbury pointed out the objection to using high temperatures and the very slight gain in output due to allowing excessive temperatures of the rotor conductors owing to their increased resistance. Is it not possible that the longitudinal expansion at high temperatures and the consequent slipping of the conductors through the insulation would constitute another limitation to temperature for any given length, or a limitation of length at the higher temperatures.

Farley Osgood: I have heard nothing said from the operators' standpoint in this matter of turbine design, the discussion, which has consumed virtually the entire evening, having been confined to the designers' arguments. It is the custom to separate the sheep from the goats, and it seems well that now we do so, in that as the discussion up to date has been held by the sheep, whereas nothing has been heard from the goats, namely, the users, who are in the very large majority, and who are the victims of the enthusiasm of the designers, as we all know. It gives me some consolation to hear less talk of 50,000- and 70,000-kw. units, the authors having confined their discussion to units approximating a 30,000 kw. limit in design.

From the standpoint of the operators there are some very important features which do not seem to have been touched on specifically this evening, and they might very properly be divided into four classes: 1. continuity of service; 2. capacity of unit out of service during periods of repair; 3. investment cost of units out of service or for spare; 4. length of time to make necessary housekeeping repairs to the units.

As to the continuity of service, I believe the designers now have sufficient experience to guarantee a very high percentage of continuous service of a unit if they do not permit their enthusiasm to take them into sizes which lead them into more or less uncertain, and certainly untried, fields of strains in the materials.

As to capacity, I think it is a matter for the decision of the operator, rather than the designer, to elect the percentage of his whole capacity which he can afford to have out of service at the time of regular or emergency repair, and the operators' enthusiasm for improved efficiency from large units should not lead him to a decision which may affect his whole service, when his large units are off the line.

The cost of these large units, as compared with their smaller relatives, should be carefully considered, not so much from the standpoint of their value when running, which question they answer themselves by their performance, but from the standpoint of idle investment when the units have to be off the line.

Finally, and most important of all, is the length of time to care for these large machines, and it seems to be clearly indicated by the papers that such machines require special tools, specially trained men, and usually factory work, when anything but the most minor repairs are made, and this certainly means long delays from every standpoint, so that a machine which can be cared for locally, either by the operating company's men, or local machine shops, has many advantages over a machine requiring specialists and special factory attention, all this from the time-out standpoint. I have said before on this floor that the manufacturers give birth to the children, and the operators raise them and, as most of us know what it means to raise children and take them through the various vicissitudes of early life, I think we can say unhesitatingly that the operating group are much more familiar with the behavior of the units turned out by the designers, than are the designers themselves. I am firmly convinced that the designers do not give enough attention to the units in operation, that they do not spend enough time on the properties of their several customers, to become intimately familiar with facts which should materially assist them in turning out a product which will be more satisfactory to both the maker and the user, and I trust that even in the rush of business, which seems to be upon everybody, the necessity of closer cooperation between the designers and the users will be so appreciated as to bring such a thing about.

I very much regret that there is no time for a detailed discussion from the operators' standpoint, as I notice a number of able operators here, who could add most usefully to the discussion of this most important subject and, in fact, I was not aware that the entire evening was to be given up to the details of design, but I take the liberty of expressing a hope that the Chairman of the meeting at some time may have another meeting on this very important subject, at which the operators and the designers may be brought together and at that time the operators may be permitted to do the shooting instead of the designers, as has been the case this evening.

Eskil Berg: I believe that Mr. Emmet has pretty nearly covered all the questions that were asked. However I would like to point out that the wheel discussed in my paper is a very conservative wheel, the elastic limit being 55,000 lb. which is as low we ever found in actual test. The great majority of wheels test 65,000 lb. or higher. We have, as a matter of fact, many wheels which are 100 inches in diameter running successfully at 1800 rev. per min.

With this wheel given, the best possible design is described, which under the steam conditions stated, is shown to be most economical at about 21,000 kw. It is also shown that by reducing the number of wheels the light load efficiency can be improved by a slight sacrifice of economy at higher loads, in which case the best point would be at about 26,000 kw.

F. D. Newbury: Mr. Foster, in his discussion, advocates radial ventilation in preference to axial ventilation. Up to the present time neither system has reached its limit of development nor become a bar to further increase in rating. It has been necessary, however, in the case of the radial system to employ a four-path design, as described by Mr. Foster, for the larger generators while with the axial system the same conditions have been successfully met with a two-path design, that is, all of the cooling air is introduced from the two ends of the core. The four-path design can be employed with the axial system with no greater complication than in the radial, so that the axial system possesses greater possibilities than does the radial. This is, of course, due to the fact that in the two-path radial system the air-gap annulus is the only air entrance while in the two-path axial system this is supplemented by the parallel axial ducts.

As to the intrinsic merits of the two systems I cannot agree with Mr. Foster that the radial system possesses any material advantage over the axial. The Westinghouse Company has used both systems extensively and has, furthermore, built experimental generators of 5000 kv-a., 3600 rev. per min. and 10,000 kv-a., 1800 rev. per min. with the type of stator ventilation described by Mr. Williamson. These experimental generators were rebuilt one with radial ventilation and one with axial ventilation so that directly comparable results were obtained. Our practise, therefore, is based on practical experience with all three systems of ventilation in use in this country and we can say that each of the three has its advantages and its defects. We prefer the axial system in those cases where the simple two-path radial system cannot be used. Contrary to Mr. Foster's opinion it is possible to obtain ample core cooling surface with axial ducts and the fact that the axial ducts are not actually in the tooth belt is compensated for by the fact that the heat flow is in the direction of the plane of the laminations instead of across the laminations, thereby making use of a conducting path having as much as ten times the heat—conductivity of the transverse path.

Mr. Foster questions the importance of armature coil eddy current losses in these large generators. This is not a serious problem in 25-cycle units but in 60-cycle generators eddy-current losses become an important factor limiting generator ratings. In 30,000 kv-a. 1800 rev. per min. 13,000-volt units designers are now using such large slot sizes—in some cases 6½ and 7 in. deep—that the attempt to increase the stator rating, or to reduce copper temperatures, by increasing the slot depth increases the total copper loss, on account of increase in eddies, in spite of the decrease in I^2R loss and increase in coil surface.

I wish to take issue with Mr. Foster's statement that these large 60-cycle generators of 30,000 and 40,000 kv- a. capacity can be designed with actual copper temperatures within 105 deg. total temperature based on 40 deg. air. This is too large a question to discuss adequately here but it is a very important one and I hope papers on this subject may be presented before the Institute in the near future.

Professor Gray gives certain figures representing limits for 1800 rev. per min. generators. These limiting figures are well in accord with practise except in one important particular. The permissible length between bearings is given as 310 in. based on the reversing stress in the neck of the journal. Critical speed, rather than stress directly, is the most important factor determining the maximum rotor length; with the proportions reached by Professor Gray the first critical speed is about two-thirds of the running speed (in coming up to speed the rotor must pass through its critical speed) and the second critical speed is only a little above the running speed. These are not conditions that would be considered permissible in a large and important unit. The largest 1800 rev. per min. rotors so far built are only half the length arrived at by Professor Gray and the prospect of any considerable increase in core length in one step must be approached with circumspection.

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APPLICABILITY OF AUTOMATIC SWITCHING TO ALL CLASSES OF TELEPHONE SERVICE

BY ARTHUR BESSEY SMITH

ABSTRACT OF PAPER

This paper is an attempt to place before engineers general information in regard to automatic telephone switching.

The subscriber's requirements are independent of the means used to satisfy them. Automatic switching is uniformly fast and involves reduced mental stress to the user because the passing of the number is positive and waiting time is reduced to the minimum. Viewed by the owner the apparatus has longer life, the service is very acceptable to the public, and but a tithe of the female employees are used.

Automatic switching apparatus has increased greatly in margin of safety. Much progress has been made toward standardization of form and toward best methods of maintenance. A few changes in structure are described and data given to show margins of safety in operation. Present practise regarding party lines and measured service (cash and credit) is stated briefly. The durability of automatic equipment is illustrated by the fact that plants have not yet worn out. Maintenance routines are essential to successful operation. Girls do routing testing with marked success.

Rural telephone lines present problems which have been solved in several ways, influenced by the greater number of telephones per line, the conditions of signalling subscribers, and the inferior insulation often encountered. Code ringing can be retained. Rural automatic service is only a little inferior to city automatic service.

The community automatic exchange serves a small group of subscribers, either isolated or part of a telephone network. The rotary line switch is used because it is simple, reliable, quick-acting, and provides 25 trunks. Eight variable factors in exchange design are presented with a discussion of each.

Toll switching in an automatic exchange gives the toll operator direct dialing to the subscriber, complete control over his line, and periodic ringing. The toll network has also been improved by applying automatic switches to intermediate points, so that the originating toll operator can set up the complete connection herself. The experience of the past twelve years shows that this increases the business-carrying capacity of toll lines at least 50 per cent to 100 per cent. A variety of schemes are available.

The automatic switching of telephone lines is adaptable to all classes of telephone service and offers a flexible means of solving problems.

ENGINEERS and others interested in the building of telephone plants often ask the question, "Will the automatic telephone system meet the needs of this particular case?

Though it may be successful elsewhere, perhaps it can not be adapted to suit the peculiar conditions found here?" To answer such questions as this is the motive underlying the preparation of this paper. Though the writer can not cover every conceivable condition, he believes that it is possible to place on record many facts which will be useful to those who have to do with automatic telephony.

The approach to automatic telephony must be made with an open mind. A highly critical attitude prevents one from appreciating such facts as are observed. The writer formerly regarded automatic switching as desirable but somewhat inflexible. His experience and observation have led him to revise his opinion. Automatic switching can be adapted to meet every need of public and private telephone service, and will do it with marked advantages.

Three kinds of service have already been discussed by the Institute. In 1908, Mr. W. Lee Campbell presented a paper entitled "A Study of Multi-Office Automatic Switchboard Telephone Systems," and again in 1910 a paper on "A Modern Automatic Telephone Apparatus." In 1912 Mr. Gerald Deakin presented "Private Automatic Exchanges in Apartment Houses." In 1910 the writer discussed "The Automatic Telephone in Relation to City Service," including suburban traffic.

The classes of telephone service which are to be treated in this paper are as follows:

The single office exchange, the basis of discussion.
Rural lines.
The Community exchange.
Toll or county line network.
Long distance toll lines.
The comprehensive system, including all kinds of service.

GENERAL CONSIDERATIONS

Before taking up the details of various classes of service, it is well to call attention to a few general factors, such as the requirements of subscribers and of the owners of telephone exchanges and of the characteristics of automatic equipment most generally used.

Subscriber's Requirements. The subscriber's requirements must be stated in terms which are independent of apparatus or methods. Much error will be avoided if we divest our minds of conditions imposed by any one means of rendering service.

The subscriber requires, without unnecessary delay, without undue stress on himself, to be connected to the telephone which he desires (and to no other), to talk to the called subscriber with ease and without interruption or eavesdropping, and to have this service available continuously. Stated concisely the requirements are

1. Speed of connection (and disconnection)
2. Ease of obtaining connection (and disconnection).
3. Accuracy of connection.
4. Voice transmission.
5. Secrecy.
6. Continuous service.

Repeated tests of a formal nature and the general experience of users of both manual and automatic equipment have settled conclusively that the latter is very much superior to the former in speed of connection. But uniformity has value as well as actual speed itself. The average automatic connection is completed in from 4 to 6 seconds, regardless of the time of day or the conditions of business. Subscribers judge the speed of a service somewhat by the average speed, but more especially by the occasions on which they suffer delay. The fact that automatic service is *uniformly* fast, greatly increases its value to the user.

The speed of connection is greatly influenced by previous conditions. If a connection exists, and one of the two subscribers desires immediately to make another call, it can be done most quickly with the automatic telephone. Hanging the receiver upon the hook for one second clears the line and permits the immediate originating of another call. A large amount of telephone business may be transacted in a short time by automatic equipment.

The stress laid upon the subscriber in originating a telephone call is mental rather than physical: It may be divided into two causes, the waiting periods and the transmission of the number of the called telephone.

With the automatic telephone, waiting is reduced to a very low factor. Things begin to happen the instant the receiver is removed from the hook. The subscriber has something to do from the very start, and when he gets through doing it, the connection has been completed and the bell of the called telephone is beginning to ring. The only waiting period is the time of ringing. Even during this time the subscriber's mind is more or less occupied by the sound of the ringing, which he can hear. Or if the called line is busy, he is immediately informed of that fact by the unmistakable "busy tone." Not only does the automatic telephone greatly reduce the waiting time, but to a certain extent it conceals the total time required.

The transmission of a directory number by the dial of the calling device produces less strain on the user than the effective transmission of the same number by the voice. It requires a distinct effort to enunciate the syllables of the various digits clearly enough to prevent mistakes. On the contrary, the sending of the number by a calling device is easy and involves no appreciable stress. For each digit, the finger is inserted in the proper hole, the dial rotated to the finger stop and let go free. The mechanical definiteness of number transmission makes automatic calling easy.

When a subscriber desires to talk to a certain telephone, he wants that telephone and no other. It is a distinct advantage to him to know that he is getting exactly that number. This is afforded by automatic switches to a much greater degree than by human beings. Automatic switches, as now made, have a large factor of safety against error. They are designed and constructed so as to be as permanent in structure as modern materials will permit. Each part is separately adjusted within limits which impose rigorous conditions. The final assembly is again tested under conditions far more severe than those found in service. The chance of getting a wrong number by fault of the apparatus is so small that it can be entirely neglected. This fact is soon discovered by habitual users.

There is nothing inherent to automatic telephone equipment to prevent the very best voice transmission.

Much has been said for and against the value of the secrecy of the automatic telephone. It still remains true that users of the system recognize and value this property. It has been said that the operator at a manual switchboard is too busy to listen to conversations—this is true only during peak loads. Some manual switchboards have "automatic listening", which excludes the operator as soon as her work is done—this prevents her from giving the connection any supervision, and defeats part of the fancied advantage of having a human intermediary. It has been said that the mechanicians employed in automatic switchrooms constitute as much menace to secrecy as the manual operator. This is not so because the man's work is laid out according to a fixed routine. He is not permitted to run about the room according to the dictates of his own judgment. What he shall do, when and how he shall do it are laid out on a schedule. His day is uniformly filled, without peak loads. Secrecy is even more evident in the private automatic exchange, where the personal interest between people is stronger and the danger of eavesdropping greater. These switchboards are without constant attendance; they run for long periods without adjustment or repair.

Absolutely continuous telephone service has come to be the standard. Continuous service is given. But it is well known that manual night service is not as prompt or as good as day service. Automatic switches work just as promptly and accurately at midnight as at noon. The public deserve as good night service as day service, for often the need of a single night call may be more acute than that of many day calls.

Requirements of Operating Company. The requirements of the operating company are determined by viewing the telephone exchange as an investment. It must yield an adequate income to the stockholders. It must be a permanent business. To be stable means that it must render satisfactory service to the subscriber and not be too difficult to manage. These requirements are not all that there are, but they have an important bearing on the business.

1. Earning ability of investment.
2. Permanence of investment.
3. Satisfactory service to subscriber.
4. Ease of management.

Telephone exchanges must pay their way. If automatic exchanges do not pay, there would not be any of them left in existence today, for this movement is twenty-seven years old.

Five exchanges, formerly manual, were converted to automatic. They ranged in size from 2300 lines and 4400 telephones to 38,000 lines and 44,000 telephones. The totals for the five were 47,000 lines and 65,000 telephones. The comparison of expenses is as follows:

	MANUAL	AUTOMATIC
Total operating and maintenance expense per year	$552,600.00	$136,800.00
Average per line per year	11.77	2.92
Average per telephone per year	8.50	2.11

At least ten of the exchanges in the United States are from fourteen to sixteen years old. Based upon the record of the existing plants and the improvements in use today, new exchanges are built every year.

The permanence of an investment depends upon the rapidity with which the equipment is destroyed, the danger of its being rendered obsolete by reason of advances in the art, and the attitude of that court of last appeal, public support.

An apparatus to be a permanent investment as to depreciation, must be capable of being kept in first class running order by ordinary maintenance, without stretching the meaning of the word.

Surely any equipment which in ten years becomes so badly worn that it can not give adequate service, or that the repairs are considered excessive, so that it must be thrown out bodily, can not be considered as being very permanent.

The life of an automatic switchboard is not known. Numerous switchboards now in operation were installed more than 15 years ago, have been in constant service since then, and are still giving the same grade of service. All the moving parts are subject to no other forces than those of the switch itself. Those forces are adequate and are always properly directed every time they act. Those parts which wear are capable of easy renewal and are taken care of by ordinary maintenance.

Although the structure of automatic telephone apparatus has been changing through a considerable period of years, most of the changes are beyond the observance of the subscriber. The oldest equipment gives nearly as quick and satisfactory service as the latest. The telephone instrument has been improved in appearance, but the old ones are still giving a service comparable to that rendered by the new. For this reason, there is little obsolescence to automatic equipment. This being true of old apparatus, it is much more true of the latest apparatus on the market.

Public support is a factor which must receive adequate consideration. Though in the past some have ignored it, the results have been anything but pleasant and have reacted unfavorably upon those who did the wrong. Any business to be classed as permanent, must have with it public support, and the more of it the better.

The public recognizes and supports the company which gives its patrons the benefits of improved service obtained from new devices. They may not clamor for something which is rather hazy or of which they do not know. But let them once experience the benefits due to the new device and their support is assured.

It is a matter of record that the great mass of the public who have habitually used both services, prefer automatic to manual. It is not a doubtful majority—the preference is as nearly unanimous as can be found. In 1911 the Automatic Electric Company sent out 3000 letters to subscribers who had been using both manual and automatic service, and 1869 replies were received of which 91.7 per cent unqualifiedly preferred automatic service. Since then others have investigated the subject with varying results, but always with the ratio in the same direction and very positive.

Other things being equal, people prefer an investment which is easy to manage. One which presents troublesome questions which seem never to come to an end is shunned, because one can never tell when something may upset the financial condition. Any factor which makes the business easier to manage, renders the investment more attractive.

The manual telephone operator is the source of exceedingly vexatious problems. She requires careful handling, adequate accessory conditions, and although her pay is rising from year to year she does not stay long in the business. At best, hers is a nerve trying occupation, and it is no wonder that conditions are as they are. They are inherent.

The automatic telephone switchboard removes about 90 per cent of the female help in central offices. The increase in male help is only about 9 per cent. The duties of the former human operators are performed by machines which require no "managing."

EQUIPMENT CHARACTERISTICS

Automatic apparatus of various kinds has been described so often that a general explanation is unnecessary. The type best known has been the subject of Institute papers, as well as of a few books.

Factor of safety is a thing of great importance. Upon it depend the maintenance expense and the dependability of the service rendered to the public. This is the principle which guided the development of automatic switching. Seemingly small details exert great influence upon the performance of apparatus. Some factors do not impress the casual observer, yet they have great influence upon results.

A brief presentation of some of the improvements which have been made in recent years will bring the record of the subject up to date. It concerns chiefly the redesign of parts of the apparatus and the standardization of the assembly, adjustment, and mode of maintenance.

The automatic switch, Fig. 1, presents a new general appearance. The relays are mounted above, the magnet mechanism in the middle, and the banks and wipers below. The relays and magnets are protected by a cover individual to each switch.

The material of the switch frame has been changed from die-cast metal to cast iron, heavily zinc plated. The die-cast frame was subject to warping. It necessitated rather frequent re-adjustment of the mechanisms mounted on it and sometimes even the chipping down of a lug which interfered with the movement of the wiper-shaft. The cast iron frame is rigid and unchanging. Apparatus mounted on it requires no attention from this cause.

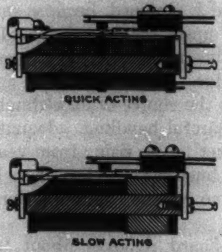


FIG. 2—HORIZONTAL AUTOMATIC RELAYS

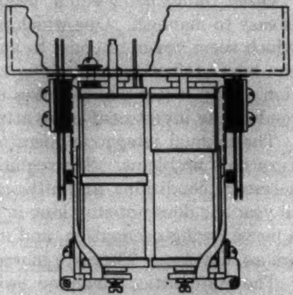


FIG. 3—AUTOMATIC RELAY MOUNTING (TOP VIEW)

The sideswitch has been abolished on nearly all switches and its work performed by relays. The sideswitch as made was the source of considerable trouble due chiefly to the wiper friction. Its elimination has greatly increased the certainty of action of the switch as a whole.

The relay has been completely redesigned (Fig. 2). The pin-pivot has replaced the point-pivot, the insulation between contact springs made permanent, and each relay rendered mechanically independent of all other relays.

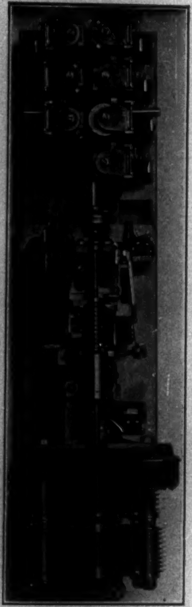


FIG. 1A—CONNECTOR SWITCH
[SMITH] (FRONT VIEW)

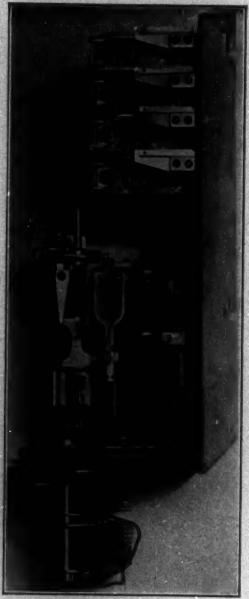


FIG. 1B—CONNECTOR SWITCH
(RIGHT VIEW)

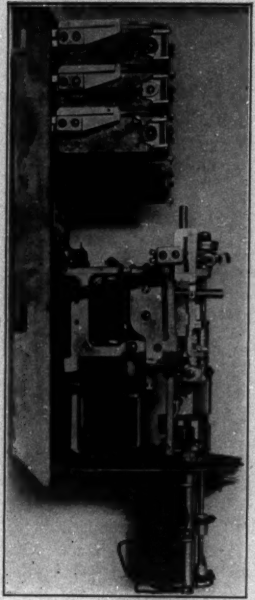


FIG. 1C—CONNECTOR SWITCH
(LEFT VIEW)

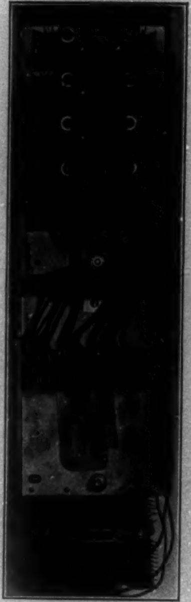


FIG. 1D—CONNECTOR SWITCH
(REAR VIEW UNCOVERED)

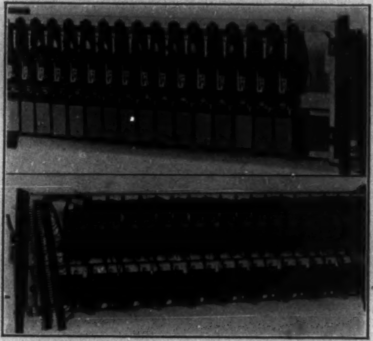
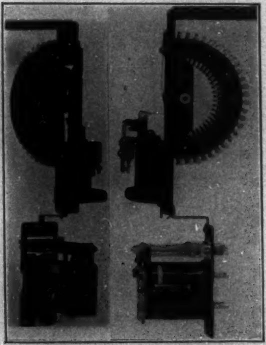


FIG. 7—ROTARY LINE-SWITCHES ON SHELF (FRONT AND REAR)



[SMITH]

FIG. 8—ROTARY LINE-SWITCH (LEFT AND RIGHT VIEWS)

The point-pivot, when adjusted without play or bind was as fine a pivot as could be made. But it was desired to have a pivot which would need no adjusting at all. The plain pin pivot has just enough play to make the armature work freely, and the wear is inappreciable. The point pivot is retained on the line relay of the plunger line switch, where the conditions are not severe.

The relay contact springs are insulated by phenol-fibre, which is a good insulator and very constant. Each assembly of springs is aged by heat treatment, so that it is practically one solid mass. Looseness and uncertainty of spring action have been practically wiped out.

The relays are mounted individually on a base plate Fig. 3. This base plate has openings through which the relay terminals (for coils and springs) extend to the rear. There the wiring is placed, connecting the relays together in a compact but accessible place. A plate covers the wiring so that none of the wiring on this part of the switch is exposed. The few wires which feed the magnets and wipers pass through a single hole in the base plate. The base plate carries also the switch frame, in which the vertical, rotary, and release magnets are mounted.

The adjustment of an automatic switch may be divided into two parts, the relay adjustment and the adjustment of the motor magnets. Every relay is adjusted to a definite armature-stroke with a fixed residual airgap. The amount of contact that each spring makes with its mate is likewise fixed. These distances are measured with thickness gauges placed between the armature and the pole of the magnet.

The contact springs of each relay are given a tension which is measured in terms of the operating and non-operating currents. Thus the spring tension is kept within limits.

The motor magnets (vertical, rotary, and release) are adjusted in a definite sequence to the proper relations to the wiper-shaft. These relations are also expressed in distances, most of them measured by thickness gages.

The final performance of the completed switch is tested by being operated over a line under conditions worse than any imposed by commercial use. The calling device delivers impulses at 14 cycles per second with an impulse ratio of 61 per cent (circuit opened 0.61 of total impulse period.) One test employs a line of zero loop resistance with a 20,000 ohm leak across it. The other test employs a 1200 ohm line without leakage.

The spring tension of the motor magnet armature is adjusted so that perfect operation is secured under these two extremes of line conditions, with the battery voltage between 46 and 49 volts.

The control of automatic switches over the subscriber line is a telegraph problem. The elements involved Fig. 4 are the line circuit, the release relay circuit, the motor magnet circuit and the release magnet circuit. All but the line circuit are wholly within the central office where the conditions are constant. Each element contributes its share toward fixing the line characteristics which are necessary for operating a switch.

The normal operation is as follows: The calling device springs (CD) are opened and closed as many times as there are units in the digit being transmitted. The line relay (LR) follows these impulses. The release relay, (Rel-Ry) having a copper collar remains energized during the series. The series relay pulls up the first time the line relay falls back and remains energized during the series of pulsations to which the magnet (VM) responds by moving the wiper shaft the required number of steps. At the end of the group of impulses, the line relay comes to rest energized, the series relay falls back and causes the local circuit to be changed to suit the next action.

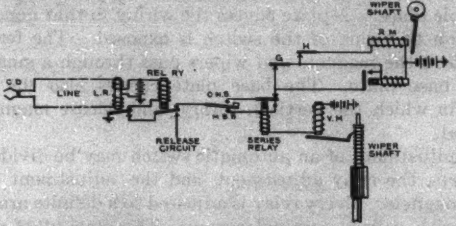


Fig. 4—Controlling Circuit for Automatic Switchboard

The period of an impulse Fig. 5 is the time from the opening of the line to the next opening of the line. It includes a "break" and a "make." The upper line represents the time that the line current is flowing. It is shown as broken three times, as in sending the digit "3." Each opening of the line produces one pulsation to the magnet. This is represented by a line just below the break in the line current. Owing to the delay of the line relay in obeying the current, there is a little time-lag at the beginning and at the end of each pulsation.

The impulse ratio is the ratio of the time that the line is opened to the total period of the impulse. Applied to the magnet, it is the ratio of the time of current flow to the total period of the impulse. Formerly a fifty per cent (0.5) impulse was used. It has been found that a longer opening of the line circuit gives greater certainty to the selection, so that sixty-one per cent (0.61) is now used. The reasons are discussed below.

The release relay must be held energized continuously, because when it falls back it will cause the switch to release. The greatest break in current which the release relay can stand without letting go is about 77 per cent. (For a group of 10 impulses at 10 per second.) At the beginning of a group of impulses, the release relay is fully energized. While the line relay armature is vibrating, the release relay is kept energized by pulsations fed to it, one pulsation each time the line relay pulls up. If these pulsations are short, the magnetism will not be fully restored each time but will gradually run down. There is always enough magnetism remaining to hold the armature to the end of ten impulses. The aging of a slow acting relay (equipped with a copper collar) always tends to make it hold better.

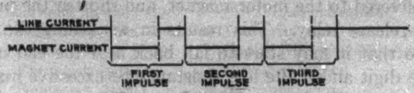


Fig. 5—Impulses

The series relay must also remain energized as long as pulsations are being sent to the motor magnet. The shortest pulsation which it can have is about 28 per cent for ten pulsations at 10 per second. The conditions which supply long pulsations to the series relay, give short ones to the release relay. But the short pulsations which each can endure give a large range through which the ratio can vary without causing either relay to cease to function properly.

The vertical magnet requires at least a 27 per cent pulsation and not more than 86 per cent for satisfactory operation under the same conditions. Being in series with the series relay, it gets the same impulse ratio. Since the magnet has considerable work to perform, it can stand a very high ratio.

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The rotary magnet of a connector can stand a variation of impulse ratio from 30 per cent to 95 per cent.

The total performance of the automatic switch is judged from the standpoint of the subscriber's line. Ordinarily a switch is adjusted to work over lines of varying length. This is because any telephone may use almost any switch. Sometimes a switch is to be used on one line or trunk only; in this case the line relay can be adjusted to operate on a 2500- or 3000-ohm loop. When adjusted for such a loop it would not work well on a low resistance loop (300 ohms and under). Loop resistance of the subscriber's line does not include the telephone instrument because in an automatic telephone the talking apparatus is cut out during the dialing period.

If the subscriber line is of zero length with no leakance or capacitance, the impulse ratio may vary from 38 per cent to 71 per cent without causing the switch to fail.

The effect of simple line resistance is to lengthen the pulsation delivered to the motor magnet, and shorten the pulsation to the release relay. This results in weakening the release relay so that it may start to fall back near the end of a ten impulse digit and if the line resistance be excessive may even cause premature release. If the loop is 500 ohms, the impulse ratio may vary from 36 per cent to 72 per cent without causing the switch to fail. If the loop is 1000 ohms, the limits are 35 per cent and 68 per cent. The margin for safety is ample.

The practical safe limit for a subscriber's line is a loop resistance of 1000 ohms. This is a greater resistance than is necessary for good common battery transmission, so that any loop which is low enough in resistance for good transmission is good enough for automatic dialing.

The effect of line leakage is to cause the line relay to be slow to fall away and prompt to pull up. This gives the motor magnet and its series relay a shorter pulsation, so that there is less power available to move the switch and a greater likelihood of the series relay not remaining energized throughout the group of pulsations. The practical limit of leakance is about 50,000 ohms for the subscriber's line, measured between wires or from either wire to earth. Since the switches are adjusted to operate perfectly with a leak of 20,000 ohms across the line, the factor of safety is ample.

Distributed capacitance such as found in telephone cable exerts no bad effect upon dialing. A capacitance between the two conductors has a tendency to delay the relay in falling back, and thus to shorten the pulsation delivered to the motor magnet. This partly neutralizes the effect of line resistance. In fact, the capacitance may be increased to seven microfarads without causing the automatic switch to fail.

Lumped capacitance, as found in telephone instruments on a party line, affects the dialing. It is more correct to say that it is the inductance of the bell in connection with the capacitance of the condenser that influences the signals. For this reason the capacitance of condensers used with harmonic bells (5 bells on one party line) is made 0.7 microfarad. With this value at least two more bells could be used without causing any failure of dialing.

In general, line faults have the same effect upon an automatic switchboard that they have upon a manual switchboard. The chief effects are compared in the table following.

	Manual	*Automatic*
Open.	No calls, no indication	No calls, no indication
+ *L* grd.	Line noisy, conversation poor	Line noisy, conversation poor Dialing bad, sometimes impossible
− *L* grd.	Permanent signal	Permanent signal
sht. ckt.	Permanent signal	Permanent signal
	Crosses between lines.	
+ *L* and + *L*	Cross talk	Cross talk, a second trunk occupied but no trouble
− *L* and − *L*	Cross talk, two line signals show when one calls	Cross talk, a second trunk is occupied when one calls
+ *L* and − *L*	Permanent signal on one line	Permanent signal on one line

A rotary line switch has been produced which beside giving access to 25 trunks possesses other advantages. It was the equipment furnished by the Automatic Electric Company for the Automatic Telephone Exchange at Orleans, France. Its chief use in America is in the Community Automatic Exchange. It will be described in this paper under that subject.

Periodic ringing is the accepted practise, on both individual and party lines. If the subscriber removes the receiver from the hook, the ringing current is cut off at once, even if it occurs during a time of ringing. The cut-off is quick enough to be complete before the receiver reaches the ear.

Party lines are segregated and served by party line boards, one hundred lines per board.

The preferred practise is to provide a party line board with four or five groups of connectors, each group being supplied with a different frequency of ringing current. Each subscriber line is multipled to all these groups of connectors. The hundreds digit of the call number selects the group of connectors and therefore fixes the frequency which shall be used. The last two digits control the vertical and rotary motions of the chosen connector as usual. Any group of connectors can equally well connect to the desired line, but the bell which rings depends upon which group of connectors is used.

An alternative method is to have but one group of connectors per hundred lines and to equip each connector with a frequency selecting device which is operated by the last digit of the call number. It makes each call number one digit longer, but is somewhat cheaper as to initial cost.

Telephone service for intermittent users (other than regular subscribers) is usually furnished by stations equipped with some form of coin collecting box. For local service, the rate is displayed near or on the telephone instrument together with directions for its use. For toll or long distance service, the user must call for the toll board to receive supervision and information.

The automatic telephone has been successfully equipped with "nickel-first" and "nickel-last" coin-boxes. With the former, the deposited coin is held until the called station answers, then a reversal of current operates a polarized relay in the coin-box which collects the money. With the latter, the user dials the number as usual, the answering of the called station reverses the battery current which operates the polarized relay in the coin box, the latter short-circuits the transmitter and shunts the receiver enough to prevent its use as a transmitter, but still permits hearing that the called station has answered. Depositing the coin clears the transmitter and receiver.

A credit meter is also in use, which records the number of completed connections. It may be attached to the instrument and worked with a push-button (instead of a coin) or may be located apart from the telephone on the subscriber's premises and require no act of the subscriber. An older type of meter is designed to be located in the central office.

If the pay station be used to call a free local number, no current reversal will take place. All free lines are connected to a group of connectors which do not reverse the current when the called station answers.

When the user of a pay station desires an out-of-town connection, he dials the toll recording desk as any local subscriber would do. When the connection is ready, the line operator calls the pay station automatically, requests the payment, and is by sound notified of the deposit of the coins as is the case in manual practise.

Durability. The durability of automatic equipment is attested by the performance of the older plants. A group of ten exchanges scattered over the Union, were installed from May 1903 to December 1904. These exchanges are still (1919) in operation. Their original joint capacity of 19,476 lines has grown to more than 104,000 lines. The original equipment is still in use. During about fifteen years of service, the regular maintenance has been able to keep the apparatus in good shape. Today the cost of maintenance material for the switchboard per line per year is from 7 to 9.5 cents, and for the calling device 2 cents per year.

Maintenance. The proper maintenance of any machine requires that it shall be prevented from deteriorating. It is cheaper from every reasonable standpoint to keep it in good working order than to neglect it until its condition is so bad that it has adversely affected service. Each function should be tested at such intervals as have been found necessary to give good service.

In a public exchange the routines involve inspections occurring daily, weekly, monthly, quarterly, twice a year and yearly. The daily, weekly and monthly routines are merely precautionary inspections, they are not adjustments.

The daily routine includes a rapid inspection of the functions of selectors. Once a week there is a rapid inspection of the line switchboards and of the functions of connector switches. Once a month the alignment of line switch plungers is examined, and the dust cleaned off the switch covers, tops of bays, etc.

Every three months the selector trunks are tested (continuity of bank and terminal wiring, particularly of infrequently used or overflow trunks and switches), wiper cords are examined and replaced if necessary, master banks and wipers are cleaned and inspected, and alarm relays are inspected and adjusted.

Twice a year the contact springs of relays and other devices are inspected for proper movement and tension and brought to standard. The switches are tested for operation under extreme line conditions. Switch shafts and master switches are cleaned and oiled. Banks are cleaned and wipers adjusted and replaced if worn. A rapid inspection is given to the rotary motion of selectors. All equipment is tested for freedom from grounds on frames and supports.

Once a year everything is checked up for conformity to standard and brought to standard. This includes the ratchet mechanism (vertical, rotary, and release) of each selector and connector, the repeaters, the line switch plunger alignment and its bank contacts and the cut-off relays.

The maintenance of an automatic exchange requires the attendance of persons who know what to do and when to do it. The average mentality required is not higher than for a manual plant of the same size.

A single office exchange having 2300 lines and 5500 subscribers telephones requires approximately the following force for maintaining the central office equipment.

One switchman.
One service clerk.
One line tester.
One information and complaint clerk.
One night man (answers toll board in addition.)

A two-office plant having 5400 lines and 10,000 telephones requires the following central office force.

One service chief.
One switchman.
Two apprentices.
Two service clerks.
One tester.
Two information clerks.
Two assistant clerks (also take care of complaints.)

The telephones can be increased to 13,900 without adding to the central office force. A service chief has charge of the switch room and does work on equipment. A switchman makes adjustments and repairs. An apprentice performs routine tests, supervises signals, and answers telephone calls to the switch room; an apprentice may be in training to become a switchman. A service clerk is a girl who performs the same duties as an apprentice. A tester makes tests on local and long distance lines; and in a small plant may act as despatcher, routing the outside troublemen. The duties of information clerk are the same as in a manual plant. The employment of girls to perform routine tests has become a factor in exchange maintenance. They learn easily, are quick and careful in their work, and are proving very satisfactory.

RURAL TELEPHONE LINES

Telephone service on rural lines differs from that in cities and towns in that dwellings are much farther apart, there is usually no common center (except the nearest town) and greater dependence is placed on the telephone.

The requirements of the subscriber are low rental, dependability of service, and the ability to make many calls to those in his own neighborhood. On the other hand, the rural subscriber is usually willing to do more than his city brother. He will gladly push buttons, turn cranks, and do anything else if necessary to get good service.

The operating company must have cheap but substantial lines, with as low an upkeep as possible, and a simple telephone requiring little maintenance.

Manual rural lines are usually operated with local battery transmission. Automatic rural party lines are also in use with local battery and with success. But a special weak current common battery transmitter has been developed which gives as good transmission on a 400 ohm loop as an ordinary transmitter does on zero loop. This has to a large extent made it possible to operate rural lines with common battery, with evident economies.

EQUIPMENT CHARACTERISTICS

The characteristics of the equipment which handles rural lines depend upon the telephone instruments, the method of operation which is desired, and the line conditions which are necessary in this class of business. Insulation can not be maintained at as high a figure as is possible in the city. Line resistance must greatly exceed the usual values. Many more telephones must be put on a single line than is necessary in town. These facts require changes in the apparatus.

Because of the number of bells and condensers (sometimes ten) bridged across the line and the lower insulation which is at times unavoidable, the control circuit and apparatus must have a greater factor of safety than is usual. To secure this a repeater is interposed between the rural line and the regular exchange apparatus.

This repeater has two line relays, Fig. 6, one in each battery lead. The impulsing is done with the relay which is connected to the positive grounded battery terminal. Also, during the time that impulses are coming in, the impedance of the negative line relay is greatly lowered. These two provisions greatly increase the reliability of the selection.

It frequently occurs that rural lines produce so much traffic that it overloads the group of first selectors belonging to the line-switch board (hundred line group). This is relieved either by subdividing the line switches into smaller groups, or by providing each rural line with its own first selector instead of a line switch; in this case the selector performs the functions of repeater as well as of selector. The relative cost of the two plans varies with conditions.

One condition incident to placing so many telephones upon one line is the chance of interference of one subscriber with the dialing of another. This is obviated by providing each telephone with a hook stop. It permits the hook to rise part way, only far enough to connect the receiver across the line (in series with a 2 microfarad condenser.) This allows the subscriber to listen without interfering with dialing which may be going on. If the line is free, the subscriber presses the hook stop; the lever then rises to the full extent of its stroke, connecting up the calling device and completing the talking circuit. Beside preventing interference with dialing, this device permits the rural subscriber to listen as much as he desires—a habit which is firmly established and warmly defended.

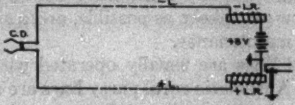

FIG. 6—RURAL LINE CONTROLLING CIRCUIT

The signalling can be either selective or code ringing. The latter is very largely employed by manual exchanges, is understood by the subscribers, and seems to be satisfactory. Selective signalling is better in some respects, but the choice properly lies with those who are closely in touch with local conditions.

To ring ten bells selectively, five bells are bridged from each line wire to earth and rung with five frequencies. Two systems of frequencies are in use. The "multiple harmonic system" employes 16.7, 25, 33.3, 50 and 66.7 cycles per second. The "non-multiple harmonic system" uses the frequencies 20, 30, 42, 54, and 66 cycles per second. This duplication resulted from the historical development of two systems, and there are many installations of both in use today. It is to be hoped that one of the two may be retained as the better, and the manufacture of the other discontinued.

Code ringing permits all the bells to be alike and to be bridged across the line. In order not to interfere with dialing, the bells are wound to 3600 ohms (*approx. 5000* ohms impedance at 10 cycles per second) and the condensers limited to 0.3 microfarad capacitance. Signals may be made of combinations of long and short rings or of two groups of short rings separated by a pause.

Rural lines are served by a line-switch board set aside for this purpose. The special apparatus required is localized to this part of the equipment. If the signalling is selective, each connector has mounted on it a minor switch which picks out the frequency to be used. If code ringing is used, each connector has a code selector and a code switch. The latter groups the code signals, the former picks out the desired signal. The code switch is a single motion (rotary) switch with a 25-point bank. The code selector is the same in form and size as a first or second selector. They involve mechanisms which have been established by experience.

In general, reverting calls are handled by providing special switches leading to reverting call connectors. Each of these connectors is equipped much as a regular rural connector is equipped, but is wired so as to ring back on the originating line. If selective ringing is used, the reverting call connector will send out alternately the frequency of the originating station and of the called station. The total period of repetition is 4 or 5 seconds. To make a reverting call, the subscriber consults a list of the subscribers on his own line and dials the number given for the desired station. He then hangs up his receiver. The alternate ringing begins and continues until the called subscriber removes his receiver from the hook and presses the hook-stop. The originating subscriber, noticing the cessation of his own signal, takes his own receiver, touches the hook-stop and proceeds to converse. Release is accomplished when both subscribers hang up their receivers. If the called station fails to respond, the originating subscriber stops the ringing by removing his receiver pressing the hook-stop, and hanging up the receiver again.

The line requirements for rural lines are as follows: maximum safe loop resistance 1000 ohms, minimum safe insulation resistance, wire to wire or either wire to earth 25,000 ohms. Any commercial telephone cable has low enough capacitance not to affect dialing.

Automatic switching offers to the rural subscriber a telephone service which is vastly superior to manual service and only slightly inferior to city automatic service. He has the same instantaneous, positive, direct calling at night as at noon, and on holidays as on work days. It is inferior to city automatic service only in the loading of the party line with so many telephones. In fact the moving force which hastened the application of automatic switching to rural lines was the pressure exerted by the farmers themselves to be given calling devices.

THE COMMUNITY EXCHANGE

The community exchange, as the term is now used, is a small unattended plant, whose toll switching and miscellaneous calls are handled from a distance and whose apparatus is often designed with the requirements of rural lines primarily in view. The community which it serves may be the farmers of a region, a small village or town, or the small suburb of a town. The traffic is chiefly local though by no means confined to the local telephones.

Miscellaneous services, such as information, complaint, dead numbers, etc., are usually cared for by the nearest public exchange where such service is maintained. Most of these community automatic exchanges are auxiliary to the ordinary type of public exchange. When the C.A.X. subscriber dials the information number, the call is trunked to the information clerk in that exchange, where records are kept for all the territory served.

Toll switching is handled in the same way as the above mentioned miscellaneous services. In fact it may pay to combine on one call number all these things, because one person can care for them all. The toll line operator is provided with a calling device and trunk connections to the community automatic exchange, so that she can complete and control any connection.

The subscribers requirements are the same as for any single office exchange with rural lines. From the standpoint of the operating company, the apparatus must require little attention and the investment must not be too large.

The characteristics of the equipment are influenced by the conditions of the particular installation. Too great rigidity of standardization would result in inefficiency. This will become apparent as we study the matter farther.

The rotary line switch Fig. 7, is the regular equipment for community automatic exchanges. It possesses simplicity of structure, individual control, minimum of motion, and greater trunk group than the plunger type.

The rotary line switch Fig. 8, is a single-motion switch, with a 25-trunk bank set vertically. The motor magnet armature has a pawl which normally locks the wipers on any trunk. The wipers rest where last used. When the magnet energizes, it withdraws the pawl so that it catches the next tooth on the ratchet wheel. When the magnet de-energizes, the armature spring drives the pawl and rotates the wipers to the next trunk contacts, and locks them there. The magnet is self vibrating, like a door bell.

Associated with the switch proper is a line relay and a cut off relay. They are mechanically interlocked so that if the line relay is de-energized, the cut off relay can pull up only part way—just enough to break contacts and clear the line of attachments. If the line relay is energized, the cut off relay can pull up all the way, connecting the subscriber's line to the wipers.

When a call comes through a connector to the subscriber, the connector energizes the cut off relay alone. This clears the line of line relay and ground, but does not connect the line to the wipers. Thus there is a clear pair of wires from the connector to the subscriber's telephone.

When the subscriber originates a call, by lifting the receiver from the hook, the line relay pulls up and causes the trunk to be tested. If the trunk is busy, the magnet rotates the wipers at a speed of 60 to 80 trunks per second until an idle trunk is found. The magnet then stops, the cut off relay pulls up, clears the line, and extends the line to the first selector. During conversation, the cut off relay alone is energized. On release the cut off relay merely falls back—the wipers remain where they are. If the trunk upon which the wipers were resting at first is idle, the cut off relay pulls up completely and without delay. The time required to seize the trunk is only that required for two relays to energize. The maximum time required, if the wipers must pass over 24 trunks, is about half a second.

Most calls take an idle trunk without moving the line switch wipers at all. Tests made on a busy private automatic exchange showed that about 16 per cent to 20 per cent of the calls moved the wipers one or more steps—the rest of the calls required no motion.

Because it possesses no common mechanism, the rotary line switch lends itself admirably to rural exchanges. There is so little to get out of order, and the factor of safety is so large, that it can be relied upon to require very little attention. And if any one part *should* fail, it affects only one subscriber line instead of a group of lines.

The most important variable factors are herewith presented and will be discussed in detail.

1.	Transmission	Local battery or common battery
2.	Line relay	Double wound or ground relay
3.	Line equipment	Lineswitch or selector
4.	Signalling	Selective or code
5.	Signal control	Push-button or periodic
6.	Reverting calls	Push-button, periodic or hand generator
7.	Battery charging means	Motor generator or gasoline engine
8.	Battery charging method	Float or hand start with automatic stop

The tendency is to use common battery transmission as far as possible. The replacement of dry cells is a great and increasing expense. It is to be incurred only if the salvage of present equipment causes it to seem more profitable to retain the local battery, or if the lines are so long that it is necessary for good transmission. As far as automatic operation is concerned, it makes no difference which is used. Most lines are long enough to require a special weak current common battery transmitter. In all cases the polarized receiver is necessary, because the subscriber must be able to listen through a condenser to see if the line is in use.

The line relay (LR-Fig. 6) which handles the impulses should be connected to the positive or grounded battery terminal. Here it has a greater factor of safety than if it has two windings, one being to each battery terminal. If the line conditions are as good as in town, the usual double-wound line relay is fully adequate.

The usual practise is to carry all lines in a community automatic exchange to line switches. All selectors and connectors are usually equipped with the ground line relay circuit, so as to render repeaters unnecessary.

In regions where rural subscribers have been using local battery magneto telephones for years and are accustomed to code signals, the code signal can still be used with automatic. The non-secret nature of the call is by some regarded as a positive advantage. It is difficult for the city-dweller to realize how the farmer uses his telephone. An instance will illustrate one point. One evening Jones repeatedly heard his bell give the signal for the Browns, who lived a quarter of a mile down the road. Knowing that the whole Brown family had gone to town for the evening, he stepped to the telephone, and gave the calling subscriber the information. More congested rural regions seem to require greater secrecy, and the selective signal is favored and sometimes even requested.

The signal may be controlled by the subscriber or by the central office apparatus. The former is called "push-button control" or "push-button ringing." If it be code signalling, the subscriber presses a button on his telephone so as to form the signals. They may be thus made more accurately and easily than by the hand generator, with its revolving armature which must be started and stopped several times in each signal.

To secure push-button ringing, the button is arranged to ground the negative line and open the positive line. The negative line relay remains energized and prevents the connection from releasing. The positive line relay controls the ringing apparatus.

Code ringing was described under the heading "Rural Telephone Lines."

Reverting calls are handled in accordance with the method of signalling used. If local battery magneto telephones are employed, the subscribers ring each other on the same line just as they did with the manual switchboard. If selective signalling is employed, the subscriber dials a special number and receives alternate signals as was described above, (Rural Telephone Lines.) If code signalling is used it may be either periodic or push-button controlled. The latter requires reverting selectors, available to the party lines.

When a rural subscriber desires to call a person on his own line, he dials a special number. He then presses his push-button to form the code signal. This action controls a relay in the reverting selector, which sends ringing current back onto the originating line.

If the code ringing is periodic, reverting calls are handled by reverting connectors which are equipped very much like the regular connectors. The subscriber consults a list of subscribers on his own line (posted on the telephone.) He then dials the special number given for the desired station and hangs up his receiver. The desired code signal is sent back on the line, repeated periodically. When the called stations answer, the ringing is cut off—the originating subscriber then comes in on the line. Both subscribers hanging up release the reverting apparatus.

The charging of the battery is done preferably by a motor generator run by commercial power. The battery bus bars are equipped with a voltage relay, which starts the motor generator whenever the pressure falls to 46 volts and stops it when it reaches 52 volts. A self-closing reverse-current circuit breaker stands between the generator and the battery. When the generator voltage is high enough to be safe, the circuit breaker closes and the charging begins. When the motor has been cut off by the high voltage, the generator voltage dies away until a slight reverse current operates the circuit breaker and cuts off the circuit.

If no commercial power is available, a gasolene unit is installed. This is started by hand as often as necessary. The regular maintainer need not do it, some one who lives near the exchange and is familiar with gasolene engines is employed to set it to going. The stopping is automatic. An ampere-hour meter is included in the battery circuit. It is compensated so as to run slowly when charging, but full speed when discharging. When the meter indicates the necessary ampere-hours have been put into the battery, it opens the ignition circuit of the engine, so that the latter stops. The circuit breaker disconnects the generator from the battery. During the time of charge, the circuit breaker inserts counter e.m.f. cells in the discharge lead to the power board—they prevent undue rise of voltage on the bus bars which would prematurely stop the charging.

The equipment is made up in three general units, the switchboard, the storage battery and the charging machine. The switchboard is made in two or three parts, which are installed side by side so as to form one unit. These parts are the local switchboard, the powerboard and the main distributing frame, and the trunking board. The latter is omitted if the exchange is isolated (no trunks to other exchanges.) All but the trunking board have been fairly well standardized. Because of the many variable conditions to be met in different places, the trunk board, containing selectors and repeaters, is made up in accordance with the needs.

A community automatic exchange needs no special building to house it. It has been installed in a store, in the bedroom of a residence—any convenient place which is available. There are signals to indicate when a fuse blows etc. The conditions of community life and the location of the apparatus are such that someone is always not far away.

This type of exchange has not been in service long enough to secure data of general worth as to the amount of maintenance labor required. In one case a group of three of these exchanges located on a straight line 26½ miles long, together with the telephones belonging to them are maintained by one man. The man is of the same general type as those who maintain ordinary automatic equipment.

The line requirements are the same as those given under "Rural Telephone Lines."

Toll Switching

Toll service between line and subscriber may be handled in any one of a number of ways. The methods described here are considered the best, as the result of much experience. Person to person calls alone are considered.

The recording toll clerk receives calls from the subscriber over trunks which come from selector banks and terminate on desks. The tickets are made out and passed to the toll line operator in the usual way.

The toll line board is provided with trunks which pass to toll selectors. Over these trunks the toll operator dials the full directory number by using a calling device which is keyed into the circuit. The selectors lead to toll connectors, the latter located in the hundred-line boards, adjacent to the local connectors.

These toll connectors may be used for local service as well.

If all the local connectors are busy, the local traffic will take one or more of the toll connectors. In this case the connector acts like a local connector, having the same current supply relays and condensers. When seized by a toll selector, the local apparatus in the toll connector is cut out, leaving a clear circuit from the last selector through the connector to the calling line. The last selector supplies current through a repeating coil.

The ringing is intermittent, but must be started by the toll operator. When the subscriber answers, the ringing stops automatically. If the subscriber hangs up his receiver, the usual supervision is given, and the toll operator may start the ringing again, if desired.

Intermittent ringing is used for toll calls because it has been found to be the more effective than manual ringing. Where local ringing is periodic and toll calls are rung by hand, much delay results. The subscriber is inclined to think that the manual ring is merely some local call which is very shortly released. Consequently he pays no attention to it. But the periodic signal gets his attention and greatly shortens the time of response to toll as well as to local calls.

The toll connector tests the called line as does any connector. If the line is busy, it sends a busy tone to the operator and also extinguishes the supervisory lamp at the toll board. When the line becomes free, the connector seizes the called line, stops the busy tone, and lights the supervisory lamp at the toll board. When the toll operator sees that the line is free, she starts the ringing by pulling a key.

The release of a toll-to-local connection is controlled by the toll operator. Merely pulling the plug out of the trunk jack causes the switches to be restored to normal.

TOLL SWITCHING BETWEEN EXCHANGES

It is desired to discuss under this heading the general considerations involved and to give some concrete illustrations of how exchanges are linked together.

General Considerations. The linkage of exchanges of any kind into a network requires, beside the toll lines, adequate means for switching these links to each other and to the lines of the subscribers who are to talk to each other. In arranging this switching, thought must be given to the cost of effecting such connections, and the effect of the means on the volume of traffic handled.

The usual manner of handling such calls is to have one or two operators in the originating exchange to connect the originating telephone and the toll line together, another operator at each switching office for toll lines, and one or two operators at the called exchange to connect the toll line to the called telephone. If each exchange has several offices and private branch exchanges, still more operators may be required.

It has been found that the manual switching of toll lines, among themselves and to the subscriber lines, entails considerable loss of time. This is chiefly due to the time required to gain the attention of operators who have other lines to serve beside the one in question. It lengthens the time required to set up a connection, it interferes with supervision, and delays the release of the circuits after both subscribers have hung up their receivers. This is so great a drag on the service that it has led to the establishment of many direct toll lines between important points, so as to do away with intermediate switching. This lowers the time-efficiency of the long direct line, because it can not be used for any intermediate traffic.

For at least twelve years toll lines have been switched by automatic means and experimenters have been at work producing apparatus which is adapted to this service. The first discovery made was that automatic switching increased the capacity of the toll line at least fifty to one hundred per cent. This remarkable increase is due to the speed of connection, the accuracy and promptness of supervision, and the quick clearing of the line after both subscribers have hung up their receivers.

No matter how the switches are controlled, the line must permit manual operation at any time, must operate automatically in both directions if both ends are at automatic exchanges, when seized must give positive indication to all stations concerned and must give supervision of both originating and called subscriber to the one operator who controls the connection.

Uniform operation is highly desirable. This point has received attention in developing automatic switching over toll lines. It is so arranged that if the operator plugs into the jack of an automatic toll line and rings, the call will go through manually as formerly. But if, after plugging into the jack, the operator throws the calling device key, she can put the call through automatically. After dialing the number, the operator rings with the regular ringing key. This operation holds no matter whether she is switching toll lines only, or if she automatically completes the connection to the called subscriber.

A few illustrations will show how automatic switching is applied to various toll traffic services. In order to avoid confusion, the examples will be very simple.

Consider the case of two adjacent exchanges which have free service from one to the other Fig. 9. Let each exchange be represented by four hundred-line-boards, having selectors and local connectors.

The lines (1 and 2) which carry the inter-exchange traffic are equipped at each end with a selector, and are in addition multipled to the banks of local selectors. A repeater is inserted between the selector banks and the lines for the purpose of holding the local switches from releasing, feeding common battery for talking, etc.

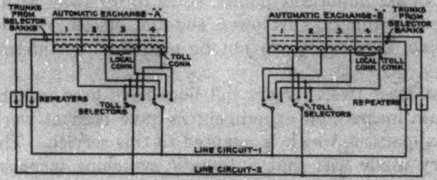
TRUNKS FROM SELECTOR BANKS
AUTOMATIC EXCHANGE-"A"
1
2
3
4
LOCAL CONN.
TOLL CONN.
REPEATERS
TOLL SELECTORS
AUTOMATIC EXCHANGE-"B"
1
2
3
4
TRUNKS FROM SELECTOR BANKS
LOCAL CONN.
TOLL CONN.
REPEATERS
TOLL SELECTORS
LINE CIRCUIT-1
LINE CIRCUIT-2

FIG. 9—AUTOMATIC EXCHANGES CONNECTED BY FREE AUTOMATIC TRUNKS

The banks of the toll selectors are run to the jacks of toll connectors, one in each hundred-line-board. These toll connectors have their banks multipled to the banks of the local connectors, so that all subscriber lines can be reached.

When a subscriber in exchange A desires to call one in B, he dials a figure which causes his first selector to seize one of the lines, making it busy on the banks of all other first selectors in both exchanges A and B. He then dials the directory number of the desired telephone, which first operated the toll selector at B and then the toll connector in the hundred-line-board chosen.

If pay service is the rule, the lines can be arranged so that the toll operator at the originating exchange can complete the connection without the aid of the toll operator at the distant exchange, Fig. 10. The toll lines will terminate at each end in toll selectors and jacks and lamps on the toll board. The trunks from local selector banks will also run to the toll board.

When a subscriber in B desires to call one in A, he dials a single figure number which causes his first selector to take an idle trunk to the toll board. The toll operator answers the call in the usual way. She has a calling device which may be associated with any pair of cords. When she plugs into the jack of the toll line, she makes it busy at both ends, showing the busy lamp at exchange A, so that the toll operator there will not try to use the line. The operator at B then dials the call through to the subscriber in A.

The operator at B controls the connection, giving it the usual supervision. When the subscribers are through, she pulls out the switchboard plug, which causes all switches to release.

The identity of the originating station may be verified by having the subscriber hang up his receiver. When the line connection is ready, the operator calls the originating subscriber automatically and permits conversation.

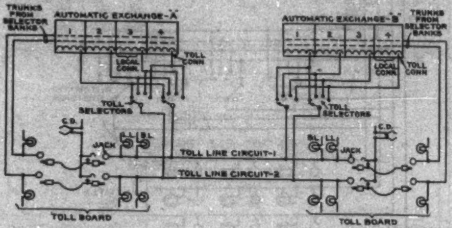
TRUNKS FROM SELECTOR BANKS
AUTOMATIC EXCHANGE-"A"
1
2
3
4
LOCAL CONN
TOLL CONN
TOLL SELECTORS
C.D.
JACK
L.L
B.L
TOLL LINE CIRCUIT-1
TOLL LINE CIRCUIT-2
TOLL BOARD
AUTOMATIC EXCHANGE-"B"
TRUNKS FROM SELECTOR BANKS
1
2
3
4
LOCAL CONN
TOLL CONN
TOLL SELECTORS
B.L
L.L
JACK
C.D.
TOLL BOARD

FIG. 10—AUTOMATIC EXCHANGES CONNECTED BY AUTOMATIC TOLL LINES

The apparatus is arranged so that the toll lines can be operated manually without change. If the operator after plugging into the toll line jack rings in the ordinary way, she will cause the customary lamp or drop to be displayed at the other toll board.

If between two such exchanges there is a toll central office, the lines can be interconnected by automatic switches with great saving in time. Consider, Fig. 11, four toll lines meeting at a toll office, which is manually and automatically operated. The former may be necessary for handling traffic to and from local subscribers.

Each toll line will terminate in three places, the toll board multiple, the automatic switch bank multiple, and the jack of the automatic switch belonging to that line.

If a distant exchange seizes a toll line, it is automatically made busy on the banks of the toll selectors and also lights the busy lamps belonging to it on the toll board. The distant exchange then dials the number of the toll line desired. When that line is seized, it is made busy on selector banks and toll board (as was the originating line.)

If the incoming call is for a subscriber in the exchange attached to the toll office, two ways are open for handling it. The distant exchange may ring on the toll line, cause the line signal to show on the toll board at the toll office and use the operator there to complete the connection manually or automatically. The second way is to run trunks from the banks of the toll selectors to the automatic switches of the local exchange and to permit the distant toll operator to dial through to the subscriber and supervise the call.

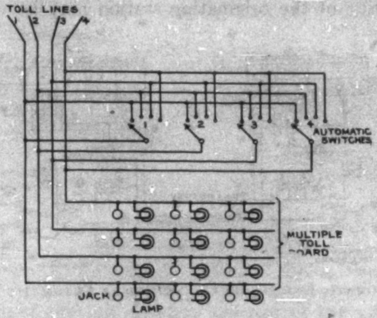

Fig. 11—Automatic Toll Switching Office

The application of automatic switching to toll lines in a small net work will illustrate some of the possibilities. For the sake of simplicity, each exchange is limited to 1000 lines.

Let us consider five automatic exchanges Fig. 12. Four of them, A, B, C and D, are located at the corners of an approximate square. Exchange M is near the center of the quadrilateral. Assume further that free service is given between all adjacent exchanges, but that a charge is made for traffic between opposite corners (A and D, B and C). The exchanges are numbered from 1 to 5 as indicated.

The exchanges are connected by lines. To keep the illustration simple, assume that there is only one line between adjacent exchanges, eight lines in all. Any link can easily be increased by adding lines and switches without changing the principles described.

Each exchange has two kinds of first selectors, local and incoming. The local first selectors (illustrated by only one, L) handle all the traffic originated by subscribers in that exchange. One bank level carries the traffic into the local switches to local subscribers. Since this is a small system, to keep the numbering uniform, each exchange has a different local level. In A it is the first level, in B it is the second level, etc. The other levels lead to adjacent exchanges, each according to the number of the exchange. From the second level, a trunk goes through a repeater (R) to the line leading to B, where it terminates on an incoming selector (I-A from A, I-D from D, etc.)

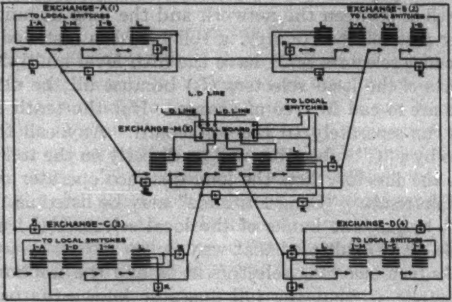
EXCHANGE-A(1)
TO LOCAL SWITCHES
I-A
I-M
I-B
L
R
EXCHANGE-B(2)
TO LOCAL SWITCHES
L
I-A
I-M
I-D
L.D. LINE
L.D. LINE
L.D. LINE
TO LOCAL SWITCHES
EXCHANGE-M(5)
TOLL BOARD
EXCHANGE-C(3)
TO LOCAL SWITCHES
I-A
I-D
I-M
L
EXCHANGE-D(4)
TO LOCAL SWITCHES
L
I-M
I-C
I-B

FIG. 12—TYPICAL NETWORK OF SMALL AUTOMATIC EXCHANGES

From the third level a trunk goes through a repeater (R) to the line leading to C, where it terminates in an incoming selector (I-A from A, I-M from M, etc.)

The repeater inserted in the outgoing trunk holds the local switches in the originating exchanges.

The incoming selectors (I–A, I–B, etc.) have the right level multipled to the banks of the local selectors (L) so that they have access through the local switches to the local subscribers.

These free lines are in reality two-way trunks. Each end of a line is attached to an incoming selector for incoming traffic and is multipled to the banks of all the local selectors (L) for outgoing traffic.

Suppose that a subscriber in exchange A calls a subscriber in exchange C, whose local number is 3225. The subscriber in A will first dial the figure "3" which causes his first selector (L) to lift its wipers to the third level and seize the line to C, making it busy at both ends so that no one else can intrude. Then he dials the figure "3" of the local number, which causes the wipers of the incoming selector (I-A) to rise to the third level and to seize an idle trunk to second selectors. The remaining three digits choose the hundreds, tens and units and complete the call.

To handle pay traffic between A and D and between B and C, as well as between the network and the long distance lines to other parts of the country, a toll board is installed at M. All incoming selectors in M have the fifth level multipled into the banks of the local selectors (L) because all the other exchanges are to call M free of charge. But the tenth or "0" level is run separately to the toll board. Any call for long distance by a "C" subscriber will go directly to the toll board, because the directory lists the long distance operator so as to secure that result. "Long Distance" may be listed as "50" or as "00", because the banks of the local selectors (L) at each exchange can be multipled that way.

Trunks from incoming selectors at M and the toll board are kept separate, so that the toll operator knows the origin of each call which comes to her.

If a subscriber at A desires to call some one at D, he is instructed by the directory to call "50" which leads his call to the toll board. The operator makes the charges and completes the call. The operator has lines leading into the local switches through which any exchange is available. If desired, special toll selectors may be set aside for her use. If the identity of the calling station is in question, the operator directs the originating subscriber to hang up. Then she calls the number which he gave, this establishes identity.

If the subscriber at A tries to avoid payment by calling through B or C, he will fail. The incoming selector (I-A) at both these places does not have any connection on its bank for the D exchange. It is impossible to get out of that place. If he tries to dial from A through M, to D, he will run against the same difficulty.

It is possible to multiple the banks of local selectors so that if a subscriber tries to evade payment, his call will be diverted to the toll board. This is done by connecting the level which will be called to the level carrying the trunk to M (see exchange C.)

Suppose that a subscriber in D has business with a subscriber in B, whom he calls frequently free of charge by dialing "22" plus the rest of the number. If that D subscriber moves to C and attempts to call "22" as before, he will find himself answered by the toll operator at M, who will make the charge and complete the call. Level "2" of local selectors in C is multipled to level "5" and the trunk to M. The incoming selector (I–C) in M has its second level multipled to the tenth and thus to the toll board.

LONG DISTANCE TOLL LINES

It seems to the writer that there is no insurmountable obstacle to prevent the extension of the automatic switching of toll lines to include long distance lines of any length. The only limit is that of desirability. Any distance over which telegraph can be worked can be covered by automatic dialing on toll lines. The problem is more difficult than straight telegraphy, for the transmission of the voice must be safeguarded. But as the advantages become more generally known, automatic switching will be extended as far as there is any advantage to be gained. The farther two points are apart, the more line there is involved in each conversation and the greater the need of economizing time.

THE COMPREHENSIVE SYSTEM

The automatic switching of telephone lines of all kinds is available for building up a comprehensive telephone system of large usefulness. It serves equally well the farmer, the small town dweller and the citizen of the metropolis and links them all together. Everywhere that telephone lines are to be joined and disconnected, and where human intelligence is not absolutely indispensible, the automatic switch materially increases the efficiency. Every year people are revising their views on what constitutes a real need for human intelligence, and this revision is not confined to any one industry.

Gradually the use of automatic switching is extending its usefulness and raising the effectiveness and the efficiency of the whole comprehensive system.

SUMMARY

Automatic switches made with reasonable care, to well known and definite limits, give superior telephone service with plenty of margin to spare. Automatic switching is not so easy that any kind of apparatus thrown together in any sort of way will give good service; neither is it so difficult that prohibitively expensive machinery barely secures commercial results.

The ways of connecting automatic switches together so as to form a smoothly operating system are many and varied, and permit adaptation to any class of telephone service. It meets the need for telephony, not as manual switching meets it, but in its own way, which in many particulars is better. This art rests upon a reasonable and scientific foundation.

DISCUSSION ON "APPLICABILITY OF AUTOMATIC SWITCHING TO ALL CLASSES OF TELEPHONE SERVICE (SMITH), NEW YORK, N. Y., DECEMBER 12, 1919.

Selby Haar: I saw an announcement in a New York paper recently that some of the new exchanges which are to be opened by the telephone company in this city will have, at least, in part, some automatic equipment. I wonder if that will be of the type described in the paper tonight.

Lyman F. Morehouse: I think perhaps I can answer the question just asked. There are several types of automatic telephone equipment, and Mr. Smith in his very interesting paper has described one of them, which has attained quite a field of usefulness. The conditions which obtain in New York City, for example, are, however, such as to require the installation of automatic equipment different from that described by Mr. Smith.

Paul S. Clapp: I had the good fortune to be in Central Europe with Mr. Hoover's Commission and assisted in the establishment of communications for that Commission. At Prague we came into rather intimate contact with the Czecho-slovak Director of Public Telegraph, and he said as soon as they could get money in that country, they would build telephone systems. Of course, the labor question is a very important one throughout Europe, and they were particularly interested in anything connected with automatic telephone systems. That statement of the Director of Postal Telegraph has two significances—one, in showing the alertness of European engineers to grasp improvements in American means of communication, and secondly, a realization of the shortness of labor and their desire to introduce machines to do work wherever possible.

Fred L. Baer: (Read by D. McNicol) In these days of disputes between employers and employees when both are so prone to ignore the third party to all such disputes, the long suffering public, it is perhaps of interest to mention that in several instances within the past year automatic exchanges continued to serve the public for periods of a month or more with practically no one in attendance in the central office. While under normal conditions it is necessary to follow maintenance routines, these are largely precautionary methods to anticipate trouble which might ensue from non-standard conditions and many of the routines can be discontinued for a more or less indefinite period in case of an emergency.

In the paper we have learned of many of the advantages of automatic operation under various conditions, but it might be of interest here to describe briefly how the condition of full automatic service, especially in large networks, is approached.

It is not often that the cut-over from manual to automatic operation is effected instantaneously in large networks except where the manual equipment which is to be displaced is of a more or less uniform condition justifying the replacement. In most cases the transition period is likely to be from two years to seven years or more, so that there is a considerable time during which there is a mixed manual and automatic service. Obviously as far as the telephone user is concerned the method of operation should be such that manual subscribers will originate all of their calls as they have been doing and automatic subscribers will originate all of their calls by means of the dial.

Various plans have been followed for affording the mixed service during the transition period, each having advantages peculiar to the particular case. For calls from a manual subscriber to an automatic subscriber the procedure is as follows:

The manual subscriber lifts his receiver and gets the operator who is known as the "A" operator. The "A" operator ascertains the number that is desired and must have means for disposing of the call. Since the subscriber is likely to require service either to a manual or to an automatic number, the "A" operator must be able to take care of either case. The control and supervision of the connection should always remain with the "A" operator. The plans for completing the calls to automatic are,

First. The "A" operator's position can be equipped with a dial and the operator can dial the desired number. Ordinarily the operator will have trunks direct to each office, and it will be necessary for her to dial only the last four digits of the number. The "A" operator's work in handling a call of this kind is practically the same as for handling a manual trunk call by order wire.

Second. The "A" operator can extend the call over a trunk to a sending operator, after having obtained a trunk assignment from the sending operator. The "A" operator's work is the same as in handling a manual trunk call. The second operator's positions, located at one or more convenient points, can be equipped in the following manner:

(a) A dial cut in key for each trunk and a common dial. After learning the number over the order wire and making the proper trunk assignment, the operator uses the dial cut-in key of the proper trunk and dials the number, then restores the dial key to normal. This operator has no supervision except busy guard lamps to prevent double assignment of trunks. When the "A" operator disconnects, the entire connection is cleared. The sending operator with this method of operation should handle from 300 to 350 calls per busy hour.

(b) A special push button sending device, so that when receiving the number over the order wire, this operator sets the call up on an idle sending machine, and when the proper trunk has been seized by the "A" operator, then the machine is started and sends the impulses. This operator has only busy guard lamp supervision and the control of the connection rests with the "A" operator. A second-sending operator with this method of operation should handle from 550 to 600 calls per busy hour.

When only a relatively small portion of a network is to be converted to automatic initially, it may be more practical to use the second plan rather than to equip all "A" positions with dials and instructing the entire staff in the dialling method of operation. Whether the sending operator would be provided with a dial or a special sending device would depend on the length of the transition period, and since the special sending positions are considerably more expensive, than the dial position, and could be justified only on the basis of operating savings. Obviously as the conversion work proceeds, there will be an economical point at which the dials can be placed on the "A" operator's positions.

For calls from automatic to manual the procedure is as follows:

First. The directory listings can differentiate between manual and automatic subscribers. Then the automatic subscriber could be instructed to call a predetermined digit, say "9", for calls to manual subscribers the call would then be routed over a trunk to a nearby manual operator, where it would terminate as a subscriber's line. An "A" operator would answer and complete the connection the same as a manual call.

Second. The directory listings can differentiate between manual and automatic subscribers. Then the automatic subscriber could be instructed to call certain digits to route the calls to the proper manual exchange where the trunks would terminate before an operator who would answer and learn the desired number and the call would be completed by plugging into the multiple. If the trunks are plug-ended and equipped for machine ringing, then the operator can complete about 300 calls during the busy hour.

Third. The directory listings would all be on an automatic basis. Then the automatic subscribers would dial the numbers as listed, and the call would be routed to the proper manual exchange where the last four figures would be displayed before a special operator who could associate the proper trunk with the number indicated, and by plugging into the proper terminal of the multiple complete the connection. An operator with this method of operation should complete from 550 to 600 calls per busy hour.

Fourth. The calling subscriber would proceed as in the previous case, but the call instead of being displayed before an operator, would be set up on selector, and connector switches. The banks of the connector switches would be connected to the corresponding hundreds of the manual multiple, and the connectors would be arranged to function properly with the manual multiple.

When only a relatively small portion of a network is to be converted to automatic initially, it may be more practical to use the first methods, as the conversion proceeds the second method will have its advantages. When a considerable portion of the plant has been put on an automatic basis, then either the third or fourth plans will be more suitable. Which of these two plans is followed would depend largely on the ability to finance the installation of the selector and connector equipment, because the saving in operating labor would be obvious. The fourth plan has many advantages to the operating company, because operating labor can be reduced to a minimum, even calls from one manual office to another manual office can be completed by the "A" operator in the originating office, through the selector and the connector equipment.

It should appear, after describing the various means of affording mixed service during a transition period, that all plans are feasible. A special study is required in each case to determine the plan best suited. The writer, however, feels that the plan which has in view the complete conversion in the shortest time is the one which will eventually be followed, because there are certain advantages which are apparent, but which cannot always be represented in a cost study.

Arthur Bessey Smith: I think it only right, that you should know that although the present automatic switch, to which, I referred, has a bank having only ten contacts on each level yet this does not by any means limit the number of trunks available for communicating between different offices, or trunks between different parts of the same office—all the way from 150 to 200 trunks in a group are available by suitable combinations of the apparatus, and I think I am safe in saying it will not be difficult to increase them to 500 trunks in one group. This has already been done in part.

In regard to the French situation, there is a company in France known as the Thompson-Houston Comany, which has been making some automatic apparatus and is in a position to make more.

I think Mr. Baer's discussion requires no answer from me, because Mr. Baer's is an expert in his own line, and knows whereof he speaks.

Presented at the 356th Meeting of the American Institute of Electrical Engineers, New York, December 12, 1919.

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THE SEARCHLIGHT IN THE U. S. NAVY

BY RALPH KELLY

Abstract of Paper

The types and uses of searchlights and signaling lights on naval ships are briefly described.

A changed form of 12-inch incandescent searchlight is suggested which will insure the lamp bulb filament always being at the focal point of the mirror and of the correct type for the application.

The present type of low-power searchlight has many faults. These faults may be corrected by supporting the carbons rigidly near the arc, the positive carbon being held at the focal point by a simple automatic control. The best size and material of carbons should be used regardless of the burning ratio.

The introduction of the high-power searchlight revolutionized the application of the searchlight to naval ships. Although great improvements have been made since that time there is still room for a material reduction in the number and complexity of the parts.

It is believed there is also a possibility of a considerable improvement in the electrodes.

The use of the dome glass door enables a searchlight to operate in close proximity to large calibre rifles and makes it possible to successfully build even larger sizes of searchlights than those at present in use.

The star shell has great possibilities, but it is doubtful if it will ever supersede the high-power searchlight.

THERE are three distinct types of searchlights commonly used in the United States Navy:

The incandescent searchlight,
The low-power searchlight,
The high-power searchlight.

These types are used for navigational work, for both daylight and night signaling and for illumination of enemy ships in night engagements. Each type has its field of usefulness in navy work.

The development of the searchlight for naval use was practically dormant for the few years prior to 1915.* In 1915 it was revived by the adoption of the high-intensity light for capital ships after a long series of tests under laboratory and sea-going conditions. Since that time the development has been so rapid that searchlights purchased in one year became almost entirely obsolete the next year.

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* "Searchlights" by C. S. McDowell. Transactions A. I. E. E. 1915.

The object of this paper is to describe the various types of searchlight equipment at present in use in the navy, their field of application and the paths of future development demonstrated by experience gained during the war. It is hoped that there will be a free discussion along constructive lines of the suggestions for future development of searchlight equipment for naval use.

THE INCANDESCENT SEARCHLIGHT

At present there are four separate forms of the incandescent searchlight used in naval service:

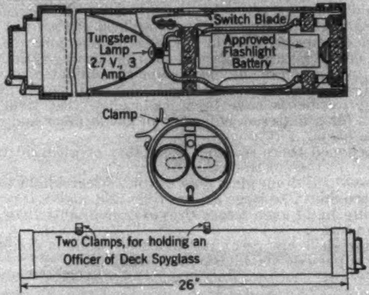
Switch Blade
Tungsten Lamp 2.7 V., 3 Amp
Approved Flashlight Battery
Clamp
Two Clamps, for holding an Officer of Deck Spyglass
26"

FIG. 1—THE PORTABLE TUBE BLINKER

The portable tube blinker,
The yard-arm blinker,
The Aldis light,
The 12-in. incandescent light.

The two blinkers cannot properly be classed as searchlights as they are used only for signaling purposes. As all sizes of searchlights are largely used for signaling, a description of these lights is included to make the article complete.

(*a*) *The Portable Tube Blinker*, Fig. 1, is extensively used for concealed local signaling at night between ships in column, between neighboring columns of ships or between landing parties on hostile coasts and their supporting ships. It is accordingly designed to throw a feeble beam of light of a very contracted area in order to avoid its being observed at long ranges or by anyone at whom it is not directly aimed.

It consists of a long brass tube with a small flash lamp bulb placed at one end at the focal point of a comparatively large parabolic reflector with a short focal length. The reflector is designed to project practically all the light from the lamp bulb in the form of a small diameter beam to the neighboring ship or station. The inside of the tube is painted black and has a long extension between the lamp bulb and the opening in the tube end. This tends to eliminate all stray light, keeping the beam to a small diameter. This form of signaling light performs its duties well and is universally used in naval service.

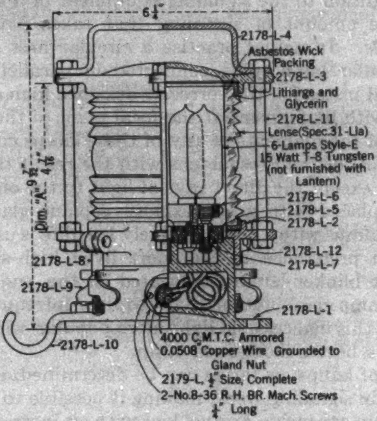
6 1/4"
Dim. "A" 9 7/16"
4 7/16"
2178-L-4
Asbestos Wick Packing
2178-L-3
Litharge and Glycerin
2178-L-11
Lense(Spec.31-Lla)
6-Lamps Style-E 15 Watt T-8 Tungsten (not furnished with Lantern)
2178-L-6
2178-L-5
2178-L-2
2178-L-12
2178-L-7
2178-L-8
2178-L-9
2178-L-1
2178-L-10
4000 C.M.T.C. Armored 0.0508" Copper Wire Grounded to Gland Nut
2179-L, 1/2" Size, Complete
2-No.8-36 R.H. BR. Mach. Screws 1/4" Long

Fig. 2—Yard-Arm Blinker Light

The only fault that has developed in its use is a minor one which may be easily corrected. When used on a small boat forming an unsteady platform, such as a destroyer or submarine chaser, it is very difficult to keep the blinker directed at the desired receiving station due to the blinker rolling on the rail. A rectangular cross section of blinker or a ball and socket rest on the rail for the present style of blinker would improve the operation.

(*b*) *The Yard-Arm Blinker*, Fig. 2, as its name implies, is a blinker lantern mounted on the yard-arm of a ship and which gives out light that is visible all around the horizon. It is widely used for night signaling in times of peace and in protected waters in times of war. The lantern is designed to contain a high-speed incandescent light with a filament which is placed at the focal point of a Fresnel lens. Signals are sent by blinking the light with a telegraphic key which alternately makes and breaks the circuit. The lamp bulb used in this lantern has been specially developed for Navy signaling work with a concentrated filament that heats and cools very rapidly.

This type of blinker has given general satisfaction and is a valuable addition to the signaling equipment of a ship in view of the large amount of signaling that takes place between naval vessels. In actual practise a circular nest of six commercial tubular lamb bulbs, connected in parallel across the line, are used instead of the correctly designed high speed signal lamp bulb with its filament at the focal point of the lens.

No part of the filaments of any of these lamps coincide with the focal point of the Fresnel lens with the result that the lens disperses all except the direct rays of light to the observer, and could be replaced with a less expensive plane glass cylinder. The nest of lamps with a comparatively long life is used by navy personnel in preference to the short lived high-speed signal lamp, as the blinker lantern is located at the end of a yard arm. Any lamp combination that requires infrequent replacements is for this inaccessible light more popular with the bluejacket than a better short-lived lamp.

The nest of lamps, as far as can be determined, gives entire satisfaction in signaling work, making it possible to replace the comparatively expensive Fresnel lens with an inexpensive plane glass cylinder.

(*c*) *The Aldis Lamp*, Fig. 3, is a projector of British design in which advantageous features are incorporated that make it superior to similar lamps previously used in naval service. While primarily designed for daylight signaling, it has been used successfully as a searchlight and for signaling in airplanes and dirigible ballons.

The projector itself is a light, well balanced, portable affair of aluminum which signals by alternately throwing a searchlight beam on and off an observer. This is accomplished by rocking the mirror back and forth by means of a trigger, the lamp bulb burning continuously. The lamp bulb is manufactured to such close limits that when it is screwed into a special socket in the lamp, its filament is in the focal point of the mirror.

This type of signaling lamp is universally used on ships, shore stations, dirigibles and airships. Its success is due to its high signaling speed, its compactness and portability, but most of all to the fact that, regardless of the personnel handling it, the lamp filament is always at the focal point of the mirror and a long range efficient beam is obtained at all times.

(*d*) *The 12-in. Incandescent Lamp* Fig. 4, is largely used in naval service on all classes of naval vessels for long range daylight signaling in fair weather, and for navigational use at night on small boats. This type of light has not been generally successful in its present form due to the fact that it requires special adjustments and care in service, for successful operation.

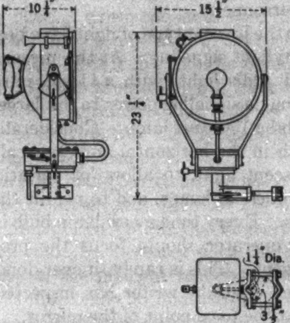

Fig. 4

It has a fixed parabolic mirror with a high-speed signaling lamp bulb which screws into a standard socket. The socket can be adjusted to place the filament of a commercial lamp bulb in the focal point of the mirror. This adjustment is made large to permit the use of several different sized lamp bulbs ranging from 150 to 400 watts. The whole is contained in a cylindrical drum which is commonly made of sheet iron. Signals are sent by blinking the filament of the lamp bulb by a telegraphic type of key.

This lamp has proved successful for navigational work on small craft and particularly so for submarines. A small 9-in. arc lamp was specially developed for submarine use, but its lack of portability, attention required in operation, and necessity for frequent recarboning caused it to give way to the incandescent searchlight. An important factor that entered into this change is that a submarine is frequently submerged without there being sufficient time to dismount the searchlight and stow it below. An incandescent searchlight can be submerged without permanent damage, while an arc lamp is placed out of commission. The small amount of care required for the incandescent type of searchlight, and the comparatively long life of a lamp bulb in comparison with a trim of carbons, makes a correctly designed light of this type ideal for use on small craft with limited personnel.

The faults in the present type of light have been demonstrated by its use in daylight signaling. At the present time the correctly designed Aldis light, with a 4½-in. mirror and 40 watt lamp, is the equal in signaling power to the larger light with its 11-in. mirror and a 150-watt lamp. Considerable adjustment is permitted, both in the horizontal and vertical planes of the 12-in. incandescent light, to allow for variations in filament location in commercial lamps, and to permit the use of various size lamp bulbs. Every time a new lamp bulb is inserted in the projector, the operator should focus the projector for that particular filament. This is rarely, if ever done, and in service in not one lamp that the writer has inspected has the lamp filament been at the focal point of the mirror. In the majority of cases it has been approximately one inch out of focus in each plane. Of course in this condition the mirror is of no value and only the direct rays of light from the filament reach a distant observer.

The high-speed concentrated filament lamp bulb specially designed for this searchlight is almost identical in appearance to a concentrated filament stereoptican type of lamp bulb designed only for searchlight work. Long life being desirable in the latter type of lamp, its filament cools and heats very slowly and it cannot be used at all for signaling work. Due to the similarity in appearance, both types of light are issued indiscriminately for the 12-in. searchlight. The personnel, not realizing that there are two entirely different types of lamps of similar appearance, condemn them both and use a commercial Mazda lamp which happens to fit the same socket, but in which no part of the filament comes any where near the focal point of the mirror.

The design of a new type of light along correct fundamental lines, is contemplated to be arranged so that it cannot be used incorrectly by naval personnel. The same type and size of mirror will be used in a brass non-corrosive drum. The lamp socket will be permanently fixed in such a position that when a 150-watt, high-speed lamp bulb is placed in it, some part of the filament will be in the focal point of the mirror. The ordinary high-speed 150-watt lamp is manufactured with a dimension across the filament of approximately ¼ inch so that in the great majority of cases with such a lamp, some part of the filament will be in the focal point of the mirror, even with ordinary commercial limits of manufacture.

To eliminate the use of lamps other than that specially designed for this class of work, the lamp bulb must be of such design that no other lamp can be used in the socket. One way of accomplishing this object is to make the signaling lamp with a bayonet base. Similar precaution will have to be taken in a 12-in. incandescent searchlight that will be used only for searchlight work to provide only for the use of a concentrated-filament, long-life, stereoptican light.

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Attempts have been made to extend the use of the incandescent searchlight to larger sizes such as 18- or 24-in. using larger wattage lamps. To obtain the larger lamp capacity; the filament area must be increased with the result that the intrinsic brilliancy of the filament remains approximately the same and there is practically no increase in light. There is no attempt being made at the present time to increase the size of the incandescent searchlight beyond the 12-in. size.

LOW-POWER ARC LIGHTS

The number of sizes of low-power arc lamps in navy service at the present time is legion, a 9-in., 12-in., 18-in., 24-in, 30-in., 36-in. and even a few 60-in. are used. The large number of sizes is a natural growth during a long period of years, new sizes having been developed for special applications.

The 9-in. size was developed specially for submarine work and is practically the 12-in. size of lamp with a few necessary modifications to fit it to the smaller drum. It has recently been superseded for submarine use by the 12-in. incandescent searchlight.

The 12-in. size is the most commonly used of all searchlights in naval service. It is used principally for long range daylight signaling on all naval vessels and for navigational lights on small boats. It has a wide field of usefulness and practically every naval ship has at least one of these lights as part of its equipment.

The 18-in. searchlight prior to the present war was used only on merchant vessels. Although the present type is not well constructed, a large number of them were purchased when the United States entered the war as there was a great shortage of searchlight material at that time, and a large number of ships to be equipped.

The 24-in. low-power light has a limited application for navigational work on naval vessels, such as hospital ships, fuel ships and tenders of all types and a certain field for long range daylight signaling.

The 30 and the 36-in. low-power searchlights were placed on battleships and cruisers prior to the development of the high-power searchlight. They are rapidly being replaced by high-intensity lights or being converted into high-intensity lights and will shortly be obsolete in naval service.

It would be of great advantage in the future to use only one size of low-power searchlight, 12 in. or 18. in., developing a searchlight along fundamentally correct lines to give a far greater beam candle power under operating conditions than the present 12-in. light. It is believed that a new 12-in. light can be developed to give as great illumination on the target as the present 24-in. low-power searchlight.

All sizes of low power searchlights in service are essentially the same in design, the mechanism differing only in size. A typical mechanism is shown in Fig. 5. The positive and negative heads are each supported on a separate movable carriage. The carriages are separately geared to a motor or solenoid which feeds the two heads together at a predetermined fixed rate, which is supposed to correspond to the burning away of the carbons. The specifications for purchasing carbons call for carbons that burn at fixed rates which, combined with proper gearing in the lamp mechanism, will theoretically result in the positive crater being in the focal point of the reflector at all times. The arc length is kept fairly constant by a balanced type of feeding mechanism that feeds the negative carbon forwards or backwards when the arc voltage is higher or lower than normal. The present size carbons are of comparatively large diameter, slow burning and with a low current density.

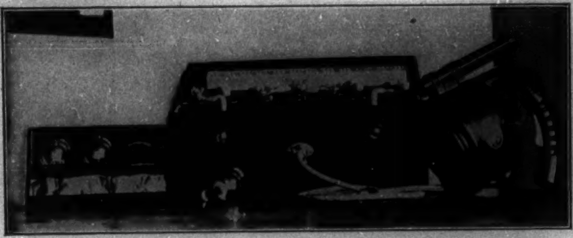

Fig. 3a—Complete Aldis Lamp Equipment

Fig. 3b—Aldis Lamp in Service

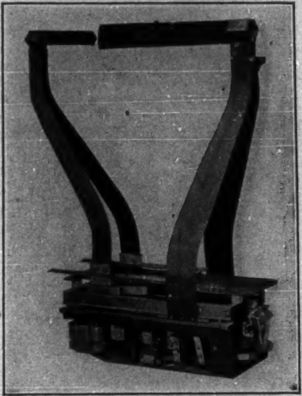

Fig. 5—Lamp Mechanism for a Low Power Searchlight

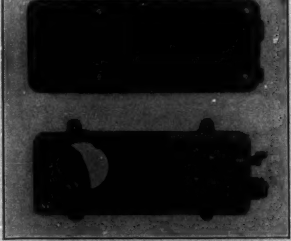

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Fig. 6—Thermostat Box and Cover

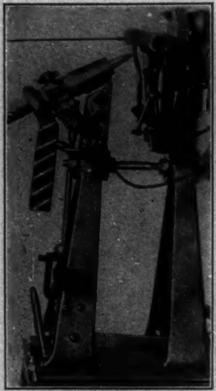

FIG. 7—SEARCHLIGHT LAMP WITH THIRD ELECTRODE CONTROL

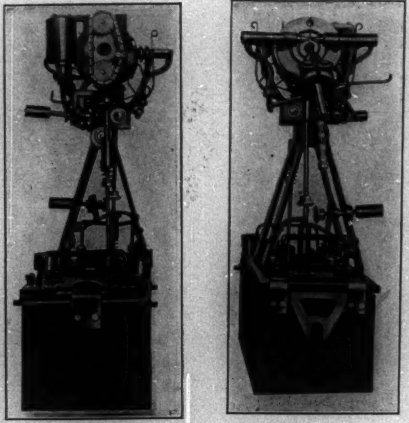

FIG. 9A—ARMA 75-AMPERE HIGH INTENSITY SEARCHLIGHT LAMP TYPE AA

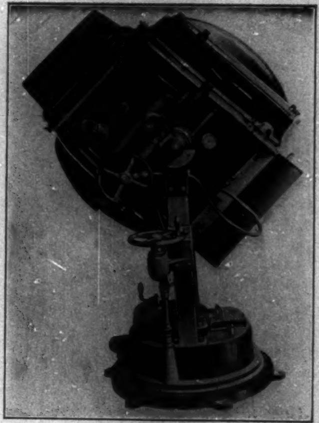

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FIG. 9B—ARMA 24-INCH HIGH INTENSITY SEARCHLIGHT TYPE B

This design of searchlight lamp has many inherent fundamental faults that may be easily corrected. Its principal fault in operation is that rarely if ever is the crater of the positive carbon at the focal point of the reflector. When it is considered that a cross-wise deviation of 1/16 in. of the crater from the focal point of the projector will give a beam of approximately double the width and one quarter the intensity of a standard searchlight beam, it is realized how important it is to maintain the crater of the positive carbon at the focal point of the reflector. Carbons cannot be manufactured commercially to give the exact burning rates specified, but vary from that rate as much as 20 per cent in each direction in extreme cases. Specifying the burning rate of carbons is a distinct handicap to a carbon manufacturer tending to hinder the development of better carbons.

The carbons themselves are of comparatively large diameter producing an arc that has a tendency to wander over the face of the positive crater, resulting in an unsteady beam and a comparatively cool, low-intensity source of light for the searchlight beam. The intensity of a searchlight beam varies with the temperature of the positive crater which makes it desirable to have a carbon current density which will give the greatest temperature at the positive crater. Each grade of carbon has a definite boiling temperature and an increase in current density beyond that necessary to produce that temperature only causes the carbon at the crater to boil away faster with no increase in crater temperature. An ideal condition, therefore, is to use a carbon with the highest boiling point and a current density that will just produce a crater temperature that is equal to the boiling point of the carbon.

The carbon heads themselves are not rigidly supported and have a marked play in all directions. This play is so marked that the rolling of the ship will change the position of the carbons in the searchlight drum. The current is fed into the carbons at their butt ends and flows through the entire length of the carbon to the arc. This tends to heat the carbon throughout its entire length which often causes it to spindle. It also results in a variable arc length, for with a short length carbon the voltage across the arc is materially greater than with new long length carbons.

A correctly designed lamp mechanism for a searchlight should produce the longest arc length possible at which the arc will burn steadily, to reduce to the lowest possible value the shadow cast on the mirror by the negative carbon. The appreciable variation in arc length in the present type of low-power searchlight results in the negative shadow being materially greater during the majority of the burning of a trim of carbons than the best obtainable condition.

A 12-in. low-power searchlight could easily be designed to overcome the faults in the present low-power searchlights. Such a light would not only give a more intense beam than the present 12-in. searchlight under laboratory conditions, but in actual service its more exact mechanism would give a searchlight beam of very materially more power than the present inexact mechanism. The carbon heads in the proposed light would be rigidly fixed in position with absolutely no play nor adjustment, would have a simple automatic control of the positive crater to maintain that crater in the focal point of the reflector at all times, and would have the current fed into the carbons close to the arc. As in the present 12-in. searchlight, the arc length could be controlled by a solenoid feed. A ground glass finder should be provided to enable an operator to check the position of the positive crater.

The automatic control of the positive and the voltage control of the arc results in the carbons being fed independently of their burning rates. Accordingly, if improved carbons with different burning rates are developed they may be used in this lamp with practically no change in the lamp mechanism, a condition which is not possible in the present type of lamp. The carbons would be manufactured with diameters in the neighborhood of one half the present size, which would result in a higher temperature of crater, a closer approach to a point source of light and a smaller negative shadow on the mirror. These carbons should be made of the best combination of materials of the highest boiling temperature that will maintain a steady arc. These searchlights should be equipped with a Venetian blind signaling shutter to permit their being used for daylight signaling as well as for navigational work. Such a light would be suitable for daylight signaling and navigational work on battleships, cruisers, auxiliary ships, destroyers, seagoing tugs, transports; in fact for all naval vessels it would supersede the numerous sizes of low-power searchlights now in naval service.

The change would be of advantage both to the manufacturers and to the navy. The manufacturers will have to develop and keep in stock but one set of tools and patterns and can concentrate on the improvement and development of that one type of searchlight. The navy will benefit by having to keep a stock supply of only one size of low-power searchlight, which will require a stock supply of only one kind of spare parts and accessories like mirrors, lamps, rheostats, switches, shutters, etc.

HIGH-POWER ARC LIGHTS

The invention of the high-intensity arc revolutionized the searchlight for naval use. The 30-in. and 36-in. low-power searchlights previously used had a maximum range of approximately 2000 yards on a dark, clear night. The present 24-in. high-power searchlight has a range of approximately 4000 yards and the 36-in. high-power searchlight has a range of approximately 7500 yards under similar conditions. It is evident that the the low-power lights are useless in gunnery work with modern, broadside defense guns, while the ranges of the high-power lights compare very favorably with those guns.

The high-power searchlight swings to the other extreme from the low-power searchlight in that it is a very complicated exact mechanism with a large number of intricate parts which are designed to operate automatically. Its success depends on the so called high-intensity phenomena obtained by the use of carbons with impregnated flame cores with high current densities, and arranged at an angle to each other. The material of the shell of the carbon is of hard amorphous carbon, the inside being a combination of special mineral salts with the necessary flame producing ingredients. At the beginning the core burns more rapidly than the shell, forming a deep crater which becomes filled with the flaming gases of the core. The combined effect of the high current density of the arc and the angle of the negative arc stream is such that these gases are imprisoned for a short period in the positive crater. These gases are heated to incandescence while imprisoned in the crater, and give a light that is far in excess in intrinsic brilliancy of that given off from the solid crater walls. This increase in intrinsic brilliancy of the source of light for the searchlight results in the superiority in range of the high power searchlight.

The carbon heads are rigidly fixed in the drum with mechanisms included in those heads for feeding the carbons. Contacts placed in the carbon heads, close to the arc, feed current in and out of the arc circuit. The arc length is kept approximately constant by a motor feed that is controlled either by the arc voltage or by arc current and feeds the negative carbon either backwards or forwards to maintain constant arc conditions.

The positive crater is kept at the focal point by automatic devices, either of the thermostatic type or of the third electrode type. Two of the companies manufacturing high power searchlights use a thermostatic control of the positive carbon while the other uses the third electrode control. In the thermostatic device, light from the positive crater is reflected by an optical arrangement of mirrors and lenses to thermostat strips placed in a box on the outside of the searchlight (Fig. 6). When the positive carbon burns back slightly from the focal point of the mirror, the reflected light strikes one of the thermostat strips, and distorts it by the heat. The thermostat strip in distorting closes an electric circuit which causes the positive carbon to feed forward. When it is fed forward to the focal point, the reflected light from the positive crater moves off the thermostat strip which cools instantly and opens up the feeding motor circuit.

This device has worked well in naval service and under the best conditions of operation will maintain the positive crater at the focal point within limits of 1/32 inch, plus or minus. The present thermostat mechanism is mounted on the outside of the drum in a box which is exposed to the weather. In every case in which the writer has inspected searchlights that have seen service, there has been either a considerable amount of water in the box or evidence of water having been recently in the box. Again, the searchlight lamp and the thermostat have a definite relation to each other and an adjustment of one requires a separate and independent adjustment of the other. A thermostat placed inside the searchlight drum and secured to the lamp standard would remedy both of these defects.

The third electrode scheme for controlling the positive crater is more simple and substantial than the thermostat and operates well under service conditions. This device (Fig. 7) consists of a carbon electrode placed above and at right angles to the positive carbon slightly behind the crater. When the positive crater burns back from the focal point, the flame of the arc strikes the third electrode completing an electric circuit of which the conducting flame of the arc forms a part. Current flowing in this circuit causes the positive carbon to feed forward. When the crater is fed to the focal point of the reflector, the flame clears the third electrode and the feeding motor circuit is broken. This type of positive carbon control has the advantage of being placed inside the searchlight drum out of the weather, and of being integral with the positive head requiring no separate adjustments.

The first type of third electrode developed was of carbon which required renewal every three trims of the searchlight carbons. There has since been developed a third electrode which is a copper disk, and which has a very much longer life than the carbons. In operation, a coating forms on this copper disk which protects the copper from burning and prolongs the life of the disk.

The intense heat of the high-intensity crater and the large amount of smoke and fumes emitted from the flame carbons make it imperative to have forced ventilation of the carbon heads and searchlight drum. An early type of high-power searchlight used the comparatively cool vapor of an alcohol flame to cool the carbon tips and the drum was exhausted by a fan connected to a separate motor. The cooling of the high-intensity searchlight was later improved by making hollow lamp standards, jacketing the carbon heads, and forcing air through these chambers and out through an outlet at the top of the searchlight drum, thus doing away with the alcohol flame method of cooling.

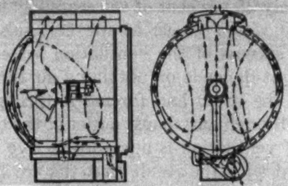

FIG. 8—DIAGRAM OF VENTILATION OF A HIGH POWER SEARCHLIGHT

In the first high-power searchlights great difficulty was experienced due to the large heat absorption and unequal heating of the mirror and a great number were smashed due to these causes. This condition was finally remedied by efficient cooling of both the front and rear of the mirror by a current of air. The ventilating system of a modern high-power searchlight, showing the paths of cooling air around the mirror, is shown in Fig. 8. The heating of searchlight mirrors has been reduced during the past year by the development of a tough white glass of pure materials which has approximately one per cent heat absorption against the five per cent absorption of the grade of glass previously used for mirrors.

Three typical searchlights developed by the three present manufacturers of high-power searchlights are shown in Figs. 9, 10 and 11. An inspection of these illustrations shows the progress in development of these exact well designed mechanisms over the less refined, inexact, low-power searchlights.

The 36-in. high-power searchlight is the largest size used on shipboard for it gives a range comparable to the largest guns and is the largest, at the present time, that will withstand the heavy concussions of the ships gunfire. It is only very recently that the searchlight has been adapted to withstand the concussion of the gunfire of a nearby 14-in. gun. Prior to this time searchlights were equipped with glass front doors made of a number of plate glass strips. This type of front door cut off approximately 25 per cent of the searchlight beam, was not watertight and would smash at the first salvo of the main battery. The success of the ventilation system of the high-power searchlight depends on obtaining air through a duct in the bottom of the drum and forcing it through well defined paths and out through an exhaust outlet at the top of the drum. This scheme of ventilation is made ineffective when the front door is smashed with the result that the arc flickers and the searchlight parts become overheated due to an insufficient supply of cooling air.

The smashing of the front door strips leaves the mirror unprotected and it is an even chance that the mirror will smash at the next salvo from the main battery. It is common practise on shipboard to dismantle searchlights equipped with plate glass front doors before a target practise, and stow them away in a safe place.

This condition has been remedied by the use of a front door made of one solid piece of arch shaped glass similar in shape to the parabolic mirror. This type of door has given excellent service in the fleet and not one of the many dome glass doors that have been installed on navy vessels has been smashed during target practise. This type of door cuts down the searchlight beam no more than the plate glass strips and makes the searchlight entirely self-contained and waterproof.

The trend of future development of the high-power searchlight should be in the simplification and the reduction of the

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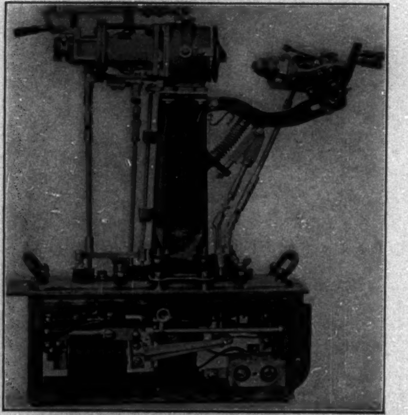

FIG. 10A—LAMP FOR 24-INCH HIGH POWER SEARCHLIGHT

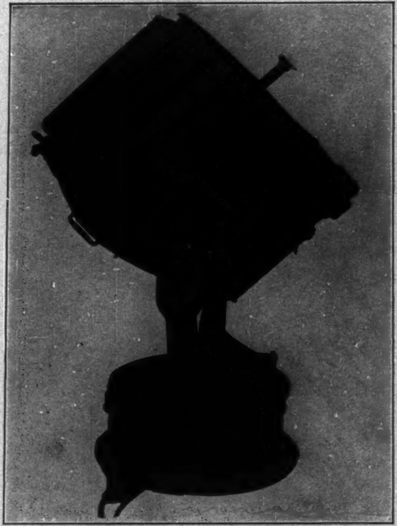

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FIG. 10B—24-INCH HIGH POWER SEARCHLIGHT

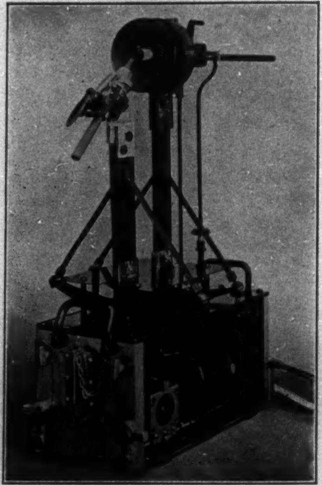

Fig. 11a—Lamp for 24-Inch High Power Searchlight

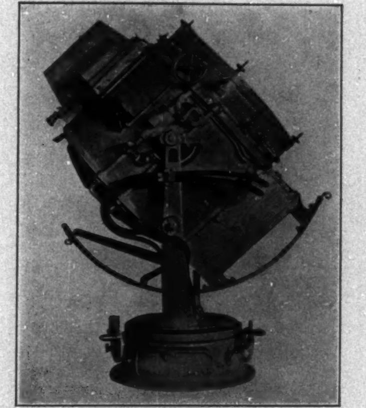

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Fig. 11b—24-Inch High Power Searchlight

large number of intricate parts on the present searchlights. A searchlight must necessarily be placed in a position exposed to the weather, and will be operated by enlisted personnel. It is a struggle to maintain the present complicated high-power searchlight mechanisms in operating condition under the severe conditions of operation on smaller boats like destroyers. After a few days at sea on a destroyer in rough weather, the present high power searchlights often become inoperative and require a number of hours overhaul to place them in operating condition. Such a searchlight soon loses its popularity with destroyer commanders and they justly complain that the high-power searchlight is more suitable for laboratory use than for destroyer use.

There is still room for a large improvement in the size and material of carbon electrodes for high-power searchlights. At the present time a carbon is used which has a hard carbon shell and a core made of a combination of flaming materials. This core gives off gases which are heated to incandescence in the arc and give the high intrinsic brilliancy light characteristic of the high-power searchlight. It also gives off a very brilliant flame which causes a correspondingly brilliant objectionable foreground illumination and gives off gases that tend to blacken the mirror and the front door.

It is a well established fact that the color and intensity of a searchlight beam is dependent only on the temperature of the source of light. The use of flame-colored salts in the electrode gives a high temperature but also burns with a large brilliant flame. It is believed that a proper selection of carbon materials, perhaps the use of a solid electrode of a very refractory carbon with a small negative carbon, will give the same amount of light that is obtained from the present high-power type of carbons. The flame from a combination like these would be practically transparent with no objectionable foreground illumination, the objectionable gases would be eliminated and the present type of very special carbon electrodes could be done away with.

The star shell for gunnery work is also a factor in the development of high-power searchlights and bids fair to rival the larger high-power searchlights for gunnery work. A searchlight, unless skilfully and carefully manipulated, is more of a debit than a credit in action as it tends to give an enemy a definite point at which to aim. A star shell that satisfactorily illuminates the target, leaving its own ship in darkness, has many decided points of advantage over a large high-power searchlight.

However, even if the star shell gives the results claimed for it, it will be a long time if ever before naval authorities will feel safe in removing the 36-in. high-power searchlight from capital ships. The 24-in. high power searchlight has had such a marked success in long range daylight signaling that it is improbable it will ever be superseded for naval use.

The use of the dome glass door for 36-in. searchlights opens up a field for the use of larger size searchlights, such as the 44-in. and 60-in. sizes for capital ships. It is believed if those larger sizes are ever required for naval service they can be successfully developed.

In closing the writer wishes to express appreciation to Major Howard C. Judson, U. S. M. C. for ideas contributed on carbon development.

DISCUSSION ON "THE SEARCHLIGHT IN THE U. S. NAVY" (KELLY), NEW YORK, N. Y., DECEMBER 12, 1919.

Donald McNicol: Mr. Kelly says that a special type of incandescent lamp is used. Is the novelty of the lamp in the design of the filament or the material of the filament?

G. A. DeGraaf: I ask Mr. Kelly whether the navy has been experimenting with the all-metallic reflector for the searchlight mirror, as the army has been using it. I think it is a glass mirror which has been used in the searchlight which is apt to be destroyed, due to the compression of the heavy gunfire.

M. L. Patterson: Under the heading of "Low power arc lights" the author of this paper has taken up the discussion of 12-in. low power searchlights, as more or less typical of a completed line of low power searchlights. It is fully understood that high power searchlights are superseding the low power searchlight at 24-in. size and above. I do not, however, agree with the author, as stated in paragraph 7, that there should be only one size of low power searchlight, for there is a large field of usefulness for at least two sizes, preferably 12 in. and 18 in.

In paragraph 7 the author states that it is believed that a new 12-in. light can be developed to give as great illumination on the target as the present 24-in. low power searchlight, and I wish to bring out a few facts to show that it is not practicable to accomplish this result. The current and the arc voltage of the two sizes referred to are as follows: 12-in. is 20 amperes and 45 volts across the arc. 24-in. is 50 amperes and 50 volts across the arc.

Since the beam candle power varies as the area of the mirror or as the square of the diameter of the mirror, other conditions being equal, it will readily be seen that for the beam of a 12-in. searchlight to equal that of a 24-in., the intrinsic brilliancy of the crater of the 12-in. must be at least four times that of the present 24-in., or over twelve times the intrinsic brilliancy of the present 12-in. light. I hesitate to quote beam candle figures as data obtained by observers under various atmospheric and operating conditions, show such a wide divergence that any figures on beam candle power, unless accompanied by a statement of the conditions under which they were obtained would be valueless. Nearly all observers, however, agree as to the relative intensity of various sizes of searchlights, and it can, therefore, be stated that the beam candle power of the 24-in. high power searchlight is approximately four times that of a 24-in. low power searchlight. From this it is evident that for the beam of a 12-in. low power searchlight to equal that of a 24-in., it would be necessary to use a 75-ampere high power arc in front of the 12-in. mirror, and it is our experience that it would be utterly impossible to do this as no glass mirror at present manufactured would stand the intense heat of a

75-ampere arc within 6 in. of the mirror, 6 in. being the focus length of the 12-in. mirror.

It should further be noted that an increase in current or an increase in size of the positive crater may not increase the beam candle power to any appreciable extent. It may sound paradoxical, but it is a fact nevertheless that the beam of a 60-in. searchlight is smaller than a beam of the 24-in. at or beyond 500 yards from the searchlight. This is due to the fact that the size of the positive crater of a 24-in. relative to the size of the mirror is greater than in the 60-in. searchlight. This would be true to a greater extent with the 12-in. searchlight, especially if an attempt is made to materially increase the power. The extra candle power would go into beam spread rather than materially increase the foot candle illumination on the target.

It is evident from this paper that the author favors the design of the 12-in. searchlight which will be entirely automatic and have fixed positive heads. If this were carried out, it would be necessary, of course, to have brushes for leading the current to the carbon, which would materially increase the size of the carbon holder. This construction would also require a mechanism for advancing the carbon through the brushes and would require considerable more power than is now required to move the carriages holding the positive and negative columns.

In reference to the current being fed in at the butt ends of the carbons, as referred to in paragraph 11, it has been our experience that there is very little tendency for the carbons of a 12-in. arc to spindle when operating under normal current. The current density in these carbons, as referred to in the last line of paragraph 8, is very low. As regards the variation in arc length, it may be stated that the amount of carbon consumed per trim is from 4 in. to 5 in. and the drop in this length of carbon with the low current density as used in these searchlights will be the only variation in arc voltage (and length). This variation is less than the variation in voltage required to operate the present feeding mechanism. This is, the variation in arc length due to the range in feeding mechanism is greater than the variation due to the drop in the carbons.

The very reason that the 12-in. low power searchlight has had such extensive use, as referred to by the author in paragraph 3, is due to its simplicity and inexpensive design, and while there is no doubt considerable improvement that can be made in the design without materially increasing the complication of the mechanism, it would be a mistake to carry this design to the extent of a completely automatic low intensity mechanism. For automatic control it would require fixed heads, feeding mechanism for advancing the carbons through the heads, a thermostatic or third electric control of positive crater, and ground glass finder, and with all this added complication, many of the advantages of the 12-in. low power searchlight would be lost.

Another point that should be noted is that with the present type of carbon holder, only short length carbon is wasted, whereas if fixed positive heads are used there will be a considerably greater length of wasted carbon per trim.

In paragraph 12, it is stated that it is quite essential to keep the size of the negative shadow at a minimum, and in this connection it should be noted that a change of negative carbon holder to a fixed negative head with feeding mechanism would necessarily increase the size of the negative shadow.

Under the heading of high power arc lights, the writer in the first paragraph has made a comparison in the ranges of the various searchlights, and gives these ranges in a definite number of yards. A statement of range without giving the illumination at the target or the kind of target that would be visible at a certain range, is meaningless.

In paragraph 4, under "high power searchlights," it is stated that two of the companies manufacturing high power searchlights use thermostatic control of the positive carbon, while the other uses the third electrode. In this connection it may be stated that at least one company manufacturing high power searchlights uses both the third electrode control and the thermostatic control. The third electrode feed being used where it is not essential to keep the positive crater in exact focus and the thermostatic feed used where more accurate control of the positive crater is required, as for United States Army and Navy uses.

E. J. Murphy: With reference to the high powered searchlight, I note that the paper* of C. S. McDowell has been quoted and there is no doubt that the type of light described in Mr. McDowell's paper has revolutionized searchlight practise especially for Naval use. Since the advent of this new light, the necessity of using the beam at high angles for defence against hostile air planes made it necessary to abandon the alcohol flame, otherwise the light has remained practically as originally designed. The principles involved have remained unchanged.

I would like to refer to the thermostat control mechanism in Mr. Kelly's paper, I note that he states that this device will hold the positive crater at the focal point of the mirror within one-thirty-second of an inch, plus or minus. He does not state, however, how close the third electrode scheme will hold the positive crater, and it might be interesting to find how accurately this method of control functions.

With reference to the ventilation, I see Fig. 8 shows a pressure system of ventilation. It might be interesting to note that two types of ventilation have been evolved, one the vacuum type, the other the pressure type, and on page 1618 he states that the ventilation will be destroyed in case the front door is broken. The company I am connected with has tried out both of these systems, and tests have been made resulting in our discarding the vacuum system on account of certain advantages of the pressure system. The pressure system enabled us to use the fan motor for driving the mechanism of the searchlight itself, thus eliminating any additional motor.

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*Trans. A. I. E. E., 1915, Vol. I, p. 263.

There is another thing which we actually tried out, and that is the effect of wind against the front of the searchlight with a broken door. We noted that with the vacuum system, the arc very frequently was ruptured, making the light practically useless, whereas with the pressure system, the pressure inside of the drum acted against the wind, and the light was operative in a thirty mile wind; under the same conditions the light with the vacuum system was practically useless.

As far as the dome glass door is concerned, I understand there is still some question as to its practicability, on account of refraction of light, etc.

I note that certain illustrations show a thermostat box and the paper mentions that this box is not watertight. This thermostat box is part of the searchlight with which I am familiar. I may say that this searchlight was developed under stress of war conditions, and it was necessary to develop the design in the shortest possible time; we did not provide gaskets for this particular box, but, notwithstanding this omission, I have not found a single case where water rendered these searchlights inoperative. In fact, I am aware that a great many of these searchlights have been at least once across the Atlantic, under heavy weather conditions, and in no case was the searchlight inoperative. Under war conditions, the personnel was not always acquainted with new apparatus such as the thermostat control and had not been previously instructed in the operation of this new searchlight. I have seen reports on at least fourteen of these lights, all of which had been subjected to heavy weather conditions, in no case did any light actually fail in service.

These lights were developed for use on destroyers under stress of war conditions and over 300 were delivered to the Navy before the armistice was signed, we kept ahead of the destroyer program, and this was extremely important, as we all know, on account of the submarine menace.

On the next to the last page of Mr. Kelly's paper there is mention as to whether the searchlights are really useful at sea. There has been some doubt of their value in the past, but reports from Admirals Beatty and Jellicoe, of the British showed that the Germans had superior equipment, and the British avoided battle at night—that is because the Germans had developed tactics that enabled them, when a hostile ship was observed to open the searchlight shutters with the light already directed on the target. This was possible due to highly developed fire control system, which is being now installed in our own ships and has already been adopted by the British

Navy. With this system the light is only shown long enough to fire a single salvo and not long enough to make a target of the vessel. There is at present no doubt that in naval practise, searchlights are absolutely necessary for night fighting.

J. C. Ledbetter: I am very much interested in the high intensity light, and I want to know whether a light of such brilliancy and such marvelous performance could possibly be used in some commercial way. Could not a light of this character be used in stage lighting, or flood lighting, or in some spectacular way, such as street light and advertising lighting in such places as Times Square. It might be that large business concerns would have use for a light of this character.

R. A. Beekman: I wish to point out one feature with regard to the difference between the third electrode and the thermostatic control. The principle object, of course, of both of them is to maintain the crater in the focus of the mirror. The mirror is mounted rigidly in the drum of the searchlight, and hence the logical thing is to have the device, controlling its position, mounted on the drum of the searchlight, which is the form used with the thermostat in the past. It is now proposed to mount the thermostat on the lamp, as is done with the third electrode, and this does not provide a rigid coupling between the position of the positive crater and the focus, as is the case when the thermostat is mounted on the drum.

B. P. Beehler: (Read by R. A. Beekman). I desire to call attention to the following points in Mr. Kelly's paper:

Paragraph 1. The range of a 24-in. high power light is at best 3000 yds.

Paragraph 2, 5th sentence. The current density has no connection whatever in the formation of the gas crater or holding the gas. This feature is entirely controlled by the direction of the arc stream. The high current density of the positive electrode may be maintained without an arc stream or gas ball formation.

Paragraph 4, last sentence. The feeding of the positive carbon in lamp operation is without exception, where automatically controlled, accomplished by independent solenoid or magnet operation.

Paragraph 7, last sentence. The formation of scale or oxidation of metallic third electrode does not increase its life. The prevailing feature permitting of the use of a metal third electrode is the fact that the temperature of flame directly back of the crater is below the melting point of the copper. The metallic electrode or any type of this control has proved unsatisfactory for any arc of over 75 amperes. This 75-ampere arc is not a true gas arc and therefore the maximum temperatures are considerably lower than high current arcs. The use of metallic third electrode has been disapproved by the U. S. Navy for all arcs over 75 amperes.

Paragraph 8, second and third sentence. The object of alcohol in the high power arc is not to cool but to surround the electrodes with an oxygen insulator, thus preventing tapering of electrodes from air burning and enabling the burning of the electrode with its tip far removed from metallic positive head and still maintain the crater at electrode diameter.

The cooling by air, later developed, is merely cooling of the metal, allowing the use of a metal head projecting over the electrode nearly to its tip, which metal insulates the electrode from atmospheric oxygen and thus eliminating the necessity of the alcohol. The air cooling also allows of use of less metal in heads, thus cutting down mirror shadows and beam holes. The high power lamp using alcohol on the electrode had very large and heavy heads, the cooling of which was accomplished by radiation.

Paragraph 11, 4th sentence. The absorption of strip glass in the front door did not exceed 15 per cent and averaged about 10 per cent absorption of light.

Paragraph 13. The dome front doors of the present parabolic and spherical types have proven inefficient. Their disadvantage is that they not only absorb about 15 per cent of light but they actually disperse about 15 per cent of light, throwing this light on the foreground, changing beam formation by virtue of the above fact and blind the observer by making a field of stray light through which he must look to see the target

Paragraph 15. It has been found that the present sizes of electrodes are probably the proper proportions to give best service. A rise in current density, at present about 1.65 amperes per sq. mm. will only add to flame and increase carbon consumption, while reducing the density by increasing the electrode size will cause loss of intensity.

Paragraph 16. It has not yet been established conclusively that the intensity of the high power arc is a factor of temperature. A theory of fluorescence of the gas ball has presented itself indicating that the supposed black body radiation of the arc is not a fact, but that a composite effect of black body radiation and fluorescence tend to give spectrum characteristics which are misleading, and indicate temperatures far in excess of those actually present.

It has been found that the use of solid electrodes will not under any condition of burning give a light equal in intensity or visibility to that of the high power arc. Solid electrodes have a critical temperature at best of 3300 deg. cent. and will not produce the gas ball phenomena as in the special cored electrode. It has been practically impossible to date to operate a solid electrode efficiently at a current density in excess of 0.66 amperes per sq. m.m. whereas the high power positive operates at a current density of 1.65 amperes successfully and efficiently.

Chester Lichtenberg: The Chief of Engineers, who is responsible for Army searchlights, has read Mr. Kelly's paper with considerable interest. He noted that no mention was made of mobile searchlights, although this variety had undergone a remarkable development during 1918 and 1919. He thought that the members of the Institute might be interested in seeing photographic reproductions of some of the more prominent recent developments of mobile searchlight equipment, and he has authorized the presentation of certain representative views.

The mobile Army of the United States was equipped in 1917 with only a few portable searchlights. These were of two varieties. One is shown in Fig. 1. It consists essentially of a 36-in. medium intensity searchlight mounted on an extensible tower, which is in turn mounted on a 3½-ton motor truck. The power unit is a 15-kw., 125-volt gasoline-electric set, mounted on a duplicate 3½-ton truck. Eight of these outfits were part of the engineer train equipments. They were used during the 1917 campaign on the Mexican border. They were not considered sufficiently mobile to accompany some of the army units. As a result, a horse drawn equipment was developed. This consists essentially of a 24 in. high intensity arc searchlight placed on mast or tower extensible to 16 feet above the ground. The searchlight and tower are mounted on a caisson. The energy is supplied by a 5-kw., 65-volt gasoline-electric set mounted on a limber. Eight of these units accompanied the first engineer train which left the United States for France on August 2, 1917.

Experience in France indicated that neither of the mobile searchlight units which formed part of the equipment of the Army at the outbreak of hostilities was suited for operation along the Western Front. The searchlights on masts were particularly unsuited since, if they remained lit more than 30 seconds within the zone of the advance, they were demolished by enemy shells. This was due to the extreme accuracy of the range finding equipment of the German armies and the rapid responsivity and excellent coordination of their artillery fire. A survey station indicated that high-power searchlights on small mobile mounts, which could be quickly emplaced, were desirable. Specifications for these were sent to the United States late in 1917.

The first attempt to meet the specifications of the A. E. F. is shown in Fig. 2. This is essentially the standard 36-in. size seacoast searchlight, with a new design of trunnion arms and with a wheeled carriage. It utilized existing tool equipment so far as this was practicable, at the same time reduced the weight of the searchlight about 50 per cent, and greatly increased its mobility. About 36 of these searchlights were in use in the 1st and 2nd American Field Armies at the time the armistice was signed. They were supplied with power from various kinds of power plants. Some were supplied by French

17-kw. sets of various makes, some by Riker trucks, the engines of which were equipped with 12-kw. generators and some with Mack trucks, the engines of which were equipped with 15-kw. generators.

A. E. F. experience indicated, however, that 36-in. searchlights were not quite powerful enough. New specifications called for 60-in. portable or semi-portable searchlights. To meet this demand, the design shown in Fig. 3 was developed. It consists essentially of the standard seacoast searchlight equipment placed on trailers. On the left is shown a 60-in. stationary type high intensity arc searchlight with its cable reel, rheostat, etc. On the right is shown a stationary 25-kw. 115-volt gasoline-electric set with its radiator, muffler, spare gasoline tanks and auxiliaries. This equipment, although mobile, was quite heavy and could only be moved at relatively low speeds. It was large in size and difficult to entrench. Only a few of these units were ordered to fill in the gap until more mobile equipments could be provided. Meantime large quantities of 60-in. coast defense searchlights, Fig. 4, and their generating equipments were shipped to France and placed in active service. One of these is shown in Fig. 5, with its emplacement and 3-meter sound detecting paraboloid. It is an installation by the 56th U. S. Engineers in the St. Mihiel Sector.

Fig. 6 shows one of the early attempts at making a very light weight, extremely mobile 60-in. high-intensity arc drum-type searchlight. It weighs, complete, about 2500 lb. Before it was completely developed, however, the design had been superseded by the open-type which was suggested in the spring of 1918 by Major R. W. Lewis, the representative in the United States of the A. E. F. Searchlight Regiment, the 56th U. S. Engineers.

The original open-type searchlight, made in accordance with Major Lewis' suggestions is shown in Fig. 7. It was made up at Ellington Field, Texas by a detachment of the 56th U. S. Engineers. They took the mirror and back door from a 36-in. standard Navy searchlight and supported it in rude trunnions and trunnion arms. It was arranged to be rotatable about a horizontal axis only. In front of the mirror was placed the automatic high-intensity arc mechanism, usually placed in the barrel of a 36-in. searchlight. The carbons and the upper part of the mechanism were enclosed in a tin can, which was intended to restrict stray light and to protect the arc from wind. The searchlight was operated for several weeks and proved unusually efficient. It had one very important characteristic. It had a very clearly defined beam, with a minimum of stray light. This was, no doubt, due to the fact that there was no barrel and so there were no reflections from the interior. Beside, it had no front glass so that there was no dispersion.

The open-type searchlight seemed to be such a feasible proposition that negotiations were immediately opened with

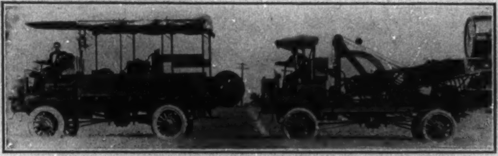

FIG. 1—TWO UNIT MOBILE SEARCHLIGHT EQUIPMENT, MOUNTED ON 3½-TON TRUCKS, SEARCHLIGHT BEING ARRANGED FOR ELEVATION BY MEANS OF AN EXTENSIBLE TOWER.

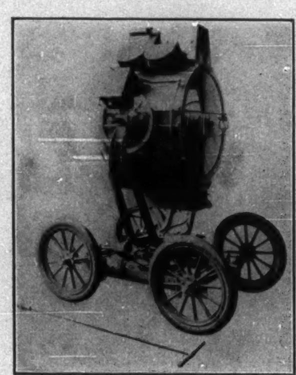

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FIG. 2—36-IN. BARREL TYPE WHEELED BASE SEARCHLIGHT, SUCH AS USED BY 1ST AND 2ND FIELD ARMIES, A. E. F.

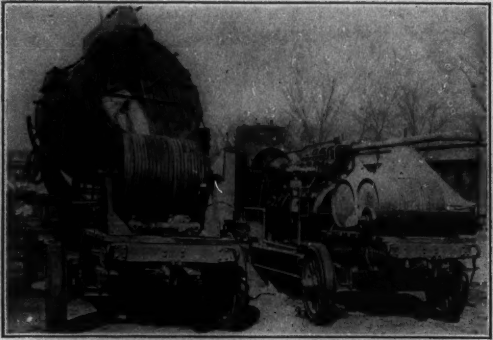

FIG. 3—TRAILER TYPE OF MOBILE SEARCHLIGHT EQUIPMENT, CONSISTING OF 60 IN. BARRELL SEARCHLIGHT AND 25-KW., 125-VOLT GASOLINE-ELECTRIC GENERATING SET.

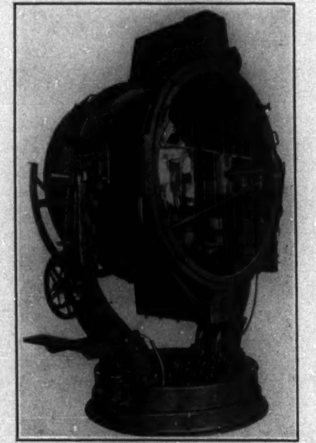

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FIG. 4—60 IN. HIGH INTENSITY ARC BARREL TYPE SEARCHLIGHT, DESIGNED FOR COAST DEFENSE INSTALLATION AND USED DURING OPERATIONS OF THE 1ST AND 2ND ARMIES, A. E. F.

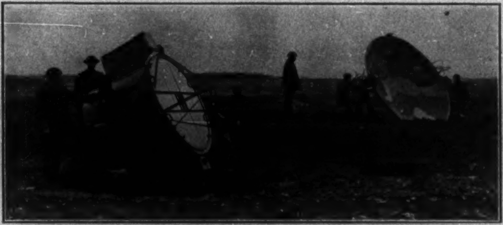

FIG. 5—A. E. F. SEARCHLIGHT EMPLACEMENT, SHOWING 60 IN. HIGH INTENSITY ARC, BARREL TYPE SEARCHLIGHT AND THREE-METER PARABOLOID.

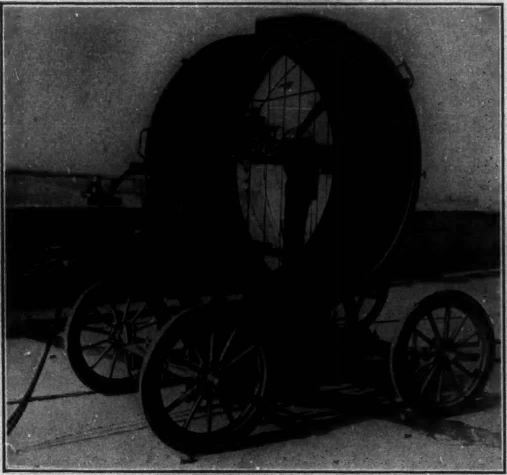

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FIG. 6—60 IN. HIGH INTENSITY ARC, LIGHT WEIGHT, BARREL SEARCHLIGHT ON WHEELED BASE

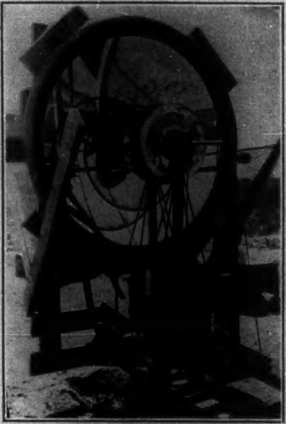

Fig. 7—Original Open Type Searchlight, with 36 in. Mirror and 150-ampere High Intensity Arc, Automatically Operated Mechanism

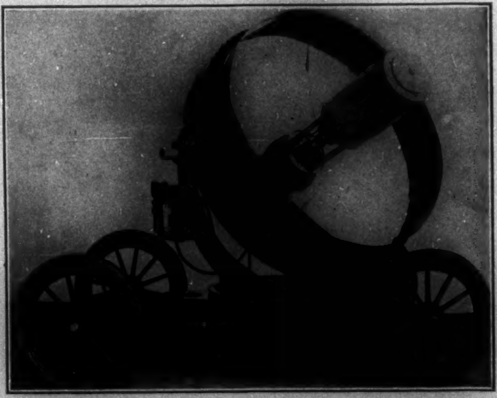

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Fig. 8—1919 Model of Open Type Searchlight, with 60 in. Mirror and 200-Ampere Medium Intensity Arc, Hand Feed Mechanism

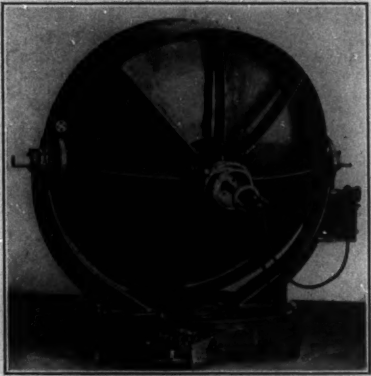

FIG. 9—1919 MODEL OF OPEN TYPE SEARCHLIGHT, WITH 60 IN. MIRROR, SHOWING WHEELS REMOVED AND BASE READY FOR EMPLACING

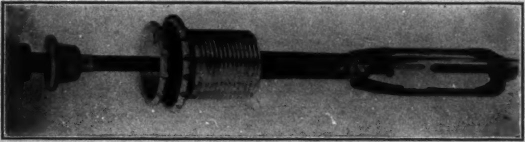

FIG. 10—MECHANISM FOR 200-AMPERE MEDIUM INTENSITY ARC, AS USED IN OPEN TYPE SEARCHLIGHT

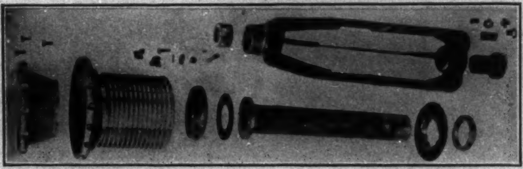

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FIG. 11—PARTS OF MECHANISM FOR OPERATING 200-AMPERE MEDIUM INTENSITY ARC IN OPEN TYPE SEARCHLIGHT

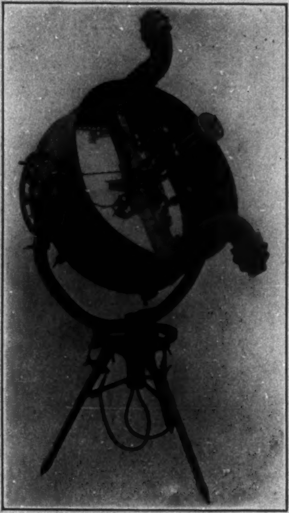

FIG. 12—VERY LIGHT WEIGHT MODEL OPEN TYPE SEARCHLIGHT, WITH 30 IN. MIRROR AND 150-AMPERE HIGH INTENSITY ARC, HAND FEED MECHANISM

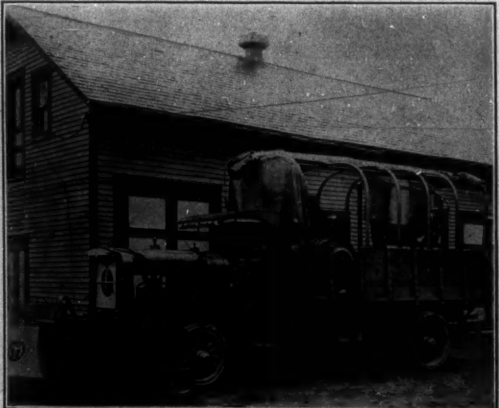

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FIG. 13—MOBILE SEARCHLIGHT POWER UNIT AND LORRY, WITH COVERS REMOVED, SHOWING GENERATOR, SWITCHBOARD, CABLE REEL AND 36 IN. WHEELED BARREL SEARCHLIGHT

several large searchlight manufacturers to produce commercial designs. Several months' effort was expended and, as a result, a very light weight, 60-in. open-type searchlight was produced. One of the highly developed designs is shown in Fig. 8. It weighs complete 1200 lb. It can be provided with either medium intensity or high intensity arc mechanisms. It has been operated in all kinds of weather and in the ordinary winds where aircraft can travel. It has proven unusually successful. It weighs about one-fifth as much as the old 60-in. searchlights and is just as efficient. Besides, it has no complicated mechanism and is very easily operated by partially skilled personnel. The running gear can be easily demounted from it, as shown in Fig. 9. This makes it a simple matter to emplace. The mechanism also is exceedingly simple. A medium intensity design is shown in Fig. 10. It consists of relatively few parts which are strong, as shown in Fig. 11. The result has been a simplified, very light weight and mobile searchlight, which is easy to move and emplace and which can be operated by a minimum number of partially skilled individuals and yet be just as effective as the very heavy, complicated searchlights previously standardized for certain classes of service. Besides it is easy to manufacture and takes previously standardized mirrors and carbons.

Another form of light weight, open-type searchlight is shown in Fig. 12. It was developed essentially for extremely mobile service where light weight is essential. It weighs 225 lb. as shown. It has a 150-ampere high-intensity arc mechanism, which is hand fed. It has a 30-in. glass mirror. It gives fully as good service up to 14,000-ft. range as do the 60-in. designs. However, it is not nearly so good beyond 14,000 ft. on account of the wide spread of its beam.

The advent of very mobile *searchlights* brought with it the need for mobile searchlight *power units.* One of the early designs is shown in Fig. 1. This was successful but not entirely satisfactory. Another of the early designs is shown in Fig. 3. This unit was not self-propelled and besides could only be moved at low speeds and then not readily. To meet the situation, a standard Engineer Department Mack 5½-ton cargo truck was modified by the addition of certain electrical auxiliaries. A generator was mounted on an extension of the crank shaft. A switchboard was placed under the body. Rheostats were located on the chassis. A cable reel was mounted in the front of the cargo space. A wheeled barrel 36-in. searchlight was placed in the back of the cargo space. The result was a complete and mobile searchlight unit which could be run over practically all ordinary roads at speeds up to 12 miles per hour and which could operate an automatic high intensity arc searchlight practically continuously. It is shown with covers removed in Fig. 13. About thirty of these units were used by the 1st and 2nd Field Armies of the A. E. F. prior to the Armistice.

The development of the open-type searchlight reduced the weight of the searchlight from about 7000 lb. to about 1200 lb. This permitted a very much lighter weight power unit to be employed. The situation was surveyed, and, based on careful studies, the power unit shown in Fig. 14 was developed. It consists essentially of a standard Cadillac ambulance chassis with a 21-kw. G. E. generator, mounted, on the propeller shaft between the transmission and the differential, as shown in Fig. 15. Mounted on this chassis is a special, very light weight body which has ramps for carrying the searchlight, as

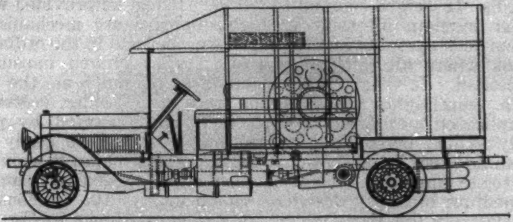

FIG. 18—50-KW. SIZE MOBILE SEARCHLIGHT POWER UNIT AND LORRY, DESIGNED FOR CARRYING 60-IN. HIGH-AMPERAGE SEARCHLIGHT, TWELVE MEN AND ALL THEIR AUXILIARIES

Length overall—21 ft., 10 in.; height—9 ft., 5 in.; width—6 ft., 2 in.; wheel base—177 in.; wheel diameter—36 in.; wheel gauge, front—62 in.; rear—67 in.

Note: Side curtains equipped with extra fly of sufficient length to extend 6 feet from side of car at ground line.

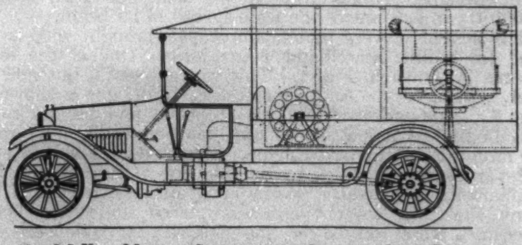

FIG. 19—6.5-KW. MOBILE SEARCHLIGHT POWER UNIT, DESIGNED FOR CARRYING 30-IN. OPEN TYPE SEARCHLIGHT AND THREE MEN

Length overall—15 ft., 10 in.; height overall—7 ft., 6 in.; width overall—5 ft., 5 in.; wheel base—134 in.; wheel diameter—33 in.; wheel gauge—56 in.

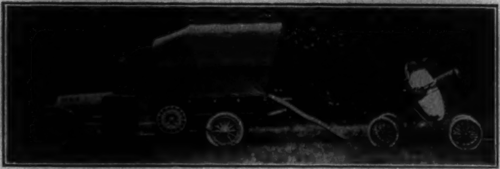

FIG. 14—LATE 1918 DESIGN MOBILE SEARCHLIGHT POWER UNIT AND LORRY, WITH 60 IN. OPEN TYPE SEARCHLIGHT

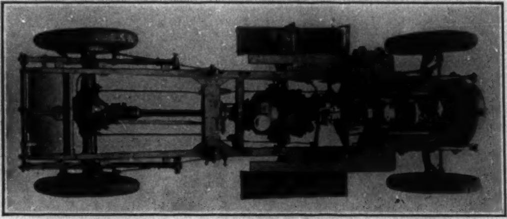

FIG. 15—TOP VIEW OF LATE 1918 DESIGN OF MOBILE SEARCHLIGHT POWER UNIT CHASSIS, SHOWING INSTALLATION OF GENERATOR BETWEEN TRANSMISSION AND DIFFERENTIAL

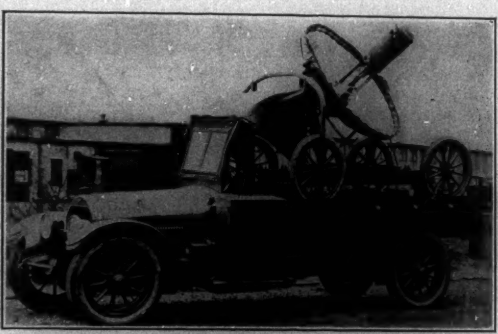

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FIG. 16—LATE 1918 DESIGN OF MOBILE SEARCHLIGHT POWER UNIT AND LORRY, WITH COVERS REMOVED, SHOWING SEARCHLIGHT IN POSITION

PLATE LXXXVIII.
A. I. E. E.
VOL. XXXVIII, 1919

FIG. 17—LATE 1918 DESIGN MOBILE SEARCHLIGHT POWER UNIT, OPEN TYPE SEARCHLIGHT, PARABOLOID AND 60 IN. BARREL TYPE SEARCHLIGHT, IN OPERATION WITH 2ND FIELD ARMY, A. E. F.

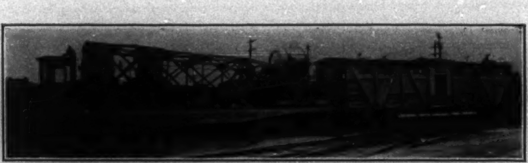

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FIG. 20—RAILWAY TYPE SEARCHLIGHT EQUIPMENT, WITH POWER UNIT IN BOX CAR AND SEARCHLIGHT MOUNTED ON HINGED TOWER

shown in Fig. 16. The entire equipment was conceived, designed and built in four months and was delivered and in operation with the 2nd American Field Army in October 1918. The complete equipment weighs 9000 lb. It has an intermittent output of 21 kw. It carries a 60-in. open-type searchlight, provided with both medium intensity and high intensity arc mechanisms and a crew of five men, with all the necessary auxiliaries for operation in the field. Another is shown in Fig. 17 operating an open-type searchlight in the 2nd A. E. F. Army sector.

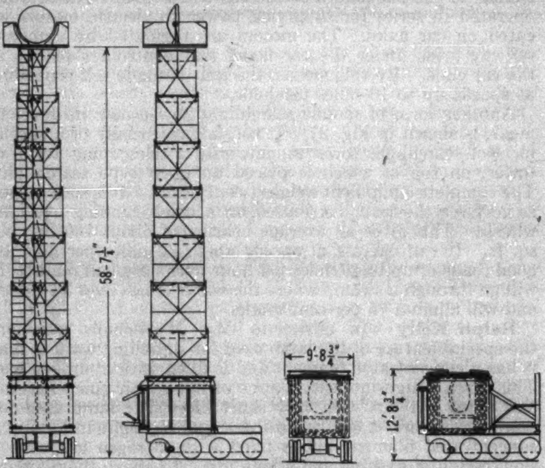

Fig. 21—Mobile Searchlight Tower and Mount, Illustrating Telescoping Type of Extensible Tower

The success of this combined design of mobile power unit and lorry gave rise to a new line of searchlight power units. A 50-kw. design is shown in Fig. 18. It employs a LaFrance Fire Engine chassis with 177-in. wheel base. It has provision for a 60-in. high-amperage searchlight, a crew of 12 men, 500 ft. of cable and all auxiliaries for maintaining this force in the field for one week. Fig. 19 shows a 6.5-kw. unit, employing a Dodge chassis. This carries a 30-in. 90-ampere high-intensity open-type searchlight, with a crew of three to four men.

Mobile searchlight units have been extended to include lights on towers for coast defense purposes. Fig. 20 shows a recently completed design. It consists essentially of two railway cars, which carry a searchlight and its tower and the power plant respectively. The tower, when raised, places the center of the searchlight 33 ft. above the top of the tracks. The searchlight is a 60-in. high-intensity coast defense design similar to the one shown in Fig. 4. It is supplied by power from either one of two standard 25-kw. 115-volt gasoline-electric sets, which are located in the box car. These are similar to the one shown at the right in Fig. 3. The sets are operated in series for supplying power to electric motors located on car axles. The motors are regulated by standard railway grids under the car floors and controllers placed at the car ends. By this means, the unit is made self-propelling at speeds up to 10 miles per hour.

Another form of mobile searchlight tower now in development, is shown in Fig. 21. It consists essentially of a special form of searchlight tower mount, with a telescoping form of tower, on top of which is placed an open type searchlight. The complete equipment weighs twenty tons. It is so arranged as to have the load distributed on a track running over the wheels. This gives an average bearing of about 700 lbs. per sq. ft. It can operate at speeds up to 20 miles per hour on good roads or up to 10 miles per hour over very bad roads. It will go through a swamp where the water is not over 3 ft. deep and will climb a 75 per cent grade.

Ralph Kelly: In answer to Mr. McNicholls question, the special feature of the lamp used for signalling work is that it has a concentrated filament and is filled with hydrogen gas. This gives a high intensity source of light with quick flashing and dying out but with very short life. The lamp used for searchlight work is similar to the signalling light in having a concentrated filament but is filled with nitrogen gas with a slow period of flashing and a long life. I believe the filament temperature of the hydrogen lamp is the highest.

The Navy experimented with metal mirrors a number of years ago but found they did not keep their shape and tarnished very quickly under sea service. The parabolic glass mirror has proven very satisfactory in Naval service with a comparatively few breaks on record that were caused by gunfire.

The army mobile Cadillac searchlight unit represents a remarkable development and among the many types of equipment that have been developed for field work, there are none that can compare with it for mobility and efficiency.

There is only one way to fight enemy planes at night and that is to illuminate them with searchlights and fight them with aircraft. The friendly aircraft have the advantage of darkness while they attack enemy planes illuminated by the searchlights. Anti aircraft guns are of little use except in protecting the searchlight equipment by preventing enemy planes from flying down the beam to a low height where they can bomb the searchlight personnel. I might say that a searchlight must frequently have its position changed to prevent being shelled by enemy artillery or by being used as a land mark for navigation of enemy planes. This is the great reason why a field searchlight should be light and mobile.

Mr. Bassett stated that the searchlight mirrors are fairly well standardized but I don't think the Army Engineer Corps will agree with him on this point. They are still in the midst of a very extensive investigation in the field of metal mirrors in which they have encountered and are still encountering tremendous difficulties.

Mr. Patterson's figures comparing the 12 in. and 24 in. low power searchlight hold good for those lights under laboratory conditions when operated by expert personnel. In actual service the present design of low power light operates very inefficiently as described in the paper. In my opinion there is no question that a well designed low power light, such as the 12 in. can be manufactured to give the same light on the target as the present 24 in. light as it is now inefficiently operated in service. The type and design of that light is a problem for the future but it is generally conceded that their is great room for improvement in the present low power light.

The 12 in. light I had in mind in writing the paper was one designed and built by Major Judson and Lieut. Thompson U. S. M. C. which used carbons of a very refractory material with an arc current of 30 amperes. The positive carbon had a diameter of ⅜ in. while the negative carbon diameter was ¼ in. which resulted in a far greater intrinsic brilliancy of light source than the present low power light. This light was equipped with a ground glass finder which added materially to its efficiency as it enabled the operator to keep the carbons in focus at all times.

In regard to Mr. Patterson's recommendation that the 12 in. and 18 in. sizes both be retained, it is not only my own opinion but that of Naval officers generally that the Navy standardize on one size of low power searchlight. That size should be either the 12 in. or the 18 in. as determined after service tests.

Commander Beehler's discussion gives the status of the third electrode control of the positive carbon for Naval work, and also answers Mr. Murphy's question on that point.

The data Mr. Murphy gave on ventilation of searchlights are extremely interesting. His company did the pioneer work in the field of searchlight ventilation and had many troublesome obstacles to overcome before they arrived at their present arrangement. This has worked admirably on the 24 in. high power searchlights under the severest of naval service conditions.

As stated in the paper, the thermostat development by Mr. Murphy has proven very successful in service. It will however be of advantage to mount the thermostat on the lamp structure out of the weather and where no special adjustments are required to maintain the positive crater in the focal position.

There was a great deal of question during the war regarding the real value of a searchlight on a naval vessel but there finally developed a strong feeling that well designed searchlight equipment, properly used will prove of immense value to a ship in a night engagement. This feeling was confirmed by both Admiral's Beatty and Jellicoe in their reports on the battle of Jutland.

As Commander Beehler brings out, it is not the high current density of the carbons alone that causes the high intensity phenomena, but the combination of that high current density and the angle of the negative arc stream. The dome glass door, in accordance with his tests, is not as efficient in transmitting light as a clean plate glass strip door. The great point in favor of the dome glass door is that it stands up against gun fire while the present type of plate glass door shatters under gun fire. Until the plate glass strip front door is developed to resist gunfire, it is only feasible to use the less efficient dome glass door for the major characteristic of a front door is that it must withstand gunfire. There is also a field of development in improving the optics of the dome glass door with a view to improving its light transmission efficiency.

The question of size, material, and current density of searchlight carbons is still one in which there are a number of conflicting opinions. If a more refractory material is used for the carbons which gives a greatly reduced tail light, the light source will unquestionably be of lower intrinsic brilliancy than is obtained with the present type of high powered carbons. However it is the light that comes back to the observer from the target that counts and with the absorption of the atmosphere in the nature of the 6th power, it is apparent that there can be a material reduction in the brilliancy of the light source and still effect but very slightly the light reflected back to the observer from the target. Is it then worth while to have a lower brilliancy light source and eliminate the objectionable foreground lighting caused by the tail light of the high power light or is the sacrifice in intrinsic brilliancy of the light source too great for the advantage gained?

Presented at a Section Meeting of the American Institute of Electrical Engineers, San Francisco, Cal., February 7, 1919.

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UNIFICATION OF THE MANUAL AND AUTOMATIC TELEPHONE SYSTEMS

BY D. E. WISEMAN

ABSTRACT OF PAPER

A description is given of the physical consolidation of the Bell manual and the Automatic Electric telephone systems of Los Angeles, Cal., which previously to June 1, 1918 operated as separate systems. While similar consolidations had been made previously they included relatively small volumes of traffic and afforded no engineering precedents as a guide to the consolidation of two systems serving 129,000 stations. The plans for the physical union of the two companies were devised by a joint committee of engineering representatives, and contained a number of novel operating and construction methods which are described.

Under the new system each subscriber has access to every other subscriber in the Los Angeles exchange and to all long-distance lines centering there. Where duplicate services were installed the subscriber was given his choice as to whether he would retain the manual or the automatic system, and about 13,000 duplicates were eliminated, the choice between the two types being equally divided.

IN telephone engineering and the resultant physical and economic accomplishment, June 1, 1918 figures prominently, for on that date the formal union of the Bell manual telephone system operated by The Pacific Telephone and Telegraph Company, and the Automatic Electric system operated by the Home Telephone and Telegraph Company in the City of Los Angeles, was effected under the management of the newly created Southern California Telephone Company, giving to every telephone user in that area a unified and unrestricted exchange telephone service and universal service over toll trunk lines to some eleven million telephones throughout the United States. Consolidations of this character have been made prior to this date but involving relatively small volumes of traffic and simple operating methods so that there were no records of actual performance or established engineering practises to serve as a precedent and guide for determining the effect of and the physical requirements necessitated by the sudden release of two large, distinct and separately bound volumes of traffic into a common channel.

Because of the novel operating and construction methods and the speculative possibilities involved in the important pioneering work of welding these fundamentally different telephone switching systems into a single eighteen million dollar plant serving 129,000 stations, a semi-technical summary of the events has been prepared for general information.

A brief reference to the conditions which brought about the consolidation will be made in order to better understand the problems that confronted the two competing telephone companies and their subscribers. Believing that competition instead of control was the automatic remedy to apply to public utilities, the City of Los Angeles invited telephone competition about sixteen years ago and then struggled along with her business firms and many of her residents paying two telephone bills for a divided and what proved to be an unsatisfactory telephone service. This condition was continued until the year 1916, when the public decided by popular vote to bring about an end to dual telephone service. Negotiations were begun and various proposals were considered by all concerned in an effort to avoid any waste or arbitrary measures. A plan was finally accepted for the organization of a local telephone company, which was to purchase the properties of the existing operating companies and unify the service, continuing with the equipments then in plant and giving the right to the telephone users to determine for themselves whether they would retain their automatic stations or manual stations. War conditions imposed restrictions in the conservation of materials and men for such projects and called for a careful weighing of the expected benefits and expenditures of materials and labor.

With this clear understanding of the requirements, a joint committee of engineering representatives was appointed to determine the methods for the physical joining of the two systems. As a result of their efforts, a fundamental plan, together with preliminary estimates of cost, was submitted and formally approved by the City of Los Angeles, the Railroad Commission of the State of California and by the Attorney General for the Federal Government, and on May 1, 1917, formal authorization was given to proceed with the project.

In order to picture the plants as they existed prior to the consolidation, I shall refer briefly to the physical properties and the operating methods of the two systems. Referring for a moment to Fig. 1, the area served by the duplicate plants was about 200 sq. mi. and lying within the corporate city limits. The open and solid circles show office and wire centers of the Home and Pacific Companies respectively, while the dotted and solid lines mark the areas of the respective districts. The Pacific Company was established first and the opening and location of its offices followed the telephone development of the city. The first office was located in the business area and as the population increased and spread to outlying districts, new offices and districts were established. Each office was located as near as practicable to the center of the wire distributing system as determined by a study of the existing plant and expected growths for 15 or more years hence. As years go by and old offices are outgrown studies very often show an economy in subdividing the original district or changing the boundaries so that this natural process tends to correct any errors in estimates of growths and locations of such expected growths.

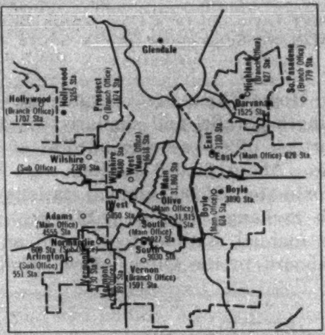
Glendale
Hollywood (Branch Office) 1707 Sta
Hollywood 3265 Sta
Prospect (Branch Office) 1674 Sta
Highland (Branch Office) 627 Sta
So. Pasadena (Branch Office) 779 Sta
Garvanza 1525 Sta
Wilshire (Sub Office) 2389 Sta
Wilshire 6480 Sta
West (Main Office) 6618 Sta
East 3130 Sta
East (Main Office) 628 Sta
Main 31,860 Sta
Boyle 3890 Sta
Olive (Main Office) 31,815 Sta
Boyle (Main Office) 624 Sta
West 5850 Sta
Adams (Main Office) 4555 Sta
South (Main Office) 4927 Sta
Normandie 608 Sta (Sub Office)
South 9030 Sta
Arlington (Sub Office) 551 Sta
Vermont
Vermont (Main Office) 891 Sta
Vernon (Branch Office) 1501 Sta

FIG. 1—AREA SERVED BY DUPLICATE PLANTS

May 1, 1917, the Pacific Company's exchange consisted of its standard outside plant and station equipment and nine manually-operated central offices serving about 69,000 stations. Bell equipment was used in units having a capacity of 9600 multiple lines. Telephone connections were established generally by the calling subscriber removing the receiver from the switch hook, causing a light to appear before an answering *A* operator, who upon receiving a request for a particular number cut-in on an order wire to the particular switchboard unit in the district, indicated by the prefix of the number called-for. An operator at the distant switchboard unit, assigned a trunk over this order wire to the calling *A* operator and completed the connection by plugging into the called-for subscriber's multiple. Fig. 2 shows a schematic transmission circuit of a typical connection. The operation of this circuit will be described later and in connection with the unified plant.

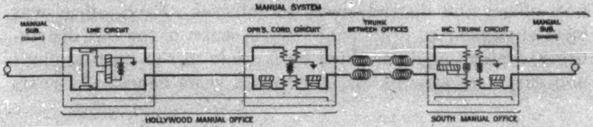
MANUAL SYSTEM
MANUAL SUB.
LINE CIRCUIT
OPR'S. CORD CIRCUIT
TRUNK BETWEEN OFFICES
INC. TRUNK CIRCUIT
MANUAL SUB.
HOLLYWOOD MANUAL OFFICE
SOUTH MANUAL OFFICE

FIG. 2—EQUIPMENT INVOLVED IN A CONNECTION BETWEEN A HOLLYWOOD OFFICE MANUAL SUBSCRIBER AND A SOUTH OFFICE MANUAL SUBSCRIBER

The former Home Company operated an automatic exchange including a parallel and similarly constructed outside plant and fourteen offices serving a total of approximately 60,000 stations. About 35,000 stations were equipped with dials and approximately 25,000 manual stations operated from private branch exchanges and as public pay stations. The lines from the manual stations terminated on a 45-position manual switchboard in the Olive Office. Calls from automatic stations to automatic stations were made by dialing five or six digits, as required, to reach the particular district and individual subscriber. In capacity the former Home Plant was one of the largest and probably the most successfully operated of any automatic system installed in the United States or abroad. Home Company private branch exchange subscribers were reached by dialing the private branch exchange operator, who completed the connection. Calls outgoing from private branch exchange subscribers were trunked to the man-

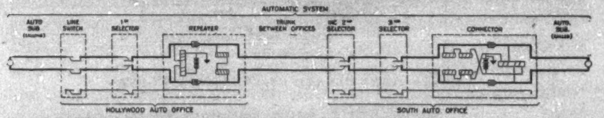
AUTOMATIC SYSTEM
AUTO SUB.
LINE SWITCH
1st SELECTOR
REPEATER
TRUNK BETWEEN OFFICES
INC. 2nd SELECTOR
3rd SELECTOR
CONNECTOR
AUTO. SUB.
HOLLYWOOD AUTO OFFICE
SOUTH AUTO OFFICE

FIG. 3—EQUIPMENT INVOLVED IN A CONNECTION BETWEEN A HOLLYWOOD OFFICE AUTOMATIC SUBSCRIBER AND A SOUTH OFFICE AUTOMATIC SUBSCRIBER

ual transfer board above mentioned, the calls coming in on an automatic traffic distributor which placed each line lamp signal before a non-busy operator. The operator upon taking up these connections would complete the call direct if to another private branch exchange station through the subscriber's multiple or dial the number required, if an automatic station was wanted. Fig. 3 shows a schematic transmission circuit of a typical full automatic connection. The operations and functions will also be described step by step later.

Los Angeles has had the distinction of having more telephones per capita than any other city in the world and is now very close to the top of the list. This general usage is reflected in the number of calls originating in each system, as shown by the records of traffic. The approximate numbers of daily average calls originating in the former Pacific plant and Home plant were in the vicinity of 430,000 and 420,000 respectively.

The engineering problems demanded, therefore, a reasonably close approximation of the volume of existing and added traffic which could be expected to flow between the groups of stations of the various districts of the two plants; a determination of the most direct and economical routing of such calls based on efficient operating methods and the use of existing facilities; and the design and development of an inter-unit trunking plant and switching circuits necessary to maintain the commercial standards for transmission and supervision between the two systems regardless of the mechanical, electrical and operating inequalities. Visual and audible signals peculiar to the separate systems required for supervisory purposes needed to be synchronized or harmonized and extended when necessary so as to afford common usage.

It is obvious that a great many plans and combinations of plans for unification were developed in sufficient detail to determine their relative capital and operating costs and advantages and disadvantages. The plan in principle that was adopted for handling the inter-office traffic between the systems was to operate all existing offices as units of the complete exchange and route the new automatic or manual trunks of each office into the adjacent office of the opposite system, where the connection could be completed by the most direct method and route.

Having referred to the facts that were most vital in shaping the project, I should like to outline the organization and schedule of work involved in the construction program estimated to cost $1,250,000. Every reasonable effort was demanded to bring about a unification of the properties without waste of time, and accordingly a complete schedule for the ordering of materials, manufacturing, assembly and installation thereof was set up after a canvas of probable material deliveries and of the labor situation. It became evident that a period of twelve months would be the shortest time possible to complete the necessary work, keeping in mind that war demands might upset the schedule, and all effort was centered on finishing the project within the minimum time. Specifications and plans were completed for each individual project, numbering all together about 110, and each one was charted with due regard to the materials involved, quantity, desired time of delivery and installation and its relation to the project as a whole. Fig. 4 shows a typical group of projects and illustrates the method of maintaining the schedule for the whole program. As items of material were, of course, duplicated in a great many of the specifications, a master chart was prepared showing the total quantities of each of the thousands of items required at specified dates.

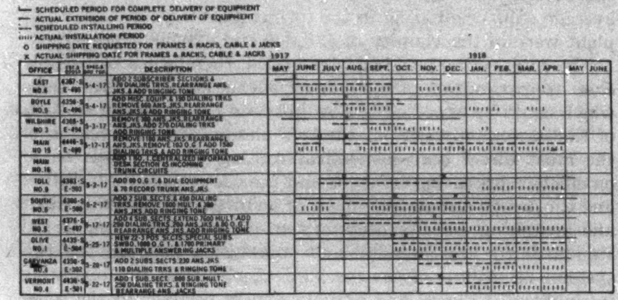

Fig. 4—Diagram Showing Method of Maintaining Schedule for the Entire Reconstruction

The largest single project consisted in the design and manufacture of the 66-position special tandem switchboard and associated equipment to be located in Olive Office for the translation and distribution of calls from automatic stations to manual stations in the downtown area. This switchboard, together with the switching circuits were designed in detail by the engineers in the general office of the telephone company, and all of the manufacturing and assembly work was performed in San Francisco. Extensive rearrangements of the Olive office building were necessary to provide space for this equipment and for retiring quarters for the large number of operators required to give the necessary 24-hour continuous service. Fig. 5 shows the floor plan arrangement of this switchboard and associated operating room space.

In the manual offices it was necessary to equip the regular subscriber positions with a dial for sending out the electrical impulses, and to provide outgoing trunks connecting directly with the automatic equipment in the adjacent office. There were approximately 375 subscriber switchboard positions in the manual system in the Los Angeles exchange that required the installation of this special equipment and associated wiring and this proved to be one of the most difficult parts of the work, because such work had to be performed on positions of switchboard that were in continuous operating service. I will refer to Fig. 6 and describe the switching circuit associated with the dials at the operators' positions. Each operator on the subscribers' switchboard is provided with 10 outgoing trunks terminating in the nearest automatic office on first selectors. These outgoing trunks are provided with twin jacks, one above the other, and electrically connected so that the dialing device can be associated with each of the trunk circuits to the distant

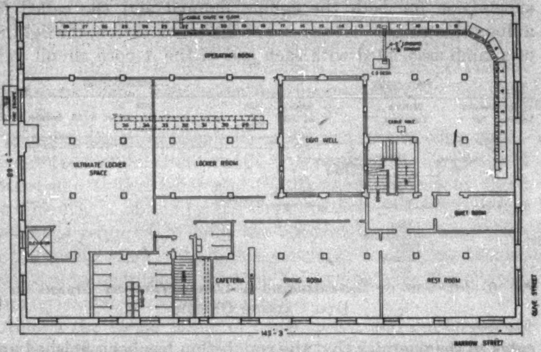
CABLE CHUTE IN FLOOR
OPERATING ROOM
C.O. DESK
CABLE HOLE
LIGHT WELL
ULTIMATE LOCKER SPACE
LOCKER ROOM
QUIET ROOM
ELEVATOR
CAFETERIA
DINING ROOM
REST ROOM
89'-6"
ALLEY
OLIVE STREET
145'-3"
NARROW STREET

Fig. 5—Plan of Second Floor Olive Office, Los Angeles, Cal.

office. The plug connecting with the dial is inserted in the lower first jack while a call originated by a manual subscriber for an automatic station is connected to the upper first trunk jack by means of the regular *A* operator's doubled-ended cord circuit. The operator then proceeds to dial the number requested and as soon as this number has been dialed, the operator moves the dial circuit plug to the lower second jack, thereby pre-selecting the trunk circuit for the next call for an automatic station. The operation of the second and third selectors and final connector is the same as described later under "Automatic to Automatic Connections." The circuit is so arranged that the manual *A* operator just referred to receives direct supervision for both the calling manual and the called for automatic station. At the end of the conversation, the lighting of a lamp associated with each end of the *A* cord circuit indicates to the operator that the connection has been finished and all cords are to be cleared from the subscriber's line and trunk jacks. The outgoing dial trunks are used in rotation and the actual number of the trunks connected by such lines to the distant office varies with the traffic requirements.

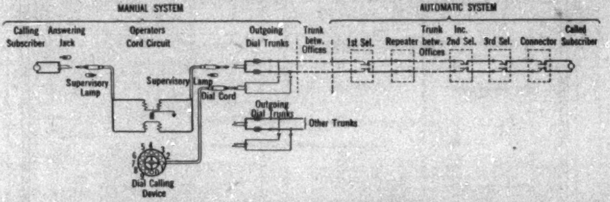
MANUAL SYSTEM
AUTOMATIC SYSTEM
Calling Subscriber
Answering Jack
Operators Cord Circuit
Outgoing Dial Trunks
Trunk betw. Offices
1st Sel.
Repeater
Trunk betw. Offices
Inc. 2nd Sel.
3rd Sel.
Connector
Called Subscriber
Supervisory Lamp
Supervisory Lamp
Dial Cord
Outgoing Dial Trunks
Other Trunks
5 4 3 2 6 7 8 9
Dial Calling Device

FIG. 6—METHOD OF ESTABLISHING A CONNECTION BY MEANS OF DIAL CALLING DEVICE

Central office telephone installation work has generally been organized for individual projects usually confined to a particular office building. The plan of scheduling the material and utilizing this material to its greatest advantage made it necessary to depart from the regular practises of the installing forces and to provide that all of the work should be treated as one project wherein the men trained for specific work were to be moved from one office to another as materials arrived and thereby facilitating the completion of the work regardless of irregularities in the arrival of materials for a particular office.

This arrangement was one of the important factors in effecting the final completion of the central office work April 30, 1918. The time intervening from the date until the beginning of the delivery of the directory and the formal announcement of consolidation on June 1, 1918 was required in placing thousands of test calls over all combinations of connections to make sure of positive operation. In a typical connection between an automatic and manual station there are 41 relays, and 79 from manual to automatic, having movable parts controlling from one to ten electrical contacts, each of which must function in the proper sequence from the start to the completion of each telephone call. I do not wish to convey an impression that telephone circuits are inherently subject to failure because they are not. The characteristics and operating requirements for each relay, for instance, are known mathematically and what the relay can be depended upon to do for a specific period of time. At regular intervals each type of relay is given its proper current adjustment using measuring instruments designed for that purpose.

The installation of telephone cables and central office equipment of the manual or automatic type required the use of highly trained and skilled labor and the Telephone Company faced the difficulty of obtaining the large number of electricians and mechanics required to hold the schedule and training them for the special work. As it was, considerable overtime became necessary to maintain a working balance between the arrivals of material and the available labor.

Among the larger items of expenditure, and one involving months of study and calculations in voice transmission were the additions and changes necessary in the cable trunking plant in order to maintain commercial standards on all local and long distance connections. These studies included the use of and application of loading coils to the former Home Company cable plant amounting to about 1000 coils and the addition and respacing of many of the coils in the portion of the Pacific Company cable plant. All together about 75,000,000 conductor-feet of various gages of underground telephone cables were ordered and installed to provide new routes and reinforce existing trunk groups required for the consolidated service.

Prior to the consolidation, practically all of the subscribers having private branch exchanges maintained duplicate switchboards and station apparatus. The problem, therefore, of consolidating this type of equipment offered no particular difficulty, as such consolidations could be and were effected by grouping the trunks formerly serving the separate systems on the particular switchboard to be retained, adding thereto the amount of line and trunk facilities desired by the subscriber and eliminating the duplicate switchboard and stations not required. The net effect was to remove about 345 single-position private exchange switchboards from the system. In the case of several of the larger commercial companies, it was necessary to order complete multiple private branch exchange switchboards of the 640-line capacity and about 30 sections of such boards were placed in service as fast as the equipment could be engineered and manufactured. It will be interesting to note that the flow of traffic to and from private exchanges was not greatly disturbed from the existing paths by this plan of consolidating; a principle which was kept well in mind and taken advantage of wherever practicable.

Where individual and party stations were duplicated, both stations were left connected until the new directories were delivered, at which time the subscriber was requested to use the telephone of the particular system that he had made application for and the other station was removed as soon as the construction forces could handle the work. Approximately 13,000 duplicate stations have been removed.

A considerable number of operators were required and for a while it looked very much as if the consolidation would have to be postponed because of the inability to obtain the needed force. Good service depends to a large extent on capable and efficient operators and you can appreciate the difficulty that confronted the operating department in the selection, employ and training of approximately 500 additional operators required for handling the special transfer switchboard installed in the Olive Office building, and for the large number of added positions of switchboard in the various manual offices. A large operating school equipment was hurriedly manufactured and installed and training of operators was started about the first of the year 1918. The schedules also provided for the early installation of dials on the subscriber positions in the manual offices for advance training of the regular operating force. Special observation equipment was designed and furnished for practise work in placing test calls and later for supervision in determining and checking the accuracy of dialing the calls placed by the subscribers. By means of automatic recording devices the numbers called for by subscribers or instructors were compared with the numbers actually dialed and in this way the operating force was gradually brought to an efficient basis by the time the construction work was completed.

Proceeding under the restrictions that each subscriber having duplicate service should determine for himself whether to retain automatic or manual substation equipment, the Commercial Department carried on a vigorous campaign to obtain these subscriber's choices and to arrange accordingly. I do not have the exact figures but I understand that of the 18,680 duplicate stations about 5200 duplicates were retained and the balance were divided evenly between the two types. The expected result of this canvass had been forecasted and was an essential factor in engineering and in construction work.

As a function of the commercial canvass the Directory Department was confronted with the necessity of recasting the entire directory scheme to fit the particular needs of a unified service. A great amount of thought was given to the determination of the most efficient arrangement of listing and numbering subscribers, and while this seems trivial yet a careless directory arrangement reflects on the quality of the telephone service and robs the public of valuable time. Many number changes were involved and a complete relisting of every subscriber's name and number into one alphabetical list introduced great possibilities of errors. (It is a matter of passing interest that 165,000 copies of the directory were issued and distributed in Los Angeles and to other exchanges for long- and short-haul toll traffic and that over 441,000 pounds of paper were required in the printing of the consolidation issue.)

In the present unified plant, local calls are divided into four main groups and are obtained in the following ways:

Calls from Automatic Stations to Automatic Stations are obtained in the same manner as under the former Home management, that is, by the subscriber dialing the number wanted as indicated by the directory listing. Such subscriber sets are provided with the familiar dialing device arranged to transmit from one to ten electrical impulses for each "pull" of the dial. In the automatic system most of the stations are reached by dialing five digits although there are a few thousand six digit numbers. The operation of the first digit of the five digit numbers selects the office district required, the second digit selects the particular thousand, the third digit selects the particular hundred, while the fourth and fifth digits select the tens and unit respectively of the number desired. Referring to Fig. 3, the calling automatic subscriber upon removing the receiver from the switchhook completes an electrical circuit through a Keith line switch, the chief function of which is to direct the simultaneous calls to a minimum number of trunks to first selectors. Each automatic line therefore terminates on a primary line switch having ten paths to as many first selectors, these trunks being multipled to other line switches, depending on the calling rate or traffic load of the particular group of lines. The operation of the first digit of the number wanted causes the mechanism of the first selector to step the brush terminals upward to one of the ten levels arbitrarily connecting to a particular 10,000-line unit. This selector is of the trunk-hunting type, that is, the subscriber, as noted above, having dialed, say, the digit 6, causes six electrical impulses to be transmitted through the selector mechanism through the stepping relay, raising the brushes to the sixth level. The selector then automatically begins a rotary movement and continues until an idle trunk is found. There are ten trunks over which this rotary trunk selection takes place and the traffic is so distributed over selector equipment that with very few exceptions an idle trunk will be found within this group of ten. The connection is then established through what is termed repeater equipment (consisting of coils and relays by the aid of which the calling impulses are repeated from the local to the distant office and talking battery is provided to the calling station) to a particular second selector in the distant office. The dialing of the second digit by the calling subscriber again operates the stepping mechanism to one of the ten levels corresponding to the thousands of the number wanted. Automatic rotary movement takes place, selecting an idle trunk to the third selector where the dialing and selecting of the hundreds is the same as for the first and second selectors. The connection now is established through to the connector which is similar in its operation to selectors with the exception that the tens digit steps the mechanism to the corresponding level while the units digit controls the rotary movement to the particular unit required. The connector also supplies talking battery to the called subscriber and sends out the necessary ringing impulses to call either the individual or party subscriber desired or sends back the busy signal if the line called is in use. The restoring of the receivers to their switchhooks automatically returns all of the connecting equipment to normal position again for use on subsequent connections.

Calls from Automatic Stations to Manual Stations are obtained by operation of an arbitrarily assigned digit, the ninth digit or level being available in this case. Trunks from these first selector ninth levels lead to the nearest manual office and terminate by the aid of the familiar line and cutoff relay on an answering jack with a lamp signal before an *A* operator. Referring to Figs. 7 and 8, the *A* operator upon receiving a line lamp signal, plugs into the corresponding answering jack and requests the subscriber to give the number (and office prefix) wanted as shown by the directory listings. The call is then completed within the office received or is trunked over the manual trunking system to the distant *B* operator in the same manner as described under calls manual to manual. In the downtown district where a considerable amount of traffic is involved, it was found impossible to add to the large number of *A* positions to the various manual units and it was necessary to install a special manual switchboard, shown in Fig. 5 in available space in the Olive office building. This special board contains only the necessary multipled terminating lines, outgoing trunks and key and supervisory equipment necessary for establishing the connections between the automatic and manual system. Automatic calls therefore in this district are completed by the dialing of the digit 9 as before, bringing the line signal in on this special *A* switchboard where such con-

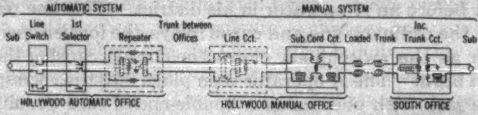
AUTOMATIC SYSTEM
MANUAL SYSTEM
Sub
Line Switch
1st Selector
Repeater
Trunk between Offices
Line Cct.
Sub.Cord Cct.
Loaded Trunk
Inc. Trunk Cct.
Sub
HOLLYWOOD AUTOMATIC OFFICE
HOLLYWOOD MANUAL OFFICE
SOUTH OFFICE

Fig. 7—Equipment Involved in a Connection Between a Hollywood Office Automatic Subscriber and a South Office Manual Subscriber

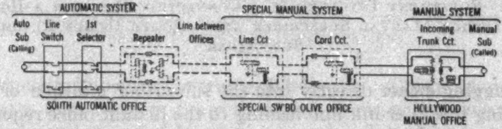
AUTOMATIC SYSTEM
SPECIAL MANUAL SYSTEM
MANUAL SYSTEM
Auto Sub (Calling)
Line Switch
1st Selector
Repeater
Line between Offices
Line Cct
Cord Cct.
Incoming Trunk Cct.
Manual Sub (Called)
SOUTH AUTOMATIC OFFICE
SPECIAL SW'BD OLIVE OFFICE
HOLLYWOOD MANUAL OFFICE

Fig. 8—Equipment Involved in a Connection Between a South Office Automatic Subscriber and a Hollywood Office Manual Subscriber

nections are completed as in regular manual practise over trunk lines assigned over order wires.

Calls from Manual Stations to Manual Stations are obtained in the same manner as existed prior to the consolidation, that is, by the agency of the *A* and *B* operators and corresponding manual switchboards. Referring to Fig. 2, the calling manual subscriber upon lifting the receiver from the hook causes a lamp to light before an *A* operator in his district. This line terminates on the ordinary line and cutoff relays, the former operating as soon as the switchhook closes the circuit, causing the line lamp to burn. The *A* operator then plugs into the corresponding line jack with one end of a double-ended cord circuit, this operation causing the cutoff relay to energize and thereby opening the circuit through the line relay and lamp. The operator then throws the listening key and requests the called for number. She then proceeds to complete the connection by plugging either directly into the subscriber multiple or by plugging into a trunk line leading to the distant office required and as assigned by the distant *B* operator over an order wire circuit. The distant trunk operator then plugs into the required subscribers' multiple and automatic ringing proceeds until the subscriber answers. Supervisory lamps associated with both the trunk cord and the double-ended *A* cord furnish the necessary indication to the operators of the establishment of the connection and end of the conversation. When both lamps associated with the double-ended *A* operator's cord relight, the connection is taken down and this operation gives a disconnect signal also to the distant trunk operator.

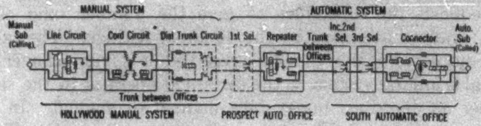
MANUAL SYSTEM
AUTOMATIC SYSTEM
Manual Sub (Calling).
Line Circuit
Cord Circuit
Dial Trunk Circuit
1st Sel.
Repeater
Trunk between Offices
Inc. 2nd Sel.
3rd Sel
Connector
Auto. Sub (Called)
Trunk between Offices
HOLLYWOOD MANUAL SYSTEM
PROSPECT AUTO OFFICE
SOUTH AUTOMATIC OFFICE

FIG. 9—EQUIPMENT INVOLVED IN A CONNECTION BETWEEN A HOLLYWOOD OFFICE MANUAL SUBSCRIBER AND A SOUTH OFFICE AUTOMATIC SUBSCRIBER

Calls from Manual Stations to Automatic Stations are obtained by direct trunk circuits. An *A* operator receiving a manual subscriber's request for a particular automatic number, as indicated by the directory listing, proceeds to dial the number in the same manner as described above for an automatic subscriber. Each *A* operator is provided with a group of trunks, as shown by Fig. 6 and described in the first part of this paper, leading to the nearest automatic office and terminating directly on first selectors.

Private Branch Exchange Calls originating from such exchanges of the former Home Company were formerly operated and are still operated on a manual basis. Calls placed by extension stations from these private exchanges are trunked to a manual switchboard in the Olive office building, this switchboard having the familiar subscriber multiple of all the private exchange trunk lines. The incoming private exchange trunk lines, however, are carried through Keith line switches, the function of which is to select a non-busy operator and to place the line lamp signal before that operator. This arrangement is commonly known as the traffic distributor. This pre-selected operator then receives the request for either an automatic or manual number and if for the former, she dials the number as described above for manual *A* operators. If the called for number is for another private exchange station, the operator completes the connection in the multiple similar to a manual-to-manual connection and if the request is for a station in a manual office, it may be obtained by the use of order wires and trunks direct to the office required or over trunk lines to the special Olive office manual *A* board. All connections to and from the former Pacific manual private branch exchange stations are completed in accordance with the above general methods of handling calls to and from manual stations

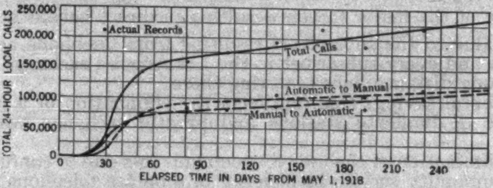

Fig. 10—Curves Showing Tendency of Originating Calls Between Former Pacific and Home Stations

Fig. 10 has been prepared to indicate the volumes of traffic expressed in calls that passed between the former automatic and manual stations as at the specified dates. The curve was developed from the following records and immediately after the record of September 13, 1918. (Later records have since been taken and are shown by dots on the original curve.)

	Automatic to Manual	Manual to Automatic	Total calls
May 31, 1918 .... (1 day before formal announcement)	16,308	31,870	48,178
June 21, 1918....................	71,883	70,921	142,804
July 19, 1918....................	83,589	76,556	160,145
August 23, 1918..................	95,059	80,285	175,344
September 13, 1918...............	105,602	88,161	193,763

So far as I know, no definite statement can be made as to the gains accruing to the subscribers at Los Angeles under the consolidated arrangement, because the most important factors are not capable of reduction to equated savings in dollars.

Some of the major factors resulting in direct benefit are:

1. Value to the subscribers resulting from telephone access to each and every subscriber in the Los Angeles exchange and to all long distance lines centering there.
2. Rental savings resulting from the elimination of a large number of duplicate stations and private exchange switchboards.
3. Elimination of the indirect economic loss due to confusion and community service inefficiency of separate telephone systems.

Some of the factors which tend to offset part of the savings are:

1. Added annual charges on the plant and equipment required to provide means for universal service. There was very little elimination of duplicate plant investment primarily because each plant was designed to care for a definite development and volume of traffic. Consolidation obviously does not reduce traffic volume, but because the telephone field is considerably increased to every subscriber, the total volume of traffic in the consolidated plant is substantially increased.
2. Added operating and maintenance costs.

From a careful weighing of these factors it is manifest that the consolidated plant offers substantial savings and benefits over the dual systems. The worth of a telephone system to any community lies, not only in its capacity for effecting good service but that such service shall be universal and available to the maximum possible number of local and distant subscribers.

Presented at a Section Meeting of the American Institute of Electrical Engineers, Pittsburgh, Pa., February 11, 1919.

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SOME PROBLEMS IN THE OPERATION OF POWER PLANTS IN PARALLEL

BY E. C. STONE

Abstract of Paper

In order to operate two power plants satisfactorily in parallel, the transmission line which ties them together must have sufficient synchronizing power, as well as sufficient carrying capacity. The "synchronizing power" of a line depends upon its resistance and reactance, the bus voltages maintained at its ends, and the maximum kilovolt-amperes it must transmit. The ability of different lines to provide satisfactory parallel operation cannot be measured by any standard which does not take account of all of these factors. Limiting values for "synchronizing power" of lines under various operating conditions are given.

The division of load between two plants in parallel is regulated by steam control; the division of wattless current associated with the load depends upon the voltages generated and may be in proportion to the division of real load, when the difference in voltage at the two busses will vary with the load transmitted, or may be arbitrary so that regardless of the load transmitted, the voltages on the two busses will be maintained constant. The latter plan generally gives better operating conditions on the system as a whole, but creates demands for wattless currents at either or both plants in excess of their normal capacity, and in so doing, involves an additional cost. The excess wattless cross-current so created can be materially reduced by varying the voltage with changes in load transmitted through the use of taps on the line transformers, by inserting additional reactance in the line at light loads, or, when the stations are tied together by several parallel lines, by cutting out one or more lines as the load decreases.

The design of a transmission line involves a consideration of load to be transmitted, voltage, reactance, resistance, losses, and charging current of the line, and of wattless generating capacity at the receiving end of the line. The wattless generating capacity at the end of the line determines how many kilowatts will be transmitted for each ampere of line current, by fixing the power factor of the load transmitted and the voltage at the receiving end of the line. When a line is to be designed for paralleling two plants, it must have sufficient "synchronizing power" to hold the two plants together.

THE advantages to be derived from the operation of all the power plants serving a territory in parallel are many and great. The maximum or peak capacity is increased because of the diversity of the load in different parts of the territory served; small plants with high operating costs are made available for peak service only, without the injurious effects of interruption to service when loading them up and shutting them down; the strain of carrying sharp and sudden peak loads is distributed over the whole generating system; customers in all parts of the system are given the benefits in steady frequency and stability of power supply which come from drawing their power from a source of supply having a capacity vastly greater than their individual requirements.

It is not possible, however, to merely synchronize all the power plants of a system together over any lines that may be available and obtain therefrom all the advantages of parallel operation without experiencing any difficulties. The steam governing devices of the various units at the different plants must be adjusted with respect to each other so as to properly divide the load at all times. The heavy concentration of power in short circuits which results from parallel operation requires that adequate circuit breakers and other protective devices must be installed. The transmission lines used for tying the plants together must have suitable characteristics and sufficient capacity. This paper deals with the latter subject.

When a number of individual machines are operated in parallel in one power plant on a single bus, the impedance in the circuit connecting them is so small as to be quite negligible in its effect on the parallel operation of the machines. When however, two separate power plants are operated in parallel, the transmission line tying them together comes in between the busses, thereby increasing the impedance of the connecting circuit to many times that of a station bus and decreasing its carrying capacity correspondingly. The impedance and carrying capacity of a transmission line thus become very important factors in determining the manner in which the power plants which it ties together will operate in parallel.

The action by which two alternating-current generators connected to a common bus are held in synchronism is commonly understood. If one machine attempts a speed different from that of the other, a difference in phase immediately develops between their voltages, which causes a cross current to flow through the local circuit made up of their armature windings and leads and the bus section between them. Because of the reactance in this circuit, the cross current is of such a phase as to transfer a part of the load carried by the lagging machine over to the leading machine. This in turn causes the former to speed up and the latter to slow down, so that their voltages are brought back into phase and synchronism is maintained.

When two power plants are operated in parallel, the action described above is modified by the insertion of the tie line in the local circuit between generators. The vector diagram for a given set of conditions as to plant voltages and line impedance, is shown in Fig. 1. When the two power plants are exactly in step so that their voltages E and E_2 are in phase, the voltage across the circuit tying them together is e_2, the current I_2 and the power transmitted from station 1 is P_2. The station delivering power to the tie line will be designated as station 1 or the transmitting station, and the station receiving power from the tie line will be designated as station 2 or the receiving station.

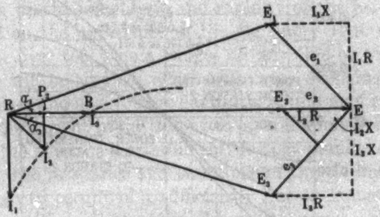

FIG. 1—VECTOR DIAGRAM

E = Voltage at Station 1.
$E_1 E_2 E_3$ = Voltages at Station 2.
$e_1 e_2 e_3$ = Voltages across tie line.
$P_1 P_2 P_3$ = Power delivered to tie line from Station 1.
R = Resistance of tie line.
X = Reactance of tie line.
Station 1 delivers power to line.
Station 2 receives power from line
X/R = f $E_2/E_1 = 0.80$

If station 2 speeds up so as to lead station 1 by the angle a_1, the voltage on the tie line is increased to e_1 and the current to I_1, but since I_1 is at right angles to E, the power transmitted from station 1 to station 2 is zero. Thus station 2, by advancing position from position E_2 to position E_1 has taken from station 1 an additional load equal to P_2. On the other hand, if station 2 lags behind the angle a_3, the current becomes I_3 and power P_3, so that station 2 has then dropped an amount of load equal to $P_3 - P_2$ which amount has been taken up by

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station 1, loading up the tie line still further until a certain critical angle of lag is reached beyond which the load transmitted over the tie line decreases instead of increases.

The curves of Fig. 2 show how the amount of power transmitted—or load transferred—over tie lines having the same resistance, but different reactances, varies with variation in the phase angle between the voltages of the plants tied together. These curves represent the theoretical transmission of power, on the assumption that the line copper losses are part of the load carried from the plant busses and are divided between the plants in proportion to their bus voltages. This was done in order to show the real transfer of load that takes place between the two plants. These curves, and all other curves in this paper are calculated for three-phase transmission, the values of power being the total power transmitted and the values of voltage, the voltage between line wires of the circuit.

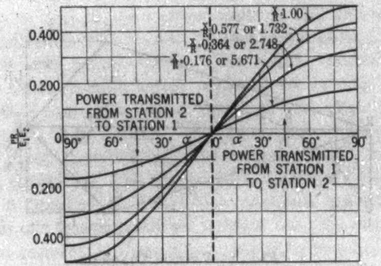

FIG. 2—POWER TRANSMISSION OVER TIE LINE

Resistance and voltages constant.
Line reactance and phase-angle between station voltages variable.
Three-phase circuit.
α = phase angle between E_1 and E_2.
$\beta = \tan^{-1} E/R$

$$\frac{XR}{E_1 E_2} = \cos\beta \sin\beta \sin\alpha = \frac{XR}{Z^2} \sin\alpha.$$

The forces tending to pull paralleled plants out of phase are produced by the variations in the supply of power to the generators and by the variations in the loads drawn from their busses. For example, assuming that the steam governing devices are set so as to make the two plants share the total load equally at all times, one-half of any increase in the load on the bus of one station would have to be carried by the other. That this could be done, the station at whose bus the increase occurred would have to drop behind by a phase angle sufficiently large to permit of the necessary additional amount of power being transmitted over the tie line to meet the new condition. The higher the impedance of the tie line, the greater will be the phase angle required; and if the increase in load takes place suddenly, the lagging plant will at first drop too far behind because of its inertia, and will perform several "vibrations" before coming stable in its new position. In practise, therefore, the phase angle is constantly varying, the degree of variation depending on the magnitude and rapidity of the load fluctuations and on the impedance of the tie line. If it varies too rapidly or over too wide a range, an unstable or "pumping" condition will be created, which will cause swinging loads on the machines and bad voltage fluctuations, and may sufficiently increase the current in the tie line to open the line breakers and separate the plants. To obtain satisfactory parallel operation, therefore, the resistance and reactance of the tie line must be low enough to permit of a sufficiently free exchange of power between plants so that under the most severe conditions of load fluctuations the phase angle between stations will never vary sufficiently to cause harmfully unstable conditions or to open line breakers. In general, practical limitations of carrying capacity of the conductor and of line losses limit the maximum permissible phase angle to from 10 to 30 degrees.

The ability of any line to hold two plants together in satisfactory parallel operation may be called its "Synchronizing Power," which is defined as the change in the amount of power that the line will transmit for each degree change in phase displacement of the voltages of the stations it ties together. Mathematically, this means the rate of change in power transmitted with respect to change in phase angle, and is represented by the "slope" of the curves of Fig. 2. The formula is as follows:

$$Ps = \frac{E_1 E_2}{1000} \times \frac{X}{Z^2} \times \cos a.$$

where

E_1 = voltage at transmitting station,

E_2 = voltage at receiving station,

X = the reactance of the tie line.
Z = the impedance of the tie line.
a = the phase angle between E_1 and E_2.
Ps = synchronizing power in kilowatts.

To be rigidly accurate, the X and Z used above should include the armature windings and bus connections of the generators at the plants, but as these generally have very low impedance compared to the impedance of the line, they can ordinarily be neglected.

On a given line, the greatest change in the amount of power transmitted for a given change in phase angle between the station voltages occurs when the phase angle is zero, while the least change occurs when the angle is 90 degrees. (See Fig. 2). The synchronizing power of a given line is therefore a maximum when the phase angle is zero and zero when the phase angle is 90 degrees. Since the phase angle increases as the load transmitted increases, the synchronizing power decreases as the load transmitted increases.

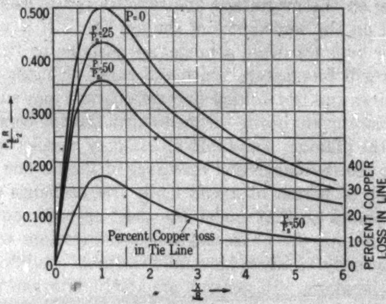

FIG. 3—SYNCHRONIZING POWER UNDER NORMAL OPERATING CONDITIONS

Power factor of load transmitted taken as unity.

$$P_S = E_1 E_2 X/Z^2 \cos \alpha$$

The curves of Fig. 3 show the synchronizing power of lines having the same resistance with varying reactances, when carrying loads of from 0 to one-half the synchronizing power of the line. It will be seen that for a line of given resistance the synchronizing power is greatest when the reactance-resistance ratio of the line is unity and decreases rapidly as this ratio changes from unity value.

While the synchronizing power of a line as defined above may be easily calculated, it is not so easy to determine the amount of synchronizing power that is necessary in a given case to secure satisfactory parallel operation. This will depend on many factors—among them the types of prime movers and their load-speed characteristics, and the character of the load delivered from the busses of the two plants. Reciprocating engine plants will require higher synchronizing power than turbine plants. Loads having rapid and large fluctuations will require more synchronizing power than steady loads. In checking up a number of actual cases, it has been found that under the conditions of fluctuating power load as found in the Pittsburgh district, the least line synchronizing power which will give satisfactory parallel operation of large turbine plants, with the relatively steady load of large systems, is a synchronizing power approximately equal to the capacity of the smaller plant to be paralleled, while with smaller plants, consisting partly of reciprocating engines and partly of turbines and the relatively high load fluctuations of the smaller systems, the tie line to give stable operation must have a synchronizing power equal to not less than 1.5 times the capacity of the smaller plant.

Inasmuch as the synchronizing power of a tie line depends upon a number of factors as explained above, it is obvious that any accurate criterion of the ability of a given line to hold two plants together must take account of all these factors. For example, it is sometimes said that a line having a capacity equal to a certain percentage of that of the smaller plant to be paralleled will give satisfactory operation. Yet two lines may be designed for just this capacity at the same voltage with widely different synchronizing powers. For similar reasons it cannot be assumed that a line which develops not more than a certain copper loss will be satisfactory, for such a statement does not take account of all of the variables.

Synchronizing power of the tie line becomes a limiting factor when the line voltage is low for the distance transmitted. When the value of volts per mile is relatively high, the synchronizing power will be found ample.

Synchronizing power must be given equal consideration whether there are generators or loaded synchronous motors at the receiving end of the line. It is unimportant in connection with synchronous condensers, however, since the energy which they take is always a very small percentage of the maximum power transmitted over the line.

The division of load between two plants in parallel depends of course upon the governor adjustments on the steam end of the prime movers; the division of wattless load associated with the energy load depends upon the voltages generated by the two plants and may be entirely different from the division of real load or energy supply. Thus the plant at one end of a line might supply all of the energy required while the plant at the other end might supply all of the wattless load.

By suitable voltage adjustments the plant which supplies power to the tie line may be made always to supply the wattless component associated with that power—under which condition the difference in voltage at the two busses will vary with the load on the line, being zero at no-load and a maximum when the maximum load is transmitted. Ordinarily, however, this operation produces too wide a range of voltage at the station busses to be satisfactory, and it becomes necessary to regulate the voltages within a closer range. This is possible, since the difference in voltage which is developed between the two ends of a transmission line when power is transmitted over it, depends not only on the amount of power transmitted but also on the amount of wattless energy. Given a certain voltage difference to be maintained between the ends of the line, there is a certain amount of load which with the wattless component associated with it, will just absorb that voltage difference. When the power transmitted is less than this value it becomes necessary in order to maintain the required voltage difference to set up a wattless cross current between the generators of the two stations, which current has the effect of lowering the power factor of the transmitting station and raising the power factor of the receiving station. When the power transmitted is more than the given value, the cross current must be in the opposite phase and has the opposite effect on the power factors of the two stations. The result is that the transmitting station has excess wattless current to take care of when the load on the tie line is light while the receiving station has excess wattless current to take care of when the load on the tie line is heavy. This is provided for by installing generators rated at a sufficiently low power factor or by installing synchronous condensers. At stations where the excessive cross current occurs only at periods of light load, more generators of normal power factor rating may be run at such periods than would be required by the energy load only—a method which is wasteful of coal, but may be more economical than the installation of additional equipment to take care of off-peak wattless current.

Fig. 4 shows how the wattless current in lines of different characteristics must be varied as the energy transmitted varies, in order to maintain constant bus voltages at both stations. Where the curves are below the horizontal zero line the wattless current in the tie line is lagging with respect to the voltage of the transmitting station and leading with respect to that of the receiving station. Where the curves are above the zero line, the wattless current is leading with respect to the voltage of the transmitting station and lagging with respect to the voltage of the receiving station.

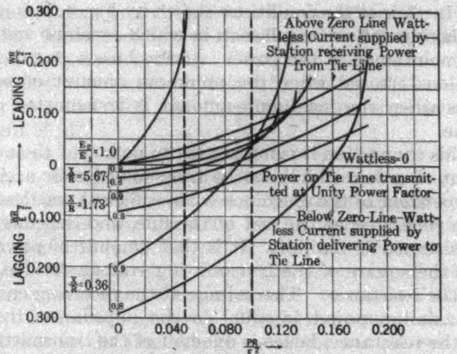

Fig. 4—Wattless Current vs. Power in Tie Line

The important points brought out by the curves of Fig. 4 are:

(1) The variation in wattless current for a given variation in load is very much greater where the reactance-resistance ratio of the line is low than where it is high. This means that the wattless cross current required to maintain constant bus voltages and the excess copper losses and wattless generating capacities arising therefrom increase very rapidly as the reactance-resistance ratio of the line decreases.

(2) When the same voltage is maintained on the busses of

both stations and load is transmitted over the tie line, the wattless cross current which is set up, always leads the voltage of the transmitting station and increases as the load transmitted increases. This means that the receiving station not only must supply all of the wattless current associated with the load received from the other station, but must also supply a certain amount of wattless cross current to the line in order to hold up the bus voltage.

(3) In order that the lagging wattless current associated with a given load as well as the load itself may be transmitted by the tie line, the voltage at the receiving station must be lower than that of the transmitting station.

(4) In view of the conditions shown by Fig. 4, it is necessary when laying out an installation in which constant voltages are to be maintained at the power station busses, to consider the conditions created when the minimum amount of power, as well as when the maximum amount is transmitted over the tie line.

In this connection it is interesting to note that the maximum amount of power which can be transmitted over a given line is independent of the reactance when sufficient wattless current can be generated at the end of the line, and depends only on the resistance of the line. It is that amount of power which equals the square of the transmitting voltage divided by four times the resistance. The voltage at the receiving end to give the maximum power is equal to the impedance divided by twice the resistance, hence is one-half of the transmitting voltage when there is no reactance, is equal to the transmitting voltage when the reactance is 1.732 times the resistance, and is greater than the transmitting voltage for a greater reactance. The very excessive line loss and wattless generating capacity required at the receiving end prohibit ever taking from a line the maximum amount of power that it can transmit.

When it becomes necessary to maintain the same bus voltages at both stations with very light loads transmitted as are maintained at full load, the wattless cross current may develop into an item of considerable cost. For this reason it becomes desirable to reduce it as much as possible. This can be done in two ways as follows:

a. By varying the voltage delivered to the tie line at the transmitting station as the load varies.

b. By varying the impedance of the tie line as the load varies.

The voltage delivered to the tie line is varied through the use of taps on the line transformers, or by means of an inductive regulator or synchronous booster. This method causes little variation in the synchronizing power of the tie lines, but involves considerable expense for special equipment.

The impedance of the tie line is varied through the use of reactance external to the lines or, when the tie circuit consists of several lines in parallel, by varying the number of lines in service. Both of these methods decrease the synchroniznig power as the impedance is increased, but involve little or no cost for special equipment, and will be found of very real value in many cases. It often happens that by a simple switching operation some of the short-circuit limiting reactances at the power stations can be switched into the lines used for paralleling, so that the impedance of these lines can be materially increased at light load periods. Changing the number of lines in service is of course accomplished by the use of the switches regularly installed with the lines, and so requires no special equipment at all.

The design of a line to transmit a given amount of power over a given distance involves a consideration of voltage, resistance, reactance, losses, and charging current of the line, and of wattless generating capacity in the form of synchronous condensers at the receiving end of the line. The selection of the proper voltage involves many factors outside of the scope of this paper, and will not be discussed. To obtain a minimum total cost of line at any given voltage, the annual cost of the copper losses in the line should equal the annual cost of the investment in copper; then, in order to obtain the minimum total cost of transmission, the proper size of synchronous condenser should be installed at the end of the minimum cost line to give the minimum total cost of line and condenser per kilowatt delivered at the end of the line. Thus the cost of copper and copper losses determine the resistance of the line, while the cost of synchronous condenser capacity in relation to the cost of the line determines how many kilowatts will be transmitted for each ampere of line current, by fixing the power factor of the load transmitted, and what is equally important, the voltage at the receiving end of the line. The charging current due to the capacitance of the line produces the effect of approximately the equivalent number of kilovolt-amperes in synchronous condenser capacity located at the middle point of the line and supplied with no additional investment.

When a line is to be designed for paralleling two plants, the minimum as well as the maximum amount of power to be transmitted, and the direction of transmission of each, must be determined from the characteristic curves of the loads supplied from the two busses, from the operating schedules desired of the prime movers at the respective stations, and to a lesser degree, from the load-speed characteristics of the machines and the fluctuating or steady nature of the load. The synchronizing power must also be given due consideration, and if constant bus voltages are to be maintained with a variable load transmitted between stations, the cost of the additional line loss and additional wattless generating capacity required because of the cross current, as well as the cost of the line and synchronous condenser capacity, must be considered in designing the line for minimum total cost of operation.

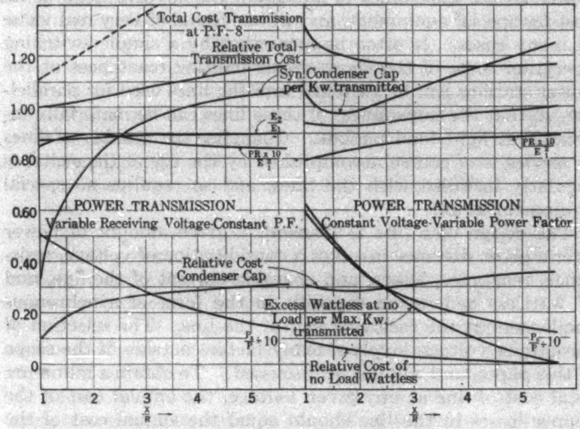

FIG. 5—POWER TRANSMISSION DATA

Transmission voltage 1000 volts per mile.
Cost of copper = Cost of copper loss.
Synchronous condenser capacity and receiving voltage to give minimum total transmission cost.
Power factor of load = 0.8.
Minimum load = 0.

In Fig. 5, curves have been calculated on the basis just outlined to determine the minimum total cost of transmission of a constant amount of power at a fixed voltage per mile, and the synchronizing power of the line resulting therefrom for various values for the ratio of reactance to resistance of the line. The following figures have been assumed as a basis:

Transmission voltage, 1000 volts per mile.
Cost of copper, 22 cents per pound.
Cost of synchronous condenser capacity, $5.00 per kv-a.
Losses of condenser, 0.035 kw. per kv-a. of capacity.
Annual cost of investment, 15 per cent.
Cost of energy, 2 mills per kilowatt-hour.
Power factor of load delivered from busses of both stations, 80 per cent.
Load factor, 50 per cent.
Range of load transmitted, 0 to maximum.

The reactance-resistance ratio of the line is determined chiefly by the frequency, size of conductors, and design of the step-up and step-down transformers. The inherent reactance of transformers goes up rapidly with their voltage, because of the increased spacing required between their high- and low-tension windings, and low reactance at high voltages is attained only by an abnormal and costly design. The reactance of a given line at given frequency, therefore, can only be reduced by splitting it up into a number of parallel lines having smaller conductors and providing abnormally designed transformers, both of which methods are very costly. On the other hand, the reactance of a line can be readily increased to almost any desired value for a few per cent of the cost of the line by the installation of reactance coils. It is therefore fortunate that good voltage regulation can be attained with high reactances.

High voltage lines are generally designed for large blocks of power, making the combined reactance of the large conductors and of the transformers inherently high. Low voltage lines, with relatively small conductors, and often no transformers, have inherently low reactance.

On the left hand side of the curve sheet, Fig. 5, are shown the conditions where the bus voltage of the receiving station varies with the load. As the reactance-resistance ratio increases from 1 to 5.5, the total cost of transmission increases 16 per cent, showing that high reactance means a slight increase in total cost of transmission, under the minimum cost conditions. The synchronous condenser capacity goes up from 0.33 kv-a. per kilowatt transmitted, corresponding to

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90 per cent power factor lagging in the line, to 1.06 kv-a. per kilowatt transmitted, corresponding to 95 per cent power factor leading in the line. Unity power factor obtains when the reactance-resistance ratio is 1.8. The cost of the condenser capacity varies from 9 per cent to 35 per cent of the cost of the line and line losses.

On the right hand side of Fig. 5 are shown conditions when the bus voltages are held constant as the load on the line is varied from zero to the maximum for which the line is designed. The wattless cross current necessary to maintain constant bus voltages introduces an additional element of cost, so that the total cost is higher than before for all values of the reactance-resistance ratio of the line. Furthermore, the total cost increases, instead of decreases, as this ratio decreases. This is because, with a given line resistance, the cross current required to maintain constant voltage increases rapidly as the reactance decreases, as is indicated by the curve showing the wattless cross current required with no power transmitted, which runs from zero for a reactance-resistance ratio of 5.7 up to 0.67 kv-a. per kilowatt of line capacity when the ratio is unity. The minimum attainable cost under these conditions is however very nearly reached when the reactance-resistance ratio is 2.5, so that the only advantage to be gained by a greater proportion of reactance is the raising of the voltage at the receiving end of the line, a feature which is only of importance when power is transmitted in both directions.

Under constant voltage conditions, more synchronous condenser capacity is used, with the result of raising the voltage at the receiving station and cutting out sufficient wattless cross current to more than pay for the additional condenser capacity. The cost of the condenser capacity runs from 30 per cent to 40 per cent of the cost of the line and line losses.

The ratio of synchronizing power to maximum load transmitted over the specified tie line varies from approximately 6, when the reactance-resistance ratio is 1, down to 1.3 when the ratio is 5.7—slightly higher than before because of the higher receiving voltages.

The curve giving values of $\frac{PR \times 10}{E_1^2}$ may be called the design curve. With the transmission voltage E_1 fixed, the resistance R of the line which will transmit a given amount of power

P at minimum cost, or the amount of power P which can be transmitted at minimum cost per kilowatt over a line having a given resistance R can be obtained immediately from this curve.

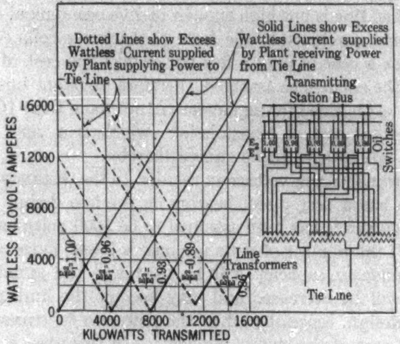
Comparing the curves under the condition of variable or constant bus voltage, it is found that for a reactance-resistance ratio in the line of 5.7, the constant bus voltages are obtained at practically no additional cost. As the ratio decreases, however, the cost of securing constant bus voltages increases, being 8 per cent of the total transmission cost for a ratio of 3, and 31 per cent for a ratio of 1. If the inherent reactance of a specified line is not high enough to give economical operation additional reactance in the form of reactance coils can be inserted at a cost of only a few per cent of the cost of the line. Under the conditions assumed, therefore, constant bus voltages should be obtained for an increase of not more than 10 per cent of the total cost of transmission—which is a very low cost for the advantages to be gained thereby.

The transmission voltage in relation to the distance of transmission has a very direct bearing on the line design problem. In the curves it has been assumed to be 1000 volts per mile of line. If the number of volts per mile is lower, more synchronous condenser capacity will be used and the synchronizing power will be decreased and may become the limiting factor of the design, especially when the power to be transmitted is a small part of the station capacity. In this case, it becomes necessary to use more copper than is required for the load, or to reduce the reactance, or to do both, in order to obtain sufficient synchronizing power.

If the number of volts per mile is greater than 1000, less synchronous condenser capacity will be used and more synchronizing power will be obtained than is indicated in Fig. 5.

In general, it will be found that where constant bus voltages are necessary, the best tie line will be the one which has the greatest reactance consistent with the necessary synchronizing power. Transmission with high reactance, however, necessitates an ample amount of wattless generating capacity at the receiving station, in order to hold up the voltage. Where this is not available, so that a considerable amount of wattless current must be supplied with the load from the transmitting station, high line reactance is impracticable and the cost of maintaining constant bus voltages is greatly increased.

In Figs. 6 and 7 are shown data worked out for specific cases, to determine how the wattless cross current could be reduced. In both cases it was necessary to maintain constant bus voltages at the two plants to be paralleled. In Fig. 6, loads varying from 0 to 15,000 kw. were to be transmitted from one station to the other. In order to keep the excess wattless current—by which is meant the wattless current which either station would be called upon to supply in addition to that associated with the real load it was delivering—below the limit of 4000 kv-a., it was necessary to provide four additional taps on the line transformers and four additional oil switches at the transmitting station to give the voltages as shown. Fig. 7 shows how in a given case, where two lines tie the two plants together, the wattless cross current was reduced at light load periods by switching in a bus limiting reactance already installed, which happened to be just large enough to double the reactance-resistance ratio of the line. Fig. 7 also shows the effect of cutting out one of the two tie lines.



Fir. 6—Effect of Varying Bus Voltages

Transmission on four No. 0 lines in parallel.
Resistance = 0.52 ohn. $X/R = 1.33$
Power factor of load 0.8. Voltage E_1 = 11,000.
Synchronizing power = 2.25 times the capacity of plant to be tied in.

Fig. 8 is a chart which was worked out to guide the system operators in regulating the cross current between the stations connected by two lines. If two lines are in service and the power factor drops below 45 per cent, one line is immediately cut out. This doubles the kilowatts transmitted by the remaining line, but does not increase its current, so that by cutting out the other line approximately 180 amperes of cross current are eliminated.

In concluding with a brief summary, it must be borne in mind that in designing a tie line to meet any specified condition many variables enter the problem. In general, costs of copper, steel, and coal go up and down together, so that absolute costs are not so important as relative costs of these items. The load factor of the power transmitted affects the overall costs of line losses, and therefore the amount of the copper used. The transmission voltage in relation to the transmission distance has a very direct bearing on the amount of synchronous condenser capacity to be used, and on the synchronizing power of the line. Since the data and curves which have been shown are based upon a series of specific assumptions, which while sufficiently correct under the conditions at hand, might have to be materially changed to meet other conditions, the data presented are of interest in showing the relative weight of the different factors rather than in determining absolute values. It is hoped that in showing the effect of the different variables on the problem as a whole, a guide may be furnished to the

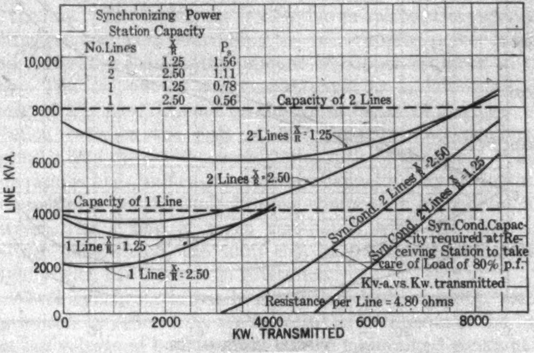


FIG. 7—REGULATION OF WATTLESS CROSS CURRENT BY VARYING THE REACTANCE AND NUMBER OF TIE LINES IN SERVICE

lines which should be followed in working out any specific case. With these limitations in mind, the following points may be emphasized:

In order to operate two power plants satisfactorily in parallel the transmission line which ties them together must have sufficient "synchronizing power," as well as sufficient carrying capacity to carry the maximum load to be transmitted between them. The synchronizing power of a given line depends upon its resistance and reactance, upon the bus voltages maintained at its ends, and upon the maximum kilovolt amperes it must transmit. The ability of different lines to provide satisfactory parallel operation cannot be measured by any standard which does not take account of all of these factors. It has been found from the writer's experience that a synchronizing power about equal to the kilowatt capacity of the smaller plant to be paralleled, for large turbine plants with load fluctuations of relatively small percentage of their capacity, and of 1.5 times the smaller plant capacity for mixed engine and turbine plants with relatively large load fluctuations, is about the least that a line can have to give satisfactory parallel operation.

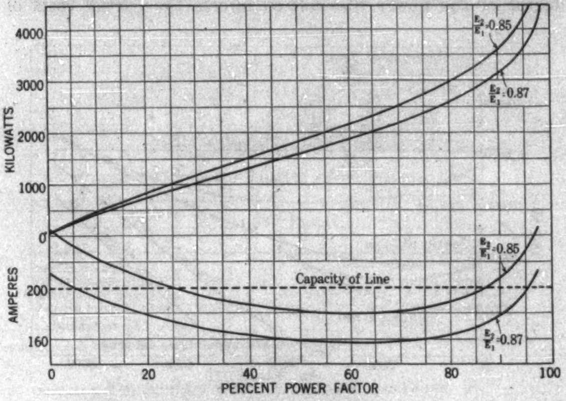


Fig. 8—System Operator's Working Diagram

Performance of one tie line.

$R = 4.8$ ohms. $X = 6.0$ ohms.

When the voltage per mile of transmission is high, line capacity, as determined by load and operating conditions at the two stations, is the limiting factor; when the voltage per mile is low, synchronizing power more often marks the limit, especially when the reactance is high and the load to be transmitted relatively small. The necessity for sufficient line synchronizing power applies equally whether generators or loaded synchronous motors are at the end of the line, but is unimportant in connection with synchronous condensers at the end of the line.

The design of equipment for transmitting a block of power over a given distance at minimum cost must take into account the use of synchronous condenser capacity at the receiving end of the line. The condenser is effective in raising the receiving voltage as well as in controlling the transmission power factor, and materially reduces the amount of copper required in the line. The amount of condenser capacity which may be economically used depends on the power factor of the load, the relation between the cost of line capacity and condenser capacity, and the reactance-resistance ratio of the line. Where this ratio is high, sufficient condenser capacity may be justified to produce a considerable leading component in the tie line current. In a line of given resistance, with the receiving voltage unregulated, the total cost of transmission increases slightly with the increase in reactance.

Where synchronous machines of sufficient capacity, either condensers or generators, are installed at both ends of the line, the bus voltage of both stations can be maintained constant as the load transmitted over the line varies, by the creation of a wattless cross current. This wattless current, however, involves an additional cost in copper loss and in the wattless generating capacity required at one or both stations to take care of it, which cost is small when the reactance-resistance ratio of the line is sufficiently high, but which rapidly increases as the reactance decreases, and for low line reactance becomes very important. This wattless current can be materially reduced under certain conditions, by varying the voltages at the ends of the line as the load varies through the use of transformer taps, by inserting additional reactance in the line at light loads, or, when the stations are tied together by several parallel lines, by cutting out one or more lines as the load decreases.

The increase in the cost of transmission with constant bus voltages caused by this wattless cross current represents the cost of eliminating the regulation of the tie line. The wattless cross current decreases as the reactance-resistance ratio of the line increases, but the synchronizing power also decreases. Hence it will generally be found that the best design of transmission equipment for constant bus voltages is that which makes the reactance of the line as high as is permissible for the synchronizing power required. With such a design, the cost of maintaining constant bus voltages—that is, of eliminating the regulation of the tie line—does not amount to more than a few per cent of the total cost of transmission, which is a very small price to pay for the operating advantages gained. It must be remembered, however, that to obtain constant bus voltages at this slight cost, the installation of sufficient wattless generating capacity at the receiving station is absolutely essential—a feature, unfortunately, that is too often lacking in existing systems.

APPENDIX

In what follows, the formulas are developed which were used in calculating the data presented in the foregoing paper.

Referring to Fig. 9,

Let E or E_1 = Voltage at transmitting station.
E_2 or kE = Voltage at receiving station.
E_0 = Voltage across tie circuit.
$k = E_2/E$
α = Angle between E and E_2
z = Impedance of tie circuit
r = Resistance of tie circuit
x = Reactance of tie circuit
$\beta = \tan^{-1} x/r$
P_1 = Power delivered to tie circuit by transmitting station in watts.
W_1 = Wattless delivered to tie circuit by transmitting station in volt-amperes
VA_1 = Total volt amperes delivered to tie circuit by transmitting station
P_2 = Power received from tie circuit by receiving station, in watts
W_2 = Wattless received from tie circuit by receiving station in volt-amperes
VA_2 = Total volt-amperes received from tie circuit by receiving station

P_s = Synchronizing power of tie circuit, in watts
I_0 = Current in tie circuit, in amperes
$\theta_1,\ \theta_2$ = Angles between voltages as shown in Fig. 9.

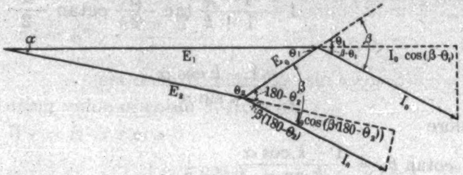


FIG. 9—VOLTAGE AND CURRENT RELATIONS WHEN POWER IS TRANSMITTED OVER TIE LINE

From the triangle of voltages,

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{\tan \frac{1}{2}(\theta_2 + \theta_1)}{\tan \frac{1}{2}(\theta_2 - \theta_1)} = \frac{1 + k}{1 - k} \qquad (1)$$

$$\tfrac{1}{2}(\theta_2 - \theta_1) = \tan^{-1}\left(\frac{1 - k}{1 + k} \operatorname{cotan} \frac{\alpha}{2}\right) \qquad (2)$$

$$\tfrac{1}{2}(\theta_2 + \theta_1) = \tfrac{1}{2}(180 - \alpha) = 90 - \alpha/2 \qquad (3)$$

Adding (**2**) and (**3**)

$$\theta_2 = 90 - \alpha/2 + \tan^{-1} \frac{1 - k}{1 + k} \operatorname{cotan} \alpha/2 \qquad (4)$$

whence

$$\tan(90 - \theta_2) = \frac{\tan \alpha/2 - \dfrac{1 - k}{1 + k} \operatorname{cotan} \alpha/2}{1 + \dfrac{1 - k}{1 + k} \tan \alpha/2 \operatorname{cotan} \alpha/2}$$

$$= \frac{k - \cos \alpha}{\sin \alpha} \qquad (5)$$

Therefore

$$\operatorname{cotan}(180 - \theta_2) = -\tan(90 - \theta_2) = \frac{\cos \alpha - k}{\sin \alpha} \qquad (6)$$

Subtracting (**2**) from (**3**)

$$\theta_1 = 90 - \frac{\alpha}{2} - \tan^{-1}\left(\frac{1 - k}{1 + k} \operatorname{cotan} \frac{\alpha}{2}\right) \qquad (7)$$

whence

$$\tan(90 - \theta_1) = \frac{\tan\frac{\alpha}{2} + \frac{1-k}{1+k}\operatorname{cotan}\frac{\alpha}{2}}{1 - \frac{1-k}{1+k}\tan\frac{\alpha}{2}\operatorname{cotan}\frac{\alpha}{2}}$$

$$= \frac{1 - k\cos\alpha}{k\sin\alpha} \qquad (8)$$

Therefore

$$\operatorname{cotan}\theta_1 = \frac{1 - k\cos\alpha}{k\sin\alpha} \qquad (9)$$

Again from the voltage triangle,

$$\frac{E_0}{E_2} = \frac{E_0}{k\,E} = \frac{\sin\alpha}{\sin\theta_1} \qquad (10)$$

whence

$$E_0 = \frac{k\,E\sin\alpha}{\sin\theta_1} \qquad (11)$$

and $$I_0 = \frac{k\,E}{z}\,.\,\frac{\sin\alpha}{\sin\theta_1} = k\,E\,.\,\frac{\cos\beta}{r}\,.\,\frac{\sin\alpha}{\sin\theta_1}\,. \qquad (12)$$

Now

$$P_1 = E\,.\,I_0\,.\cos(\beta - \theta_1) \qquad (13)$$

$$= E^2\,.\,\frac{k\cos\beta}{r}\,.\,\frac{\sin\alpha}{\sin\theta_1}\,.\cos(\beta - \theta_1) \qquad (14)$$

$$= E^2\,.\,\frac{k\cos\beta}{r}\,.\,\frac{\sin\alpha}{\sin\theta_1}$$

$$.\,(\cos\beta\cos\theta_1 + \sin\beta\sin\theta_1) \qquad (15)$$

$$= E^2\,.\,\frac{k\cos\beta}{r}\,(\cos\beta\sin\alpha\operatorname{cotan}\theta_1 + \sin\alpha\sin\beta)$$

$$(16)$$

By substituting value of cotan θ_1 from (**9**)

$$P_1 = E^2/r\,k\cos\beta$$

$$\left(\sin\alpha\sin\beta + \sin\alpha\cos\beta\left\{\frac{1 - k\cos\alpha}{k\sin\alpha}\right\}\right)$$

$$(17)$$

$$= E^2/r\,k\cos\beta$$

$$\left(\sin\alpha\sin\beta + \frac{\cos\beta}{k} - \cos\alpha\cos\beta\right) \qquad (18)$$

$$= E^2/r\,k\cos^2\beta\,(\sin\alpha\tan\beta + 1/k - \cos\alpha) \qquad (19)$$

Again, $W_1 = E \,.\, I_0 \,.\, \sin(\beta - \theta_1)$ **(20)**

$$= E^2 \,.\, \frac{k \cos \beta}{r} \,.\, \frac{\sin \alpha}{\sin \theta_1} \left(\sin \beta \cos \theta_1 - \cos \beta \sin \theta_1\right) \qquad \textbf{(21)}$$

$$= E^2 \,.\, \frac{k \cos \beta}{r} \left(\sin \alpha \sin \beta \operatorname{cotan} \theta_1 - \sin \alpha \cos \beta\right) \qquad \textbf{(22)}$$

Substituting value of cotan θ_1 from (**9**)

$$W_1 = E^2/r\, k \cos \beta \left(\sin \alpha \sin \beta \left\{\frac{1 - k \cos \alpha}{k \sin \alpha}\right\} - \sin \alpha \cos \beta\right) \qquad \textbf{(23)}$$

$$= -\, E^2/r\, k \sin \beta \cos \beta \left(\frac{\sin \alpha}{\tan \beta} + \cos \alpha - \frac{1}{k}\right) \qquad \textbf{(24)}$$

$$= -\, E^2/r\, k \cos^2 \beta \tan \beta \left(\frac{\sin \alpha}{\tan \beta} + \cos \alpha - \frac{1}{k}\right) \qquad \textbf{(25)}$$

$$V A_1 = E \,.\, I_0 = E \,.\, k E \,.\, \frac{\cos \beta}{r} \,.\, \frac{\sin \alpha}{\sin \theta_1} \qquad \textbf{(26)}$$

$$= \frac{E^2}{r} k \cos \beta \frac{\sin \alpha}{(1 + \operatorname{cotan}^2 \alpha)^{\frac{1}{2}}} \qquad \textbf{(27)}$$

$$= \frac{E^2}{r} k \cos \beta \,.\, \sin \alpha \,.\, \frac{(k^2 + 1 - 2\, k \cos \alpha)^{\frac{1}{2}}}{k \sin \alpha} \qquad \textbf{(28)}$$

$$V A_1 = E^2/r \cos \beta \,(1 + k^2 - 2\, k \cos \alpha)^{\frac{1}{2}} \qquad \textbf{(29)}$$

Similarly,

$$P_2 = E_2 I_2 \cos\left(\beta - \{180 - \theta_2\}\right) \qquad \textbf{(30)}$$

$$W_2 = E_2 I_2 \sin\left(\beta - \{180 - \theta_2\}\right) \qquad \textbf{(31)}$$

$$V A_2 = E_2 \,.\, I_0 = k E \,.\, k E \,.\, \frac{\cos \beta}{r} \,.\, \frac{\sin \alpha}{\sin(180 - \theta_2)} \qquad \textbf{(32)}$$

From which are obtained by substituting for cotan $(180 - \theta_2)$ its equal $\dfrac{\cos \alpha - k}{\sin \alpha}$ as obtained in (**6**).

$$P_2 \quad = E^2/r\ k \cos^2 \beta\ (\sin \alpha \tan \beta + \cos \alpha - k) \qquad \mathbf{(33)}$$

$$W_2 \quad = \frac{E^2}{r} k \cos^2 \beta \tan \beta \left(\frac{\sin \alpha}{\tan \beta} + k - \cos \alpha \right) \qquad \mathbf{(34)}$$

$$V\ A_2 = E^2/r\ k \cos \beta\ (1 + k^2 - 2\ k \cos \alpha)^{1/2} \qquad \mathbf{(35)}$$

Copper Loss in tie Circuit. It is obvious that the copper losses in the tie circuit are equal to the power delivered to one end of the tie circuit minus the power received at the other end. That is, **(36)**

$$I_0^2\ r = P_1 - P_2 = E^2/r\ k \cos^2 \beta\ (\sin \alpha \tan \beta + 1/k - \cos \alpha) - E^2/r\ k \cos^2 \beta\ (\sin \alpha \tan \beta + \cos \alpha - k)$$

$$= E^2/r\ k \cos^2 \beta\ (k + 1/k - 2 \cos \alpha) \qquad \mathbf{(37)}$$

Synchronizing Power. It may be assumed that the elements of line loss in P_1 and P_2 are variable loads supplied from the busses of transmitting and receiving stations, respectively. On this basis, the power actually transferred from one station to the other for a given phase angle α between their voltages as obtained from (**19**) to (**33**) is found to be

$$E^2/r\ k \cos^2 \beta \sin \alpha \tan \beta \qquad \mathbf{(38)}$$

$$\text{or } E^2/r\ k \sin \beta \cos \beta \sin \alpha \qquad \mathbf{(39)}$$

The synchronizing power P_s, is defined as the change in the amount of power transferred over the tie circuit for one degree change in phase between bus voltages.

Therefore $$P_s = \frac{d}{d\ \alpha} (E^2/r\ k \sin \beta \cos \beta \sin \alpha) \qquad \mathbf{(40)}$$

$$= E^2/r\ k \sin \beta \cos \beta \cos \alpha) \qquad \mathbf{(41)}$$

which is equivalent to $\frac{E_1\ E_2}{r} \cdot \frac{x}{z} \cdot \frac{r}{z} \cdot \cos \alpha$ **(42)**

$$\text{or } E_1\ E_2 \cdot x/z^2 \cdot \cos \alpha \qquad \mathbf{(43)}$$

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SOME NEW FORMULAS FOR REACTANCE COILS

BY H. B. DWIGHT

ABSTRACT OF PAPER

Formulas are presented and derived, which have not been previously published, for mutual inductance of coils with parallel axes, repulsion of coils with parallel axes, and self-inductance of long cylindrical coils.

These formulas apply to practically all cases of reactance coils in common use. They are very convergent and accurate, and will give results to a given degree of accuracy with a minimum amount of labor.

For many engineering problems, precise accuracy is not required, and sets of curves are given from which approximate readings may be taken.

PART I—MUTUAL INDUCTANCE OF COILS WITH PARALLEL AXES

THE common method of mounting reactance coils is with parallel axes, that is, side by side, and it is desirable to have formulas which apply to coils in this position. The values of mutual inductance derived in the following paragraphs are useful for calculating the unbalance in voltage and the means for correcting it, when three coils are placed side by side and connected in a three-phase circuit. The formulas are also useful in the design of radio apparatus. The calculation is very exact for widely separated coils, and it has an accuracy within a small percentage for coils placed as close together as it is usual to mount reactance coils. For usual engineering problems where precise accuracy is not required, it is not necessary to calculate the results, but values may be read from the curves of Fig. 1, thus saving the labor of computation.

A formula for the mutual inductance of two circles formed of one turn of infinitesimally thin wire has been given by S. Butterworth,* for the case when the axes of the circles are parallel, and when the distance between the centers of the circles is somewhat larger than their diameters. In order to obtain a formula suitable for commercial reactance coils of many turns, the author has integrated the above formula four

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*Scientific Paper No. 320 of the Bureau of Standards, Washington, D. C., June, 1918, Eq. 10 A, and *Philosophical Magazine*, Vol. 31, 1916, page 443.

times over the rectangular section of a cylindrical coil, with the following result:—

M, in henrys,

$$= \frac{2\,\pi^2\,a^3\,N^2}{0.3937\,b^2\,10^9}\Bigg[\frac{a}{r}\left(\frac{r}{s}-1\right)\left(1+\frac{c^2}{6\,a^2}+\frac{c^4}{144\,a^4}\right)$$

$$+\frac{a^3}{2\,r^3}\left(1-\frac{3}{2}\,\frac{s^2}{r^2}+\frac{r^3}{2\,s^3}\right)$$

$$\left(1+\frac{7}{12}\,\frac{c^2}{a^2}+\frac{13}{240}\,\frac{c^4}{a^4}+\frac{c^6}{960\,a^6}\right)$$

$$-\frac{5}{8}\,\frac{a^5}{r^5}\left(1-\frac{5\,s^2}{r^2}+\frac{35}{8}\,\frac{s^4}{r^4}-\frac{3}{8}\,\frac{r^5}{s^5}\right)$$

$$\left(1+\frac{17}{15}\,\frac{c^2}{a^2}+\frac{169}{600}\,\frac{c^4}{a^4}+\frac{41}{2800}\,\frac{c^6}{a^6}+\frac{113}{672{,}000}\,\frac{c^8}{a^8}\right)$$

$$+\frac{35}{32}\,\frac{a^7}{r^7}\left(1-\frac{21}{2}\,\frac{s^2}{r^2}+\frac{189}{8}\,\frac{s^4}{r^4}-\frac{231}{16}\,\frac{s^6}{r^6}+\frac{5}{16}\,\frac{r^7}{s^7}\right)$$

$$\left(1+\frac{155}{84}\,\frac{c^2}{a^2}+\frac{2167}{2520}\,\frac{c^4}{a^4}+\frac{541}{4704}\,\frac{c^6}{a^6}\right.$$

$$\left.+\frac{2129}{3136\times180}\,\frac{c^8}{a^8}+\frac{197}{1960\times3456}\,\frac{c^{10}}{a^{10}}\right)+\text{etc.}\Bigg] \qquad (1)$$

where $r^2 = s^2 + b^2$ and $a = \frac{1}{2}\,d$ = the mean radius of the coil. N is the number of turns in each coil. The two coils are alike and placed as in Fig 1.

The dimensions are assumed to be given in inches; if they are given in centimeters, the factor 0.3937 should be omitted. In measuring the coil, each dimension should be taken as the number of wires or cables in that dimension times their pitch. The actual measured dimension over the copper should not be used, as it is slightly too small.

It will be noted that formula (1) is indeterminate when $b = 0$ and therefore $r = s$. For such a case, the following formula should be used:

$$M = \frac{\pi^2\,a^4\,N^2}{0.3937\,s^3\,10^9}\Bigg[\left(1+\frac{c^2}{6\,a^2}+\frac{c^4}{144a^4}\right)$$

$$+\frac{9}{4}\,\frac{a^2}{s^2}\left(1+\frac{7}{12}\,\frac{c^2}{a^2}+\frac{13}{240}\,\frac{c^4}{a^4}+\frac{c^6}{960\,a^6}\right)$$

$$+\frac{375}{64}\frac{a^4}{s^4}\left(1+\frac{17}{150}\frac{c^2}{a^2}+\frac{169}{600}\frac{c^4}{a^4}+\frac{41}{2800}\frac{c^6}{a^6}+\frac{113}{672{,}000}\frac{c^8}{a^8}\right)$$

$$+\frac{8575}{512}\frac{a^6}{s^6}\left(1+\frac{155}{84}\frac{c^2}{a^2}+\frac{2167}{2520}\frac{c^4}{a^4}+\frac{541}{4704}\frac{c^6}{a^6}+\frac{2129}{3136\times 180}\frac{c^8}{a^8}+\frac{197}{1960\times 3456}\frac{c^{10}}{a^{10}}\right)+\text{etc.}\Bigg] \quad (2)$$

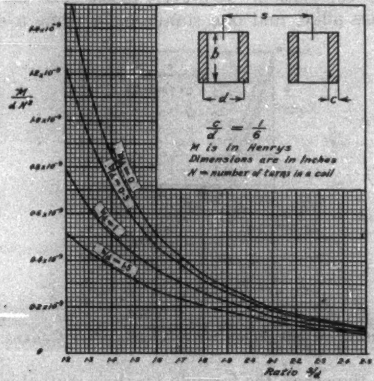

FIG. 1—MUTUAL INDUCTANCE OF REACTANCE COILS WITH PARALLEL AXES

This gives the mutual inductance of two flat disks in the same plane.

An approximate formula for the mutual inductance of two reactance coils with parallel axes has been previously published by the writer.* This was derived by making a certain distortion of one of the coils which, as was stated, tended to make the result slightly too small. The results of this formula for one shape of coil are shown in Curve II, Fig. 2. In order to show that the results were accurate enough for engineering

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*"Repulsion and Mutual Inductance of Reactors", by H. B. Dwight, *The Electrical World*, p. 1148, June 16, 1917.

work, another formula was derived using a greater distortion which tended to make the results distinctly too large, as shown in Curve III. It is interesting to note that Curve I, which shows the result of the formula now given, lies just a little above Curve II.

The new formula is more convenient to use than the older approximate formula, and gives more accurate results for any given position of the coils. Neither formula is very convergent for s/d less than about 1.2.

The case outlined in Fig. 1, in which the two coils with parallel axes are alike and stand on the same plane, is the simplest for calculation. However, if the coils have parallel axes and are alike, and one stands on a plane a distance e higher than the other, the formula for their mutual inductance is not very complicated. It is as follows:

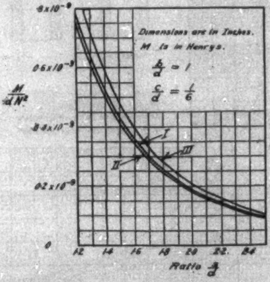

FIG. 2—COMPARISON WITH APPROXIMATE CURVES

M, in henrys,

$$= \frac{\pi^2 N^2}{0.3937\, b^2 \times 10^9} \Bigg[a^4 \left(\frac{1}{p} - \frac{2}{q} + \frac{1}{r} \right)$$

$$\left(1 + \frac{c^2}{6\, a^2} + \frac{c^4}{144\, a^4} \right) - \frac{1}{2}\, a^6 \left\{ \left(\frac{1}{p^3} - \frac{2}{q^3} + \frac{1}{r^3} \right) \right.$$

$$\left. - \frac{3\, s^2}{2} \left(\frac{1}{p^5} - \frac{2}{q^5} + \frac{1}{r^5} \right) \right\}$$

$$\left(1 + \frac{7}{12} \frac{c^2}{a^2} + \frac{13}{240} \frac{c^4}{a^4} + \frac{c^6}{960\, a^6} \right)$$

$$+ \frac{5}{8} a^8 \left\{ \left(\frac{1}{p^5} - \frac{2}{q^5} + \frac{1}{r^5} \right) - 5\, s^2 \left(\frac{1}{p^7} - \frac{2}{q^7} + \frac{1}{r^7} \right) \right.$$

$$\left. + \frac{35\, s^4}{8} \left(\frac{1}{p^9} - \frac{2}{q^9} + \frac{1}{r^9} \right) \right\}$$

$$\left(1 + \frac{17}{15} \frac{c^2}{a^2} + \frac{169}{600} \frac{c^4}{a^4} + \frac{41}{2800} \frac{c^6}{a^6} + \frac{113}{672{,}000} \frac{c^8}{a^8} \right)$$

$$- \frac{35}{32} a^{10} \left\{ \left(\frac{1}{p^7} - \frac{2}{q^7} + \frac{1}{r^7} \right) - \frac{21\, s^2}{2} \left(\frac{1}{p^9} - \frac{2}{q^9} + \frac{1}{r^9} \right) \right.$$

$$+ \frac{189\, s^4}{8} \left(\frac{1}{p^{11}} - \frac{2}{q^{11}} + \frac{1}{r^{11}} \right)$$

$$\left. - \frac{231\, s^6}{16} \left(\frac{1}{p^{13}} - \frac{2}{q^{13}} + \frac{1}{r^{13}} \right) \right\}$$

$$\left(1 + \frac{155}{84} \frac{c^2}{a^2} + \frac{2167}{2520} \frac{c^4}{a^4} + \frac{541}{4704} \frac{c^6}{a^6} + \frac{2129}{3136 \times 180} \frac{c^8}{a^8} \right.$$

$$\left. + \frac{197}{1960 \times 3456} \frac{c^{10}}{a^{10}} \right) + \text{etc.} \Big] \qquad (3)$$

where $p^2 = s^2 + (b - e)^2$

$q^2 = s^2 + e^2$

and $r^2 = s^2 + (b + e)^2$

This formula includes the case of two coaxial coils, when the coils are not near to each other. When the coils are coaxial, the mutual inductance is opposite in sign to the value when the coils are side by side. There is a position, when two coils with parallel axes are diagonally from each other, for which the mutual inductance is zero.

The formula for the mutual inductance of two unequal coils with parallel axes, in any relative position, is similar to the above, but is, of course, more complicated.

Example I. Find the voltage drop due to mutual inductance in each of three coils placed side by side in a row, and carrying three-phase, 60-cycle current, 400 amperes per phase, the data being as follows:—

Mean radius of coils $= a = 12.76$ inches.
Length of coils $= b = 30.87$ "
Thickness of winding $= c = 4.87$ "
Number of turns $= N = 114$
Distance between centers $= 45$ inches.

Let the three coils in a row be called A, B and C. For A and B, $s = 45$ inches.

$$M_{AB} = 0.00142\,(0.0508 + 0.0061 + 0.0007 + 0.00008)$$
$$= 8.2 \times 10^{-5} \text{ henry.}$$

or using Fig. 1, $\qquad \dfrac{s}{d} = \dfrac{45}{25.52} = 1.76$

and $\qquad \dfrac{b}{d} = \dfrac{30.87}{25.52} = 1.21$

$$M_{AB} = 0.25 \times 10^{-9} \times 114^2 \times 25.52 = 8.3 \times 10^{-5} \text{ henry.}$$

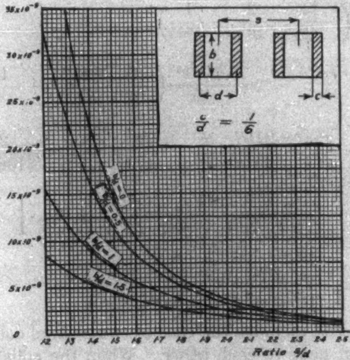

FIG. 3—MECHANICAL FORCE BETWEEN REACTANCE COILS WITH PARALLEL AXES

Average force in pounds = F (from curves) $I_1 I_2 N^2 \cos\theta$ where N is the number of turns in each coil and where θ is the phase angle between I_1 and I_2 which are in amperes.

Using a spacing $s = 90$ inches

$$M_{AC} = 1.1 \times 10^{-5} \text{ henry.}$$

Let the currents in the three coils A, B and C, which are 120 deg. apart in phase, be

$$I_A = 400 \text{ amperes}$$
$$I_B = -200 + j\,200\sqrt{3} \text{ amperes}$$

and $\qquad I_C = -200 - j\,200\sqrt{3}$ amperes

The drop in A due to mutual inductance is

$$2\pi \times 60j\,(I_B\,M_{AB} + I_C\,M_{AC}) = -9.3 - j\,7.0 \text{ volts.}$$

The drop in B due to mutual inductance is

$2\pi \times 60\, j\, (I_A M_{AB} + I_C M_{BC}) = 10.7 + j\, 6.2$ volts.

The drop in C due to mutual inductance is

$2\pi \times 60\, j\, (I_A M_{AC} + I_B M_{BC}) = 10.7 - j\, 4.5$ volts.

PART II—REPULSION OF COILS WITH PARALLEL AXES

The formula for calculating the mechanical force exerted between two equal reactance coils placed in the usual way, side by side with parallel axes, may be derived by differentiating the expression for the mutual inductance of the coils in the same position. Curves are given in Fig. 3 which should be more convenient in most cases than the formula, for solving problems.

The differential with respect to s of formula (**1**) for mutual inductance, using absolute units, is equal to the mechanical force in dynes between the coils when one absampere is flowing in each coil, the currents being in phase. Changing the units, this gives the force in pounds for one ampere turn, which is equal to:

F, in pounds,

$$\begin{aligned}
&= \frac{2\pi^2 a^2}{4.45 \times 10^7 b^2}\Bigg[\frac{a^2}{r^2}\left(\frac{r^2}{s^2} - \frac{s}{r}\right)\left(1 + \frac{c^2}{6a^2} + \frac{c^4}{144a^4}\right) \\
&+ \frac{3}{4}\frac{a^4}{r^4}\left(\frac{4s}{r} - \frac{5s^3}{r^3} + \frac{r^4}{s^4}\right) \\
&\qquad \left(1 + \frac{7}{12}\frac{c^2}{a^2} + \frac{13}{240}\frac{c^4}{a^4} + \frac{c^6}{960a^6}\right) \\
&- \frac{75}{64}\frac{a^6}{r^6}\left(\frac{8s}{r} - \frac{28s^3}{r^3} + \frac{21s^5}{r^5} - \frac{r^6}{s^6}\right) \\
&\quad \left(1 + \frac{17}{15}\frac{c^2}{a^2} + \frac{169}{600}\frac{c^4}{a^4} + \frac{41}{2800}\frac{c^6}{a^6} + \frac{113}{672{,}000}\frac{c^8}{a^8}\right) \\
&+ \frac{245}{256}\frac{a^8}{r^8}\left(\frac{32s}{r} - \frac{216s^3}{r^3} + \frac{396s^5}{r^5} - \frac{429}{2}\frac{s^7}{r^7} + \frac{5}{2}\frac{r^8}{s^8}\right) \\
&\left(1 + \frac{155}{84}\frac{c^2}{a^2} + \frac{2167}{2520}\frac{c^4}{a^4} + \frac{541}{4704}\frac{c^6}{a^6} + \frac{2129}{3136 \times 180}\frac{c^8}{a^8}\right. \\
&\qquad\qquad \left. + \frac{197}{1960 \times 3456}\frac{c^{10}}{a^{10}}\right) + \text{etc.}\Bigg] \qquad (\mathbf{4})
\end{aligned}$$

where $r^2 = s^2 + b^2$ and $a = \frac{1}{2} d$ = the mean radius of the coil. The dimensions may be given in either inches or centimeters, since only ratios of dimensions appear.

It results therefore, that if all the dimensions, including spacing, of a group of coils be increased by a certain ratio, but the currents be left the same, the mechanical force in pounds is not changed at all. However, large coils are generally subject to large forces, since they carry proportionately large currents.

When different alternating currents I_1 and I_2, at a phase angle θ, flow in the two coils, the average force in pounds is

$$F\, I_1\, I_2\, N^2 \cos\theta$$

where I_1 and I_2 are the effective values of the currents in amperes and where N is the number of turns in each coil.

As in the case of the expression for mutual inductance, formula (**4**) becomes indeterminate when $b = 0$ and therefore $r = s$. In such a case, which is that of two flat disks in the same plane, the following formula may be used:

$$\begin{aligned} F, \text{ in pounds, } = {} & \frac{3\,\pi^2\,a^4}{4.45 \times 10^7\, s^4} \Bigg[\left(1 + \frac{c^2}{6\,a^2} + \frac{c^4}{144\,a^4}\right) \\ & + \frac{15}{4}\,\frac{a^2}{s^2} \left(1 + \frac{7}{12}\,\frac{c^2}{a^2} + \frac{13}{240}\,\frac{c^4}{a^4} + \frac{c^6}{960\,a^6}\right) \\ & + \frac{875}{64}\,\frac{a^4}{s^4} \Bigg(1 + \frac{17}{15}\,\frac{c^2}{a^2} + \frac{169}{600}\,\frac{c^4}{a^4} + \frac{41}{2800}\,\frac{c^6}{a^6} \\ & \qquad\qquad\qquad\qquad + \frac{113}{672{,}000}\,\frac{c^8}{a^8}\Bigg) \\ & + \frac{25{,}725}{512}\,\frac{a^6}{s^6} \Bigg(1 + \frac{155}{84}\,\frac{c^2}{a^2} + \frac{2167}{2520}\,\frac{c^4}{a^4} + \frac{541}{4704}\,\frac{c^6}{a^6} \\ & \qquad + \frac{2129}{3136 \times 180}\,\frac{c^8}{a^8} + \frac{197}{1960 \times 3456}\,\frac{c^{10}}{a^{10}}\Bigg) + \text{etc.} \Bigg] \end{aligned} \qquad (\mathbf{5})$$

The mechanical force between two coils side by side with parallel axes is a repulsion when the currents flow in the same direction around both coils, and it is an attraction when the currents are in opposite directions.

The value of force as calculated above is the average force, that is, the effective steady sustained pressure, due to alternating current. It is the same in value as the force due to direct current. At the peak of the alternating-current wave, when the currents are in phase, the force rises to double the above value, and it becomes zero when the momentary current becomes zero.

A comparison is shown in Fig. 4 between calculated values and a test curve.* The approximate data for the test curve are given in example II.

The approximate formula previously published by the author in the *Electrical World* of June 16, 1917, gives results which are slightly less than those of formula (**4**) which are more accurate. The curves have practically the same relative position as in Fig. 2.

Example II—Find the average mechanical force acting on each of two coils carrying single-phase currents which are in phase, the conditions being as follows:

Mean radius of coils $= a =$ 12.76 inches.
Length of coils $= b =$ 30.87 "
Thickness of winding $= c =$ 4.87 "
Number of turns $= N = 114$
Current in amperes $= I_1 = I_2 = 400$ amperes.

The coils are placed side by side, with the parallel axes 45 inches apart.

$$\text{Force} = 158\,(0.0362 + 0.0065 + 0.0010 + 0.0001) = 6.89 \text{ pounds.}$$

Using the curves of Fig. 3,

$$\frac{b}{d} = \frac{30.87}{25.52} = 1.21, \text{ and } \frac{s}{d} = \frac{45}{25.52} = 1.76$$

$$\text{Force} = 3.3 \times 10^{-9} \times 400^2 \times 114^2 = 6.8 \text{ pounds.}$$

Example III. Find the average mechanical force on each of three coils of the same size and spacing as those in Example II, placed side by side in a row, and carrying three-phase current, 400 amperes per phase. In determining the average mechanical forces due to three-phase currents, the forces due to the products of the instantaneous values of two alternating currents, which are 120 deg. out of phase, are calculated in the same way as the watts due to the products of the instantaneous values of

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*The test curve was published in an article on "The Mechanical Stresses in Reactance Coils" by W. M. Dann, *The Electric Journal*, page 206, April 1914.

an alternating current and an alternating voltage which are out of phase.

Let the three coils in a row be called *A*, *B*, and *C*. The currents in them are 120 deg. apart, in phase. The average force on *A* caused by I_B is

$$6.89 \cos 120 \text{ deg.} = 3.44 \text{ pounds,}$$

the calculation being similar to Example II.

To find the force on *A* caused by I_C, formula (4) is used, the axial spacing being 90 in. The average force on *A* caused by I_c is

$$0.50 \cos 120 \text{ deg.} = 0.25 \text{ pounds.}$$

This is to be added to the force caused by I_B, making a total average force on *A* of 3.69 pounds, tending to move it toward the center coil. Coil *C* is also under an average attraction of 3.69 pounds. Coil *B* is attracted by both *A* and *C*, and although there is a momentary force first in one direction and then in the other, the average force on *B* is zero.

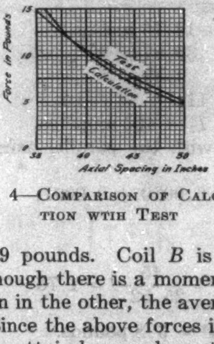

FIG. 4—COMPARISON OF CALCULATION WTIH TEST

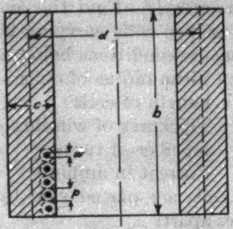

FIG. 5—SECTION OF REACTANCE COIL

Since the above forces increase as the square of the current they attain large values at times of short circuit.

PART III—SELF-INDUCTANCE OF LONG REACTANCE COILS

Reactance coils, as generally manufactured, are cylindrical in shape and have an axial length practically as long as, or longer than, their mean diameter. The radial thickness of the winding is usually considerable. The self-inductance of a thick coil with usual radial thickness of winding, may be as much as 15 per cent less than the self-inductance, L_s, of a very thin solenoid of the same mean diameter. The self-inductance of the above described thick coils may be calculated very precisely by formula (**13**) given below, which may be used to a fair degree of approximation for coils somewhat shorter than their mean diameter. Curves are given in Figs. 6 and 7 from which readings may be taken which will be correct within about one per cent for most shapes of reactance coils.

An alternative method for determining the self-inductance of a cylindrical coil is to obtain the value of L_s from Table XXI, Scientific Paper No. 169 of the Bureau of Standards, and obtain the correction for the thickness of the coils from Dr. E. B. Rosa's equations (91) and (93) and Tables IX and X of the same paper. This method is especially useful for coils just shorter than their mean diameter, since other formulas are then not very convergent.

As indicated in Fig. 5, all the dimensions of a coil should be measured to the pitch lines of the conductors. Thus the length of the coil should be taken as the number of conductors per layer multiplied by the pitch of the conductors. The exact dimensions over the copper are slightly too small and should not be used.

In order to obtain an expression for the self-inductance of a thick cylindrical coil, it is first necessary to know the mutual inductance of two coaxial and concentric thin solenoids of equal length. For long solenoids this has been given in a useful form by T. H. Havelock.*

By using the form in elliptic integrals given in equation (**4**) of Havelock's paper in the *Philosophical Magazine*, and by changing to the dimensions shown in Fig. 5, the expression for mutual inductance becomes:

$$M = \frac{\pi^2 d^2 N^2}{b^2} \Big[b - \frac{2\,D}{3\,\pi\, d^2} \{ (D^2 + d^2)\, E - (D^2 - d^2)\, F \}$$

$$- D \Big\{ - \frac{1}{8}\,\frac{D}{b} + \frac{1}{128} \Big(1 + \frac{d^2}{D^2} \Big) \frac{D^3}{b^3}$$

$$- \frac{1}{1024} \Big(1 + 3\,\frac{d^2}{D^2} + \frac{d^4}{D^4} \Big) \frac{D^5}{b^5}$$

$$+ \frac{5}{2^{15}} \Big(1 + 6\,\frac{d^2}{D^2} + 6\,\frac{d^4}{D^4} + \frac{d^6}{D^6} \Big) \frac{D^7}{b^7} - \ldots \Big\} \Big]$$

abhenrys. (**6**)

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*[*] *The Philosophical Magazine*, March 1908, Page 340, equation 25: and *Bulletin* of the Bureau of Standards, Vol. 8, No. 1, 1911, Page 56 equation 38.

where d and D are the diameters of the two thin solenoids, where N is the number of turns on each solenoid, and where F and E are complete elliptic integrals of the first and second kinds, to modulus d/D.

The two elliptic integrals can be expanded in terms of the complementary modulus k' as follows:*

$$E = 1 + \frac{1}{2} k'^2 \left(\operatorname{logh} \frac{4}{k'} - \frac{1}{1.2} \right) + \frac{1^2\,3}{2^2\,4} k'^4 \left(\operatorname{logh} \frac{4}{k'} - \frac{2}{1.2} - \frac{1}{3.4} \right) + \ldots$$

$$F = \operatorname{logh} \frac{4}{k'} + \frac{1^2}{2^2} k'^2 \left(\operatorname{logh} \frac{4}{k'} - \frac{2}{1.2} \right) + \frac{1^2\,3^2}{2^2\,4^2} k'^4 \left(\operatorname{logh} \frac{4}{k'} - \frac{2}{1.2} - \frac{2}{3.4} \right) + \ldots \qquad (7)$$

where $k' = \sqrt{1 - k^2} = \sqrt{1 - d^2/D^2}$

The expression for mutual inductance can now be integrated twice over the section of the thick coil indicated in Fig. 5, and the following formula for the self-inductance of a long thick coil is obtained:

$$L = L_s + \Delta L \qquad (8)$$

where L_s, in henrys, $= \dfrac{\pi^2 d^2 N^2}{0.3937 \times 10^9\, b} \left[1 - \dfrac{4\,d}{3\,\pi\,b} + \dfrac{1}{8} \dfrac{d^2}{b^2} - \dfrac{1}{64} \dfrac{d^4}{b^4} + \dfrac{5}{1024} \dfrac{d^6}{b^6} - \ldots \right] \qquad (9)$

and where ΔL, in henrys,

$$= \frac{\pi^2 d^2 N^2}{0.3937 \times 10^9\, b} \left[- \frac{2}{3} \frac{c}{d} + \frac{1}{3} \frac{c^2}{d^2} + \frac{4\,d}{3\,\pi\,b} \left\{ \frac{1}{4} \frac{c^2}{d^2} \left(\operatorname{logh} \frac{4\,d}{c} - \frac{23}{12} \right) - \frac{1}{80} \frac{c^4}{d^4} \left(\operatorname{logh} \frac{4\,d}{c} - \frac{1}{20} \right) + \ldots \right\} \right.$$

***Bulletin* of the Bureau of Standards, Vol. 8, No. 1, by E. B. Rosa and F. W. Grover, equation 3, page 8.

$$
\begin{aligned}
&+\frac{c^2}{d^2}\left\{\frac{1}{12}\,\frac{d^2}{b^2}-\frac{7}{192}\,\frac{d^4}{b^4}+\frac{17}{768}\,\frac{d^6}{b^6}\right.\\
&\qquad\qquad\left.-\frac{775}{32\times 1536}\,\frac{d^8}{b^8}+\ldots\right\}\\
&+\frac{c^4}{d^4}\left\{\frac{1}{72}\,\frac{d^2}{b^2}-\frac{13}{960}\,\frac{d^4}{b^4}+\frac{169}{7680}\,\frac{d^6}{b^6}\right.\\
&\qquad\qquad\left.-\frac{2167}{32\times 2304}\,\frac{d^8}{b^8}+\ldots\right\}+\ldots\\
&\left.+\frac{2b}{\pi\, q\, d\, N}\left(\operatorname{logh}\frac{p}{w}+0.14\right)\right] \qquad (\mathbf{10})
\end{aligned}
$$

In the above formulas, N is the number of turns of effective conductor in the coil, q is the number of wires or cables in parallel, and the dimensions are in inches. If the dimensions are in centimeters, the factor 0.3937 should be omitted, and if the inductance is to be given in abhenrys instead of henrys the factor 10^9 should be omitted. Note that

$$
\begin{aligned}
\operatorname{logh} m &= \text{the hyperbolic or natural logarithm of } m\\
&= 2.3026 \log_{10} m.
\end{aligned}
$$

The quantity L_s is the self-inductance of an infinitely thin solenoid of diameter d. Formula (**9**) is the same as (79), Bulletin of the Bureau of Standards, Vol. 8, No. 1 or (21A), Scientific Paper No. 320 of the Bureau of Standards. The general term of the series may be obtained from either of these references. There are other formulas (See 11) which are more convergent than (**9**). The value of L_s may also be accurately and conveniently found from Table XXI Bulletin of the Bureau of Standards, Vol. 8, No. 1.

The quantity ΔL, given by (**10**), is the change in the inductance due to the radial thickness of the winding. Formula (**10**) should not be used for practical problems, because formula (**13**) to be given later, is more convergent, and therefore more convenient and accurate, for any coil with which (**10**) can be used. Formula (**10**) was published by the author in the *Electrical World*, Feb. 9, 1918, page 300. It gives the same result, except for the term $\frac{2167}{32\times 2304}\,\frac{d^8}{b^8}$, as a method published by S. Butterworth in the *Proceedings* of the Physical Society of London, Vol. 27, 1915, page 371.

It is assumed that the coil is wound with round wire or cable. The last term of (**10**) represents the effect of the air space between the wires.

A very convergent formula for the self-inductance of an infinitely thin solenoid which has not been previously published, may be derived directly from (36) or (39), Bulletin of the Bureau of Standards, Vol. 8, No. 1, by making the dimensions of the two solenoids equal, thus obtaining the self-inductance of one solenoid instead of the mutual inductance of two. Both (**36**) and (**39**) give the same result:

$$L_s \text{ in henrys, } = \frac{\pi^2 d^2 N^2}{0.3937 \times 10^9 b} \times \frac{d}{2b} \left[\frac{1}{m} - \frac{8}{3\pi} - \frac{m^3}{8} + \frac{m^5}{16} - \frac{15}{128} m^7 + \frac{21}{128} m^9 - \frac{315}{1024} m^{11} + \frac{297}{512} m^{13} - \frac{39{,}039}{32{,}768} m^{15} + \ldots \right] \qquad \mathbf{(11)}$$

where $m^2 = \dfrac{d^2}{d^2 + 4b^2}$

This is much more convergent than (9). It is somewhat more convergent than (20 A), Scientific Paper No. 320 of the Bureau of Standards, since $m^2 = \dfrac{d^2}{d^2 + 4b^2}$ is smaller than $k^2 = \dfrac{d^2}{d^2 + b^2}$. When $b = d$, 9 terms of (11) give six significant figures, but 13 terms of (20 A) are required to give this result.

Formula (11) need not necessarily be used for obtaining the value of L_s, because formulas (76), (77), and (78), Bulletin of the Bureau of Standards, Vol. 8, No. 1, are extremely convergent, and very accurate values of L_s can also be easily obtained from Table XXI of the same Bulletin.

Series in m for thick coils can be derived (see formula 13) which are similar to those in d/b of formula (**10**), but which are much more convergent than they are, and which therefore supersede them for practical calculations. These series in m have not been previously published. The series in m will give

5 or 6 significant figures for the self inductance of a thick cylindrical coil whose length is as short as its diameter, but formula (**10**) is not nearly so accurate for such a short coil. That the two formulas (**10**) and (**13**) are equivalent may be shown by the fact that they give the same results for any problem for which they are both convergent. It is also proved by expanding the terms of one formula by the binomial theorem, and this is found to produce the other formula. The derivation of (**13**) is as follows.

It is first necessary to establish a proposition regarding the mutual inductance of two solenoids, similar to Maxwell's proposition regarding two coils, in paragraph 700 of "Electricity and Magnetism".* In the same manner as in the paragraph referred to, it may be shown that, if M_s is the mutual inductance of the central solenoids of two cylindrical coils of mean radii a_1 and a_2 and of thicknesses of winding c_1 and c_2, then the mutual inductance of the two thick coils is

$$M = M_s + \frac{c_1^2}{2^2 \underline{/3}} \frac{\partial^2 M_s}{\partial a_1^2} + \frac{c_2^2}{2^2 \underline{/3}} \frac{\partial^2 M_s}{\partial a_2^2}$$
$$+ \frac{c_1^4}{2^4 \underline{/5}} \frac{\partial^4 M_s}{\partial a_1^4} + \frac{c_1^2 c_2^2}{2^4 \underline{/3} \ \underline{/3}} \frac{\partial^4 M_s}{\partial a_1^2 \partial a_2^2} + \frac{c_2^4}{2^4 \underline{/5}} \frac{\partial^4 M_s}{\partial a_2}$$
$$+ \frac{c_1^6}{2^6 \underline{/7}} \frac{\partial^6 M_s}{\partial a_1^6} + \frac{c_1^4 c_2^2}{2^6 \underline{/3} \ \underline{/5}} \frac{\partial^6 M_s}{\partial a_1^4 \partial a_2^2}$$
$$+ \frac{c_1^2 c_2^4}{2^6 \underline{/3} \ \underline{/5}} \frac{\partial^6 M_s}{\partial a_1^2 \partial a_2^4} + \frac{c_2^6}{2^6 \underline{/7}} \frac{\partial^6 M_s}{\partial a_2^6} + \ldots \qquad (\mathbf{12})$$

This formula is most accurate for coils that are far apart, and it does not give good results for coils that are close together. It may be useful for giving extensions to present formulas for mutual inductance of solenoids.

If formula (17), Bulletin of the Bureau of Standards Vol. 8, No. 1, for the mutual inductance of two circles far apart, be integrated to give the mutual inductance of two coaxial solenoids whose centers are far apart measured along the axis, terms are obtained which are of the same form as the second half of formula (38) of the above bulletin, which forms a series in A/l. See formula (3) when $s = 0$. Now proposition (**12**) gives exactly the effect of the thicknesses c_1 and c_2 of the coils for the formula for coils far apart, and it is to be expected that it will give the effect of c_1 and c_2 for the same terms which appear in the last half of (38, Bulletin, Vol. 8, No. 1).

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*See also Scientific Paper No. 169 of the Bureau of Standards by E. B. Rosa and F. W. Grover, page 33.

This is found to be the case, and the series in c^2 and c^4 in the last half of (**10**), can be derived by means of (**12**). The latter method is much easier than the method of straight integration of (38, Bulletin Vol. 8, No. 1), since differentiation is in general a simpler process than integration, and fewer and smaller terms are required to be handled.

Now it is noticeable that formula (36) of the Bulletin, Vol. 8, No. 1, contains the first line of (38) of the same bulletin. Therefore the terms in A/r of (36, Vol. 8, No. 1), give the same total result as the terms in A/l of (38, Vol. 8, No. 1). This may be proved definitely by expanding one set of terms by the binomial theorem, and they are found to produce the other set. Therefore, since proposition (**12**) applies to the terms in A/l of (38 Vol. 8, No. 1), it applies to the terms in A/r of (36, Vol. 8, No. 1). A similar statement applies to (39, Vol. 8, No. 1) since that formula is the same as (36, Vol. 8, No. 1) when the two solenoids are of equal length.

The differentiation of (36) or (39) (Vol. 8, No. 1), is not difficult, and then proposition (**12**) can be applied. It is found by expanding some of the terms by the binomial theorem that when $A = a$

$$\frac{\partial^2 M}{\partial A^2} = \frac{\partial^2 M}{\partial a^2}$$

as would be expected, and this produces a certain simplification in the series. Terms from formula (**10**) extended are then included to give the effect of integrating the terms in a/A of (36) or (38) (Vol. 8, No. 1), and the following complete result is obtained:

$$\Delta L, \text{ in henrys} = \frac{\pi^2 d^2 N^2}{0.3937 \times 10^9 b} \left[- \frac{2}{3} \frac{c}{d} + \frac{1}{3} \frac{c^2}{d^2} \right.$$

$$\left. + \frac{4d}{3\pi b} \left\{ \frac{1}{4} \frac{c^2}{d^2} \left(\operatorname{logh} \frac{4d}{c} - \frac{23}{12} \right) - \frac{1}{80} \frac{c^4}{d^4} \right. \right.$$

$$\left(\operatorname{logh} \frac{4\,d}{c} - \frac{1}{20}\right) - \frac{1}{896}\,\frac{c^6}{d^6}\left(\frac{23}{20} \operatorname{logh} \frac{4\,d}{c} - \frac{4547}{5600}\right)$$

$$+ \ldots \Bigg\}$$

$$+ \frac{c^2}{d^2} \times \frac{d}{b} \left\{ \frac{m}{6} - \frac{5}{24}\,m^3 + \frac{m^5}{3} - \frac{95}{128}\,m^7 + \frac{217}{128}\,m^9 \right.$$

$$\left. - \frac{2135}{512}\,m^{11} + \frac{11 \times 1961}{2048}\,m^{13} - \frac{13 \times 68{,}915}{32{,}768}\,m^{15} \right.$$

$$+ \ldots \Bigg\}$$

$$+ \frac{c^4}{d^4} \times \frac{d}{b} \left\{ \frac{m}{36} - \frac{17}{180}\,m^3 + \frac{53}{96}\,m^5 - \frac{1265}{576} \right.$$

$$m^7 + \frac{38{,}857}{4608}\,m^9 - \frac{3913}{128}\,m^{11} + \frac{231 \times 9551}{20{,}480}\,m^{13}$$

$$\left. - \frac{143 \times 10{,}625}{4096}\,m^{15} + \ldots \right\}$$

$$+ \frac{c^6}{d^6} \times \frac{d}{b} \left\{ - \frac{m^3}{120} + \frac{15}{112}\,m^5 - \frac{1117}{672}\,m^7 + \frac{1183}{96}\,m^9 \right.$$

$$- \frac{21 \times 3641}{1024}\,m^{11} + \frac{11 \times 367{,}621}{10{,}240}\,m^{13}$$

$$\left. - \frac{143 \times 109{,}353}{8192}\,m^{15} + \ldots \right\} + \ldots$$

$$\left. + \frac{2\,b}{\pi\,q\,d\,N}\left(\operatorname{logh} \frac{p}{w} + 0.14\right)\right] \qquad (\mathbf{13})$$

where $m^2 = \dfrac{d^2}{d^2 + 4\,b^2}$

This formula is to be used in conjunction with (**8**). It should be used instead of (**10**) for practical problems, since it is more convenient and accurate. It appears longer than (**10**), but that is merely because more terms have been worked out. In reality, fewer terms of (**13**), than of (**10**), are needed to obtain the self-inductance of a given coil to a certain number of significant figures. The terms which are not needed can easily be neglected. For most coils made, the terms in c^6/d^6 will be found to be practically negligible, even for precision work.

The last term of formula (**13**) represents the effect of the air and insulation space between the conductors. It is assumed that the coil is wound with round wire or cable. If square wire is used, the constant 0.14 in the last term should be omitted. Therefore, for uniform current distribution over the section of the coil, as, for example, with square wire with infinitely thin insulation, the last term becomes zero. For precise work, the constant 0.14, which is an average value, should be changed in accordance with Dr. E. B. Rosa's results for various shapes of coils, given in the Bulletin of the Bureau of Standards, Vol. 3, 1907, page 37, and Vol. 8, No. 1, 1911, page 141.

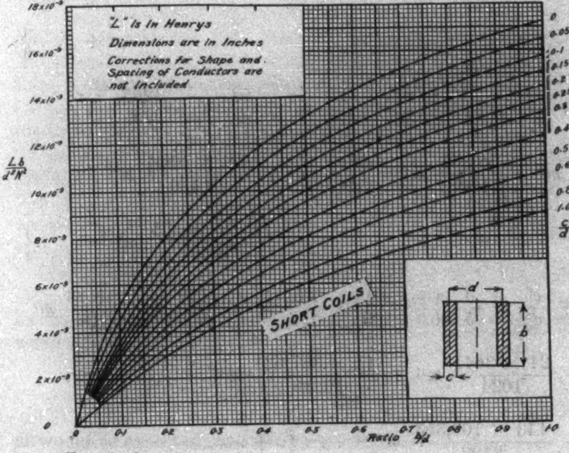

FIG. 6—SELF-INDUCTANCE OF SHORT REACTANCE COILS

A set of curves showing the self-inductance of cylindrical coils of various shapes, is given in Figs. 6 and 7. This should be useful for many engineering problems where precise accuracy is not required. The curves can give quicker and more accurate results for many cases than most approximate formulas.

The values of inductance of long coils in which c is almost as great as d, were calculated by a formula derived by the writer.* The values of inductance of coils considerably shorter than their mean diameter were derived from Tables C and D, by Prof. T. R. Lyle, Scientific Paper No. 320 of the Bureau of Standards, pages 569 and 570.

Example IV. To find the self-inductance of a coil wound with 1000 turns of round wire in 10 layers of 100 turns each.

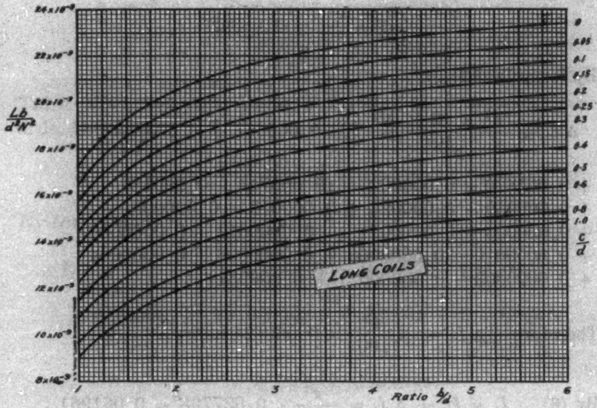

FIG. 7—SELF-INDUCTANCE OF LONG REACTANCE COILS

The diameter of the insulated wire is $p = 0.1$ cm. and the diameter of the bare wire is $w = 0.08$ cm. The mean diameter of the coil is $d = 10$ cm., the length is $b = 10$ cm., and the thickness is $c = 1$ cm. (Example I, Bulletin of the Bureau of Standards Vol. 4, No. 3, page 374.)

By formula (**11**), $m = \frac{1}{\sqrt{5}}$

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*Formula (5), "Self Inductance of Long Reactance Coils", by H. B. Dwight, *The Electrical World*, Feb. 9, 1918, page 300.

and L_s, in henrys, $= \dfrac{\pi^2 \times 10 \times N^2}{10^9}$ [1.118034 − 0.424413

$$- 0.005590 + 0.000559 - 0.000210 + 0.000059$$
$$- 0.000022 + 0.000008 - 0.000003]$$

$$= \frac{\pi^2 \times 10 \times N^2}{10^9} \times 0.688422$$

Table XXI, Scientific Paper No. 169, of the Bureau of Standards, gives

$$\frac{\pi^2 \times 10 \times N^2}{10^9} = 0.688423$$

By formula (**13**),

$$\Delta L = \frac{\pi^2 \times 10 \times N^2}{10^9} [- 0.066667 + 0.003333$$

$$+ 0.001880 - 0.000002 + \{0.000745 - 0.000186$$
$$+ 0.000060 - 0.000026 + 0.000012 - 0.000006$$
$$+ 0.000003 - 0.000002\} + \{0.0000012 - 0.0000008$$
$$+ 0.0000010 - 0.0000008 + 0.0000006 - 0.0000004$$
$$+ 0.0000003 - 0.0000002\} + 0.000231]$$

Therefore, $\Delta L = - \dfrac{\pi^2}{100} \times 0.060624$ henry.

By (**8**), $L = L_s + \Delta L = \dfrac{\pi^2}{100} \times 0.627798 = 0.061961$ henry.

The value given by (**9**) and (**10**) is 0.06194 henry.

The value given in the Bulletin of the Bureau of Standards, Vol. 4, No. 3, Exam. 1, page 374, is 0.061865 henry, which is less than 1/5 of 1 per cent different from the value given by (**13**). The result given in the above bulletin was obtained by Dr. E. B. Rosa's formula (91) Scientific Paper No. 169, of the Bureau of Standards. This formula has the advantage that it is equally applicable to coils of all lengths, whereas (**13**) is not very convergent for coils which are shorter than their mean diameter. Where (**13**) is convergent, however, it is more accurate than the other formula.

Example V. Find the self-inductance of a coil of square wire with very thin insulation in which

$$\frac{b}{d} = 2, \; \frac{c}{d} = \frac{1}{10} \text{ and } \frac{\pi^2 N^2 d^3}{8 b^2} = 10^6 \text{ abhenrys.}$$

(Example 9, Scientific Paper No. 320 of the Bureau of Standards, page 559).
By formula (11),

$$L_s = \frac{\pi^2 d^2 N^2}{b} [1.0307764 - 0.2122066 - 0.0004458 + 0.0000131 - 0.0000015 + 0.0000001]$$

$$= \frac{\pi^2 d^2 N^2}{b} \times 0.8181357$$

Table XXI, Scientific Paper No. 169 of the Bureau of Standards, gives

$$L_s = \frac{\pi^2 d^2 N^2}{b} \times 0.818136$$

By formula (**13**),

$$\Delta L = \frac{\pi^2 d^2 N^2}{b} [- 0.066667 + 0.003333 + 0.000940 - 0.000001 + \{0.000202 - 0.000015 + 0.000001\} + \{0.0000003\}\,]$$

$$\Delta L = - \frac{\pi^2 d^2 N^2}{b} \times 0.062205 \text{ abhenrys.}$$

$$\Delta L = - 0.995286 \text{ millihenry.}$$

The value given by formula (29 A), Scientific Paper No. 320 of the Bureau of Standards, is

$$\Delta L = - 0.99526 \text{ millihenry.}$$

Example VI. A problem relating to a coil which is more similar to usual current-limiting reactors may be of interest. Find the self-inductance of a coil of mean diameter $d = 50$ inches, wound with 10 layers of cable, 75 turns per layer, two cables in parallel. The diameter of the cable is $w = 0.5$ in., and the distance from center to center of cables is $p = 1$ in. Therefore,

$$b = 75 \text{ in}, \quad c = 10 \text{ in.}, \quad q = 2, \text{ and } \quad N = 375.$$

By formula (**11**),

$$L_s\text{, in henrys, } = \frac{\pi^2 \times 50^2 \times 375^2}{0.3937 \times 10^9 \times 75} \times \frac{50}{150}[3.1623 - 0.8488 - 0.0040 + 0.0002]$$

$$= \frac{\pi^2 \times 50^2 \times 375^2}{0.3937 \times 10^9 \times 75} \times 0.7699$$

By formula (**13**),

$$\Delta L = \frac{\pi^2 \times 50^2 \times 375^2}{0.3937 \times 10^9 \times 75} \times [-0.1333 + 0.0133 + 0.0031 + \{0.0014 - 0.0002\} + 0.0011]$$

Therefore, $L = \frac{\pi^2 \times 50^2 \times 375^2}{0.3937 \times 10^9 \times 75} \times [0.7699 - 0.1146]$

$$= 0.07700 \text{ henry.}$$

The result, when calculated by (**9**) and (**10**) is the same.

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INHERENT LIMITATIONS ON TRANSFORMATIONS POSSIBLE BY STATIONARY APPARATUS

BY JOSEPH SLEPIAN

ABSTRACT OF PAPER

Expressions for electrostatic and electromagnetic energies, Joulian heat dissipation and power are given in complex quantities. The pure imaginary part of the expression for power in a static network is shown to be equal to 2 ω times the difference between the mean electromagnetic energy and mean electrostatic energy. Use is made of this new principle in considering the problems of power-factor correction and phase splitting. It is shown that in general for phase transforming by static apparatus both magnetic and electrostatic storage of energy are necessary, and it is shown how the minimum amounts of each are determined by the load.

The symmetry of the coefficients in the general equations for the steady state in a static network is demonstrated, and it is shown that limitations upon voltage and current transformations follow. The voltage regulation of any phase-splitting arrangement is considered.

CONTINUED failures in attempts to construct networks of stationary apparatus, which shall have certain characteristics, lead to the conviction that such characteristics are unattainable by static means. Examples are power-factor correction without condensers, single-phase to balanced polyphase transformations by condensers only or inductances only, or single-phase to polyphase transformations with inherent regulation for variable load. However convincing such failures may be, it is always satisfying to obtain general demonstrations of the impossibility of attainment of the characteristics desired, especially as the demonstrations generally reveal what characteristics can be obtained.

Considering the requirements of energy storage in the transforming apparatus, the problems mentioned above fall into two classes. In the problem of balanced power-factor correction, the instantaneous power drawn from any one phase is exactly equal to the instantaneous power supplied to the other phases. The instantaneous power supplied to the power-factor correcting apparatus is always zero. Hence there is no *a priori* reason for supposing a relation between the wattless power supplied by the apparatus and the electromagnetic energy stored up in the apparatus. In fact, there is no such relation when rotating machines are considered. The wattless power supplied by a synchronous condenser may be varied over a wide range without changing the kinetic energy or magnetic energy as determined by the flux, which is stored in the machine. For purely static apparatus, however, such a relation does exist as is shown in III. Even when the wattless power it supplies is balanced, a static network must have a minimum of energy storage, and this minimum varies with the magnitude of the wattless load.

In the problem of transformation from single phase to balanced polyphase, there is an obvious requirement for energy storage irrespective of the character of the transforming apparatus. The power supplied by the single-phase line has a double-frequency component and is pulsating. The power drawn by the balanced polyphase load, however, is continuous. Hence it is clear that the transforming apparatus must store up energy while the single-phase power is at its maximum, to be given out again while the single-phase power is at its minimum. This clearly calls for momentary energy storage, and therefore for a minimum mean energy storage dependent on the load. In rotating phase-converters, such as are in general use, the pulsations are in the kinetic energy of the rotor. The mean kinetic energy on the rotor is in general far in excess of the minimum mean energy called for by the nature of the transformation. This is because the heating limitations upon transforming electrical energy into mechanical are what determine the size of the rotor, rather than the energy considerations. If the heating limitations could be removed, the most economical rotating phase converter would be one in which the rotor would come to rest twice per cycle, and where the maximum velocity of the rotor would vary with the load. It is remarkable that networks of static apparatus can be devised which will effect the phase transformation with the minimum amount of energy storage called for above, provided that the polyphase power factor exceeds 0.707. This is shown in section 3. In such networks the condensers are without charge, and the reactors without flux simultaneously twice every cycle.

In all which follows, by static network is meant a network constructed of self and mutual inductors, condensers and resistors.

I. POWER AND ENERGY

Power and energy in an alternating-current circuit are periodic and may be expanded into a Fourier series. If harmonics of current and voltage are absent, these Fourier developments consist only of two terms, a constant and a double-frequency term. The constant may in particular be zero. Thus strictly speaking, it is erroneous to speak of power and energy as double-frequency quantities. Actually they consist respectively of two components, one of zero frequency and one of double frequency. These two components are easily expressed in terms of complex voltage and current.

If $E = e_1 + j\, e_2$, a complex quantity, denotes an alternating e. m. f. in the usual way, the instantaneous e. m. f., e, is given by

$$e = \text{Real part of } (\sqrt{2}\, E\, \epsilon^{j\omega t}) = \sqrt{2}/2\, (E\, \epsilon^{j\omega t} + \hat{E}\, \epsilon^{-j\omega t})$$

where $\hat{E} = e_1 - j\, e_2$ is the complex number conjugate to E, that is having the same real part but with pure imaginary part of opposite sign.

Similarly, if I denotes an alternating current, its instantaneous value i is given by:

$$i = \text{real part of } (\sqrt{2}\, I\, \epsilon^{-j\omega t}) = \sqrt{2}/2\, (I\, \epsilon^{j\omega t} + \hat{I}\, \epsilon^{-j\omega t})$$

The instantaneous power p is given by the product of the instantaneous e. m. f. and the instantaneous current. Thus:

$$\begin{aligned} p &= 1/2\, (E\, \epsilon^{j\omega t} + \hat{E}\, \epsilon^{-j\omega t})\, (I\, \epsilon^{j\omega t} + \hat{I}\, \epsilon^{-j\omega t}) \\ &= 1/2\, (E\, \hat{I} + E\, I\, \epsilon^{2j\omega t} + \hat{E}\, I + \hat{E}\, \hat{I}\, \epsilon^{-2j\omega t}) \\ &= \text{real part of } (E\, \hat{I} + E\, I\, \epsilon^{2j\omega t}) \end{aligned}$$

Thus the instantaneous power consists of a constant component, real part of $E\, \hat{I}$, and a double-frequency component, real part of $E\, I\, \epsilon^{2j\omega t}$.

In the complex plane, the quantity $E\, \hat{I} + E\, I\, \epsilon^{2j\omega t}$ is represented by a point describing synchronously the perimeter of a circle whose center is the point $E\, \hat{I}$, and whose radius is the absolute value of $E\, I$. Since $E\, \hat{I}$ and $E\, I$ have the same absolute values, the circle passes through the origin.

It becomes clear at once from Fig. 1 that when $E\, \hat{I}$ is real and positive, the circle lies entirely to the right of the j axis, so that the power is always positive; when $E\, \hat{I}$ is pure imaginary, the circle has the j axis for a diameter so that the power is positive and negative respectively, half the time.

By the angle of a vector in the complex plane is meant the angle which it makes with the real axis. In Fig. 1, θ is the angle of $E\,\hat{I}$. The angle of the product of two vectors is the sum of the angles of the two vectors respectively. The angle of the conjugate of a vector is the negative of the angle of the vector. Hence, the angle of $E\,\hat{I}$ being the difference between angle of E and angle of I, is the angle of lag of I behind E. Hence:

$$\cos\theta = \text{power factor.}$$

$$\tan\theta = \frac{\text{Pure imaginary part of } E\,\hat{I}}{\text{Real part of } E\,\hat{I}}$$

In any balanced polyphase system, the e. m. fs. in the successive phases are given by $E,\ a\,E,\ a^2\,E,\ \ldots\ldots\ a^{n-1}\,E$,

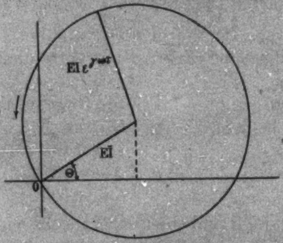

FIG. 1

where a is a primitive complex nth root of unity. If $n = 3$, $a = -\,1/2 + j\,\sqrt{3}/2$.

It is readily seen that a is conjugate to a^{n-1}, a^2 is conjugate to a^{n-2}, etc.

If the phases are carrying balanced currents, they will be given by $I,\ a\,I,\ a^2\,I,\ \ldots\ a^{n-1}\,I$.

To obtain the power we need the conjugates of the preceding quantities, and since the conjugate of a product is the product of the conjugates we get for the conjugate values, $\hat{I},\ a^{n-1}\,\hat{I},\ a^{n-2}\,\hat{I},\ \ldots\ a\,\hat{I}$.

The instantaneous power in the successive phases is, therefore, real part of $(E\,\hat{I} + E\,I\,\epsilon^{2j\omega t})$, real part of $(a\,E\,a^{n-1}\,\hat{I} + a\,E\,a\,I\,\epsilon^{2j\omega t})$, real part of $(a^2\,E\,a^{n-2}\,\hat{I} + a^2\,E\,a^2\,I\,\epsilon^{2j\omega t})$, etc.

Summing these up and noticing that $1 + a^2 + a^4 + \ldots\ a^{2(n-1)} = 0$ we get for the total power,

$$p = \text{real part of } (n\,E\hat{I}).$$

In a balanced polyphase system, the instantaneous power input is constant in time.

The instantaneous magnetic energy contained in a system of circuits magnetically coupled is:

$$t_m = 1/2\,(L_{11}\,i_1^2 + L_{22}\,i_2^2 + 2\,M_{12}\,i_1\,i_2 + \ldots)$$

where L_{11}, L_{22}, M_{12} etc. are the coefficients of self and mutual induction of the parts of the system. Substituting

$$i_1 = \sqrt{2}/2\,(I_1\,\epsilon^{j\omega t} + \hat{I}_1\,\epsilon^{-j\omega t}),\ i_2 = \sqrt{2}/2\,(I_2\,\epsilon^{j\omega t} + \hat{I}_2\,\epsilon^{-j\omega t}),\ \text{etc.,}$$

$$\begin{aligned} t_m = {} & 1/2\,(L_{11}\,I_1\,\hat{I}_1 + L_{22}\,I_2\,\hat{I}_2 + M[I_1\,\hat{I}_2 + \hat{I}_1\,I_2] + \ldots) \\ & + 1/2\,(1/2\,[L_{11}\,I_1^2 + L_{22}\,I_2^2 + 2\,M_{12}\,I_1\,I_2 + \ldots]\,\epsilon^{2j\omega t} \\ & + 1/2\,[L_{11}\,\hat{I}_1^2 + L_{22}\,\hat{I}_2^2 + 2\,M_{12}\,\hat{I}_1\,\hat{I}_2 + \ldots]\,\epsilon^{-2j\omega t}) \\ = {} & T_m + \text{real part of } 1/2\,[L_{11}\,I_1^2 + L_{22}\,I_2^2 + 2\,M_{12}\,I_1\,I_2 + \ldots]\,\epsilon^{2j\omega t} \end{aligned}$$

$$T_m = 1/2\,(L_{11}\,I_1\,\hat{I}_1 + L_{22}\,I_2\,\hat{I}_2 + M\,[I_1\,\hat{I}_2 + \hat{I}_1\,I_2] + \ldots)$$

therefore represents the mean magnetic energy. It is always real and positive.

The instantaneous electrostatic energy in a group of condensers is $t_e = 1/2\left(\frac{q_1^2}{c_1} + \frac{q_2^2}{c_2} + \ldots\right)$

$$\begin{aligned} q_1 = \int i_1\,dt &= \int \sqrt{2}/2\,(I_1\,\epsilon^{j\omega t} + \hat{I}_1\,\epsilon^{-j\omega t})\,dt \\ &= \sqrt{2}/2\left(\frac{I_1}{j\,\omega}\,\epsilon^{j\omega t} + \frac{\hat{I}_1}{-j\,\omega}\,\epsilon^{-j\omega t}\right) \end{aligned}$$

Similarly for $q_2\,q_3 \ldots$ etc. Substituting in t_e.

$$\begin{aligned} t_e = {} & 1/2\left(\frac{1}{c_1}\,\frac{I_1\,\hat{I}_1}{\omega^2} + \frac{1}{c_2}\,\frac{I_2\,\hat{I}_2}{\omega_2^2} + \ldots\right) \\ & + 1/2\left(1/2\left[-\frac{1}{c_1}\,\frac{I_1^2}{\omega^2} - \frac{1}{c_2}\,\frac{I_2^2}{\omega^2}\ldots\right]\epsilon^{2j\omega t}\right. \\ & \left. + 1/2\left[-\frac{1}{c_1}\,\frac{\hat{I}_1^2}{\omega^2} - \frac{1}{c_2}\,\frac{\hat{I}_2^2}{\omega^2}\ldots\right]\epsilon^{-2j\omega t}\right) \\ = {} & T_e + \text{real part of } 1/2[-1/c_1\,\frac{I_1^2}{\omega} - 1c_2\,\frac{I_2^2}{\omega^2} \ldots]\,\epsilon^{2j\omega t} \end{aligned}$$

$T_e = 1/2\left(\frac{1}{c_1}\frac{I_1\hat{I}_1}{\omega^2} + \frac{1}{c_2}\frac{I_2\hat{I}_2}{\omega^2} + \dots\right)$ therefore represents the mean electrostatic energy. It is always real and positive.

The instantaneous rate of heat generation in a system of resistors is $d = r_1 i_1^2 + r_2 i_2^2 + \dots$ Substituting the equivalent complex quantities:

$$\begin{aligned} d &= (r_1 I_1 \hat{I}_1 + r_2 I_2 \hat{I}_2 + \dots) + 1/2\,([r_1 I_1^2 + r_2 I_2^2 \\ &\qquad + \dots]\,\epsilon^{2j\omega t} + [r_1 \hat{I}_1^2 + r_2 \hat{I}_2^2 + \dots]\,\epsilon^{-2j\omega t}) \\ &= D + \text{real part of } 1/2\,[r_1 I_1^2 + r_2 I_2^2 + \dots]\,\epsilon^{2j\omega t} \end{aligned}$$

$D = r_1 I_1 \hat{I}_1 + r_2 I_2 \hat{I}_2 + \dots$ gives the mean rate of energy dissipation into heat. It is always real and positive.

II. THE RELATION BETWEEN POWER, ELECTROSTATIC AND MAGNETIC ENERGY AND HEAT GENERATION

Suppose an arbitrary system of inductances, capacities and resistances given, having n terminals. Let $V_1, V_2 \dots V_n$, n complex numbers denote the potentials at these terminals, and $I_1, I_2, \dots I_n$ the currents at the terminals, taken positive when they flow into the system. Let $V_{n+1}, V_{n+2}, \dots$ etc., be the potentials at the nodes or junctions of the branches of the network. Let I_{kl} be the current *from* the node k *to* the node l. Thus $I_{kl} = -I_{lk}$. In what follows we shall include the n terminals among the nodes of the system.

Consider the expression

$\sum_k V_k (\hat{I}_{k1} + \hat{I}_{k2} + \dots)$. Since by Kirchhoff's laws $I_{k1} + I_{k2} + \dots = 0$ for every internal node, the whole must reduce to $V_1 \hat{I}_1 + V_2 \hat{I}_2 + \dots V_n \hat{I}_n$

$$\sum_k V_k (\hat{I}_{k1} + \hat{I}_{k2} + \dots) = V_1 \hat{I}_1 + V_2 \hat{I}_2 + \dots V_n \hat{I}_n \tag{1}$$

Take any branch a, joining two nodes, k and l. The current in the branch gives two terms in the above expression, namely:

$$\begin{aligned} V_k \hat{I}_{kl} + V_l \hat{I}_{kl} &= V_k \hat{I}_{kl} - V_l \hat{I}_{kl} = (V_k - V_l)\,\hat{I}_{kl} \\ &= (V_k - V_l)\,\hat{I}_a \end{aligned}$$

But $V_k - V_l = r_a I_a + j\,\omega\,(L_{aa} I_a + M_{ab} I_b + M_{ac} I_c$

$$+ \dots) + \frac{1}{j\,\omega}\,\frac{1}{C_a}\,I_a$$

Hence $(V_k - V_e)\,\hat{I}_a = r_a I_a \hat{I}_a + j\,\omega\,(L_a I_a \hat{I}_a + M_{ab} I_b \hat{I}_a$

$$+ M_{ac} I_c \hat{I}_a + \ldots\ldots) - j\,\omega \left(\frac{1}{C_a}\,\frac{I_a \hat{I}_a}{\omega^2} \right)$$

Now write similar equations for the other branches. The left-hand sides will add up to the left-hand side of (1). The right-hand sides will add up to $D + 2\,j\,\omega\,(T_m - T_e)$

Hence:

$$V_1 \hat{I}_1 + V_2 \hat{I}_2 + \ldots V_n \hat{I}_n = D + 2\,j\,\omega\,(T_m - T_e) \qquad \textbf{(2)}$$

Hence the real part of $V_1 \hat{I}_1 + V_2 \hat{I}_2 + \ldots\ldots V_n \hat{I}_n$ is the Joulian heat generated per second, and the pure imaginary part is $2\,j\,\omega$ times the mean magnetic energy minus the mean electrostatic energy.

III. APPLICATION OF THE RELATION, POWER FACTOR COMPENSATION

The principle enunciated in II is frequently useful in determining the possibilities of solution of certain problems, and the minimum amounts of magnetic and electrostatic energy storage necessary.

Let us consider first the problem of power factor correction in a polyphase system by static means. Because of the possibility of introducing arbitrary quadrature e.m.fs. in series with any phase by means of transformers placed across the other phases, many of us have tried to compensate for lagging power factor by means of transformers only. The constant failure of the schemes tried has convinced most of us that the idea is inherently impossible, but it is satisfying to have a definite proof of this fact.

The proposition is to insert a network between a polyphase supply and a lagging load so as to give unity power factor at the generator end. Sum up $V\,\hat{I}$ for all the terminals of the network. For the supply terminals, since $\cos\theta = 1$, $\Sigma\, V\,\hat{I}$ is real; the coefficient of j is zero. For the load terminals since leading currents are supplied *to* the network θ is negative. Hence the coefficient of j in $V\,\hat{I}$ for a load terminal must be negative. Its value is the wattless power supplied to the load through that terminal. Thus $V\,\hat{I}$ summed up for all the terminals must have for the coefficient of j the negative of the wattless power supplied the load. Hence by equation **(2)**

$$2\,\omega\,(T_e - T_m) = \text{wattless power of load.}$$

Hence in any such network there must be electrostatic storage of energy. In fact the minimum amount possible is $\frac{1}{2\,\omega}$ times the wattless power of the load.

IV. PHASE SPLITTING

The use of condensers and reactances for changing from single phase to polyphase is well known. We shall here determine the minimum amounts of condenser and reactance energy storage necessary for this purpose. The impossibility of supplying a balanced polyphase load of lagging or unity power factor from a single-phase source by means of reactances alone will also be shown here.

Suppose then a network inserted between a single-phase source and a polyphase load such that the polyphase is balanced.

Summing up $V\,\hat{I}$ for the load terminals we get $-\,n\,E_2\,\hat{I}_2$ where E_2 is the polyphase voltage to neutral, and I_2 is the current per phase supplied *to* the load. Summing $V\,\hat{I}$ up for the supply terminals gives $E_1\,\hat{I}_1$ where E_1 is the single-phase voltage and I_1 is the current taken from the supply. Hence for the network,

$$\Sigma\,V\,\hat{I} = E_1\,\hat{I}_1 - n\,E_2\,\hat{I}_2 = D + 2\,j\,\omega\,(T_m - T_e). \qquad (2')$$

The double-frequency component of power supplied by the single-phase source is real part of $E_1\,I_1\,\epsilon^{2j\omega t}$; that withdrawn by the load is zero as shown in II. This double-frequency power must be supplying and removing energy from the magnetic and electrostatic fields in the network. The integral with respect to time of this double frequency component, real part of $\frac{1}{2\,j\,\omega}\,E_1\,I_1\,\epsilon^{2j\omega t}$ must give the double frequency component of the total energy stored. The mean value of all the energy stored added to the double frequency component must never be less than zero as energy can never be negative. Hence we must have

$$T_m + T_e \text{ is not less than absolute value of } \frac{1}{2\,j\,\omega}\;E_1\,I_1$$

$$2\,\omega(T_m + T_e) \text{ is not less than absolute value of } E_1\,I_1.$$

Now absolute value of $E_1\,I_1$ is the same as absolute value of

$E_1 \hat{I}_1$, and this, of course, is not less than the real part of $E_1 \hat{I}_1$. Using the symbol $\geqq$ for "greater than or equals":

$$2\,\omega\,(T_m + T_e) \geqq \text{absolute value of } E_1 \hat{I}_1 \geqq \text{real part of } E_1 I_1 \tag{3}$$

From equation (**2**)

$$2\,\omega\,(T_m - T_e) = \text{imaginary part of } E_1 \hat{I}_1 - \text{imaginary part of } n\,E_2 \hat{I}_2 \tag{4}$$

Subtracting (**4**) from (**3**)

$$4\,\omega\,T_e \geqq \text{absolute value of } E_1 \hat{I}_1 - \text{imaginary part of } E_1 \hat{I}_1 + \text{imaginary part of } n\,E_2 \hat{I}_2 \tag{5}$$

Adding (**3**) and (**4**) we get

$$4\,\omega\,T_m \geqq \text{absolute value of } E_1 \hat{I}_1 + \text{imaginary part of } E_1 \hat{I}_1 - \text{imaginary part of } n\,E_2 \hat{I}_2 \tag{6}$$

In equation (**5**) the first two terms on the right must clearly combine to give a positive quantity. If the polyphase load has a lagging power factor, imaginary part of $n\,E_2 \hat{I}_2$ will be positive. Hence in this case the right hand side of E will be positive, and not less than imaginary part of $n\,E_2 \hat{I}_2$. Thus with a lagging polyphase load, the minimum condenser energy storage for which a network can be constructed is given by:

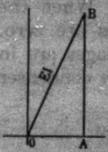

FIG. 2

$$4\,\omega\,T_e = \text{wattless power of load.} \tag{6'}$$

Actually, however, this limit cannot be reached, for to attain it we must make imaginary part of $E_1 \hat{I}_1$ equal absolute value of $E_1 \hat{I}_1$. Since, however, the real part of $E_1 \hat{I}_1$ is determined by the true polyphase power, this equality can be attained only as $E_1 \hat{I}_1$ becomes infinite. (See Fig. 2). Equation (**6**) shows that at the same time, T_m would have to become infinite.

Similarly for a leading polyphase load, the magnetic energy T_m cannot be less than $\frac{1}{4\,\omega}$ times the wattless polyphase power. To actually attain this value, however, would require zero single-phase power factor and infinite electrostatic energy.

A case of special interest is where the single-phase power factor is unity. In this case the imaginary part of $E_1 \hat{I}_1$ is zero, and the absolute value of $E_1 \hat{I}_1$ equals the true single-phase power. Hence (**5**) and (**6**) become:

$$4\ \omega\ T_e \geqq \text{Single-phase power} + \text{polyphase wattless power (lagging).} \qquad (5'')$$

$$4\ \omega\ T_m \geqq \text{Single-phase power} - \text{polyphase wattless power (lagging).} \qquad (6'')$$

Networks are well known, which effect phase splitting with the minimum of condenser and reactance energy storage indicated above. We now see that it is impossible to do better in this respect.

When the polyphase load has a power factor less than cos 45 deg. then if network losses are negligible, equation (**6''**) shows that reactance may be dispensed with while still keeping unity power factor on the single phase.

If a phase splitting network employs the minimum amount of energy storage, the total instantaneous energy stored must be zero twice every cycle. When the inductance energy is zero, the inductance voltages are a maximum. When the condenser energy is zero, the condenser voltages are zero. Hence in such a network all the inductance voltages are in phase with each other and in quadrature with the condenser voltages.

IV. THE RECIPROCAL RELATION

Suppose a system of terminal potentials $V_1 V_2 \ldots V_n$ applied to the terminals of a network produces the terminal currents $I_1, I_2 \ldots I_n$. Suppose a different system of terminal potentials $V_1', V_2', \ldots V_n'$ produces the terminal currents $I_1', I_2', \ldots I_n'$. Then

$$V_1 I_1' + V_2 I_2' + \ldots V_n I_n' = V_1' I_1 + V_2' I_2 + \ldots V_n' I_n \qquad (7)$$

Such reciprocal relations are well known in mathematical physics. The proof as given below follows the lines of the proofs usually given.

Using the notation of II, we have at once:

$$\sum_k V_k (I_{k1}' + I_{k2}' + I_{k3}' + \ldots) = V_1 I_1' + V_2 I_2' + \ldots V_n I_n' \qquad (8)$$

Any branch a joining the nodes k and l contributes two terms to the left hand member of (**8**), namely: $V_k I_{kl}' + V_l I_{lk}'$.

But

$$V_k I_{kl}' + V_l I_{lk}' = V_k I_{kl}' - V_l I_l' = I_a' (V_k - V_l)$$

$$= I_a' \Big[r_a I_a + j\omega (L_{aa} I_a + M_{ab} I_b + \dots) + \frac{1}{j\omega} \frac{I_a}{C_a} \Big]$$

$$= r_a I_a I_a' + j\omega (L_{aa} I_a I_a' + M_{ab} I_a' I_b + M_{ac} I_a' I_c + \dots) + \frac{1}{j\omega} \frac{I_a I_a'}{C_a}$$

Summing up for all the branches gives

$$\Sigma V_k (I_{k1}' + I_{k2}' + \dots) = r_a I_a I_a' + r_b I_b I_b' + \dots + j\omega (L_{aa} I_a I_a' + L_{bb} I_b I_b' + M_{ab} I_a I_b' + M_{ba} I_a' I_b + \dots) + \frac{1}{j\omega} \left(\frac{I_a I_a'}{C_a} + \frac{I_b I_b'}{C_b} + \dots \right) \tag{9}$$

Similarly, by summing up $\sum\limits_k V_k' (I_{k1} + I_{k2} + \dots)$ first by nodes and then by branches,

$$\Sigma V_k' (I_{k1} + I_{k2} + \dots) = V_1' I_1 + V_2' I_2 + \dots V_n' I_n \tag{8'}$$

$$\sum_k V_k' (I_{k1} + I_{k2} + \dots) = r_a I_a I_a' + r_b I_b I_b' + \dots + j\omega (L_{aa} I_a' I_a + L_{bb} I_b' I_b + M_{ab} I_a' I_b + M_{ba} I_a I_b' + \dots) + \frac{1}{j\omega} \left(\frac{I_a' I_a}{C_a} + \frac{I_b' I_b}{C_b} + \dots \right) \tag{9'}$$

Hence comparing (**8**) (**9**) (**8′**) and (**9′**) we see that

$$V_1 I_1' + V_2 V_2' + \dots V_n I_n' = V_1' I_1 + V_2' I_2 + \dots V_n' I_n \tag{7}$$

From equation (**7**) follows at once the well known reciprocal properties of a network. For example; suppose we supply one volt to the terminal 1 of a network keeping the other terminals at zero voltage. A certain current I_2 will flow at terminal 2. Now suppose we apply one volt at terminal 2 keeping the other terminals at zero potential. A certain current I_1' will flow at terminal 1. Now substituting for V_1, $V_2, \ldots V_n$ the values 1, 0, 0, . . . 0, and for $V_1', V_2', \ldots V_n'$ the values 0, 1, 0. . . . 0, in equation (7) we find:

$$I_1' = I_2$$

That is, a volt placed at terminal 1 produces the same current at terminal 2 as one volt placed at terminal 2 would produce at terminal 1.

In the next section we shall be concerned principally with four terminal networks in which the currents in pairs of terminals are constrained to be equal and opposite. For this special case, putting $n = 4$ in equation (7) and $I_3 = - I_1$, $I_4 = - I_2$, $I_3' = - I_1'$, $I_4' = - I_2'$, we get:

$$(V_1 - V_3) I_1' + (V_2 - V_4) I_2' = (V_1' - V_3') I_1 + (V_2' - V_4') I_2$$

Setting $V_1 - V_3 = E_1$, $V_2 - V_4 = E_2$, $V_1' - V_3' = E_1'$ and $V_2' - V_4' = E_2'$. We find,

$$E_1 I_1' + E_2 I_2' = E_1' I_1 + E_2' I_2 \tag{7'}$$

From this equation we deduce at once this following reciprocal relation. One volt difference of potential placed at one pair of terminals will circulate the same amount of current through a short circuit at the other pair of terminals as one volt difference of potential placed at the second pair will circulate through a short circuit at the first pair.

V. THE GENERAL SINGLE-PHASE TRANSFORMING NETWORK

Any network having four terminals may be used to receive current at some voltage at one pair of terminals and delivering current at some other voltage or phase at the other pair of terminals. We propose to investigate here what are the limitations on such transformations by purely static means.

The solution in detail for the steady state in any network can always be obtained by means of Kirchhoff's Laws generalized for complex quantities. Given the currents I_1 and I_2 in the two pairs of terminals, which we shall now call the primary and secondary terminals, E_1 and E_2 the primary and secondary voltages may be calculated. Now the equations given by Kirchhoff's laws are all linear in the unknowns. The process of solving them only involves linear operations. Hence E_1 and E_2 must be linear functions of I_1 and I_2. Thus:

$$E_1 = a\,I_1 + b\,I_2$$
$$E_2 = b'\,I_1 + cI_2 \tag{8}$$

Here the quantities a, b, b', c are complex numbers. However, it is easily seen that they are not independent.

In fact, applying the reciprocal relation (**7**′) we find:

$$E_1 I_1' + E_2 I_2' = a\,I_1 I_1' + b\,I_2 I_1' + b'\,I_1 I_2' + c\,I_2 I_2'$$

and $E_1' I_1 + E_2' I_2 = a\,I_1 I_1' + b\,I_2' I_1 + b'\,I_1' I_2 + c\,I_2' I_2$

Hence $b\,I_2 I_1' + b'\,I_1 I_2' = b\,I_2' I_1 + b'\,I_1' I_2$

Hence $b\,(I_2 I_1' - I_2' I_1) = b'\,(I_1' I_2 - I_1 I_2')$

Therefore: $b = b'$

Thus the primary and secondary voltages and currents of any network of the kind we are considering always satisfy relations of the form:

$$E_1 = (a_1 + j\,a_2)\,I_1 + (b_1 + j\,b_2)\,I_2$$
$$E_2 = (b_1 + j\,b_2)\,I_2 + (c_1 + j\,c_2)\,I_2 \tag{8'}$$

What is the significance of the real and pure imaginary parts of the coefficients in (**8**′). Forming the power expression we have:

$$\begin{aligned} E_1 \hat{I}_1 + E_2 \hat{I}_2 &= [a_1 I_1 \hat{I}_1 + b_1 (I_2 \hat{I}_1 + I_1 \hat{I}_2) + c_1 I_2 \hat{I}_2] \\ &\quad + j[a_2 I_1 \hat{I}_1 + b_2 (I_2 \hat{I}_1 + I_1 \hat{I}_2) + c_2 I_2 \hat{I}_2] \\ &= D + 2j\,\omega\,(T_m - T_e) \end{aligned}$$

Since the two brackets are real, the first must equal D the rate of generation of Joulian heat. If there are no losses in the network the first bracket must be zero, and the coefficients a, b, c must be pure imaginary.

Since we are interested in the inherent limitations upon the transformation possibilities of an ideal network, that is, one in which the losses are zero, we shall assume in all that follows that a, b, and c are pure imaginary.

$$a = j\,A$$
$$b = j\,B$$
$$c = j\,C$$

where A B C are real. The network equations then are:

$$E_1 = j\,A\,I_1 + j\,B\,I_2$$
$$E_2 = j\,B\,I_1 + j\,C\,I_2 \tag{8''}$$

Let us first consider the possibility of a constant potential constant potential transformation. Given E_1 constant, we require E_2 to be independent of the load, I_2.
Eliminating I_1 in (**8''**) we obtain:

$$E_2 = B/A\,E_1 + \left(\frac{B^2 - A\,C}{j\,A}\right) I_2 \qquad \textbf{(9)}$$

E_2 can be independent of I_2 only if $B^2 - A\,C = 0$. This is the condition for perfect coupling. Furthermore, the coefficient of E is real. Hence we conclude, in any constant potential to constant potential transformation without losses, the transformed potential must agree in phase with the primary potential.

Next consider a constant potential constant current transformation. Solving (**8''**) for I_2 we get:

$$I_2 = -\,j\,B\,E_1 + \left(\frac{j\,A}{B^2 - A\,C}\right) E_2 \qquad \textbf{(10)}$$

I_2 is independent of E_2 only if $A = 0$. Furthermore the coefficient of E_1, is pure imaginary. Hence in any constant potential constant current transformation without losses the secondary current is in quadrature with the primary voltage.

Let us now discuss the problem of determining a network such that for a given secondary load the secondary voltage shall be in quadrature with the primary voltage, and the primary power factor shall have a predetermined value.

The load can be given in terms of its admittance, $y_2 = g_2 - j\,b_2$. The primary power factor can be determined by giving the apparent primary admittance, $I_1 E_1 = y_1 = g_1 - j\,b_1$ Since there are no losses in the network we must have $g_1 = g_2 = g$.

What restrictions do the above conditions impose upon A, B, C? Substituting $I_1 = (g - j\,b_1)\,E_1$, $I_2 = -\,(g - j\,b_2)\,E_2$, and $E_2 = j\,E_1$ in (8''), we get:

$$E_1 = j\,(g - j\,b_1)\,A\,E_1 + j(-\,g + j\,b_2)\,j\,B\,E_1$$

$$j\,E_1 = j\,(g - j\,b_1)\,B\,E_1 + j\,(-\,g + j\,b_2)\,j\,C\,E_1$$

Simplifying:

$$1 = b_1\,A + g\,b + j\,(g\,A - b_2\,B)$$

$$1 = g\,B - b_2\,C + j\,(-\,b_1\,B - g\,C)$$

Separating the real and imaginary parts of these two equations, we get:

$$1 = b_1 A + g B$$
$$0 = g A - b_2 B$$
$$1 = g B - b_2 C$$
$$0 = b_1 B + g C$$

Solving we find:

$$A = \frac{b_2}{b_1 b_2 + g^2} \quad B = \frac{g}{b_1 b_2 + g^2} \quad C = \frac{-b_1}{b_1 b_2 + g^2}$$

Thus we see that A, B, and C are completely determined by the conditions imposed on the network. But A, B, C determine entirely the secondary voltage in terms of the secondary current. Hence we conclude, all networks without losses which effect phase quadrature for a given load with given primary power factor have the same inherent regulation of secondary voltage as the secondary load is varied.

REFERENCES

The relation between wattless power and electrostatic and magnetic energies given in II, was discovered independently by the author. Since this paper was submitted to the Institute the same relation was proved in a different manner by J. B. Pomey, *Revue Generale de l'electricite*, Vol. III, 9, pp. 315–319, March 2, 1918.

The relation has subsequently been generalized by the author to include rotating commutatorless machines, with many interesting conclusions.

Examples of reciprocal relations such as are given in IV may be found in Maxwell, "Electricity and Magnetism", third edition, pp. 105 and 405.

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STARTING CONDITIONS OF SYNCHRONOUS MACHINES

BY ALFRED HAY AND F. N. MOWDAWALLA

INTRODUCTION

IN view of the growing popularity and importance of the synchronous motor as one of the standard types of motor available for power distribution purposes, and its increasing use as a means of improving the power factor of a load, no apology seems necessary for a paper containing a detailed study of the behavior of such a motor during what has always been regarded as a somewhat critical period—*viz.*, the period when the motor is being accelerated from rest to synchronous speed. The subject is by no means a new one, and has already been dealt with by several writers, not, however, in a manner sufficiently thorough and exhaustive to make further contributions to it superfluous, and one of the main objects of the present paper is to explain a number of hitherto somewhat obscure points, and to draw attention to others which have not previously been noticed.

REVIEW OF PREVIOUS WORK ON THE SUBJECT

The earliest paper specially devoted to a study of the starting conditions in synchronous machines appears to be one read in 1912 before the American Institute of Electrical Engineers by C. J. Fechheimer.[1] In this the author, after some general introductory remarks, gives an account of experiments made to determine (1) the relation connecting the starting torque with the impressed potential difference under various conditions and (2) the variation of torque, current and power factor with speed, while the rotor is accelerated from rest, a constant potential difference being maintained across the stator terminals. The experimental results are embodied in an interesting series of curves. Among the questions discussed by the author are the desirability or otherwise of keeping the field circuit open during the acceleration period, and the tendency of certain synchronous machines to run in the neighborhood of half the speed of synchronism. Fechheimer's paper gave rise to a very interesting discussion.

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[1] C. J. Fechheimer, TRANS. A. I. E. E. VOL. XXXI, 1912, p. 529.

In 1913 E. Rosenberg[2] read a paper before the Institution of Electrical Engineers, on Synchronizing machines in which he considered, among other things, the occurrences during the starting period, and discussed in some detail the behavior of the machine towards the end of the acceleration period, just before it is pulled into synchronism. It may be mentioned that in the discussion on Fechheimer's paper, it was stated by B. G. Lamme that it was "difficult to see just what is going on in the motor at the instant it pulls into synchronism." So far as the authors are aware, Rosenberg's was the first attempt to furnish a detailed explanation of the action in question.

The next contribution to the subject is one by F. D. Newbury, in the form of a paper read before the American Institute of Electrical Engineers in June 1913.[3] The main interest of this paper lies in the oscillographic records which are given of the starting period.

In December 1917 the authors of the present paper published, in the *Journal* of the Indian Institute of Science, an account of some experimental investigations of the occurrences during the starting period of a synchronous machine, and a full theoretical discussion of the type of induction motor whose stator is supplied with polyphase currents, and whose rotor is provided with a single-magnetic-axis winding. As will be seen later the theoretical results obtained explain in a satisfactory manner certain striking peculiarities exhibited by the machine during the starting period.

The most recent addition to the literature of the subject is an article by Theo. Schou in the *Electrical World* of April 6th, 1918 (Vol. 71, p. 714). In this article the author points out that a satisfactory selfstarting synchronous motor should partake of the characteristics of both an induction motor and an alternator, and should present features of design intermediate between these two classes of machines. He accordingly advocates the use of a shorter air gap and longer polar arc than are customary in alternators of standard design. He further suggests the use of materials having a pronounced skin effect for the squirrel-cage windings of self-starting synchronous motors[4] and points out the advantages of fractional-pitch windings in reducing the troubles arising from dead points.

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[2] E. Rosenberg, "Self-synchronizing Machines." *Journal* of the Institution of Electrical Engineers, Vol. 51, p. 62 (1913).

[3] TRANSACTIONS of the American Institute of Electrical Engineers, Vol. 32, p. 1509.

TORQUES CONCERNED IN ACCELERATING THE MOTOR AND IN PULLING IT INTO SYNCHRONISM

The self-starting synchronous motor is accelerated and finally pulled into synchronism by the action of a number of torques, differing widely from each other, and the resultant effect of which will largely depend on their relative importance. Cases may arise where, owing to the preponderance of a certain type of component torque, it may be impossible to get the machine to run up to synchronous speed. Again, the torques concerned in the initial acceleration of the rotor are quite distinct from the torque which finally pulls it into synchronism. The authors are of opinion that no really clear understanding of the occurrences during the starting period is attainable without a detailed study of the various torques which act on the rotor during that period. It will accordingly be necessary to consider the nature of the various torques concerned. These torques may be classified as follows:

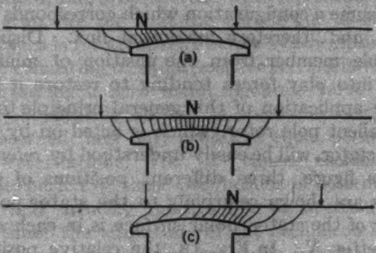

FIG. 1—FLUX DISTRIBUTIONS FOR DIFFERENT RELATIVE POSITIONS OF STATOR FIELD AND MAGNET POLES

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[4] The utilization of the skin effect in the rotor conductors of induction motors was patented by H. M. Hobart in 1900.

1. Torque due to varying magnetic reluctance (synchronous torque.)
2. Torque due to hysteresis.
3. Torque due to starting squirrel-cage or damping coils if present.
4. Torque due to currents induced in the field winding, if this winding is closed.
5. Torque due to eddy currents.

1. TORQUE DUE TO VARYING MAGNETIC RELUCTANCE

It is a well known general principle of electromagnetism that any electromagnetic system which includes a movable member tends to assume a configuration which corresponds to minimum reluctance and therefore maximum flux. Displacements of the movable member from the position of minimum reluctance call into play forces tending to restore it to that position. The application of this general principle to the special case of a salient pole rotor, which is acted on by the rotating field of the stator, will be easily understood by reference to Fig. 1. In this figure, three different positions of one of the rotor poles are shown relatively to the stator polar surfaces. The center of the stator polar surface is in each case marked with the letter *N*. In Fig. 1B, the relative positions of the rotor and stator polar surfaces correspond to minimum reluctance, and from the symmetry of flux distribution it is immediately obvious that there is no tangential pull on the rotor. In Fig. 1A, the rotor pole is shown displaced from the position of minimum reluctance in one direction, and in Fig. 1C in the opposite direction. If we bear in mind that the dynamical stresses correspond to a tension along the lines of force and a pressure at right angles to them, it is easy to see that in Fig. 1A there is a tangential force acting on the pole from right to left, while in Fig. 1C it acts from left to right. If we now suppose that the stator polar surfaces travel past the pole in a direction from left to right, then the successive positions will be those shown in Fig. 1. For every displacement of the stator polar surface to one side of a rotor pole, there will be an equal displacement to the other side, and the forces corresponding to equal displacements in opposite directions will be equal and opposite. Thus the rotating field will exert an alternating torque on the rotor, and the positive and negative half-waves of this torque will be equal. The frequency of the torque will be equal to twice the supply frequency multiplied by the motor slip, and so long as the motor slips, the mean value of the alternating torque due to varying magnetic reluctance will be zero. The only effect of this torque is to throw the rotor into forced vibrations, having a frequency equal to that of the torque. Owing to the large moment of inertia of the rotor the vibrations will be imperceptible for large values of the slip, *i. e.*, during the greater part of the acceleration period. When, however, the slip has become sufficiently small and in consequence the frequency of the torque sufficiently low, the amplitude of the rotor oscillations will become marked, and will increase with decreasing slip.

Summing up, we see that the torque due to variable magnetic reluctance is for all speeds below synchronism an alternating torque consisting of equal positive and negative half-waves, and hence having a zero mean algebraic value. It is therefore quite inoperative so far as steady acceleration of the rotor is concerned, and only produces equal periodic accelerations and retardations, *i. e.*, it causes oscillations of the rotor. The period of these oscillations is determined by the rotor slip, and steadily increases with decreasing slip. At the same time, the amplitude of the oscillation increases.

The graph of the varying magnetic reluctance torque expressed as a function of the speed is shown in Fig. 2A. For all speeds below synchronism its mean value is zero, while at synchronism it is capable of assuming any positive or negative value between definite limits. Since the speed of synchronism is the only speed at which this torque has a value differing from zero, we may conveniently refer to it as the *synchronous torque.*

2. TORQUE DUE TO HYSTERESIS

In dealing with the torque due to varying magnetic reluctance, we have neglected the effect of hysteresis. It now becomes necessary to take this into account. Owing to hysteresis the rotor will tend to retain more or less strongly the effects of previous magnetizations. Thus referring to Figs. 1A and 1C, if hysteresis were absent, the magnitudes of the torques in these two cases would be equal. Owing, however, to the fact that in the position of the rotating field corresponding to Fig. 1A the magnetization of the rotor is increasing from a lower to a higher value, while in position Fig. 1C it is decreasing from a higher to a lower value, the actual flux in Fig. 1C will be higher than that in Fig. 1A. The same applies to each pair of corresponding or equidistant positions on opposite sides of the position of maximum flux shown

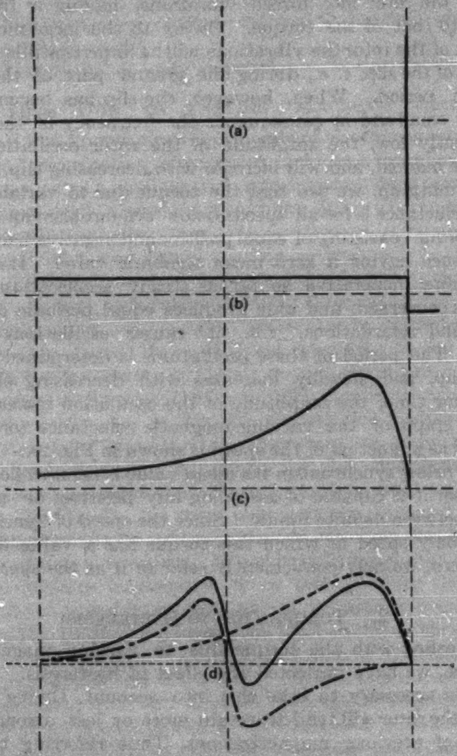

FIG. 2—TORQUE-SPEED CURVES

in Fig. 1B. For any such pair of positions, the driving torque is greater than the retarding torque. The effect of hysteresis is thus seen to be the production of a disparity between the positive and negative half-waves of the varying magnetic reluctance torque, the positive half-waves being uniformly larger than the negative ones. This effect is equivalent to raising the magnetic reluctance torque waves above the axis of time, *i. e.*, to the addition of a steady driving torque to the alternating magnetic reluctance torque. The torque due to hysteresis is thus seen to be a steady driving torque and is instrumental in producing acceleration of the rotor. It is to be noted that, assuming the flux per pole to remain constant during the acceleration period, the hysteresis torque has the same value for all speeds below synchronism. If the speed were made to pass through synchronism to higher vaules, the hysteresis torque would undergo reversal at synchronism.

The graph of the hysteresis torque as a function of the speed is shown in Fig. 2B.

3. Torque Due to Starting Squirrel-Cage or Damping Coils if Present

Little need be said about this torque, which may be called the induction motor torque, as everybody is familiar with the relation connecting the torque and speed of an induction motor. The squirrel-cage of a self-starting synchronous machine forms the rotor winding of an induction motor whose stator windings are represented by the armature; the relation connecting torque and speed will be of precisely the same nature as in an induction motor.

The graph of this torque as a function of the speed is of the well-known form shown in Fig. 2C.

The method of varying the torque-speed curve of an induction motor by the introduction of resistance into the rotor is also well known. The effect of introducing resistance is to cause a shearing of the torque speed curves backwards towards the origin, the maximum torque remaining unaffected in value, but occurring at a lower speed. If a very powerful torque is necessary at starting, it is advisable to use a high-resistance squirrel-cage. On the other hand, with such a squirrel cage the speed to which the motor finally settles down corresponds to a large slip, and this, as will be seen later on, makes it more difficult to pull the rotor into synchronism. The ideal arrangement would be one in which the squirrel-cage resistance at starting is such as to give maximum torque, the resistance then automatically decreasing with increase of speed in such a manner that at each speed maximum torque is maintained, until finally the lowest possible resistance is reached, corresponding to a very small slip. The use of the skin effect in conductors for automatically decreasing the rotor resistance with increasing speed has recently been proposed by Theo. Schou.[5]

4. TORQUE DUE TO CURRENTS INDUCED IN FIELD WINDING, IF THIS WINDING IS CLOSED

If the field winding be closed, the currents induced in it will give rise to a torque, and a careful study of the nature of this torque is essential to a clear understanding of the occurrences during the starting period. The armature of the machine may again be regarded as forming the stator winding of an induction motor, of which the rotor winding is represented by the field coils. There is, however, this very important difference between the starting squirrel-cage and the field winding: the currents induced in the squirrel-cage are capable, according to their distribution in space, of giving rise to a field whose magnetic axes may occupy any positions whatsoever relatively to the center lines of the field poles; but the currents induced in the field winding can only produce a field whose magnetic axes are coincident with the center lines of the field poles. This is conveniently expressed by saying that the field winding is a "single magnetic axis" winding; because it can only produce a field having a single definite set of magnetic axes, namely, those corresponding to the center lines of the salient poles. Now a motor having a polyphase stator, but a single-phase or single-magnetic-axis rotor, exhibits certain striking peculiarities which differentiate it sharply from a motor in which both stator and rotor windings are polyphase. The earliest reference to this type of motor which the authors have been able to find occurs in a paper by H. Gorges[6]. Since such motors are not ordinarily used in practise, their characteristics do not seem to be very generally known, and have only occasionally been referred to. A complete analytical theory of this type of motor is given in the Appendix. The torque-speed curve of such a motor is shown in Fig. 2D, and its most striking characteristic is the torque reversal which occurs over a certain range of speed in the neighborhood of half-synchronism. The analytical theory of this motor, is somewhat complicated, but the following general explanation based on a paper published in 1898 by F. Eichberg[7] may be useful. The currents induced in the single-phase rotor winding by the rotating field of the stator give rise to a field which, relatively to the rotor core, is a simple alternating or oscillating field. By a well-known transformation this oscillating field may be replaced by two equal and (relatively to the rotor core) oppositely rotating fields, the crest value of each rotating field being half the maximum crest value of the oscillating field. Now if the slip of the motor be s and if its speed of synchronism be denoted by n, the frequency of the rotor currents will be $s\,f$, where f is the frequency of supply, and the speed of its component rotating fields relatively to the rotor core will be $s\,n$, the speed of the rotor in space being $(1 - s)\,n$. Regarding the direction of rotation of the rotor as positive, the speed of one of the rotating fields relatively to the rotor core is $+\ s\,n$, while that of the other is $-\ s\,n$. Hence the speeds of the rotor rotating fields *in space* are $s\,n + (1 - s)\,n = n$, and $-\ s\,n + (1 - s)\,n = (1 - 2\,s)\,n$. The interaction between the stator field, and the first rotating component of the rotor field, whose speed n in space is the same as that of the stator field, gives rise to a torque in every respect similar to that of an ordinary induction motor with polyphase windings on both stator and rotor. The second rotating component of the rotor field, whose speed in space is $(1 - 2\,s)\,n$, is clearly incapable of reacting with the stator field in such a manner as to give rise to a resultant torque; for, owing to the difference of speed, the relative position of the fields is constantly changing, periodically passing through a succession of cycles during each of which the average algebraic value of the torque is zero. Although incapable of torque production by interaction with the stator field, the second rotating component of the rotor field is capable of giving rise to a torque by different kind of action. In sweeping across the stator conductors, it induces in them e.m.fs. of frequency $(1 - 2\,s)\,f$, and these produce currents of the same frequency in the stator windings and the circuit external to them (represented by the mains and everything connected across them, generators, motors, lamps, etc.). Since the total impedance external to the stator windings is extremely small in comparison with that of the windings themselves, the result, so far as the currents of frequency $(1 - 2\,s)\,f$ are concerned, is nearly the same as if the stator windings were short-circuited. The currents give rise to a rotating field whose speed $(1 - 2\,s)\,n$ in space is the same as that of the inducing rotor field, and the interaction of these two fields whose relative space position is invariable, results in the production of a torque. To fix ideas, we may think of the second component of the rotor field as produced by a polyphase winding on the rotor supplied with suitable polyphase currents having a frequency $s\,f$, and of the rotating field due to this as inducing currents of frequency $(1 - 2\,s)\,f$ in the stator windings. The arrangement would then be equivalent to a polyphase motor whose primary is represented by the rotor and whose secondary is represented by the stator. The slip of this imaginary motor would be $(1 - 2\,s)$ and so long as s is less than ½, the slip and torque would be positive. Zero slip would occur at $s = ½$, *i. e.*, at half the speed of synchronism. Beyond this point the slip and torque would assume negative values.

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[5] Loc. cit.

[6] H. Gorges: "Ueber Drehstrommotoren mit verminderter Tourenzahl, *Elektrotechnische Zeitschrift*, Vol. 17, p. 517 (1896.)

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[7] F. Eichberg, *Zeitschrift fur Elektrotechnik* (Wein), Vol. 16, p. 578 (1898).

The resultant torque of the motor would be obtained by taking the algebraic sum of the torques due to the two oppositely rotating components of the oscillating rotor field. This torque is represented by the full line curve in Fig. 2D, the dotted and chain-dotted curves corresponding to the component torques due to the two rotating components of the rotor field.

If we suppose that the torque arising from the current in the field winding is large in comparison with the other torques acting on the rotor, so that the dominant effect is that due to the field winding, then it is evident that the torque reversal which occurs at half-synchronous speed will tend to make the machine run in the neighborhood of that speed, and it will then be impossible to run the machine up to full synchronism.

In the discussions which have taken place regarding the tendency of the machine to settle down to a speed in the neighborhood of half-synchronism, erroneous views have frequently been expressed. Some engineers appear to hold the opinion that the machine *locks* into exact half-synchronism. As we have seen, the speed to which it settles down, although near half-synchronism, is not definite, and may, according to the special circumstances of each case, be anywhere in the neighborhood of that speed. It is no more correct to say that the machine locks into half-synchronism, than it would be to say that an ordinary induction motor locks into full synchronism.

5. TORQUE DUE TO EDDY CURRENTS

If the field structure is laminated throughout, the torque due to eddy-currents will be insignificant. The case is otherwise, however, with solid field poles in which large eddy-currents may arise. If we were to imagine the rotor replaced by a solid cylinder of conducting material, then the rotating field of the stator would give rise to rotating eddy-current sheets in the conducting cylinder, and the axes of such current sheets[8] would follow the axes of the rotating stator field. There would in this case be perfect freedom of motion of the axes of the current sheets relatively to the rotor, and this condition is closely approximated to in an ordinary squirrel-cage winding. If we next suppose that the conducting cylinder is cut up into a number of sectors by radial barriers of insulating material, then the freedom of motion of the axes of the current sheets relatively to the rotor would be largely destroyed, and these axes could only swing through an angular distance not exceeding the angular width of a sector. Now this is approximately the case corresponding to a salient pole rotor with solid poles. In such a rotor, owing to the restriction imposed on the free development of eddy currents by the relatively large spaces between the field poles, the axes of the eddy currents can only travel through a relatively short distance. If the axes could not travel at all, the arrangement would be identical with that of a rotor having a single magnetic axis winding; while if the axes could travel with perfect freedom, it would be identical with that of a squirrel-cage rotor. Hence we see that the torque due to eddy currents in the solid field cores will partake partly of the nature of the torque due to a single-magnetic-axis rotor winding, and partly of that due to an ordinary polyphase rotor. The single-magnetic-axis effect is, however, in many cases found to predominate, and considerable difficulty may then be experienced in getting the rotor to pass well beyond half-synchronous speed.

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[8] By the axes of the current sheets are meant the lines along which the current density is zero or the lines with which all the individual current filaments are linked.

PULLING INTO SYNCHRONISM

Of the various torques concerned in accelerating the rotor from rest to synchronism, three, namely the induction motor, the single-magnetic-axis and the eddy current torque, are functions of the speed, and may hence conveniently be referred to as the speed torques. Let us suppose that by the action of the speed and hysteresis torques the rotor has been brought to a speed not far removed from synchronism. Since in the neighborhood of synchronism, the speed torques rapidly decrease with decreasing slip—as shown in Fig. 2—and assume zero values at synchronism, it is evident that these torques would never be able to bring the rotor up to full synchronism; and the hysteresis torque is generally much too weak to effect this. The rotor is finally pulled into synchronism by the varying magnetic reluctance torque, and is maintained at synchronous speed by the same torque, all the other torques vanishing at that speed. As already explained, the varying magnetic reluctance torque may for this reason be conveniently termed the synchronous torque, and we shall in what follows refer to it as such.

We have already seen that the synchronous torque is an alternating torque having a zero mean value for all speeds other than that of synchronism, and is thus incapable of exerting any steady driving or accelerating effect so long as the speed of the rotor is below synchronism. The frequency of the synchronous torque is given by $2\,s\,f.$, and the forced oscillations of the rotor to which the synchronous torque gives rise have the same frequency as the torque itself. Now the rotor speed oscillations call into play a further alternating or oscillating torque, owing to the fact that the speed torques change with the speed of the rotor. The speed torques may for small values of the slip be taken to be proportional to the slip, and hence their changes to be proportional to the changes in the speed. The effect is the same as if we were to substitute for the fluctuating speed torques a constant torque equal to the sum of the mean values of the speed torques, together with an oscillating torque whose amplitude is proportional to that of the speed fluctuations.

For the sake of simplicity we shall assume the speed fluctuations to obey the simple harmonic law. They may then be graphically represented in a vector diagram by the projections on the vertical axis of the vector $O\,V$ in Fig. 3, this vector rotating at $s\,f_r$ revolutions per second. The instantaneous projection of $O\,V$ gives the difference between the instantaneous speed and the mean speed. Since the oscillating component of the speed torques may, as we have seen, in the neighborhood of synchronism be taken to be proportional at every instant to the difference between the instantaneous and the mean speed, and since increase of speed produces decrease of speed torques, it is evident that the oscillating or fluctuating component of the speed torques may be represented by a vector $O\,F$ in direct phase opposition to $O\,V$. Next, if we assume that the alternating synchronous torque is also a simple harmonic function of the time, then the resultant of the synchronous torque and the oscillating component of the speed torques will give us the alternating torque which gives rise to the periodic accelerations and retardations of the rotor. The phase of this resultant torque is easily determined; for since its zero value must occur at the instant of maximum speed, it is evident that the vector $O\,R$, which represents the resultant torque, must be 90 deg. ahead of $O\,V$, as shown in Fig. 3. Lastly, the synchronous torque vector $O\,S$ is obtained by subtracting from the resultant accelerating torque $O\,R$ the oscillating component $O\,F$ of the speed torques. The angular velocity of all the vectors in the diagram of Fig. 3 is directly proportional to the slip, being, in fact, equal to $4\,\pi\,s\,f$.

Let ω denote the excess of the instantaneous rotor speed over the mean speed (corresponding to the vertical projection of $O\,V$ in Fig. 3), and let y stand for the instantaneous resultant accelerating torque (vertical projection of $O\,R$). Then, if K is the moment of inertia of the rotor,

$$y = K\,\frac{d\,\omega}{d\,t},$$

or

$$d\,\omega = 1/K\,y\,d\,t$$

and hence, taking as the origin of time the instant at which ω is zero,

$$\omega = 1/K \int y\,d\,t,$$

from which it is seen that the amplitude of the speed fluctuation $O V$ is proportional to the time-integral over a quarter-period of the resultant accelerating torque; or, since the period of this torque varies inversely as the slip, $O V$ is proportional to $\frac{O R}{s}$

By means of the vector diagram of Fig. 3 we can easily show that the amplitude of the speed fluctuations must increase with decreasing rotor slip. For, assuming the diagram to represent the conditions prevailing at a given mean speed, if the mean speed increases, $O R$ must decrease; for if it were to remain constant, then owing to the increase of its period due to the decrease of slip, its time-integral over a quarter-period would be increased, and $O V$, which is proportional to this time integral, would increase. This again would cause $O F$ ($S R$), which is proportional to $O V$, to increase; but since the length $O S$ is constant, an increase of $S R$ could only be brought about by a decrease of $O R$ (as shown by the dotted lines $O S'$ and $R' S'$ in the figure). It follows *a fortiori* that $O R$ could not increase with increase of mean speed.

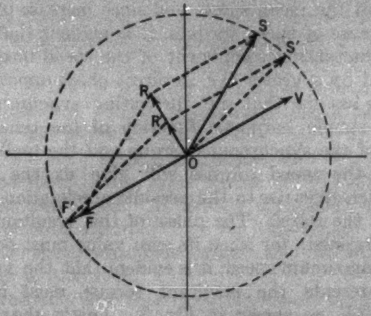

Fig. 3—Vector Diagram of Speed Fluctuations and Torques

We thus see that as the mean speed of the rotor gradually increases, the vector $O V$ undergoes steady elongation, the vector $O F = S R$ a similar steady elongation ($O F$ is proportional to $O V$) and the vector $O R$ a steady contraction. The vector $O\,S$ remains fixed in magnitude, but gradually approaches $O\,V$. At the same time the angular velocity of all the vectors in the diagram steadily decreases. If we were to consider the actual paths traced out by the extremities of the vectors during the last few cycles preceding synchronism, we should find that V traces out a spiral path opening outwards, F a similar path, R a spiral path contracting inwards, while S continues to move in its original circular path. Just before synchronism is reached, $O\,V$ is moving with extreme slowness and $O\,S$ is only very slightly in advance of it. As $O\,V$, having passed through the horizontal position, moves into the first quadrant, its projection gradually increases until the value of this projection when added to the mean rotor speed gives the speed of synchronism. At this instant all the torques have disappeared with the exception of the synchronous torque.

It is clear that at the instant when synchronism is first reached the synchronous torque cannot be less than the total torque resisting the motion. For, if such were the case, then balance of the total driving and resisting torques must have taken place at some instant preceding synchronism, and such balance would have prevented any further increase of speed, *i. e.*, it would have prevented the rotor from reaching synchronism. Hence at the instant when synchronism is reached, the synchronous torque must either equal or exceed the total resisting torque. In the first case, the rotor will steadily maintain synchronous speed. In the second, further acceleration will take place, and the rotor will settle down to the steady speed of synchronism only after a number of oscillations, the final position which it takes up relatively to the stator poles being such that the synchronous torque arising from the displacement of the magnetic axes of the stator and rotor is exactly equal to the total resisting torque. Whether the rotor comes up to synchronous speed quietly without oscillations, or whether such oscillations take place before it finally settles down to the steady speed of synchronism, the running will correspond to stable conditions. For in either case a momentary increase of speed results in decrease of driving torque, and a momentary decrease of speed in increase of driving torque. The momentary changes in the driving torque which arise during speed fluctuations, are due partly to changes in the synchronous torque, which tend to check such fluctuations, and partly to the reappearance of the speed and hysteresis torques, which have a similar effect.

OPEN VERSUS CLOSED FIELD WINDINGS AT STARTING

The advisability or otherwise of closing the field windings at starting has been repeatedly discussed. The danger of breaking down the insulation by the high voltage induced in the field windings when the stator circuits are first connected to the mains must be taken into account. Although this danger is entirely avoided by short-circuiting the windings before the stator is connected to the mains, there is no doubt that, from the point of view of initial torque and rapidity of starting, it is inadvisable to have the field circuit closed. The effect of closing field windings is similar to that of reducing the resistance of the squirrel-cage or eddy current path,—a procedure which is well-known to lower the initial torque. Again, as the neighborhood of half-synchronism is approached, the powerful single-axis rotor effect may seriously affect the acceleration of the rotor, and may frequently entirely prevent the machine from attaining any speed greatly exceeding that of half-synchronism. In order therefore to increase the acceleration of the rotor during the early stages of the starting process, the field should be kept open; any risk of breaking down the insulation may be guarded against by the use of a suitable field break-up switch.

Now although it is advisable to keep the field circuit open during the initial stages of the starting operation it by no means follows that it would be equally advantageous to keep it open until the machine has been pulled into synchronism. The slip with which the rotor ultimately tends to run under the action of the speed torques will depend on the resistance of the circuits in which the currents giving rise to the speed torques circulate. By lowering this resistance the torque will be momentarily raised and the speed increased. Now closing the field circuit would be equivalent to such reduction of resistance, so that the short-circuiting of the field during the final stages of the starting operation will cause the mean rotor speed to approach more closely to the speed of synchronism than would otherwise be the case. There is thus a distinct advantage in closing the field circuit during the final stages of the starting operation, after the rotor speed has reached a value not differing greatly from synchronism.

Cases may in fact arise where a machine with its field open might refuse to pull into synchronism, but could be made to do so by closing the field circuit. This conclusion has been verified experimentally. A certain machine was started with its field open, the rotor potential difference being so low that the rotor settled down to a speed below synchronism, and refused to pull into synchronism. The moment, however, that the field circuit was closed, the rotor locked into synchronism.

OSCILLATIONS IN STATOR CURRENT DURING THE PERIOD IMMEDIATELY PRECEDING SYNCHRONISM

It is well known that as the speed of synchronism is approached violent fluctuations in the stator current gradually become noticeable. These are indicated in Fig. 5 of Rosenberg's paper, and are easily accounted for. So long as the speed is below synchronism, the field poles are slipping past the stator poles, and periodic fluctuations are taking place in the reluctance accompanied by corresponding fluctuations of reactance which throw the stator current into oscillations. The frequency of these oscillations being $2 s f$ (since the reluctance returns to the same instantaneous value after the pole has moved through a distance equal to the pole-pitch) they are not noticeable at low speeds, and only become apparent when the slip has become sufficiently small.

SOME EXPERIMENTAL RESULTS

(a) *Relations Connecting Stator Potential Difference with Stator Current, Stator Input and Power-Factor, Rotor Speed and Field E. M. F. when Field is Open-Circuited.* The experiments embodied in the series of curves given were carried out on a four-pole, five-kw. three-phase converter designed for a continuous current voltage of 100–130 volts at a speed of 750 rev. per min. This machine had laminated main poles, was fitted with commutating poles, but had no special starting devices. While the experiments about to be described were being carried out, the brushes were entirely removed from the commutator. Before each set of readings the machine was allowed to run light for a sufficiently long time to get the bearings into a steady state.

In the first set of experiments, the results of which are exhibited graphically in Figs. 4 and 5, a number of gradually increasing potential differences were applied to the rotor slip rings, and after the speed corresponding to any given potential difference had settled down to a constant value, readings of the speed, current, power, etc., were taken when the potential difference had been raised sufficiently to enable the machine to lock into synchronism, it was still further increased, and then a second series of readings, corresponding to decreasing values of the potential difference was obtained. In the illustrations both the ascending and descending branches of the various curves are given.

Fig. 4 shows the relations connecting speed and field e. m. f. with stator potential difference. Below a potential difference of about 15 volts across the slip rings the machine would not run at all. The speed then gradually increased with the potential difference, the increase becoming much slower beyond a certain point, and at a slip ring-potential difference of about 45 volts the machine was able to lock into synchronism. During the descending set of readings, synchronism was maintained down to a voltage of about 35 volts. Below this point the speeds obtained with given voltages were found to be uniformly higher than those corresponding to the ascending branch of the curve. Since, as shown by Fig. 5, the power supplied to the machine was found to be lower for decreasing values of the potential difference. in spite of the higher value of the speed, it is to be inferred that for decreasing values of the potential difference the resisting torque was uniformly less. This would indicate a decrease in the frictional resistances, probably due to the temperature of the bearings being higher during the descending set of readings than during the ascending set.

The changes in the field e. m. f. are related to those in the speed. The field e. m. f. may be regarded as proportional to the product of two factors, namely the maximum flux per pole and the slip. At first the field e. m. f. rises with increase of potential difference, indicating that the increase of flux is more important than the decrease of slip. Beyond a certain point the decrease of slip is more important than the increase of flux, and the field e. m. f. begins to decrease. It does not vanish at synchronism, indicating that there is either swaying or pulsation of the flux which enters the main poles. Since for descending values of the potential difference the speed is uniformly higher and hence the slip lower than for ascending values, we should expect the field e. m. f.

to be uniformly lower in the former case, and the curve of field e. m. f. shows that such is the case.

Fig. 5 shows the relations connecting stator current, stator power and power-factor with potential difference. The difference between the ascending and descending branches of the power or input curve has already been referred to. It must be remembered that when the machine is not running synchronously, its behavior is similar to that of an induction motor. Hence, owing to the lower resisting torque during the descending set of readings we should expect a smaller current and also a lower power-factor (as is at once evident from consideration of the circle diagram) than during the ascending set; and the curves of Fig. 5 fully confirm this.

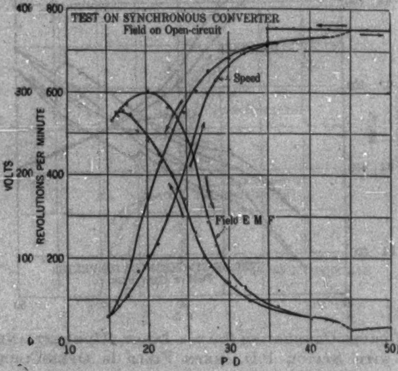

FIG. 4—CURVES CONNECTING SPEED AND FIELD E.M.F. WITH STATOR P.D. WHEN FIELD IS OPEN-CIRCUITED

(*b*) *Relations Connecting Stator Potential Difference with Speed and Field Current, when the Field Circuit is Closed.* Figs. 6 and 7 give the connection between potential difference and speed when the field circuit is closed through various resistances, and in Fig. 6 the curve corresponding to the field on open-circuit, previously shown in Fig. 4, is repeated for the sake of comparison.

When the field was on dead short-circuit, the machine refused to run up to anything like synchronous speed, and seemed to approach asymptotically a speed somewhat above half-synchronism.[9] The explanation of this fact has already been given (reference may be made in this connection to Fig. 2D).

The curves of Fig. 7 show that by the introduction of a suitable amount of resistance into the field circuit the tendency of the machine to settle down to a speed in the neighborhood of half-synchronism may be overcome, and that the machine may be made to lock into synchronism. This result may be explained as follows. Considering the complete torque-speed curve of an induction machine over the entire range of slip, positive and negative, we may regard the point of zero slip as dividing this curve into two branches, one of which corresponds to positive values of the slip, and the other to negative values. If we now suppose resistance to be introduced into the rotor circuit, then as is well known, the result is to produce a shearing of the two branches of the torque speed curve in opposite directions from the point of zero slip. Referring now to the torque speed curves of Fig. 2D, it must be noticed that the point of zero slip for the chain-dotted curve

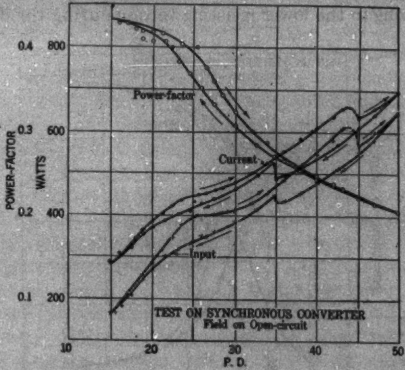

Fig. 5—Curves Connecting Stator Input, Current and Power Factor with Stator P.D. when Field Is Open-Circuited

corresponds to half-synchronism, while for the dotted curve the point of zero slip is at full synchronism. From this it follows that the introduction of resistance will in the region between half and full synchronism cause a shearing of the dotted and chain-dotted curves in opposite directions, the dotted curve being sheared from left to right, while the chain-dotted one is sheared from right to left. It is easy to see that this will cause a rise of the minimum in the resultant curve (the full line curve of Fig. 2D), and if the resistance remains sufficiently large the minimum resultant torque will assume a positive value, so that the driving torque will be positive over the entire range of speed from zero to synchronism. The shearing of the dotted and chain-dotted curves in opposite directions in the region between half and full synchronism is, however, only one of the causes concerned in suppressing the negative portion of the resultant torque-speed curve, and besides this there is another cause. The dotted curve is the torque-speed curve of an induction motor whose stator is supplied at constant potential difference and frequency; whereas the chain-dotted curve is the curve of an imaginary induction motor whose stator is supplied at variable

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[9] Incidentally the fact that the machine reached a speed in excess of half-synchronism definitely disposes of the erroneous view previously referred to that the machine tends to lock into exact half-synchronism.

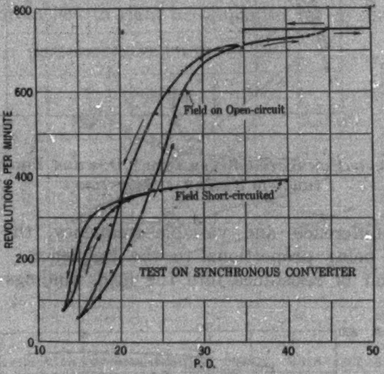

FIG. 6—CURVES CONNECTING SPEED WITH P.D. WHEN FIELD IS (a) OPEN-CIRCUITED, AND (b) SHORT-CIRCUITED

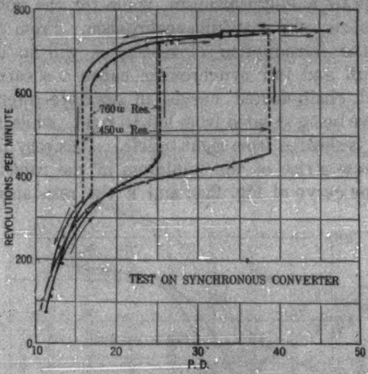

FIG. 7—CURVES CONNECTING SPEED WITH P.D. WHEN FIELD IS CLOSED THROUGH EXTERNAL RESISTANCE

potential difference and variable frequency, the potential difference being proportional to the frequency. Now the introduction of resistance into the field windings is equiv-

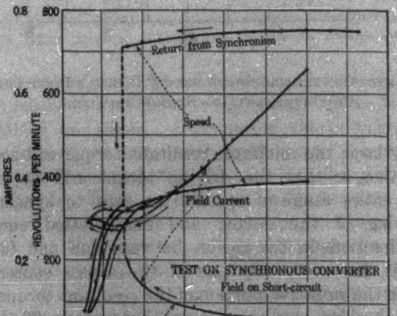

FIG. 8—CURVES CONNECTING SPEED AND FIELD CURRENT WITH P. D. WHEN FIELD IS SHORT-CIRCUITED

alent to the introduction of resistance into the primary winding of the imaginary motor (since the primary winding of this imaginary motor is represented by the field winding) and is thus equivalent to a reduction of the potential difference across its terminals. This will result in a reduction of all the ordinates of the chain-dotted curve. While therefore, the introduction of resistance into the field circuit results in a simple shearing of the dotted curves from right to left unaccompanied by any change in the values of the ordinates, the effect on the chain-dotted curve is a two-fold one, namely a shearing from left to right accompanied by a shrinkage of the ordinates. This shrinkage of the ordinates will further help to suppress the negative portion of the resultant curve.

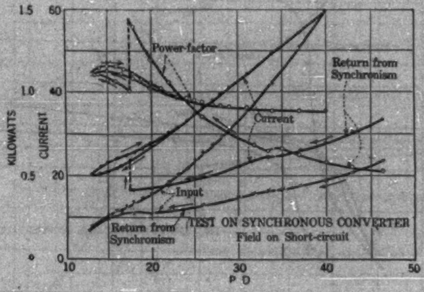

FIG. 9—CURVES CONNECTING STATOR INPUT, CURRENT AND POWER FACTOR WITH P.D. WHEN FIELD IS SHORT-CIRCUITED

It will be noticed that in the curves of Fig. 6 which refer to the open-circuit and short-circuit conditions of the field, there are no discontinuities in the speed curves (except that which occurs at the instant of breaking from synchronism in the case of the open-circuit curve); whereas the curves of Fig. 7 show two well-marked discontinuities (one on each of the curves), in addition to the discontinuities at break from synchronism. These discontinuities are readily accounted for by considering the shape of the resultant or full-line torque-speed curve of Fig. 2D. In the cases to which Fig. 7 refers the resultant torque-speed curve lies, as already explained, wholly above the axis of speed, all its ordinates being positive; but the curve has two maxima separated by a minimum. It is the existence of this minimum which causes the discontinuities in the speed curves. Stability of running can only be secured by working on a portion of the torque-speed curve which has a downward slope from left to right. With increasing potential difference and speed the point on the (varying) torque-speed curve corresponding to the stable running condition for the given potential difference gets displaced further and further to the right, until finally it reaches the minimum point on the curve. An increase of potential difference beyond the value corresponding to this minimum point results in a passage into the unstable region which lies between the minimum and the second maximum, and no stable running is possible in this region. It is only after the speed has passed beyond the second maximum of the torque-speed curve that stability can again be reached. The point where the discontinuity occurs along the ascending branch gives approximately the speed corresponding to minimum torque; while the discontinuity on the descending branch marks approximately the second maximum of torque. The first maximum of torque would correspond roughly to the lowest speed at which the machine will run.

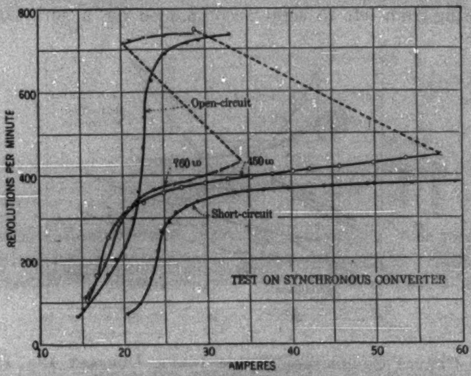

FIG. 10—CURVES CONNECTING SPEED WITH STATOR CURRENT UNDER VARIOUS CONDITIONS

In Fig. 8 are shown the relations connecting speed and potential difference, and field current and potential difference, with the field on dead short circuit. Each curve is shown as having three branches. Two of these correspond to the values obtained by first increasing the speed to a certain value and then decreasing it. The third branch, marked "Return from synchronism" was obtained by first open-circuiting the field and raising the potential difference to a value sufficient to enable the machine to lock into synchronism, then short-circuiting the field and taking a set of readings while the potential difference was being decreased. The first two branches of the speed curve are identical with those shown in Fig. 6. The relations connecting current and power factor with potential difference are given by Fig. 9,

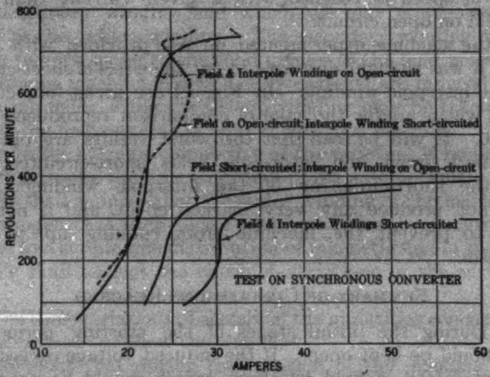

Fig. 11—Curves Connecting Speed with Stator Current under Various Conditions

Returning to Fig. 6, it will be seen that the machine starts with a lower potential difference when the field is short-circuited than when it is on open circuit; and this might at first sight appear to contradict the statement previously made regarding the advantage of starting with the field circuit open. Such, however, is not the case; for the real basis of comparison is not the potential difference applied to the stator, but the *current* taken by it. The relation connecting speed with current for the various arrangements tried is shown in Fig. 10, and it will be seen at once that the machine starts up with a considerably lower current when the field is open-circuited. If the field is closed through a resistance, there will be a certain value of this resistance which gives the best initial results, but even with this value the current during the intermediate stages rises to a higher value than when the field is open-circuited. On the other hand, it will be noticed that with the field closed through a resistance high enough to allow of the machine being pulled into synchronism, a higher value of speed is reached with a given current than with the field on open circuit.

As the machine experimented on was provided with interpoles, it was thought desirable to try the effect of short-circuiting the interpole winding. In Fig. 11 are given four curves, two of which are for the sake of comparison reproduced from Fig. 10. It will be seen that the worst results are obtained with both main and inter-pole fields short-circuited, and that the short-circuiting of the interpole windings alone does not produce any very marked effect, and is not sufficient to prevent the machine from running up to synchronism.

SUMMARY OF CONCLUSIONS REACHED

1. During the initial stages of the starting period the field should be kept open. If the induced voltage exceeds the limit of safety, a field break-up switch should be provided. Closing the field circuit not only largely increases the current during the initial stages of the starting period, but may entirely prevent the machine from running up to synchronous speed. This is due to the single-magnetic-axis effect of the field winding.

2. If the field is kept closed and the machine only reaches a speed in the neighborhood of half-synchronism, there is no tendency to lock into exact half-synchronism.

3. There is a distinct advantage in short-circuiting the field after the field has reached a value not differing greatly from synchronism. This will greatly facilitate the final locking into synchronism.

APPENDIX

ANALYTICAL THEORY OF POLYPHASE INDUCTION MOTOR HAVING SINGLE-PHASE WINDING ON THE ROTOR

The type of motor under consideration is one whose stator is provided with a polyphase winding, but whose rotor carries only a single-phase or single-magnetic-axis winding. We may speak of the stator winding as the *primary* and of the rotor winding as the *secondary* winding of the machine. We may further suppose that the rotor carries a supplementary single-phase winding whose magnetic axes are coincident with those of the secondary winding and which we may conveniently speak of as the *tertiary* winding. The secondary winding may be taken to correspond to the solid masses of the field poles of a synchronous machine in which eddy currents are induced by the rotating field; while the *tertiary* winding may be taken to represent the field winding of the synchronous machine.

Owing to the extreme complexity of the problem, it becomes necessary to make the following simplifying assumptions:—

(1) We shall suppose that the stator winding of the machine is so arranged that each phase when traversed by a current gives rise to a simple sine distribution of magnetic flux in space.

(2) We shall neglect the effect of variations in the permeability.

(3) The time-variations of certain of the quantities concerned will be supposed to follow the sine law.

It is obvious that during the rotation of the machine the mutual inductance of the secondary with any one phase of the primary winding will undergo periodic fluctuations, and from supposition (1) it follows—as can be easily shown—that such fluctuations will obey the sine law if the speed of rotation is constant. Let M_0 denote the maximum value of the mutual inductance of the secondary and a certain phase of the primary winding. Then if m_a, m_b and m_c denote the instantaneous mutual inductances of the three phases with the secondary at time t we may write, since the space displacement of the three oscillating magnetic flux waves due to the armature current is 120 electrical degrees,

$$m_a = M_0 [\, (1 - S)\, wt + \alpha]$$

$$m_b = M_0 \sin\left[\, (1 - S)\, w\, t + \alpha + \frac{2\,\pi}{3}\right]$$

$$m_c = M_0 \sin\left[(1 - S)\, w\, t + \alpha + \frac{4\,\pi}{3} \right]$$

where $w = 2\,\pi \times$ frequency of armature current, and s = slip of rotor $= \left(\dfrac{\text{synchronous speed} - \text{rotor speed}}{\text{synchronous speed}}\right)$.

Let the currents in the three primary phases be represented by

$$i_a = I_m \sin w\, t$$

$$i_b = I_m \sin\left(w\, t + \frac{2\,\pi}{3}\right)$$

$$i_c = I_m \sin\left(w\, t + \frac{4\,\pi}{3}\right)$$

If we provisionally assume that both the secondary and tertiary circuits are open, so that they are incapable of reacting on the primary circuits, then in order to maintain the currents i_a, i_b and i_c in the three phases of the primary we must provide impressed e. m. f's. which are given by

$$e_a = Z_1\, I_m \sin\, (w\, t + \theta_1)$$

$$e_b = Z_1\, I_m \sin\left(w\, t + \frac{2\,\pi}{3} + \theta_1\right)$$

$$e_c = Z_1\, I_m \sin\left(w\, t + \frac{4\,\pi}{3} + \theta_1\right)$$

In the above expressions, Z_1 denotes the equivalent impedance of each phase when the three windings are supplied with currents differing 120 deg. in phase. If r_1 = resistance of each phase, and L_1 = true self-inductance of each phase (*i. e.*, flux linkage with phase when unit current is flowing through it and when remaining two phases are devoid of current), then

$$Z_1 = \sqrt{r_1^2 + (3/2\, w\, L_1)^2}$$

The fact that the equivalent reactance of a phase is 3/2 times its true reactance is due to mutual inductance between phases, and is a consequence of the assumption that the flux distribution in space due to any one phase follows the sine law.

The angle θ_1 in the above expressions is such that $\tan\theta_1 = \dfrac{3/2\, w\, L_1}{r_1}$.

The total instantaneous flux linked with the secondary due to the primary currents i_a, i_b and i_c is

$$\Phi = m_a\, i_a + m_b\, i_b + m_c\, i_c = 3/2\, M_0\, I_m \cos\,(S\, w\, t - \alpha)$$

This flux gives rise to an e. m. f. in the secondary of amount

$$e_2 = -\frac{d\,\Phi_2}{dt} = \frac{3}{2}\, S\, w\, M_0\, I_m \sin\,(S\, w\, t - \alpha)$$

We shall now assume that the secondary is closed, and that the e. m. f. e_2 is allowed to produce a current in it.

Let r_2, L_2 be the resistance and self-inductance respectively of the secondary. Its impedance is then

$$Z_2 = \sqrt{r_2^2 + (s\, w\, L_2)^2}$$

and the e. m. f. e_2 gives rise to a secondary current

$$i_2 = \frac{3}{2\, z_2}\, s\, w\, M_0\, I_m \sin\,(s\, w\, t - \alpha - \theta_2)$$

where $$\tan\,\theta_2 = \frac{s\, w\, L_2}{r_2}$$

For the sake of simplicity, we shall put

$$k_2 = \frac{3/2\, s\, w\, M_0}{z_2}$$

so that we may write

$$i/2 = k/2\, I_m \sin\,(s\, w\, t - \alpha - \theta_2)$$

The secondary current reacts on the inducing primary circuits, and gives rise to magnetic fluxes linked with them. The flux linkage with the first phase, whose instantaneous mutual inductance with the secondary is m_a, is given by $m_a\, i_2$, and the e. m. f. to which this flux gives rise is

$$-\frac{d}{dt}\,(m_1\, i_2) =$$

$$-\frac{d}{d\,t}\,\{M_0 \sin\,[\,(1 - s)\, w\, t + \alpha]\, k_2\, I_m \sin\,(s\, w\, t - \alpha - \theta_2)\,\} =$$

$$(1/2 - s)\, w\, M_0\, k_2\, I_m\; \sin\,[\,(1 - 2\, s)\, w\, t + 2\,\alpha + \theta_2]$$
$$- 1/2\, w\, M_0\, k_2\, I_m \sin\,(w\, t - \theta_2)$$

Similar expressions (with suitable changes of time-phase) hold good for the remaining two phases.

It will be noticed that each of these e. m. fs. consists of two components of different frequency. If we wished to maintain the original currents i_a, i_b and i_c unaltered, then in addition to the original impressed e. m. fs., e_a, e_b and e_c we should have to provide e. m. fs., equal and opposite in phase to those induced in the primaries by the secondary current i_2. Now since these latter e. m. fs. consist of two components, one [such as

$$-\,1/2\, w\, M_0\, k_2\, I_m \sin\,(w\,t - \theta_2)$$

in the first phase of the primary of fundamental frequency], and the other [such as $(1/2 - s)\; w\; M_0\, k_2\, I_m \sin\,[\,(1 - 2\,s)\, w\,t + 2\,\alpha + \theta_2]$ in the first phase] of a frequency which is a function of the slip, the source of impressed e. m. f. would have to be capable of a continuous variation of wave-shape in order that the currents i_a, i_b and i_c might remain unaltered. It is needless to point out that such special continuous variation of wave-shape could not be secured in practise. We shall therefore make an assumption which is much more likely to conform to actual conditions. We shall suppose that the source of impressed e. m. f. continues to supply a pure sine wave, but that the excitation of this source is varied so that it not only provides the original e. m. fs. e_a, e_b and e_c but, in addition, any other sine-wave components of *fundamental frequency* which may be necessary to balance fundamental frequency e. m. fs [such as the e. m. f. $-\,1/2\, w\, M_0\, k_2\, I_m \sin\;(w\,t - \theta_2)$ in the first phase] induced in the primaries by the secondary. The remaining components in the induced e. m. fs. whose frequency differs from the fundamental, and which are not balanced by corresponding components in the impressed e. m. f. waves, give rise to additional primary currents whose magnitude we proceed to determine.

For the sake of simplicity, we shall assume the impedance of the source of impressed e. m. f. to be negligible in comparison with the impedance of the primaries of the machine under consideration, so that the total impedance of the circuits on which the induced e. m. fs. of frequency other than the fundamental act will be represented by the impedance of the primaries. The equivalent reactance of each phase of the primary corresponding to e. m. fs. such as $(1/2 - s)\, w\, M_0\; k_2\;\, I_m \sin\,[\,(1 - 2\,s)\; w\,t + 2\;\alpha\; + \;\theta_2]$ in the first phase, is $3/2\;(1 - 2\,s)\; w\,L_1$.

Hence the corresponding current in the first phase is given by

$$i_a{}' = \frac{(1/2 - s)\; w\, M_0\, k_2\, I_m}{\sqrt{r_1{}^2 + [3/2\;(1 - 2\,s)\; w\, L_1]^2}}\;\sin\,[\,(1 - 2\,s)\; w\,t + 2\,\alpha + \theta_2 - \theta_3]$$

If for the sake of simplicity we put

$$k_1 = \frac{(1/2 - s)\, w\, M_0}{z_3}$$

then

$$i_a' = k_1\, k_2\, I_m \sin\left[\,(1 - 2\,s)\, w\, t + 2\,\alpha + \theta_2 - \theta_3\right]$$

Similar expressions hold good, with suitable changes of time-phase, for the corresponding currents in the remaining two phases.

Before proceeding further, it will be convenient to collect all the results so far obtained.

In the first phase of the primary, we have the following components of impressed e. m. f.:—

$$e_a = z_1\, I_m \sin\,(w\, t + \theta_1) \tag{1}$$

and

$$e_a' = 1/2\; w\, M_0\, k_2\, I_m \sin\,(w\, t - \theta_2) \tag{2}$$

where

$$z_1 = \sqrt{r_1^2 + (3/2\; w\, L_1)^2} \tag{3}$$

$$\tan\,\theta_1 = \frac{3/2\; w\, L_1}{r_1} \tag{4}$$

$$k_2 = \frac{3/2\; s\, w\, M_0}{z_2} \tag{5}$$

$$z_2 = \sqrt{r_2^2 + (s\, w\, L_2)^2} \tag{6}$$

$$\tan\,\theta_2 = \frac{s\, w\, L_2}{r_2} \tag{7}$$

Next, in the first phase of the primary we have the current components

$$i_a = I_m \sin w\, t \tag{8}$$

and

$$i_a' = k_1\, k_2\, I_m \sin\left[\,(1 - 2\,s)\, w\, t + 2\,\alpha + \theta_2 - \theta_3\right] \tag{9}$$

where

$$k_1 = \frac{(1/2 - s)\, w\, M_0}{z_3} \tag{10}$$

$$z_3 = \sqrt{r_1^2 + [3/2\,(1 - 2\,s)\, w\, L_1]^2} \tag{11}$$

$$\tan\,\theta_3 = \frac{3/2\,(1 - 2\,s)\, w\, L_1}{r_1} \tag{12}$$

Again, considering the secondary, we have in the induced e. m. f.

$$e_2 = 3/2\; s\, w\, M_0\, I_m \sin\,(s\, w\, t - \alpha) \tag{13}$$

which gives rise to the secondary current

$$i_2 = k_2 I_m \sin (s w t - \alpha - \theta_2) \qquad (14)$$

We may now proceed with our investigation.

In addition to the e. m. f. e_2 in the secondary, which is due to the original currents i_a, i_b and i_c in the primaries, and which produces the current i_2, we now have another e. m. f., due to currents of the type i_a', in the primaries. This second e. m. f., as in the case of e_2, may be shown to be

$$e_2' = 3/2\, s w M_0 k_1 k_2 I_m \sin (s w t - \alpha - \theta_2 + \theta_3) \qquad (15)$$

If we were to provide an e. m. f. in the secondary equal and opposite to e_2' this latter e. m. f. would be neutralized, and the only currents in the primaries and the secondary would be those already considered and given by (8), (9) and (14).

Since, however, no such neutralization actually takes place, the e. m. f. e_2' is free to act, and gives rise to a current in the secondary.

$$i_2' = k_1 k_2^2 I_m \sin (s w t - 2\,\theta_2 + \theta_3) \qquad (16)$$

This current, in turn, reacts on the primary, and, proceeding as before, we can show that it induces in the first phase of the primary an e. m. f. given by

$$(1/2 - s)\, w M_0 k_1 k_2^2 I_m \sin [\,(1 - 2\,s)\, w t + 2\,\alpha + 2\,\theta_2 - \theta_3] \\ - 1/2\, w M_0 k_1 k_2^2 I_m \sin (w t - 2\,\theta_2 + \theta_3)$$

This e. m. f. is seen to consist of two terms of different frequency. We shall assume, as before, that the term of fundamental frequency is balanced by an equal and opposite component

$$e_a'' = 1/2\, w M_0 k_1 k_2^2 I_m \sin (w t - 2\,\theta_2 + \theta_3) \qquad (17)$$

in the impressed e. m. f. wave of the primary, while the other term, being unbalanced, gives rise to a current in the first phase of the primary

$$i_a'' = k_1^2 k_2^2 I_m \sin [\,(1 - 2\,s)\, w t + 2\,\alpha + 2\,\theta_2 - 2\,\theta_3] \qquad (18)$$

Again, the current i_1'', like i_1 and i_1', induces an e. m. f. in the secondary, given by

$$e_2'' = 3/2\, s w M_0 k_1^2 k_2^3 I_m \sin (s w t - \alpha - 2\,\theta_2 + 2\,\theta_3) \qquad (19)$$

The e. m. f. e_2'' gives rise to a secondary current

$$i_2'' = k_1^2 k_2^3 I_m \sin (s w t - \alpha - 3\,\theta_2 + 2\,\theta_3) \qquad (20)$$

The current in its turn, reacts on the primary, inducing in the first phase an e. m. f.

$$(1/2 - s)\, w M_0 k_1^2 k_2^3 I_m \sin [(1 - 2\,s)\, w t + 2\,\alpha + 3\,\theta_2 - 2\,\theta_3] \\ - 1/2\, w M_0 k_1^2 k_2^3 I_m \sin (w t - 3\,\theta_2 + 2\,\theta_3),$$

the fundamental frequency component of which we shall assume, as before, to be balanced by a component in the impressed e. m. f. equal to

$$e_a''' = 1/2\, w\, M_0\, k_1^2\, k_2^2\, I_m \sin(w\,t - 3\,\theta_2 + 3\,\theta_3) \qquad (21)$$

while the other component is left free to produce in the first phase of the primary a current

$$i_a''' = k_1^3\, k_2^3\, I_m \sin[\,(1 - 2\,s)\,w\,t + 2\,\alpha + 3\,\theta_2 - 3\,\theta_3] \qquad (22)$$

Proceeding in this way, we ultimately obtain, for each of the quantities under consideration—primary impressed e. m. f., primary current, secondary induced e. m. f., and secondary current—an infinite series of terms.

For the sake of simplicity, we shall put

$$k = k_1\, k/2 \qquad (23)$$

and

$$\theta = \theta_2 - \theta_3 \qquad (24)$$

Considering first the primary impressed e. m. f. the first phase we see from (1), (2), (17) and (21), that this is given by

$$z_1\, I_m \sin(w\,t - \theta_1) + 1/2\, w\, M_0\, k_2\, I_m \sin(w\,t - \theta_2) + k \sin$$
$$(w\,t - \theta_2 - \theta) + k^2 \sin(w\,t - \theta_2 - 2\,\theta) + k_3 \sin$$
$$(w\,t - \theta_2 - 3\,\theta) +]\,\ldots\ldots \qquad (25)$$

Similar expressions, with suitable time-phase differences, hold good for the impressed e. m. fs., in the other two phases.

Taking next the primary current in the first phase, and using (8), (9), (18) and (22), we find that this current is given by the infinite series.

$$I_m \sin w\,t + k\,I_m\, \{\sin[\,(1 - 2\,s)\,w\,t + 2\,\alpha + \theta] + k \sin$$
$$[\,(1 - 2\,s)\,w\,t + 2\,\alpha + 2\,\theta]$$
$$+ k^2 \sin[\,(1 - 2\,s)\,w\,t + 2\,\alpha + 3\,\theta] + \} \qquad (26)$$

Similarly, using (13), (15) and (19), we find for the secondary induced e. m. f.

$$3/2\, s\, w\, M_0\, I_m\, [\sin(s\,w\,t - \alpha) + k \sin(s\,w\,t - \alpha - \theta)$$
$$+ k \sin(s\,w\,t - \alpha - 2\,\theta) +] \qquad (27)$$

Lastly, using (14), (16) and (20), we obtain the secondary current.

$$k_2\, I_m\, [\sin(s\,w\,t - \alpha - \theta_2) + k \sin(s\,w\,t - \alpha - \theta_2 - \theta)$$
$$+ k_2 \sin(s\,w\,t - \alpha - \theta_2 - 2\,\theta) +] \qquad (28)$$

Each of the four expressions (25), (26), (27) and (28) involves an infinite series of the type.

$$\Sigma = \sin \varphi + k \sin (\varphi \pm \theta) + k^2 \sin (\varphi \pm 2\,\theta) + k^3 \sin (\varphi \pm 3\,\theta) +$$

This series may be summed as follows:

Putting $\sin \varphi = \dfrac{1}{2\,j}\,(e^{j\varphi} - e^{j\varphi})$

and $\sin (\varphi \pm n\,\theta) = \dfrac{1}{2\,j}\,[e_1^{j}\,(\varphi + n\,\theta) - e^{-j}\,(\varphi \pm n\,\theta)\,]$

where $j^2 = -1$, we may write

$$\Sigma = \frac{1}{2\,j}\,\{e^{j\varphi}\,[1 + k\,e^{\pm j\theta} + k^2\,e^{\pm 2j\theta} + k^3\,e^{\pm 3j\theta} + \ldots .] - e^{\pm j\theta}$$
$$[\,1 + k\,e^{\pm j\theta} + k^2\,e^{\mp 2j\theta} + \,]\,\}$$

$$= \frac{1}{2\,j}\left(\frac{e^{j\varphi}}{1 - k\,e^{\pm j\theta}} - \frac{e^{-j\varphi}}{1 - k\,e^{\mp j\theta}}\right)_1$$

which, after a number of transformations, may be reduced to the simple form

$$\Sigma = \frac{\sin \varphi - k \sin (\varphi \mp \theta)}{(1 - k \cos \theta)^2 + (k \sin \theta)^2}$$

If we put

$$a = \sqrt{(1 - k \cos \theta)^2 + (k \sin \theta)^2} \qquad \textbf{(29)}$$

and

$$\tan\,\beta = \frac{k \sin \theta}{1 - k \cos \theta} \qquad \textbf{(30)}$$

then, after a further transformation, we may write

$$\Sigma = 1/a \sin (\varphi \pm \beta) \qquad \textbf{(31)}$$

Primary P. D. Applying this result to (**25**), we find for the primary impressed e. m. f. in the first phase.

$$z_1\,I_m \sin (w\,t + \theta_1) + \frac{1/2\,w\,M_0\,k_2\,I_m}{a} \sin (w\,t - \theta_2 - \beta)$$

This may be thrown into the form

$$E \sin (w\,t + \varphi) \qquad \textbf{(32)}$$

where

$$E = I_m \left\{ \left[r_1 + \frac{1/2\,w\,M_0\,k_2}{a} \cos (\theta_2 + \beta) \right]^2 + \left[3/2\,w\,L_1\,\frac{1/2\,w\,M_0\,k_2}{2} \sin (\theta_2 + \beta) \right]^2 \right\}^{\frac{1}{2}} \qquad \textbf{(33)}$$

and

$$\tan \psi = \frac{3/2\, w\, L_1 - \dfrac{1/2\, w\, M_0\, k_2}{a} \sin\,(\theta_2 + \beta)}{r_1 + \dfrac{1/2\, w\, M_0\, k_2}{a} \cos\,(\theta_2 + \beta)} \qquad \textbf{(34)}$$

Primary Current. Similarly, using (**31**) in (**26**), we obtain for the primary current in the first phase

$$I_m \,\{\sin w\, t + k/a \sin\,[\,(1 - 2\, s)\, w\, t + 2\, \alpha + \theta + \beta]\,\} \qquad \textbf{(35)}$$

It will be noticed that, with a pure sine wave of impressed e. m. f., the primary current wave is a distorted one.

Secondary Current. Proceeding in a similar manner with (**27**) and (**28**), we obtain the following expressions for the secondary induced e. m. f. and secondary current:—

Secondary induced e. m. f.

$$= I_m \frac{3/2\, s\, w\, M_0}{a} \sin\,(s\, w\, t + \alpha - \beta) \qquad \textbf{(36)}$$

Secondary current

$$= I_m\, k_2/a \sin\,(s\, w\, t - \alpha - \theta_2 - \beta) \qquad \textbf{(37)}$$

Both the above waves are pure sine waves.

Tertiary E. M. F. We next proceed to determine the e. m. f. induced in the open tertiary circuit. This e. m. f. is the resultant of the two e. m. fs. induced, by the primary and secondary currents. Let m_a', m_b' and m_c' denote the instantaneous mutual inductances between the tertiary and the three phases of the primary. Then we may write

$$m_a' = M_1 \sin\,[\,(1 - s)\, w\, t + \alpha]$$

$$m_b' = M_1 \sin\left[\,(1 - s)\, w\, t + \alpha + \frac{2\,\pi}{3}\right]$$

$$m_c' = M_1 \sin\left[\,(1 - s)\, w\, t + \alpha + \frac{4\,\pi}{3}\right]$$

The total instantaneous flux linkage with the tertiary due to the joint action of the currents in the three phases of the primary is

$$M_1\, I_m \sin\,[\,(1 - s)\, w\, t + \alpha]\,\{\sin w\, t + k/a \sin\,[\,(1 - 2\, s)\, w\, t$$

$$+ 2\, \alpha + \beta]\,\} + M_1\, I_m \sin\left[\,(1 - s)\, w\, t + \alpha + \frac{2\,\pi}{3}\right]$$

$$\Big\{ \sin\left(w\,t + \frac{2\,\pi}{3}\right) + k/a \sin\Big[(1 - 2\,s)w\,t + \frac{2\,\pi}{3}$$

$$+ 2\,\alpha + \beta \Big]\Big\} + M_1\,I_m \sin\left[(1 - s)\,w\,t + \alpha + \frac{4\,\pi}{3} \right]$$

$$\Big\{ \sin\left(w\,t + \frac{4\,\pi}{3}\right) + k/a \sin\Big[(1 - 2\,s)\,w\,t + \frac{4\,\pi}{3}$$

$$+ 2\,\alpha + \beta \Big]\Big\}$$

This, after simple transformations, is reducible to the form

$$3/2\,M_1\,I_m \cos\,(s\,w\,t - \alpha) + 3/2\,k/a\,M_1\,I_m \cos\,(s\,w\,t - \alpha - \beta),$$

and the tertiary e. m. f. due to this flux is

$$3/2\,s\,w\,M_1\,I_m \sin\,(s\,w\,t - \alpha) + 3/2\,s\,w\,M_1\,I_m\,k/a \sin\,(s\,w\,t - \alpha - \beta)$$

Next, if we denote by M_2 the mutual inductance between the tertiary and the secondary, the flux linkage with the tertiary due to the current in the secondary is

$$M_2\,I_m\,k_2/a \sin\,(s\,w\,t - \alpha - \theta_2 - \beta)$$

and the tertiary e. m. f. due to it is

$$-\,s\,w\,M_2\,I_m\,k_2/a \cos\,(s\,w\,t - \alpha - \theta_2 - \beta)$$

Thus the total tertiary e. m. f. is given by

$$s\,w\,I_m\,\{3/2\,M_1 \sin\,(s\,w\,t - \alpha) + 3/2\,M_1\,k/a \sin\,(s\,w\,t - \alpha - \beta) - M_2\,k_2/a \cos\,(s\,w\,t - \alpha - \theta_2 - \beta)\,\}$$

This may be exhibited in the form

$$e_3 = E_3 \sin\,(s\,w\,t - \alpha - j) \tag{38}$$

where

$$E_3 = I_m\,s\,w\,\{\,[3/2\,M_1\,(1 + k/a \cos\,\beta) - M_2\,k_2/a \sin\,(\theta_2 + \beta)\,]^2 + [3/2\,M_1\,k/a \sin\,\beta + M_2\,k_2/a \cos\,(\theta_2 + \beta)\,]^2\,\}^{1/2} \tag{39}$$

and

$$\tan j = \frac{1/a\,[3/2\,k\,M_1 \sin\,\beta + k_2\,M_2 \cos\,(\theta_2 + \beta)\,]}{3/2\,M_1\,(1 + k/a \cos\,\beta) - M_2\,k_2/a \sin\,(\theta_2 + \beta)} \tag{40}$$

Primary Power. We shall next consider the power impressed on the primary. Since the variable frequency component in the primary current wave (**35**) is incapable of contributing to the power, the only effective component is that of fundamental frequency, and this yields an amount of power in the first phase given by

$$1/2\, E\, I_m \cos \varphi$$

so that the total primary power in all three phases is

$$3/2\, E\, I_m \cos \varphi \qquad (41)$$

Power Factor. The r. m. s. value of the primary current (35) being

$$\frac{I_m}{\sqrt{2}} \sqrt{1 + (k/a)^2}$$

the volt-amperes per phase are given by

$$1/2\, E\, I_m \sqrt{1 + (k/a)^2}$$

so that the power factor of the primary is

$$\frac{\cos \varphi}{\sqrt{1 + (k/a)^2}} \qquad (42)$$

Mechanical Power. We shall now consider the total *mechanical power* transmitted to the rotor of the machine. If the losses due to hysteresis and eddy-currents are neglected, the mechanical power is easily obtained by subtracting the primary and secondary copper losses from the total primary power. A consideration of the mechanical power is, it is needless to point out, of very great interest in connection with the self-starting qualities of the machine.

Torque. The value of the torque exerted by the rotor is easily determined from the mechanical power by dividing the latter by the angular velocity.

Up to the present, we have supposed the tertiary to be on open circuit. If now we imagine it to be closed, tertiary currents will appear which will react on both primaries and secondary. The same general principle as that already used might be employed for investigating the reactions of the various circuits on each other. But if we are concerned only with the general nature of the results obtained, it is simpler to suppose the secondary and tertiary circuits replaced by a single equivalent secondary. Thus the general law of variation of the tertiary current will be the same as that of the secondary, which we have already investigated.

The method used in the above investigation is, it will be observed identical in principle with Lord Kelvin's method of electrical images as applied to problems in electrostatics. It may possibly be found to be of great use in connection with certain types of alternating-current problems.

We shall apply the above theory to the study of a particular case which corresponds approximately to a certain synchronous machine for which experimental results are available. The following numerical values (at a frequency of 12.5) have been assumed for the various constants:—

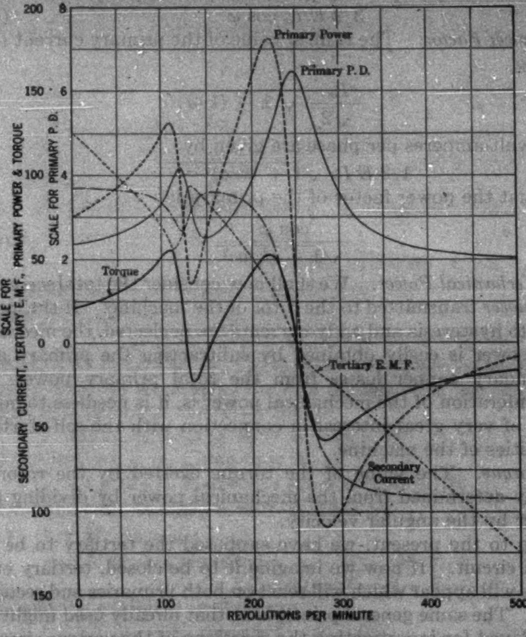

FIG. 12—CURVES CONNECTING PRIMARY P. D., PRIMARY POWER, SECONDARY CURRENT, TERTIARY E. M. F. AND TORQUE WITH SPEED WHEN THE PRIMARY CURRENT IS MAINTAINED AT A CONSTANT VALUE

$r_1 = 0.006 \quad r_2 = 0.004 \quad w\,M_0 = .043 \quad w\,M_1 = 4.2$

$w\,L_1 = 0.0513 \quad w\,L_2 = 0.046 \quad I_m = 48.3 \quad w\,M_2 = 2.3$

By the aid of the formulas established above, values of the primary p. d., primary current, secondary current, tertiary e. m. f., primary power, mechanical power, torque, power factor and phase angle were calculated for various values of *s* and a constant value of 48.3 for I_m. Since all the variables involving I_m are directly proportional to it, we can immediately find their values for any other value of I_m. Accordingly, we can find the values corresponding to those values of I_m for

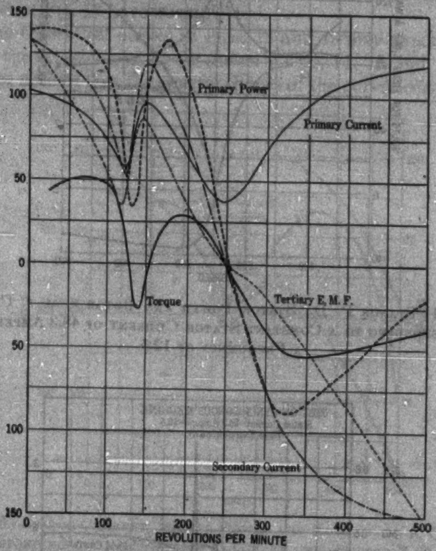

Fig. 13—Curves Connecting Primary Current, Primary Power, Secondary Current, Tertiary e. m. f. and Torque With Speed When the Primary P. D. is Maintained Constant

For primary current curve multiply vertical scale readings by 2
" " power " " " " " " 10
" secondary current " " " " " " 3
" tertiary e. m. f. " " " " " " 4
" torque " " " " " " 10

various values of which yield a constant value for the primary current. This will give us the variations of the quantities with s when the *primary current is maintained constant.* The values so calculated are given in Table I, and some of them are

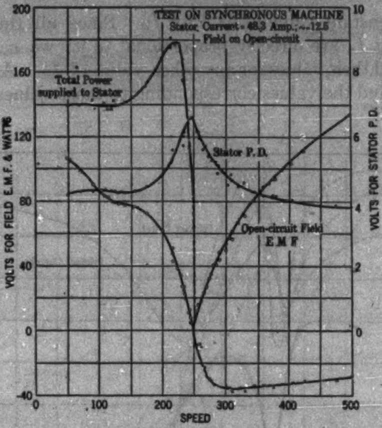

FIG. 14—STATOR P. D., STATOR POWER AND ROTOR E. M. F. CURVES CORRESPONDING TO A CONSTANT STATOR CURRENT OF 48.3 AMPERES AT A FREQUENCY OF 12.5

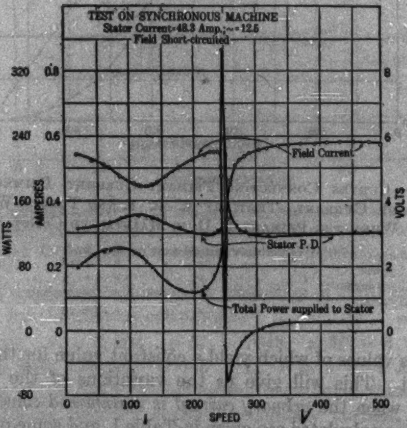

FIG. 15—STATOR P. D., STATOR POWER AND ROTOR CURRENT CURVES CORRESPONDING TO A CONSTANT STATOR CURRENT OF 48.3 AMPERES AT A FREQUENCY OF 12.5

plotted in Fig. 12. Similarly, we may obtain the values of the variables for different values of *s* when the values of I_m are so chosen as to yield a constant value of the primary p. d. We thus obtain the results given in Table II, which corresponds to a *constant primary line P. D.* of volts, and from which the curves of Fig. 13 have been plotted.

The theoretical curves of Figs. 12 and 13 may be compared with the experimental curves shown in Figs. 14, 15 and 16. It will be noticed that the general shapes of the curves correspond in the two cases, and that the peculiarities of shape are satisfactorily accounted for by theory. The experimental curves, however, do not exhibit such very pronounced peaks and valleys as do the theoretical ones. In this connection it must be remembered that the theoretical curves are based on a number of assumptions which are not quite realized in practise. Among others, the assumption has been made that the primary p. d. retains its pure sine wave form, and remains unaffected by the variable frequency component of the primary current. In actual experiments, the generator was a machine of smaller output than the synchronous motor, and its e. m. f. would certainly suffer distortion.

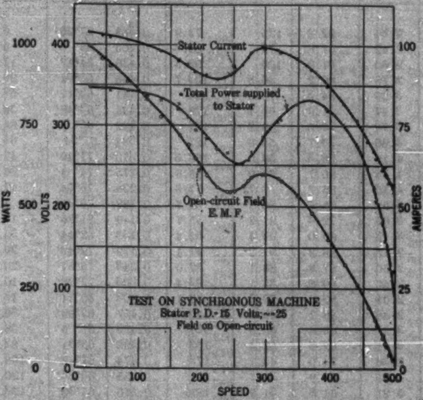

Fig. 16—Stator Current, Stator Power and Rotor e. m. f. Curves Corresponding to a Constant Value of the Stator P. D. = 15 Volts at a Frequency of 25

TABLE I.

$r_1 = 0.006$; $w L_1 = 0.0513$; $r_2 = 0.004$; $w L_2 = 0.046$; $w M_0 = 0.043$; $w M_1 = 4.2$; $w M_2 = 2.3$; frequency = 12.5.

Primary Current Maintained Constant at 48.3 amperes.

Speed 7 p. m.	Primary terminal P. D.	Secondary current.	Tertiary e. m. f.	Power supplied to primary.	Power factor.	Phase angle.*	Mechanical power	Torque, in 107 c. g. s. units.
0	2.34	92.2	125	75.9	0.388	67°26′	0	
25	2.42	91.8	118	82.3	0.405	66° 5′	6.6	25.05
50	2.60	91.0	100	92.5	0.426	64°47′	17.4	33.2
75	2.78	89.2	86.8	103.7	0.446	63°31′	29.8	37.9
100	3.29	84.3	70.6	124	0.451	63°11′	53.8	51.4
105	3.47	81.9	65.7	128	0.441	63°49′	59.4	54.0
110	3.70	78.5	60.2	131	0.422	65° 3′	64.1	55.6
115	3.96	73.7	55.8	127	0.384	67°24′	63.6	52.8
120	4.16	68.3	59.9	108.5	0.312	71°49′	47.8	38.0
122.5	4.18	66.7	67.9	95.9	0.275	74° 4′	36.1	28.2
125	4.10	66.7	7.77	77.6	0.226	76°55′	17.8	13.6
127.5	3.92	68.8	86.1	59.7	0.182	79°30′	− 1.2	− 9.3
130	3.67	72.3	91.2	45.7	0.149	81°26′	− 17.2	−12.6
135	3.13	80.0	93.4	35.4	0.135	82°14′	− 32.1	−22.7
140	2.75	85.2	91.2	41.1	0.179	79°42′	− 29.9	−20.4
145	2.56	87.9	88.1	52.0	0.243	75°57′	− 20.9	−13.8
150	2.50	89.8	85.1	63.4	0.303	72°22′	− 10.4	− 6.6
162.5	2.61	89.8	77.3	88.1	0.403	66°14′	13.8	8.1
175	2.86	88.7	69.2	110	0.457	62°98′	36.1	19.7
187.5	3.23	86.4	60.9	131	0.483	61° 5′	58.8	29.9
200	3.71	82.3	52.4	153	0.492	60°31′	83.5	39.9
212.5	4.36	74.9	43.1	173	0.474	61°42′	108.3	48.7
220	4.85	67.9	36.7	181	0.455	63°33′	120.3	52.2
225	5.21	61.3	31.9	181	0.515	65°30′	123.5	52.4
230	5.58	53.0	26.4	174	0.373	68° 6′	120.8	50.1
235	5.93	42.6	20.3	157	0.317	71°31′	108.2	44.0
240	6.23	29.9	13.5	129	0.248	75°30′	83.6	33.3
245	6.42	15.5	6.5	89.2	0.166	80°26′	46.2	18.0
250	6.46	0	0	42.1	0.078	85°32′	0	0
255	6.33	15.3	5.5	− 5.2	−0.0098	90°34′	− 48.1	−18.0
260	6.08	29.2	9.7	− 45.6	−0.0897	95° 9′	− 91.0	−33.4
265	5.74	41.2	12.9	− 75.6	−0.157	99° 3′	−124.4	−44.8
270	5.37	51.0	15.4	− 94.7	−0.211	102°10′	−147.1	−52.0
275	5.00	58.8	17.7	−105	−0.251	104°34′	−161.0	−55.9
280	4.66	65.0	19.8	−109	−0.281	106°18′	−168.2	−57.4
287.5	4.20	71.8	22.9	−108	−0.308	107°58′	−171.0	−56.8
300	3.62	79.2	28.3	− 98.3	−0.325	108°57′	−165.4	−53.0
325	2.82	86.1	39.8	− 71.3	−0.303	107°37′	−242.9	−42.0
350	2.55	89.2	51.5	− 54.7	−0.256	104°50′	−128.5	−35.0
400	2.21	91.4	76.1	− 30.9	−0.167	99°36′	−106.4	−25.4
450	2.09	92.3	100.5	− 17.1	−0.098	95°38′	− 93.2	−19.8
500	2.03	92.7	124.8	− 8.4	−0.0496	92°51′	− 84.8	−16.2

*Angle of lag of primary current behind primary p. d.

TABLE II.

$r_1 = 0.006$; $\omega L_1 = 0.0513$; $r_2 = 0.004$; $\omega L_2 = 0.046$; $\omega M_0 = 0.043$; $\omega M_1 = 4.2$; $\omega M_2 = 2.3$; frequency = 12.5.

Primary P. D. Maintained Constant at 10 Volts.

Speed, 2 p. m.	Primary current	Secondary current	Tertiary e. m. f.	Power supplied to primary.	Power factor.	Phase angle.*	Mechanical power.	Torque in 107 c. g. s. units.
0	206	394	534	1385	0.388	67°26′	0	
25	199	378.5	465	1400	0.405	66° 5′	112	426
50	186	350.5	386	1370	0.426	64°47′	258	493
75	174	321	312	1340	0.446	63°31′	386	492
100	147	256	214	1146	0.451	63°11′	496	474
105	139	236	189	1060	0.441	63°49′	492	448
110	130.4	212	163	952	0.422	65° 3′	467	405
115	122	186	141	812	0.384	67°24′	405	337
120	116	164	144	625	0.312	71°49′	276	219
122.5	115.6	160	163	550	0.275	74° 4′	207	161
125	118	163	190	462	0.226	76°55′	106	81
127.5	123	176	220	389	0.182	79°30′	− 8	—
130	132	197	249	340	0.149	81°26′	− 128	− 93.9
135	154	255	298	361	0.135	82°14′	− 327	−231.5
140	176	310	332	545	0.179	79°42′	− 395	−270
145	189	343	344	793	0.243	75°57′	− 319	−210
150	193	357	340	1013	0.303	72°22′	− 166	−106
162.5	185	344	296	1290	0.403	66°14′	203	119
175	169	310	241	1335	0.457	62°48′	440	240
187.5	150	267.5	189	1250	0.483	61° 5′	563	287
200	130	222	141	1110	0.492	60°31′	608	290
212.5	111	172	99	910.5	0.474	61°42′	571	256
220	99.6	140	76	769	0.445	63°33′	512	222
225	92.8	118	61	666	0.415	65°30′	456	194
230	86.5	95	47.4	559	0.373	68° 6′	388	161
235	81.4	71.7	34	447	0.317	71°31′	307	125
240	77.5	48.1	21.7	332	0.248	75°39′	215	85.6
245	75.2	24.1	10.2	217	0.166	80°26′	112	43.8
250	74.8	0	0	101	0.078	85°32′	0	0
255	76.3	24.1	8.7	− 13	−0.0098	90°34′	− 120	− 45
260	79.4	48	16	−123	−0.0897	95° 9′	− 246	− 90.4
265	84.1	71.7	22.5	−229	−0.157	99° 3′	− 377	−136
270	89.9	96.9	28.7	−328	−0.211	102°10′	− 509	−180
275	96.5	118	35.3	−420	−0.251	104°34′	− 644	−223
280	104	135.5	42.4	−504	−0.281	106°18′	− 775	−264
287.5	115	171	54.5	−614	−0.308	107°58′	− 969	−322
300	133.5	219	78.2	−750	−0.325	108°57′	−1260	−402
325	171	306	141	−898	−0.303	107°37′	−1800	−529
350	189	349	202	−838	−0.256	104°50′	−1970	−537
400	218	413	344	−630	−0.167	99°36′	−2170	−518
450	231	442	481	−393	−0.098	95°38′	−2140	−454
500	238	456	614	−204	−0.050	92°51′	−2055	−392

*Angle of lag of primary current behind primary p. d.

DISCUSSION ON "STARTING CONDITIONS OF SYNCHRONOUS MACHINES (HAY AND MOWDAWALLA), CALCUTTA, INDIA, JANUARY 2, 1919.

Quentin Graham: In considering the various torques which act upon the rotor during the starting and synchronizing periods the authors have confined themselves to those torques which are active when the field winding is either open-circuited or short-circuited. They have not considered the conditions which exist when the field is excited before synchronism is reached. In this case there is another torque to be taken into account which is the predominant one during the period of pulling into synchronism.

When the motor is running in synchronism its torque, due to the interaction between the ampere turns of the field winding and those of the armature winding, depends upon the position of the poles relative to the revolving m. m. f. of the armature. The torque rises from zero to a maximum as the relative shift of the rotor passes through one half a pole pitch. This is the limit of stable operation as a synchronous motor. With a further relative shift of the rotor the torque falls off to zero and then goes through a negative half cycle. The complete cycle extends over two pole pitches. Now if we have the machine running as an induction motor with a small slip and then excite the field winding the rotor will be subjected to this cycle of torque every time it slips one pair of poles. The result is that the rotor is made to oscillate about its mean position at slip frequency. It will be noticed that the frequency of oscillation of an open-circuited or short-circuited rotor, due to the non-uniformity of the magnetic circuit, is twice slip frequency as has been brought out in the paper.

The diagram and the analysis given by Messrs. Hay and Mowdawalla for the case of a rotor oscillating at twice slip frequency is applicable to the case of the excited rotor which oscillates at slip frequency. It has been shown by Rosenberg, in the article to which reference is made in the present paper, that the torque due to the excited field is usually large compared to the torque component which is dependent upon the oscillating speed for its existence. Or, referring to Fig. 3, of the present paper, the vector $O\,S$ should be large in comparison with the vector $O\,F$ if the diagram is to be used to represent conditions when the field is excited. This would justify the neglecting of the torque represented by the vector $O\,F$ in calculating the pulling-in ability of a machine as has been done by Rosenberg.

The pulling into synchronism with a fully excited field is of particular interest since it is this condition which nearly always exists in practical work. The motor is brought up to within a few percent of synchronous speed as an induction motor and if it is carrying an appreciable load it will not be able to pull into step of its own accord. The field must then be excited in order to cause the rotor to come up to synchronous speed. From the analysis which the authors have given it can be seen that if the motor does not pull in to step during the first half cycle of positive or accelerating torque it will not pull in at all. That is, during the following half cycle of torque it will be retarded an equal amount and will therefore continue to oscillate in speed but will not reach synchronous speed.

It may be deduced also that the certainty of pulling in to step depends upon the position of the rotor poles relative to the stator m. m. f. wave at the instant that the field is excited. If the rotor is in such a position that it must pass through the whole of the accelerating half cycle of torque before entering the zone of retarding torque the likelihood of its reaching synchronous speed is greater than if the field were excited a little later so that only a part of the accelerating wave is passed through before retardation commences. It is somewhat analogous to the rise in flux in a transformer that is switched on to the line, the maximum flux reached being dependent on the point of the voltage wave at which the switch is closed. Experiments have proved that the pulling-in ability of a motor does vary considerably with the point on the slip cycle at which the field is closed.

The conclusion which the authors reach that the field should be kept open-circuited during the early part of the starting period is open to question. While it is true that the starting torque is higher with open-circuited than with short-circuited field, as shown by theory and confirmed by experiment, the difference between the two is not very great in motors having well designed starting windings. Furthermore the complicated construction which would result from applying field break-up switches to revolving field synchronous motors would more than offset the gain in starting torque. Closing the field circuit during starting has been found to be the most practicable method.

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It may be deduced also that the certainty of pulling in to step depends upon the position of the rotor poles relative to the stator, or field, poles at the instant that the field is excited. If the rotor is in such a position that it must pass through the whole of the decelerating half cycle of torque before entering the zone of attracting torque the likelihood of its reaching synchronous speed is greater than if the field were excited a little later so that only a part of the accelerating wave is passed through before retardation commences. It is somewhat analogous to the case of flux in a transformer that is switched on to the line, the maximum flux reached being dependent on the point of the voltage wave at which the switch is closed. Experiments have proved that the pulling-in ability of a motor does vary considerably with the point on the slip cycle at which the field is closed.

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REPORT OF THE BOARD OF DIRECTORS

FOR THE FISCAL YEAR ENDING APRIL 30, 1919

The Board of Directors of the American Institute of Electrical Engineers presents herewith to the membership its Thirty-fifth Annual Report, for the fiscal year ending April 30, 1919. A general balance sheet showing the condition of the Institute's finances on April 30, 1919, together with other detailed financial statements, is included herein.

Directors' Meetings.—The Board of Directors held ten regular meetings during the year; seven of these were held in New York, one in Atlantic City in June, one in Philadelphia in December, and one in Boston in March.

The practise has been continued of printing from month to month in Section I of the Institute PROCEEDINGS information regarding the more important activities of the Institute which have been under consideration of the Board of Directors, the committees, and the various officers.

Annual Meeting.—The Annual Business Meeting was held at Institute headquarters, New York, on May 17, 1918. The Annual Report of the Board of Directors for the fiscal year ending April 30, 1918, was presented as published in full in the June 1918 issue of the PROCEEDINGS. The Tellers Committee presented its report upon the election of officers for the year beginning August 1, 1918.

Directly following the business meeting came the ceremony of the presentation of the Edison Medal to Colonel John J. Carty.

Annual Convention.—The Thirty-fourth Annual Convention was held at Atlantic City on June 26 to 28, 1918. This was strictly a business meeting at which 15 technical papers were presented at five sessions. Several meetings of the Section Delegates were held. Major General Black delivered an address on "Engineers and the War."

October Meeting.—The 342nd Institute meeting, scheduled to be held at Philadelphia, October 11 and 12, 1918, was cancelled on account of the influenza epidemic. Presentation of the technical papers was postponed until the meeting of December 13, 1918.

New York Meeting.—The 343rd Institute meeting was held in New York on November 8, 1918 under auspices of the Industrial and Domestic Power Committee.

Toronto Meeting.—The 344th Institute meeting was held in Toronto, Ontario on November 22 and 23, 1918. Three technical papers were presented on power installations and developments in Ontario and Quebec.

Philadelphia Meeting.—The 345th Institute meeting was held in Philadelphia on December 13, 1918. Two technical papers were presented and two addresses on Research.

New York Meeting.—The 346th Institute meeting was held in New York on January 10, 1919. The evening was devoted to an address by Major General Squier on aeronautics.

Mid-Winter Convention.—The Seventh Mid-Winter Convention was held in New York on February 19 to 21, 1919. Attendance exceeded 1300. Four technical sessions were held and fifteen papers presented. The first session was held jointly with the A. I. M. E. A complete report of this convention was published in the March 1919 PROCEEDINGS.

Boston Meeting.—The 348th Institute meeting was held at Boston on March 14, 1919. Morning, afternoon and evening sessions were held at which three technical papers and several addresses were presented on widely divergent topics.

New York Meeting.—The 349th Institute meeting was a joint meeting with the Illuminating Engineering Society held at New York on April 11, 1919. One paper was presented on the subject of Lighting Codes.

Meetings and Papers Committee.—The Meetings and Papers Committee has held meetings every month at the time and place of the regular meetings of the Institute held during the past year as referred to above. At these Committee meetings the programs of the Institute meetings were planned and the various papers offered for presentation and the policy of the Committee were discussed. The policy of the Committee has been to continue the practise of holding some meetings outside of New York; one was held in Toronto in November, one in Philadelphia in December, and one in Boston in March.

At the beginning of the fiscal year the Committee found that owing to the war activities of the membership it was very difficult to get good papers, but with the cessation of war activities a considerable number of desirable papers were offered. After completing plans for all the meetings through and including November 1919, a considerable number of papers is still available. In fact there is material available for some Section meetings, which the Committee would like to encourage in order that certain papers well worthy of presentation may be presented at an earlier date than would be the case if they had to wait for the regular Institute meetings.

A Pacific Coast Convention, to be held in Los Angeles in September 1919, has been authorized.

Technical Activities.—The Board of Directors' Committee on Technical Activities was appointed for the purpose of co-relating the work of the various Technical Committees and to bring about the adoption of programs of work for the Committees to undertake, which might prove of advantage to the Institute. In the past the Technical Committees have been mainly concerned with securing papers for meetings and with few exceptions there have been no other activities. It has been realized for a long time that there were other fields that the Technical Committees could work in to the advantage of the Institute. Some committees had already undertaken programs mainly with the object of preparing statements of existing successful practise for the guidance of people interested in various industries, but in the past it has not always been possible to carry out such programs, as there was no system for insuring the continuity of such action. The Board's Committee has communicated with the Chairmen of Technical Committees during the year and there has been an increase in the amount of work undertaken by these committees. The Industrial Power Committee has had for sometime a large program which is being worked out, and the other committees have started work along somewhat similar lines, such as the Iron and Steel Committee, which is preparing to summarize the successful practise of the Iron and Steel Industry with regard to electrical installations; the Marine Committee which has been engaged in preparing proposed installation rules for which there is at present a great need, and others of the Committees are engaged on similar useful programs which will be presented to the Institute either in the form of papers or reports.

The Annual Reports of the sixteen technical committees of the Institute will be presented at the Annual Convention of the Institute in June 1919, and will probably be printed in the July or August issues of the PROCEEDINGS.

Editing Committee.—The Editing Committee has jurisdiction over the publication of the annual TRANSACTIONS and has been engaged with Volume XXXVII covering the year 1918. This volume will be published in two parts, Part I is completed and Part II is now in the bindery. The volume contains approximately 2000 pages and 87 plates of half-tone illustrations, comprising papers and discussions presented at Institute meetings during the year.

Sections Committee.—The year ending May 1st witnessed the crest of the wave of American activity in the great war. It is then not surprising that it should include the slight resulting depression in attendance at the meetings of the various sections of the Institute indicated in the table below.

Whether drawn from the lessons of the hour regarding the value of co-operation or led by some other inspiration, the tendency in section affairs during the year has been that of continued development in the direction of affiliation with the sections of other technical societies. This has resulted not only in the highly desirable objective of greater co-operation and unity among engineers, but has also placed the engineer in higher standing with his community by his greater participation in civic and municipal affairs in those centers where such affiliation has been undertaken.

The Sections Committee, encouraged by the well established policy of the Institute which has long aimed to develop this co-operation, appointed at their conference at Atlantic City last June, a committee to study existing methods and formulate suggestions for the guidance of all sections so located as to make such affiliation profitable.

This committee, now working in conjunction with the general committee on the development of Institute activities, should be able to tender a helpful report at the meeting of the sections delegates next month. The sections at Atlanta, Chicago, Milwaukee, St. Louis, Spokane, Indianapolis, Philadelphia, Portland, Ore., and San Francisco, now well established in this plan of affiliation should find numerous additions to their ranks as time progresses.

The activities of the Student Branches have continued to be materially modified by the war conditions, notably by the organization of the Students Army Training Corps. A considerable number of the Branches have been inactive during the greater portion of the year, but since January 1, many have reorganized and are now active; all will probably resume normal activity by next Fall.

Two new Branches were authorized during the year at the University of Pennsylvania and Drexel Institute.

	For Fiscal Year Ending						
	May 1 1913	May 1 1914	May 1 1915	May 1 1916	May 1 1917	May 1 1918	May 1 1919
SECTIONS							
Number of Sections.........	29	30	31	32	32	34	34
Number of Section meetings held....................	244	233	246	251	265	245	217
Total Attendance..........	22,825	22,626	23,507	28,553	31,299	34,614	25,837
BRANCHES							
Number of Branches	47	47	52	54	59	59	61
Number of Branch meetings held....................	357	306	328	360	368	268	156
Attendance................	11,808	11,617	12,712	15,166	16,107	10,683	6,441

Standards Committee.—The Standards Committee has held during the year six regular monthly meetings, one special meeting, and a joint meeting with the U. S. National *Reorganization of Work.*

In order to expedite the work of the Committee, the entire system of subcommittees has been reorganized so that all detail consideration of changes in standards and other technical affairs is now referred in each case to a specific sub-committee. Action by the Standards Committee is taken only after receiving recommendations from a subcommittee. A large amount of detail discussion has thus been eliminated. There are at present twenty-four of these sub-committees.

Revision of the General Form of the Standardization Rules.—A large share of the time of the Standards Committee during the past year has been devoted to a consideration of changes in the form and arrangement of the Standardization Rules. This matter has been in the hands of the Subcommittee on the *Form but not the Substance* for some time past and a general system of revision by which the Standardization Rules are divided into chapters was reported and has been adopted.

Changes in Substance of the Present Rules.—It is hoped to complete the full text of this revision of the form of the Standardization Rules this year. A few minor changes of substance will be included.

A revised chapter covering Lighting and Illumination and based on the latest rules of the Illuminating Engineering Society has been recommended by the Subcommittee on Lighting and Illumination and substituted for the present chapter.

Cooperation with the General Committee on Standardization of Electrical Equipment and Supplies of the War Department.—At the request of Major N. J. Neall, Chairman of the above Committee, that the Standards Committee should cooperate with the General Committee of the War Department in the preparation of specifications, the Standards Committee voted to render all possible assistance through its subcommittees. Dr. P. G. Agnew was appointed a special subcommittee to further this work which has been carried on throughout the year. Very effective assistance has been rendered the War Department Committee.

Joint Meeting with U. S. National Committee of the I. E. C.—At a joint meeting of the U. S. National Committee of the I. E. C. and the Standards Committee it was decided to recommend the early reappointment of the subcommittee of the I. E. C. on Rating, to meet preferably in Paris. Professor C. A. Adams, Mr. H. M. Hobart and C. O. Mailloux will represent the Institute. These representatives while abroad will also take up with the Rating Panel of the British Engineering Standards Association the actions taken and suggestions made by the Standards Committee.

American Engineering Standards Committee.—The consideration of suggested changes in the Constitution and Rules of Procedure made it necessary to continue the organization committee until May 1918, when a revised draft of the Constitution and Rules of Procedure was agreed upon and sent to the five appointing societies (A. S. C. E., A. I. M. M. E., A. S. M. E., A. I. E. E., and A. S. T. M.) for their approval. This approval was received from all of the societies, and representatives appointed by each met on October 19th, 1918, and organized the American Engineering Standards Committee.

Soon after organization it became evident that the Committee must be enlarged in order to secure the cooperation of several important organizations engaged in standardization work. This fact was emphasized by the action of a conference held in Washington, January 15, 1919, to consider the method of developing and approving Industrial Safety Codes. At this conference it developed that the principal organizations would not cooperate with this Committee unless they had direct representation on it. Such representation is not possible under the present Constitution, so after careful consideration the Committee has decided that an increase in its membership is desirable. It will therefore be necessary to make further changes in the Constitution. Such a revision has been drafted and approved by the Committee and will be submitted to the five Founder

Societies for their approval during the month of May 1919. It is expected that this revision will enable the Committee to begin active work in the near future.

Public Policy Committee.—The Public Policy Committee, during the administrative year about to close, has held itself in readiness to take up any matters which might be referred to it by the Institute authorities, but has not initiated any recommendations. The reason for the inactivity of the Committee is that for the last two years the several matters which would have naturally been acted upon by it have been considered by Engineering Council.

Committee on Safety Codes.—The Committee on Safety Codes has continued to represent the Institute on the Electrical Committee of the National Fire Protection Association, and representatives attended the meeting of the N. F. P. A. Committee in New York in March, this being the intermediate meeting, at which was discussed the various features of the Electrical Code, in preparation for the regular biennial meeting in 1920, at which time changes in the Code are officially made.

The Committee was represented at the conference in Washington with the Bureau of Standards, for the purpose of discussing matters relating to the American Engineering Standards Committee, and the desirability of the writing of an Industrial Safety Code.

The Committee has been active during the year on routine matters of a code nature, with the N. E. L. A.

The first part of the fiscal year the Committee was more or less inactive, due to war conditions, but since the signing of the armistice inter-association matters have revived, and almost all the activities of the Committee occurred in the last quarter of the year.

Board of Examiners.—The Board of Examiners during the year held sixteen meetings, averaging about two and one-half hours each. It considered and referred to the Board of Directors a total of 2111 applications for admission or transfer to the higher grades.

Some applicants for admission were objected to on ethical grounds. Although the number of such cases was small, considerable time was required for the necessary investigations; adverse action was taken in a few such cases. The Board continued the practise of requiring questionable applicants to furnish proof of loyalty to the ideals of the United States.

The result of the Board's work for the year is given in the following tabulated statement:

APPLICATIONS FOR ADMISSION

Recommended for grade of Associate	1213	
Not recommended	9	1222
Recommended for grade of Member	87	
Not recommended for admission to this grade	23	110
Recommended for grade of Fellow	4	
Not recommended for admission to this grade	1	5
Recommended for enrolment as Students		659

Applications for Transfer

Recommended for grade of Member....................	81	
Not recommended for transfer to this grade............	19	100
Recommended for grade of Fellow....................	12	
Not recommended for transfer to this grade............	3	15
Total number of applications considered...............		2111
Applications reconsidered...........................		6
Total..		2117

Membership.—Sixteen hundred and ninety-five applications for membership in the Institute have been received this year, which is by far the largest number ever received in any one year. The nearest approach to this was for the year ending May 1, 1918, when 1235 applications were received.

A printed list of reasons why a man should become a member of the Institute was printed for distribution to desirable candidates. Several thousand of these were called for by the Sections.

The Membership Committee feels that new members should be obtained at an increasing rate to keep pace with the industry.

The 10,000 mark has been passed, the membership of the Institute as of May 1, 1919, being 10,352. This shows a net increase of 1070 members since May 1, 1918, approximately twice the net increase of the previous year, which was 572.

During the five years ending May 1, 1917, the net increase was only 1251; the total applications received in these five years, 2440. These figures emphasize the greatly increased activity of the past two years.

	Honorary Member	Fellow	Member	Associate	Total
Membership, April 30, 1918.	6	464	1332	7480	9282
Additions:					
Transferred..............	....	24	88	....	....
New Members Qualified...		6	84	1221	....
Reinstated...............	....	4	7	55	....
Deductions:					
Died....................	....	4	7	66	....
Resigned................	....	1	4	77	....
Transferred..............	....	....	18	94	....
Dropped.................	....	4	15	229	....
Membership, April 30, 1919..	6	489	1467	8290	10252

Net increase in membership during the year.................................... 970

Deaths.—The following deaths have occurred during the year:

Fellows—A. O. Benecke, M. H. Collbohm, W. L. Hooper, C. R. McKay.

Members—George E. Claflin, *F. J. Duffy, Charles H. Hile, W. J. Jenks, Etna Kuhlman, Frank I. Porter, O. O. Rider.

Associates—E. G. Anderson, H. F. Anderson, Arthur S. Andrews, Samuel Avis, G. R. Baker, *Remson Bishop, John Bottomley, Ralph E. Brown, W. B. Burbeck, *E. W. Caldwell, J. J. Campbell, Eric Carlson, *Lucian Carr, *W. W. Crawford, E. W. Currier, Harry L. Curtis, *R. F. Day, Edward D. Donald, *John J. Donnohue, Hugh H. Emery, E. B. Fahnestock, I. Fujioka, John D. Gaboury, W. K. Greenwood, John Hanover, Vernon E. Hess, W. Lesniewski, Edward Lineberry, Claudius B. Little, *George Mac Indoe, J. A. MacQueen, Glen W. Merrill, *Charles J. Moore, A. F. Moray, *N. I. Moultrop, H. P. Myers, Archie Oakes, B. A. Ordonez, C. H. Parmly, D. H. Plank, James M. Poyner, H. E. Randall, Jr., O. B. Reynolds, E. M. Rhett, Arnold Ruegg, Owen Samsel, Charles P. Seeger, S. V. Setti, *D. A. Shanks, C. F. Shipman, Charles F. Sise, *J. C. M. Small, Herbert E. Smith, Alan Smout, C. W. Steiner, John F. Stevens, S. Sugiyana, T. R. Taltavall, Ralph E. Thomas, *A. R. Thompson, Stuart Thomson, L. J. Vogel, H. Webber, F. S. Wheeler, Richard M. Wilson, Minor I. Woodward, N. R. Work.

Total deaths, 77. *Died in U. S. Service.

Employment Service.—An employment service has been maintained for many years at Institute headquarters for the purpose of assisting members in obtaining positions. The services of the Institute consist principally in acting as a medium for bringing together the employer and the employee. No charge is made.

The engineering service bulletin is published each month in the Institute PROCEEDINGS and it has served to place many men in positions of responsibility, both in this country and abroad. The bulletin is subdivided into two parts: one containing announcements of vacancies, and the other containing lists of men available, with condensed records of their experience. The value of this service to the membership will continue to increase from year to year as men of engineering and executive responsibility realize the opportunity afforded by it to reach technical men whom it would be difficult to find through other channels.

Demobilization greatly increased the number of men available, while the number of positions open has greatly decreased. The service has been extended without charge to non-members who are or have been in the service of the Army or Navy. An effort has been made to keep those men on the available file in closer touch with positions open than heretofore possible when dependence was alone placed on the Service Bulletin of the PROCEEDINGS, by issuing to them when deemed advisable, a list of the opportunities on file.

The Institute's employment service is coordinated with that of other organizations, through the medium of the Engineering Societies Employment Bureau, organized under the auspices of the Engineering Council and conducted under the direction of the secretaries of the national societies of Civil, Mining, Mechanical, and Electrical Engineers.

Development of the Institute.—In October 1918 the Board of Directors authorized the appointment of a Committee on Development to consider suggested or proposed modifications in the activities and methods of work of the Institute, including the relations of the Institute to other organizations. This action was taken with a view to making a survey of, and adjusting, the work of the Institute, having in mind the rapidly changing social, economic, and industrial conditions.

Each Section of the Institute was requested to designate a member of this Committee; and these representatives of Sections, together with seven additional members appointed by the President, constitute the Committee, which has been actively at work for several months. The membership was urged to submit suggestions and most of the Sections have held at least one meeting for the purpose of discussing the subject. On account of the wide geographical distribution of the membership, no meetings of the entire Committee have been held, but the representatives of the Sections have made reports to an Executive Committee of seven members of the Development Committee. This Executive Committee has held two meetings, at which numerous suggestions from the membership throughout the country have been considered. The principal features of these suggestions representing the consensus of the views expressed, are being compiled by the Executive Committee for further consideration by the members of the general Committee; and the whole subject will be thoroughly considered and discussed in conferences to be held under the auspices of the Development Committee at the Annual Convention of the Institute in June, after which it is expected that a report, embodying definite recommendations, will be presented to the Board of Directors.

War Activities.—The Institute continued during the past year in active cooperation with the various government departments and agencies engaged in war work as listed in the Report of the Board of Directors for 1918.

The work in furtherance of Allied victory in which the Institute aided during the period from May 1918 to the signing of the armistice included cooperation in obtaining personnel for many branches of service, as follows: candidates for the 4th series of Officers Training Schools, for Submarine Service, for the Coast Artillery Corps and the Forest Service; specially trained men for work in the Bureau of Mines and to act as instructors in the various army training schools; for the U. S. Navy Steam Engineering School at Stevens Institute and for commissions in the Ordnance Dept.

The compilation and correction of the Institute's Honor Roll has been continued until from all information at hand it shows a total of 1417 members have served in the uniformed forces of the United States and the Allies of which number 21 died in service. Practically all of the 1417 members voluntarily entered the service and it is particularly gratifying to note the large proportion of officers, as given in the summary. A large proportion of these officers received their commissions, or were inducted into the service as a result of calls made upon the Institute by various departments of the Army and Navy.

SUMMARY

U. S. ARMY	
Major Generals	7
Brigadier Generals	5
Colonels	9
Lieut. Colonels	23
Majors	110
Captains	219
First Lieutenants	199
Second Lieutenants	162
Sergeants	34
Corporals	17
Miscellaneous	267
Total	1052

U. S. NAVY	
Rear Admirals	1
Captains	1
Commanders	6
Lieut. Commanders	9
Lieutenants	45
Lieutenants (j. g.)	72
Ensigns	75
Miscellaneous	75
Total	284

U. S. MARINE CORPS	
Lieutenants	3
Private	1
Total	4

ALLIED FORCES	
British Army	63
British Navy	6
French Army	6
Italian Army	1
Japanese Navy	1
Total	77

Total on Honor Roll..............1417

U. S. National Committee, International Electrotechnical Commission.— The U. S. National Committee of the I. E. C. held a joint meeting with the Standards Committee in Philadelphia, December 12, 1918, at which resolutions were adopted favoring the resumption of I. E. C. activities, and recommending the early reappointment of a subcommittee of the I. E. C. on Rating, to meet preferably in Paris, for the purpose of considering certain communications from the French Committee, particularly in regard to I. E. C. rules on Temperature Elevations of Machinery.

At this meeting, the printing of the I. E. C. Rating Rules as an appendix to the 1918 edition of the A. I. E. E. Standardization Rules, was suggested and authorized.

The following documents have been received from the General Secretary and distributed among the members of the U. S. National Committee:

(1) The Annual Report of the Honorary Secretary (Colonel Crompton) for the year 1918.

(2) A communication from the French Committee regarding a term for the unit of "Reactive Power."

(3) A list of Statutes of the I. E. C.

(4) A communication from the Dutch Committee in regard to an international standard of comparison of aluminum.

The U. S. National Committee will be represented by Messrs. C. A. Adams, H. M. Hobart, and C. O. Mailloux at a meeting of the I. E. C. Special Committee on Rating, at Paris, May 5-7, 1919.

It is believed that the conventions of I. E. C., interrupted for more than four years by the war, will now be resumed.

American Engineers Go To France.—At the invitation of the French Society of Civil Engineers and a committee of the French Engineers' Congress, and with the approval of the French government, a delegation of American engineers was appointed to study with French engineers problems involved in the industrial rehabilitation of France; such as, utilization of commercial ports, the development of navigable waterways, the development of water power, improvement of road systems, and many other questions of development.

Immediately on receipt of the invitation a delegation was organized representing the American national societies. The Executive Committee of the Institute selected Lewis B. Stillwell as the A. I. E. E. representative. The complete delegation, which sailed from New York on December 5, 1918, was, as follows: Messrs. James F. Case, Chairman, Geo. W. Fuller, A. M. Hunt, Nelson P. Lewis, Charles T. Main, E. Gybbon Spilsbury, Lewis B. Stillwell, George F. Swain, and George W. Tillson.

On the evening of February 10, 1919 a joint meeting of the four Founder Societies was held in the Engineering Societies Building, New York, at which the delegates presented informal reports, abstracts of which were published in the PROCEEDINGS for March, 1919.

United Engineering Society.—This Society performs for the national societies of Civil, Mining, Mechanical, and Electrical Engineers, certain specific acts which are governed by contracts; the primary function of the United Society being to hold in trust and to administer for these societies the Engineering Societies Building, in which the headquarters of the national societies are located.

Extracts from the annual financial report of the United Engineering Society were published in the April 1919 PROCEEDINGS.

Library.—The library of the Institute is combined with the libraries of the national societies of civil, mining, and mechanical engineers, administered as the "Engineering Societies Library" under the direction of the Library Board of the United Engineering Society; this board is composed of representatives of each of the four societies referred to above.

In order to place the facilities of the library at the disposal of persons residing at a distance from New York, a Library Service Bureau has been established, and a staff of expert searchers and translators is employed to cover almost any engineering topic, in the following manner: abstracting, translating, bibliographing, statistical searches and reports, searches for patent purposes, copying, preparing reference cards, etc.

An abstract of the annual report of the Engineering Societies Library covering the calendar year 1918, was published in the April 1919 PROCEEDINGS.

Engineering Council.—The Engineering Council represents the result of an organized effort inaugurated in the latter part of 1916 by the national societies of Civil, Mining, Mechanical, and Electrical Engineers, to establish an instrument through which united action could be brought about upon matters of common interest to engineers and which would serve as a connecting medium between the engineer and the public welfare in matters of interest to the engineering profession. The first meeting of the Council was held on June 27, 1917.

For details regarding the field, aims, and activities of the Council, members are referred to the numerous statements published from time to time in the monthly Institute PROCEEDINGS. A resume of the annual report covering the activities of the Council for the year ending February 13, 1919, was published in the April 1919 PROCEEDINGS.

Engineering Foundation.—The Engineering Foundation is a fund established by the United Engineering Society on January 27, 1915, through the generosity of Mr. Ambrose Swasey, of Cleveland.

The purpose of the Engineering Foundation is the advancement of the engineering arts and sciences in all their branches to the greatest good of the engineering profession and the good of mankind, which it is proposed to accomplish largely through the promotion of engineering research.

The Engineering Foundation is administered by a board upon which the Institute and other national engineering societies are represented. The annual report of the Engineering Foundation was also published in the April 1919 PROCEEDINGS.

Representatives.—The Institute has continued its representation upon various national committees and other local and national bodies with which it has been affiliated in past years, and in addition has appointed representatives upon a number of new organizations, some of the more recent being the Engineering Division of the National Research Council, and the American Bureau of Welding.

Edison Medal.—The Edison Medal for 1917, which had been awarded to Col. John J. Carty "for his work in the science and art of telephone engineering" was presented to Col. Carty with appropriate ceremonies at the Annual Meeting of the Institute held in New York on May 17, 1918.

The Edison Medal for 1918, has been awarded to Benjamin G. Lamme "for invention and development of electrical machinery," and the presentation will take place at the Annual Meeting of the Institute which will be held in New York on May 16, 1919.

John Fritz Medal.—The John Fritz Medal Board of Award which is composed of representatives of the national societies of Civil, Mining, Mechanical and Electrical Engineers awarded the 1919 medal to General George W. Goethals, the builder of the Panama Canal. The presentation will take place at the Engineering Societies Building in New York on May 22, 1919.

Finance Committee.—During the year the committee has held monthly meetings, has passed upon the expenditures of the Institute for various purposes, and otherwise performed the duties prescribed for it in the Constitution and By-laws.

Haskins and Sells, certified public accountants, have audited the books, and their report is included herein.

NEW YORK
CHICAGO
DETROIT
SAINT LOUIS
BOSTON
CLEVELAND
BALTIMORE
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HASKINS & SELLS
CERTIFIED PUBLIC ACCOUNTANTS
CABLE ADDRESS "HASKSELLS"
30 BROAD STREET
NEW YORK

SAN FRANCISCO
LOS ANGELES
NEW ORLEANS
SEATTLE
DENVER
ATLANTA
WATERTOWN
LONDON

ol. 38
1919

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

CERTIFICATE OF AUDIT

We have audited the books and accounts of the American Institute of Electrical Engineers for the year ended April 30, 1919, and

WE HEREBY CERTIFY that the accompanying General Balance Sheet properly sets forth the financial condition of the Institute on April 30, 1919, that the Statement of Income and Profit & Loss for the year ended that date is correct, and that the books of the Institute are in agreement therewith.

HASKINS & SELLS,
Certified Public Accountants.

NEW YORK,
May 13, 1919.

AMERICAN INSTITUTE OF

EXHIBIT A.

GENERAL BALANCE SHEET

ASSETS

REAL ESTATE:			
One-fourth Interest in United Engineering Society's Real Estate, 25 to 33 West 39th Street:			
Land and Building		$472,500.00	
Real Estate Equipment		14,292.79	
Total Real Estate			$486,792.79
EQUIPMENT:			
Library—Volumes and Fixtures		$ 39,847.55	
Works of Art, Paintings, etc.		3,001.35	
Office Furniture and Fixtures		12,428.68	
Total		$ 55,277.58	
Less Reserve for Depreciation		10,840.47	
Remainder—Equipment			44,437.11
INVESTMENTS:			
Bonds—City of Wilmington, Delaware, 4½%, 1934, Par Value $15,000.00		$ 15,782.05	
United States Third Liberty Loan, 4¼% Bonds		10,000.00	
Total Investments			25,782.05
WORKING ASSETS:			
Publications Entitled "Transactions," etc.		$ 12,599.00	
Paper and Cover Paper		2,162.05	
Paper for Volume 37 and Advertising		336.69	
Badges		1,166.74	
Total Working Assets			16,264.48
CURRENT ASSETS:			
Cash		$ 14,916.68	
Accounts Receivable:			
Members for Past Dues		9,728.15	
Advertisers		678.80	
Miscellaneous Sales		388.09	
Miscellaneous Printing and Subscriptions		1,034.00	
Accrued Interest on Investments		109.38	
Accrued Interest on Bank Balances		284.59	
Total Current Assets			27,139.69
FUNDS:			
Life Membership Fund			
Cash	$ 438.67		
Chicago, Burlington & Quincy Railroad Company Bonds, 4%, 1958, Par Value $5,000.00	4,868.75		
Accrued Interest	33.33	$ 5,340.75	
International Electrical Congress of St. Louis—Library Fund:			
Cash	$ 137.61		
New York City Bonds, 4½%, 1957, Par Value $2,000.00	2,242.33		
New York Telephone Company Bond, 4½%, 1939, Par Value $1,000.00	878.75		
Accrued Interest	67.50	3,326.19	
MAILLOUX FUND:			
Cash	$ 142.35		
New York Telephone Company Bond, 4½%, 1939	1,000.00		
Accrued Interest	22.50	1,164.85	
Midwinter Convention Fund—Cash		135.89	
Total Funds			9,967.68
DUES PAID IN ADVANCE—INTERNATIONAL ELECTROTECHNICAL COMMISSION, LONDON, ENGLAND			250.00
Total			$610,633.80

ELECTRICAL ENGINEERS.

April 30, 1919.

Liabilities

Current Liabilities:		
Accounts Payable—Subject to Approval by the Finance Committee	$ 6,362.87	
Due United Engineering Society, Account Building Addition, Including Accrued Interest	7,625.75	
Dues Received in Advance	2,805.45	
Entrance Fees and Dues Advanced by Applicants for Membership	455.50	
Total Current Liabilities		$ 17,249.57
Fund Reserves:		
Life Membership Fund	$ 5,340.75	
International Electrical Congress of St. Louis—Library Fund	3,326.19	
Mailloux Fund	1,164.85	
Midwinter Convention Fund	135.89	
Total Fund Reserves		9,967.68
Surplus: Per Exhibit "B"		583,416.55
Total		$610,633

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

STATEMENT OF INCOME AND PROFIT AND LOSS.

FOR THE YEAR ENDED APRIL 30, 1919.

EXHIBIT B.

REVENUE:			
Entrance Fees		$ 7,225.00	
Dues		102,908.43	
Students Dues		4,044.00	
Transfer Fees		1,060.00	
Advertising		10,641.33	
Subscriptions		3,144.92	
Sales of "Transactions," etc		2,954.80	
Badges Sold	$3,285.75		
Less Cost	2,387.83	897.92	
Interest on Investments		1,089.61	
Interest on Bank Balances		936.40	
Exchange		41.91	
Total			$134,944.32
EXPENSES:			
Meetings and Papers Committee:			
Salaries		$ 5,940.00	
Binding and Mailing "Proceedings"		5,887.20	
Printing "Proceedings"		8,947.43	
Engraving "Proceedings"		2,549.13	
Paper and Cover Paper		8,342.44	
Envelopes		882.18	
Stationery and Miscellaneous Printing		378.53	
General Expense		348.03	
Meetings		6,115.33	
Total			$ 39,390.27
EDITING COMMITTEE:			
Volume No. 34		$ 56.19	
Volume No. 35		48.63	
Volume No. 36		4,721.88	
Volume No. 37		7,062.51	
Total		$ 11,889.21	
Add Decrease in Inventory:			
April 30, 1918	$14,049.00		
April 30, 1919	12,599.00	1,450.00	13,339.21
EXECUTIVE DEPARTMENT:			
Salaries		$ 19,805.00	
General Expense		2,097.24	
Express		371.17	
Postage		2,479.30	
Advertising		3,371.90	
Stationery and Miscellaneous Printing		4,019.83	
Year Book and Catalogue		4,597.61	
Total			36,742.05
FORWARD			$ 89,471.53
TOTAL REVENUE—(Forward)			$134,944.32

TOTAL REVENUE—(Forward)		$134,944.32
EXPENSES—(Forward)		$ 89,471.53
Sections Committee:		
Sections Meeting	$ 6,230.07	
Branch Meetings	81.11	
Delegates Convention Expense	3,286.44	
Salaries, New York Office	2,340.00	
Stationery and Printing, New York Office	768.46	
Total		12,706.08
United Engineering Society:		
Assessment	$ 4,800.00	
Library Committee	4,000.00	
Engineering Council	3,733.36	
Total		12,533.36
General:		
Membership Committee	$ 1,647.34	
Finance Committee	150.00	
Standards Committee	1,005.49	
Code Committee	30.00	
American Engineering Standards Committee	600.00	
Committee on Institute Development	71.00	
Annual Dues—International Electrotechnical Commission	250.00	
President's Special Appropriation	331.72	
Honorary Secretary	4,000.00	
John Fritz Medal Award	55.89	
European Standards Committees—Conferences	1,500.00	
Interest on United Engineering Society Building Addition Loan	556.95	
Miscellaneous Printing	252.73	
Membership Classification	771.75	
Amortization of Premium on City of Wilmington, Delaware, 4¼% Bonds of 1934	52.14	11,275.01
Total		$125,985.98
Add:		
Increase in Accounts Payable—Subject to Approval by the Finance Committee, Expenses Undistributed at:		
May 1, 1918	$ 6,304.01	
April 30, 1919	6,362.87	58.86
Total Expenses		$126,044.84
NET REVENUE		$ 8,899.48
PROFIT AND LOSS CREDITS:		
Proceeds from Sale of Old Electrotypes	$ 440.30	
Refund of Payment to Pan American Engineering Society	15.00	
Total		455.30
GROSS SURPLUS FOR THE YEAR		$ 9,354.78
PROFIT AND LOSS CHARGES:		
Uncollectible Dues Written Off	$ 1,985.00	
Dues of Members in Military Service and in Countries Affected by the War, Written Off	1,685.50	
Provision for Depreciation of Furniture and Fixtures	1,489.35	
Adjustment of Inventory of April 30, 1918, of Library Volumes and Fixtures	184.00	
Total		5,343.85
NET SURPLUS FOR THE YEAR		$ 4,010.93
SURPLUS, MAY 1, 1918		579,405.62
SURPLUS, APRIL 30, 1919		$583,416.55

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

STATEMENT OF CASH RECEIPTS AND DISBURSEMENTS FOR DESIGNATED PURPOSES, FOR THE YEAR ENDED APRIL 30, 1919.

EXHIBIT C.

RECEIPTS:	
Life Membership Fund	$216.68
International Electrical Congress of St. Louis Library Fund—Interest and Royalties	94.50
Mailloux Fund—Interest	45.00
Midwinter Convention Fund—Interest	6.54
Total	$362.72
DISBURSEMENTS:	
Life Membership Fund	216.68
Midwinter Convention Fund	34.23
Mailloux Fund	70.00
International Electrical Congress, San Francisco, 1915—refund of subscriptions	40.50
Total	$361.41

RECEIPTS AND DISBURSEMENTS PER MEMBER.

During each fiscal year for the past eight years.

Year ending April 30	1912	1913	1914	1915	1916	1917	1918	1919
Membership, April 30, each year	7450	7654	7876	8054	8212	8710	9282	10352
Receipts per Member	$13.19	$13.45	$14.08	$14.06	$13.62	$13.30	$13.17	13.05
Disbursements per Member	12.44	15.57	12.86	13.54	13.74	12.75	11.99	12.79
Credit Balance per Member	$ .75	*$2.12	$1.22	$ .52	*$ .12	$ .55	$1.18	$ .26

*Deficit.

Respectfully submitted for the Board of Directors,

F. L. HUTCHINSON, *Secretary.*

New York, May 16, 1919.